Leptonic & semileptonic exclusive decays

(form factors and HQET constraints)

Christoph Bobeth

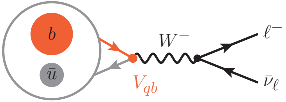
Technical University Munich

GDR Lectures
29 September, 2020

Outline

- Leptonic CC and FCNC decays
- Semileptonic CC decays
- Form factor determination
- z-Expansion & unitarity bounds
- Heavy-to-heavy form factors in HQET

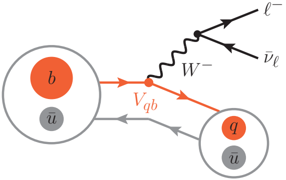
Main motivation: Determination of $|V_{cb}|$ and $|V_{ub}|$



Leptonic $|V_{ub}|$

$$\mathcal{B} \propto |V_{ub}|^2 f_B^2 \frac{m_\ell^2}{m_B^2}$$

- f_B = B-meson decay constant
- helicity-suppression → difficult experimentally



Exclusive $|V_{ub}|$

$\overline{B} \to \pi\ell\overline{\nu}_\ell$, $\Lambda_b \to p\ell\overline{\nu}_\ell$

future $B_s \to K\ell\overline{\nu}_\ell$

Exclusive $|V_{cb}|$

$\overline{B} \to (D, D^*)\ell\overline{\nu}_\ell$, $\Lambda_b \to \Lambda_c\ell\overline{\nu}_\ell$

future $B_s \to (D_s, D_s^*)\ell\overline{\nu}_\ell$

$$\frac{d\mathcal{B}}{dq^2} \propto |V_{qb}|^2 f^2(q^2) \otimes d\Pi(q^2)$$

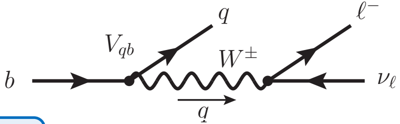
- $f^2(q^2) \otimes d\Pi(q^2)$ = form factors $\otimes$ phase space
- exclusive = low background in experiment

EFT for $b \to q\,\ell\bar{\nu}_\ell$

... from previous results of $\mu \to e\bar{\nu}_e\,\nu_\mu$ and $b \to s\,c\bar{c}$ follows in SM analogously:

EFT for $b \to q\,\ell\bar{\nu}_\ell$

with $q = u, c$ and $\ell = e, \mu, \tau$



$$\mathcal{L}_{\text{EFT}} = \mathcal{L}_{\text{QCD}\times\text{QED}} - \frac{4\mathcal{G}_F}{\sqrt{2}} V_{qb}\, C_{V_L}\, Q_{V_L}$$

$$Q_{V_L} \equiv [\bar{q}\gamma_\mu P_L b][\bar{\ell}\gamma^\mu P_L \nu_\ell]$$

!!! in principle each $C_{V_L} Q_{V_L} \to C_{V_L}^{q\ell\nu} Q_{V_L}^{q\ell\nu}$ should carry indices for q, ℓ, $\nu_{\ell'}$

- show here also gauge interactions $\mathcal{L}_{\text{QCD}\times\text{QED}}$ of quarks and leptons
- in SM only a single operator Q_{V_L}
- in SM the result $C_{V_L}^{\text{SM}}(\mu_W) = 1$ is lepton-flavor-universal
- no RG running under QCD $\Rightarrow$ $C_{V_L}^{\text{SM}}(\mu_b) = C_{V_L}^{\text{SM}}(\mu_W) + \mathcal{O}(\alpha_e)$
- EW matching corrections and QED RG evolution from $\mu_W \to \mu_b$

  "Sirlin correction": $C_{V_L}^{\text{SM}}(\mu_b) = 1 + \frac{\alpha_e}{\pi} \ln \frac{m_Z}{\mu_b} \approx 1.007$ [Sirlin NPBB 196 (1982) 83]

Leptonic CC and FCNC

$\Delta B = 1$ decays

Leptonic decays and B_q-decay constant

Matrix element at leading order in EW interactions:

$$i\mathcal{A}_{\text{EFT}} \equiv \langle \ell\bar{\nu}_\ell | i\mathcal{L}_{\text{EFT}} | \overline{B}_u \rangle \rightarrow \left\langle \ell\bar{\nu}_\ell \right| - i\,\frac{4\mathcal{G}_F}{\sqrt{2}} V_{qb}\, C_{V_L} Q_{V_L} \left| \overline{B}_u \right\rangle$$

$\Rightarrow$ the notation $\langle \ldots | \mathcal{L}_{\text{QCD}\times\text{QED}} + \sum_i C_i Q_i | \ldots \rangle$ denotes a Green function / S-matrix element,

where the path integral — do not show explicitly $\mathcal{L}_{\text{QCD}\times\text{QED}}$

- is meant to be fully evaluated w.r.t. QCD → requires nonperturbative methods
- usually QED treated perturbatively, restricted to lowest order

  (real radiation treatement left to experimentalists via generators/simulation)
- only single insertion of dim-6 operators: $C_i Q_i$ $\Rightarrow$ lowest order in EW interactions

Leptonic decays and B_q-decay constant

Matrix element at leading order in EW interactions:

$$i\mathcal{A}_{\rm EFT} \equiv \langle \ell\overline{\nu}_\ell | i\mathcal{L}_{\rm EFT} | \overline{B}_u \rangle \to \left\langle \ell\overline{\nu}_\ell \right| -i\frac{4\mathcal{G}_F}{\sqrt{2}} V_{qb}\, C_{V_L} Q_{V_L} \left| \overline{B}_u \right\rangle$$

$\Rightarrow$ the notation $\langle \ldots | \mathcal{L}_{\rm QCD\times QED} + \sum_i C_i Q_i | \ldots \rangle$ denotes a Green function / S-matrix element,

where the path integral

do not show explicitely $\mathcal{L}_{\rm QCD\times QED}$

- is meant to be fully evaluated w.r.t. QCD $\to$ requires nonperturbative methods
- usually QED treated perturbatively, restricted to lowest order

  (real radiation treatement left to experimentalists via generators/simulation)
- only single insertion of dim-6 operators: $C_i Q_i \Rightarrow$ lowest order in EW interactions

For leptonic decay $\overline{B}_u \to \ell\overline{\nu}_\ell$

$$i\mathcal{A}_{\rm EFT} \propto \langle \ell\overline{\nu}_\ell | Q_{V_L} | \overline{B}_u \rangle \propto \langle \ell\overline{\nu}_\ell | \overline{\ell}\gamma_\mu P_L \nu_\ell | 0 \rangle \times \langle 0 | \overline{q}\gamma^\mu P_L b | \overline{B}_u(p_B) \rangle \quad \leftarrow \text{only LO QED}$$

$$\propto \left[\overline{u}(p_\ell)\gamma_\mu P_L v(p_\nu)\right] \times f_{B_u} p_B^\mu \quad \leftarrow \text{decay constant}$$

$$\propto f_{B_u} m_\ell \left[\overline{u}(p_\ell)\gamma_5 v(p_\nu)\right] \quad \leftarrow \text{use } p_B = p_\ell + p_\nu \text{ \& EOM}$$

B_q meson decay constant

$$\langle 0 | \overline{q}\gamma^\mu\gamma_5 b | \overline{B}_q(p_B) \rangle \equiv i f_{B_q} p_B^\mu$$

Leptonic CC decays $B_u \to \ell\overline{\nu}_\ell$

Can calculate **Branching Ratio ($\mathcal{B}$)** with hadronic matrix element $\langle \ell\overline{\nu}_\ell | Q_{V_L} | \overline{B}_u \rangle \to f_{B_q}\, m_\ell \, [\overline{\ell}\, \gamma_5\, \nu_\ell]$

$$\mathcal{B}^\ell_{\rm SM} = \tau_{B_u}\, \Gamma[B_u \to \ell\overline{\nu}_\ell] = \frac{\tau_{B_u}\, m_{B_u}}{8\pi}\, m_\ell^2 \beta_\ell^2\, (f_{B_u})^2\, \left|\mathcal{G}_F\, V_{ub}\, C_{V_L}\right|^2$$

