Introduction to Effective Theories in Flavor Physics

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GDR Lectures
29 September, 2020

4 Lectures

1) **29/09/20 10:00-11:00**

   EFT of weak interactions in the SM

2) **29/09/20 14:00-15:00**

   Exclusive leptonic and semileptonic charged-current decays

3) **01/10/20 10:00-11:00**

   Inclusive semileptonic decays

4) **01/10/20 11:00-15:00**

   B-anomalies

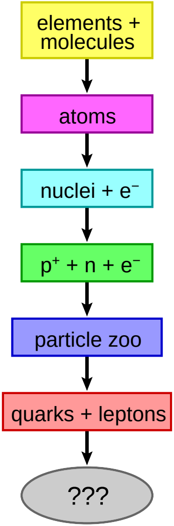
Outline

- Flavor in the SM
- Flavor transitions in SM
- Introduction to EFT (muon decay)
- $\Delta B = 1$ EFT: operators, matching, mixing, ...

Flavor in the Standard Model

Effective theories (EFT) and the Standard Model (SM)

Notions of matter changed within last 100 years



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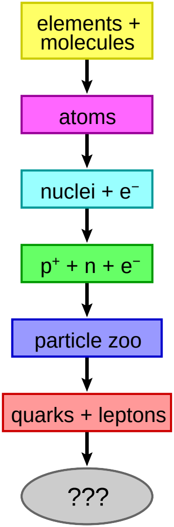
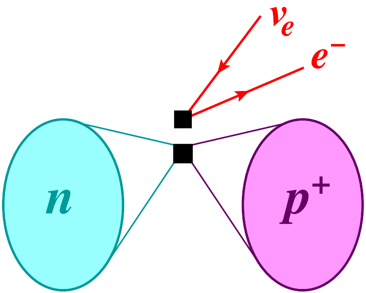


- β-decay: $n \to p^+ + e^- + \bar{\nu}_e$
- 4-Fermi-theory (1933/34)

  $\sim \mathcal{G}_F [\overline{\Psi}(p^+)\,\Gamma\,\Psi(n)][\overline{e}\,\Gamma'\,\nu_e]$

  Fermi coupling $\mathcal{G}_F \sim 1/M^2$



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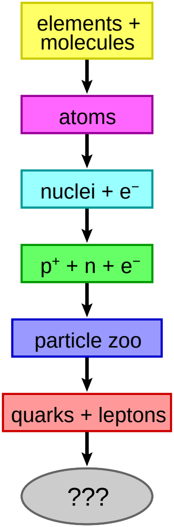

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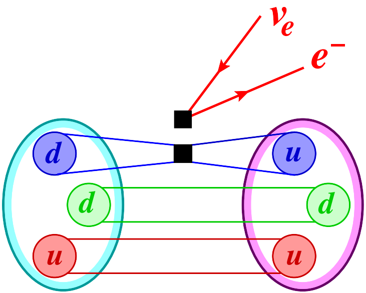

- up- and down-Quarks are constituents of n and p
- Quarks are bound by strong force (Gluons) to hadrons
- Quarks have fractional electric charges $Q_u = +2/3$ and $Q_d = -1/3$

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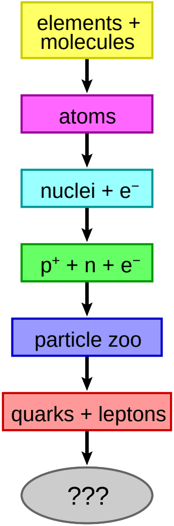

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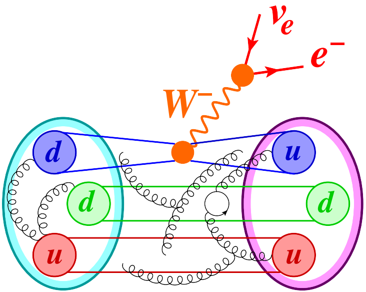

- Quarks have fractional electric charges $Q_u = +2/3$ and $Q_d = -1/3$
- Conservation of charges in weak and strong interactions
  → described by symmetries (local gauge invariance)
- forces are transmitted by spin-1 gauge bosons
  - strong interaction: Gluons
  - weak interaction: massive charged W and neutral Z bosons
- Fermi constant $\mathcal{G}_F \propto g_2^2/m_W^2$ is an effective coupling

“General” principles employed in the SM

We try to test known principles and to find new ones at microscopic length scales and high energy densities

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1) **relativistic quantum field theory** + **S-Matrix**

   ▶ Lorentz symmetry imposes restrictions on interactions of fields

2) **local gauge invariance** provides fundamental interactions

   ▶ gauge fields + interactions are introduced automatically,
   BUT gauge bosons are predicted to be massless

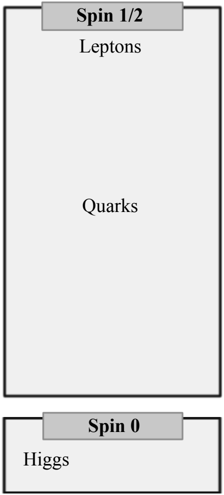
3) **Spontaneous symmetry breaking (SSB)**

   (Englert/Brout-Higgs-Guralnik/Hagen/Kibble mechanism)

   ▶ requires postulation of (at least one) Higgs field (not strongly interacting)
   ▶ mass generation of gauge bosons and Quarks/Leptons
   ▶ masses of Quarks and Leptons $\propto$ to their coupling to Higgs

⇒ Interaction with Higgs gives rise to different flavors

. . . the “current” SM



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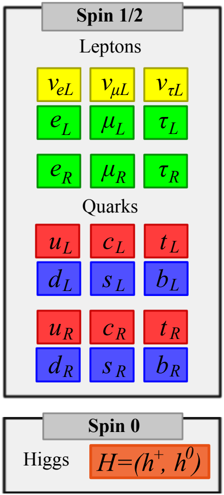

Relativistic invariance + renormalizability ($\leq$ dim 4)

- 3 generations of massless Lepton’s and Quark’s
- Higgs potential:

$$V(H) \sim \mu^2 (H^\dagger H) - \Lambda (H^\dagger H)^2$$

- Yukawa potential:

$$\mathcal{L}_{\text{Yukawa}} \sim \overline{Q}_L \left(Y_U \widetilde{H} u_R + Y_D H d_R \right) + \overline{L}_L Y_L H \ell_R$$

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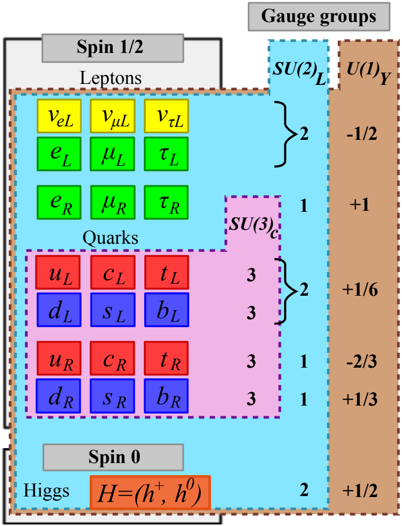

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Local gauge invariance $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$

- 3 gauge couplings: g_s, g_2, g_1
- massless gauge fields

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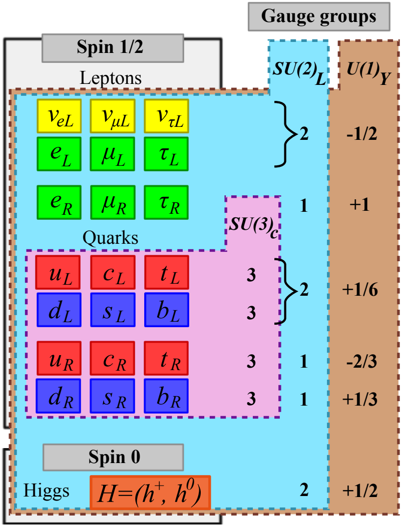

