# Prospects for $b \rightarrow s \nu \bar{\nu}$ and implications from b-anomalies 

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September 30, 2020



## $b \rightarrow s \ell^{+} \ell^{-}$transition



- FCNC process, which is loop supressed in the SM (potential sensitivity to NP)
- We don't actually see quarks, we see hadrons (mesons or baryons).
- Several studies have been done for this transition on the meson side $\left(B \rightarrow K^{(*)} \ell^{+} \ell^{-}, B_{s} \rightarrow \phi \ell^{+} \ell^{-}\right)$.



## B-physics anomalies

Deviations from the SM expectations in $b \rightarrow s \ell^{+} \ell^{-}$:

- Around $2 \sigma-3 \sigma$ for branching ratios and angular observables in $b \rightarrow s \mu \mu\left(B \rightarrow K \mu \mu, B \rightarrow K^{*} \mu \mu\right.$, $\left.B_{s} \rightarrow \phi \mu \mu\right)$



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$$
R_{K^{(*)}}=\frac{\mathcal{B}\left(B \rightarrow K^{(*)} \mu \mu\right)}{\mathcal{B}\left(B \rightarrow K^{(*)} e e\right)}
$$

- Around $2.5 \sigma$ for LFU ratio comparing $b \rightarrow s \mu \mu$ and $b \rightarrow \operatorname{see}\left(B \rightarrow K \ell^{+} \ell^{-}, B \rightarrow K^{*} \ell^{+} \ell^{-}\right)$

[1903.09252]

[1705.05802]


## $b \rightarrow$ sll: Effective Hamiltonian

Local operator effective theory for $b \rightarrow s$ transitions. Non-local high energy processes are reduced to local operators as in Fermi Theory.

$$
\mathcal{H}_{\text {eff }}\left(b \rightarrow s \ell^{+} \ell^{-}\right)=-\frac{4 G_{F}}{\sqrt{2}} V_{t b} V_{t s}^{*} \sum_{i} \mathcal{C}_{i} \mathcal{O}_{i}
$$

With the SM operators relevant for this analysis

$$
\mathcal{O}_{7}=\frac{e}{g^{2}} m_{b}\left(\bar{s} \sigma_{\mu \nu} P_{R} b\right) F^{\mu \nu} \quad \mathcal{O}_{9}=\frac{e^{2}}{g^{2}}\left(\bar{s} \gamma_{\mu} P_{L} b\right)\left(\bar{\ell} \gamma^{\mu} \ell\right) \quad \mathcal{O}_{10}=\frac{e^{2}}{g^{2}}\left(\bar{s} \gamma_{\mu} P_{L} b\right)\left(\bar{\ell} \gamma^{\mu} \gamma_{5} \ell\right)
$$

- Wilson coefficients $\left(\mathcal{C}_{i}\right)$ contain short distance dynamics. They are accurately computed in SM and would deviate in presence of NP. Operators absent or suppressed in the SM, can be introduced by NP.
- We want to constrain these Wilson coefficients from data (done for mesons, now for baryons).
- We will also consider contributions in chirally flipped operators $\left(\mathcal{O}_{7^{\prime}}, \mathcal{O}_{9^{\prime}}, \mathcal{O}_{10^{\prime}}\right)$.



## Global fits of $b \rightarrow$ sौl

- (LFU) NP hints in rare semileptonic B decays indicate significant non-standard effects in muonic final states.
- Smaller effect in electrons is not excluded but not required to fit data.
- $b \rightarrow s \tau \tau$ transitions are at present only poorly constrained.