- **short-distance V_{ub}** ← we like to determine ($\mathcal{G}_F$ known from muon decay)
- **long-distance f_{B_u}** ← nowadays from lattice $= (189.4 \pm 1.4)$ MeV [FNAL/MILC 1712.09262]
- **helicity-suppression m_ℓ** ← makes it difficult for experiments $\beta_\ell \equiv \sqrt{1 - m_\ell^2/m_{B_u}^2}$
- **B_u lifetime τ_{B_u}** $= (1638 \pm 4) \cdot 10^{-15}$ s

Leptonic CC decays $B_u \to \ell\overline{\nu}_\ell$

Can calculate **Branching Ratio ($\mathcal{B}$)** with hadronic matrix element $\langle \ell\overline{\nu}_\ell | Q_{V_L} | \overline{B}_u \rangle \to f_{B_q}\, m_\ell \left[\overline{\ell}\,\gamma_5\, \nu_\ell\right]$

$$\mathcal{B}^\ell_{\rm SM} = \tau_{B_u}\, \Gamma[B_u \to \ell\overline{\nu}_\ell] = \frac{\tau_{B_u}\, m_{B_u}}{8\pi}\; m_\ell^2 \beta_\ell^2\; (f_{B_u})^2\; \left|\mathcal{G}_F\, V_{ub}\, C_{V_L}\right|^2$$

- **short-distance V_{ub}** ← we like to determine ($\mathcal{G}_F$ known from muon decay)
- **long-distance f_{B_u}** ← nowadays from lattice = (189.4 ± 1.4) MeV [FNAL/MILC 1712.09262]
- **helicity-suppression m_ℓ** ← makes it difficult for experiments $\beta_\ell \equiv \sqrt{1 - m_\ell^2/m_{B_u}^2}$
- **B_u lifetime τ_{B_u}** $= (1638 \pm 4) \cdot 10^{-15}$ s

SM predictions (neglecting uncertainties from m_ℓ, m_{B_u}, $\mathcal{G}_F$): for $|V_{ub}| = 3.615 \cdot 10^{-3}$

$$\mathcal{B}^\ell_{\rm SM} = 2.551 \left(1 \pm 0.002|_{\tau_B} \pm 0.015|_{f_B}\right) \times |V_{ub}|^2 \times m_\ell^2 \beta_\ell^2 = \begin{cases} 8.71 \cdot 10^{-12} & \ell = e \\ 3.72 \cdot 10^{-7} & \ell = \mu \\ 9.34 \cdot 10^{-5} & \ell = \tau \end{cases}$$

⇒ current hadronic uncertainty allow for $\delta|V_{ub}| \sim 1\%$, provided experimental uncertainty $< 1\%$

$\mathcal{B}^\tau_{\rm exp} = (10.9 \pm 2.4) \cdot 10^{-5}$, $\mathcal{B}^\mu_{\rm exp} \in [2.9, 1.1] \cdot 10^{-7}$ @ 90% CL, $\mathcal{B}^e_{\rm exp} < 9.8 \cdot 10^{-7}$ @ 90% CL

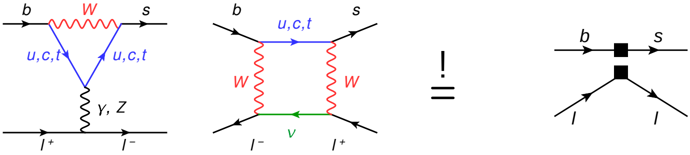
Leptonic FCNC decays $B_q \to \ell\bar{\ell}$ $(q = d, s)$

In SM one-loop **Matching**

[Inami/Lim Prog.Theor.Phys. 65 (1981) 297]

$\mathcal{G}_F \dfrac{\alpha_e}{s_W^2}$

LO



- at LO EW & all orders in QCD, only one operator has **non-zero** hadronic matrix element

$$\mathcal{L}_{\text{EFT}} = \mathcal{L}_{\text{QCD}\times\text{QED}} + \frac{4\mathcal{G}_F}{\sqrt{2}} \frac{\alpha_e}{4\pi} V_{tb} V_{tq}^* C_{10} Q_{10}, \qquad Q_{10} = [\bar{q}\gamma_\mu P_L b][\bar{\ell}\gamma^\mu \gamma_5 \ell]$$

similar to $b \to q\ell\bar{\nu}_\ell$: $\langle \ell^+\ell^- | Q_{10} | \bar{B}_q \rangle \to f_{B_q} (2m_\ell) [\bar{\ell}\gamma_5 \ell]$

- other semileptonic operator $Q_9 = [\bar{q}\gamma_\mu P_L b][\bar{\ell}\gamma^\mu \ell]$

$$\langle \ell^+\ell^- | Q_9 | \bar{B}_q(p_B) \rangle \propto f_{B_q} p_B^\mu \times [\bar{u}(p_\ell)\gamma_\mu v(p_{\bar{\ell}})] \propto f_{B_q} [\bar{u}(p_\ell)(\not{p}_\ell + \not{p}_{\bar{\ell}}) v(p_{\bar{\ell}})] \overset{\text{EOM}}{=} 0$$

- $C_{10} = C_{10}(m_t/m_W) + \mathcal{O}(\alpha_s, \alpha_s^2, \alpha_e)$ higher orders are known + no RG under QCD

$B_{s,d} \to \ell\bar{\ell}$ – theory status

Branching ratio

$$\mathcal{B}_{q\ell} \;\propto\; \tau_{B_q}\, \mathcal{G}_F^2\, \alpha_e^2 \times \left(\frac{2 m_\ell}{m_{B_q}}\right)^2 \times \boxed{\left|V_{tb} V_{tq}^*\right|^2}\; \boxed{\left(f_{B_q}^{(0)}\right)^2} \times \left|\, \boxed{C_{10}} + \frac{\alpha_e}{4\pi} \boxed{\mathcal{A}_{\rm NLO}} \,\right|^2$$

- helicity suppression
- **CKM** to be determined
- ***B_q* decay constant** in pure QCD from lattice $\quad f_{B_d} = (189.4 \pm 1.4)$ MeV $\quad f_{B_s} = (230.7 \pm 1.2)$ MeV
  [FNAL/MILC 1712.09262]
- **LO amplitude** $\propto C_{10}$ at NNLO QCD & NLO EW
  [Hermann/Misiak/Steinhauser 1311.1347, CB/Gorbahn/Stamou 1311.1348]

- **NLO QED amplitude** $\propto C_{7,9}^{\rm eff}$ $\quad$!!! restricted to $\ell = \mu$, assuming $m_\mu \sim \Lambda_{\rm QCD}$
  - $\Rightarrow$ power-enhanced $m_b/\Lambda_{\rm QCD}$ from spectator-quark dynamics [Beneke/CB/Szafron 1708.09157]
  - $\Rightarrow$ factorization in SCET$_{1+2}$ and resummation between $\mu \sim m_b \;\to\; \mu \sim m_\mu, \Lambda_{\rm QCD}$
  - $+$ $f_{B_q}^{(0)}$ sufficient for power-enhanced $\mathcal{A}_{\rm NLO}$, beyond new $f_{B_q}^{(n)}$ required
  - $+$ combination with soft real-radiation for $\Delta E \ll m_\mu, \Lambda_{\rm QCD}$ [Beneke/CB/Szafron 1908.07011]