The SM has 2 + 3 + 9 + 4 = 18 parameters

omitting massive neutrino's and θ_{QCD}

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SSB = Mass generation

- residual symmetry with massless photon:

$$SU(2)_L \otimes U(1)_Y \to U(1)_{em}$$

- massive gauge fields: m_W, m_Z
- massive Leptons and Quarks: (but $m_\nu = 0$)

$$Y_L \to m_{e,\mu,\tau}, \quad Y_D \to m_{d,s,b}, \quad Y_U \to m_{u,c,t}$$

- Quark-mixing: $V_{\text{CKM}} = 3 \times 3$ unitary
  4 parameters: λ, A, ρ, η
  [Cabibbo/Kobayashi/Maskawa]

Three generations in the SM

3 copies of matter fields ($i = 1, 2, 3$) postulated as $SU(2)_L$ **doublets** (Q, L) and **singlets** (u, d, ℓ)

Quarks: $Q_{L,i} = \begin{pmatrix} u_{L,i} \\ d_{L,i} \end{pmatrix}, \quad u_{R,i}, \quad d_{R,i}$ **Leptons:** $L_{L,i} = \begin{pmatrix} \nu_{L,i} \\ \ell_{L,i} \end{pmatrix}, \quad \ell_{R,i}$

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Local gauge invariance implemented with the help of **covariant derivative** (same for all 3 copies)

$$D_\mu \Phi = \Big(\partial_\mu + \underbrace{i g_1 Y_\Phi B_\mu}_{U(1)_Y} + \underbrace{i g_2 \tau^a W^a_\mu}_{SU(2)_L} + \underbrace{i g_s T^A G^A_\mu}_{SU(3)_c} \Big) \Phi$$

- some group-indices have been suppressed here
- hypercharges: Y_H fixed by requirement to have massless photon after EWSB

$$Y_Q = +\frac{1}{6}, \quad Y_u = +\frac{2}{3}, \quad Y_d = -\frac{1}{3}, \quad Y_L = -\frac{1}{2}, \quad Y_\ell = -1, \quad Y_H = +\frac{1}{2}$$

$$Y_Q = Y_d + Y_H = Y_u - Y_H \quad \text{and} \quad Y_L = Y_\ell + Y_H$$

- electric charge: $Q \equiv Y + \tau^3$

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- acting on $\Phi = \{Q_{L,i},\ u_{R,i},\ d_{R,i}, \ldots\}$
- $\Phi_{\alpha,a}$ in fundamental representation: $\alpha \to SU(2)_L,\ a \to SU(3)_c$
- **gauge fields:** $B_\mu, \quad W^a_\mu, \quad G^A_\mu$ (transform as adjoint representation)
- **gauge couplings:** $g_1, \quad g_2, \quad g_s$
- generators of $SU(2)_L$: $\tau^a = \sigma^a/2$ (σ^a : 2×2 Pauli matrices, $a = 1, 2, 3$)
  $SU(3)_c$: $T^A = \lambda^A/2$ (λ^A : 3×3 Gellman matrices, $A = 1, \ldots 8$)

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Gauge interactions of matter fields

$$\mathcal{L}_{\text{gauge}} = \sum_{i=1}^{3} \left(\overline{Q}_{L,i}\, i \not{D}\, Q_{L,i} + \overline{u}_{R,i}\, i\not{D}\, u_{R,i} + \overline{d}_{R,i}\, i\not{D}\, d_{R,i} + \text{Leptons} \right), \qquad \not{D} \equiv D_\mu \gamma^\mu$$

- local $SU(2)_L$ invariance forbids mass terms $\sim \; -m_\Phi \left[\overline{\Phi}_L \Phi_R + \overline{\Phi}_R \Phi_L\right]$
- $\mathcal{L}_{\text{gauge}}$ is diagonal in generations
- can rotate with unitary 3×3 matrices $\qquad V_\chi^a\, V_\chi^{a\dagger} = \mathbb{1}_{3\times3} \quad (a = Q, u, d)$

$$Q_L' = V_L^Q\, Q_L, \qquad u_R' = V_R^u\, u_R, \qquad d_R' = V_R^d\, d_R$$

and $\mathcal{L}_{\text{gauge}}$ remains diagonal $\Rightarrow$ **Q_L, u_R and d_R are weak eigenstates**

- huge global flavor symmetry of $\mathcal{L}_{\text{gauge}}$: $\qquad G_{\text{SM}} \equiv U(1)_Y \otimes U(1)_B \otimes U(1)_L$

$$G_{\text{flavor}} \equiv SU(3)_{Q_L} \otimes SU(3)_{U_R} \otimes SU(3)_{D_R} \otimes SU(3)_{L_L} \otimes SU(3)_{E_R} \otimes U(1)_{\text{PQ}} \otimes G_{\text{SM}}$$

Yukawa couplings → origin of Flavor

Yukawa interactions of Higgs-doublet with quarks & leptons $\widetilde{H} = i\sigma^2 H^*$

$$\mathcal{L}_{\text{Yukawa}} = -\sum_{i,j=1}^{3} \Big(Y_{U,ij}\, [\overline{Q}_{L,i} \widetilde{H}]\, u_{R,j} + Y_{D,ij}\, [\overline{Q}_{L,i} H]\, d_{R,j} + \text{Leptons} \Big) + \text{h.c.}$$

- 3×3 complex-valued **Yukawa couplings** $Y_{U,D}$ ⇒ not generation-diagonal !!!
- invariant under global $G_{SM} = U(1)_Y \otimes U(1)_B \otimes U(1)_L$, but not under G_{flavor} of $\mathcal{L}_{\text{gauge}}$

  ⇒ accidental global symmetries of SM (at dim-4 only): B = baryon number, L = lepton number

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Quark & Lepton masses when breaking the $\text{SU}(2)_L \times \text{U}(1)_Y \to \text{U}(1)_{\text{em}}$

Higgs-field acquires **vacuum expectation value** v (VEV)

(in R_ξ-gauge)

$$H = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} + \begin{pmatrix} G^+ \\ (h^0 + iG^0)/\sqrt{2} \end{pmatrix}$$

⇒ $[\overline{Q}_L \cdot (0, v)] = v\, \bar{d}_L$ and $[\overline{Q}_L\, i\sigma^2\, (0, v)] = v\, \bar{u}_L$

$$\mathcal{L}_{\text{Yukawa}} \simeq -\sum_{i,j=1}^{3} \left(\frac{v\, Y_{U,ij}}{\sqrt{2}} [\bar{u}_{L,i}\, u_{R,j}] + \frac{v\, Y_{D,ij}}{\sqrt{2}} [\bar{d}_{L,i}\, d_{R,j}] + \ldots \right) + \text{h.c.} + \text{terms}(h^0, G^{0,\pm})$$

⇒ Quark masses are "generation-non-diagonal":

!!! distinguish generations → **Flavor**

$$[M_U]_{ij} \equiv \frac{v\, Y_{U,ij}}{\sqrt{2}} \quad \text{and} \quad [M_D]_{ij} \equiv \frac{v\, Y_{D,ij}}{\sqrt{2}}$$

From weak → mass eigenstates

After EWSB mass terms of quarks are “generation-non-diagonal”

$$\mathcal{L}_{\text{Yukawa}} \simeq -\sum_{i,j=1}^{3} \left([M_U]_{ij}\, \overline{u}_{L,i}\, u_{R,j} + [M_D]_{ij}\, \overline{d}_{L,i}\, d_{R,j} \right) + \text{h.c.} + \ldots$$

Requires separate rotations for u_L and d_L to **mass eigenstates u', d'**

$$u'_L = V_L^u\, u_L, \qquad d'_L = V_L^d\, d_L, \qquad u'_R = V_R^u\, u_R, \qquad d'_R = V_R^d\, d_R,$$

such that **mass matrices** are diagonal **but** each generation has different mass ⇒ flavor

$$M_a^{\text{diag}} = V_L^a\, M_a\, V_R^{a\dagger} = \frac{v}{\sqrt{2}} V_L^a\, Y_a\, V_R^{a\dagger} \qquad a = U, D$$

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Remember that gauge interactions $\mathcal{L}_{\text{gauge}}$ are invariant under $Q'_L = V_L^Q\, Q_L$,
but not under separate trafo of u_L and d_L

$$\mathcal{L}_{\text{gauge}} = \sum_{i=1}^{3} \overline{Q}_{L,i}\, i\not{D}\, Q_{L,i} + \ldots = \sum_{i,j,k=1}^{3} \begin{pmatrix} \overline{u'}_{L,i}\, [V_L^u]_{ik} \\ \overline{d'}_{L,i}\, [V_L^d]_{ik} \end{pmatrix}^T i\not{D} \begin{pmatrix} [V_L^{u\dagger}]_{L,kj}\, u'_{L,j} \\ [V_L^{d\dagger}]_{L,kj}\, d'_{L,j} \end{pmatrix} + \ldots$$

