Lattice computation of nucleon sigma terms

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BMW collaboration


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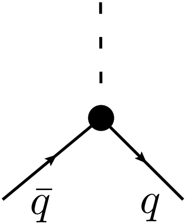
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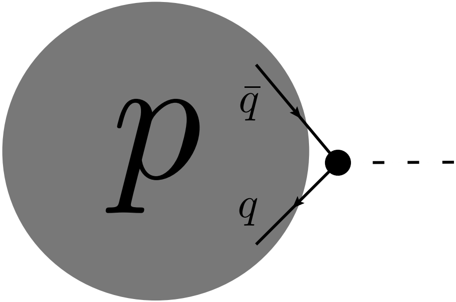
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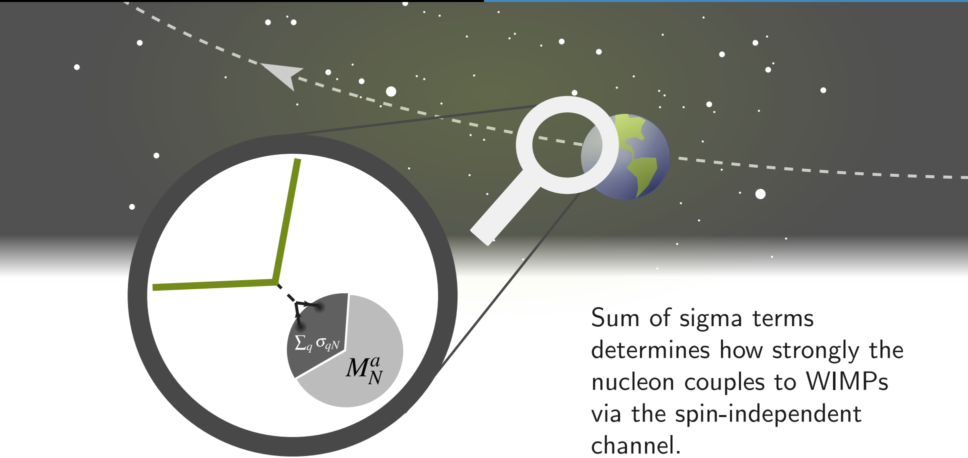


Sigma terms and dark matter detection

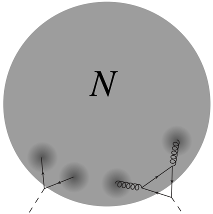
The Higgs field and WIMP dark matter interacts with the nucleon via the scalar quark density.

Sum of sigma terms determines how strongly the nucleon couples to WIMPs via the spin-independent channel.

Sigma terms are required for the interpretation of direct dark matter detection experiments. [1]

[1] J. Ellis, N. Nagata and K. A. Olive, Eur. Phys. J. C **78** (2018) no.7, 569 [arXiv:1805.09795].

Nucleon sigma terms?



Nucleon couples to the external Higgs field. That coupling is mediated by the couplings g_q of the quark flavors q to the Higgs field.

Elementary fermions have a mass, which is proportional to the fermion-Higgs coupling g_f. As a consequence $m_f = g_f \frac{\partial m_f}{\partial g_f}$.

We can define this logarithmic derivative also for non-elementary particles, like the nucleon:

$$M_N f_{qN} = \sigma_{qN} := g_q \frac{\partial M_N}{\partial g_q} = m_q \frac{\partial M_N}{\partial m_q}$$

The Feynman-Hellmann theorem states

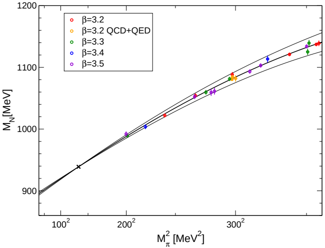
$$\sigma_{qN} = m_q \frac{\partial M_N}{\partial m_q} = m_q \langle N \mid \bar{q}q \mid N \rangle \quad , \quad \langle N \mid N \rangle = 1$$

$$M_N = \sum_q \underbrace{m_q \langle N \mid \bar{q}q \mid N \rangle}_{\sigma_{qN}} + \langle N \mid O_{\text{rest}} \mid N \rangle$$

However, the nucleon states do also depend on the quark mass. Therefore

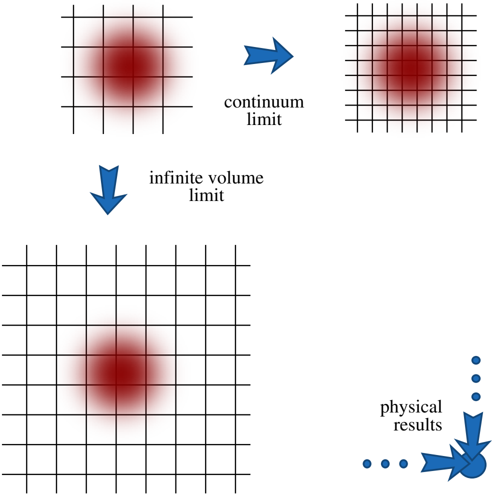
$$M_N = \langle N(m_q) \mid H(m_q) \mid N(m_q) \rangle,$$

$$M_N^{(\phi)} - M_N^{(\chi)} = \sigma_{qN} + \mathcal{O}(m_q^{(\phi)2}).$$



Lattice QCD

Calculations can only be done with a finite number of lattice sites $\rightarrow$ Extrapolations to small lattice spacing and large volumina necessary.