$B_s \to \mu\bar{\mu}$ – uncertainty budget

Non-radiative rate time-integrated, use $|V_{cb}|_{\text{incl}}$ [Beneke/CB/Szafron 1908.07011]

$N_f = 2 + 1 + 1$ [FLAG 1902.08191]

$$\overline{\mathcal{B}}_{s\mu}^{(0)} = 3.660 \left(1 \pm 1.1\%\Big|_{f_{B_s}} \pm 3.1\%\Big|_{\text{CKM}} \pm 1.1\%\Big|_{m_t} \pm 0.6\%\Big|_{\text{pmr}} \pm 1.2\%\Big|_{\text{non-pmr}} {}^{+0.3}_{-0.5}\%\Big|_{\text{LCDA}}\right) \cdot 10^{-9}$$

- **main parametric** long-distance (f_{B_s}) and short-distance (CKM and m_t)
- **non-QED:** parametric (τ_{B_s}, α_s) and non-parametric (μ_W, μ_b and higher order)
- ***B*-meson LCDA:** λ_B and $\sigma_{1,2}$ entering power-enhanced QED crr'n

World average: $\overline{\text{Br}}_{s\mu}^{(0)}\Big|_{\text{exp}} = \left(2.69^{+0.37}_{-0.35}\right) \cdot 10^{-9}$ [LHCb+CMS+ATLAS, Run 1+2, LHCb-CONF-2020-002 + therein]

$B_s \to \mu\bar{\mu}$ – uncertainty budget

Non-radiative rate time-integrated, use $|V_{cb}|_{\text{incl}}$ [Beneke/CB/Szafron 1908.07011]

$N_f = 2+1+1$ [FLAG 1902.08191]

$$\overline{\mathcal{B}}^{(0)}_{s\mu} = 3.660 \left(1 \pm 1.1\%\big|_{f_{B_s}} \pm 3.1\%\big|_{\text{CKM}} \pm 1.1\%\big|_{m_t} \pm 0.6\%\big|_{\text{pmr}} \pm 1.2\%\big|_{\text{non-pmr}} {}^{+0.3}_{-0.5}\%\big|_{\text{LCDA}}\right) \cdot 10^{-9}$$

- **main parametric** long-distance (f_{B_s}) and short-distance (CKM and m_t)
- **non-QED:** parametric (τ_{B_s}, α_s) and non-parametric (μ_W, μ_b and higher order)
- ***B*-meson LCDA:** λ_B and $\sigma_{1,2}$ entering power-enhanced QED crr'n

World average: $\overline{\text{Br}}^{(0)}_{s\mu}\big|_{\text{exp}} = \left(2.69^{+0.37}_{-0.35}\right) \cdot 10^{-9}$ [LHCb+CMS+ATLAS, Run 1+2, LHCb-CONF-2020-002 + therein]

Sensitivity to $|V_{tb}V^*_{ts}|$ with $N_f = 2+1+1$ & assuming **LHCb:** 4% uncertainty with 300/fb

[A. Puig @ LHCb Upgrade WS, LAPP, Annecy, 03/2018, LHCb 1208.3355]

$$\delta\overline{\text{Br}}^{(0)}_{s\mu}\Big|_{\text{theory}} \approx 2.1\% \quad + \quad \delta\overline{\text{Br}}^{(0)}_{s\mu}\Big|_{\text{LHCb 300/fb}} \approx 4.0\% \quad \Rightarrow \quad \delta\left|V_{tb}V^*_{ts}\right| \approx 2.5\%$$

for comparison from $b \to c\ell\bar{\nu}_\ell$: $\delta|V_{cb}|_{\text{incl}} = 1.5\%$ [Gambino/Healey/Turczyk 1606.06174]

$\delta|V_{cb}|_{\text{excl}} = 2.2\%$ [Bordone/Jung/van Dyk 1908.09398]

Semileptonic CC decays

$|V_{qb}|$ from exclusive $B \to (P, V)\, \ell\overline{\nu}_\ell$

Exclusive processes

$b \to u$	$b \to c$
$B \to \pi$	$B \to D$
$B_s \to K$	$B_s \to D_s$
$B \to \rho, \omega$	$B \to D^*$
$B_s \to K^*$	$B_s \to D_s^*$
$\Lambda_b \to p$	$\Lambda_b \to \Lambda_c^{(*)}$

- experiment + theory: nonproblematic
- theory finite-width approximation
- $b \to u$ decays in SM suppressed by $\left|\frac{V_{ub}}{V_{cb}}\right|^2 \sim 1 \cdot 10^{-2}$
- ν-reconstruction favors B-factories over LHC
- B_s decays at LHC suppressed by f_s/f_d (similar for Λ_b)

$|V_{qb}|$ from exclusive $B \to (P, V)\, \ell\overline{\nu}_\ell$

Exclusive processes

$b \to u$	$b \to c$
$B \to \pi$	$B \to D$
$B_s \to K$	$B_s \to D_s$
$B \to \rho, \omega$	$B \to D^*$
$B_s \to K^*$	$B_s \to D_s^*$
$\Lambda_b \to p$	$\Lambda_b \to \Lambda_c^{(*)}$

- experiment + theory: nonproblematic
- theory finite-width approximation
- $b \to u$ decays in SM suppressed by $\left|\frac{V_{ub}}{V_{cb}}\right|^2 \sim 1 \cdot 10^{-2}$
- ν-reconstruction favors B-factories over LHC
- B_s decays at LHC suppressed by f_s/f_d (similar for Λ_b)

Observables

$B \to P\, \ell\overline{\nu}_\ell$

$$\frac{d\mathcal{B}}{dq^2}$$

$B \to V(\to P_1 P_2)\, \ell\overline{\nu}_\ell$

$$\frac{d^4\Gamma}{dq^2\, d\cos\theta_\ell\, d\cos\theta_V\, d\chi}$$

$\Lambda_b \to \Lambda_c(\to \Lambda\pi)\, \ell\overline{\nu}_\ell$

$$\frac{d^4\Gamma}{dq^2\, d\cos\theta_\ell\, d\cos\theta_\Lambda\, d\chi}$$

$\Rightarrow$ **angular distributions** provide further observables $F_L(q^2)$, $\mathcal{A}_{\text{FB}}(q^2)$, ...

LFU ratios $$R^{\ell\ell'}(M) \equiv \frac{\mathcal{B}(B \to M\, \ell\, \overline{\nu}_\ell)}{\mathcal{B}(B \to M\, \ell'\, \overline{\nu}_{\ell'})}$$ hadronic uncertainties cancel (especially in SM)

Form Factors (FF)

Matrix element at leading order in EW interactions:

$$i\mathcal{A}_{\text{EFT}} \equiv \langle \ell\overline{\nu}_\ell M | \, i\mathcal{L}_{\text{EFT}} \, | \overline{B} \rangle \rightarrow \left\langle \ell\overline{\nu}_\ell M \right| - i \frac{4\mathcal{G}_F}{\sqrt{2}} V_{qb} \, C_{V_L} Q_{V_L} \left| \overline{B} \right\rangle$$

For semileptonic decay $\overline{B} \to M \, \ell\overline{\nu}_\ell$

$$i\mathcal{A}_{\text{EFT}} \propto \langle \ell\overline{\nu}_\ell M | Q_{V_L} | \overline{B} \rangle \propto \langle \ell\overline{\nu}_\ell | \overline{\ell}\gamma_\mu P_L \nu_\ell | 0 \rangle \times \langle M | \overline{q}\gamma^\mu P_L b | \overline{B} \rangle \quad \leftarrow \text{only LO QED}$$

$$\propto \left[\overline{u}(p_\ell)\gamma_\mu P_L v(p_\nu) \right] \times \quad \text{FF}(q^2) \quad \leftarrow \text{form factor}$$

Form Factors (FF)

