Calibration of a gravitational-wave detector

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JOGLy
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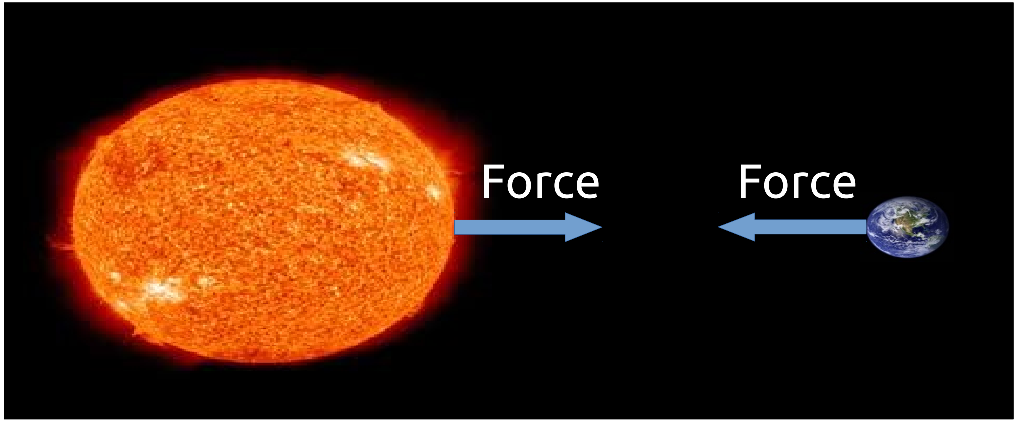
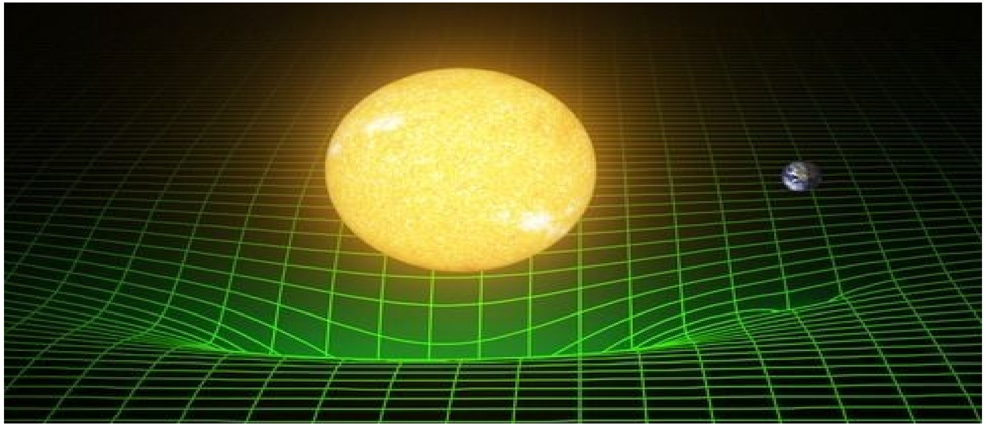
What is gravity?





Newton's theory – 1687:

A massive object attracts another massive object by a force acting along the line intersecting both centers of mass.

Einstein's theory – 1915:

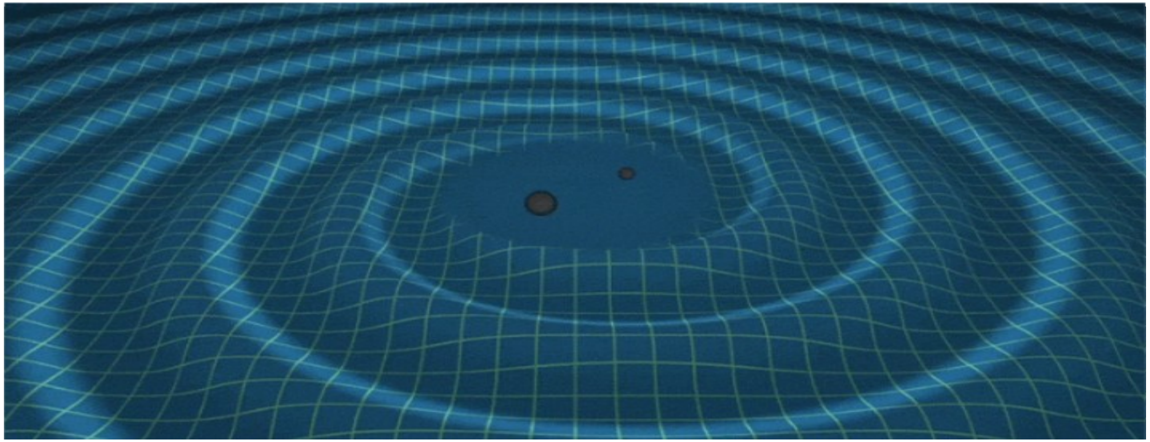
Gravity is due to spacetime curvature.
Matter tells spacetime how to bend and spacetime tells matter how to move.

Gravitational Waves (GW)

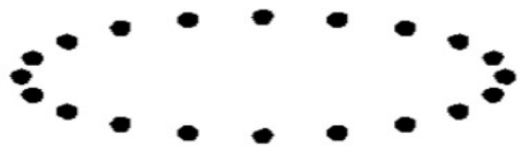
Albert Einstein – 1916:

- Massive accelerating objects disturb spacetime creating waves of spacetime propagating at the speed of light in the Universe.