Main 1D scenarios for $b \rightarrow s \mu \mu$
These prefered scenarios show pulls from the SM of around $\sigma \sigma$

$$
\begin{gathered}
C_{9}^{\mu, \mathrm{NP}} \quad C_{9}^{\mu, \mathrm{NP}}=-C_{10}^{\mu, \mathrm{NP}} \\
C_{9}^{\mu, \mathrm{NP}}=-C_{9^{\prime}}^{\mu, \mathrm{NP}}
\end{gathered}
$$



# Connecting $b \rightarrow s \ell \ell$ with $b \rightarrow s \nu \bar{\nu}$ and $s \rightarrow d \nu \bar{\nu}$ 

 Based on: arXiv:2005.03734S. Descotes-Genon, S. Fajfer, J. F. Kamenik, M. Novoa-Brunet

## $b \rightarrow s \nu \bar{\nu}$ and $s \rightarrow d \nu \bar{\nu}$

- Why are these modes interesting?
- Also FCNC!
- Neutrinos and charged leptons in $S U(2)_{L}$ doublets in SM
- Not affected by $c \bar{c}$ contributions.
- Currently being measured and we expect soon future measurements! (NA62, Belle, KOTO, KLEVER, Belle 2)


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- If we are interested in $s \rightarrow d \nu \bar{\nu}$, why not $s \rightarrow d \ell \ell$ ?
- Dominated by long distances.
- Summation over neutrinos of all families (compared to processes with charged leptons of a single family).
- We can try to connect $b \rightarrow s \ell \ell$ with $b \rightarrow s \nu \bar{\nu}$ and $s \rightarrow d \nu \bar{\nu}$ following the work done for $b \rightarrow c \ell \nu$ [Bordone et al. 2017].


## Connecting $b \rightarrow s \ell \ell$ with $b \rightarrow s \nu \bar{\nu}$ and $s \rightarrow d \nu \bar{\nu}$

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## Connecting $b \rightarrow s \ell \ell$ with $b \rightarrow s \nu \bar{\nu}$ and $s \rightarrow d \nu \bar{\nu}$

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- If not, what are the most general assumptions we can make to connect them?
- What are the implications of the $b \rightarrow s \ell \ell$ constraints on these modes?


## NP from an EFT Lagrangian

Writting the operators in the down-quark and charged lepton mass basis $Q_{L}^{i}=\left(V_{j i}^{\mathrm{CKM} *} u_{L}^{j}, d_{L}^{i}\right)^{T}$ and $L_{L}^{\alpha}=\left(U_{\beta \alpha}^{\mathrm{PMNS}} \nu_{L}^{\beta}, \ell_{L}^{\alpha}\right)^{T}$

$$
\begin{aligned}
& \mathcal{L}_{\text {eff. }}=\mathcal{L}_{\mathrm{SM}}-\frac{1}{v^{2}} \lambda_{i j}^{q} \lambda_{\alpha \beta}^{\ell}\left[C_{T}\left(\bar{Q}_{L}^{i} \gamma_{\mu} \sigma^{a} Q_{L}^{i}\right)\left(\bar{L}_{L}^{\alpha} \gamma^{\mu} \sigma^{a} L_{L}^{\beta}\right)\right. \\
& +C_{S}\left(\bar{Q}_{L}^{i} \gamma_{\mu} Q_{L}^{i}\right)\left(\bar{L}_{L}^{\alpha} \gamma^{\mu} L_{L}^{\beta}\right)+C_{R L}^{\prime}\left(\bar{d}_{R}^{i} \gamma_{\mu} d_{R}^{i}\right)\left(\bar{L}_{L}^{\alpha} \gamma^{\mu} L_{L}^{\beta}\right) \\
& \left.+C_{L R}^{\prime}\left(\bar{Q}_{L}^{i} \gamma_{\mu} Q_{L}^{i}\right)\left(\bar{\ell}_{R}^{\alpha} \gamma^{\mu} \ell_{R}^{\beta}\right)+C_{R R}^{\prime}\left(\bar{d}_{R}^{i} \gamma_{\mu} d_{R}^{i}\right)\left(\bar{\ell}_{R}^{\alpha} \gamma^{\mu} \ell_{R}^{\beta}\right)\right]
\end{aligned}
$$

We assume that the same flavour structure encoded in $\lambda_{i j}^{q}$ and $\lambda_{\alpha \beta}^{\ell}$ holds for all operators.

