To break or not to break: strongly-coupled QCD-like theories in the Infrared

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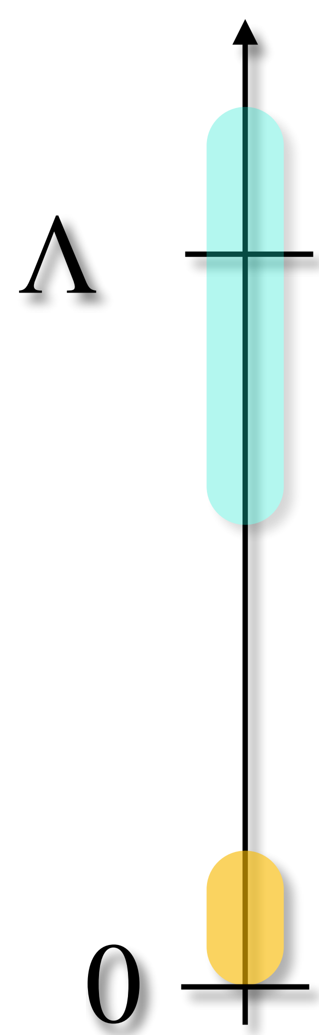
Strong dynamics in the Infrared

In the infrared, theories become strongly-coupled, and (assuming) confinement happens if number of flavor is below conformal window.

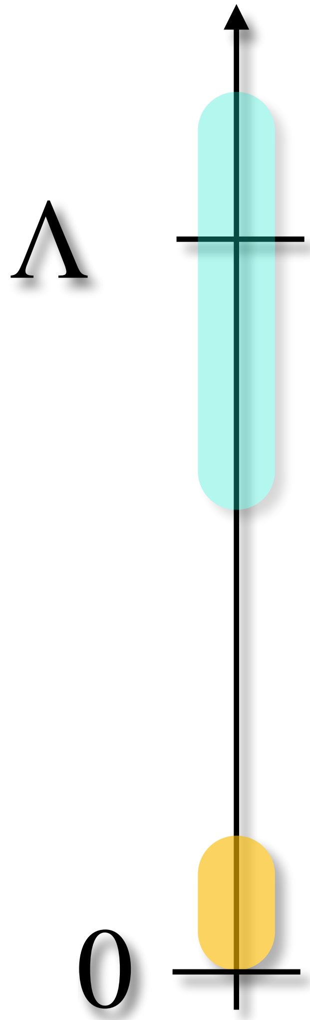
Bound states are color singlets and classified under global symmetry, which in QCD-like theories (focus of this talk) is $SU(n_f)_L \times SU(n_f)_R \times U(1)_V$.

Typical spectrum:



massive composite resonances

(almost) massless states,
protected by (approximate) symmetries,
either linearly or nonlinearly realized.



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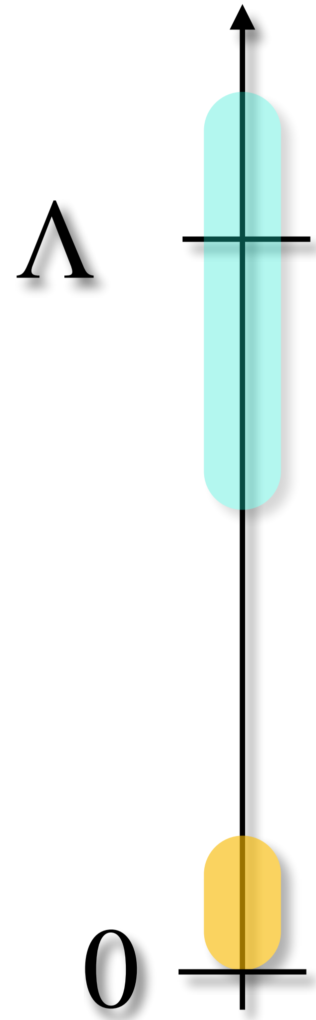
The existence of these massless states

is implied by 't Hooft anomaly matching, i.e.

"**Anomalies at IR = Anomalies at UV**".



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To break or not to break:

If chiral symmetry breaking happens, (almost) massless states would be (pseudo) Nambu-Goldstone bosons;

Otherwise, instead there are (almost) massless composite <u>chiral</u> fermions.

