Heavy Quark Expansion
inclusive decays

$B \to X_{u,c} \ell \overline{\nu}_\ell$ and $B \to X_s(\gamma, \ell\bar{\ell})$

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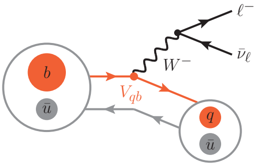
Outline

- Inclusive $B \to X_c \ell \overline{\nu}_\ell$
- Remarks on Heavy Quark Expansion

Main motivation: Determination of $|V_{cb}|$ and $|V_{ub}|$



Inclusive $|V_{ub}|$

$\overline{B} \to X_u \ell \bar{\nu}_\ell$

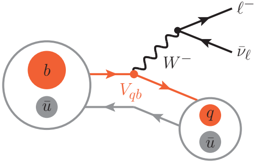
Inclusive $|V_{cb}|$

$\overline{B} \to X_c \ell \bar{\nu}_\ell$

$$Br \propto |V_{qb}|^2 \left[\Gamma(b \to q \ell \bar{\nu}_\ell) + \frac{1}{m_{b,c}^2} + \alpha_s \ldots \right]$$

- $\Gamma(b \to q \ell \bar{\nu}_\ell)$ = free quark decay
- $1/m_{b,c}$ = subleading HQE corrections
- higher background in experiment

Main motivation: Determination of $|V_{cb}|$ and $|V_{ub}|$



Inclusive $|V_{ub}|$

$\overline{B} \to X_u \ell \bar{\nu}_\ell$

Inclusive $|V_{cb}|$

$\overline{B} \to X_c \ell \bar{\nu}_\ell$

$$Br \propto |V_{qb}|^2 \left[\Gamma(b \to q\ell\bar{\nu}_\ell) + \frac{1}{m_{b,c}^2} + \alpha_s \dots \right]$$

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- $1/m_{b,c}$ = subleading HQE corrections
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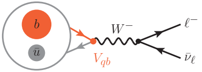
. . . as alternative with different theoretical and experimental systematics to

Leptonic $|V_{ub}|$

$$Br \propto |V_{ub}|^2 \, f_B^2 \, \frac{m_\ell^2}{m_B^2}$$

- f_B = B-meson decay constant



Exclusive $|V_{ub}|$

$\overline{B} \to \pi\ell\bar{\nu}_\ell$, $\Lambda_b \to p\ell\bar{\nu}_\ell$

future $B_s \to K\ell\bar{\nu}_\ell$

Exclusive $|V_{cb}|$

$\overline{B} \to (D, D^*)\ell\bar{\nu}_\ell$, $\Lambda_b \to \Lambda_c\ell\bar{\nu}_\ell$

future $B_s \to D_s\ell\bar{\nu}_\ell$

$$Br \propto |V_{qb}|^2 \, f^2(q^2) \otimes d\Pi(q^2)$$

- $f^2(q^2) \otimes d\Pi(q^2)$ = form factors & phase space
- low background in experiment

Inclusive $b \to c\ell\overline{\nu}$

Inclusive semileptonic decays

Final hadronic X_q contains quark with flavor q, focus here on $q = c \Rightarrow X_c = D, D\pi, D\pi\pi, D^*, \ldots$

$$\overline{B}(p_B) \to X_c(p_X) + \ell(p_\ell) + \overline{\nu}_\ell(p_\nu) \qquad q \equiv p_B - p_X = p_\ell + p_\nu$$

$$\frac{d^3\Gamma}{dq^2\, dE_\ell\, dE_\nu} = \frac{1}{8m_B} \sum_{X_c} \sum_{\text{spins}} \left| \langle X_c \ell \overline{\nu}_\ell | \mathcal{L}_{\text{EFT}} | \overline{B} \rangle \right|^2 \delta^4[p_B - q - p_X]$$

$$= \frac{\mathcal{G}_F^2 |V_{cb}|^2}{2\pi^3} \Big[W_1\, q^2 + W_2\, (2E_\ell E_\nu - q^2/2) + W_3\, q^2 (E_\ell - E_\nu) \Big]\, \theta\left[E_\nu - \frac{q^2}{4E_\ell} \right]$$