Generation and propagation of GW



Effect of GW on test mass particles

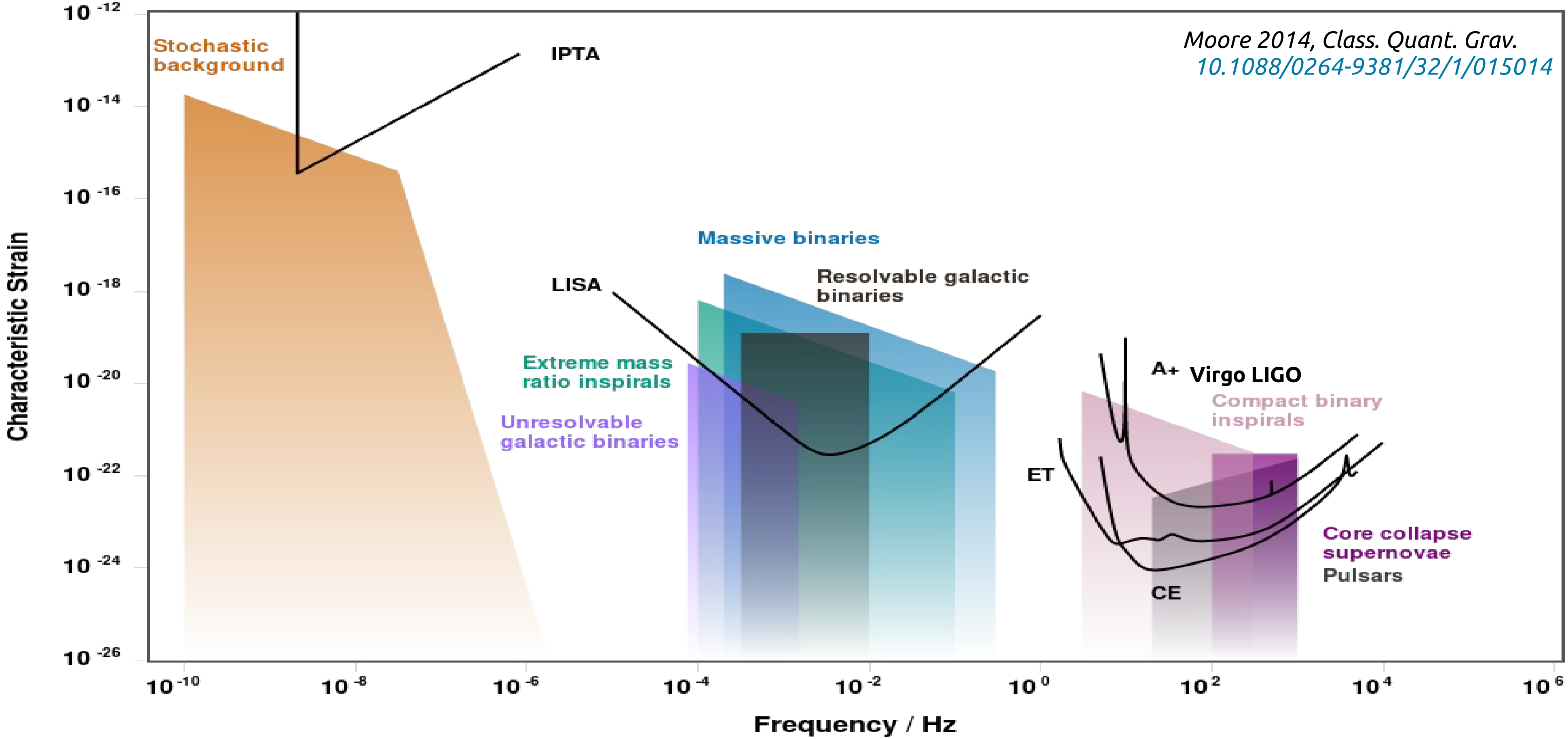


GW strain

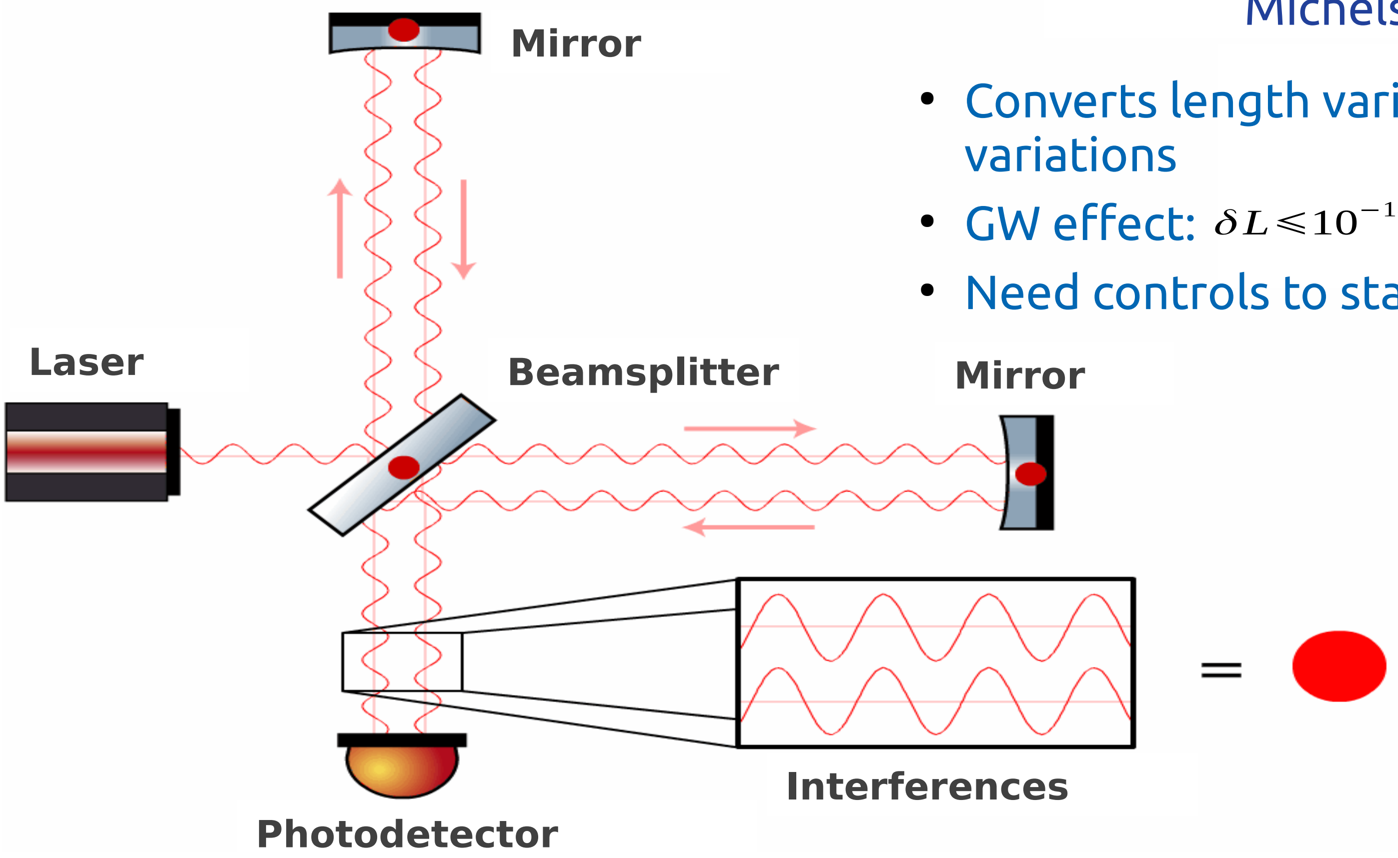
Length at rest

$$\delta L = \frac{1}{2} h \cdot L$$

GW sources and detectors



Interferometric detection



Michelson interferometer

- Converts length variations into power variations
- GW effect: $\delta L \leqslant 10^{-18} m$
- Need controls to stay close to dark fringe

Advanced Virgo – Cascina, Italy

Advanced Virgo (AdV)

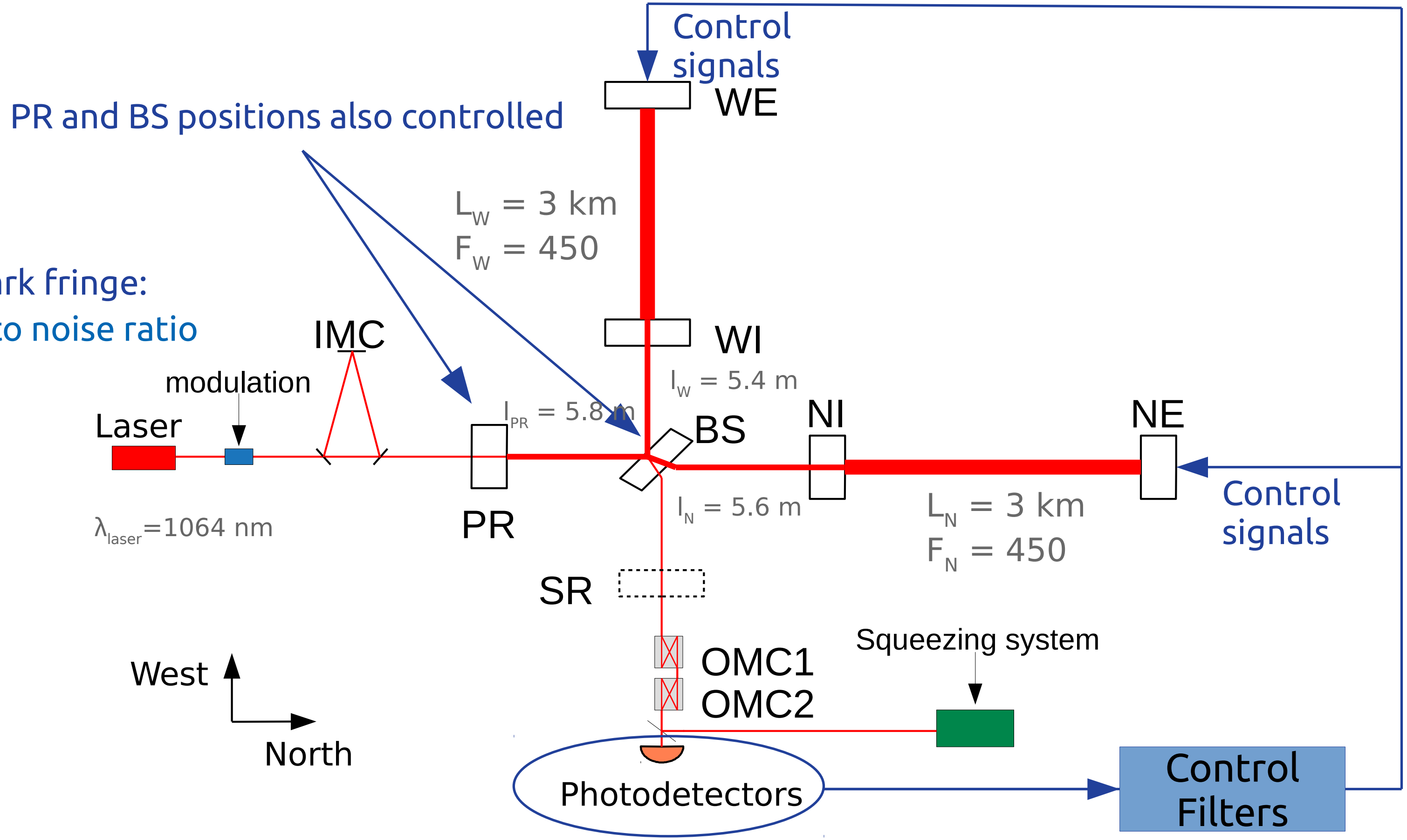
Suspended mirrors
→ free test masses

Interferometer close to dark fringe:
→ better photon signal to noise ratio

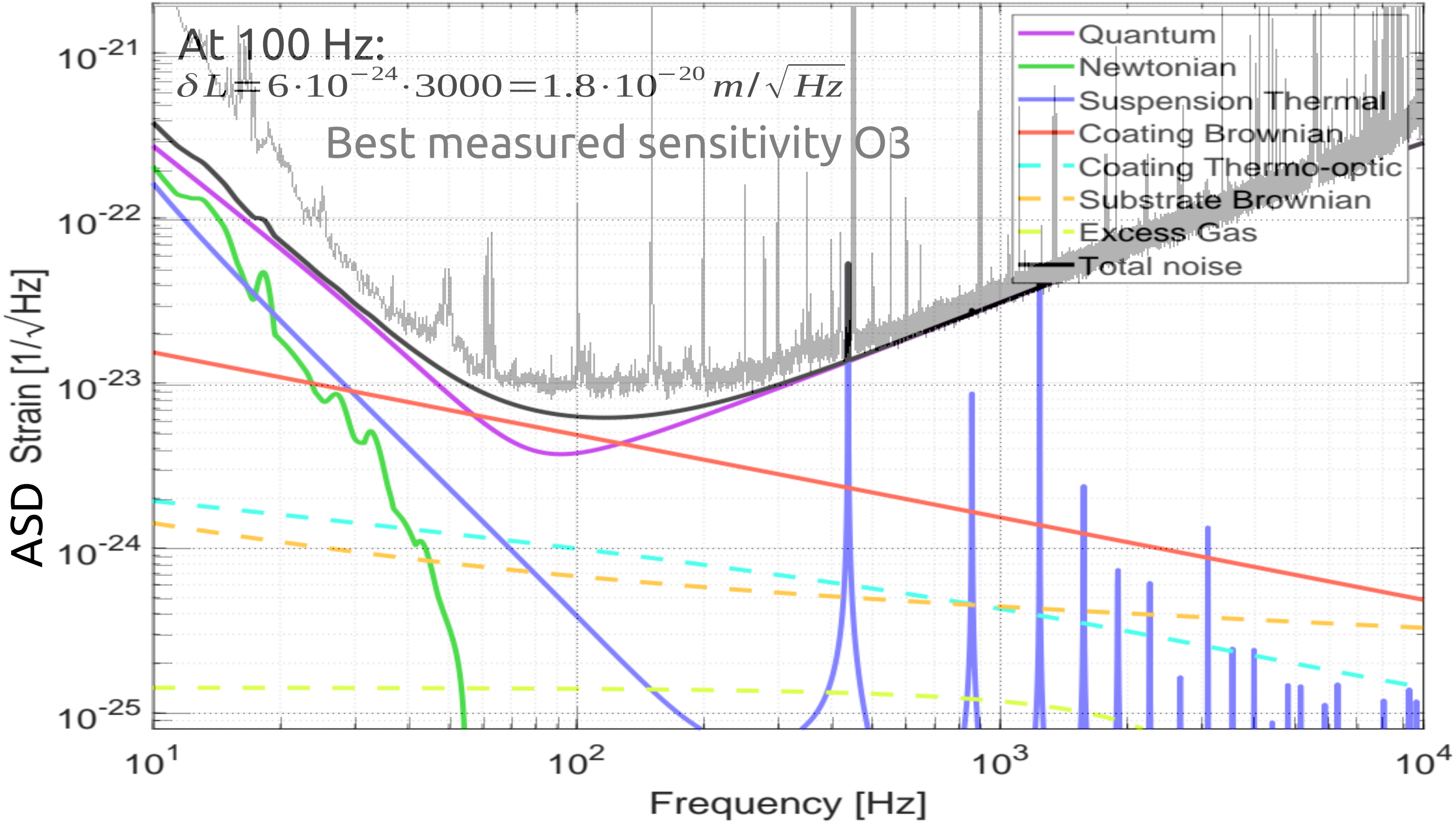
Sensitivity:

$$\tilde{h} \propto \frac{1}{L\sqrt{P}}$$

→ long arm-length L
→ high laser power P



Sensitivity of AdV (O3)



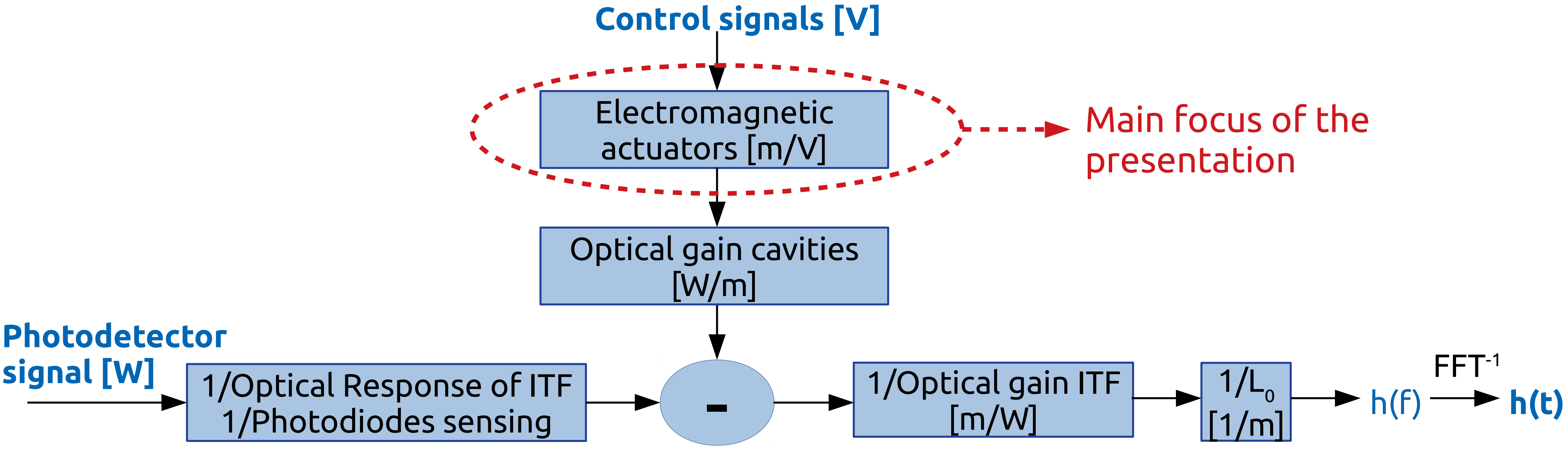
Principle of reconstruction of *h(t)*

ITF « locked » close to dark fringe → most of the GW signal is in control signals