⇒ expanding $SU(2)_L$ indices:

charged flavor-non-diagonal gauge interactions

$$\propto \overline{u}_{L,i} \left[V_L^u\, V_L^{d\dagger} \right]_{ij} d_{L,j} \;\to\; \overline{u}_L\, V_{\text{CKM}}\, d_L$$

Cabibbo-Kobayashi-Maskawa (CKM)

Flavor changes in SM → CKM matrix

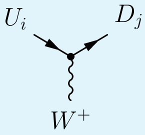
$U_i = \{u, c, t\}$:

$Q_u = +2/3$

$D_j = \{d, s, b\}$:

$Q_d = -1/3$

$$\mathcal{L}_{udW^\pm} \simeq \frac{g_2}{\sqrt{2}} \begin{pmatrix} \bar{u} & \bar{c} & \bar{t} \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \gamma^\mu P_L \begin{pmatrix} d \\ s \\ b \end{pmatrix} W^+_\mu$$



~ Cabibbo-Kobayashi-Maskawa (CKM) matrix

- determined by Yukawa-couplings $V_{\text{CKM}} \equiv V^u_L \, V^{d\,\dagger}_L$
- **CP violation** realized via complex phase in V_{CKM}

  [Kobayashi/Maskawa Prog.Theor.Phys. 49 (1973) 652]
- **unitary** matrix: $V_{\text{CKM}} V^\dagger_{\text{CKM}} = \mathbb{1}_{3\times3}$ → in principle $18 - 9 = 9$ real parameters
- phase transformations of five quark fields allow to remove unphysical dof's (degrees of freedom)

  ⇒ **only 4 real parameters**

⇒ All information on quark Yukawa couplings $\in \mathbb{C}$ is given by $6 + 4 = 10$ real parameters: they are the **6 quark masses** and **4 CKM parameters**

Testing the SM search for all flavor-changing processes predicted *and not* predicted by the SM *and* to (over-) determine CKM parameters

The CKM matrix

- Cabibbo-Kobayashi-Maskawa matrix:

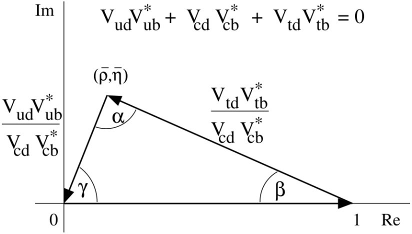
$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- unitarity $V^\dagger_{\mathrm{CKM}} V_{\mathrm{CKM}} = \mathbb{1}_{3\times 3}$

  of i-th and j-th rows/columns gives

  6 Unitarity triangles (UT)

  $\Rightarrow$ most common $i = 1$, $j = 3$:



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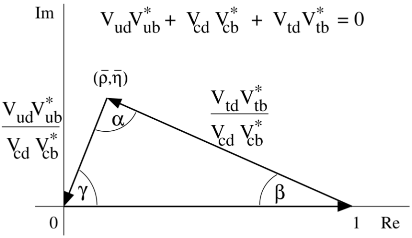


- there are many parametrizations of unitary 3×3 matrix with 4 param's

  $\Rightarrow$ convention dependence

- some things are convention independent

  (invariant under quark-field rephasing)

  Plaquettes

$$J_{ij;kl} \equiv \pm \,\mathrm{Im}[V_{ik} V_{jl} V^*_{il} V^*_{jk}]$$

  with $i \neq j$ and $k \neq l$

  $\Rightarrow$ for 3×3 all the $J_{ij;kl}$ are equivalent

  $\Rightarrow$ a measure of CP violation

  [Jarlskog PRL 55 (1985) 1039]

- **Jarlskog invariant** $J \equiv J_{ij;kl}$

  is twice the area of unitarity triangles:

  "$J = 2 \times \triangle_{\mathrm{UT}}$"

  $\Rightarrow$ measured $|J| \approx 2.8 \times 10^{-5}$

Parametrizations of the CKM matrix

Standard parametrization from PDG (Particle Data Group)

$$V_{\rm CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$\Rightarrow$ uses 3 angles + 1 phase: $s_{ij} \equiv \sin\theta_{ij}$ $\qquad (c_{ij})^2 = 1 - (s_{ij})^2$

Wolfenstein parametrization expansion in $\lambda \approx V_{us} \sim 0.2$ [Wolfenstein Phys.Rev.Lett. 51 (1983) 1945]

$$V_{\rm CKM} \approx \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & \lambda^3 A(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & \lambda^2 A \\ \lambda^3 A(1-\rho-i\eta) & -\lambda^2 A & 1 \end{pmatrix} + \mathcal{O}\left(\lambda^4\right)$$

$\Rightarrow$ uses Wolfenstein parameters λ, A, ρ and η:

$$s_{12} = \lambda = \frac{|V_{us}|}{\sqrt{|V_{ud}|^2 + |V_{us}|^2}}, \qquad s_{23} = A\lambda^2 = \lambda\left|\frac{V_{cb}}{V_{us}}\right|, \qquad s_{13}e^{i\delta} = V_{ub}^* = A\lambda^3(\rho + i\eta) = \frac{A\lambda^3(\overline{\rho} + i\overline{\eta})\sqrt{1-A^2\lambda^4}}{\sqrt{1-\lambda^2}[1 - A^2\lambda^2(\overline{\rho} + i\overline{\eta})]}$$

$\Rightarrow$ ensures $\overline{\rho} + i\overline{\eta} = -(V_{ud}V_{ub}^*)/(V_{cd}V_{cb}^*)$ independent of phase convention

$\Rightarrow$ CKM in terms of λ, A, $\overline{\rho}$ and $\overline{\eta}$ unitary to all orders in λ: $\quad \overline{\rho} = \rho\,(1 - \lambda^2/2 + \ldots), \quad \overline{\eta} = \eta\,(1 - \lambda^2/2 + \ldots)$

Now we know what “Flavor” means in the SM

$\downarrow$

What flavor transitions does the SM predict?

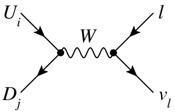
Tree (CC) versus Loops (FCNC)

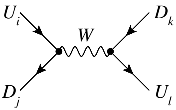
charged current (CC) $\quad Q_i \neq Q_j$

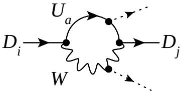
Tree: only $U_i \to D_j$ & $D_i \to U_j$

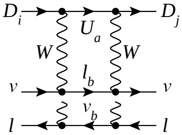
neutral current (FCNC) $\quad Q_i = Q_j$

Loop: $D_i \to D_j$ (& $U_i \to U_j$)









$M_1 \to \ell\bar{\nu}_\ell$

$M_1 \to M_2 + \ell\bar{\nu}_\ell$

$M_1 \to M_2 M_3$

$M_1 \to M_2 + \{\gamma, Z, g\}$

$\{\gamma, Z, g\} \to \{\ell\bar{\ell}, \nu\bar{\nu}, M_3\}$

$M_1 \to \ell\bar{\ell}$

$M_1 \to M_2 + \{\ell\bar{\ell}, \nu\bar{\nu}, M_3\}$

$M^0 \leftrightarrow \overline{M}^0$ (= mixing)

Amp $\sim \mathcal{G}_F V_{ij}$

$\sim \mathcal{G}_F V_{ij} V_{lk}^*$

$\sim \mathcal{G}_F g \sum_a V_{ai} V_{aj}^* f(m_a)$

$\sim \mathcal{G}_F g^2 \sum_{a,b} V_{ai} V_{aj}^* f(m_{a,b})$

Tree (CC) versus Loops (FCNC)

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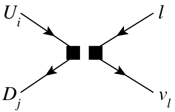

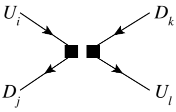

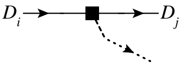

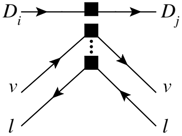

$M_1 \to \ell\bar{\nu}_\ell$

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$M_1 \to M_2 + \{\ell\bar{\ell}, \nu\bar{\nu}, M_3\}$