$\Rightarrow$ used here factorization at LO in QED into **hadronic $W_{\alpha\beta}$** and **leptonic $L_{\alpha\beta}$** tensors

$$2\,\mathcal{G}_F^2 |V_{cb}|^2 W_{\alpha\beta} L^{\alpha\beta} \equiv \frac{(2\pi)^3}{2m_B} \sum_{X_c} \sum_{\text{spins}} \left| \langle X_c \ell \overline{\nu}_\ell | \mathcal{L}_{\text{EFT}} | \overline{B} \rangle \right|^2 \delta^4[p_B - q - p_X]$$

where

$$L^{\alpha\beta} = 4\left[p_\ell^\alpha p_\nu^\beta + p_\ell^\beta p_\nu^\alpha - g^{\alpha\beta} p_\ell p_\nu - i\varepsilon^{\alpha\beta\rho\delta} p_{\ell\rho} p_{\nu\delta} \right]$$

$$W_{\alpha\beta} \equiv \frac{(2\pi)^3}{2m_B} \sum_{X_c} \delta^4[p_B - q - p_X] \langle \overline{B} | J_\alpha^\dagger | X_c \rangle \langle X_c | J_\beta | \overline{B} \rangle \qquad J_\mu \equiv [\overline{c}\, \gamma_\mu P_L\, b]$$

$$= -g_{\alpha\beta}\, W_1 + v_\alpha v_\beta\, W_2 - i\varepsilon_{\alpha\beta\mu\nu} v^\mu q^\nu\, W_3 + q_\alpha q_\beta\, W_4 + (v_\alpha q_\beta - v_\beta q_\alpha)\, W_5$$

with scalar functions $W_j = W_j(q^2, v \cdot q)$

$W_{4,5}$ do not contribute since $q^\alpha L_{\alpha\beta} = q^\beta L_{\alpha\beta} = 0$

$W_{\alpha\beta}$ from $\overline{B} \to \overline{B}$

Hadronic tensor $W_{\alpha\beta}$ parametrizes strong dynamics for $\overline{B} \to X_c$ transition $\Rightarrow$ can be related to

$$iT_{\alpha\beta} = \int d^4x\, e^{-iqx} \frac{\langle \overline{B} |\, \mathrm{T}\{J^\dagger_\alpha(x),\, J_\beta(0)\}\, | \overline{B} \rangle}{2m_B}$$

$\leftarrow$ insert complete set of states

$$= \int d^4x \frac{e^{-iqx}}{2m_B} \left\{ \theta[x^0] \sum_{X_c} \langle \overline{B} | J^\dagger_\alpha(x) | X_c \rangle \langle X_c | J_\beta(0) | \overline{B} \rangle + \theta[-x^0] \sum_{X_{\bar{c}bb}} \langle \overline{B} | J_\beta(0) | X_{\bar{c}bb} \rangle \langle X_{\bar{c}bb} | J^\dagger_\alpha(x) | \overline{B} \rangle \right\}$$

$\downarrow$ in B rest frame $p_B^\mu = m_B v^\mu = m_B(1, \vec{0})^\mu$ and used $J_\mu(x) = e^{i\hat{P}x} J_\mu(0) e^{-i\hat{P}x}$ and write $J_\mu \equiv J_\mu(0)$

$$= \frac{i(2\pi)^3}{2m_B} \left\{ \underbrace{\sum_{X_c} \frac{\langle \overline{B} | J^\dagger_\alpha | X_c \rangle \langle X_c | J_\beta | \overline{B} \rangle}{m_B - E_{X_c} - q^0 + i\varepsilon}}_{\text{resembles } W_{\alpha\beta}} \delta^3[\vec{p}_X + \vec{q}] - \sum_{X_{\bar{c}bb}} \frac{\langle \overline{B} | J_\beta | X_{\bar{c}bb} \rangle \langle X_{\bar{c}bb} | J^\dagger_\alpha | \overline{B} \rangle}{E_{X_{\bar{c}bb}} - m_B - q^0 - i\varepsilon} \delta^3[\vec{p}_X - \vec{q}] \right\}$$

$W_{\alpha\beta}$ from $\overline{B} \to \overline{B}$

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- at fixed $\vec{q}$ cuts along real axis in plane of $q^0 = v \cdot q$ (B rest frame)