→ Need to recontruct the GW signal $h=\frac{\Delta(L_N-L_W)}{L_0}=\frac{\Delta L_{free}}{L_0} \quad (L_0=3\,km)$

Photodetector signal	-	Control signals	⇒	Online reconstructed GW amplitude *h(t)*
↓		↓		↓
Real differential arm length variation of the ITF		Controlled differential arm length variation of the ITF		Free differential arm length variation of the ITF
ΔL_{real}		ΔL_{ctrl}		$\Delta L_{free} \equiv L_0 \cdot h$

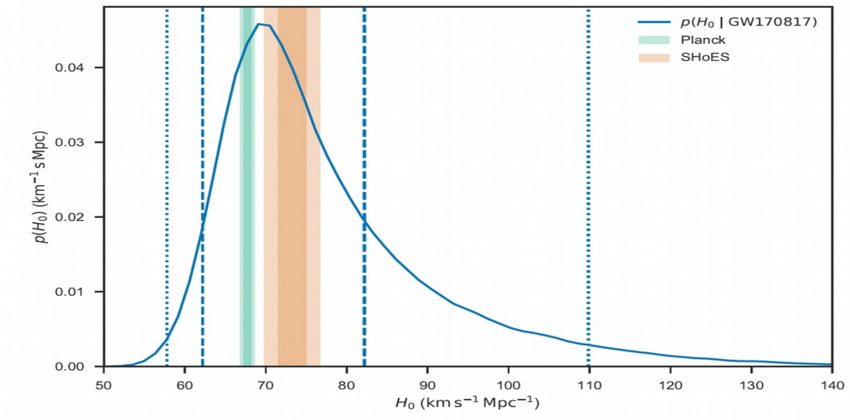
Reconstruction of *h(t)*



- Frequency domain computation
- All blue boxes need to be calibrated to get a correct *h(t)*
- Latency production of *h(t)* ~8 s

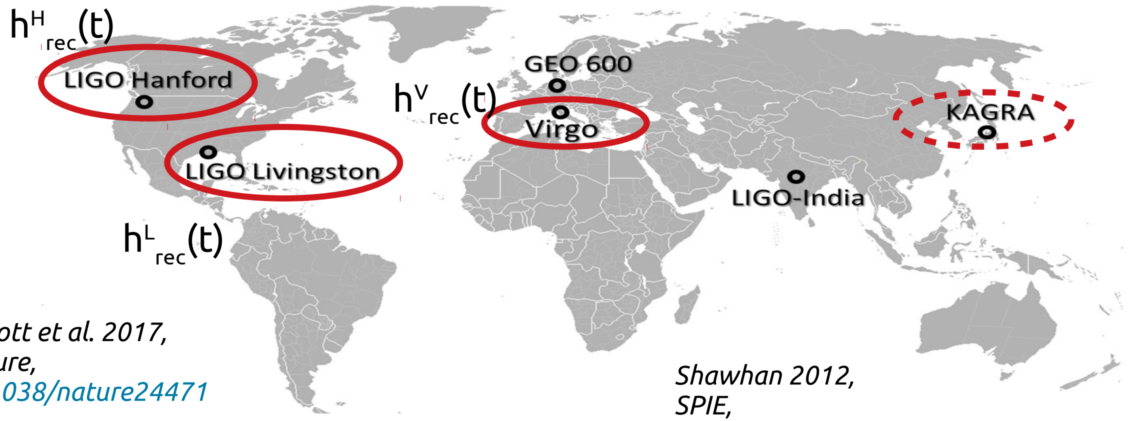
Calibration motivations

Many results from GW detections, one example:
Hubble constant with GW170817



Abbott et al. 2017, Nature, *10.1038/nature24471*

LIGO-Virgo (soon KAGRA) coincident detections/analyses:



Shawhan 2012, SPIE, *10.1117/12.926372*

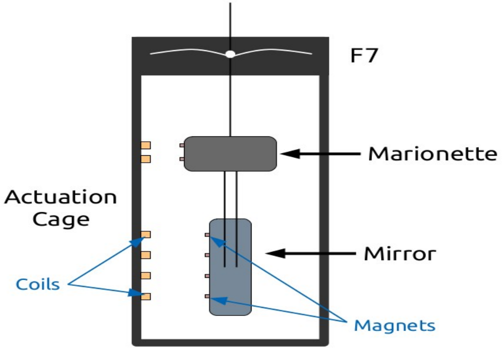
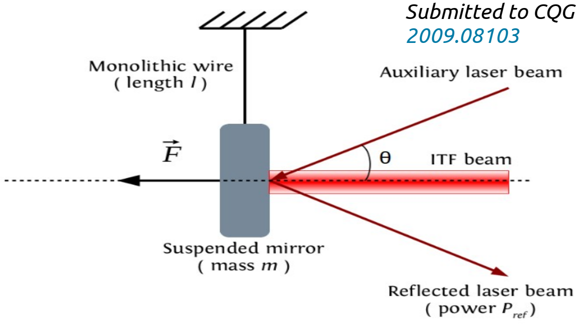
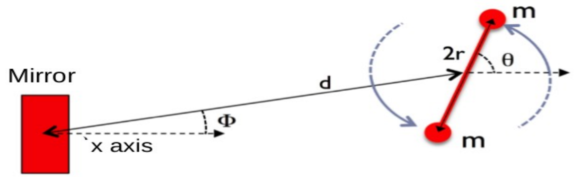
- Hubble constant requires a precise measurements of luminosity distance of a GW source
- Parameters estimation of GW sources should not be biased by calibration errors

- Absolute intercalibration of the detectors network in amplitude (based on NIST)
- Absolute timing (GPS) between the detectors is also crucial for sky-localization

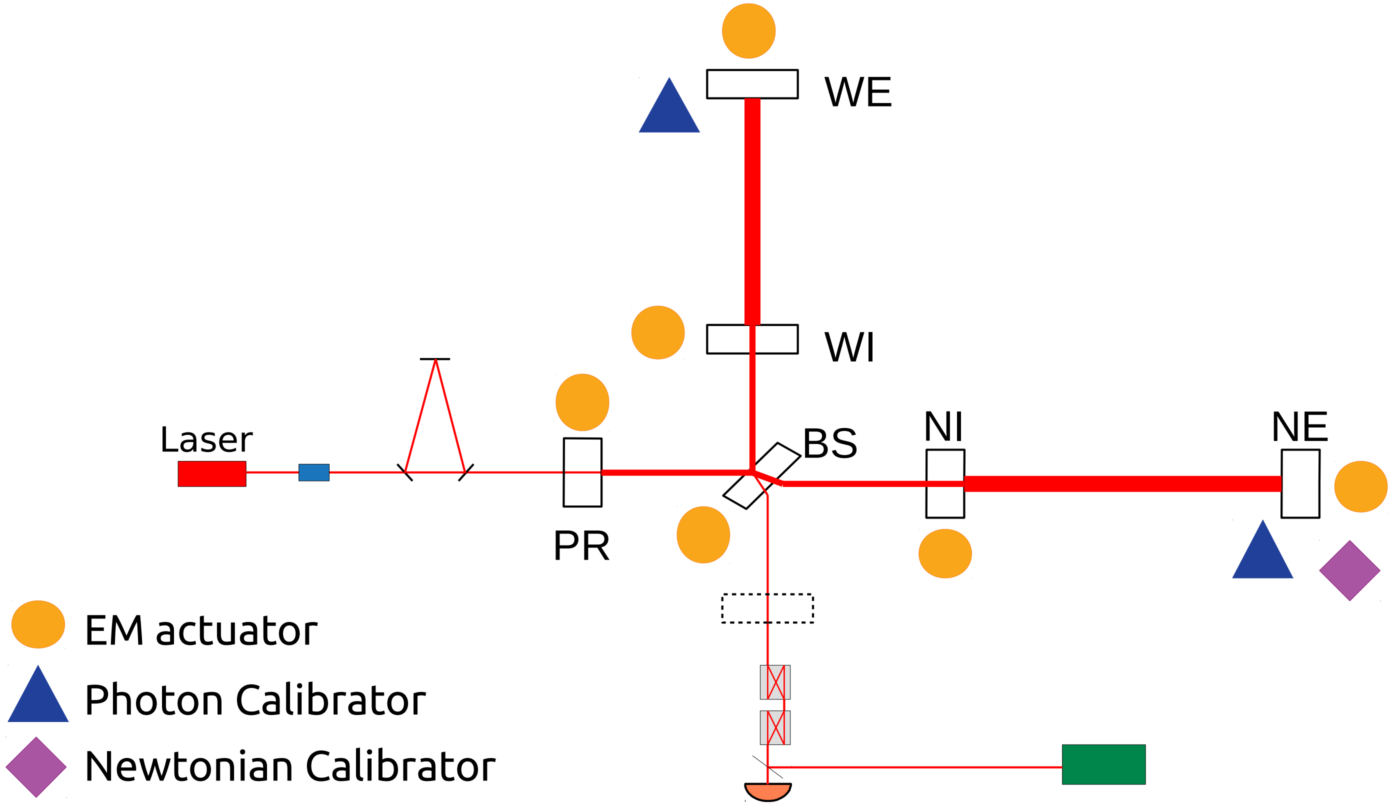
Compact Binary Coalescence (CBC) analysis uses matched filtering:

- Following the GW signal in time and in frequency
- Data of detectors network need to be calibrated from ~20 Hz to ~2 kHz