’t Hooft anomaly matching

Strategy of ‘t Hooft: Given the two possibilities, i.e.

- Chiral symmetry is non-linearly realized;
- Chiral symmetry is linearly realized,

one assumes chiral symmetry is not broken and tries to match anomalies with massless chiral fermions.

’t Hooft anomaly matching

Strategy of ’t Hooft: Given the two possibilities, i.e.

- Chiral symmetry is non-linearly realized;
- Chiral symmetry is linearly realized,

one assumes chiral symmetry is not broken and tries to match anomalies with massless chiral fermions.

→ If one fails, i.e. no sensible spectrum of chiral fermions is possible, chiral symmetry must be broken.

→ Otherwise, we know nothing.

A quick example from ’t Hooft’s original paper and Weinberg’s QFT vol.2:

’t Hooft (1979 Cargese lecture)

e.g. QCD $(N_c=3,\ n_f=3)$

Global symmetry : $SU(3)_L \times SU(3)_R \times U(1)_V$

$$B:\ (\ydiagram{3}, \cdot)_1,\quad (\ydiagram{2,1}, \cdot)_1,$$

$$(\ydiagram{1,1,1}, \cdot)_1,\quad (\ydiagram{1,1}, \ydiagram{1})_1,\quad (\ydiagram{2}, \ydiagram{1})_1$$

$$q_L:\ (\ydiagram{1}, \cdot)_{1/3}$$

Anomalies : $SU(3)_L^2\, U(1)_V\ ,\ SU(3)_L^3$

$$\sum_B l(B)A(B) = \sum_q l(q)A(q)$$

l(B) are nonzero integers for chiral fermions.

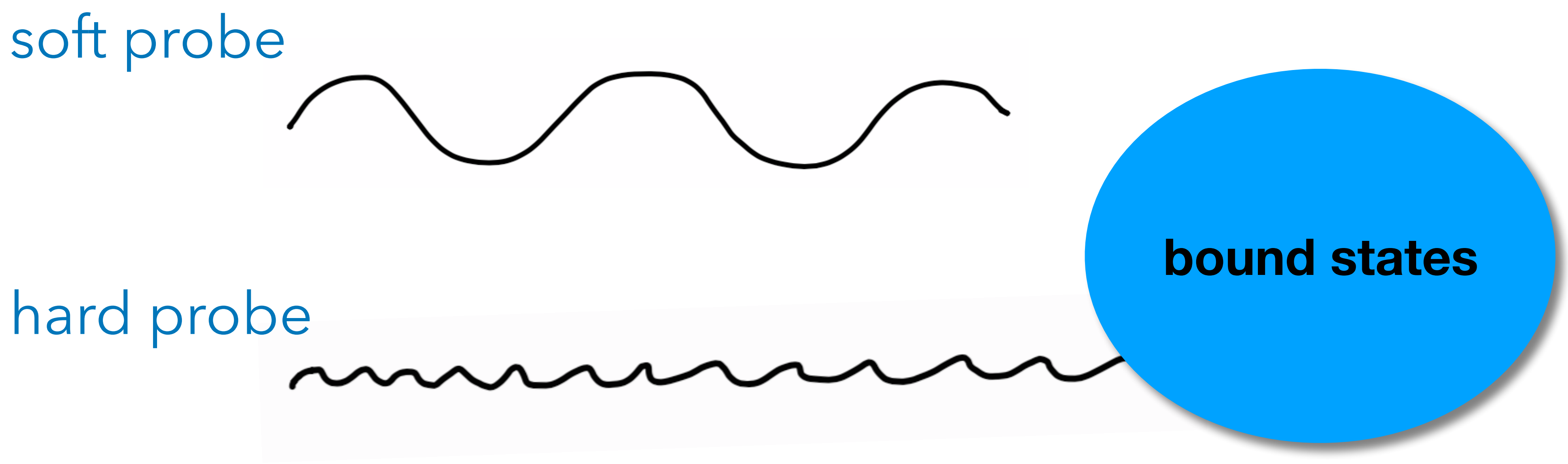
A(B) are polynomials of nf, e.g.

$$A_3\left\{(\ydiagram{3}, \cdot)_1\right\} = \frac{(n_f+6)(n_f+3)}{2}$$

Conclusion of ’t Hooft:

no integral {l(B)} are admissible, such that chiral symmetry is broken in QCD.