  X_c $\quad -\infty < q^0 < m_B - \sqrt{\vec{q}^2 + m^2_{X_{c,\mathrm{min}}}}$ $\quad$ and $\quad$ $X_{\bar{c}bb}$ $\quad \sqrt{\vec{q}^2 + m^2_{X_{\bar{c}bb,\mathrm{min}}}} - m_B < q^0 < \infty$

  where intermediate states go on-shell

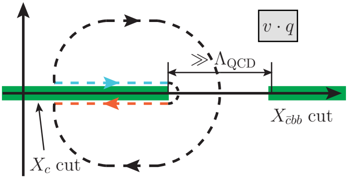
- even for $\vec{q}^2 = 0$, the two cuts are well-separated: ($X_{c,\mathrm{min}} \simeq D$-meson)

  from $(m_B - m_D)$ to $(m_B + m_D)$ $\;\Rightarrow\;$ $2m_D \gg \Lambda_{\mathrm{QCD}}$

$W_{\alpha\beta}$ from $\overline{B} \to \overline{B}$

Now integrate along semileptonic region = cut (blue line), using

$$\frac{1}{\omega - \omega' \pm i\varepsilon} = P\frac{1}{\omega - \omega'} \mp i\pi\delta(\omega - \omega')$$



i.e. calculate $\widetilde{T}_{\alpha\beta}(q^2) \equiv \int dq^0\, T_{\alpha\beta}(q^2, q^0)$

$$\frac{1}{\pi}\,\mathrm{Im}\,\widetilde{T}_{\alpha\beta} = -\frac{(2\pi)^3}{2m_B}\sum_{X_c}\delta^4[p_B - q - p_X]\langle\overline{B}|J_\alpha^\dagger|X_c\rangle\langle X_c|J_\beta|\overline{B}\rangle - \frac{(2\pi)^3}{2m_B}\sum_{X_{\bar{c}bb}}\delta^4[p_B + q - p_X]\ldots$$

$\Rightarrow$ $\delta^4[p_B + q - p_X]$ of $X_{\bar{c}bb}$ vanishes if q in semileptonic region for physical X_c

$$\widetilde{W}_{\alpha\beta} = -\frac{1}{\pi}\,\mathrm{Im}\,\widetilde{T}_{\alpha\beta}$$

Note: the $\int dq^0 \ldots$ implies

$$\frac{d^3\Gamma}{dq^2\, dE_\ell\, dE_\nu} \to \frac{d^2\Gamma}{dq^2\, dE_\ell}$$

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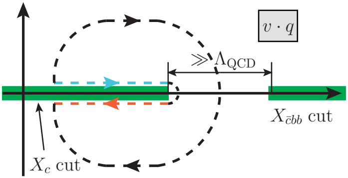

$$\frac{1}{\pi}\,\mathrm{Im}\,\widetilde{T}_{\alpha\beta} = -\frac{(2\pi)^3}{2m_B}\sum_{X_c}\delta^4[p_B - q - p_X]\langle\overline{B}|J^\dagger_\alpha|X_c\rangle\langle X_c|J_\beta|\overline{B}\rangle - \frac{(2\pi)^3}{2m_B}\sum_{X_{\bar{c}bb}}\delta^4[p_B + q - p_X]\ldots$$

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Note: the $\int dq^0 \ldots$ implies

$$\frac{d^3\Gamma}{dq^2\,dE_\ell\,dE_\nu} \to \frac{d^2\Gamma}{dq^2\,dE_\ell}$$

- have related $\widetilde{W}_{\alpha\beta}$ to imagingary part of $\overline{B} \to \overline{B}$ matrix elements $\widetilde{T}_{\alpha\beta}$ (optical theorem at work)
- can not calculate $\widetilde{T}_{\alpha\beta}$ close to resonances along cut, BUT

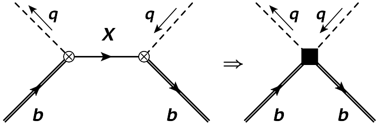
"blue + red" contour = "black" contour

- on black contour mostly far ($\gg \Lambda_{\rm QCD}$) away from resonances $\Rightarrow$ use OPE for $T_{\alpha\beta}$

OPE for $b \to b$ at tree-level

OPE expresses $T_{\alpha\beta}$ in terms of matrix elements of local operators

$$-i\int d^4x e^{-iqx}\mathrm{T}\{J_\alpha^\dagger(x), J_\beta(0)\} \;\to\; \sum_i z_i\,\mathcal{O}_i$$