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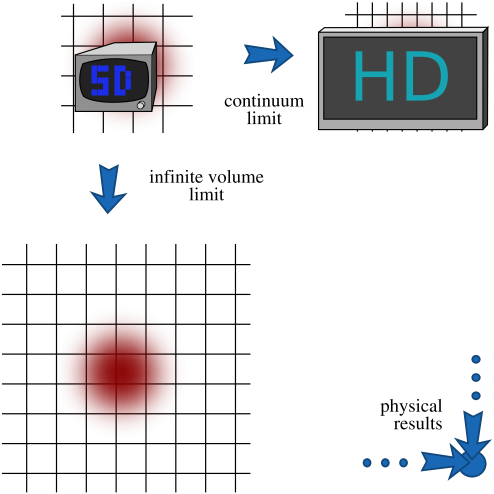

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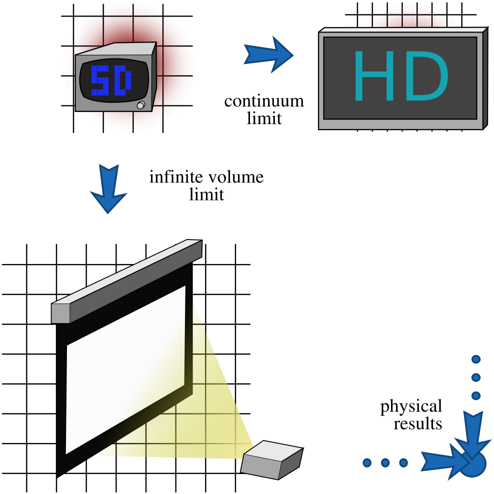
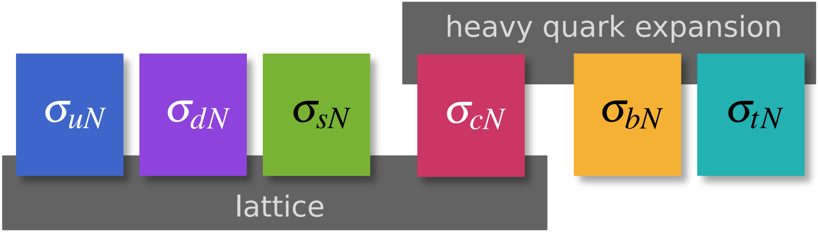


How to determine the sigma terms?



Up, down, strange, and charm sigma terms can be determined on the lattice.

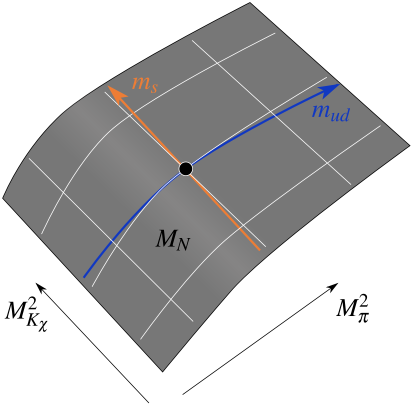
For the top, bottom, and charm sigma terms an expansion in $1/m_q$ can be employed. The validity can be checked in case of σ_{cN}, where the $\mathcal{O}(m_q^{-2})$ effects are biggest.

σ_{uN} σ_{dN} σ_{sN}

Strategy for the light and strange sigma terms:

1. Determine $M_N(M_\pi^2, M_{K_\chi}^2)$ from fit to lattice data. Use Wilson fermions.
2. Calculated $J = \partial \log M_{\text{meson}}^2 / \partial \log m_{\text{quarks}}$ to get light and strange sigma term. Use staggered fermions.
3. Use isospin splitting relationships to disentangle up and down sigma terms.

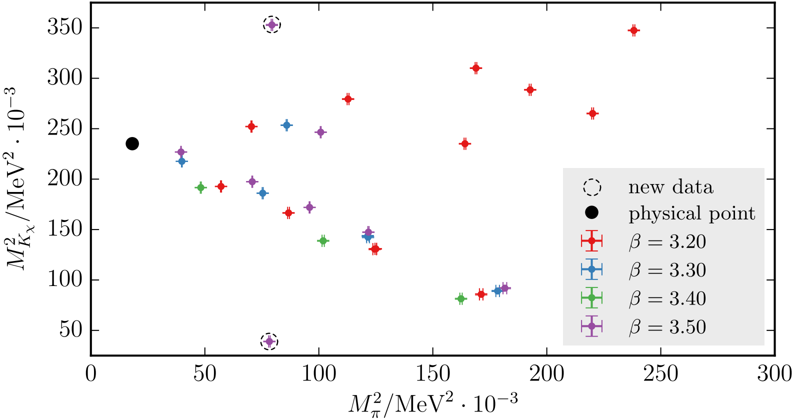
Note that we, because of the logarithmic derivative, do not need to determine the renormalization factors for the quark masses.