$M^0 \leftrightarrow \overline{M}^0$ (= mixing)

Amp $\sim \mathcal{G}_F\, C(V_{ij})$ $\qquad \sim \mathcal{G}_F\, C(V_{ij})$ $\qquad \sim \mathcal{G}_F\, C(V_{ij}, m_a)$ $\qquad \sim \mathcal{G}_F\, C(V_{ij}, m_a, m_b)$

- **decoupling for $m_Q \ll m_W$ $\Rightarrow$ effective theory à la Fermi** [Fermi 1933/34]

  works for all quarks except top quark ($m_W < m_t$)

- short-distance (SD) couplings: ***C* = Wilson coefficients**

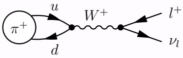
  depend on SD-parameters $\Rightarrow$ in SM: CKM and heavy masses: m_W, m_Z, m_t

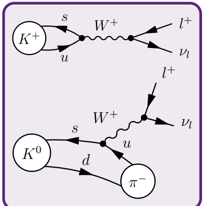
  $\Rightarrow$ extract in measurement and calculate in specific UV completions

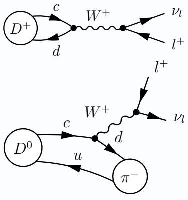
- overall rescaling factor **Fermi's constant $\mathcal{G}_F \sim$ GeV^{-2}**, measured in $\mu \to e\bar{\nu}_e\nu_\mu$

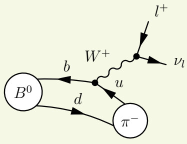
Overview of decay channels for CKM determination

Nuclear β-decay

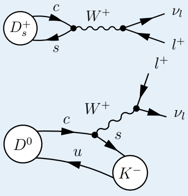


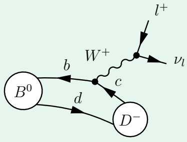


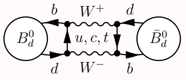


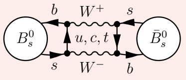


$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$









Also many strategies with hadronic B decays $B \to M_1 M_2$

[Figures from Lellouch 1104.5484]

So far "CKM-picture" of SM works

⇒ fit of CKM-Parameters . . .

CKM matrix in terms of

4 Wolfenstein parameters

$$\lambda \sim 0.22, \quad A, \quad \bar{\rho}, \quad \bar{\eta}$$

⇒ nowadays a sophisticated fit:

"combine and overconstrain"

!!! numerous *b*-physics measurements

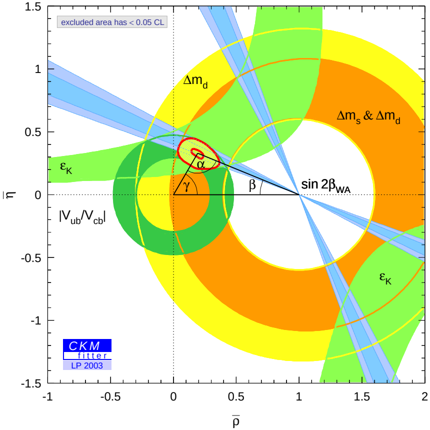
[experimental input from CKMfitter homepage]

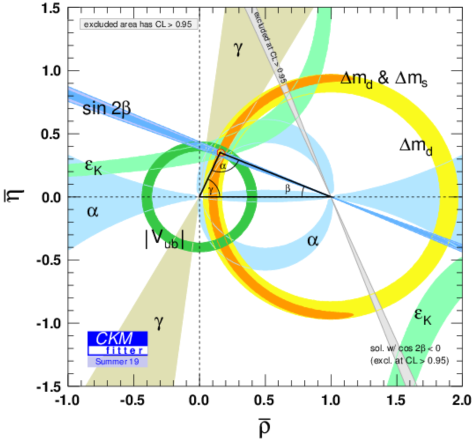
$\vert V_{ud}\vert$ (nuclei)	$0.97425 \pm 0 \pm 0.00022$
$\vert V_{us}\vert f_+^{K\to\pi}(0)$	0.2163 ± 0.0005
$\vert V_{cd}\vert$ (νN)	0.230 ± 0.011
$\vert V_{cs}\vert$ ($W \to c\bar{s}$)	$0.94^{+0.32}_{-0.26} \pm 0.13$
$\vert V_{ub}\vert$ (semileptonic)	$(4.01 \pm 0.08 \pm 0.22) \times 10^{-3}$
$\vert V_{cb}\vert$ (semileptonic)	$(41.00 \pm 0.33 \pm 0.74) \times 10^{-3}$
$\mathcal{B}(\Lambda_p \to p\mu^-\bar{\nu}_\mu)_{q^2>15}/\mathcal{B}(\Lambda_p \to \Lambda_c\mu^-\bar{\nu}_\mu)_{q^2>7}$	$(1.00 \pm 0.09) \times 10^{-2}$
$\mathcal{B}(B^- \to \tau^-\bar{\nu}_\tau)$	$(1.08 \pm 0.21) \times 10^{-4}$
$\mathcal{B}(D_s^- \to \mu^-\bar{\nu}_\mu)$	$(5.57 \pm 0.24) \times 10^{-3}$
$\mathcal{B}(D_s^- \to \tau^-\bar{\nu}_\tau)$	$(5.55 \pm 0.24) \times 10^{-2}$
$\mathcal{B}(D^- \to \mu^-\bar{\nu}_\mu)$	$(3.74 \pm 0.17) \times 10^{-4}$
$\mathcal{B}(K^- \to e^-\bar{\nu}_e)$	$(1.581 \pm 0.008) \times 10^{-5}$
$\mathcal{B}(K^- \to \mu^-\bar{\nu}_\mu)$	0.6355 ± 0.0011
$\mathcal{B}(\tau^- \to K^-\bar{\nu}_\tau)$	$(0.6955 \pm 0.0096) \times 10^{-2}$
$\mathcal{B}(K^- \to \mu^-\bar{\nu}_\mu)/\mathcal{B}(\pi^- \to \mu^-\bar{\nu}_\mu)$	1.3365 ± 0.0032
$\mathcal{B}(\tau^- \to K^-\bar{\nu}_\tau)/\mathcal{B}(\tau^- \to \pi^-\bar{\nu}_\tau)$	$(6.431 \pm 0.094) \times 10^{-2}$
$\mathcal{B}(B_s \to \mu\mu)$	$(2.8^{+0.7}_{-0.6}) \times 10^{-9}$
$\vert V_{cd}\vert f_+^{D\to\pi}(0)$	0.148 ± 0.004
$\vert V_{cs}\vert f_+^{D\to K}(0)$	0.712 ± 0.007
$\vert\varepsilon_K\vert$	$(2.228 \pm 0.011) \times 10^{-3}$
Δm_d	(0.510 ± 0.003) ps^{-1}
Δm_s	(17.757 ± 0.021) ps^{-1}
$\sin(2\beta)_{[c\bar{c}]}$	0.691 ± 0.017
$(\phi_s)_{[b\to c\bar{s}s]}$	-0.015 ± 0.035
$S_{\pi\pi}^{+-}$, $C_{\pi\pi}^{+-}$, $C_{\pi\pi}^{00}$, $\mathcal{B}_{\pi\pi}$ all charges	Inputs to isospin analysis
$S_{\rho\rho,L}^{+-}$, $C_{\rho\rho,L}^{+-}$, $S_{\rho\rho}^{00}$, $C_{\rho\rho}^{00}$, $\mathcal{B}_{\rho\rho,L}$ all charges	Inputs to isospin analysis
$B^0 \to (\rho\pi)^0 \to 3\pi$	Time-dependent Dalitz analysis
$B^- \to D^{(*)}K^{(*)-}$	Inputs to GLW analysis
$B^- \to D^{(*)}K^{(*)-}$	Inputs to ADS analysis
$B^- \to D^{(*)}K^{(*)-}$	GGSZ Dalitz analysis