Matrix element at leading order in EW interactions:

$$i\mathcal{A}_{\text{EFT}} \equiv \langle \ell\overline{\nu}_\ell M | \, i\mathcal{L}_{\text{EFT}} \, | \overline{B} \rangle \rightarrow \left\langle \ell\overline{\nu}_\ell M \right| - i \frac{4\mathcal{G}_F}{\sqrt{2}} V_{qb} \, C_{V_L} Q_{V_L} \left| \overline{B} \right\rangle$$

For semileptonic decay $\overline{B} \to M \, \ell\overline{\nu}_\ell$

$$i\mathcal{A}_{\text{EFT}} \propto \langle \ell\overline{\nu}_\ell M | Q_{V_L} | \overline{B} \rangle \propto \langle \ell\overline{\nu}_\ell | \bar{\ell}\gamma_\mu P_L \nu_\ell | 0 \rangle \times \langle M | \overline{q}\gamma^\mu P_L b | \overline{B} \rangle \qquad \leftarrow \text{only LO QED}$$

$$\propto \left[\overline{u}(p_\ell)\gamma_\mu P_L v(p_\nu) \right] \times \quad \text{FF}(q^2) \qquad \leftarrow \text{form factor}$$

$B \to P$ seudoscalar FF's → depend on momentum transfer $q \equiv p - k = p_\ell + p_\nu$

$$\langle P(k) | \, \overline{q}\,\gamma_\mu \, b \, | B(p) \rangle = f_+ \, (p+k)_\mu + \left[f_0 - f_+ \right] \frac{m_B^2 - m_P^2}{q^2} q_\mu , \qquad \langle P | \, \overline{q}\,\gamma_\mu \gamma_5 \, b \, | B \rangle = 0$$

q^2-differential branching ratio

$$\frac{d\mathcal{B}}{dq^2} \propto \tau_B |V_{qb}|^2 \, \beta_\ell^2 \, |\vec{p}| \left[m_B^2 |\vec{p}|^2 \left(1 - \frac{m_\ell^2}{2\,q^2} \right)^2 (f_+)^2 + \frac{3 m_\ell^2}{8\,q^2} (m_B^2 + m_P^2)^2 (f_0)^2 \right]$$

⇒ only $f_+(q^2)$ relevant if $m_\ell \ll q^2$ $(\ell = e, \mu)$, f_0 important for $\ell = \tau$ $\qquad \beta_\ell \equiv \sqrt{1 - m_\ell^2/q^2}$

Form Factor definitions

$B \to P$ seudoscalar FF's

$$\left\langle P(k)\middle|\overline{q}\,\gamma_\mu\, b\middle|B(p)\right\rangle = f_+ (p+k)_\mu + \left[f_0 - f_+\right]\frac{m_B^2 - m_P^2}{q^2} q_\mu$$

$$\left\langle P(k)\middle|\overline{q}\,\sigma_{\mu\nu} q^\nu\, b\middle|B(p)\right\rangle = \frac{i\, f_T}{m_B + m_P}\left[q^2 (p+k)_\mu - (m_B^2 - m_P^2) q_\mu\right]$$

- 3 $B \to P$ form factors f_+ = vector FF f_0 = scalar FF f_T = tensor FF
- kinematical constraint at $q^2 = 0$: $f_+ = f_0$
- in SM there is no $b \to q\ell\overline{\nu}_\ell$ operator with tensor structure $[\overline{q}\,\sigma_{\mu\nu}\ldots b]$

Form Factor definitions

$B \to P$ seudoscalar FF's

$$\langle P(k)|\bar{q}\gamma_\mu b|B(p)\rangle = f_+(p+k)_\mu + [f_0 - f_+]\frac{m_B^2 - m_P^2}{q^2} q_\mu$$

$$\langle P(k)|\bar{q}\sigma_{\mu\nu}q^\nu b|B(p)\rangle = \frac{i f_T}{m_B + m_P}\left[q^2(p+k)_\mu - (m_B^2 - m_P^2)q_\mu\right]$$

- 3 $B \to P$ form factors f_+ = vector FF f_0 = scalar FF f_T = tensor FF
- kinematical constraint at $q^2 = 0$: $f_+ = f_0$
- in SM there is no $b \to q\ell\bar{\nu}_\ell$ operator with tensor structure $[\bar{q}\,\sigma_{\mu\nu}\ldots b]$

Approximate relations among FFs in the **Heavy-Quark limit $m_b \to \infty$** valid up to Λ_{QCD}/m_b

[Isgur/Wise PRD 42 (1990) 2388]

- different sets for a) **Heavy-to-heavy** ($b \to c$) (heavy = "$\Lambda_{QCD} \ll m_c \lesssim m_b$")
  b) **Heavy-to-Light** ($b \to u, d, s$) (light = "$m_q \lesssim \Lambda_{QCD}$")
- for heavy-to-light further "symmetries" in **Large Recoil limit** $E_M \sim \frac{m_B}{2} \Leftrightarrow q^2 \to 0$

[Charles/Le Yaouanc/Oliver/Pene/Raynal hep-ph/9812358]

Form Factor definitions

$B \to V$ector FF's

$$\langle V(k,\eta)|\bar{q}\gamma_\mu b|B(p)\rangle = \varepsilon_{\mu\nu\alpha\beta}\eta^{*\nu}p^\alpha k^\beta \frac{2V}{m_B+m_V}$$

$$\langle V(k)|\bar{q}\gamma_\mu\gamma_5 b|B(p)\rangle = i\eta^{*\nu}\left\{q_\mu q_\nu \frac{2m_V}{q^2}A_0 + (m_B+m_V)\left[g_{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right]A_1 + (m_B-m_V)\left[q^\mu - \frac{q^2}{m_B^2-m_V^2}(p+k)_\mu\right]\frac{q_\nu}{q^2}A_2\right\}$$

$$\langle V(k,\eta)|\bar{q}i\sigma_{\mu\nu}q^\nu b|B(p)\rangle = \varepsilon_{\mu\nu\alpha\beta}\eta^{*\nu}p^\alpha k^\beta 2T_1$$

$$\langle V(k,\eta)|\bar{q}i\sigma_{\mu\nu}q^\nu\gamma_5 b|B(p)\rangle = i\eta^{*\nu}\left\{[g_{\mu\nu}(m_B^2-m_V^2)-(p+k)_\mu q_\nu]T_2 + \left[q^\mu - \frac{q^2}{m_B^2-m_V^2}(p+k)_\mu\right]q_\nu T_3\right\}$$

- 7 $B \to V$ FFs: V = vector FF, $A_{1,2}$ = axial-vector FFs, A_0 = scalar FF, $T_{1,2,3}$ = tensor FFs
- kinematical constraint at $q^2 = 0$: $A_0 = \frac{m_B+m_V}{2m_V}A_1 - \frac{m_B-m_V}{2m_V}A_2$ and $T_1 = T_2$
- in SM there is no $b \to q\ell\bar{\nu}_\ell$ operators with tensor structure $[\bar{q}\,\sigma_{\mu\nu}\ldots b]$

Form Factor definitions

$B \to V$ector FF's

$$\langle V(k,\eta)|\bar{q}\gamma_\mu b|B(p)\rangle = \varepsilon_{\mu\nu\alpha\beta}\eta^{*\nu}p^\alpha k^\beta \frac{2V}{m_B+m_V}$$

$$\langle V(k)|\bar{q}\gamma_\mu\gamma_5 b|B(p)\rangle = i\eta^{*\nu}\left\{q_\mu q_\nu \frac{2m_V}{q^2}A_0 + (m_B+m_V)\left[g_{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right]A_1 + (m_B-m_V)\left[q^\mu - \frac{q^2}{m_B^2-m_V^2}(p+k)_\mu\right]\frac{q_\nu}{q^2}A_2\right\}$$

$$\langle V(k,\eta)|\bar{q}i\sigma_{\mu\nu}q^\nu b|B(p)\rangle = \varepsilon_{\mu\nu\alpha\beta}\eta^{*\nu}p^\alpha k^\beta 2T_1$$

$$\langle V(k,\eta)|\bar{q}i\sigma_{\mu\nu}q^\nu\gamma_5 b|B(p)\rangle = i\eta^{*\nu}\left\{[g_{\mu\nu}(m_B^2-m_V^2) - (p+k)_\mu q_\nu]T_2 + \left[q^\mu - \frac{q^2}{m_B^2-m_V^2}(p+k)_\mu\right]q_\nu T_3\right\}$$