$$M_{K_\chi}^2 = 2M_K^2 - M_\pi^2$$

σ_{uN} σ_{dN} σ_{sN}

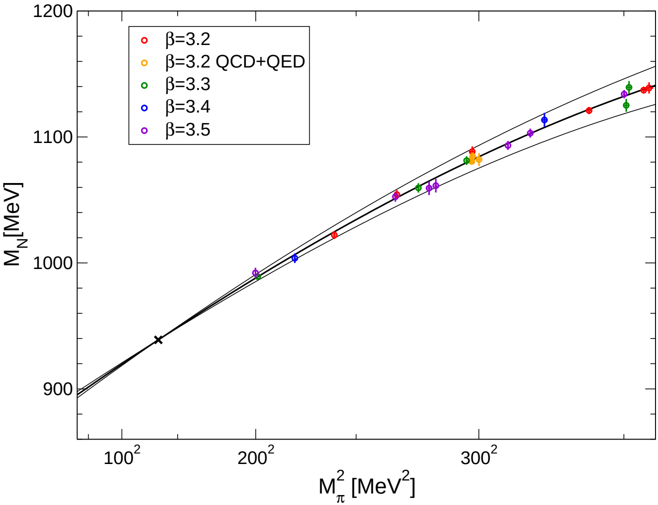
Wilson configurations



Datapoints at four different lattice spacings $a = (0.060.10)$ **fm**.

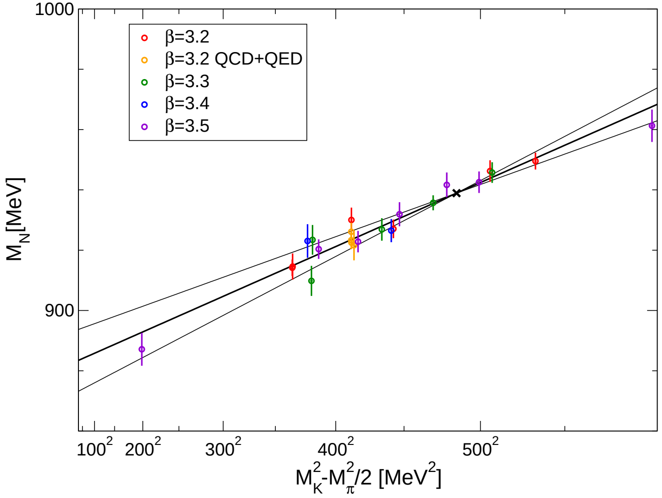
σ_{uN} σ_{dN} σ_{sN}

Nucleon fits:



σ_{uN} σ_{dN} σ_{sN}

Nucleon fits:



σ_{uN} σ_{dN} σ_{sN}

Mixing matrix: We can use a simple quadratic expansion around the physical point

$$M^2_{\text{meson}} = c_0 + (c'_{1,ud} + d_{1,ud}a^2)\Delta_{ud} + (c'_{1,s} + d_{1,s}a^2)\Delta_s + c_{2,ud,s}\Delta_{ud}\Delta_s + c_{2,ud}\Delta^2_{ud} + c_{2,s}\Delta^2_s + c_{c/s}\Delta_{c/s}$$

where we used

$$\Delta_{ud} = \frac{m_{ud}(r_0 + r_1 a^2)}{m_s^{(\phi)}[\beta]} - 1,$$

$$\Delta_s = \frac{m_s}{m_s^{(\phi)}[\beta]} - 1$$

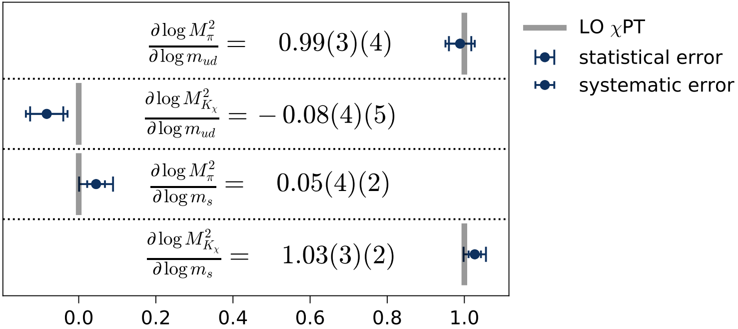
$$\Delta_{c/s} = \frac{m_c}{m_s} - \left(\frac{m_c}{m_s}\right)^{(\phi)}$$

Note that $m_x^{(\phi)}[\beta]$ is an additional fit parameter per β.

Mixing matrix:

σ_{uN} σ_{dN} σ_{sN}

The results for the mixing matrix are:



The result for the strange to light quark mass ratio are

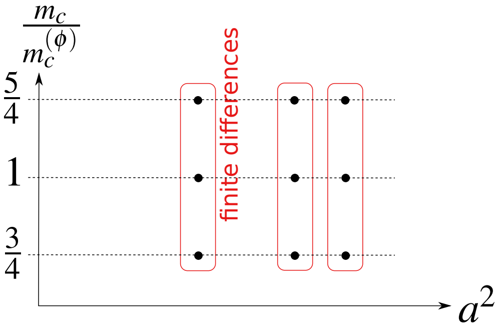
$$\frac{m_s}{m_{ud}} = 27.293(33)(08)$$

(FLAG result: $\frac{m_s}{m_{ud}} = 27.30(34)$).