## Quark Sector: $U(2)_{q}$ and (General) Minimal Flavour

## Violation

- We classify the NP flavour structure in terms of an approximate $U(2)_{q=Q, D}$ flavour symmetry.

$$
\mathbf{q} \equiv\left(q_{L}^{1}, q_{L}^{2}\right) \sim(\mathbf{2}, \mathbf{1}) \quad \mathbf{d} \equiv\left(d_{R}^{1}, d_{R}^{2}\right) \sim(\mathbf{1}, \mathbf{2}) \quad d_{R}^{3}, q_{L}^{3} \sim(\mathbf{1}, \mathbf{1})
$$

- In the exact $U(2)_{q}$ limit only $\lambda_{33}^{q}$ and $\lambda_{11}^{q}=\lambda_{22}^{q}$ are non-vanishing.
- Departures from the $U(2)_{q}$ limit manifest through non-diagonal terms $\left(\lambda_{i \neq j}^{q}\right)$.
- We may impose the leading NP $U(2)_{q}$ breaking to be aligned with the SM Yukawas, yielding a (G)MFV structure [D'Ambrosio et al. 2002; Kagan et al. 2009]

$$
d_{L}^{3}=b_{L}+\theta_{q} e^{i \phi_{q}}\left(V_{t d} d_{L}+V_{t s} s_{L}\right)
$$

- In (G)MFV, the chirally flipped $\mathcal{C}_{i^{\prime}}$ are suppressed.


## Lepton Sector: $U(1)_{\ell}^{3}$ symmetry

We assume an approximate $U(1)_{\ell}^{3}$ symmetry (broken only by the neutrino masses) yielding $\lambda_{i \neq j}^{\ell} \simeq 0$ in order to fulfill LFV limits. We then consider three possible scenarios:
(1) The simplest $\lambda_{\mu \mu}^{\ell} \neq 0 ; \lambda_{e e}^{\ell}=\lambda_{\tau \tau}^{\ell}=0$
(2) The anomaly-free assignment $\lambda_{\mu \mu}^{\ell}=-\lambda_{\tau \tau}^{\ell} ; \lambda_{e e}^{\ell}=0$
(3) The hierarchical charge scenario $\lambda_{e e}^{\ell} \ll \lambda_{\mu \mu}^{\ell} \ll \lambda_{\tau \tau}^{\ell}$ (For concreteness we consider $\lambda_{\alpha \alpha}^{\ell} / \lambda_{\mu \mu}^{\ell}=m_{\alpha} / m_{\mu}$ )

## Relating (G)MFV Lagrangian with WET

Leading order in $U(2)_{q}$ breaking

$$
\begin{aligned}
& b \rightarrow s \ell \ell\left\{\begin{array}{lll}
C_{9}^{\mu, \mathrm{NP}} & \propto & \lambda_{\mu \mu}^{\ell} l_{33}^{q}\left(\left(C_{T}+C_{S}\right)+C_{L R}^{\prime}\right)+\mathcal{O}\left(\lambda_{23}^{q}\right) \\
C_{1, \mathrm{NP}}^{\mu} & \propto & \lambda_{\mu \mu}^{\ell} \mu_{33}^{q}\left(-\left(C_{T}+C_{S}\right)+C_{L R}^{\prime}\right)+\mathcal{O}\left(\lambda_{23}^{q}\right) \\
C_{9}^{\mu, N P} & \propto & \lambda_{\mu \mu}^{l} \mu_{23}^{q}\left(C_{R R}^{\prime}+C_{R L}^{\prime}\right) \\
C_{10^{\prime}}^{\mu, N P} & \propto & \lambda_{\mu \mu}^{\ell} \lambda_{23}^{q}\left(C_{R R}^{\prime}-C_{R L}^{\prime}\right)
\end{array}\right. \\
& \qquad b \rightarrow s \nu \bar{\nu}\left\{\begin{array}{lll}
C_{L}^{\nu_{\alpha}, \mathrm{NP}} & \propto & \lambda_{\alpha \alpha}^{\ell} \lambda_{33}^{q}\left(C_{T}-C_{S}\right)+\mathcal{O}\left(\lambda_{23}^{q}\right) \\
C_{R}^{\nu_{\alpha}, \mathrm{NP}} & \propto & \lambda_{\alpha \alpha}^{\ell} \lambda_{23}^{q} C_{R L}^{\prime}
\end{array}\right. \\
& s \rightarrow d \nu \bar{\nu}\left\{\begin{array}{lll}
C_{s d}^{\nu_{\alpha}, \mathrm{NP}} & \propto & \lambda_{\alpha \alpha}^{\ell} \lambda_{33}^{q}\left(C_{T}-C_{S}\right)+\mathcal{O}\left(\lambda_{23}^{q}\right)+\mathcal{O}\left(\lambda_{13}^{q}\right)
\end{array}\right.
\end{aligned}
$$