- 7 $B \to V$ FFs: V = vector FF, $A_{1,2}$ = axial-vector FFs, A_0 = scalar FF, $T_{1,2,3}$ = tensor FFs
- kinematical constraint at $q^2 = 0$: $A_0 = \frac{m_B+m_V}{2m_V}A_1 - \frac{m_B-m_V}{2m_V}A_2$ and $T_1 = T_2$
- in SM there is no $b \to q\ell\bar{\nu}_\ell$ operators with tensor structure $[\bar{q}\,\sigma_{\mu\nu}\ldots b]$

$\Lambda_b \to \Lambda_c$ FF's

3×vector FFs 3×axial-vector FFs and tensor FFs

Form factor determinations

Determination of FF's

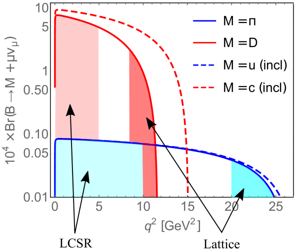
FFs calculated with nonperturbative methods

Light-Cone Sum Rules (LCSR)

- low q^2 = large recoil
- two setup's with:

  a) light-meson LCDA's

  b) B-meson LCDA's



Lattice QCD (LQCD)

- high q^2 = low recoil
- 1st principle for $B \to P$
- some appr. for $B \to V$ assume stable V

- calculate hadronic corr-function via:

  a) using unitarity $\Rightarrow$ dispersive representation involving form factor

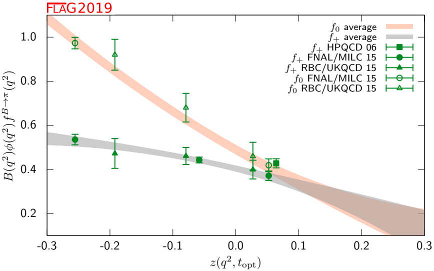
  b) light-cone OPE at q^2 where applicable ($q^2 \lesssim 0$) $\Rightarrow$ partons & perturbative
- sum rule obtained by maching both results and using (semi-global) quark-hadron duality

- numerical evaluation in discretized and finite space-time volume
- achieves nowadays uncertainties below 10 % for $B \to P$

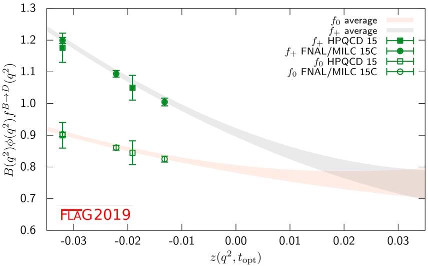
Example for $B \to P$ of FFs

LQCD averaged by FLAG (Flavour Lattice Averaging Group) **http://flag.unibe.ch/2019/MainPage**

$B \to \pi$



$B \to D$



- FLAG-averages provided in **FF-parametrization:** “constrained BCL with $N = 3$”

$$f_+(q^2) = \frac{1}{1 - q^2/m_{B*}^2} \sum_{n=0}^{N-1} a_n^+ \left[z^n - (-1)^{n-N} \frac{n}{N} z^N \right], \qquad f_0(q^2) = \sum_{n=0}^{N-1} a_n^0 z^n$$

LQCD use $f_+(0) = f_0(0)$ [Bourrely/Caprini/Lellouch 0807.2722]

- mapping to

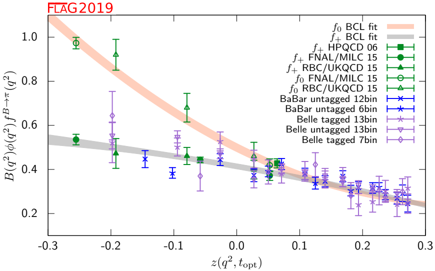
$$z(q^2, t_{\rm opt}) \equiv \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_{\rm opt}}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_{\rm opt}}}, \qquad t_\pm = (m_B \pm m_P)^2, \qquad t_{\rm opt} = \sqrt{t_+}(\sqrt{m_B} - \sqrt{m_P})^2$$

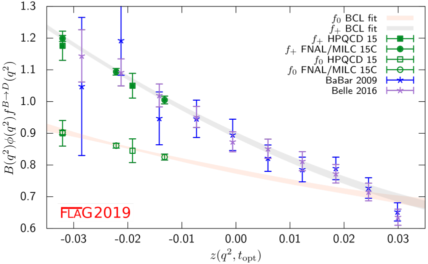
- combined fit in SM-framework with V_{qb} $\Rightarrow$ fitting FF shape from data

Example for $B \to P$ of FFs

LQCD averaged by FLAG (Flavour Lattice Averaging Group) **http://flag.unibe.ch/2019/MainPage**

$B \to \pi$ **$B \to D$**





- FLAG-averages provided in **FF-parametrization:** "constrained BCL with $N = 3$"

$$f_+(q^2) = \frac{1}{1 - q^2/m^2_{B*}} \sum_{n=0}^{N-1} a_n^+ \left[z^n - (-1)^{n-N} \frac{n}{N} z^N \right], \qquad f_0(q^2) = \sum_{n=0}^{N-1} a_n^0 z^n$$

LQCD use $f_+(0) = f_0(0)$ [Bourrely/Caprini/Lellouch 0807.2722]

- mapping to

$$z(q^2, t_{opt}) \equiv \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_{opt}}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_{opt}}}, \qquad t_\pm = (m_B \pm m_P)^2, \qquad t_{opt} = \sqrt{t_+}(\sqrt{m_B} - \sqrt{m_P})^2$$

- combined fit in SM-framework with V_{qb} $\Rightarrow$ fitting FF shape from data

$B \to V$ form factors

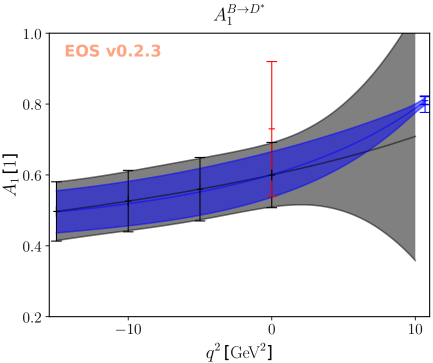
Much “less solid” calculations for $B \to V$ FFs:
(more appropriate for experimental detection via $V \to P_1 P_2$ would be $B \to P_1 P_2$ FFs)

- LQCD has to assume Vector meson to be stable (approx.)
- similar issues for LCSR

Experimentally $B \to D^* \ell \bar{\nu}_\ell$ favoured
⇒ for V_{cb} the $B \to D^*$ FFs are important

- currently LQCD provides only A_1 at $q^2_{max} = (m_B - m_D^*)^2$
  [FNAL/MILC 1403.0635, HPQCD 1711.11013]
- LCSR calculation done at $q^2 = (-15, -10, -5, 0)\,\mathrm{GeV}^2$ for all V, $A_{0,1,2}$, $T_{1,2,3}$
- fitted to z-expansion:

LCSR only
LCSR + LQCD
without [Khodjamirian/Mannel/Offen hep-ph/0611193]