Strategy for the charm sigma term:

σ_{cN}

We measured the nucleon mass on 9 staggered, Symmanzik improved, 4 times stout smeared ensembles:



We cover

- Three lattice spacings: $\beta = 3.75$, $\beta = 3.7753$, and $\beta = 3.84$.
- Three charm masses: $\frac{3}{4}m_c^{(\phi)}$, $m_c^{(\phi)}$, $\frac{5}{4}m_c^{(\phi)}$.

We use finite differences to determine the charm sigma term.

σ_{cN}

Finite difference approximation: For each lattice spacing we used finite differences to estimate the charm sigma term. We used two differences:

$$\Delta^+ M_N = M_N(m_c = {}^5\!/\!_4 m_c^{\text{central}}) - M_N(m_c = m_c^{\text{central}})$$
$$\Delta^- M_N = M_N(m_c = m_c^{\text{central}}) - M_N(m_c = {}^3\!/\!_4 m_c^{\text{central}})$$

We combined them in two ways:

- The standard finite difference formula (error: $\mathcal{O}((\delta m_c/m_c)^2) = \mathcal{O}(1/16)$)

$$\sigma_{cN} = m_c \frac{\partial M_N}{\partial m_c} = 2 \frac{\Delta^+ M_N + \Delta^- M_N}{M_N^{(\phi)}}.$$

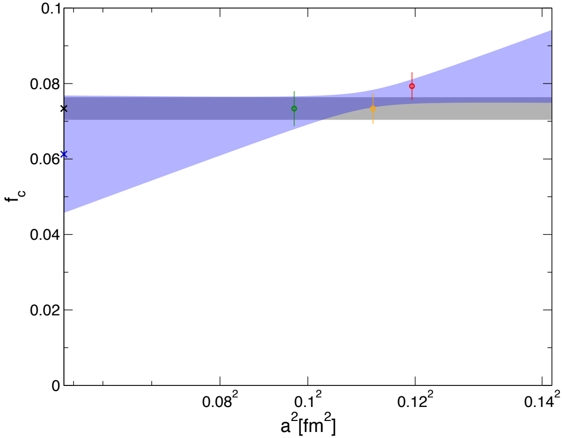
- Based on the HQ behaviour of sigma terms (error: $\mathcal{O}((\sigma_{cN}/M_N^{(\phi)})^3) = \mathcal{O}(3 \times 10^{-4})$)

$$\sigma_{cN} = \frac{1}{\log \frac{5}{4} \log \frac{4}{3} \log \frac{5}{3}} \left(\log^2 \frac{4}{3} \Delta^+ M_N + \log^2 \frac{5}{4} \Delta^- M_N \right)$$

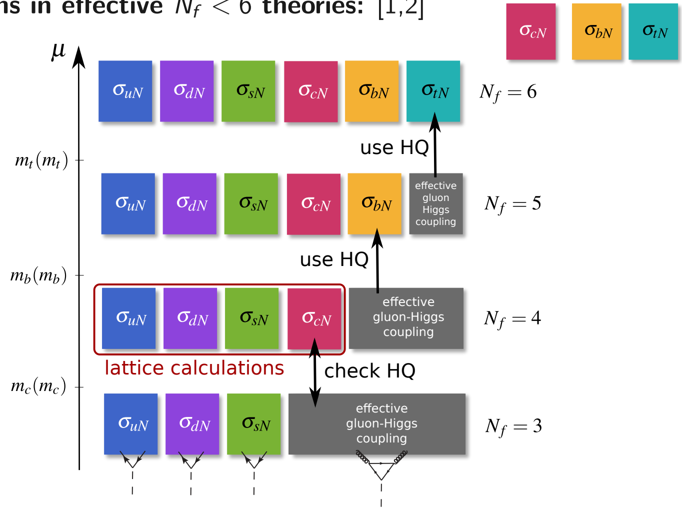
σ_{cN}

For the continuum extrapolation we used:

1. A constant fit in a^2 with only the two finest lattice spacings included.
2. A constant fit in a^2 with all lattice spacings included.
3. A linear fit in a^2 with all lattice spacings included.