At parton level and lowest order in QCD

$$t_{\alpha\beta} = \bar{b}\gamma_\alpha P_L \frac{\not{p}_b - \not{q} + m_c}{(p_b-q)^2 - m_c^2 + i\varepsilon}\gamma_\beta P_L\, b = \bar{b}\gamma_\alpha \frac{m_b\not{v} - \not{q} + \not{k}}{(m_b v - q + k)^2 - m_c^2 + i\varepsilon}\gamma_\beta P_L\, b$$

↓ expand in $k/m_b \ll 1$ & use Chisholm identity & $\Delta \equiv (m_b v - q)^2 - m_c^2 + i\varepsilon$

$$\approx \frac{1}{\Delta}\bar{b}\left[(m_b v - q)_\alpha\gamma_\beta + (m_b v - q)_\beta\gamma_\alpha - g_{\alpha\beta}(m_b\not{v} - \not{q}) - i\varepsilon_{\alpha\beta\mu\nu}(m_b v - q)^\mu\gamma^\nu\gamma_5\right]\frac{\mathbb{1} - \gamma_5}{2}\, b$$

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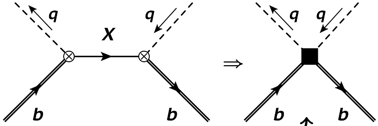

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↓ expand in $k/m_b \ll 1$ & use Chisholm identity & $\Delta \equiv (m_b v - q)^2 - m_c^2 + i\varepsilon$

$$\approx \frac{1}{\Delta}\bar{b}\left[(m_b v - q)_\alpha\gamma_\beta + (m_b v - q)_\beta\gamma_\alpha - g_{\alpha\beta}(m_b\not{v} - \not{q}) - i\varepsilon_{\alpha\beta\mu\nu}(m_b v - q)^\mu\gamma^\nu\gamma_5\right]\frac{\mathbb{1} - \gamma_5}{2}\, b$$

Then this corresponds to **local dim-3 operators** $\mathcal{O}_i^{(3)} \simeq \bar{b}\gamma^\mu(\mathbb{1}, \gamma_5)_i b$ with

$$\langle \bar{B}|\bar{b}\gamma^\mu b|\bar{B}\rangle = 2p_B^\mu = 2m_B v^\mu, \qquad \langle \bar{B}|\bar{b}\gamma^\mu\gamma_5 b|\bar{B}\rangle = 0$$

← here exact, no transition to HQET needed

and find

$$T_{\alpha\beta} = \frac{\langle \bar{B}|t_{\alpha\beta}|\bar{B}\rangle}{2m_B} = \frac{1}{2\Delta}\left[-g_{\alpha\beta}(m_b - vq) + 2m_b v_\alpha v_\beta - i\varepsilon_{\alpha\beta\mu\nu}v^\mu q^\nu - (v_\alpha q_\beta + v_\beta q_\alpha)\right]$$

$$\equiv -g_{\alpha\beta}T_1 + v_\alpha v_\beta T_2 - i\varepsilon_{\alpha\beta\mu\nu}v^\mu q^\nu T_3 + q_\alpha q_\beta T_4 + (v_\alpha q_\beta + v_\beta q_\alpha)T_5$$

Back to $W_{\alpha\beta}$

Can read off that

$$T_1 = \frac{m_b - vq}{2\Delta}, \qquad T_2 = \frac{2m_b}{2\Delta}, \qquad T_3 = \frac{1}{2\Delta}, \qquad T_4 = 0, \qquad T_5 = -\frac{1}{2\Delta}$$

Remember $\Delta = (m_b v - q)^2 - m_c^2 + i\varepsilon$ and $vq = q^0$

$$\Delta = -2m_b\left[q^0 - \underbrace{\frac{m_b^2 + q^2 - m_c^2}{2m_b}}_{\equiv q_*^0} - i\varepsilon\right] \qquad \Rightarrow \qquad \frac{1}{\Delta} \to \frac{i\pi\delta[q^0 - q_*^0]}{-2m_b}$$

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Remember $\Delta = (m_b v - q)^2 - m_c^2 + i\varepsilon$ and $vq = q^0$

$$\Delta = -2m_b\Big[q^0 - \underbrace{\frac{m_b^2 + q^2 - m_c^2}{2m_b}}_{\equiv q^0_*} - i\varepsilon\Big] \qquad \Rightarrow \qquad \frac{1}{\Delta} \to \frac{i\pi\delta[q^0 - q^0_*]}{-2m_b}$$

