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GDR lectures on EFT

Lectures given for the GDR Intensity Frontier

21-25 September 2020

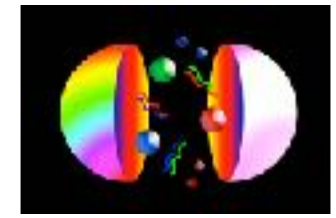
Standard Model

100 GeV

$\gamma, g, \nu_i, e, \mu, \tau$ + u, d, s, c, b

5 GeV

$\gamma, g, \nu_i, e, \mu, \tau$ + u, d, s, c

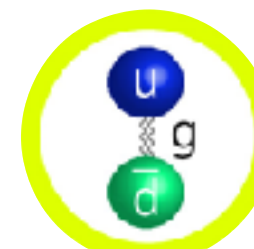


2 GeV

γ, ν_i, e, μ + hadrons

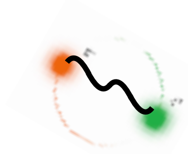
1 GeV

γ, ν_i, e, μ + pions and kaons



100 MeV

γ, ν_i, e

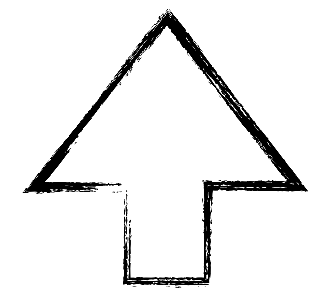


1 MeV

γ, ν_i

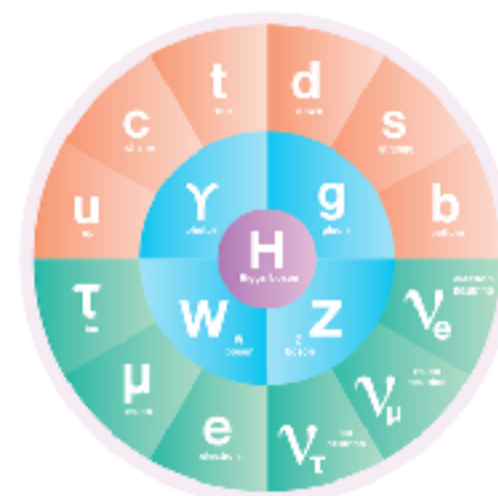
100 TeV

10 TeV



1 TeV

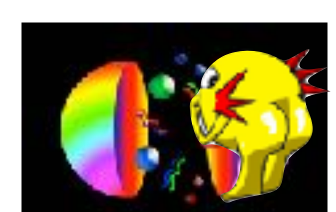
Standard Model



100 GeV

10 GeV

$\gamma,g,\nu_i,e,\mu,\tau$ + u, d, s, c, b



Lecture 4

Effective Theory above the electroweak scale, or SMEFT et al

Standard Model

$$\mathcal{L}_{\mathrm{SM}} = -\frac{1}{4}\sum_{V\in B,W^i,G^a} V_{\mu\nu}V^{\mu\nu} + \sum_{f\in q,u,d,l,e} i\bar{f}\gamma^\mu D_\mu f$$
$$-\left(\bar{u}Y_u qH + \bar{d}Y_d H^\dagger q + \bar{e}Y_e H^\dagger l + \mathrm{h.c.}\right)$$
$$+D_\mu H^\dagger D^\mu H + \mu_H^2 H^\dagger H - \lambda (H^\dagger H)^2$$

$$D_\mu f = \partial_\mu f - i g_s G_\mu^a T^a f - i g_L W_\mu^i \frac{\sigma^i}{2} f - i g_Y B_\mu Y f$$
$$V_{\mu\nu}^a = \partial_\mu V_\nu^a - \partial_\nu V_\mu^a + g f^{abc} V_\mu^b V_\nu^c$$

18 free parameters (19 together with θ_{QCD}) all of them measured with a good precision

Motivation to go beyond the Standard Model

- The Standard Model has been totally successful in describing all collider and low-energy experiments. Discovery of the 125 GeV Higgs boson was the last piece of puzzle to fall into place

- On the other hand, we know for a fact that physics beyond the SM exists (neutrino masses, dark matter, inflation, baryon asymmetry). There are also some theoretical hints for new physics (strong CP problem, flavor hierarchies, gauge coupling unification, naturalness problem)

- But there isn't one model or class of models that is strongly preferred, at this moment. We need to keep an open mind on many possible forms of new physics that may show up in experiment. This requires a model-independent approach

- Currently, the leading model-independent tool to parametrize the possible effects of heavy new physics is effective field theory

EFT approach to BSM

$$\mathscr{L}_{\mathrm{SM}} = -\frac{1}{4} \sum_{V \in B, W^i, G^a} V_{\mu\nu} V^{\mu\nu} + \sum_{f \in q,u,d,l,e} i\bar{f}\gamma^\mu D_\mu f$$

$$- \left(\bar{u} Y_u q H + \bar{d} Y_d H^\dagger q + \bar{e} Y_e H^\dagger l + \mathrm{h.c.} \right)$$

$$+ D_\mu H^\dagger D^\mu H + \mu_H^2 H^\dagger H - \lambda (H^\dagger H)^2$$

In the EFT approach, we assume that the particle spectrum is that of the SM, in some energy regime between the weak scale and the cutoff Λ, where Λ >> m_Z .

$$\mathscr{L}_{\mathrm{EFT}} = \mathscr{L}_{\mathrm{SM}} + \Delta \mathscr{L}_{\mathrm{BSM}}$$

Known SM Lagrangian

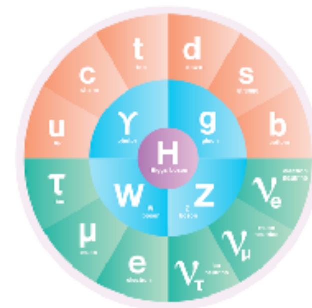
All possible interactions between the SM fields not present in the SM

Remains to choose some power counting to organize $\Delta\mathscr{L}_{\mathrm{BSM}}$ in a systematic expansion

Linear vs non-linear

Two mathematical formulations for effective theories with SM spectrum

Linearly realized electroweak symmetry	Non-linearly realized electroweak symmetry
$SU(3)_C \times SU(2)_L \times U(1)_Y$	**$SU(3)_C \times U(1)_{em}$**



$$L \in SU(2)_L \quad R \in U(1)_Y$$

$$H \to LH$$

$$U \to LUR^\dagger \qquad h \to h$$

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} iG_1 + G_2 \\ \text{v} + h + iG_3 \end{pmatrix}$$

$$U = \exp\left(\frac{i\pi^a \sigma^a}{\text{v}}\right)$$

125 GeV Higgs boson

Goldstone bosons eaten by W and Z

Higgs VEV $v \approx 246$ GeV

Expansion parameter $v \approx 246$ GeV

In general, the two formulations lead to two distinct effective theories

SMEFT $\subset$ **HEFT**

Linear vs non-linear: Higgs self-couplings

In the SM self-coupling completely fixed...

$$\mathscr{L}_{\rm SM} \supset m^2|H|^2 - \lambda|H|^4$$

$$\to -\frac{1}{2}m_h^2h^2 - \frac{m_h^2}{2{\rm v}}h^3 - \frac{m_h^2}{8{\rm v}^2}h^4$$

...but they can be deformed by BSM effects

SMEFT

$$\mathscr{L}_{\rm SMEFT} = \mathscr{L}_{\rm SM} - \frac{c_6}{\Lambda^2}|H|^6 + \mathcal{O}(\Lambda^{-4})$$

$$\mathscr{L}_{\rm SMEFT} \supset -\frac{m_h^2}{2{\rm v}}(1+\delta\lambda_3)h^3 - \frac{m_h^2}{8{\rm v}^2}(1+\delta\lambda_4)h^4 - \frac{\lambda_5}{\rm v}h^5 - \frac{\lambda_6}{{\rm v}^2}h^6$$

$$\delta\lambda_3 = \frac{2c_6{\rm v}^4}{m_h^2\Lambda^2},\ \delta\lambda_4 = \frac{12c_6{\rm v}^4}{m_h^2\Lambda^2},\ \lambda_5 = \frac{3c_6{\rm v}^2}{4\Lambda^2},\ \lambda_6 = \frac{c_6{\rm v}^2}{8\Lambda^2}$$