- A scenario for NP in $b \rightarrow s \ell \ell$ defines curves or regions in neutrino modes $\left(C_{S}-C_{T}\right.$ vs $\left.C_{S}+C_{T}\right)$.


## Constraints on $b \rightarrow s \nu \bar{\nu}$ and $s \rightarrow d \nu \bar{\nu}$ from $b \rightarrow s \ell l$

We consider the limit of (linear) MFV (only $\mathcal{C}_{9}$ and $\mathcal{C}_{10}$ allowed) in which $b \rightarrow s \nu \bar{\nu}$ and $s \rightarrow d \nu \bar{\nu}$ FCNC transitions are rigidly correlated driven by the combination of Wilson coefficients $C_{S}-C_{T}$. In this limit the NP contribution is the same for all $B \rightarrow h_{s} \nu \bar{\nu}$ modes.

$$
R(X)=\frac{B(X)}{B(X)_{\mathrm{SM}}}
$$

- Shaded: Completely free parameters affecting 2 or 3 neutrinos.
- Purple: Opposite sign $\lambda_{\mu \mu}^{\ell}=-\lambda_{\tau \tau}^{\ell}$
- Red: Only muons $\lambda_{\mu \mu}^{\ell}$
- Brown: Hierarchical $\left(\lambda_{\mu \mu}^{\ell}=\frac{m_{\mu}}{m_{\tau}} \lambda_{\tau \tau}^{\ell}\right)$



## Constraints on $b \rightarrow s \nu \bar{\nu}$ from $b \rightarrow s \ell l$

Out of the (G)MFV limit we allow $\mathcal{C}_{9}, \mathcal{C}_{9}$, NP and we take the other coefficients to 0 . Deviations from the diagonal are driven by $C_{9^{\prime}}$

- Blue: (G)MFV Limit $\left(\mathcal{C}_{9^{\prime}}=0\right)$
- Purple: Opposite sign $\lambda_{\mu \mu}^{\ell}=-\lambda_{\tau \tau}^{\ell}$
- Green: Only muons $\lambda_{\mu \mu}^{\ell}$
- Red: Hierarchical $\left(\lambda_{\mu \mu}^{\ell}=\frac{m_{\mu}}{m_{\tau}} \lambda_{\tau \tau}^{\ell}\right)$
- Grey: Completely free parameters affecting 1, 2 or 3 neutrinos.



## $b \rightarrow s \nu \bar{\nu}$ : experimental prospects at Belle II

## Experimental prospects : $B \rightarrow K^{(*)} \nu \bar{\nu}$

- B meson decay with $b \rightarrow s \nu \bar{\nu}$ transition
- This process has yet to be observed
- Actively searched for in the Belle II experiment.



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Main $b \rightarrow s \nu \bar{\nu}$ processes studied in Belle II:

- $B^{+} \rightarrow K^{+} \nu \bar{\nu}$
- $B^{0} \rightarrow K_{S}^{0} \nu \bar{\nu}$
- $B^{+} \rightarrow K^{*+} \nu \bar{\nu}$
- $B^{0} \rightarrow K^{* 0} \nu \bar{\nu}$


## Experimental prospects : $B \rightarrow K^{(*)} \nu \bar{\nu}$

- B meson decay with $b \rightarrow s \nu \bar{\nu}$ transition
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- Actively searched for in the Belle II experiment.