Then find structure functions via $W_i = -\frac{1}{\pi}\,\mathrm{Im}\,T_i$

$$W_1 = \frac{m_b - vq}{4m_b}\delta[q^0 - q^0_*], \quad W_2 = \frac{\delta[q^0 - q^0_*]}{2}, \quad W_3 = \frac{\delta[q^0 - q^0_*]}{2m_b}, \quad W_4 = 0, \quad W_5 = -\frac{\delta[q^0 - q^0_*]}{2m_b}$$

- with W_i can perform q^0, or equivalently E_ν integration in $\dfrac{d^3\Gamma}{dq^2\,dE_\ell\,dE_\nu}$
- have found that leading order result is given by decay of free b-quark
- know how to improve systematically for higher orders in Λ_{QCD}/m_b and α_s

$\Lambda_{\rm QCD}/m_b$ corrections

Starting again with tree-level expression

$$t_{\alpha\beta} = \bar{b}\gamma_\alpha \frac{m_b \not{v} - \not{q} + \not{k}}{(m_b v - q + k)^2 - m_c^2 + i\varepsilon}\gamma_\beta P_L b = \bar{b}\gamma_\alpha \frac{m_b \not{v} - \not{q} + \not{k}}{\Delta - 2qk + k^2}\gamma_\beta P_L b$$

$\downarrow$ keep 1st order in $k/m_b \ll 1$ & $(\Delta - 2kq + k^2)^{-1} \approx 1/\Delta + 2qk/\Delta^2$

$$\approx \frac{1}{\Delta}\left[\bar{b}\gamma_\alpha \not{k}\gamma_\beta P_L b\right] + \frac{2qk}{\Delta^2}\left[\bar{b}\gamma_\alpha (m_b \not{v} - \not{q})\gamma_\beta P_L b\right]$$

1) go to HQET by $b \to h_v$: given by **dim-4 operators** $\mathcal{O}_i^{(4)} = \bar{h}_v \gamma_\alpha \left(\mathbb{1}, \gamma_5\right)_i iD_\beta h_v$

2) treat Dirac-algebra again with Chisholm identity and go to $T_{\alpha\beta}$ by taking $\langle \overline{B}_v | \ldots | \overline{B}_v \rangle$

- $\langle \overline{B}_v | \bar{h}_v \gamma_\alpha \gamma_5 iD_\beta h_v | \overline{B}_v \rangle = 0$ vanishes by parity
- from $\not{v} h_v = h_v \quad \Rightarrow \quad \bar{h}_v \gamma_\alpha iD_\beta h_v = v_\alpha \bar{h}_v iD_\beta h_v$
- then parametrize $\langle \overline{B}_v | \bar{h}_v iD_\alpha h_v | \overline{B}_v \rangle = X\, v_\alpha$ and contract with v^α

  $X = \langle \overline{B}_v | \bar{h}_v iv \cdot D h_v | \overline{B}_v \rangle = 0$, because of EOM from $\mathcal{L}_0 = \bar{h}_v (iv \cdot D) h_v$

There are no $\Lambda_{\rm QCD}/m_b$ corrections to $B \to X_c \ell \bar{\nu}$

[Chay/Georgi/Grinstein PLB247 (1990) 399]

The problem with m_b

Have see that the decay of free b-quark provides first terms $\hat{m}_c \equiv \frac{m_c}{m_b}$

$$\Gamma[B \to X_c \ell \overline{\nu}] = \frac{\mathcal{G}_F^2 |V_{cb}|^2 \, m_b^5}{192\pi^3} \left(1 - 8\hat{m}_c^2 + 8\hat{m}_c^6 - \hat{m}_c^8 - 12\hat{m}_c^4 \ln \hat{m}_c^2\right) + \mathcal{O}\left(1/m_b^2, \alpha_s\right)$$

- depends on 5th power of m_b $\Rightarrow$ which value should be used?
- pole mass m_b^{pole} not well defined $\Rightarrow$ renormalon issue (="series in α_s not well behaved")
- closer investigation shows: also $\Gamma[B \to X_c \ell \overline{\nu}]$ has renormalon issue (in α_s)
- "both issues cure each other"