So far “CKM-picture” of SM works

⇒ **fit of CKM-Parameters . . . 2003 → 2019**

Unitarity: $V_{ub}V_{ud}^* + V_{cb}V_{cd}^* + V_{tb}V_{td}^* = 0$





More on CKM fits **http://ckmfitter.in2p3.fr/www/html/ckm_main.html**
http://www.utfit.org/UTfit/

Hierarchies in masses and CKM

The determinations in framework of SM show huge hierarchies that can not be explained in the SM

- masses within each generation
- CKM matrix

$\lambda \approx 0.225$

Cabibbo angle

$$V_{\mathrm{CKM}} \approx \begin{pmatrix} 1 & \lambda & \lambda^3 A \\ -\lambda & 1 & \lambda^2 A \\ \lambda^3 A & -\lambda^2 A & 1 \end{pmatrix}$$

- in down-type FCNCs *top*-, *charm*- and *up*-contributions

$b \to s$

$$V_{tb}V_{ts}^* \approx -V_{cb}V_{cs}^* \sim \lambda^2 A$$

$$V_{ub}V_{us}^* \sim \lambda^4 A$$

$b \to d$

$$V_{tb}V_{td}^* \sim V_{cb}V_{cd}^* \sim V_{ub}V_{ud}^* \sim \lambda^3 A$$

$s \to d$

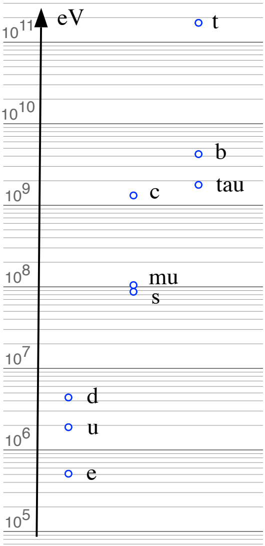
$$V_{cs}V_{cd}^* \approx -V_{us}V_{ud}^* \sim \lambda$$

$$V_{ts}V_{td}^* \sim \lambda^5 A$$

$\Rightarrow$ in $s \to d$ *top* part enhanced by m_t^2, but CKM-suppressed

$\lambda^4 A \approx 0.0021$ versus $(m_c/m_W)^2 \approx 0.0003$

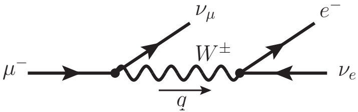
$\Rightarrow$ CKM suppresses dim-6, such that dim-8 phenomenologically not negligible in ΔM_K, ε_K, $K^+ \to \pi + \nu\bar{\nu}$



Effective theories:
Example muon decay

Fermi theory for $\mu \to e \overline{\nu}_e \nu_\mu$

In SM $\mu^- \to e^- \overline{\nu}_e \nu_\mu$ at tree-level
via $W^\pm$-boson exchange



$$q \equiv p_\mu - p_{\nu_\mu} = p_e + p_{\nu_e}$$

$$i\mathcal{A}_{\rm SM} = i\left(-i\frac{g_2}{\sqrt{2}}\right)^2 \left[\overline{u}(p_{\nu_\mu})\gamma_\mu P_L u(p_\mu)\right] \frac{-i\,g^{\mu\nu}}{q^2 - m_W^2} \left[\overline{u}(p_e)\gamma_\nu P_L v(p_{\nu_e})\right]$$

$$\approx \frac{g_2^2}{2m_W^2}[\overline{\nu}_\mu\gamma_\mu P_L\,\mu][\overline{e}\gamma^\mu P_L\nu_e] + \mathcal{O}\left(m_\mu^2 \big/ m_W^2\right) \qquad P_{L(R)} \equiv \frac{1}{2}(1 \mp \gamma_5)$$

!!! Expansion in the μ-rest frame $q^2 \ll m_W^2$ ($m_\mu \approx 0.1$ GeV and $m_W \approx 80$ GeV)

$\Rightarrow$ this corresponds to an **OPE (operator product expansion)**, keeping only dim-6

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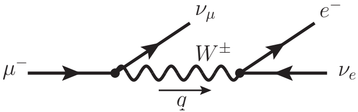
In SM $\mu^- \to e^- \bar{\nu}_e \nu_\mu$ at tree-level via $W^\pm$-boson exchange



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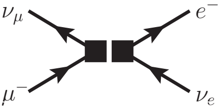
Can reproduce with an **Effective Theory** (as Fermi anticipated)

$$\mathcal{L}_{\text{EFT}} = -\frac{4}{\sqrt{2}} C_{VLL}\, Q_{VLL}$$

$$Q_{VLL} \equiv [\bar{\nu}_\mu \gamma_\mu P_L \mu][\bar{e}\gamma^\mu P_L \nu_e]$$

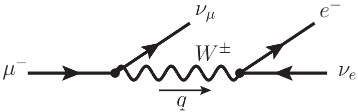


- C_{VLL} = **Wilson coefficient** $\Rightarrow$ effective coupling constant
- Q_{VLL} = 4-Fermi **Operator** (contact interaction)

$$i\mathcal{A}_{\text{EFT}} = i\left(-i\frac{4}{\sqrt{2}} C_{VLL}\right)[\bar{\nu}_\mu \gamma_\mu P_L \mu][\bar{e}\gamma^\mu P_L \nu_e] = \frac{4C_{VLL}}{\sqrt{2}} Q_{VLL}$$

Fermi theory for $\mu \to e \bar{\nu}_e \nu_\mu$

In SM $\mu^- \to e^- \bar{\nu}_e \nu_\mu$ at tree-level via $W^\pm$-boson exchange



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$\Rightarrow$ this corresponds to an **OPE (operator product expansion)**, keeping only dim-6

There is a **full theory** (the SM) and an **effective theory** that reproduces it for $q^2 \ll m_W^2$

Determine C_{VLL} from **Matching** both amplitudes (due to renormalization beyond tree-level at scale $\mu_W \sim m_W$)

$$\mathcal{A}_{SM} \stackrel{!}{=} \mathcal{A}_{EFT} \qquad \Rightarrow \qquad C_{VLL}^{SM} = \frac{\sqrt{2}\,g_2^2}{8\,m_W^2} = \frac{1}{\sqrt{2}\,v^2}$$

!!! $C_{VLL} \sim \text{GeV}^{-2}$ carries information on full theory

Fermi's constant $\mathcal{G}_F$ from μ-lifetime

Can determine C_{VLL} from precise measurement of $\tau_\mu = (2.196\,981\,1 \pm 0.000\,002\,2)\mu\mathrm{s}$

Calculate **μ-lifetime** from $\mathcal{A}_{\mathrm{EFT}}$, neglecting QED corrections from photons

$$\frac{1}{\tau_\mu} \equiv \Gamma_\mu = \frac{1}{2m_\mu}\sum d\Pi_3 \left|\mathcal{A}_{\mathrm{EFT}}\,\mathcal{A}^\dagger_{\mathrm{EFT}}\right|^2$$

$$= \frac{m_\mu^5}{192\pi^3}|C_{VLL}|^2\left[1+\Delta q^{(0)}(x)\right], \qquad x = \frac{m_e^2}{m_\mu^2} \sim 2\cdot 10^{-5}$$

- $\Delta q^{(0)}(x)$ tiny phase-space corrections from e^- mass (m_{ν_e} and m_{ν_μ} neglected)

Fermi's constant $\mathcal{G}_F$ from μ-lifetime

Can determine C_{VLL} from precise measurement of $\tau_\mu = (2.1969811 \pm 0.0000022)\mu s$

Calculate **μ-lifetime** from $\mathcal{A}_{\text{EFT}}$, with QED corrections

$$\frac{1}{\tau_\mu} \equiv \Gamma_\mu = \frac{1}{2m_\mu}\sum d\Pi_3 \left|\mathcal{A}_{\text{EFT}}\,\mathcal{A}^\dagger_{\text{EFT}}\right|^2 + \frac{1}{m_\mu}\sum d\Pi_4 \ldots \text{real emission} + \ldots$$

$$= \frac{m_\mu^5}{192\pi^3}\Big[1 + \Delta q(\alpha_e, x)\Big]\,|C_{VLL}|^2$$

with $\Delta q(\alpha_e, x) = \sum_{n=0}^{\infty}\left(\frac{\alpha_e}{\pi}\right)^n \Delta q^{(n)}(x)$, which depends on α_e and x $y \approx 0$