## Experimental challenges

Two main challenges with the observation of $B \rightarrow K^{(*)} \nu \bar{\nu}$ :

- Neutrinos do not interact with our detector
- Rare decay $\mathcal{B}\left(B \rightarrow K^{(*)} \nu \bar{\nu}\right) \simeq 10^{-6} / 10^{-5}$


## Experimental challenges

Two main challenges with the observation of $B \rightarrow K^{(*)} \nu \bar{\nu}$ :

- Neutrinos do not interact with our detector
$\rightarrow$ Belle II : $e^{+} e^{-}$collisions + clean environment + hermeticity
- Rare decay $\mathcal{B}\left(B \rightarrow K^{(*)} \nu \bar{\nu}\right) \simeq 10^{-6} / 10^{-5}$
$\rightarrow$ Belle II : highest instantaneous luminosity


Collision event @ Belle II


SuperKEKB luminosity calendar

## Background rejection

A typical analysis consists of several selections:

- Reject non- $b \bar{b}$ events using mainly event shape variables
- To distinguish signal from $b \bar{b}$ background exploit mainly :
- Presence of neutrinos (missing 4-momentum)
- Absence of extra particles in the event (veto on extra tracks and cut or fit to extra energy in the calorimeter)



## $B \rightarrow K^{(*)} \nu \bar{\nu}$ reconstruction

- In Belle II, collisions produce pairs of $B$ mesons : $B_{\text {sig }}$ and $B_{\text {tag }}$.
- Collisions of interest : $B_{\text {sig }} \rightarrow K^{(*)} \nu \bar{\nu}$ and $B_{\text {tag }} \rightarrow$ reconstructible final state.
How to "observe" $B_{\text {sig }}$ ?


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- Collisions of interest : $B_{s i g} \rightarrow K^{(*)} \nu \bar{\nu}$ and $B_{t a g} \rightarrow$ reconstructible final state.
How to "observe" $B_{\text {sig }}$ ?
$\rightarrow$ Reconstruct $B_{\text {tag }}+K^{(*)}$ and infer neutrino presence with missing 4-momentum.



## Full Event Interpretation

To reconstruct the tag-side, use of the Full Event Interpreation (FEI) algorithm.

- Key tool in the $B \rightarrow K^{(*)} \nu \bar{\nu}$ search.
- Sequential reconstruction of the exclusive $B_{\text {tag }}$ decay chain.
- Trained on simulation.

tag-side



## Reconstruction efficiency

The total number of reconstructed candidates can be expressed as:

$$
N=\mathcal{L} \times \sigma \times \mathcal{B}\left(B \rightarrow K^{(*)} \nu \bar{\nu}\right) \times \epsilon_{\text {sig }} \times \epsilon_{\text {tag }}
$$

$\epsilon_{t a g}=$ Tag-side reconstruction efficiency $=\Sigma_{i} \epsilon_{i} \times \mathcal{B}_{i}$ (sum over the $\mathcal{O}(10000)$ covered $B_{\text {tag }}$ decay channels)
$\rightarrow$ main experimental challenge : enhancing $\epsilon_{i}$ and $\mathcal{B}_{i}$

## Tagging methods

Two exclusive tagging methods :

- Hadronic tag: The $B_{\text {tag }}$ is required to decay in a fully hadronic channel
- Semileptonic tag: The $B_{\text {tag }}$ is required to decay in a semileptonic channel



## In the Future

Several paths to continue looking for $b \rightarrow s$ processes in Belle II :

- Untagged analysis
- Refinement of standard tagged analysis
- Upgrades/replacements of the FEI algorithm (Deep learning)


## Stay tuned !

## Conclusions

Connecting $b \rightarrow s \ell \ell$ to neutrino modes

- Neutrino FCNC modes like $b \rightarrow s \nu \bar{\nu}$ and $s \rightarrow d \nu \bar{\nu}$ are interesting probes of NP.
- These modes can be related to and constrained by $b \rightarrow s \ell \ell$ through simple assumptions.