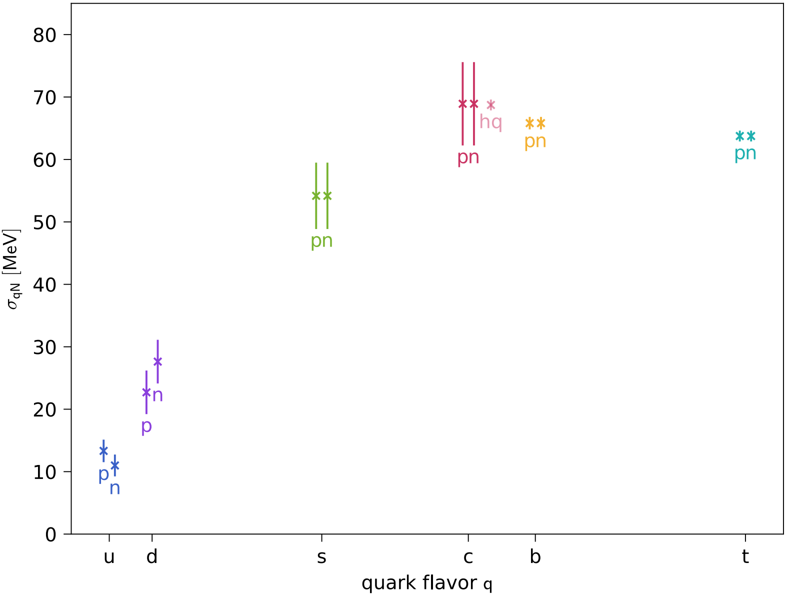
Sigma terms in effective $N_f < 6$ theories: [1,2]



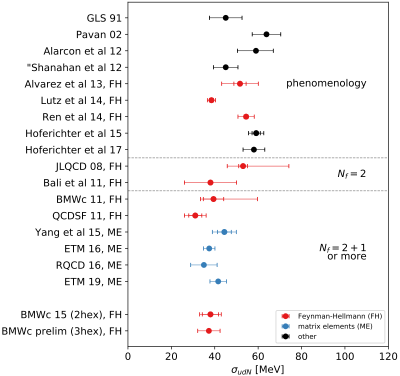
[1] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Phys. Lett. **78B** (1978) 443.

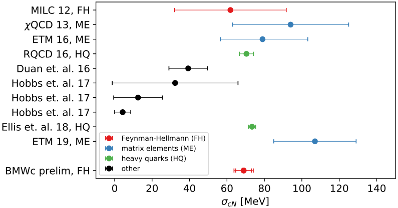
[2] R. J. Hill and M. P. Solon, Phys. Rev. D **91** (2015) 043505 [arXiv:1409.8290].

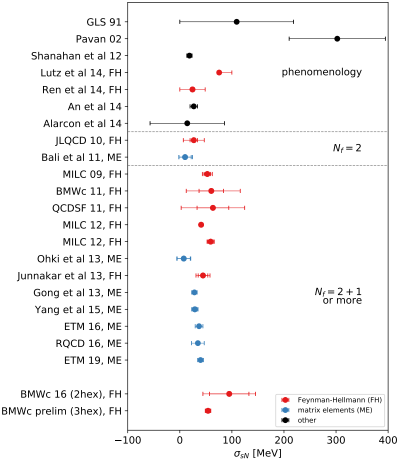
Results


80
70
60
50
40
30
20
10
0
σ_qN [MeV]
p
n
p
n
pn
pn
hq
pn
pn
u
d
s
c
b
t
quark flavor q

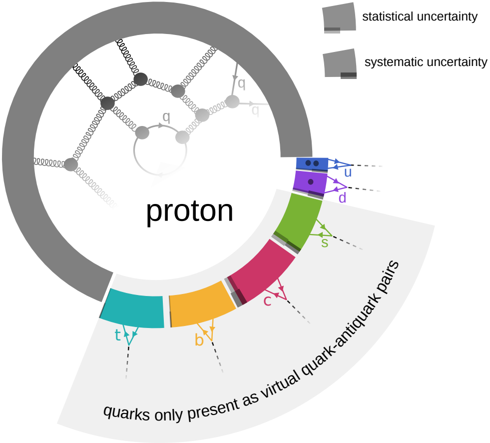
Other determinations







Higgs coupling of the nucleon

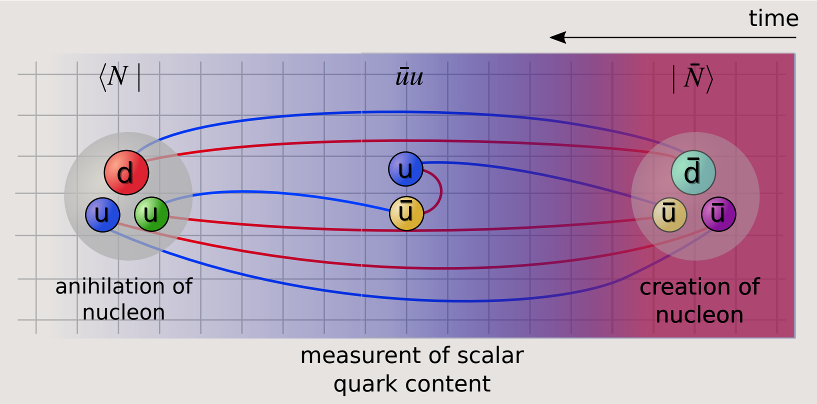


The nucleons couple only $\sim \frac{1}{3}$ as strongly to the Higgs field as fundamental fermions with the same mass would.

backup slides

σ_{uN} σ_{dN} σ_{sN} σ_{cN}

Direct evaluation of matrix elements $\langle N \mid m_q \bar{q} q \mid N \rangle$ requires disconnected contributions:

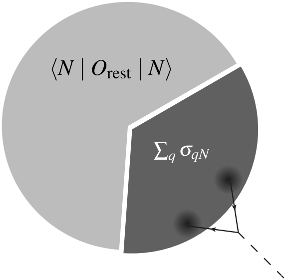


We use the Feynman-Hellmann method: $\frac{\partial M_N}{\partial m_q} = m_q \langle N \mid \bar{q} q \mid N \rangle$

Sigma terms and the mass of the nucleon

The Hamiltonian of QCD can be decomposed like [1]

$$H = \sum_q m_q \bar{q}q + O_{\text{rest}} \quad , \quad \frac{\partial O_{\text{rest}}}{\partial m_q} = 0$$



The Feynman-Hellmann theorem states

$$\sigma_{qN} = m_q \frac{\partial M_N}{\partial m_q} = m_q \langle N \mid \bar{q}q \mid N \rangle \ , \ \ \langle N \mid N \rangle = 1$$

so that $M_N = \sum_q \sigma_{qN} + \langle N \mid O_{\text{rest}} \mid N \rangle$.

[1] X. D. Ji, Phys. Rev. D **52** (1995) 271 [hep-ph/9502213].

Sigma terms and the mass of the nucleon

What is $\langle N \mid O_{\text{rest}} \mid N\rangle$?

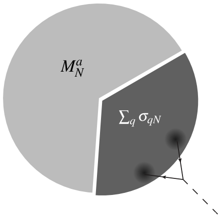
We look at the trace $\theta^{\mu}{}_{\mu}$ of the energy momentum tensor [1]:

$$\langle N \mid \theta^{\mu}{}_{\mu} \mid N\rangle = \underbrace{\langle N \mid \sum_q m_q \bar{q}q \mid N\rangle}_{\sum_q \sigma_{qN}} + \underbrace{\langle N \mid \gamma_m \sum_q m_q \bar{q}q + \frac{\beta}{g} G^2 \mid N\rangle}_{\text{anomaly contribution } M_N^a} = M_N$$

Note: $\gamma_m \sum_q m_q \bar{q}q + \frac{\beta}{g} G^2$ does not depend on m, because the $\gamma_m m$-term cancels with a contribution in the renormalized G^2-term.

We find $\langle N \mid O_{\text{rest}} \mid N\rangle = M_N^a$ and

$$M_N = \sum_q \sigma_{qN} + M_N^a$$



[1] X. D. Ji, Phys. Rev. D **52** (1995) 271 [hep-ph/9502213].

Is $M_N = \sum_q \sigma_{qN} + M_N^a$ **the** decomposition of the nucleon mass?

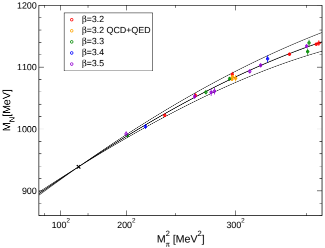
It is **a** decomposition in the following sense:

- It is a sum of *positive contributions*.
- Each contribution is a *scale and scheme independent* observable in QCD.
- The individual contributions have a *clear physical meaning*. (experiments!)
- The contributions are properties of the *physical nucleon state*, making no references to unphysical theories.

However, the nucleon states do also depend on the quark mass. Therefore

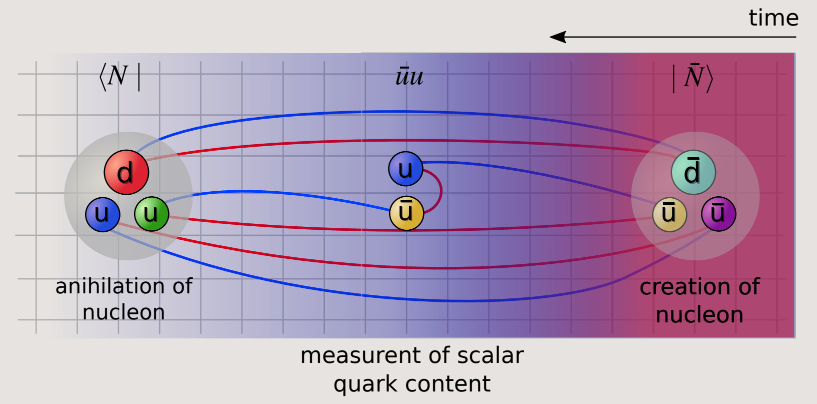
$$M_N = \langle N(m_q) \mid H(m_q) \mid N(m_q) \rangle,$$

$$M_N^{(\phi)} - M_N^{(\chi)} = \sigma_{qN} + \mathcal{O}(m_q^{(\phi)2}).$$



σ_{uN} σ_{dN} σ_{sN} σ_{cN}

Direct evaluation of matrix elements $\langle N \mid m_q \bar{q} q \mid N \rangle$ requires disconnected contributions:



We use the Feynman-Hellmann method: $\frac{\partial M_N}{\partial m_q} = m_q \langle N \mid \bar{q} q \mid N \rangle$

σ_{uN} σ_{dN} σ_{sN}

Nucleon fits:

To determined finite volume effects, we had to use our ensembles with electromagnetic interactions, which feature the same parameters at several volumes. There, we used the neutron mass M_n instead of the nucleon mass $1/2(M_p + M_n)$.