SMEFT: Predicts correlations between self-couplings as long as Λ >> v

HEFT

$$\mathscr{L}_{\rm HEFT} \supset -c_3\frac{m_h^2}{2{\rm v}}h^3 - c_4\frac{m_h^2}{8{\rm v}^2}h^4 - \frac{c_5}{\rm v}h^5 - \frac{c_6}{{\rm v}^2}h^6 + \ldots$$

HEFT: no correlations between self-couplings

Linear vs non-linear

- SMEFT and HEFT lead to a vastly different phenomenology at the electroweak scale

- Choosing SMEFT or HEFT implicitly entails an assumption about a class of BSM theories that we want to characterize

- SMEFT is appropriate to describe BSM theories which can be parametrically decoupled, that is to say, where the mass scale of the new particles depends on a free parameter(s) that can be taken to infinity

- Conversely, HEFT is appropriate to describe non-decoupling BSM theories, where the masses of the new particles vanish in the limit $v \to 0$

Example: cubic Higgs deformation

Consider a toy EFT model where Higgs cubic (and only that) deviates from the SM

$$\mathscr{L} = \mathscr{L}_{\rm SM} - \Delta_3 \frac{m_h^2}{2v} h^3$$

$$V(h) = \frac{m_h^2}{2} h^2 + \frac{m_h^2}{2v} \left(1 + \Delta_3\right) h^3 + \frac{m_h^2}{8v^2} h^4$$

This EFT belongs to the HEFT but not SMEFT parameter space

HEFT = Non-analytic Higgs potential

$$V(h) = \frac{m_h^2}{2} h^2 + \frac{m_h^2}{2\mathrm{v}} \left(1 + \Delta_3\right) h^3 + \frac{m_h^2}{8\mathrm{v}^2} h^4 \qquad \textbf{(1)}$$

Given a Lagrangian for Higgs boson h, one can always uplift it to a manifestly SU(2)xU(1) invariant form by replacing

$$h \to \sqrt{2H^\dagger H} - \mathrm{v}$$

After this replacement, Higgs potential contains terms non-analytic at H=0

$$V(H) = \frac{m_h^2}{8\mathrm{v}^2} \left(2H^\dagger H - \mathrm{v}^2\right)^2 + \Delta_3 \frac{m_h^2}{2\mathrm{v}} \left(\sqrt{2H^\dagger H} - \mathrm{v}\right)^3 \qquad \textbf{(2)}$$

(1) and (2) are equal in the unitary gauge

$$H \to \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \mathrm{v} + h \end{pmatrix}$$

<u>Thus, (1) and (2) describe the same physics</u>

Non-analytic Higgs potential

$$V(H) = \frac{m_h^2}{8\mathrm{v}^2}\left(2H^\dagger H - \mathrm{v}^2\right)^2 + \Delta_3 \frac{m_h^2}{2\mathrm{v}}\left(\sqrt{2H^\dagger H} - \mathrm{v}\right)^3$$

In the unitary gauge, the Higgs potential looks totally healthy and renormalizable…

Going away from the unitary gauge:

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} iG_1 + G_2 \\ \mathrm{v} + h + iG_3 \end{pmatrix} \quad \Rightarrow \quad V \supset \Delta_3 \frac{m_h^2}{2\mathrm{v}}\left(\sqrt{(h+\mathrm{v})^2 + G^2} - \mathrm{v}\right)^3$$

$$G^2 \equiv \sum_i G_i^2$$

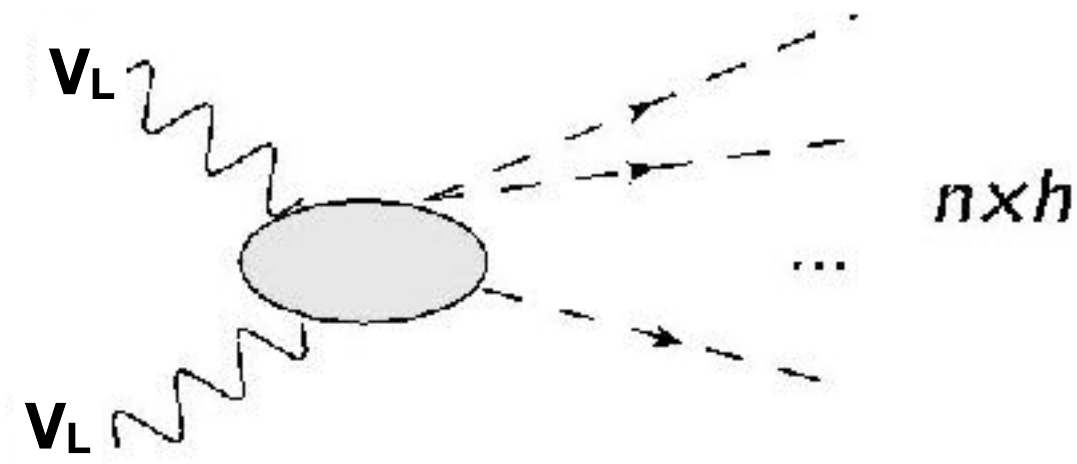
Away from the unitary gauge, it becomes clear that the Higgs potential contains non-renormalizable interactions suppressed only by the EW scale v

$$V \supset \Delta_3 \frac{3m_h^2}{4\mathrm{v}}\frac{G^2 h^2}{h + \mathrm{v}} + \mathcal{O}(G^4) = \Delta_3 \frac{3m_h^2}{4} G^2 \sum_{n=2}^{\infty}\left(\frac{-h}{\mathrm{v}}\right)^n + \mathcal{O}(G^4)$$

Multi-Higgs production

Consider VBF production of n ≥ 2 Higgs bosons: $V_L V_L \to n \times h$

By the equivalence theorem, at high energies the same as $GG \to n \times h$



Expanded potential contains interactions

$$V \supset = \Delta_3 \frac{3m_h^2}{4} G^2 \sum_{n=2}^{\infty} \left(\frac{-h}{\mathrm{v}}\right)^n$$

leading to interaction vertices with arbitrary number of Higgs bosons

$$\mathcal{M}(GG \to \underbrace{h \ldots h}_{n}) \sim \Delta_3 \frac{n! m_h^2}{\mathrm{v}^n}$$

Amplitudes for multi-Higgs production in W/Z boson fusion are only suppressed by the scale v and do not decay with growing energy, leading to unitarity loss at some scale right above v

Unitarity primer

S matrix unitarity $S^\dagger S = 1$

symmetry factor for n-body final state

implies relation between forward scattering amplitude, and elastic and inelastic production cross sections

$$2\mathrm{Im}\mathscr{M}(p_1p_2 \to p_1p_2) = S_2\int d\Pi_2\,|\,\mathscr{M}^{\mathrm{elastic}}(p_1p_2 \to k_1k_2)\,|^2 + \sum S_n \int d\Pi_n\,|\,\mathscr{M}^{\mathrm{inelastic}}(p_1p_2 \to k_1\ldots k_n)\,|^2$$

Equation is "diagonalized" after initial and final 2-body state are projected into partial waves

$$a_l(s) = \frac{S_2}{16\pi}\sqrt{1-\frac{4m^2}{s}}\int_{-1}^{1} d\cos\theta P_l(\cos\theta)\mathcal{M}(s,\cos\theta),$$

$$2\mathrm{Im}a_l = a_l^2 + \sum S_n \int d\Pi_n\,|\,\mathscr{M}_l^{\mathrm{inelastic}}\,|^2$$

This can be rewritten as the Argand circle equation

$$(\mathrm{Re}a_l)^2 + (\mathrm{Im}a_l - 1)^2 = R_l^2, \qquad R_l^2 = 1 - \sum S_n \int d\Pi_n\,|\,\mathscr{M}_l^{\mathrm{inelastic}}\,|^2$$

Unitarity primer

Argand circle equation

$$(\mathrm{Re}\,a_l)^2 + (\mathrm{Im}\,a_l - 1)^2 = R_l^2, \qquad R_l^2 = 1 - \sum S_n \int d\Pi_n \,|\, \mathscr{M}_l^{\mathrm{inelastic}} \,|^2$$