## Prospects for $b \rightarrow s \nu \bar{\nu}$

- Specific methods developed to observe $b \rightarrow s \nu \bar{\nu}$ processes in Belle II
- Precise measurements expected by the end of Belle II data taking


## Thank You!

# Prospects for $b \rightarrow s \nu \bar{\nu}$ and implications from b-anomalies 

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## Back up

## Possible Grossman-Nir bound violation at KOTO

- Cannot be explained without invoking isospin breaking NP (We can't explain it with (G)MFV) and additional long-lived neutral final states in the $K_{L}$ decay beyond the three SM neutrinos.
- We note that new CP phases in $s \rightarrow d$ transitions only appear beyond the (G)MFV limit.
- Little can be said about the implications of $b \rightarrow s \mu \mu$ data model independently in this part of parameter space.
- A potential future experimental confirmation of $C_{9,}^{\mu, N P} \neq 0$ could at best provide circumstantial evidence for the presence of $U(2)_{q}$ breaking beyond (G)MFV


## Relating (G)MFV Lagrangian with WET

The rare B decays $B \rightarrow K^{(*)} \nu \bar{\nu}$

$$
\begin{aligned}
\mathcal{B}(B \rightarrow K \nu \bar{\nu}) & =(4.5 \pm 0.7) \times 10^{-6} \frac{1}{3} \sum_{\nu}\left(1-2 \eta_{\nu}\right) \epsilon_{\nu}^{2} \\
\mathcal{B}\left(B \rightarrow K^{*} \nu \bar{\nu}\right) & =(6.8 \pm 1.1) \times 10^{-6} \frac{1}{3} \sum_{\nu}\left(1+1.31 \eta_{\nu}\right) \epsilon_{\nu}^{2} \\
\mathcal{B}\left(B \rightarrow X_{s} \nu \bar{\nu}\right) & =(2.7 \pm 0.2) \times 10^{-5} \frac{1}{3} \sum_{\nu}\left(1+0.09 \eta_{\nu}\right) \epsilon_{\nu}^{2}
\end{aligned}
$$

where $\left\langle F_{L}\right\rangle$ is the longitudinal $K^{*}$ polarisation fraction in $B \rightarrow K^{*} \nu \bar{\nu}$ decays. For each flavour of neutrino $\nu=\nu_{e}, \nu_{\mu}, \nu_{\tau}$, the two NP parameters can in turn be expressed as

$$
\epsilon_{\nu}=\frac{\sqrt{\left|C_{L}^{\nu}\right|^{2}+\left|C_{R}^{\nu}\right|^{2}}}{\left|C_{S M}^{\nu}\right|}, \eta_{\nu}=\frac{-\operatorname{Re}\left(C_{L}^{\nu} C_{R}^{\nu *}\right)}{\left|C_{L}^{\nu}\right|^{2}+\left|C_{R}^{\nu}\right|^{2}},
$$

where $C_{L, R}^{\nu}=C_{L, R}^{\nu, \mathrm{SM}}+C_{L, R}^{\nu, \mathrm{NP}}$ and $C_{L}^{\nu, \mathrm{SM}}=-6.38$ and $C_{R}^{\nu, \mathrm{SM}}=0$ at $\mu=m_{b}$. Including leading $U(2)_{q}$ breaking effects we can write again

$$
\begin{aligned}
C_{L}^{\nu_{\alpha}, \mathrm{NP}} & =-\frac{\pi}{\alpha_{e m} V_{t b} V_{t s}^{*}} \lambda_{33}^{q} \lambda_{\alpha \alpha}^{\ell}\left[V_{t s}^{*} \theta_{q} e^{-i \phi_{q}}+r_{23}\right]\left[C_{S}-C_{T}\right] \\
C_{R}^{\nu_{\alpha}, \mathrm{NP}} & =-\frac{\pi}{\alpha_{e m} V_{t b} V_{t s}^{*}} \lambda_{33}^{q} \lambda_{\alpha \alpha}^{\ell} r_{23} C_{R L}^{\prime}
\end{aligned}
$$

with $\alpha=e, \mu, \tau$.