[Gubernari/Kokulu/van Dyk 1811.00983]

z-Expansions
and unitarity bounds

Further reading: Textbook by

Irinel Caprini

"Functional Analysis and Optimization Methods in Hadron Physics"

[https://doi.org/10.1007/978-3-030-18948-8]

FFs & dispersion relation

Introduced FFs as $\overline{B} \to M$ matrix element

$$\langle M | J^\mu | \overline{B} \rangle \equiv (\ldots)^\mu F(q^2)$$

with quark currents $J^\mu \equiv [\overline{q}\gamma_\mu \ldots b]$

$\Rightarrow$ by **crossing symmetry** same function $F(q^2)$

$$\langle 0 | J^\mu | \overline{B} M \rangle \equiv (\ldots)^\mu F(q^2)$$

describes also $\overline{B} + M$ production/annihilation

FFs & dispersion relation

Introduced FFs as $\overline{B} \to M$ matrix element $\Rightarrow$ by **crossing symmetry** same function $F(q^2)$

$$\langle M | J^\mu | \overline{B} \rangle \equiv (\ldots)^\mu F(q^2) \qquad \langle 0 | J^\mu | \overline{B} M \rangle \equiv (\ldots)^\mu F(q^2)$$

with quark currents $J^\mu \equiv [\overline{q}\gamma_\mu \ldots b]$

describes also $\overline{B} + M$ production/annihilation

They are part of 2-point correlation function

$$\Pi_{\mu\nu}(q^2) = (q_\mu q_\nu - q^2 g_{\mu\nu})\,\Pi_T(q^2) + g_{\mu\nu}\,\Pi_L(q^2) \equiv i \int d^4x\, e^{i\,q\cdot x} \langle 0 | \mathrm{T}\{J_\mu(x), J_\nu(0)\} | 0 \rangle$$

that fulfills a (n-subtracted, $n = 1$ or 2 in practice) dispersion relation

$$\chi_A(q^2) = \frac{1}{n!} \frac{d\Pi_A(t)}{d\,t^n}\Big|_{t=q^2} = \frac{1}{\pi}\int_0^\infty dt\, \frac{\mathrm{Im}\,\Pi_A(t)}{(t - q^2)^n} \qquad (A = L, T)$$

- dispersion relation allows to calculate $\chi_A(q^2)$ at some q^2 from knowledge of $\mathrm{Im}\,\Pi_A$ or vice versa
- can calculate $\boldsymbol{\chi_A(q^2)}$ **perturbatively** at q^2 far from where J_μ can create resonances

  for $B \to D^{(*)}$ need $(m_b + m_c)\Lambda_{\mathrm{QCD}} \ll (m_b + m_c)^2 - q^2 \quad \Rightarrow \quad q^2 = 0$ sufficient

  "Standard OPE" $\Pi_{A,\mathrm{OPE}}(q^2) = \sum_{k=1}^\infty C_{A,k}\,\langle \mathcal{O}_k \rangle$ Wilson coeff's $C_{A,k}$ depend on $m_{b,q}$

[(Novikov)/Shifman/Vainshtein/Zakharov NPB147 (1979) 385 & 448, (NPB174 (1980) 378)]

Unitarity bounds on FFs

Hadronic representation:

Insert complete set of on-shell states $|n\rangle = \{|\overline{B}M\rangle, \ldots\}$ with correct quantum numbers (unitarity)

$$\Pi_{\mu\nu} = i \int d^4x\, e^{i\,q\cdot x} \sum\!\!\!\!\!\!\int_n d\mu_n \langle 0|J_\mu(x)|N\rangle \langle N|J_\nu(0)|0\rangle \qquad J_\mu(x) = e^{i\hat{P}x} J_\mu(0) e^{-i\hat{P}x}$$

$$= i(2\pi)^4 \sum\!\!\!\!\!\!\int_n d\mu_n\, \delta^{(4)}[q - p_n] \langle 0|J_\mu|N\rangle \langle N|J_\nu|0\rangle$$

if choose $\mu = \nu$, then on r.h.s $|\langle 0|J_\mu|N\rangle|^2 \geq 0$ is positive, such that

$$\mathrm{Im}\,\Pi_{\mu\mu} = (2\pi)^4 \sum\!\!\!\!\!\!\int_n d\mu_n\, \delta^{(4)}[q - p_n] \left|\langle 0|J_\mu|N\rangle\right|^2 \qquad \leftarrow \text{keep only first state } |\overline{B}M\rangle \text{ in sum}$$

$$\geq \underbrace{(2\pi)^4 \int d\mu_{BM}\, \delta^{(4)}[q - p_{BM}]}\, |F(q^2)|^2 \qquad \leftarrow \text{remember } \langle 0|J^\mu|\overline{B}M\rangle \propto F(q^2)$$

From dispersion relation obtain a bound on $|F(t)|$ in terms of perturbative result of $\chi_A(q^2)$

$$1 \geq \frac{1}{\chi_A(q^2)\,\pi} \int_{p^2_{BM}}^{\infty} dt\, \frac{\widetilde{\phi}(t)\,|F(t)|^2}{(t - q^2)^n}$$

- lower intergration boundary $t = 0 \rightarrow t = p^2_{BM}$ since $d\mu_{BM} \propto \theta[q^2 - p^2_{BM}]$
- these bounds are on FF on the real axis $q^2 > p^2_{BM}$

Mapping to unit disk

Considered $B \to M$ FFs $F(q^2)$ extended to complex plane

$q^2 \mapsto t \in \mathbb{C}$ from **semileptonic region $m_\ell^2 \leq q^2 \leq t_-$**

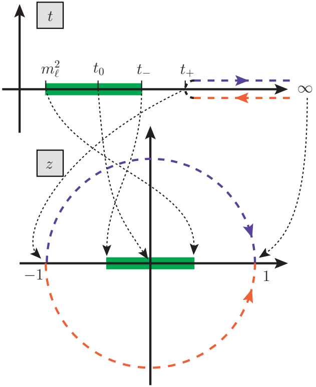
- $t_- \equiv (m_B - m_M)^2 = q^2_{max}$
- $t_+ \equiv (m_B + m_M)^2$ is **threshold for $|\bar{B}M\rangle$ production**
- choose freely $t_0 < t_+$

and transform to z-plane into unit-circle

$$z(t, t_0) \equiv \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$

⇒ "semileptonic" region mapped to $|z| \ll 1$

$|z| \leq 0.035$ for $B \to D$ and $|z| \leq 0.29$ for $B \to \pi$



Mapping to unit disk

Considered $B \to M$ FFs $F(q^2)$ extended to complex plane

$q^2 \mapsto t \in \mathbb{C}$ from **semileptonic region $m_\ell^2 \leq q^2 \leq t_-$**

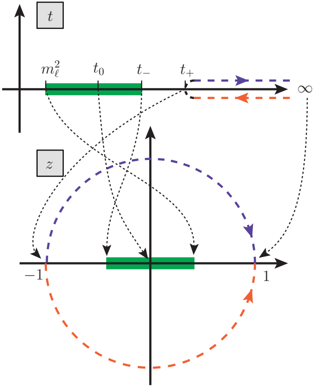
- $t_- \equiv (m_B - m_M)^2 = q^2_{\max}$
- $t_+ \equiv (m_B + m_M)^2$ is **threshold for $|\overline{B}M\rangle$ production**
- choose freely $t_0 < t_+$

and transform to z-plane into unit-circle

$$z(t, t_0) \equiv \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$

$\Rightarrow$ "semileptonic" region mapped to $|z| \ll 1$

$|z| \lesssim 0.035$ for $B \to D$ and $|z| \lesssim 0.29$ for $B \to \pi$



Unitarity bound becomes

$$\frac{1}{2\pi i} \oint \frac{dz}{z} \left|\phi(z)\, B(z)\, F(z)\right|^2 \leq 1$$

- **outer function** $\phi(z) \propto \sqrt{\widetilde{\phi}(z)\, /\, \chi_A(q^2)}$
- $F(t)$ is mostly analytic. If there are known **subthreshold resonances** at $t_- \leq t \leq t_+$ (stable under QCD, e.g. $B_c^{(*)}$ etc.) they are removed with a **Blascke factor** $B(z)$ (sufficient to know positions z_n)

z–Parametrization

FF parametrization in $z = z(t, t_0)$ ansatz

$$F(z) = \frac{1}{B(z)\,\phi(z)} \sum_{k=0}^{\infty} a_k\, z^k$$

- $B(z) \times F(z)$ is analytic function
- $\phi(z)$ has no zeros inside unit disc

Ansatz into unitarity bound & z-integration

$$\sum_{k=0}^{\infty} |a_k|^2 \le 1$$

!!! constraint on coefficients as well

Having in mind truncation in k after few terms, because $|z| \ll 1$ in "semileptonic region",

+ unitarity bound on $|a_k|$ provides model-independent and powerful parametrization!