Our fit function is of the form

$$M_N(v_i) = M_N^{(\phi)} \prod_i (1 + c_i \Delta v_i)^{t_i}$$

with

$$\Delta v_i = \left(v_i - v_i^{(\phi)}\right)$$

i	v_i	possible t_i values
1	M_π^2	$+1$
2	M_{K_χ}	$+1$/-1
3	$M_\pi^{\frac{1}{2}} L^{-\frac{3}{2}} e^{-M_\pi L}$	$+1$
4	M_π^3	$+1$
5	M_π^4	$+1$
6	$\alpha_s a M_\pi^2$	$+1$
7	$\alpha_s a M_{K_\chi}^2$	$+1$
8	$a^2 M_\pi^2$	$+1$
9	$a^2 M_{K_\chi}^2$	$+1$

σ_{uN} σ_{dN} σ_{sN}

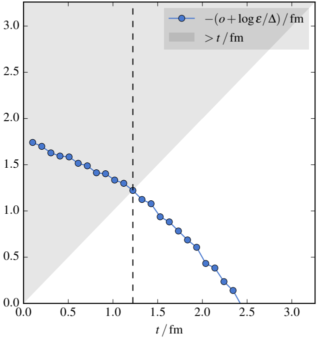
Nucleon fits:

Data at large M_π^2 is more precise then then at low M_π^2. $\Rightarrow$ We had to determine the Plateau-Range as a function of the statistical accuracy of the data. Effective mass behaves as

$$M^{\text{eff}}(t) = M - \frac{1}{\Delta t} \ln\left(1 + c_1 \exp(-(M' - M)t)\right).$$

Demanding that the shift to $M^{\text{eff}}(t)$ due to the excited state is smaller then some factor of the relative error $\epsilon(t)$ of the effective mass, we arrive at

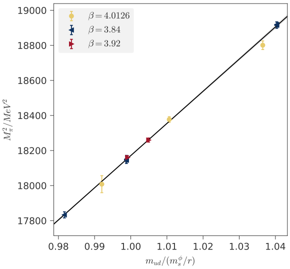
$$t > -\frac{\ln\left(\frac{\epsilon(t)}{\mu}\right)}{M' - M} + T$$

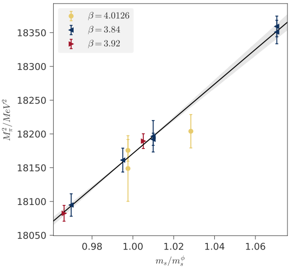


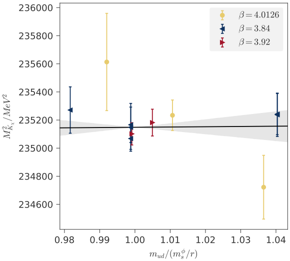
σ_{uN} σ_{dN} σ_{sN}

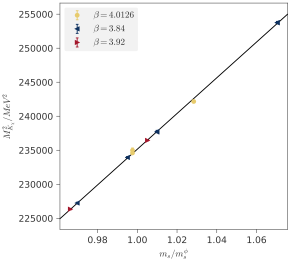
Mixing matrix:

preliminary









σ_{cN}

Determination of the nucleon mass:

With staggered fermions the nucleon propagator behaves like:

$$C_O(t) = \sum_{i=0}^{N_{\text{states}}-1} a_i p_i^{t+1}(\exp(-m_i t) + (-1)^{t+1} \exp(-m_i(N_t - t)))$$

We need not only the mass, but a propagator with the parity partner removed. To that end we construct operators of the form

$$O' = \sum_{\tau=0}^{m} b_j \exp(H\tau) O \exp(-H\tau)$$

and apply a variational method.

σ_{cN}

We construct the matrix [1-3]

$$M(t) = \begin{pmatrix} C_O(t) & C_O(t+1) & \dots & C_O(t+m) \\ C_O(t+1) & C_O(t+2) & \dots & C_O(t+m+1) \\ \vdots & \vdots & \ddots & \vdots \\ C_O(t+m) & C_O(t+m+2) & \dots & C_O(t+2m) \end{pmatrix}.$$

and solve the GEVP [4]

$$M(t_0)\vec{v}_i(t_0,t_1) = M(t_1)\lambda_i(t_0,t_1)\vec{v}_i(t_0,t_1).$$

We get the correlation functions of the O' via

$$C_i'(t;t_0,t_1) = \vec{v}_i^{\dagger}(t_0,t_1)M(t)\vec{v}_i(t_0,t_1)$$

[1] C. Aubin and K. Orginos, "A new approach for Delta form factors," AIP Conf. Proc. **1374** (2011) 621 doi:10.1063/1.3647217 [arXiv:1010.0202 [hep-lat]]. CITATION = doi:10.1063/1.3647217;18 citations counted in INSPIRE as of 23 Jan 2018