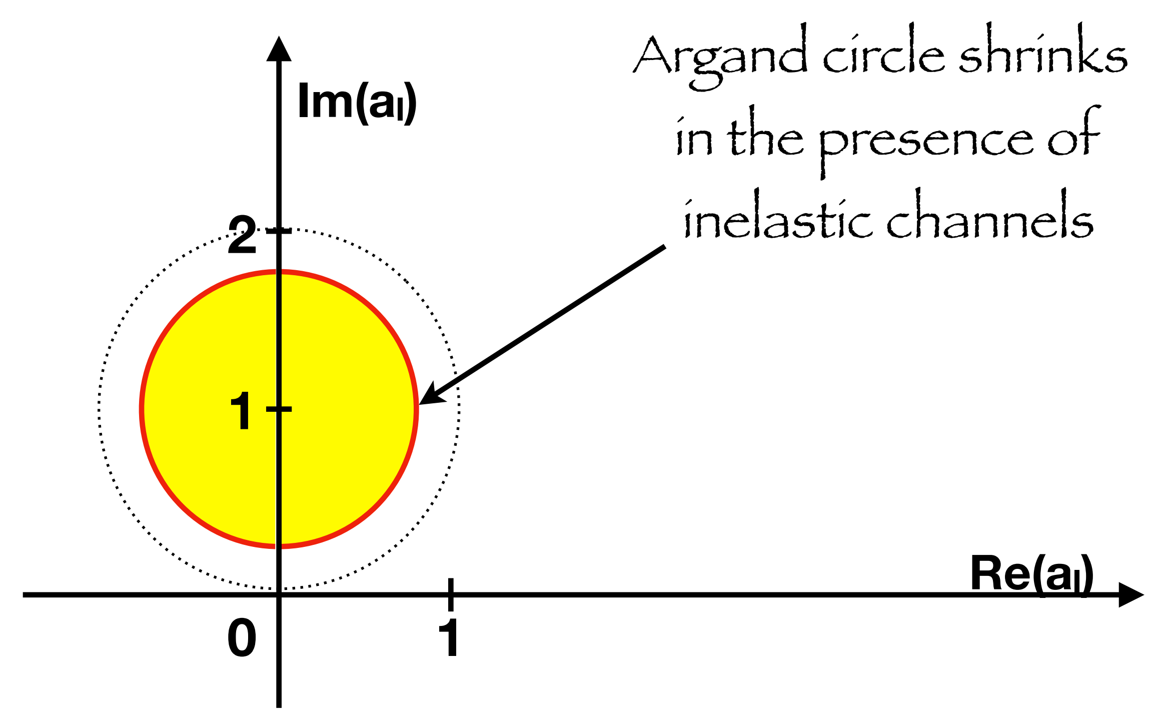
implies constraints on both elastic and inelastic amplitudes

Often used

$$|\,\mathrm{Re}\,a_l\,| \leq 1$$

$$\sum S_n \int d\Pi_n \,|\, \mathscr{M}_l^{\mathrm{inelastic}} \,|^2 \leq 1$$

Often forgotten



Unitarity constraints on inelastic channels

Unitarity (strong coupling) constraint on inelastic multi-Higgs production

$$\sum_{n=2}^{\infty} \frac{1}{n!} \int d\Pi_n \, |\mathcal{M}(GG \to h^n)|^2 = \sum_{n=2}^{\infty} \frac{1}{n!} V_n(\sqrt{s}) \, |\mathcal{M}(GG \to h^n)|^2 \lesssim \mathcal{O}(1)$$

Volume of phase space in the massless limit:

$$V_n(\sqrt{s}) = \int d\Pi_n = \frac{s^{n-2}}{2(n-1)!(n-2)!(4\pi)^{2n-3}} \sim \frac{s^{n-2}}{(n!)^2(4\pi)^{2n}}$$

In a fundamental theory,

2 → n amplitude must decay as $1/s^{n/2-1}$

in order to maintain unitarity up to arbitrary high scales

Process	*Unitarity limit*
2 → 2	**1**
2 → 3	**$1/s^{1/2}$**
2 → 4	**$1/s$**
...	...

Unitarity constraints on HEFT

Unitarity equation

$$\sum_{n=2}^{\infty} \frac{1}{n!} V_n(\sqrt{s}) \, |\mathcal{M}(GG \to h^n)|^2 \lesssim \mathcal{O}(1)$$

Our amplitude

$$\mathcal{M}(GG \to \underbrace{h \ldots h}_{n}) \sim \Delta_3 \frac{n! m_h^2}{\mathrm{v}^n}$$

$$\mathcal{O}(1) \gtrsim \sum_{n=2}^{\infty} \frac{1}{n!} V_n(\sqrt{s}) \, |\mathcal{M}(GG \to h^n)|^2 \sim \sum_{n=2}^{\infty} \frac{1}{n!} \frac{s^{n-2}}{(n!)^2 (4\pi)^{2n}} \Delta_3^2 \frac{(n!)^2 m_h^4}{\mathrm{v}^{2n}} \sim \frac{\Delta_3^2 m_h^4}{s^2} \exp\left[\frac{s}{(4\pi \mathrm{v})^2}\right]$$

In model with deformed Higgs cubic, multi-Higgs amplitude do not decay with energy leading to unitarity loss at a finite value of energy

$$\Lambda \lesssim (4\pi \mathrm{v}) \log^{1/2}\left(\frac{4\pi \mathrm{v}}{m_h |\Delta_3|^{1/2}}\right)$$

Unless Δ_3 is unobservably small, unitarity loss happens at the scale 4 π v ~ 3 TeV !

Linear vs non-linear summary

- EFT with non-linearly realized electroweak symmetry (aka HEFT) is equivalent to EFT with linearly realized electroweak symmetry but whose Lagrangian is a non-polynomial function of the Higgs field that is non-analytic at $H=0$

- This non-analyticity leads to explosion of multi-Higgs amplitudes at the scale $4 \pi v$. For this reason, the validity regime of HEFT is limited below the scale of order $4 \pi v \sim 3$ TeV

- HEFT is useful to approximate BSM theories where new particles' masses vanish in the limit $v \to 0$, e.g. SM + a 4th generation of chiral fermions

- On the other hand, an EFT with linearly realized electroweak symmetry and the Lagrangian polynomial in the Higgs field (aka SMEFT) is useful to approximate BSM theories where new particles' masses do not vanish in the limit $v \to 0$, and thus can be parametrically larger than the electroweak scale, e.g. SM + vector-like fermions

- In the following we forget HEFT and focus on SMEFT

SMEFT

<u>Assumptions:</u>

1) At energies $E < \Lambda$ no other degrees of freedom than those of the SM
2) Masses of BSM particles entering at the scale Λ do not vanish in the limit $v \to 0$

Then we can organize the EFT as an expansion in $1/\Lambda$, where each term is a linear combination of SU(3)xSU(2)xU(1) invariant operators of a given canonical dimension D

$$\mathscr{L}_{\mathrm{SMEFT}} = \mathscr{L}_{\mathrm{SM}} + \frac{1}{\Lambda}\mathscr{L}_{D=5} + \frac{1}{\Lambda^2}\mathscr{L}_{D=6} + \frac{1}{\Lambda^3}\mathscr{L}_{D=7} + \frac{1}{\Lambda^4}\mathscr{L}_{D=8} + \ldots$$

Known SM Lagrangian

Higher-dimensional $SU(3)_C \times SU(2)_L \times U(1)_Y$ invariant interactions added to the SM

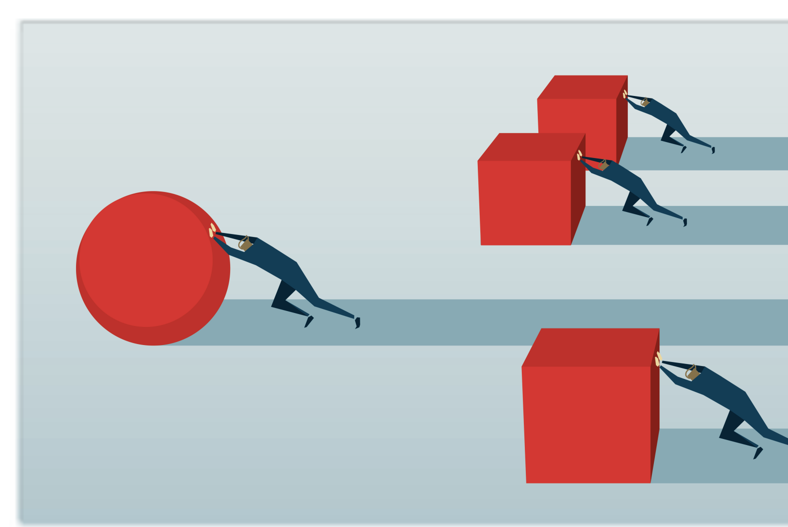
At each order we should include a <u>complete</u> and <u>non-redundant</u> set of operators eventually subject to some additional global symmetries

SMEFT

- In a sense, the future of particle physics is about determining the Wilson coefficients of all these higher-dimensional operators

- More optimistically, probing an operator suppressed by the scale Λ corresponds, in a way, to performing an experiment at an experiment at the energy scale Λ. The exciting point is that in many cases Λ >> TeV, thus we are not limited by the LHC reach in exploring high energies!