(There can be some caveates, depending on $B \to M$, and issues with asymptotic limits)

z–Parametrization

FF parametrization in $z = z(t, t_0)$ ansatz

$$F(z) = \frac{1}{B(z)\,\phi(z)} \sum_{k=0}^{\infty} a_k\, z^k$$

- $B(z) \times F(z)$ is analytic function
- $\phi(z)$ has no zeros inside unit disc

Ansatz into unitarity bound & z-integration

$$\sum_{k=0}^{\infty} |a_k|^2 \leq 1$$

!!! constraint on coefficients as well

Having in mind truncation in k after few terms, because $|z| \ll 1$ in "semileptonic region",
+ unitarity bound on $|a_k|$ provides model-independent and powerful parametrization!

(There can be some caveates, depending on $B \to M$, and issues with asymptotic limits)

BGL = Boyd/Grinstein/Lebed [hep-ph/9412324, 9508211, 9705252]

- often used for $B \to D^{(*)}$ FFs
- use $\phi(z)$ and $B(z)$ such that unitarity bound takes form $\sum_k |a_k|^2 \leq 1$

BCL = Bourrely/Caprini/Lellouch [0807.2722]

- used by FLAG and often LQCD collaborations for $B_{(s)} \to \pi, K, D$ FFs
- replace $\phi(z) \to$ simple pole, e.g. $1 - q^2/m_{B^*}$, differently for each FF
- unitarity bound takes complicated form $\sum_{j,k} B_{jk}(t_0)\, a_j(t_0)\, a_k(t_0) \leq 1$

Heavy-to-heavy FFs
Constraints from HQET

Further reading:

Textbook by

Aneesh Manohar and Mark Wise

"Heavy Quark Physics" [Camb.Monogr.Part.Phys.Nucl.Phys.Cosmol. 10 (2000) 1-191]

Reviews/Lectures

M. Neubert [Phys. Rept. 245, 259 (1994)]

M. Wise [Les Houches Summer School 1997, hep-ph/9805468]

Heavy-quark systems

Consider here bound state with single heavy quark $H^{(Q)} = (Q\bar{q})$

- **heavy** means $m_q \sim \Lambda_{\rm QCD} \ll m_Q,$ in practice $m_Q = m_b$ or m_c
- interaction of Q with light quarks and gluons **(= brown muck)** (in or close to it's restframe) described with **Heavy Quark Effective Theory (HQET)**
- heavy mass implies that changes of velocity due to brown muck

$$v^\mu = \frac{p^\mu}{m_Q} \qquad \Rightarrow \qquad \delta v^\mu = \frac{\delta p^\mu}{m_Q} \sim \frac{\Lambda_{\rm QCD}}{m_Q} \to 0 \qquad \text{for} \qquad m_Q \to \infty$$

  $\Rightarrow$ Q has constant velocity
- since QCD flavorblind all heavy quarks alike for $m_Q \to \infty$
  $\Rightarrow$ **heavy-quark flavor symmetry**, broken by $(1/m_b - 1/m_c)$
- acts in restframe as static color source
  $\Rightarrow$ **heavy-quark spin symmetry**, broken at $1/m_Q$ by chromo-magnetic interactions
- HQET Lagrangian is a series in

$$\mathcal{L}_{\rm HQET} = \mathcal{L}_0 + \frac{1}{m_Q}\mathcal{L}_1 + \frac{1}{m_Q^2}\mathcal{L}_2 + \ldots$$

  where only $\mathcal{L}_0$ has spin-flavor symmetry

Heavy quark field

Start by splitting QCD-field, where momentum $p^\mu = m_Q v^\mu + k$ with $k \sim \Lambda_{\rm QCD}$ and $v \cdot v = 1$

$$Q(x) = e^{-im_Q v\cdot x}\left[h_v(x) + \mathcal{H}_v(x)\right], \qquad \not{v} h_v = h_v, \qquad \not{v}\mathcal{H}_v = -\mathcal{H}_v$$

Insert into QCD-Lagrangian (full theory)

$$\begin{aligned}
\mathcal{L}_{\rm QCD} &= \overline{Q}(i\not{D} - m_Q)Q \\
&= \left[\overline{h}_v + \overline{\mathcal{H}}_v\right] e^{im_Q v\cdot x}\,(i\not{D} - m_Q)\, e^{-im_Q v\cdot x}\left[h_v + \mathcal{H}_v\right] \\
&= \left[\overline{h}_v + \overline{\mathcal{H}}_v\right](i\not{D} - m_Q + m_Q\not{v})\left[h_v + \mathcal{H}_v\right] \\
&= \left[\overline{h}_v + \overline{\mathcal{H}}_v\right]\left[i\not{D}h_v + (i\not{D} - 2m_Q)\mathcal{H}_v\right] \\
&= \overline{h}_v(iv\cdot D)h_v - \overline{\mathcal{H}}_v(iv\cdot D + 2m_Q)\mathcal{H}_v + \overline{h}_v i\not{D}_\perp \mathcal{H}_v + \overline{\mathcal{H}}_v i\not{D}_\perp h_v
\end{aligned}$$

Use EOM $(iv\cdot D + 2m_Q)\mathcal{H}_v = i\not{D}_\perp h_v$ to arrive at $\quad V_\perp^\mu \equiv V^\mu - (v\cdot V)v^\mu$

$$\mathcal{L}_{\rm QCD} = \overline{h}_v(iv\cdot D)h_v + \overline{h}_v\, i\not{D}_\perp \frac{1}{2m_Q + iv\cdot D} i\not{D}_\perp\, h_v$$

Derivatives acting on $h_v(x)$ yield residual momentum $D_\mu h_v(x) \to k_\mu u_v(p) \ll 2m_Q h_v(x)$

$$\Rightarrow \text{expansion} \qquad \frac{1}{2m_Q + iv\cdot D} \approx \frac{1}{2m_Q} - \frac{1}{4m_Q^2}(iv\cdot D) + \ldots$$

HQET

The **HQET Lagrangian** (at tree-level)

$$\mathcal{L}_{\text{HQET}} = \bar{h}_v(iv \cdot D)h_v + \frac{1}{2m_Q}\bar{h}_v i\not{D}_\perp i\not{D}_\perp h_v + \mathcal{O}\left((m_Q)^{-2}\right)$$

$$\approx \bar{h}_v(iv \cdot D)h_v + \underbrace{\frac{1}{2m_Q}\bar{h}_v(i\not{D}_\perp)^2 h_v}_{\text{kinetic energy}} - \underbrace{c_F \frac{g_s}{4m_Q}\bar{h}_v \sigma_{\mu\nu} G^{\mu\nu} h_v}_{\text{magnetic moment}}$$