- $\Delta q^{(1)}(x) = -1.8076$ [Kinoshita/Sirlin Phys. Rev. 113 (1959) 1652, Nir, PLB221 (1989) 184]
- $\Delta q^{(2)}(x) = (6.700 \pm 0.002)$ [Ritbergen/Stuart hep-ph/9904240]

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The EFT allows to conveniently separate QED dynamics from C_{VLL} into Δq

!!! QED renormalization of Δq requires to choose scale $\mu \sim m_\mu$ to avoid large log's $\ln \mu/m_\mu$

$\Rightarrow$ Formally $C_{VLL}(\mu)$ at low-energy scale, but trivial evolution to scale $\mu_W \sim m_W$

$\Rightarrow$ $\mathcal{G}_F \equiv C_{VLL}$ is also called **Fermi's constant**, and it is best defined by

one finds from τ_μ that

$$\mathcal{G}_F = |C_{VLL}| = 1.1663787(6) \cdot 10^{-5}\,\text{GeV}^{-2}$$

Fermi's constant in the SM

Determination of C_{VLL} can be used to determine short-distance parameters of SM:

Tree-level matching of the SM: $$C_{VLL}^{SM} = \frac{\sqrt{2}\, g_2^2}{8\, m_W^2} = \frac{1}{\sqrt{2} v^2} \quad \Rightarrow \quad \boxed{v = 246.2\,\text{GeV}}$$

Beyond tree-level matching: $$C_{VLL}^{SM} = \frac{\sqrt{2}\, g_2^2}{8\, m_W^2}\left[1 + \boxed{\Delta r(\alpha_e, m_W, m_Z, m_t, m_H)}\right]$$

- radiative corrections to tree-level $W^\pm$ exchange in $\Delta r(\alpha_e, m_W, m_Z, m_t, m_H)$
- μ-lifetime important measurement to fix SM parameters like m_W, m_Z, m_H

  in electroweak-precision fits of SM
- if **New Physics (NP)** only contributes to $C_{VLL} = C_{VLL}^{SM} + C_{VLL}^{NP}$

  $\Rightarrow$ constraints from muon-liftime apply to sum $\mathcal{G}_F = \left|C_{VLL}^{SM} + C_{VLL}^{NP}\right|$

  $\Rightarrow$ C_{VLL}^{NP} depends on fundamental parameters of NP scenario

Fermi's constant beyond the SM

Let's assume only left-handed ν's $\Rightarrow$ then only one additonal $\Delta L = 0$ operator

$$\mathcal{L}_{\text{EFT}} = -\frac{4}{\sqrt{2}}\left[\left(C_{VLL}^{\text{SM}} + C_{VLL}^{\text{NP}}\right) Q_{VLL} + C_{SRL}\, Q_{SRL}\right] \qquad Q_{SRL} \equiv [\overline{\nu}_\mu P_R\, \mu][\overline{e}\, P_L \nu_e]$$

leads to modification of μ-lifetime

$$\frac{1}{\tau_\mu} = \frac{m_\mu^5}{192\pi^3}\left[1 + \Delta q^{(0)}(x)\right] \underbrace{\left(|C_{VLL}|^2 + \frac{|C_{SRL}|^2}{4} + \frac{18}{5} \overbrace{\frac{m_e}{m_\mu}}^{\approx 1/200} \text{Re}\left(C_{VLL} C_{SRL}^*\right) \times \left[1 + \mathcal{O}(x)\right]\right)}_{\equiv \left(\mathcal{G}_F^{(0)}\right)^2}$$

- $\mathcal{G}_F^{(0)}$ denotes that only $\Delta q^{(0)}(x)$ is used when additional Q_{SRL} included

  $\Rightarrow$ theory less precisely known compared to only Q_{VLL}
- one observable not enough to fix two complex-valued numbers

  $\Rightarrow$ measure other observables in $d^2\Gamma/(dE_e\, d\cos\vartheta)$ $\rightarrow$ Michel parameters
- in SMEFT ($v \ll \Lambda$): $C_{VLL}^{\text{SM}} \sim 1/v^2$ and additional suppression of v^2/Λ^2 for C_{VLL}^{NP} and C_{SRL}

  $\Rightarrow$ in τ_μ the $|C_{SRL}|^2 \sim v^4/\Lambda^4$ compared to v^2/Λ^2 $\rightarrow$ negligible

  $\Rightarrow$ one might neglect $\text{Re}\left(C_{VLL} C_{SRL}^*\right) \sim v^2/\Lambda^2$, because helicity-suppressed

[for example Jenkins/Manohar/Stoffer 1709.04486]

Michel parameters

More observables to discriminate SM and NP effects $\Rightarrow$ measure angular distribution

$$\frac{d^2\Gamma}{dx\, d\cos\vartheta} \propto x^2 \left\{3(1-x) + \frac{2\rho}{3}(4x-3) + 3\eta\,\frac{x_0}{x}(1-x) \pm P_\mu\,\xi\,\cos\vartheta\left[1-x+\frac{2\delta}{3}(4x-3)\right]\right\}$$

- in restframe of muon & electron polarisation insensitive detector
- maximum electron energy $E_e^{\max} = (m_\mu^2 + m_e^2)/(2m_\mu)$
- reduced electron energy $x = E_e/E_e^{\max}$ and $x_0 = m_e/E_e^{\max}$
- ϑ is direction of electron w.r.t. muon polarization $\vec{P}_\mu$
- degree of muon polarisation $P_\mu = |\vec{P}_\mu|$

Angular observables ρ, η, ξ, δ known as **Michel parameters**

[Michel ProcPhysSocA63 (1950) 514, Bouchiat/Michel PR106 (1957) 170, Kinoshita/Sirlin (1957) PR107 593 & PR108 844]

in SM: $\rho = \xi\delta = 3/4, \quad \xi = 1, \quad \eta = 0$

$\Rightarrow$ measurements with electron polarisation depend on further Michel parameters

Michel parameters

More observables to discriminate SM and NP effects $\Rightarrow$ measure angular distribution

$$\frac{d^2\Gamma}{dx\, d\cos\vartheta} \propto x^2 \left\{3(1-x) + \frac{2\rho}{3}(4x-3) + 3\eta\,\frac{x_0}{x}(1-x) \pm P_\mu\,\xi\,\cos\vartheta\left[1-x+\frac{2\delta}{3}(4x-3)\right]\right\}$$

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in SM: $\rho = \xi\delta = 3/4, \quad \xi = 1, \quad \eta = 0$

$\Rightarrow$ measurements with electron polarisation depend on further Michel parameters

!!! SM particularly simple $\Rightarrow$ few parameters and correlations between many observables

- few parameters $\Rightarrow$ theory control needed only for few observables for good determinations
- correlations $\Rightarrow$ allow stringent tests of SM
- more parameters/operators in new physics scenarios lead to less predictivity
  $\Rightarrow$ less stringent tests possible and more measurements needed

Effective theory

for $\Delta B = 1$ decays

B-Hadron decays are a Multi-scale problem . . .

. . . with hierarchical interaction scales

electroweak IA	$\gg$	ext. mom'a in B restframe	$\gg$	QCD-bound state effects
$m_W \approx 80$ GeV $m_Z \approx 91$ GeV		$m_B \approx 5$ GeV		$\Lambda_{\text{QCD}} \approx 0.5$ GeV

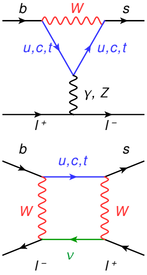
B-Hadron decays are a Multi-scale problem ...

... with hierarchical interaction scales

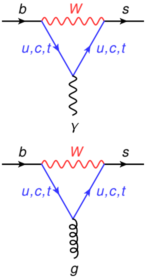
electroweak IA	$\gg$	ext. mom'a in B restframe	$\Rightarrow$	decoupling heavy particles
$m_W \approx 80$ GeV $m_Z \approx 91$ GeV		$m_B \approx 5$ GeV		W, Z-boson, top-quark

$$\mathcal{L}_{\text{EFT}} \sim \mathcal{G}_F\, V_{\text{CKM}} \times \left[\sum_{9,10} C_i^{\ell\bar\ell}\, Q_i^{\ell\bar\ell} + \sum_{7\gamma,8g} C_i\, Q_i + \text{CC} + (\text{QCD \& QED-peng}) \right]$$

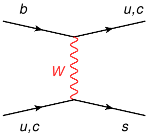
semi-leptonic



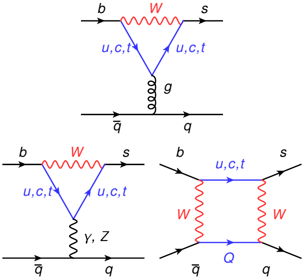
el- & chr-mgn dipole



charged current



QCD & QED -penguin



B-Hadron decays are a Multi-scale problem . . .