- EFT language does not describe all possible form of new physics. However it is a very universal language that allows us to systematize our thinking and better plan and design future experiments



SMEFT at dimension-5

$$\mathscr{L}_{\rm SMEFT} = \mathscr{L}_{\rm SM} + \frac{1}{\Lambda}\mathscr{L}_{D=5} + \frac{1}{\Lambda^2}\mathscr{L}_{D=6} + \frac{1}{\Lambda^3}\mathscr{L}_{D=7} + \frac{1}{\Lambda^4}\mathscr{L}_{D=8} + \dots$$

$$\frac{c_{ij}}{\Lambda}(L_i H)(L_j H) + \mathrm{h.c.} \to c_{ij}\frac{\mathrm{v}^2}{\Lambda}\nu_i \nu_j + \mathrm{h.c.}$$

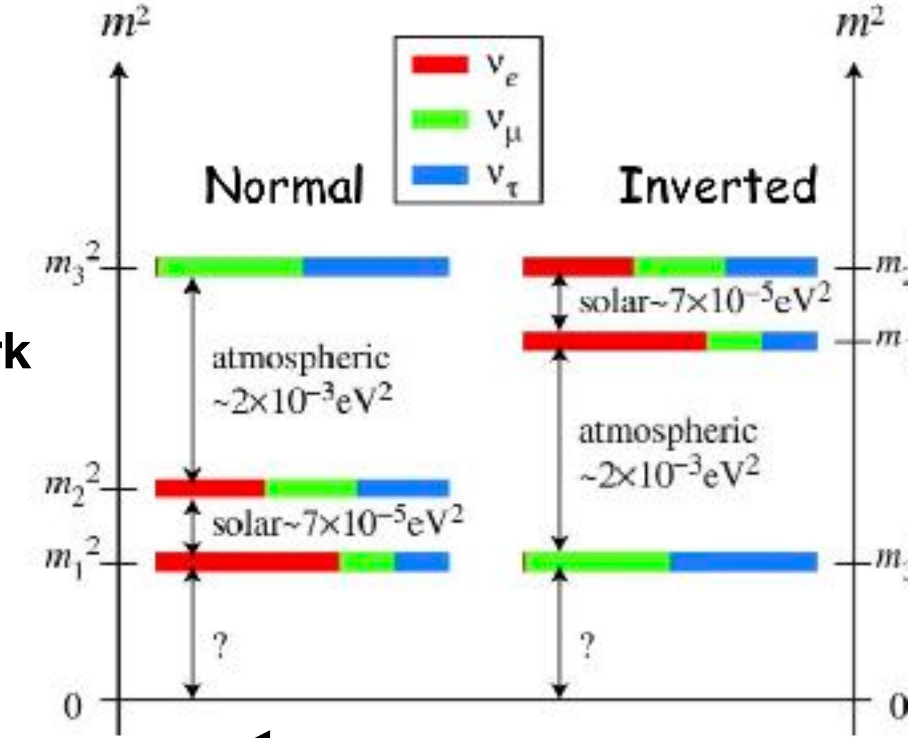
$$H \to \begin{pmatrix} 0 \\ \mathrm{v}/\sqrt{2} \end{pmatrix} \qquad L_i \to \begin{pmatrix} \nu_i \\ e_i \end{pmatrix}$$

- At dimension 5, the only operators one can construct are the so-called Weinberg operators, which break the lepton number
- After electroweak symmetry breaking they give rise to Majorana mass terms for the SM (left-handed) neutrinos
- Neutrino oscillation experiments strongly suggest that these operators are present (unless neutrino masses are of the Dirac type)

This is a huge success of SMEFT: corrections to the SM Lagrangian predicted at the leading order in the EFT expansion, are indeed observed in experiment!

SMEFT at dimension-5

$$\mathscr{L}_{\rm SMEFT} \supset c_{ij}\frac{v^2}{\Lambda}\nu_i\nu_j + {\rm h.c.}$$



Neutrino masses or most likely in the 0.01 eV - 0.1 eV ballpark (while the lightest neutrino may even be massless)

It follows that $\Lambda / c_{ij} \sim 10^{15}$ GeV

One problem now:

$$\mathscr{L}_{\rm SMEFT} = \mathscr{L}_{\rm SM} + \frac{1}{\Lambda}\mathscr{L}_{D=5} + \frac{1}{\Lambda^2}\mathscr{L}_{D=6} + \frac{1}{\Lambda^3}\mathscr{L}_{D=7} + \frac{1}{\Lambda^4}\mathscr{L}_{D=8} + \ldots$$

If this is really the correct expansion, then we will never see any other effects of higher-dimensional operators, except possibly of baryon-number violating ones :/

However, it is possible that there is more than one mass scale of new physics

Dimension-5 interactions are special because they violate lepton number L. If we assume that the mass scale of new particles with L-violating interactions is Λ_L, and there is also L-conserving new physics at the scale $\Lambda << \Lambda_L$, then the expansion is

$$\mathscr{L}_{\rm SMEFT} = \mathscr{L}_{\rm SM} + \frac{1}{\Lambda_L}\mathscr{L}_{D=5} + \frac{1}{\Lambda^2}\mathscr{L}_{D=6} + \frac{1}{\Lambda_L^3}\mathscr{L}_{D=7} + \frac{1}{\Lambda^4}\mathscr{L}_{D=8} + \ldots$$

SMEFT at dimension-6

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda_L}\mathcal{L}_{D=5} + \frac{1}{\Lambda^2}\mathcal{L}_{D=6} + \frac{1}{\Lambda_L^3}\mathcal{L}_{D=7} + \frac{1}{\Lambda^4}\mathcal{L}_{D=8} + \dots$$

$$v \ll \Lambda \ll \Lambda_L$$

Bosonic CP-even	
O_H	$(H^\dagger H)^3$
$O_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
O_{HD}	$\left\lvert H^\dagger D_\mu H\right\rvert^2$
O_{HG}	$H^\dagger H\, G^a_{\mu\nu} G^a_{\mu\nu}$
O_{HW}	$H^\dagger H\, W^i_{\mu\nu} W^i_{\mu\nu}$
O_{HB}	$H^\dagger H\, B_{\mu\nu} B_{\mu\nu}$
O_{HWB}	$H^\dagger \sigma^i H\, W^i_{\mu\nu} B_{\mu\nu}$
O_W	$\epsilon^{ijk} W^i_{\mu\nu} W^j_{\nu\rho} W^k_{\rho\mu}$
O_G	$f^{abc} G^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu}$

Bosonic CP-odd

$O_{H\widetilde{G}}$	$H^\dagger H\, \widetilde{G}^a_{\mu\nu} G^a_{\mu\nu}$
$O_{H\widetilde{W}}$	$H^\dagger H\, \widetilde{W}^i_{\mu\nu} W^i_{\mu\nu}$
$O_{H\widetilde{B}}$	$H^\dagger H\, \widetilde{B}_{\mu\nu} B_{\mu\nu}$
$O_{H\widetilde{W}B}$	$H^\dagger \sigma^i H\, \widetilde{W}^i_{\mu\nu} B_{\mu\nu}$
$O_{\widetilde{W}}$	$\epsilon^{ijk} \widetilde{W}^i_{\mu\nu} W^j_{\nu\rho} W^k_{\rho\mu}$
$O_{\widetilde{G}}$	$f^{abc} \widetilde{G}^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu}$

Yukawa	
$[O^\dagger_{eH}]_{IJ}$	$H^\dagger H e^c_I H^\dagger \ell_J$
$[O^\dagger_{uH}]_{IJ}$	$H^\dagger H u^c_I \widetilde{H}^\dagger q_J$
$[O^\dagger_{dH}]_{IJ}$	$H^\dagger H d^c_I H^\dagger q_J$