[Bigi/Shifman/Uraltsev/Vainshtein hep-ph/9402360, Luke/Manohar/Savage hep-ph/9407407]

One needs other observables that depend on m_b^{pole}

- higher moments in lepton energy $\langle E_\ell \rangle, \langle E_\ell^2 \rangle, \ldots$
- higher moments in invariant mass of hadronic states $\langle m_X^2 - \overline{m}_D^2 \rangle, \langle (m_X^2 - \overline{m}_D^2)^2 \rangle, \ldots$

In practice $|V_{cb}|$ determination from $B \to X_c \ell \overline{\nu}$ requires simultaneous fit of Γ and moments

Status of $B \to X_c \ell \overline{\nu}$

Status of $\overline{B} \to X_c \ell \overline{\nu}_\ell$ prediction

Decay width and **moments in lepton energy and hadronic mass**

$\Rightarrow$ double expansion in $a_s \equiv \alpha_s(\mu)/\pi$ and $\Lambda_{\rm QCD}/m_b$

$$
\begin{aligned}
M_i &= M_i^{(0)} + a_s M_i^{(1)} + a_s^2 M_i^{(2)} \\
&+ \left(M_i^{(\pi,0)} + a_s M_i^{(\pi,0)}\right) \frac{\mu_\pi^2}{m_b^2} + \left(M_i^{(G,0)} + a_s M_i^{(G,0)}\right) \frac{\mu_G^2}{m_b^2} \\
&+ M_i^{(D)} \frac{\rho_D^3}{m_b^3} + M_i^{(LS)} \frac{\rho_{LS}^3}{m_b^3} + \mathcal{O}\left(a_s^3, \frac{a_s^2}{m_b^2}, \frac{a_s}{m_b^3}, \frac{1}{m_b^4}, \frac{1}{m_b^3 m_c^2}\right)
\end{aligned}
$$

[see refs. in review Gambino arXiv:1501.00314]

- $M_i^{(j)}$ depend on m_c, m_b, $E_{\rm cut}$ and μ (renormalization schemes: “kinetic” or “1S”)
- dim-5: $\mu_\pi^2 \propto \langle B|\bar{b}_v(\vec{D})^2 b_v|B\rangle$, $\quad \mu_G^2(\mu) \propto \langle B|\bar{b}_v \sigma_{\mu\nu} G^{\mu\nu} b_v|B\rangle$
- dim-6: ρ_D^3 Darwin term, ρ_{LS}^3
- dim-7,8: $\mathcal{O}(1/m_Q^{4,5})$ contributions estimated 1.3% effect to Γ, less on moments

[Mannel/Turzcyk/Uraltsev 1009.4622, Heinonen/Mannel 1407.4384]

Latest fits

Combined fit of V_{cb} with

$$m_{b,c}, \quad \mu^2_{\pi,G}, \quad \rho^3_{D,LS}$$

- use data on (total + partial) width + moments from Babar, Belle, CLEO, DELPHI, CDF
- include effects of higher-dim matrix elements (dim-7: 9 MEs, dim-8: 18 MEs) with some assumptions
- renormalization m_b^{kin} in kinetic scheme
- use external input on $m_c^{\overline{\text{MS}}}(3\,\text{GeV})$, hyperfine splitting $(M_{B^*} - M_B)$ for μ_G, ...

[Alberti/Gambino/Healey/Nandi 1411.6560, Gambino/Healey/Turczyk 1606.06174]

	$\lvert V_{cb}\rvert$ $\times 10^3$	m_b^{kin} [GeV]	$m_c^{\overline{\text{MS}}}$ [GeV]	μ_π^2 [GeV^2]	μ_G^2 [GeV^2]	ρ_D^3 [GeV^3]	ρ_{LS}^3 [GeV^3]
w/o dim-7,8	42.21(78)	4.553(20)	0.987(13)	0.465(68)	0.332(62)	0.170(39)	−0.150(96)
w dim-7,8	42.11(74)	4.546(21)	0.987(13)	0.432(68)	0.355(60)	0.145(61)	−0.169(97)

Authors scrutinize stability of fits, using also external input for m_b

$$\lvert V_{cb}\rvert_{\text{incl}} = (42.00 \pm 0.64) \times 10^{-3}$$

only 2% uncertainty from combination of exp. and th. uncertainties

General remarks

Heavy Quark Expansion (HQE)