Bound states and their properties



Qualitative picture:

soft probes are only sensitive to the quantum numbers of bound states under the global symmetry in infrared.

At low energies, bound states with different constituents but the same quantum number are identical to each other. As in 't Hooft anomaly matching equations, they are degenerate.

To break this degeneracy, one needs hard probes in order to be sensitive to inner structure.

Bound states and their properties

Bound states are color singlets:

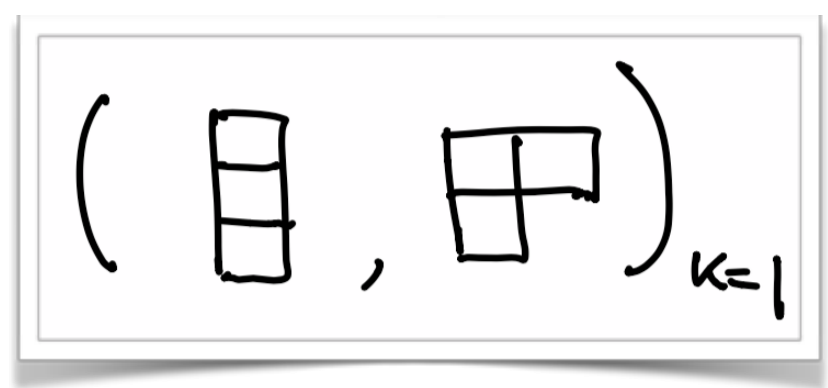
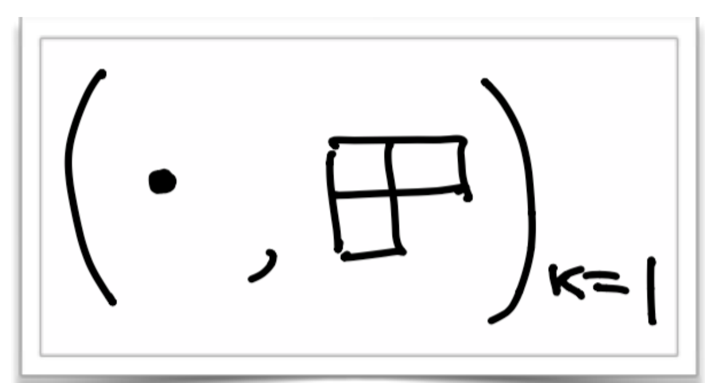
$$n(q_L) + n(q_R) - n(\bar{q}_L) - n(\bar{q}_R) = \kappa N_c$$

Massless composites are spin-1/2 fermions (Weinberg-Witten):

κ and N_c are both odd integers

For example: exotics states v.s. baryonic states

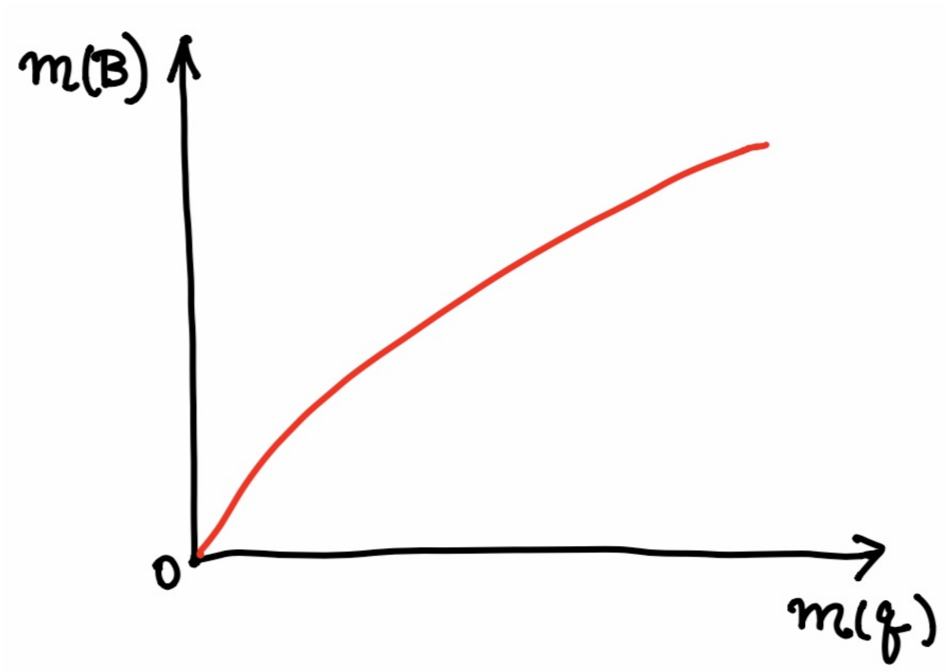
$$\bar{q}_{i_1} \sim \bar{q}_{i_1}\epsilon^{i_1 i_2 \cdots i_{n_f}} \qquad q^i$$