Vertex		Dipole	
$[O^{(1)}_{H\ell}]_{IJ}$	$i\bar\ell_I \bar\sigma_\mu \ell_J H^\dagger \overleftrightarrow{D}_\mu H$	$[O^\dagger_{eW}]_{IJ}$	$e^c_I \sigma_{\mu\nu} H^\dagger \sigma^i \ell_J W^i_{\mu\nu}$
$[O^{(3)}_{H\ell}]_{IJ}$	$i\bar\ell_I \sigma^i \bar\sigma_\mu \ell_J H^\dagger \sigma^i \overleftrightarrow{D}_\mu H$	$[O^\dagger_{eB}]_{IJ}$	$e^c_I \sigma_{\mu\nu} H^\dagger \ell_J B_{\mu\nu}$
$[O_{He}]_{IJ}$	$i e^c_I \sigma_\mu \bar e^c_J H^\dagger \overleftrightarrow{D}_\mu H$	$[O^\dagger_{uG}]_{IJ}$	$u^c_I \sigma_{\mu\nu} T^a \widetilde{H}^\dagger q_J\, G^a_{\mu\nu}$
$[O^{(1)}_{Hq}]_{IJ}$	$i\bar q_I \bar\sigma_\mu q_J H^\dagger \overleftrightarrow{D}_\mu H$	$[O^\dagger_{uW}]_{IJ}$	$u^c_I \sigma_{\mu\nu} \widetilde{H}^\dagger \sigma^i q_J\, W^i_{\mu\nu}$
$[O^{(3)}_{Hq}]_{IJ}$	$i\bar q_I \sigma^i \bar\sigma_\mu q_J H^\dagger \sigma^i \overleftrightarrow{D}_\mu H$	$[O^\dagger_{uB}]_{IJ}$	$u^c_I \sigma_{\mu\nu} \widetilde{H}^\dagger q_J\, B_{\mu\nu}$
$[O_{Hu}]_{IJ}$	$i u^c_I \sigma_\mu \bar u^c_J H^\dagger \overleftrightarrow{D}_\mu H$	$[O^\dagger_{dG}]_{IJ}$	$d^c_I \sigma_{\mu\nu} T^a H^\dagger q_J\, G^a_{\mu\nu}$
$[O_{Hd}]_{IJ}$	$i d^c_I \sigma_\mu \bar d^c_J H^\dagger \overleftrightarrow{D}_\mu H$	$[O^\dagger_{dW}]_{IJ}$	$d^c_I \sigma_{\mu\nu} \bar H^\dagger \sigma^i q_J\, W^i_{\mu\nu}$
$[O_{Hud}]_{IJ}$	$i u^c_I \sigma_\mu \bar d^c_J \tilde H^\dagger D_\mu H$	$[O^\dagger_{dB}]_{IJ}$	$d^c_I \sigma_{\mu\nu} H^\dagger q_J\, B_{\mu\nu}$

Table 2.3: Two-fermion D=6 operators in the Warsaw basis. The flavor indices denoted by I, J. For complex operators (O_{Hud} and all Yukawa and dipole operat the corresponding complex conjugate operator is implicitly included.

$(\bar RR)(\bar RR)$		$(\bar LL)(\bar RR)$	
O_{ee}	$\eta(e^c\sigma_\mu \bar e^c)(e^c\sigma_\mu \bar e^c)$	$O_{\ell e}$	$(\bar\ell\bar\sigma_\mu\ell)(e^c\sigma_\mu \bar e^c)$
O_{uu}	$\eta(u^c\sigma_\mu \bar u^c)(u^c\sigma_\mu \bar u^c)$	$O_{\ell u}$	$(\bar\ell\bar\sigma_\mu\ell)(u^c\sigma_\mu \bar u^c)$
O_{dd}	$\eta(d^c\sigma_\mu \bar d^c)(d^c\sigma_\mu \bar d^c)$	$O_{\ell d}$	$(\bar\ell\bar\sigma_\mu\ell)(d^c\sigma_\mu \bar d^c)$
O_{eu}	$(e^c\sigma_\mu \bar e^c)(u^c\sigma_\mu \bar u^c)$	O_{eq}	$(e^c\sigma_\mu \bar e^c)(\bar q\bar\sigma_\mu q)$
O_{ed}	$(e^c\sigma_\mu \bar e^c)(d^c\sigma_\mu \bar d^c)$	O_{qu}	$(\bar q\bar\sigma_\mu q)(u^c\sigma_\mu \bar u^c)$
O_{ud}	$(u^c\sigma_\mu \bar u^c)(d^c\sigma_\mu \bar d^c)$	O'_{qu}	$(\bar q\bar\sigma_\mu T^a q)(u^c\sigma_\mu T^a \bar u^c)$
O'_{ud}	$(u^c\sigma_\mu T^a \bar u^c)(d^c\sigma_\mu T^a \bar d^c)$	O_{qd}	$(\bar q\bar\sigma_\mu q)(d^c\sigma_\mu \bar d^c)$
		O'_{qd}	$(\bar q\bar\sigma_\mu T^a q)(d^c\sigma_\mu T^a \bar d^c)$

$(\bar LL)(\bar LL)$		$(\bar LR)(\bar LR)$	
$O_{\ell\ell}$	$\eta(\bar\ell\bar\sigma_\mu\ell)(\bar\ell\bar\sigma_\mu\ell)$	O_{quqd}	$(u^c q^j)\epsilon_{jk}(d^c q^k)$
O_{qq}	$\eta(\bar q\bar\sigma_\mu q)(\bar q\bar\sigma_\mu q)$	O'_{quqd}	$(u^c T^a q^j)\epsilon_{jk}(d^c T^a q^k)$
O'_{qq}	$\eta(\bar q\bar\sigma_\mu\sigma^i q)(\bar q\bar\sigma_\mu\sigma^i q)$	$O_{\ell equ}$	$(e^c\ell^j)\epsilon_{jk}(u^c q^k)$
$O_{\ell q}$	$(\bar\ell\bar\sigma_\mu\ell)(\bar q\bar\sigma_\mu q)$	$O'_{\ell equ}$	$(e^c\bar\sigma_{\mu\nu}\ell^j)\epsilon_{jk}(u^c\bar\sigma^{\mu\nu} q^k)$
$O'_{\ell q}$	$(\bar\ell\bar\sigma_\mu\sigma^i\ell)(\bar q\bar\sigma_\mu\sigma^i q)$	$O_{\ell edq}$	$(\bar\ell\bar e^c)(d^c q)$

Table 2.4: Four-fermion D=6 operators in the Warsaw basis. Flavor indices a suppressed here to reduce the clutter. The factor η is equal to 1/2 when all flav indices are equal (e.g. in $[O_{ee}]_{1111}$), and $\eta = 1$ otherwise. For each complex operat the complex conjugate should be included.

$$O_{duq} = (d^c u^c)(\bar Q \bar\ell)$$
$$O_{qqu} = (qq)(\bar u^c \bar e^c)$$
$$O_{qqq} = (qq)(q\ell)$$
$$O_{duu} = (d^c u^c)(u^c e^c)$$

From operators to observables

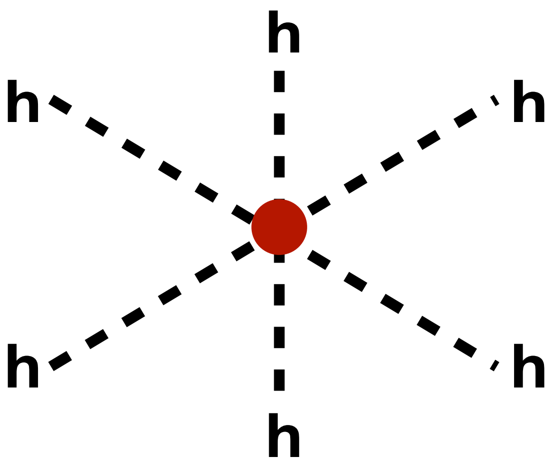
Two kinds of effects

New interactions not present in SM Lagrangian

Corrections to strength of SM interactions

New interactions

1. New vertices e.g. $\frac{1}{\Lambda^2}|H|^6 \to \frac{h^6}{8\Lambda^2} + \frac{3vh^5}{4\Lambda^2} + \ldots$



in particular, violation of global symmetries of the SM e.g. $\frac{1}{\Lambda^2}u^c d^c u^c e^c$

2. New Lorentz structures e.g. $\frac{1}{\Lambda^2}|H|^2 W^i_{\mu\nu}W^i_{\mu\nu} \to \frac{2v}{\Lambda^2}hW^+_{\mu\nu}W^-_{\mu\nu} + \ldots$

in addition to $\frac{h}{v}2m_W^2 W^+_\mu W^-_\mu$ **present in the SM**

in particular, violation of CP e.g. $\frac{1}{\Lambda^2}\epsilon_{ijk}W^i_{\mu\nu}W^j_{\nu\rho}\tilde{W}^k_{\rho\mu} \to -\frac{3i\sin\theta_W}{2\Lambda^2}W^+_{\mu\nu}W^-_{\nu\rho}\tilde{F}_{\rho\mu} + \ldots$