## Relating (G)MFV Lagrangian with WET

Similarly, the rare kaon decays $K^{+} \rightarrow \pi^{+} \nu \bar{\nu}$ and $K_{L} \rightarrow \pi^{0} \nu \bar{\nu}$ can be conveniently expressed in presence of NP

$$
\begin{aligned}
& \mathcal{B}\left(K^{+} \rightarrow \pi^{+} \nu \bar{\nu}(\gamma)\right)=(8.4 \pm 1.0) \times 10^{-11} \\
& \times \frac{1}{3} \sum_{\nu}\left|1+\frac{C_{s d}^{\nu, \mathrm{NP}}}{V_{t s} V_{t d}^{*} X_{t}+\left(X_{c}+\delta X_{c, u}\right) V_{c s} V_{c d}^{*}}\right|^{2} \\
& \mathcal{B}\left(K_{L} \rightarrow \pi^{0} \nu \bar{\nu}\right)=(3.4 \pm 0.3) \times 10^{-11} \\
& \times \frac{1}{3} \sum_{\nu}\left[1+\operatorname{Im}\left(\frac{C_{s d}^{\nu, \mathrm{NP}}}{V_{t s} V_{t d}^{*} X_{t}}\right)\right]^{2},
\end{aligned}
$$

Numerically, $X_{t}=1.469(17)$ and $\left(X_{c}+\delta X_{c, u}\right)=0.00106(6)$. For each neutrino flavour $\nu=\nu_{e}, \nu_{\mu}, \nu_{\tau}, C_{s d}^{\nu, N P}$ receives contributions from three operators of the weak effective Hamiltonian yielding

$$
\begin{aligned}
& C_{s d}^{\nu_{\alpha}, \mathrm{NP}}=\frac{\pi s_{W}^{2}}{\alpha_{e m}} \lambda_{33}^{q} \lambda_{\alpha \alpha}^{\ell}\left[\theta_{q}^{2} V_{t s} V_{t d}^{*}\left(C_{S}-C_{T}\right)\right. \\
& +\theta_{q}\left(V_{t s} e^{i \phi_{q}} r_{13}^{*}+V_{t d}^{*} e^{-i \phi_{q}} r_{23}\right)\left(C_{S}-C_{T}\right) \\
& \left.+r_{12}\left(C_{S}-C_{T}+C_{R L}^{\prime}\right)\right]
\end{aligned}
$$