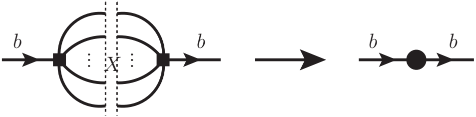
When focusing on **total inclusive decay width**, optical theorem

$$\Gamma[H \to X] = \frac{1}{m_H} \text{Im} \langle H | \, i \int d^4x \, \text{T}\{\mathcal{L}_{\text{EFT}}(x), \mathcal{L}_{\text{EFT}}(0)\} \, | H \rangle \qquad \leftarrow \text{insert complete set of states}$$

$$= \frac{1}{2m_H} \sum_X (2\pi)^4 \delta^4[p_H - p_X] \left| \langle H | \, \mathcal{L}_{\text{EFT}} \, | X \rangle \right|^2$$

relates to $H \to H$ scattering (For $B \to X_c \ell \bar{\nu}$ the derivation was carried out at level of differential decay width)

Assuming large energy release from H to intermediate on-shell states X allows local OPE



with

- dim-3 : $\bar{b}b$
- dim-4 : $\bar{b} i \not{D} b \;\to\; m_b \bar{b} b$ via EOM back to dim-3
- dim-5 : $\bar{b}\, g_s \sigma_{\mu\nu} G^{\mu\nu}\, b$

$$\Gamma[H \to X_f] \;\propto\; z_3^f \langle H | \bar{b} b | H \rangle + z_5^f \frac{\langle H | \bar{b}\, g_s \sigma_{\mu\nu} G^{\mu\nu}\, b | H \rangle}{m_b^2} + \ldots$$

$\Rightarrow$ eventually systematic expansion in $1/m_b$ by transition to HQET fields $H \to H_v$ and $b \to h_v$

Applications of HQE

Semileptonic CC and FCNC decays

- semileptonic CC decays: $B \to X_{c,u} \ell \overline{\nu}$ for $\ell = e, \mu, \tau$
- semileptonic FCNC decays: $B \to X_{d,s} \gamma$, $B \to X_{s,d} \ell \bar{\ell}$ and $B \to X_{s,d} \nu \overline{\nu}$

When calculating total width, the integration over $\ell \overline{\nu}$, γ or $\ell \bar{\ell}$ phase spaces provides a "smearing" over m_X^2 of final state (averaging over bound-state effects of individual hadrons)

$\Rightarrow$ hypotheses of **global quark-hadron duality**

Nonleptonic decays

Here "smearing" works only if sum includes sufficiently many hadronic states

$\Rightarrow$ hypotheses of **local quark-hadron duality**

- lifetime ratios of b-hadrons: $\tau(B_u)/\tau(B_d)$, $\tau(B_s)/\tau(B_d)$ and $\tau(\Lambda_b)/\tau(B_d)$
- lifetime differences for neutral $B_{d,s}$ mesons: $\Delta\Gamma_{d,s}$
- hadronic $\mathcal{B}(B \to X_{c\bar{c}s})$ etc.

Limitations from Phase space cuts . . .

. . . usually required by experiment to suppress backgrounds (bkg):

- $B \to X_u \ell \bar{\nu}_\ell$ complicated due to huge charm-bkg, shape functions from $B \to X_s \gamma$ moments
- $B \to X_c \ell \bar{\nu}_\ell$ E_ℓ lepton energy
- $B \to X_s \gamma$ $E_\gamma \in [1.7, 2.0]$ GeV (photon energy in B-meson restframe)
- $B \to X_s \ell \bar{\ell}$ $M_{X_s} < [1.8, 2.0]$ GeV to remove double semileptonic bkg
  practically irrelevant at high q^2, but not at low q^2

!!! extrapolations beyond cuts introduce model-dependent uncertainties in measurements

OR . . .

. . . introduce new scales in theory

$\Rightarrow$ rate less inclusive $\Rightarrow$ additional non-perturbative effects (shape functions etc.)