Modified interaction strength

There are 3 ways higher-dimensional operators may modify SM interaction strength

1. **Directly: after electroweak symmetry breaking, an operator contributes to a gauge or Yukawa interaction already present in the SM**

2. **Indirectly: after electroweak symmetry breaking, an operator contributes to the kinetic term of a SM field, thus effectively shifting the strength of all interactions of that field**

3. **Stealthily: after electroweak symmetry breaking, an operator contributes to an experimental observable from which some SM parameter is extracted**

Modified interaction strength: directly

Example: $$\frac{i}{\Lambda^2}\bar{e}_R\gamma^\mu e_R(H^\dagger D_\mu H - D_\mu H^\dagger H)$$

After electroweak symmetry breaking $$i(H^\dagger D_\mu H - D_\mu H^\dagger H) \to -\frac{\mathrm{v}^2}{2}\sqrt{g_L^2+g_Y^2}Z_\mu + \dots$$

$$\frac{ic_{He}}{\Lambda^2}\bar{e}_R\gamma^\mu e_R(H^\dagger D_\mu H - D_\mu H^\dagger H) \to -c_{He}\frac{\mathrm{v}^2\sqrt{g_L^2+g_Y^2}}{2\Lambda^2}\bar{e}_R\gamma^\mu e_R Z_\mu$$

This adds up to the weak interaction in the SM $$\sqrt{g_L^2+g_Y^2}\left(T_f^3 - \sin^2\theta_W Q_f + \delta g^{Zf}\right)\bar{f}\gamma^\mu f Z_\mu$$

$$\boxed{\delta g_R^{Ze} = -c_{He}\frac{\mathrm{v}^2}{2\Lambda^2}}$$

Thus c_{He} can be constrained, e.g., form LEP-1 Z-pole data

Modified interaction strength: indirectly

Example: $$(H^\dagger H)\Box(H^\dagger H)$$

This contributes to the kinetic term of the Higgs boson

$$\frac{c_{H\Box}}{\Lambda^2}(H^\dagger H)\Box(H^\dagger H) \rightarrow -\frac{c_{H\Box}\mathrm{v}^2}{\Lambda^2}(\partial_\mu h)^2$$

Together with the SM kinetic term:

$$\mathscr{L}_{\text{SMEFT}} \supset \frac{1}{2}(\partial_\mu h)^2\left(1 - \frac{2c_{H\Box}\mathrm{v}^2}{\Lambda^2}\right)$$

To restore canonical normalization, we need to rescale the Higgs boson field:

$$h \rightarrow h\left(1 + \frac{c_{H\Box}\mathrm{v}^2}{\Lambda^2}\right)$$

This restore canonical normalization of the Higgs boson field, up to terms of order $1/\Lambda^4$, which we ignore here

Modified interaction strength: indirectly

$$h \to h\left(1 + \frac{c_{H\Box}\mathrm{v}^2}{\Lambda^2}\right)$$

After this rescaling, the dimension-6 contributio vanishes from the Higgs boson kinetic term

However, it resurfaces in all Higgs boson couplings present in the SM !

$$\frac{h}{\mathrm{v}}\left[2m_W^2 W_\mu^+ W_\mu^- + m_Z^2 Z_\mu Z_\mu\right] \to \frac{h}{\mathrm{v}}\left(1 + \frac{c_{H\Box}\mathrm{v}^2}{\Lambda^2}\right)\left[2m_W^2 W_\mu^+ W_\mu^- + m_Z^2 Z_\mu Z_\mu\right]$$

$$\frac{h}{\mathrm{v}} m_f \bar{f} f \to \frac{h}{\mathrm{v}}\left(1 + \frac{c_{H\Box}\mathrm{v}^2}{\Lambda^2}\right) m_f \bar{f} f$$

Hence, the Higgs boson interaction strength predicted by the SM is universally shifted

LHC measurements of the Higgs signal strength provide a bound on the Wilson coefficient

$$\mu = 1.09 \pm 0.11 \quad \Rightarrow \quad \frac{c_{H\Box}\mathrm{v}^2}{\Lambda^2} = 0.09 \pm 0.11$$

or, equivalently

$$\frac{c_{H\Box}}{\Lambda^2} = \frac{1}{(820\mathrm{GeV})^2} \pm \frac{1}{(740\mathrm{GeV})^2}$$

Higgs measurements only probe new physics scale of order a TeV

Modified interaction strength: stealthily

Consider the dimension-6 operator $|H^\dagger D_\mu H|^2$

After electroweak symmetry breaking:

$$\frac{c_{HD}}{\Lambda^2}|H^\dagger D_\mu H|^2 \to \frac{c_{HD}\mathrm{v}^2}{2\Lambda^2}\frac{(g_L^2+g_Y^2)\mathrm{v}^2}{8}Z_\mu Z_\mu + \dots$$

Thus it modifies the Z boson mass:

$$m_Z^2 = \frac{(g_L^2+g_Y^2)\mathrm{v}^2}{4}\left(1+\frac{c_{HD}\mathrm{v}^2}{2\Lambda^2}\right)$$

We have this very precise O(10^{-4}) measurement of the Z boson mass

$$m_Z = (91.1876 \pm 0.0021)\ \mathrm{GeV}$$

From which we find the very stringent constraint

$$\frac{|c_{HD}|}{\Lambda^2} \leq \frac{1}{(26\ \mathrm{TeV})^2}$$

Modified interaction strength: stealthily

Consider the dimension-6 operator $|H^\dagger D_\mu H|^2$

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Thus it modifies the Z boson mass: $$m_Z^2 = \frac{(g_L^2+g_Y^2)\mathrm{v}^2}{4}\left(1+\frac{c_{HD}\mathrm{v}^2}{2\Lambda^2}\right)$$

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From which we find the very stringent constraint

$$\frac{|c_{HD}|}{\Lambda^2} \leq \frac{1}{(26\ \mathrm{TeV})^2}$$

No! Non! Nein! Nie! Нет! Ni!

Modified interaction strength: stealthily

Consider the dimension-6 operator $|H^\dagger D_\mu H|^2$

After electroweak symmetry breaking:

$$\frac{c_{HD}}{\Lambda^2}|H^\dagger D_\mu H|^2 \to \frac{c_{HD}\mathrm{v}^2}{2\Lambda^2}\frac{(g_L^2+g_Y^2)\mathrm{v}^2}{8}Z_\mu Z_\mu + \dots$$

Thus it modifies the Z boson mass: $m_Z^2 = \frac{(g_L^2+g_Y^2)\mathrm{v}^2}{4}\left(1+\frac{c_{HD}\mathrm{v}^2}{2\Lambda^2}\right)$

We cannot use the Z-boson mass measurement to constrain new physics because, it is one of the inputs to determine the electroweak parameters of the SM

In the SM:

$$G_F = \frac{1}{\sqrt{2}\mathrm{v}^2}$$

$$\alpha = \frac{g_L^2 g_Y^2}{4\pi(g_L^2+g_Y^2)}$$

$$m_Z^2 = \frac{(g_L^2+g_Y^2)\mathrm{v}^2}{4}$$

$$g_L = 0.6485$$

$$g_Y = 0.3580$$

$$\mathrm{v} = 246.22\ \mathrm{GeV}$$

with very small errors

Modified interaction strength: stealthily

$|H^\dagger D_\mu H|^2$

In the presence of our dimension-6 operators, the relation between electroweak couplings and observables is disrupted

$$G_F = \frac{1}{\sqrt{2}\mathrm{v}^2} \qquad \alpha = \frac{g_L^2 g_Y^2}{4\pi(g_L^2 + g_Y^2)} \qquad m_Z^2 = \frac{(g_L^2 + g_Y^2)\mathrm{v}^2}{4}\left(1 + \frac{c_{HD}\mathrm{v}^2}{2\Lambda^2}\right)$$

Now we cannot assign numerical values to the electroweak parameters, because they depend on c_{HD}

A useful trick is to get rid of the dimension-6 pollution in the input equations by redefining the SM electroweak parameters

$$g_L \to \tilde{g}_L\left(1 - \frac{c_{HD} g_L^2 \mathrm{v}^2}{4(g_L^2 - g_Y^2)\Lambda^2}\right) \qquad g_Y \to \tilde{g}_Y\left(1 + \frac{c_{HD} g_Y^2 \mathrm{v}^2}{4(g_L^2 - g_Y^2)\Lambda^2}\right)$$