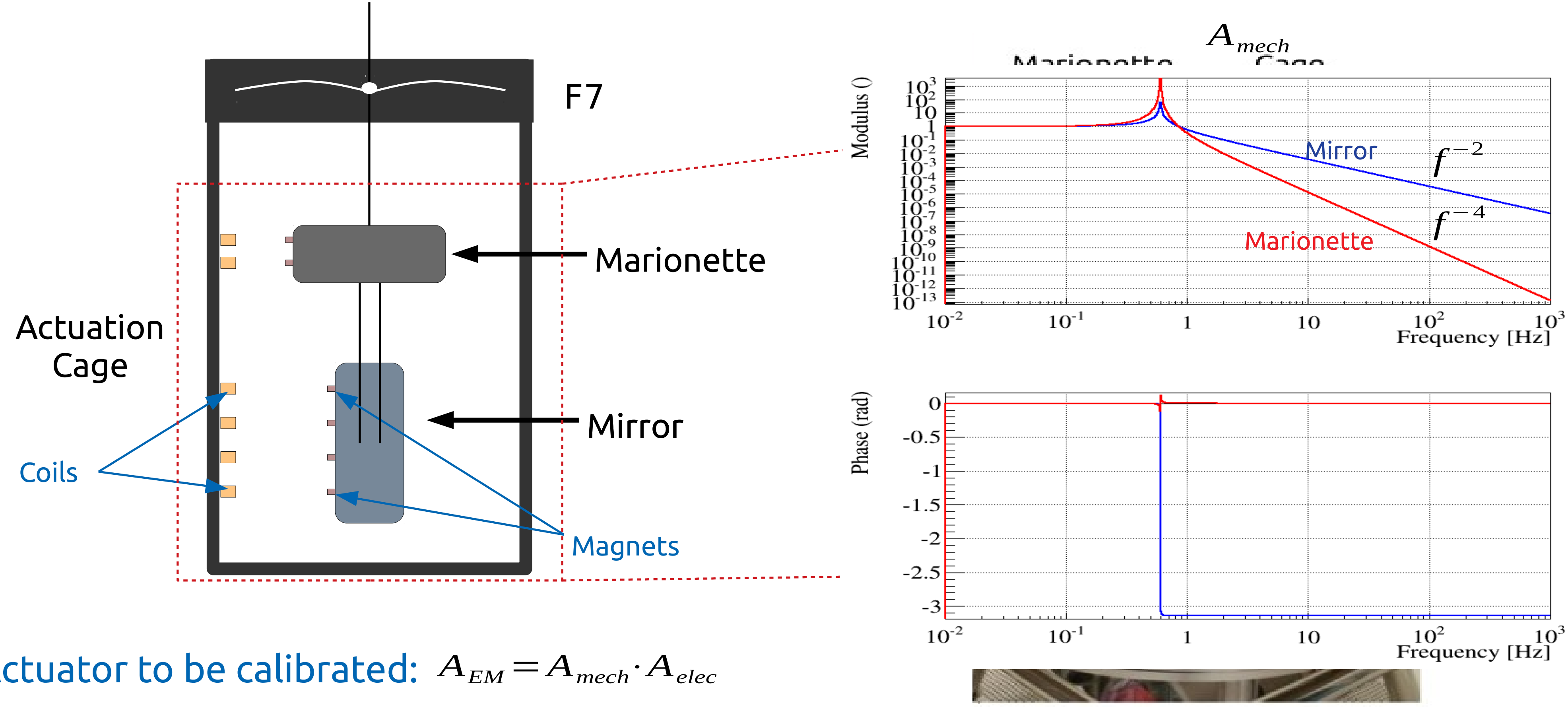
Past, present, future calibration

Electromagnetic actuator (EM)	**Photon calibrator** (PCal)	**Newtonian calibrator** (NCal)
	*Estevez et al. 2020, Submitted to CQG* *2009.08103* 	*Estevez et al. 2018, Class. Quant. Grav.* *10.1088/1361-6382/aae95f* *Estevez et al. 2020, in preparation* 
Method (up to O2): → Move the mirror with EM force and reconstruct the motion using fringes onto the photodetector	Method (O3 + future): → Induce a mirror motion by radiation pressure and compare it to an EM motion	Method (future): → Induce a mirror motion by variations of the local gravitational field (Second order effect of Newton's law in d^{-4})
Calibration reference: → Laser wavelength of the ITF	Calibration reference: → *Absolute* laser power (NIST)	Calibration reference: → Gravitational Constant G
Calibration range: → ~10 Hz to ~1 kHz	Calibration range: → ~10 Hz to ~10 kHz	Calibration range: → ~10 Hz to ~200 Hz (maybe more)
ITF not in observing configuration	ITF in observing configuration	ITF in observing configuration

Several EM actuators to calibrate

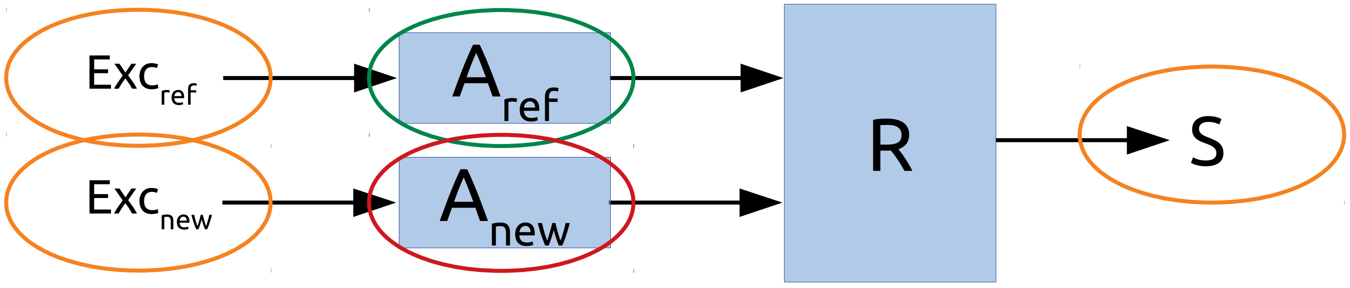


Electromagnetic actuators



Actuator to be calibrated: $A_{EM} = A_{mech} \cdot A_{elec}$

Principle of calibration



R: Response of the system
A: Actuator
S: Output of the system
Exc: Excitations/Perturbations

$$\frac{S}{Exc_{ref}} = A_{ref} R$$

$$\frac{S}{Exc_{new}} = A_{new} R$$

$$\Rightarrow \quad A_{new} = \left[\frac{S}{Exc_{new}}\right]\left[\frac{S}{Exc_{ref}}\right]^{-1} A_{ref}$$

We need A_{ref} → Photon Calibrators (PCal)

Photon Calibrator (PCal)

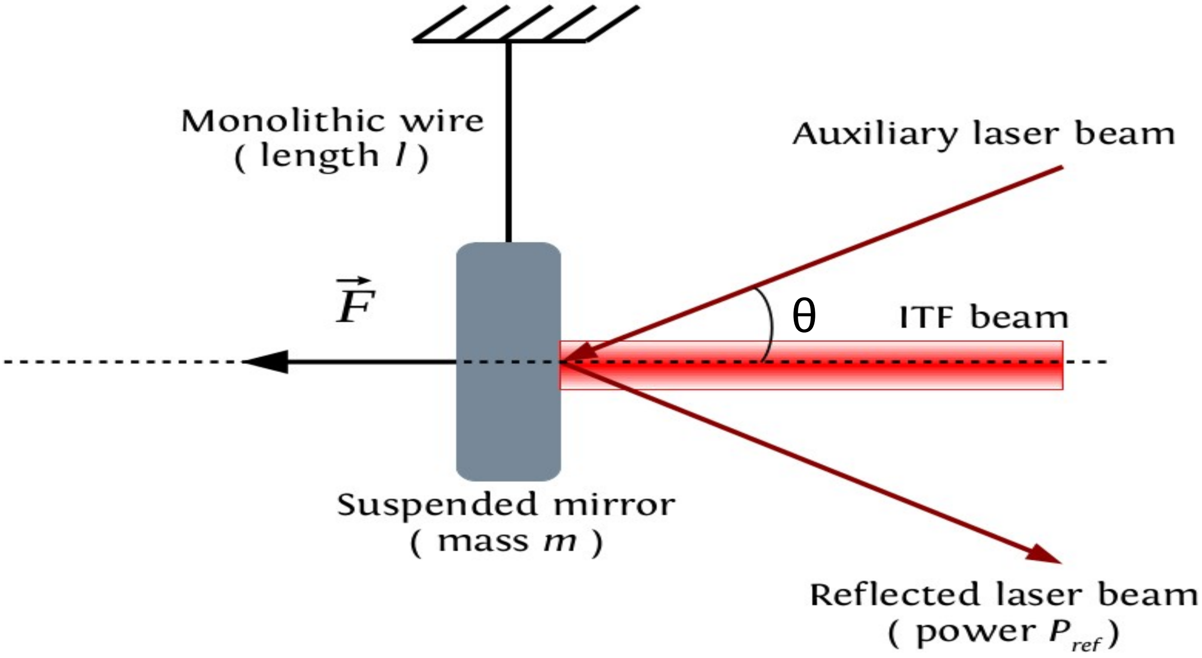
From laser power to force:

$$F = \frac{2\cos(\theta)}{c} P_{end}$$

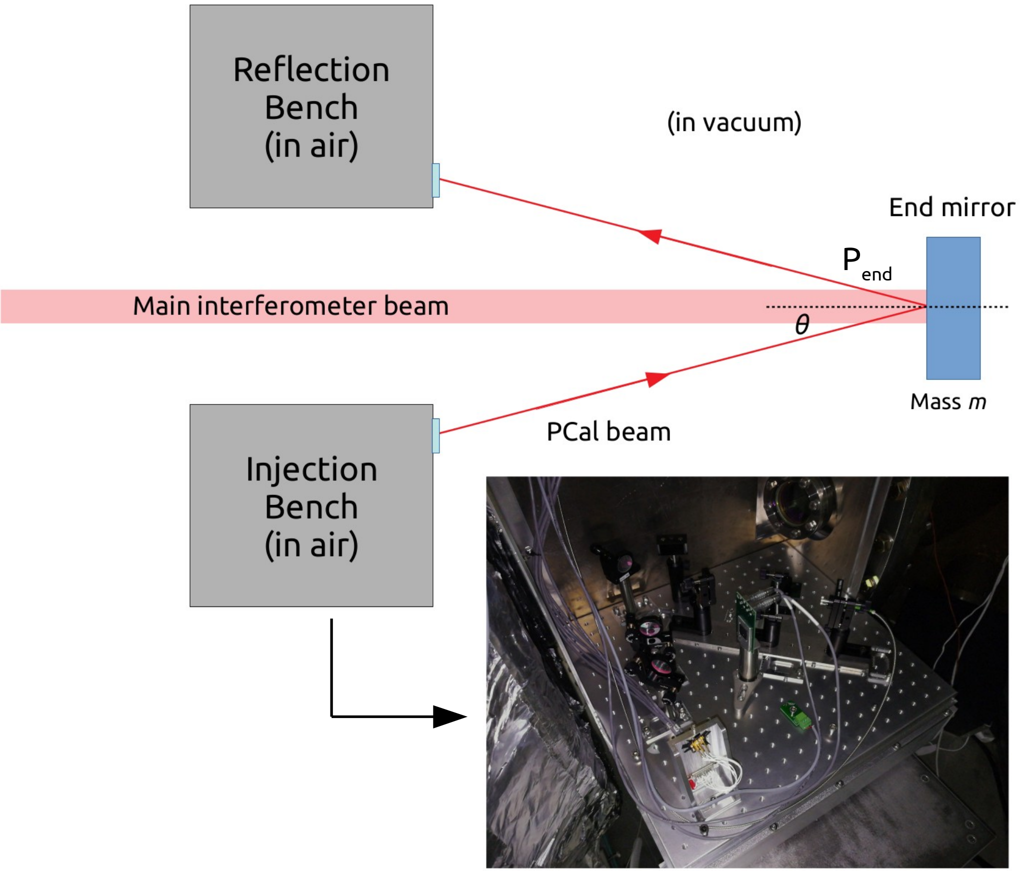
From force to mirror motion assuming a rigid body:

$$x = -\frac{1}{m(2\pi f)^2} F$$

with f the modulation frequency of the laser power



Experimental setup



$$x_{pcal}^{free}(f) = -\frac{1}{m(2\pi f)^2}\frac{2\cos(\theta)}{c}P_{end}(f)$$

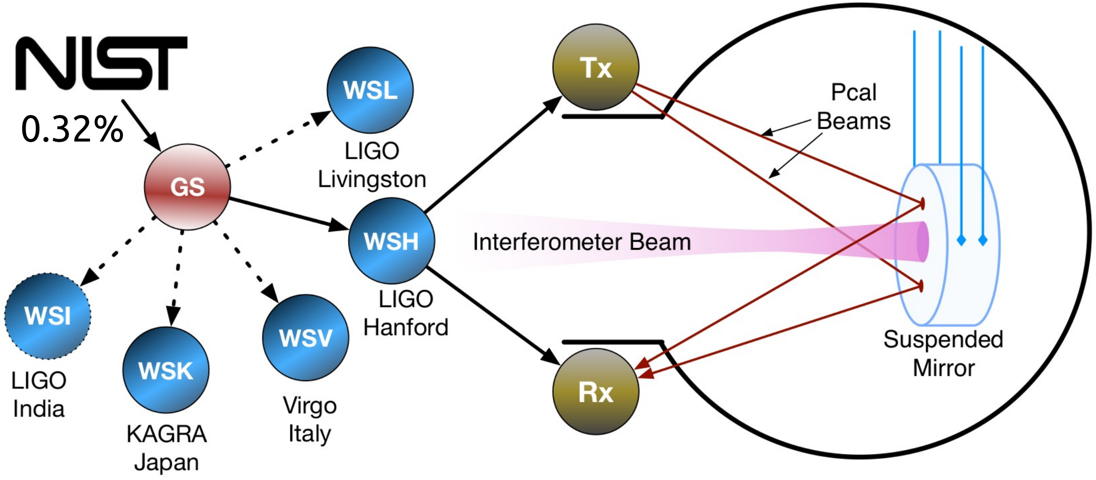
Geometrical parameters:

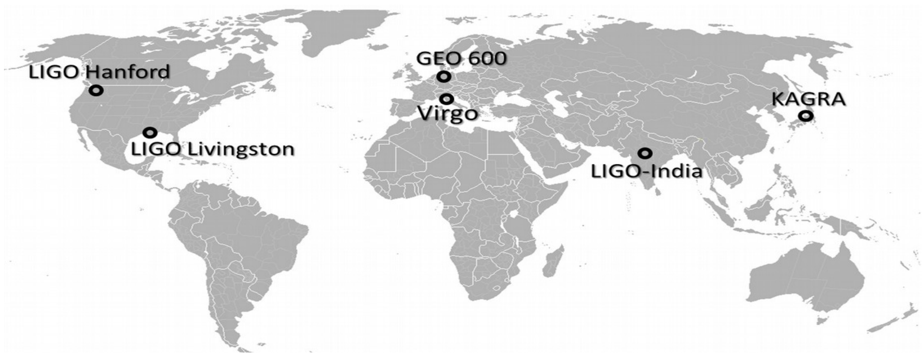
- Angle of incidence θ known with opto-mechanical constraints and design
- Mass of the mirror m known with density of mirror material and volume

Need to precisely estimate P_{ref} (~2 W):

- Using a laser pick-off sent to photodiodes (~3 mW)
- Photodiodes are calibrated with an integrating sphere (derive a conversion factor in V/W)
- Estimation of optical efficiency (viewports + end mirror)
- **Does Virgo integrating sphere measure the same laser power as LIGO ones?**

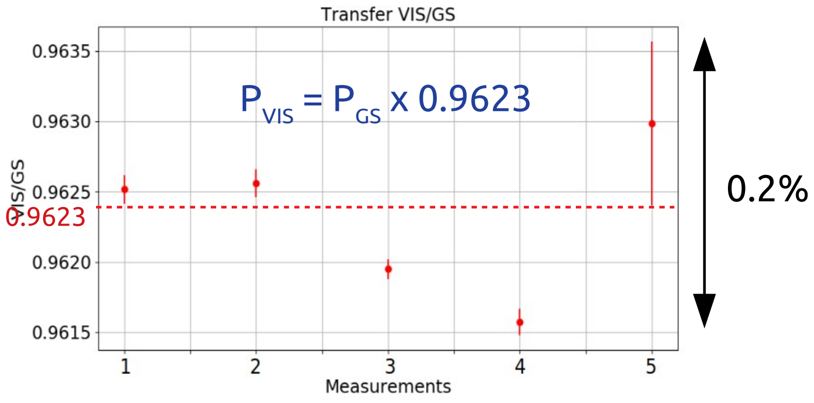
Worldwide intercalibration





Calibration transfer at LIGO Hanford between Virgo Integrating Sphere and Gold Standard

P_{VIS} has to be corrected by +3.92%



Only stat. errors

Uncertainty on PCal simple model

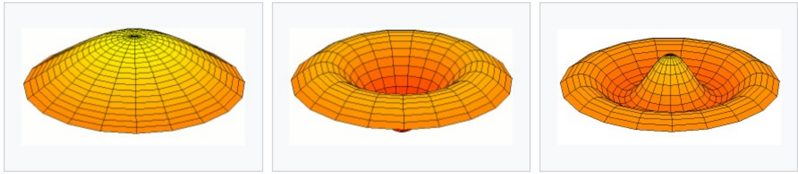
Variable	1σ Uncertainty
GS responsivity (2018)	0.32%
VIS linearity	0.4%
VIS/GS responsivity ratio	0.1%
VIS/WSV responsivity ratio	0.5%
Voltage calibrator	0.007%
Conversion factor [V/W]	1%
Angle cosine	0.12%
Rotation of ETM	0.001%
Mass of ETM	0.05%
PD stability w.r.t temperature (O3a)	0.1%
PD stability in time (O3a)	0.5%
Total	1.34%

This table is only valid for a "free test mass" response, below 400 Hz,
but we need to calibrate up to ~ 2 kHz

More realistic PCal model

The mirror is not a rigid body:

→ Excitation of internal axisymmetric high order modes due to the laser beam hitting the center of the mirror

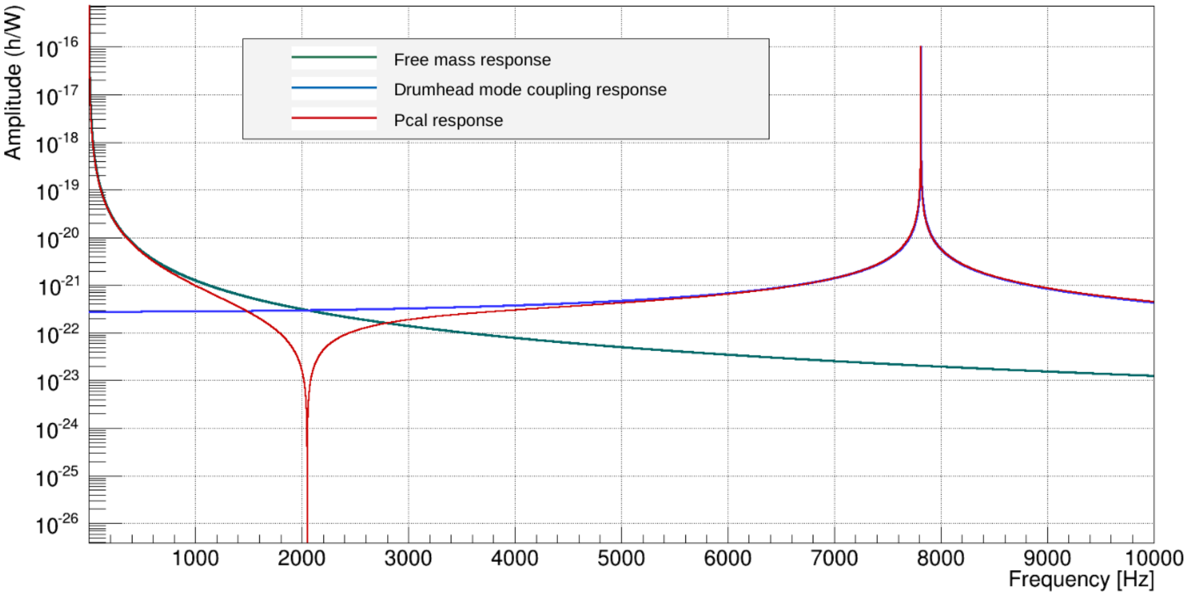


$$x_{pcal}(f) = \left[-\frac{1}{m(2\pi f)^2} + H_d(f) \right] \frac{2\cos(\theta)}{c} P_{end}(f)$$

$$H_d(f) = \frac{G_d}{1 + \frac{j}{Q_d}\frac{f}{f_d} - \left(\frac{f}{f_d}\right)^2}$$

Need to precisely measure G_d and f_d ($Q_d > 10^6$)

PCal	ΔG_d	Δf_d
WE	$\pm 0.35\%$	$\pm 0.014\%$
NE	$\pm 0.37\%$	$\pm 0.006\%$



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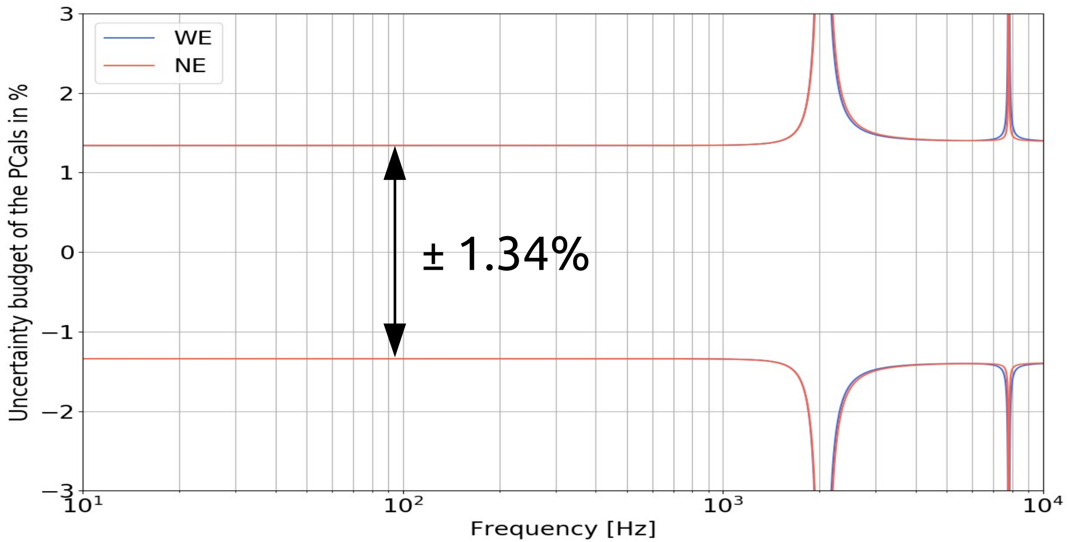
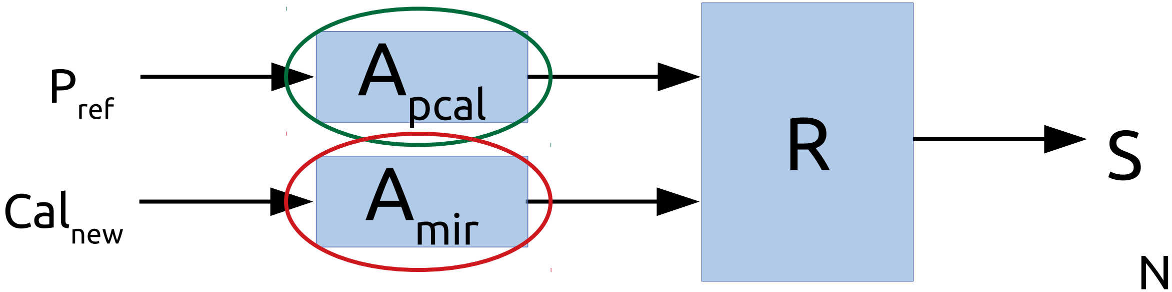