. . . so far extrapolation beyond cuts mostly left to experimentalists

Status of $B \to X_s\gamma$ theory

$$\Gamma(B \to X_q\gamma) = \Gamma(b \to q\gamma)_{\rm p} + \delta\Gamma_{\rm np}$$
$$\propto (|C_7|^2 + |C_7'|^2)$$

$$O_{7\,(7')} \propto m_b \left[\bar{s}\sigma_{\mu\nu} P_{R(L)} b\right] F^{\mu\nu}$$

- $\Gamma(b \to q\gamma)_{\rm p}$ = perturbatively calculable part @ NNLO
- $\delta\Gamma_{\rm np}$ = non-perturbative part
  around 5% uncertainty @ $E_\gamma \geq 1.6$ GeV
  [Benzke/Lee/Neubert/Paz arXiv:1003.5012]
- $b \to du\bar{u}\gamma$ sizeable in $b \to d\gamma$
  [Asatrian/Greub et al. arXiv:1305.6464]

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Latest SM updates @ NNLO QCD for $E_\gamma \geq 1.6$ GeV [Misiak et al. arXiv:1503.01789]

$$\mathcal{B}(B \to X_s\gamma)|_{\rm SM} = (3.36 \pm 0.23) \cdot 10^{-4}$$

$$\mathcal{B}(B \to X_d\gamma)|_{\rm SM} = (1.73^{+0.12}_{-0.22}) \cdot 10^{-5}$$

uncertainty budget due to:

- 5% non-perturbative
- 3% higher order
- 3% interpolation of m_c-dep. in NNLO corr.
- 2% parametric

Better adopted for actual measurement without strange tagging $\Rightarrow X_{s+d}$:

$$R_\gamma \equiv \frac{\mathcal{B}(B \to X_s\gamma) + \mathcal{B}(B \to X_d\gamma)}{\mathcal{B}(B \to X_x\ell\bar{\nu}_\ell)} = (3.31 \pm 0.22) \times 10^{-3}$$

Current world averages

$$\mathcal{B}(B \to X_s\gamma)|_{\rm exp} = (3.43 \pm 0.22) \cdot 10^{-4}$$

$$\mathcal{B}(B \to X_d\gamma)|_{\rm exp} = (1.41 \pm 0.57) \cdot 10^{-5}$$

$\Rightarrow$ bound on charged Higgs mass in 2HDM (type-II) $m_{H^\pm} > 480$ GeV @ 95% CL

Status of $B \to X_{d,s}\,\ell\bar{\ell}$ theory

3 angular observables in differential decay width $\qquad \mathcal{B} \propto (H_L + H_T)$ and $A_{\rm FB} \propto H_A$

$$\frac{8}{3}\,\frac{d^2\Gamma}{dq^2\,d\cos\theta_\ell} = \left(1+\cos^2\theta_\ell\right)H_T(q^2) + 2\left(1-\cos^2\theta_\ell\right)H_L(q^2) + 2\cos\theta_\ell H_A(q^2)$$

different dependence on short-distance $C_{7,9,10}$ – **complementary to $B \to K^{(*)}\bar{\ell}\ell$ at low q^2**

$(\hat{s} = q^2/m_b^2)$

$$H_T \propto \hat{s}\,(1-\hat{s})^2\left[|C_9 + \frac{2}{\hat{s}}C_7|^2 + |C_{10}|^2\right] \qquad H_L \propto (1-\hat{s})^2\left[|C_9 + 2C_7|^2 + |C_{10}|^2\right]$$

$$H_A \propto -4\,\hat{s}(1-\hat{s})^2 \mathrm{Re}\left[(C_9 + \frac{2}{\hat{s}}C_7)C_{10}^*\right]$$

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SM predictions @ NNLO QCD and NLO QED [Huber/Hurth/Lunghi 1503.04849]

- theory unc. for $\mathcal{B}$ and $H_{L,T}$: 6 – 9 % in $q^2 \in [1, 3.5], [3.5, 6], [1, 6]$ GeV^2
- theory unc. for H_A: from 5 – 70 %, depend strongly on q^2-binning around zero-crossing
- zero-crossing of H_A predicted with $\lesssim 4\,\%$
- QED corrections lead to **pronounced differences** for $\ell = e$ and $\ell = \mu$
- at high-q^2 uncertainties larger: $\mathcal{B}$ about 30 %

Effects of M_{X_s}-cuts analysed in SCET at level of subleading shape functions
$\Rightarrow$ require combination of $B \to X_s\gamma$, $B \to X_s\bar{\ell}\ell$ and $B \to X_u \ell\bar{\nu}_\ell$

[Lee et al. hep-ph/0511334, 0512191, 0812.0001, Bernlocher et al.1101.3310, Bell et al. 1007.3758]