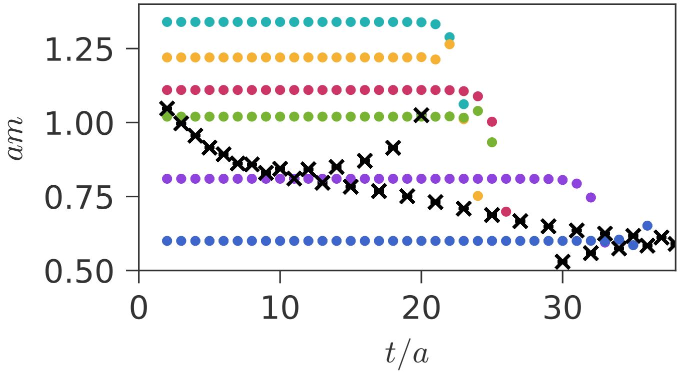
[2] Y. Hua, and T. Sarkar, IEEE transactions on antennas and propagation **37**, 229–234 (1989)

[3] T. Sarkar, and O. Pereira, IEEE Antennas and Propagation Magazine **37**, 48–55 (1995)

[4] C. DeTar and S. H. Lee, "Variational method with staggered fermions," Phys. Rev. D **91** (2015) no.3, 034504 [arXiv:1411.4676 [hep-lat]]. CITATION = doi:10.1103/PhysRevD.91.034504;3 citations counted in INSPIRE as of 03 Jun 2019

σ_{cN}

We performed a study with mock data:



Black: Original correlation function. Colored: "Projected" correlation functions.

σ_{cN}

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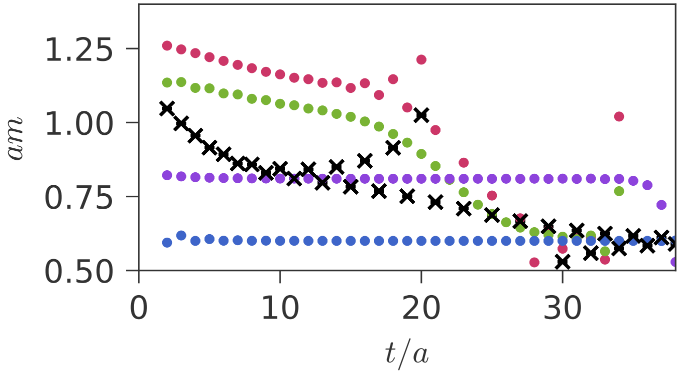
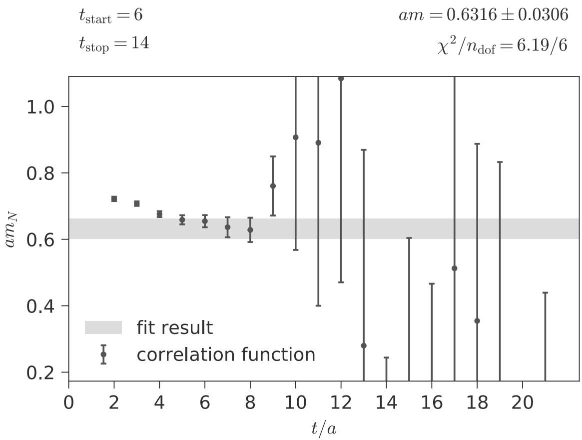


Black: Original correlation function. Colored: "Projected" correlation functions.

σ_{cN}

This method is well suited for the extraction of ground states in the presence of many excited (oscillating) states.

For the nucleon the "projected" correlation function behaves like



σ_{cN}

As an alternative we used

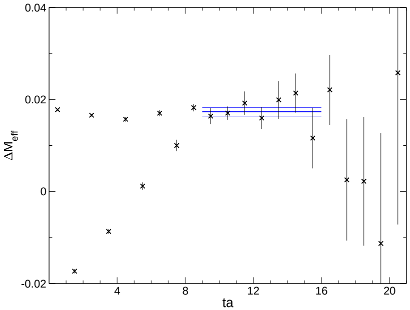
$$D_O(t) = C_O(t) + e^{\tilde{m}} C_O(t+1)$$

We defined the effective mass

$$m^{\text{eff}}(t) = -\log\left(\frac{D_O(t)}{D_O(t+1)}\right)$$

and its average between the times t_a and t_b. We then minimized the deviation of the effective mass from the averaged effective mass.

The results agree very well with the GEVP based method.



σ_{bN} σ_{tN}

To leading order [1,2] the

$$f_{hN} = \frac{1}{b_0}(1-\lambda) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{m_h}, \alpha_s\right)$$

where h denotes the heavy $(n+1)$-th flavor and $\lambda = \sum_{q=1}^{n} f_{qN}$. In [2] corrections up to order $\mathcal{O}(\alpha_s^3)$ are calculated. They take the form

$$f_{hN} = \sum_{i=0}^{3}(b_i^n - c_i^n\lambda)\alpha_s^i.$$

[1] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Phys. Lett. **78B** (1978) 443.

[2] R. J. Hill and M. P. Solon, Phys. Rev. D **91** (2015) 043505 [arXiv:1409.8290].