This is the limiting calibration uncertainty for the reconstructed *h(t)*

Calibration of EM actuators

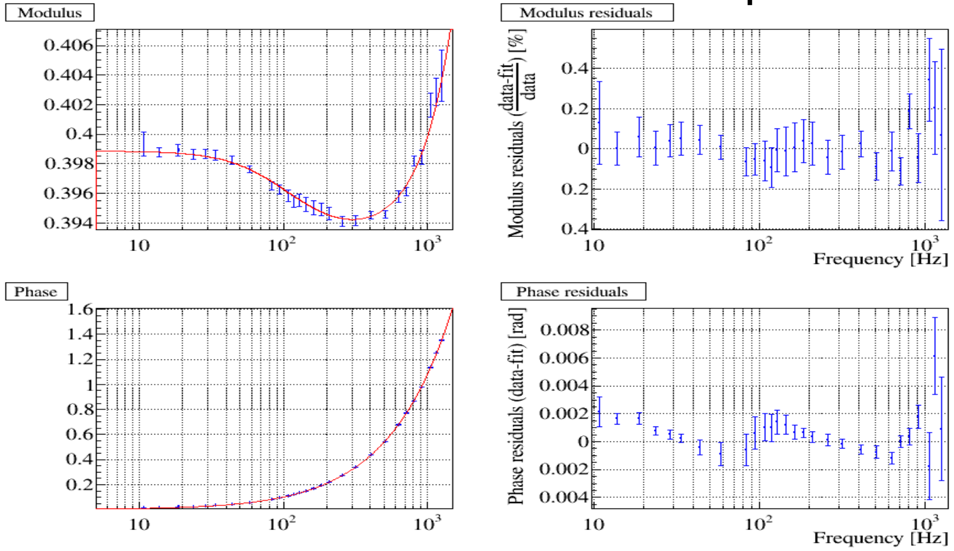


- A_{pcal} has been calibrated
- A_{mir} has to be calibrated

$$A_{mir} = \left[\frac{S}{Cal_{new}}\right]\left[\frac{S}{P_{ref}}\right]^{-1} A_{pcal}$$

$$A_{mir} = A_{mech} \cdot A_{elec} \quad \text{with} \quad A_{mech} \propto f^{-2}$$

Normalized EM actuator response

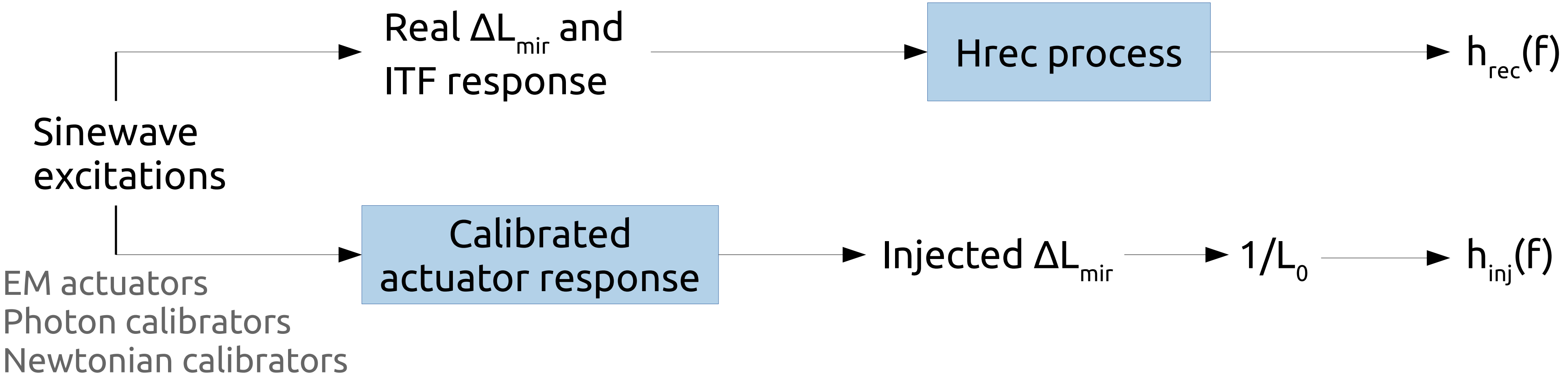


Normalize A_{mir} by the simple pendulum response:

➔ Better see the fine effects of the electronics

Check *h(t)* reconstruction

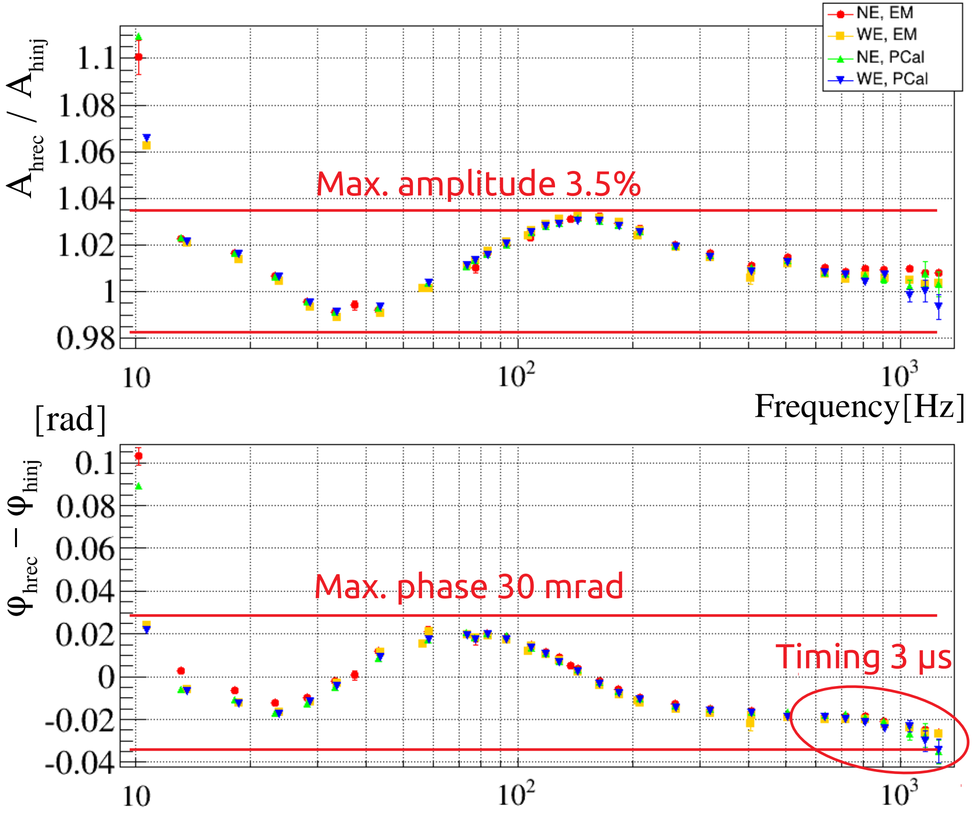
EM actuators are calibrated → *h(t)* can be reconstructed
→ Need to assess how well we reconstruct *h(t)*



Transfer function from h_{inj} to h_{rec}

- Perfect case → amplitude 1, phase 0 rad
- Real case → frequency dependent bias

Uncertainty on the reconstructed *h(t)*



Adding uncertainty from calibration:

Amplitude uncertainty → **δA = ±5%**

Phase uncertainty → **δΦ = ±35 mrad**

Timing uncertainty → **δτ = ±10 µs**

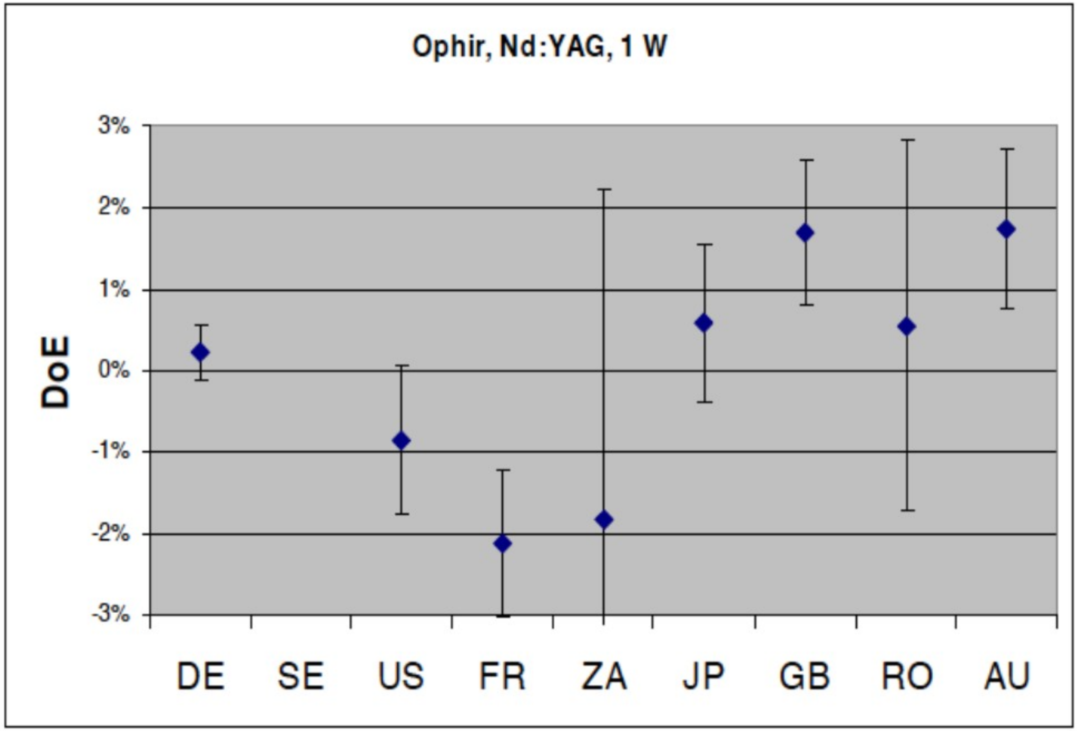
Uncertainty provided with online *h(t)*

EM actuators and PCal measurements:

- Same bias in *h(t)*
- Something is not accurate in the reconstruction

Absolute calibration issue

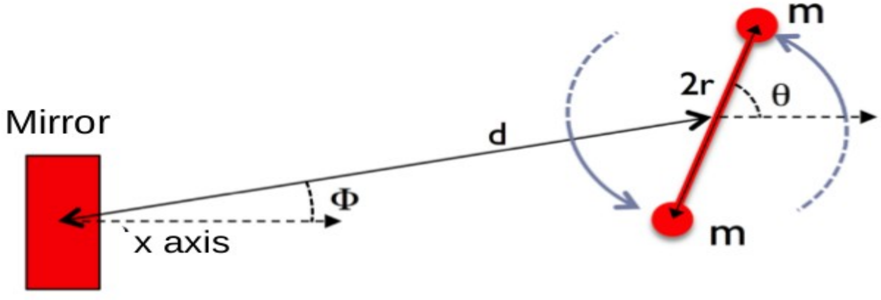
2009 EUROMET Comparison Project no. 156



Reference: Gold Standard calibrated by NIST at the level of 0.32%

Need another calibration method to check the absolute calibration
→ Newtonian Calibrator (NCal)

Newtonian Calibrator (NCal)



$$F = \frac{9}{2}\frac{GMmr^2}{d^4}\left[\cos(2\theta)\right]$$

For frequency well above pendulum resonance (0.6 Hz):

$$\Delta x_{ncal}(\theta, f_h) = \frac{F(\theta)}{M(2\pi f_h)^2}$$ $f_h = 2f_{rotor}$

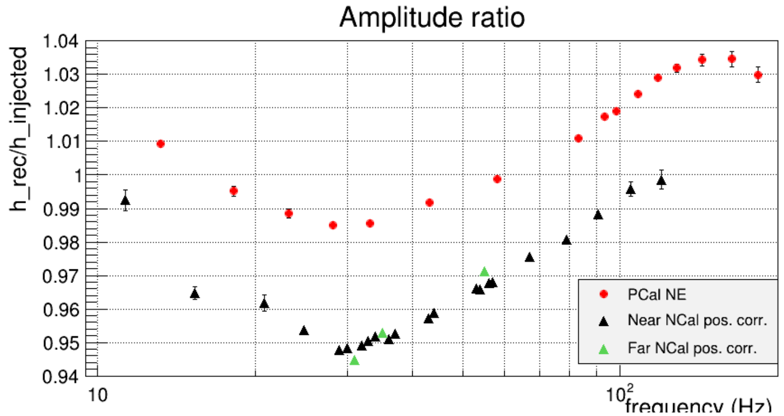
Mirror motion amplitude:

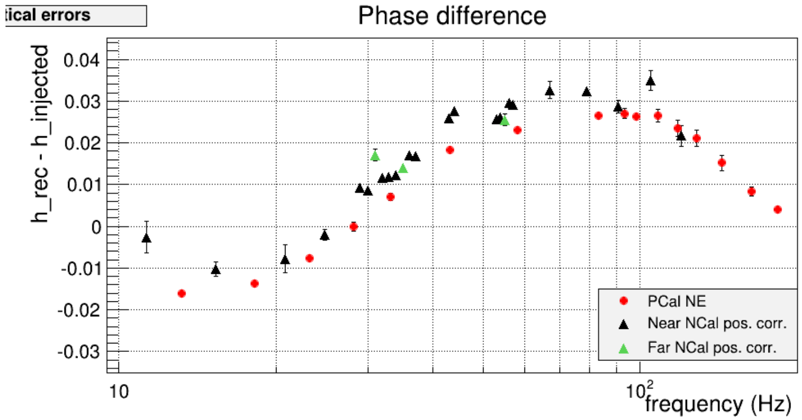
$$\alpha_h(f_h) = \frac{RP}{f_h^2} \qquad R = \frac{Gmr^2}{8\pi^2} \qquad P = \frac{9}{d^4}$$

The rotor geometry is not just point masses:
→ We developed and used a finite element analysis model to analyse the O3 NCal data

Tests with the NCal

Verification of h(t) reconstruction





NCal independent method:

→ Same frequency dependent shape as with PCal and EM actuators

→ Offset on amplitude (~3%)

→ Nice agreement on the phase

Parameter	h_{rec}/h_{inj} near [%]	h_{rec}/h_{inj} far [%]
NCal to mirror distance d	2.02	1.31
NCal to mirror angle Φ	0.23	0.23
NCal vertical position z	1.6 e-4	0.7 e-4
Rotor geometry	0.53	0.53
Modeling method	0.018	0.017
Mirror torque from NCal	0.05	0.03
Total	2.1	1.4

(Uncertainty on G is ~0.002%)

Not incompatible with the PCal systematic uncertainty

Takeaway

- Calibration of the gravitational-wave detectors is (will be) of prime importance not to bias the scientific results
  - Current level of uncertainty is between 2% and 5% in amplitude and ~2° in phase
  - "Good enough" for the current sensitivity of the detectors
  - Expect to go below 1% in amplitude in the future

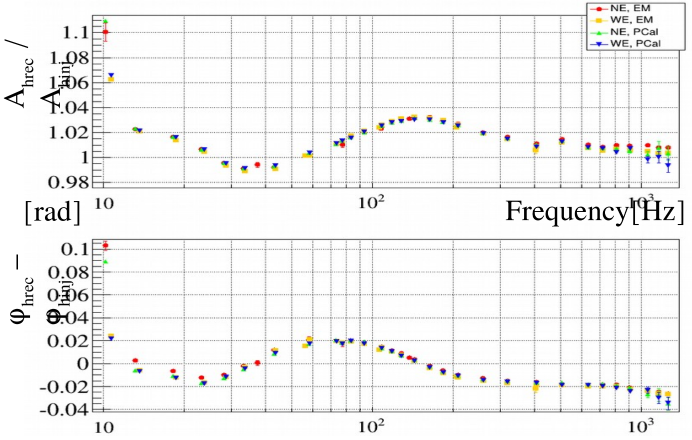
- Method: compare an injected known signal to the output of the interferometer

- Photon Calibrators are the reference calibration tools for the detectors network
  - Possibility to intercalibrate the integrating spheres on a common "Gold Standard" calibrated by NIST
  - Fastest way to validate the reconstructed *h(t)* from 10 Hz to 2 kHz (and beyond)
  - Measurement of laser power is not that simple, dependence on temperature, humidity etc…

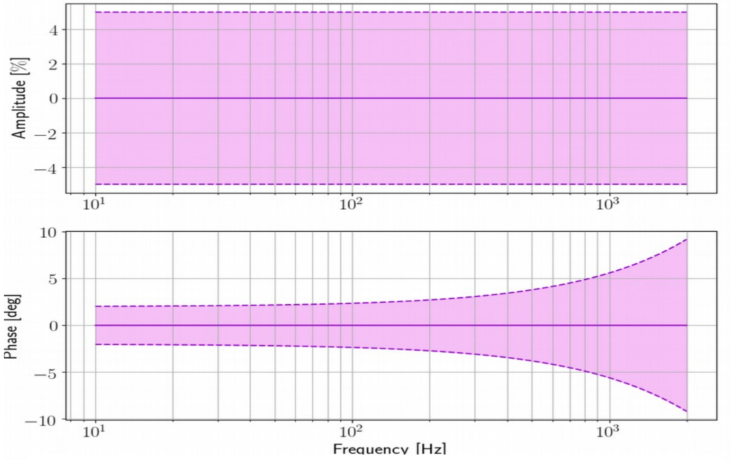
- Relative calibration between the detectors has been tackled but is the calibration absolute?
  - Development of Newtonian Calibrators with "simpler" parameters to control (distance and geometry)
  - Difficult to check the reconstructed *h(t)* at high frequency
  - Possibility to calibrate the PCals at low frequency and extend the calibration with the PCals at high frequency