where $\alpha=e, \mu, \tau$

## Relating (G)MFV Lagrangian with WET

Linear order in $U(2)_{q}$ breaking

$$
\begin{aligned}
& \left\{C_{9}^{\mu, \mathrm{NP}}=-\frac{\pi}{\alpha_{e m} V_{t b} V_{t s}^{*}} \lambda_{33}^{q} \lambda_{\mu \mu}^{\ell}\left[V_{t s}^{*} \theta_{q} e^{-i \phi_{q}}+r_{23}\right]\right. \\
& \times\left(C_{T}+C_{S}+C_{L R}^{\prime}\right) \\
& b \rightarrow s \ell \ell\left\{C_{10}^{\mu, \mathrm{NP}}=-\frac{\pi}{\alpha_{e m} V_{t b} V_{t s}^{*}} \lambda_{33}^{q} \lambda_{\mu \mu}^{\ell}\left[V_{t s}^{*} \theta_{q} e^{-i \phi_{q}}+r_{23}\right]\right. \\
& \times\left(-C_{T}-C_{S}+C_{L R}^{\prime}\right), \\
& C_{9^{\prime}}^{\mu, \mathrm{NP}}=-\frac{\pi}{\alpha_{e m} V_{t b} V_{t s}^{*}} \lambda_{33}^{q} \lambda_{\mu \mu}^{\ell} r_{23}\left(C_{R R}^{\prime}+C_{R L}^{\prime}\right) \\
& C_{10^{\prime}}^{\mu, \mathrm{NP}}=-\frac{\pi}{\alpha_{e m} V_{t b} V_{t s}^{*}} \lambda_{33}^{q} \lambda_{\mu \mu}^{\ell} r_{23}\left(C_{R R}^{\prime}-C_{R L}^{\prime}\right) \\
& b \rightarrow s \nu \bar{\nu} \begin{cases}C_{L}^{\nu_{\alpha}, N P} & =-\frac{\pi}{\alpha_{e m} V_{t b} V_{t s}^{*}} \lambda_{33}^{q} \lambda_{\alpha \alpha}^{\ell}\left[V_{t s}^{*} \theta_{q} e^{-i \phi_{q}}+r_{23}\right]\left[C_{S}-C_{T}\right] \\
C_{R}^{\nu_{\alpha}, \mathrm{NP}} & =-\frac{\pi}{\alpha_{e m} V_{t b} V_{t s}^{*}} \lambda_{33}^{q} \lambda_{\alpha \alpha}^{\ell} r_{23} C_{R L}^{\prime}\end{cases} \\
& s \rightarrow d \nu \bar{\nu}\left\{\begin{array}{ccc}
C_{s d}^{\nu_{\alpha}, \mathrm{NP}} & = & \frac{\pi s_{W}^{2}}{\alpha_{e m}} \lambda_{33}^{q} \lambda_{\alpha \alpha}^{\ell}\left[\theta_{q}^{2} V_{t s} V_{t d}^{*}\left(C_{S}-C_{T}\right)\right. \\
& + & \theta_{q}\left(V_{t s} e^{i \phi_{q} r_{13}^{*}}+V_{t d}^{*} e^{-i \phi_{q} r_{23}}\right)\left(C_{S}-C_{T}\right) \\
& + & \left.r_{12}\left(C_{S}-C_{T}+C_{R L}^{\prime}\right)\right]
\end{array}\right.
\end{aligned}
$$

- A scenario for NP in $b \rightarrow s \ell \ell$ defines curves or regions in neutrino modes $\left(C_{S}-C_{T}\right.$ vs $\left.C_{S}+C_{T}\right)$.


## Expected sensitivities

| Observables | Belle 0.71 $\mathrm{ab}^{-1}\left(0.12 \mathrm{ab}^{-1}\right)$ | Belle II $5 \mathrm{ab}^{-1}$ | Belle II $50 \mathrm{ab}^{-1}$ |
| :--- | :---: | :---: | :---: |
| $\operatorname{Br}\left(B^{+} \rightarrow K^{+} \nu \bar{\nu}\right)$ | $<450 \%$ | $30 \%$ | $11 \%$ |
| $\operatorname{Br}\left(B^{0} \rightarrow K^{* 0} \nu \bar{\nu}\right)$ | $<180 \%$ | $26 \%$ | $9.6 \%$ |
| $\operatorname{Br}\left(B^{+} \rightarrow K^{*+} \nu \bar{\nu}\right)$ | $<420 \%$ | $25 \%$ | $9.3 \%$ |
| $F_{L}\left(B^{0} \rightarrow K^{* 0} \nu \bar{\nu}\right)$ | - | - | 0.079 |
| $F_{L}\left(B^{+} \rightarrow K^{*+} \nu \bar{\nu}\right)$ | - | - | 0.077 |
| $\operatorname{Br}\left(B^{0} \rightarrow \nu \bar{\nu}\right) \times 10^{6}$ | $<14$ | $<5.0$ | $<1.5$ |
| $\operatorname{Br}\left(B_{s} \rightarrow \nu \bar{\nu}\right) \times 10^{5}$ | $<9.7$ | $<1.1$ | - |

Figure 1: From The Belle II physics book