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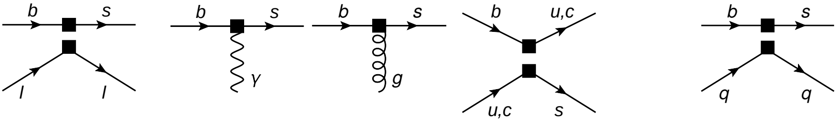
$m_W \approx 80$ GeV $m_Z \approx 91$ GeV		$m_B \approx 5$ GeV		at scales below m_B

$$\mathcal{L}_{\text{EFT}} \sim \mathcal{G}_F\, V_{\text{CKM}} \times \left[\sum_{9,10} C_i^{\ell\bar{\ell}}\, Q_i^{\ell\bar{\ell}} + \sum_{7\gamma,8g} C_i\, Q_i + \text{CC} + (\text{QCD \& QED-peng}) \right]$$

semi-leptonic | el- & chr-mgn dipole | charged current | QCD & QED -penguin



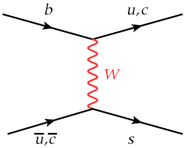
C_i = **Wilson coefficients** contain short-dist. pmr's (heavy masses $m_t, \ldots$ – CKM factored out) and leading logarithmic QCD-corrections to all orders in α_s

$\Rightarrow$ in SM known up to NNLO QCD and NLO EW/QED

Q_i = **dim-6 operators** flavor-changing coupling of light quarks

Tree-level = "current-current" op's in the SM

SM = **Full theory**: in b-rest frame external momenta $q^2 \sim m_b^2 \ll m_W^2 \Rightarrow$ expand W-propagator



$$i\mathcal{A}_{\text{SM}} = -\frac{g_2^2}{2} V_{cb} V_{cs}^* \frac{1}{q^2 - m_W^2} [\bar{s}\gamma_\mu P_L c][\bar{c}\gamma^\mu P_L b]$$

$$\overset{q^2 \ll m_W^2}{\approx} \frac{4\mathcal{G}_F}{\sqrt{2}} V_{cb} V_{cs}^* [\bar{s}\gamma_\mu P_L c][\bar{c}\gamma^\mu P_L b] + \mathcal{O}\left(\frac{m_b^2}{m_W^2}\right)$$

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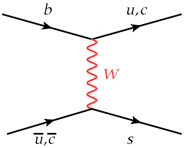

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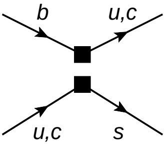
The same result can be obtained from an **EFT** Lagrangian



$$\mathcal{L}_{\rm EFT} = c_2 Q_2 = \frac{4\mathcal{G}_F}{\sqrt{2}} V_{cb} V_{cs}^* C_2 Q_2$$

$$Q_2 \equiv [\bar{s}\gamma_\mu P_L c][\bar{c}\gamma^\mu P_L b]$$

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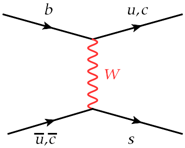

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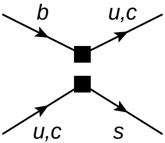


$$\mathcal{L}_{EFT} = c_2 Q_2 = \frac{4\mathcal{G}_F}{\sqrt{2}} V_{cb} V_{cs}^* C_2 Q_2 \qquad Q_2 \equiv [\bar{s}\gamma_\mu P_L c][\bar{c}\gamma^\mu P_L b]$$

$$i\mathcal{A}_{EFT} = -c_2 [\bar{s}\gamma_\mu P_L c][\bar{c}\gamma^\mu P_L b]$$

Requiring equality of amplitudes (Greens funct’s) = **Matching**

$$\mathcal{A}_{SM} \overset{!}{=} \mathcal{A}_{EFT} \quad \Rightarrow \quad c_2 = -\frac{4\mathcal{G}_F}{\sqrt{2}} V_{cb} V_{cs}^* \quad (\text{or } C_2 = -1)$$

$$V_{cb} V_{cs}^* \approx -V_{tb} V_{ts}^* + \ldots \quad \Rightarrow \quad c_2 = +\frac{4\mathcal{G}_F}{\sqrt{2}} V_{tb} V_{ts}^* \quad (\text{or } C_2 = +1)$$

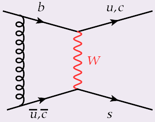
used here $V_{ub} V_{us}^* \ll V_{tb} V_{ts}^*$ and $V_{ub} V_{us}^* \ll V_{cb} V_{cs}^*$

Matching at higher orders

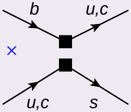
Benefit of EFT's $\Rightarrow$ can resum large log's to all orders in perturbation theory (PT)

$$\alpha_s^n \ln^n\left(\frac{m_b}{m_W}\right) = \alpha_s^n \left[\ln\left(\frac{m_b}{\mu_0}\right) + \ln\left(\frac{\mu_0}{m_W}\right)\right]^n, \qquad \ln\left(\frac{m_b}{m_W}\right) \approx -2.8$$

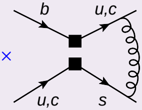
Matching $C_i(\mu_0, m_W) = C_i^{(0)} + \frac{\alpha_s}{4\pi} C_i^{(1)} + \ldots$ order by order **μ_0 = factorisation scale**



+ Counter Term (=CT) $\stackrel{!}{=}$ $\frac{\alpha_s}{4\pi} C^{(1)} \times$



$+ \; C^{(0)} \times$



+ CT

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- allows to separate log's of full theory side into Wilson coefficients $C^{(1)}$ and ...
- 1-loop matrix element $\propto C^{(0)}$ of EFT has same $\ln(m_b/\mu_0)$ since EFT should reproduce IR of full theory (otherwise wrong EFT)
- $C^{(1)}(\mu_0)$ can be determined perturbatively only with choice: $\mu_0 \sim m_W$ otherwise large log's will enter $C^{(1)}(\mu_0)$

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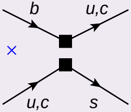
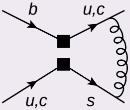
$$\frac{\alpha_s}{4\pi}\left(\ldots \ln\frac{\mu_0}{m_W} + \ldots \ln\frac{m_b}{\mu_0} + \text{remainder}\right) + \text{CT} \stackrel{!}{=} \frac{\alpha_s}{4\pi} C^{(1)} \times$$



$$+\ C^{(0)} \times$$



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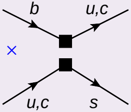

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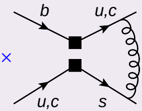

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Matching determines Wilson coefficients at high scale $\mu_0 \sim m_W$

Renormalization Group (RG) equation

- main purpose of RG eq.: relating couplings (Wilson coefficients) at different scales
- effect of RG eq.: resummation of large log's to all orders in coupling (α_s or α_e)

RG equation derived from requirement that "bare" (effective) couplings are μ-independent

$$\mu \frac{d}{d\mu} C_i(\mu) = \left[\gamma^T(\mu)\right]_{ij} C_j(\mu) \qquad \gamma_{ij} = \textbf{anomalous dimension matrix (=ADM)}$$

Formal solution of system of coupled 1st order ordinary differential equations (ODE)

$$C_i(\mu) = [U(\mu, \mu_0)]_{ij} \, C_j(\mu_0), \qquad [U(\mu, \mu_0)]_{ij} = T_{\mu'} \exp\left[\int_{\mu_0}^{\mu} \gamma^T(\mu') d\mu'\right]$$

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In case of two operators Q_1 and Q_2, leading order RG equation

$$\mu \sim m_b, \quad \mu_0 \sim m_W, \quad \eta \equiv \alpha_s(\mu_0)/\alpha_s(\mu) \approx 0.55, \quad \eta_\pm \equiv (\eta^{6/23} \pm \eta^{-12/23})/2$$

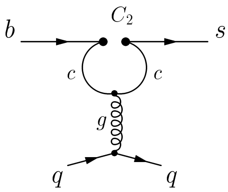
$$\begin{aligned} C_1(\mu) &= \eta_+ C_1(\mu_0) + \eta_- C_2(\mu_0) &\approx& +1.11\,C_1(\mu_0) - 0.26\,C_2(\mu_0) \\ C_2(\mu) &= \eta_- C_1(\mu_0) + \eta_+ C_2(\mu_0) &\approx& -0.26\,C_1(\mu_0) + 1.11\,C_2(\mu_0) \end{aligned}$$

SM matching: $C_1^{\rm SM}(\mu_0) = 0 + \mathcal{O}(\alpha_s)$ and $C_2^{\rm SM}(\mu_0) = 1 + \mathcal{O}(\alpha_s)$

$\Rightarrow$ non-zero C_1 at scales $\mu < \mu_0$ from **"Mixing of Q_2 into Q_1"**

Examples of mixing of $Q_{1,2}$ into ...