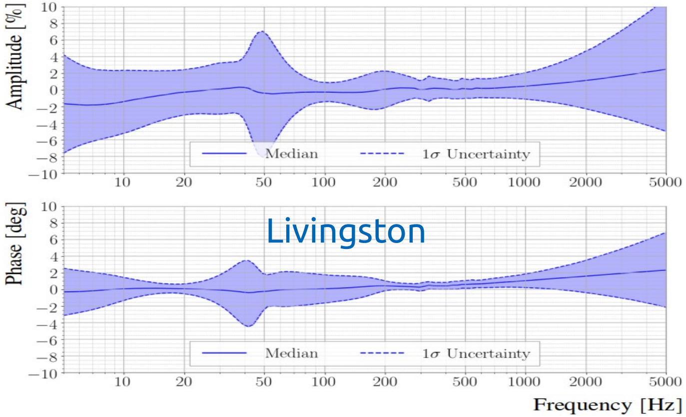
EXTRA SLIDES

Uncertainty on online *h(t)*

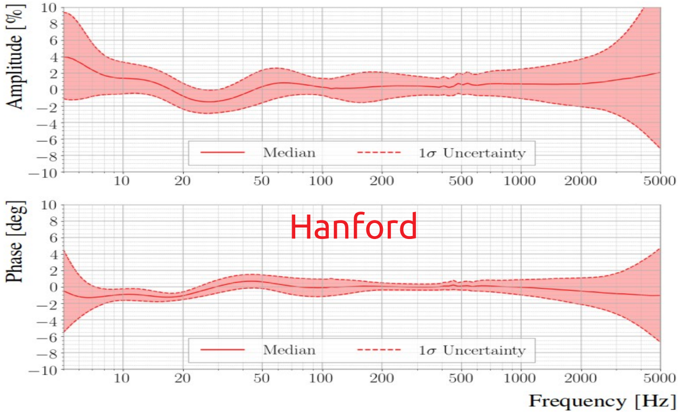


Uncertainty on online *h(t)* for Virgo

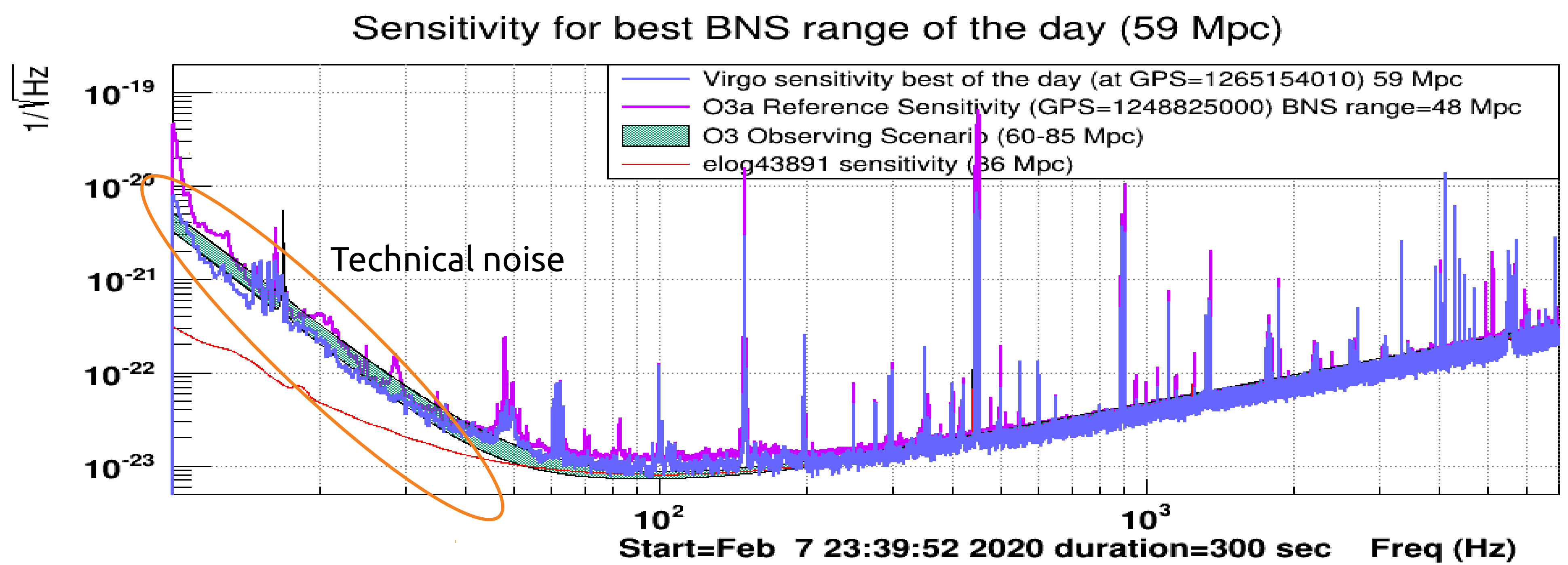




Uncertainty on online *h(t)* for LIGO

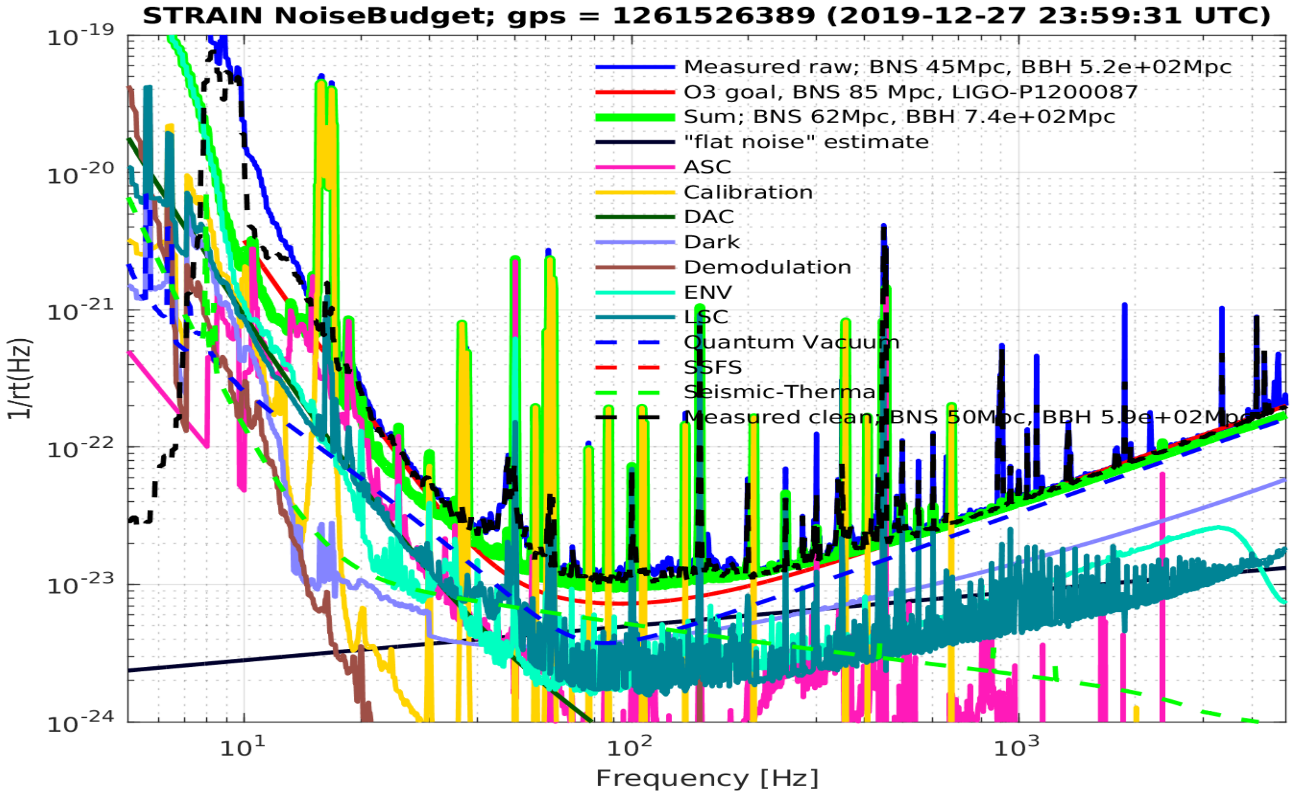


Advanced Virgo sensitivity O3

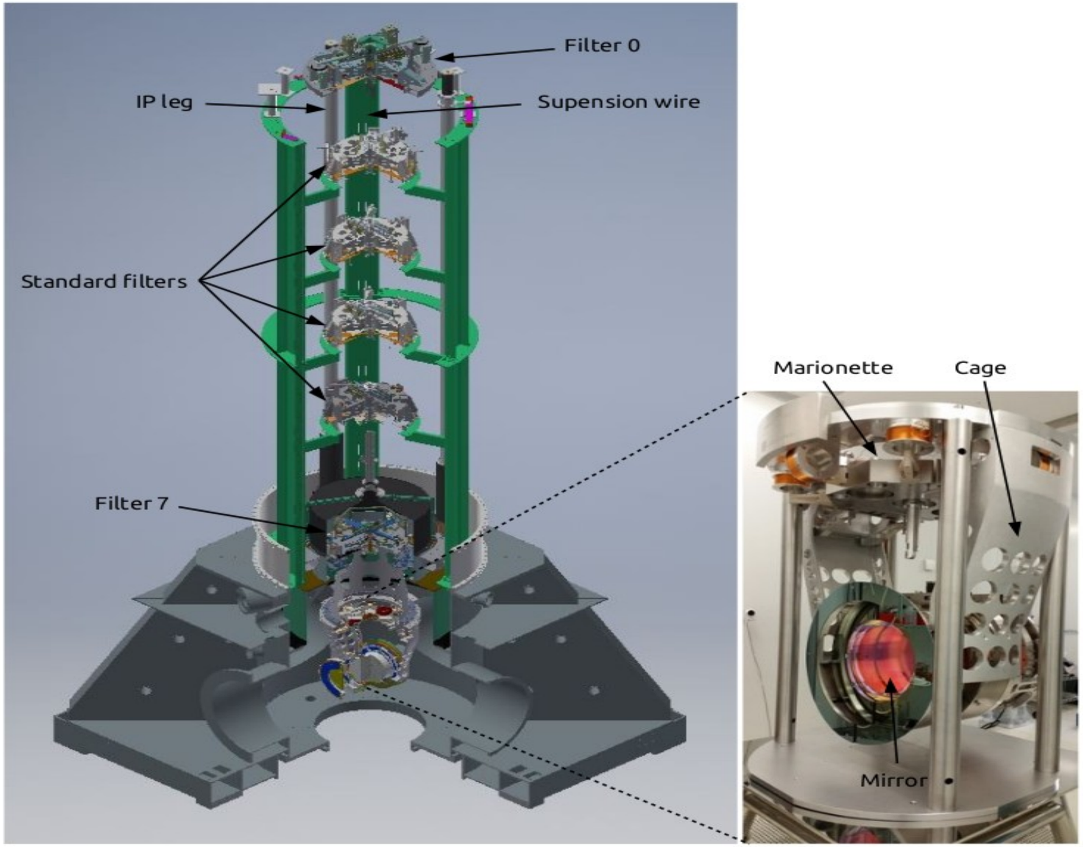


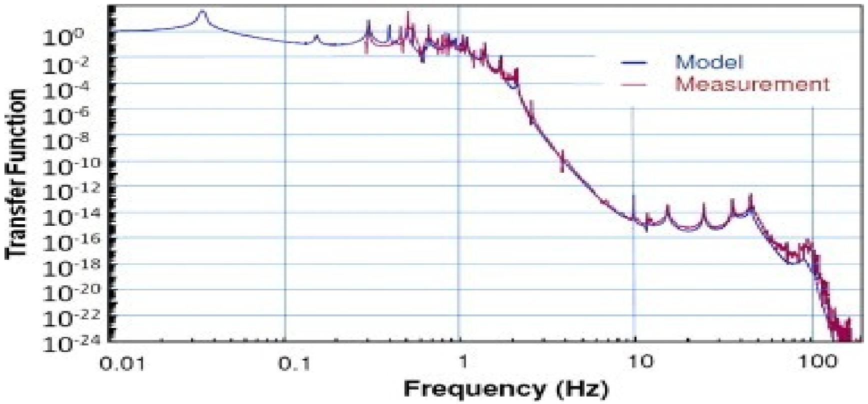
Noise budget O3

Not only “fundamental noise”



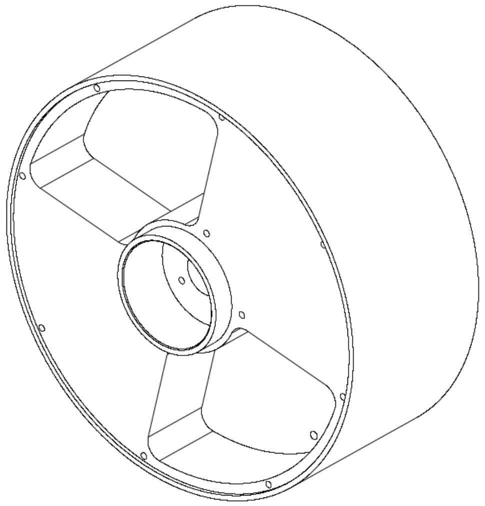
Advanced Virgo super-attenuator





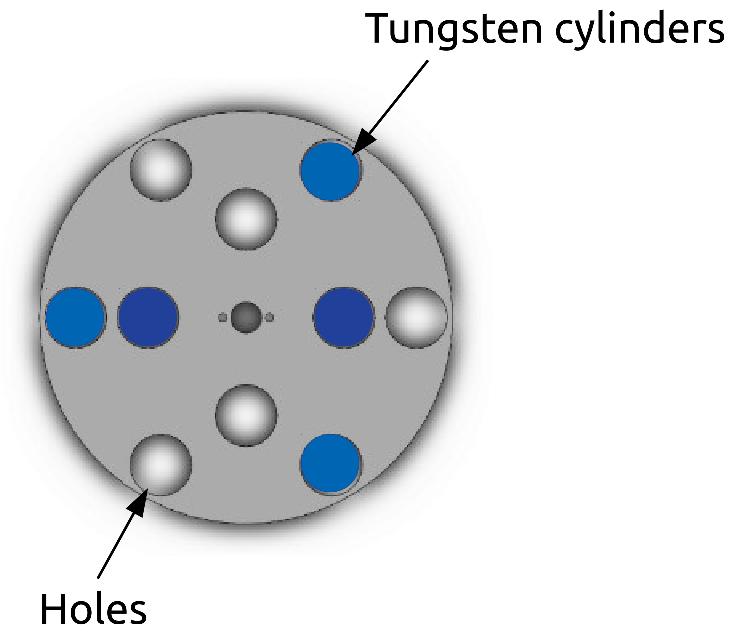
NCal geometry

Virgo:



Two aluminum 90° cylindrical sectors
Expected force at 2f
Spins up to f ~100 Hz

LIGO:



Expected forces at 2f, 3f, 4f and 6f
(but forces > 2f are very small in practice…)
Spins up to f ~10 Hz