For the twiddle electroweak parameter, we can now assign numerical values

$$G_F = \frac{1}{\sqrt{2}\mathrm{v}^2}$$

$$\alpha = \frac{\tilde{g}_L^2 \tilde{g}_Y^2}{4\pi(\tilde{g}_L^2 + \tilde{g}_Y^2)}$$

$$m_Z^2 = \frac{(\tilde{g}_L^2 + \tilde{g}_Y^2)\mathrm{v}^2}{4}$$

$$\tilde{g}_L = 0.6485$$

$$\tilde{g}_Y = 0.3580$$

$$\mathrm{v} = 246.22 \ \mathrm{GeV}$$

same as in the SM

Modified interaction strength: stealthily

Z mass cannot be used to constrain new physics, because it was already used to set numerical values for the twiddle electroweak parameter

But new physics emerges now in other observables, e.g. in the W mass

$$m_W = \frac{g_L \mathrm{v}}{2} = \frac{\tilde{g}_L \mathrm{v}}{2}\left(1 - \frac{c_{HD} g_L^2 \mathrm{v}^2}{4(g_L^2 - g_Y^2)\Lambda^2}\right) = \frac{\tilde{g}_L \mathrm{v}}{2}\left(1 - \frac{c_{HD} \tilde{g}_L^2 \mathrm{v}^2}{4(\tilde{g}_L^2 - \tilde{g}_Y^2)\Lambda^2}\right)$$

We can now use the experimental measurement of the W mass

$$m_W = \left(80.379 \pm 0.012\right) \ \mathrm{GeV}$$

to constrain the Wilson coefficients

$$-\frac{1}{(7 \ \mathrm{TeV})^2} \leq \frac{c_{HD}}{\Lambda^2} \leq -\frac{1}{(12 \ \mathrm{TeV})^2}$$

at 1 sigma

Numerically very different constraint than what one would (incorrectly) obtain from Z mass!

Modified interaction strength: stealthily

Corollary: relation between Wilson coefficients and interaction strength in the Lagrangian depends on the input scheme

Sector	Electroweak	Flavor
SM parameters	$g_L\, g_Y\, \text{v}\, \lambda$	$\lambda\, A\, \rho\, \eta$
Example Input	$G_F\, \alpha(0)\, m_Z\, m_h$	$\Gamma(K \to \mu\nu_\mu)/\Gamma(\pi \to \mu\nu_\mu), \quad \Gamma(B \to \tau\nu_\tau), \quad \Delta M_d, \quad \Delta M_s.$

Modified interaction strength

All of these effects: new vertices and Lorentz structures
+ direct, indirect, and stealthy shifts of the SM interaction strength,
often operate simultaneously

Example, Higgs interactions with gauge bosons in dimension-6 SMEFT:

$$
\begin{aligned}
\mathcal{L}_{\rm hvv} = & \frac{h}{v}[2(1+\delta c_w)m_W^2 W_\mu^+ W_\mu^- + (1+\delta c_z)m_Z^2 Z_\mu Z_\mu \\
& + c_{ww}\frac{g_L^2}{2}W_{\mu\nu}^+ W_{\mu\nu}^- + \tilde{c}_{ww}\frac{g_L^2}{2}W_{\mu\nu}^+ \tilde{W}_{\mu\nu}^- + c_{w\Box} g_L^2 \left(W_\mu^- \partial_\nu W_{\mu\nu}^+ + \text{h.c.}\right) \\
& + c_{gg}\frac{g_s^2}{4}G_{\mu\nu}^a G_{\mu\nu}^a + c_{\gamma\gamma}\frac{e^2}{4}A_{\mu\nu}A_{\mu\nu} + c_{z\gamma}\frac{e g_L}{2c_\theta}Z_{\mu\nu}A_{\mu\nu} + c_{zz}\frac{g_L^2}{4c_\theta^2}Z_{\mu\nu}Z_{\mu\nu} \\
& + c_{z\Box} g_L^2 Z_\mu \partial_\nu Z_{\mu\nu} + c_{\gamma\Box} g_L g_Y Z_\mu \partial_\nu A_{\mu\nu} \\
& + \tilde{c}_{gg}\frac{g_s^2}{4}G_{\mu\nu}^a \tilde{G}_{\mu\nu}^a + \tilde{c}_{\gamma\gamma}\frac{e^2}{4}A_{\mu\nu}\tilde{A}_{\mu\nu} + \tilde{c}_{z\gamma}\frac{e g_L}{2c_\theta}Z_{\mu\nu}\tilde{A}_{\mu\nu} + \tilde{c}_{zz}\frac{g_L^2}{4c_\theta^2}Z_{\mu\nu}\tilde{Z}_{\mu\nu}]
\end{aligned}
$$

Important to properly evaluate all these effects, to correctly capture the correlations between various couplings predicted by SMEFT

Warsaw Basis

Bosonic CP-even	
O_H	$(H^\dagger H)^3$
$O_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
O_{HD}	$\left\lvert H^\dagger D_\mu H\right\rvert^2$
O_{HG}	$H^\dagger H\, G^a_{\mu\nu}G^a_{\mu\nu}$
O_{HW}	$H^\dagger H\, W^i_{\mu\nu}W^i_{\mu\nu}$
O_{HB}	$H^\dagger H\, B_{\mu\nu}B_{\mu\nu}$
O_{HWB}	$H^\dagger\sigma^i H\, W^i_{\mu\nu}B_{\mu\nu}$
O_W	$\epsilon^{ijk}W^i_{\mu\nu}W^j_{\nu\rho}W^k_{\rho\mu}$
O_G	$f^{abc}G^a_{\mu\nu}G^b_{\nu\rho}G^c_{\rho\mu}$

Bosonic CP-odd	
$O_{H\widetilde{G}}$	$H^\dagger H\, \widetilde{G}^a_{\mu\nu}G^a_{\mu\nu}$
$O_{H\widetilde{W}}$	$H^\dagger H\, \widetilde{W}^i_{\mu\nu}W^i_{\mu\nu}$
$O_{H\widetilde{B}}$	$H^\dagger H\, \widetilde{B}_{\mu\nu}B_{\mu\nu}$
$O_{H\widetilde{W}B}$	$H^\dagger\sigma^i H\, \widetilde{W}^i_{\mu\nu}B_{\mu\nu}$
$O_{\widetilde{W}}$	$\epsilon^{ijk}\widetilde{W}^i_{\mu\nu}W^j_{\nu\rho}W^k_{\rho\mu}$
$O_{\widetilde{G}}$	$f^{abc}\widetilde{G}^a_{\mu\nu}G^b_{\nu\rho}G^c_{\rho\mu}$

Table 2.2: Bosonic D=6 operators in the Warsaw basis.

Yukawa	
$[O^\dagger_{eH}]_{IJ}$	$H^\dagger H e^c_I H^\dagger \ell_J$
$[O^\dagger_{uH}]_{IJ}$	$H^\dagger H u^c_I \widetilde{H}^\dagger q_J$
$[O^\dagger_{dH}]_{IJ}$	$H^\dagger H d^c_I H^\dagger q_J$

Vertex	
$[O^{(1)}_{H\ell}]_{IJ}$	$i\bar\ell_I\bar\sigma_\mu\ell_J H^\dagger \overleftrightarrow{D}_\mu H$
$[O^{(3)}_{H\ell}]_{IJ}$	$i\bar\ell_I\sigma^i\bar\sigma_\mu\ell_J H^\dagger\sigma^i \overleftrightarrow{D}_\mu H$
$[O_{He}]_{IJ}$	$ie^c_I\sigma_\mu\bar e^c_J H^\dagger \overleftrightarrow{D}_\mu H$
$[O^{(1)}_{Hq}]_{IJ}$	$i\bar q_I\bar\sigma_\mu q_J H^\dagger \overleftrightarrow{D}_\mu H$
$[O^{(3)}_{Hq}]_{IJ}$	$i\bar q_I\sigma^i\bar\sigma_\mu q_J H^\dagger\sigma^i \overleftrightarrow{D}_\mu H$
$[O_{Hu}]_{IJ}$	$iu^c_I\sigma_\mu\bar u^c_J H^\dagger \overleftrightarrow{D}_\mu H$
$[O_{Hd}]_{IJ}$	$id^c_I\sigma_\mu\bar d^c_J H^\dagger \overleftrightarrow{D}_\mu H$
$[O_{Hud}]_{IJ}$	$iu^c_I\sigma_\mu\bar d^c_J \widetilde{H}^\dagger D_\mu H$