- **kinetic energy** of heavy quark breaks flavor symmetry $\propto 1/m_Q$
  $\Rightarrow$ reparametrization invariance shows that no α_s corrections to all orders
- **magnetic moment** interaction $(\mu_Q \cdot B_c)$ breaks heavy quark spin & flavor symmetry
  $\Rightarrow$ Wilson coefficient $c_F = 1 + \mathcal{O}(\alpha_s)$

Normalization of hadron states in QCD

- in QCD $\langle H(p', \varepsilon') | H(p, \varepsilon) \rangle = 2E(\vec{p})\,(2\pi)^3 \delta^{(3)}[\vec{p} - \vec{p}']\delta_{\varepsilon\varepsilon'}$
- in HQET $\langle H_{v'}(k', \varepsilon') | H_v(k, \varepsilon) \rangle = 2v^0\,(2\pi)^3 \delta_{v'v}\,\delta^{(3)}[\vec{k} - \vec{k}']\delta_{\varepsilon\varepsilon'}$ (labelled by v and residual k)
- QCD $\leftrightarrow$ HQET $|H(p, \varepsilon)\rangle = \sqrt{m_H}\,|H_v(k, \varepsilon)\rangle + \mathcal{O}\left((m_Q)^{-1}\right)$

$\overline{B} \to D^{(*)}$ FFs

Use instead of q^2: $$w = \frac{p_B \cdot p_D}{2 m_B m_{D^{(*)}}} = v \cdot v' = \frac{m_B^2 + m_{D^{(*)}}^2 - q^2}{2 m_B m_{D^{(*)}}}$$ with $1 \leq w \leq 1.6\ldots$

Consider B and D^* as heavy: $p_B = m_B v$ and $p_D = m_{D^{(*)}} v'$

In the following convenient to use FFs $F(w)$ relevant for $\overline{B} \to D^{(*)} \ell \overline{\nu}_\ell$ are

$$\frac{\langle D(p') | \bar{c} \gamma^\mu b | \overline{B}(p) \rangle}{\sqrt{m_B m_D}} = \langle D_{v'} | \bar{c}_{v'} \gamma^\mu b_v | \overline{B}_v \rangle = h_+ \, [v + v']^\mu + h_- \, [v - v']^\mu$$

$$\frac{\langle D^*(p', \epsilon) | \bar{c} \gamma^\mu b | \overline{B}(p) \rangle}{\sqrt{m_B m_{D^*}}} = \langle D^*_{v'}(\epsilon) | \bar{c}_{v'} \gamma^\mu b_v | \overline{B}_v \rangle = h_V \, i \varepsilon^{\mu\nu\alpha\beta} \epsilon^*_\nu v'_\alpha v_\beta$$

$$\frac{\langle D^*(p', \epsilon) | \bar{c} \gamma_\mu \gamma_5 b | \overline{B}(p) \rangle}{\sqrt{m_B m_{D^*}}} = \langle D^*_{v'}(\epsilon) | \bar{c}_{v'} \gamma_\mu \gamma_5 b_v | \overline{B}_v \rangle = h_{A_1} (w + 1) \epsilon^{*\mu} - \epsilon \cdot v \left[h_{A_2} v^\mu + h_{A_3} v'^\mu \right]$$

6 FFs that describe $\overline{B} \to D^{(*)} \ell \overline{\nu}_\ell$, in terms of initial and final velocities

Hadronic matrix elements

Technique of calculating matrix elements (ME) of operators with "hadronic fields"

- fields $P = B, D$ and $V = B^*, D^*$ in ground-state doublets (4×4 matrix, bispinor)

$$H_v^{(Q)} = \frac{1+\slashed{v}}{2}\Big(\overbrace{\slashed{P}_v^{*(Q)}}^{\equiv V(v,\epsilon)} - \gamma_5 \overbrace{P_v^{(Q)}}^{\equiv P(v)} \Big), \qquad \slashed{v} H_v^{(Q)} = H_v^{(Q)}, \qquad v \cdot P_v^{*(Q)} = 0$$

$\Rightarrow$ transforms under heavy-quark symmetry as $H_v^{(Q)} \to D(R)_Q H_v^{(Q)}$

- quark currents replaced by field-products of B and $D^{(*)}$ that represent their MEs

$$\bar{c}_{v'} \Gamma b_v \;\to\; \mathrm{Tr}\left\{ \overline{H}_{v'}^{(c)} \Gamma H_v^{(b)} X \right\} \qquad \text{where} \qquad X = X_0 + X_1 \slashed{v} + X_2 \slashed{v}' + X_3 \slashed{v} \slashed{v}'$$

transform in same way under heavy-quark symmetry

- X and $X_i = X_i(w)$ contain dynamics due to light degrees of freedom
  $s_{\text{light}} = \pm 1/2 \Rightarrow$ depends only on initial and final velocities v and v', no Lorentz indices

- with $\slashed{v} H_v^{(Q)} = H_v^{(Q)}$ and $\slashed{v}\slashed{v} = v^2 = 1 \quad \Rightarrow \quad X \to -\xi(w)$ just a scalar function

Isgur-Wise function [PLB 232 (1990) 113; PLB 237 (1990) 527]

Form factor relations

Performing traces and comparing with definitions of FFs yields

$$h_+(w) = h_V(w) = h_{A_1}(w) = h_{A_3}(w) = \xi(w), \qquad h_-(w) = h_{A_2}(w) = 0$$

Normalization of $\xi(w)$ at zero-recoil $w \to 1$ given by ME of b-number current in QCD

$$2p_B^\mu = 2m_B v^\mu = \langle B(p)|\bar{b}\gamma^\mu b|B(p)\rangle$$

$$= m_B\langle B_v|b_v\gamma^\mu b_v|B_v\rangle = m_B\, h_+(1)\,[v+v]^\mu = 2m_B v^\mu\, \xi(1) \quad \Rightarrow \quad \xi(1) = 1$$

Lukes theorem [Luke, PLB252 (1990) 247]

There are no $1/m_Q$ corrections to form factor relations at zero recoil

QCD corrections to $(\bar{c}\Gamma b)$ currents when matching on HQET:

$$\xi(1) \;\to\; \eta_A \left[1 + \delta_{1/m_Q^2}\right] \xi(1), \qquad \eta_A = 1 + \frac{\alpha_s}{\pi}\left(\frac{m_b + m_c}{m_b - m_c}\ln\frac{m_b}{m_c} - \frac{8}{3}\right) + \mathcal{O}\left(\alpha_s^2\right)$$

at 2-loop $\eta_A = 0.960 \pm 0.007$ [Czarnecki hep-ph/9603261]

Another FF parametrization

The combination of unitarity bounds with FF relations from HQET is due to

CNL = Caprini/Lellouch/Neubert [hep-ph/9712417]

- combine all spin-parity cannels ($J^P = 0^+, 0^-, 1^-, 1^+$) and include B_c poles in unitarity bounds
- exploit spin symmetry of HQET in ground-state doublets of $B^{(*)}$ and $D^{(*)}$, including subleading $1/m_Q$ and α_s corrections
- use z-expansion up to $\mathcal{O}\left(z^3\right)$ for $B \to D$ vector-FF $V_1(w)$, i.e. 3 parameters $\to \rho_1^2, c_1, d_1$

  $\Rightarrow$ unitarity bounds lead to strong correlation between ρ_1^2 and c_1, such that d_1 quasi-fixed
- other $B \to D^*$ and $B^* \to D^{(*)}$ FFs expressed via HQET relations as

$$\frac{F_j(w)}{V_1(w)} = A_j \left[1 + B_j(w-1) + C_j(w-1)^2 + D_j(w-1)^3 + \ldots\right]$$

where A, B, C, D known from HQET (including uncertainties)