QCD penguin operators: $b \to s\, q\bar{q}$



$$Q_{3(5)} = \left[\bar{s}\gamma_\mu P_L b\right] \sum_q \left[\bar{q}\gamma^\mu P_{L(R)} q\right]$$

$$Q_{4(6)} = \left[\bar{s}_\alpha\gamma_\mu P_L b_\beta\right] \sum_q \left[\bar{q}_\beta\gamma^\mu P_{L(R)} q_\alpha\right]$$

$\mu \sim m_b, \quad \mu_0 \sim m_W,$ using $C^{\rm SM}_{3,4,5,6}(\mu_0)$

$$C_3(\mu) = +0.0010\left[1 - 1.5\, C_1(\mu_0) + 12.6\, C_2(\mu_0)\right]$$

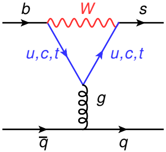
$$C_4(\mu) = -0.0017\left[1 - 2.0\, C_1(\mu_0) + 16.0\, C_2(\mu_0)\right]$$

$$C_5(\mu) = +0.0004\left[1 - 1.5\, C_1(\mu_0) + 19.2\, C_2(\mu_0)\right]$$

$$C_6(\mu) = -0.0027\left[1 - 1.5\, C_1(\mu_0) + 12.6\, C_2(\mu_0)\right]$$

- 1 is contribution from $C^{\rm SM}_{3,4,5,6}(\mu_0)$
- because $C_2^{\rm SM}(\mu_0) = 1$, main contr'n from mixing with Q_2
- $C_1^{\rm SM}(\mu_0) = \mathcal{O}(\alpha_s) \ll C_2^{\rm SM}(\mu_0)$

at μ_0: $-3\, C^{\rm SM}_{3,5} = C^{\rm SM}_{4,6} = \dfrac{\alpha_s(\mu_0)}{8\pi}\widetilde{E}_0(x_t)$ and $\widetilde{E}_0(x_t) \approx -0.39$



These operators are most relevant for
$B \to K + (\pi, \rho, \ldots)$ or $B \to K^* + (\pi, \rho, \ldots)$

Examples of mixing of $Q_{1,2}$ into ...

Electro- and chromo-magnetic dipole operators: $b \to s\gamma$ and $b \to sg$

$$Q_{7\gamma} = \frac{e}{(4\pi)^2} m_b \big[\bar{s}\sigma^{\mu\nu} P_R b\big] F_{\mu\nu} \qquad Q_{8g} = \frac{g_s}{(4\pi)^2} m_b \big[\bar{s}_\alpha \sigma^{\mu\nu} P_R \mathbf{T}^a_{\alpha\beta} b_\beta\big] G^a_{\mu\nu}$$

$\mu \sim m_b$, $\mu_0 \sim m_W$, using $C^{\rm SM}_{7\gamma}(\mu_0) = -0.19$ and $C^{\rm SM}_{8g}(\mu_0) = -0.05$

$$C_{7\gamma}(\mu) \approx -0.13 + 0.02\, C_1(\mu_0) - 0.19\, C_2(\mu_0) \overset{\rm SM}{=} -0.32$$

$$C_{8g}(\mu) \approx -0.03 + 0.10\, C_1(\mu_0) - 0.09\, C_2(\mu_0) \overset{\rm SM}{=} -0.12$$

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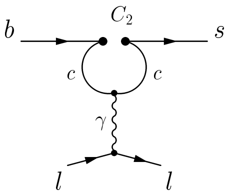
$\Rightarrow$ a 10 % change in $C_2^{\text{NP}}(\mu_0) \approx 0.1$ w.r.t. SM gives

- 6% effect on $C_{7\gamma}(\mu)$
- 12 % on $Br(B \to X_s\gamma) \propto |C_7(\mu)|^2$

example of strong "indirect constraints" on $C_2(\mu_0)$ from $Br(B \to X_s\gamma)$

Examples of mixing of $Q_{1,2}$ into ...



Semileptonic: $b \to s \ell^+ \ell^-$

$$Q_9 = [\bar{s}\gamma_\mu P_L b] \sum_\ell [\bar{\ell}\gamma^\mu \ell]$$

In SM (μ = 5 GeV, μ_0 = 160 GeV)

$$\widetilde{C}_1^{(0)}(\mu_0) = 0, \quad \widetilde{C}_2^{(0)}(\mu_0) = 1, \quad \widetilde{C}_1^{(1)}(\mu_0) = 23.3, \quad \widetilde{C}_4^{(1)}(\mu_0) = 0.5, \quad C_9^{(1)}(\mu_0) = 1.5$$

The LL + NLL piece

$$C_9(\mu) = 4.50\,\widetilde{C}_1^{(0)}(\mu_0) + 1.89\widetilde{C}_2^{(0)}(\mu_0) \; + \; 0.04\,\widetilde{C}_1^{(1)}(\mu_0) - 0.03\,\widetilde{C}_4^{(1)}(\mu_0) + C_9^{(1)}(\mu_0)$$

$$\overset{\text{SM}}{=} 0. + 1.89 \; + \; 0.92 - 0.02 + 1.47 = 4.26$$

Note: Here used Chetyrkin/Misiak/Münz [hep-ph/9612313] operator definition of $Q_1, \ldots, Q_6$

$$\widetilde{Q}_1 \equiv [\bar{s}\gamma_\mu P_L \mathbf{T}^a c][\bar{c}\gamma^\mu P_L \mathbf{T}^a b] \quad \text{and} \quad \widetilde{Q}_2 \equiv [\bar{s}\gamma_\mu P_L c][\bar{c}\gamma^\mu P_L b]$$

$$\text{@ LO:} \quad \widetilde{C}_1^{(0)} = 2C_1^{(0)} \quad \text{and} \quad \widetilde{C}_2^{(0)} = C_1^{(0)}/3 + C_2^{(0)}$$

Have EFT Lagrangian!

What next?

Outlook . . .

What is achieved via EFT:

- decoupled heavy degrees of freedom for process $\ll m_W$
- restricted to most relevant dim ≤ 6 operators
- RG equation resums large log's $\alpha_s^n \ln(m_b/m_W)^n$ to all orders in α_s
- EFT allows to include BSM effects via new operators model-independently

Need predictions of observables

- example of muon decay is "trivial" as only QED involved $\Rightarrow$ in principle perturbative
- processes with quarks involve QCD: quarks are not free, but confined at $\ll m_W$
  !!! external states are mesons/baryons $\Rightarrow$ nonperturbative

$\Rightarrow$ hadronic matrix elements needed — examples for $b \to s$

- decay constants $\langle 0 | \bar{s}\, \gamma^\mu \ldots b | B(p) \rangle$
- local form factors $\langle M(p') | \bar{s}\, \gamma^\mu \ldots b | B(p) \rangle$
- nonlocal objects $\int dx\, e^{ikx} \langle M(p') | \mathrm{T}\{[\bar{q}\gamma^\alpha q](x),\ [\bar{s}\, \gamma^\mu \ldots b](0)\, | B(p) \rangle$

Nonperturbative methods and/or reliable parametrizations + phenomenology required