Dipole	
$[O^\dagger_{eW}]_{IJ}$	$e^c_I\sigma_{\mu\nu}H^\dagger\sigma^i\ell_J W^i_{\mu\nu}$
$[O^\dagger_{eB}]_{IJ}$	$e^c_I\sigma_{\mu\nu}H^\dagger\ell_J B_{\mu\nu}$
$[O^\dagger_{uG}]_{IJ}$	$u^c_I\sigma_{\mu\nu}T^a\widetilde{H}^\dagger q_J\, G^a_{\mu\nu}$
$[O^\dagger_{uW}]_{IJ}$	$u^c_I\sigma_{\mu\nu}\widetilde{H}^\dagger\sigma^i q_J\, W^i_{\mu\nu}$
$[O^\dagger_{uB}]_{IJ}$	$u^c_I\sigma_{\mu\nu}\widetilde{H}^\dagger q_J\, B_{\mu\nu}$
$[O^\dagger_{dG}]_{IJ}$	$d^c_I\sigma_{\mu\nu}T^a H^\dagger q_J\, G^a_{\mu\nu}$
$[O^\dagger_{dW}]_{IJ}$	$d^c_I\sigma_{\mu\nu}\bar H^\dagger\sigma^i q_J\, W^i_{\mu\nu}$
$[O^\dagger_{dB}]_{IJ}$	$d^c_I\sigma_{\mu\nu}H^\dagger q_J\, B_{\mu\nu}$

Table 2.3: Two-fermion D=6 operators in the Warsaw basis. The flavor indices are denoted by I, J. For complex operators (O_{Hud} and all Yukawa and dipole operators) the corresponding complex conjugate operator is implicitly included.

$(\bar R R)(\bar R R)$	
O_{ee}	$\eta(e^c\sigma_\mu\bar e^c)(e^c\sigma_\mu\bar e^c)$
O_{uu}	$\eta(u^c\sigma_\mu\bar u^c)(u^c\sigma_\mu\bar u^c)$
O_{dd}	$\eta(d^c\sigma_\mu\bar d^c)(d^c\sigma_\mu\bar d^c)$
O_{eu}	$(e^c\sigma_\mu\bar e^c)(u^c\sigma_\mu\bar u^c)$
O_{ed}	$(e^c\sigma_\mu\bar e^c)(d^c\sigma_\mu\bar d^c)$
O_{ud}	$(u^c\sigma_\mu\bar u^c)(d^c\sigma_\mu\bar d^c)$
O'_{ud}	$(u^c\sigma_\mu T^a\bar u^c)(d^c\sigma_\mu T^a\bar d^c)$

$(\bar L L)(\bar R R)$	
$O_{\ell e}$	$(\bar\ell\bar\sigma_\mu\ell)(e^c\sigma_\mu\bar e^c)$
$O_{\ell u}$	$(\bar\ell\bar\sigma_\mu\ell)(u^c\sigma_\mu\bar u^c)$
$O_{\ell d}$	$(\bar\ell\bar\sigma_\mu\ell)(d^c\sigma_\mu\bar d^c)$
O_{eq}	$(e^c\sigma_\mu\bar e^c)(\bar q\bar\sigma_\mu q)$
O_{qu}	$(\bar q\bar\sigma_\mu q)(u^c\sigma_\mu\bar u^c)$
O'_{qu}	$(\bar q\bar\sigma_\mu T^a q)(u^c\sigma_\mu T^a\bar u^c)$
O_{qd}	$(\bar q\bar\sigma_\mu q)(d^c\sigma_\mu\bar d^c)$
O'_{qd}	$(\bar q\bar\sigma_\mu T^a q)(d^c\sigma_\mu T^a\bar d^c)$

$(\bar L L)(\bar L L)$	
$O_{\ell\ell}$	$\eta(\bar\ell\bar\sigma_\mu\ell)(\bar\ell\bar\sigma_\mu\ell)$
O_{qq}	$\eta(\bar q\bar\sigma_\mu q)(\bar q\bar\sigma_\mu q)$
O'_{qq}	$\eta(\bar q\bar\sigma_\mu\sigma^i q)(\bar q\bar\sigma_\mu\sigma^i q)$
$O_{\ell q}$	$(\bar\ell\bar\sigma_\mu\ell)(\bar q\bar\sigma_\mu q)$
$O'_{\ell q}$	$(\bar\ell\bar\sigma_\mu\sigma^i\ell)(\bar q\bar\sigma_\mu\sigma^i q)$

$(\bar L R)(\bar L R)$	
O_{quqd}	$(u^c q^j)\epsilon_{jk}(d^c q^k)$
O'_{quqd}	$(u^c T^a q^j)\epsilon_{jk}(d^c T^a q^k)$
$O_{\ell equ}$	$(e^c\ell^j)\epsilon_{jk}(u^c q^k)$
$O'_{\ell equ}$	$(e^c\bar\sigma_{\mu\nu}\ell^j)\epsilon_{jk}(u^c\bar\sigma^{\mu\nu}q^k)$
$O_{\ell edq}$	$(\bar\ell\bar e^c)(d^c q)$

Table 2.4: Four-fermion D=6 operators in the Warsaw basis. Flavor indices are suppressed here to reduce the clutter. The factor η is equal to 1/2 when all flavor indices are equal (e.g. in $[O_{ee}]_{1111}$), and $\eta = 1$ otherwise. For each complex operator the complex conjugate should be included.

Full set has 2499 distinct operators, including flavor structure and CP conjugates

Other options are the SILH basis, HISZ basis, Higgs basis, …
though all of the above much less used

Alonso et al 1312.2014, Henning et al 1512.03433

To square or not to square

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{\text{SM}} + \frac{1}{\Lambda}\mathscr{L}_{D=5} + \frac{1}{\Lambda^2}\mathscr{L}_{D=6} + \frac{1}{\Lambda^3}\mathscr{L}_{D=7} + \frac{1}{\Lambda^4}\mathscr{L}_{D=8} + \dots$$

Amplitude truncated at dimension 6:

$$M = M_{\text{SM}} + \frac{1}{\Lambda^2}M_6$$

Observables depend on amplitude squared:

$$|M|^2 = |M_{\text{SM}}|^2 + \frac{1}{\Lambda^2}\left(M_6\bar{M}_{\text{SM}} + \bar{M}_6 M_{\text{SM}}\right) + \frac{1}{\Lambda^4}|M_6|^2$$

SM prediction **dimension-6 correction** **Should I keep this???**

Yes, if SM contribution is zero, as for example in flavor-violating Higgs decays
Yes, if you can argue that dimension-8 contribution is suppressed wrt to dimension-6 squared
Otherwise, no, except to evaluate you uncertainty due to higher orders in SMEFT.

Beyond dimension-6 ?

$$\mathscr{L}_{\mathrm{SMEFT}} = \mathscr{L}_{\mathrm{SM}} + \frac{1}{\Lambda}\mathscr{L}_{D=5} + \frac{1}{\Lambda^2}\mathscr{L}_{D=6} + \frac{1}{\Lambda^3}\mathscr{L}_{D=7} + \frac{1}{\Lambda^4}\mathscr{L}_{D=8} + \cdots$$

As of now:

- **The size of a SMEFT operator basis is known for any reasonable dimension DC**
- **A concrete basis of operators has been constructed up to dimension 9**

When it makes sense to include operators with dimension higher than six, in a phenomenological analysis ?

Beyond dimension-6 ?

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{\text{SM}} + \frac{1}{\Lambda}\mathscr{L}_{D=5} + \frac{1}{\Lambda^2}\mathscr{L}_{D=6} + \frac{1}{\Lambda^3}\mathscr{L}_{D=7} + \frac{1}{\Lambda^4}\mathscr{L}_{D=8} + \dots$$

Sometimes, a qualitatively new phenomenon arises at higher dimensions

At tree level, light-by-light scattering receives contribution from dimension-8, which in some situations may compete with lower order loop contributions

$$\mathscr{L}_{D=8} \supset (B_{\mu\nu}B_{\mu\nu})^2 + \dots$$

Neutron-antineutron oscillations arise at dimension-9

$$\mathscr{L}_{D=9} \supset \epsilon_{abc}\epsilon_{def}(\bar{d}_a\bar{d}_d)(q_b q_e)(q_c q_f) + \dots$$

In all such cases however, you need to argue why you don't expect any larger effects of new physics from operators of lower dimensions

Thank You