composite operators $(N_c = 3,\ n_f = 3,\ \kappa = 1)$	Young tableaux	irreps
$n(q_L) = 1,\ n(\bar{q}_L) = 1,\ n(q_R) = 3,\ n(\bar{q}_R) = 0$		$(1{,}8)_{\kappa=1}$
$n(q_L) = 0,\ n(\bar{q}_L) = 0,\ n(q_R) = 3,\ n(\bar{q}_R) = 0$		

Persistent mass condition

Vafa, Witten (1984)

As implied by Vafa-Witten, in QCD-like theories bound states that contain massive constituents must also be massive.



Turning on a "tiny" mass for one flavor, global symmetry is reduced:

$$\mathcal{G} = SU(n_f)_L \times SU(n_f)_R \times U(1)_V$$

$$\Downarrow$$

$$\mathcal{G}' = SU(n_f - 1)_L \times SU(n_f - 1)_R \times U(1)_H \times U(1)_V$$

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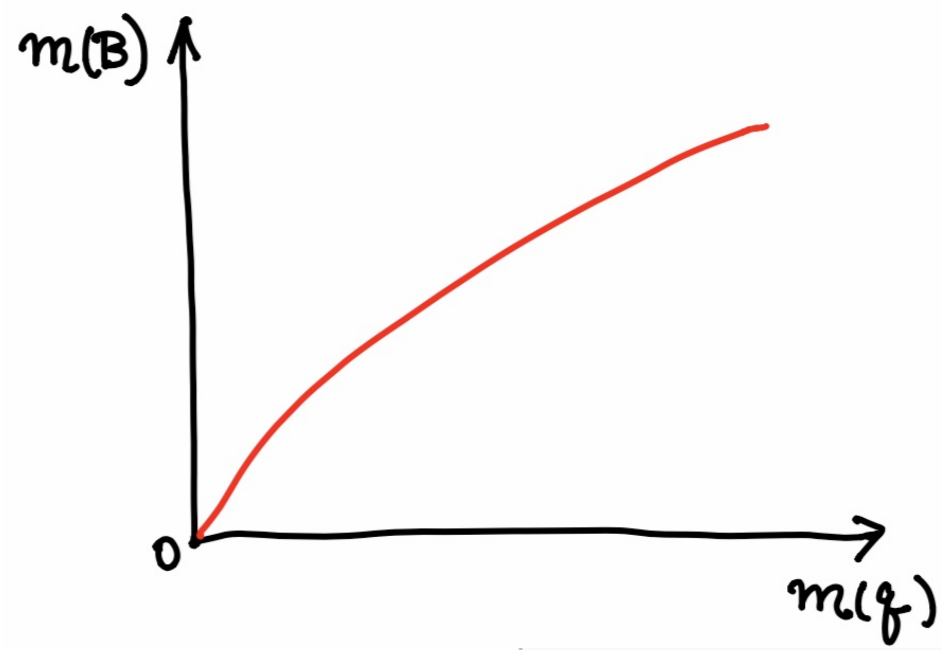

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For example:

$SU(3)_L \times SU(3)_R \times U(1)_V \qquad\qquad SU(2)_L \times SU(2)_R \times U(1)_V \times U(1)_H$

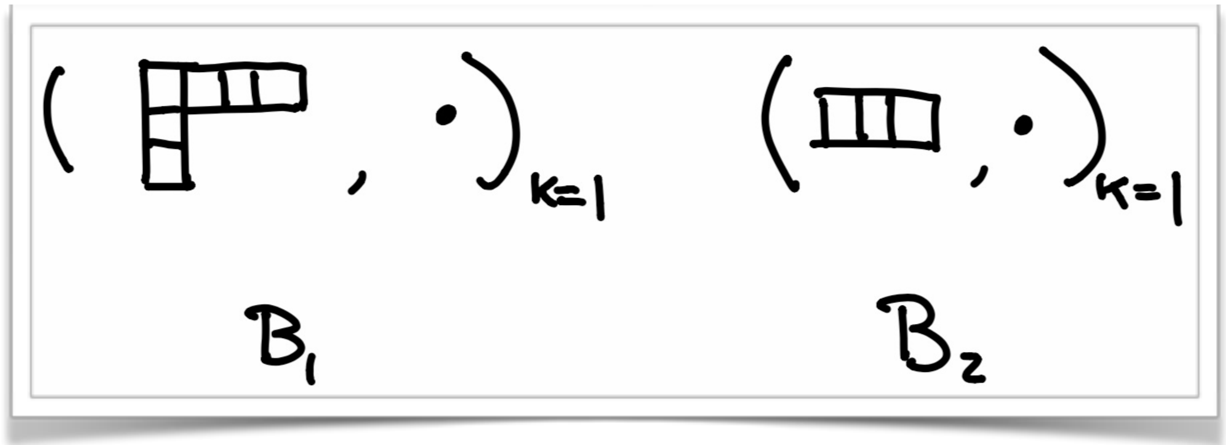
$$(\square\square\square, \cdot)_{k=1} \longrightarrow (\square\square, \cdot)_{k=1,h=1} + (\square, \cdot)_{k=1,h=2} + (\cdot, \cdot)_{k=1,\, h=3} + (\square\square\square, \cdot)_{k=1,\, h=0}$$

Massive states become <u>vectorial</u> (except flavor singlets):

$$\ell(\square\square, \cdot)_{k=1} = \ell(\square, \cdot)_{k=1} = 0$$

Bound states with different Young tableaux, but still in the same irrep, are distinguishable in persistent mass conditions.

e.g.



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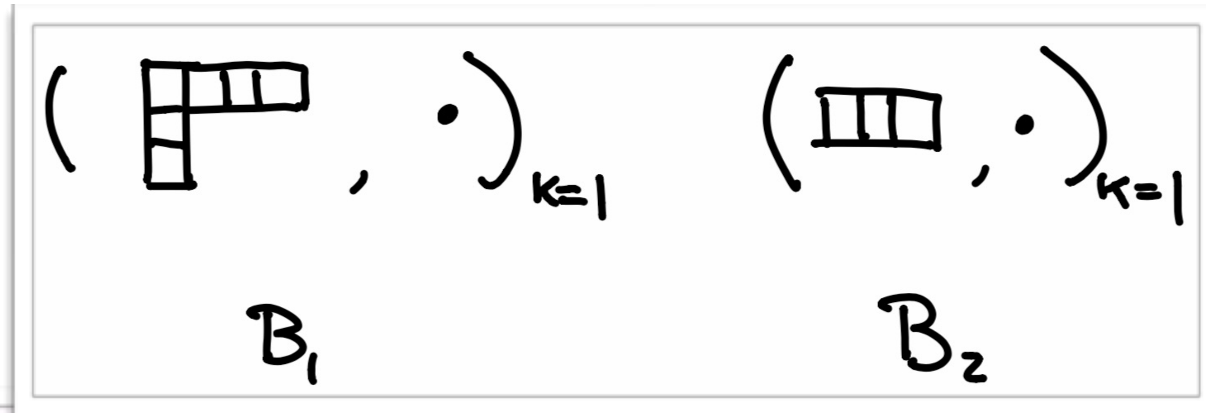

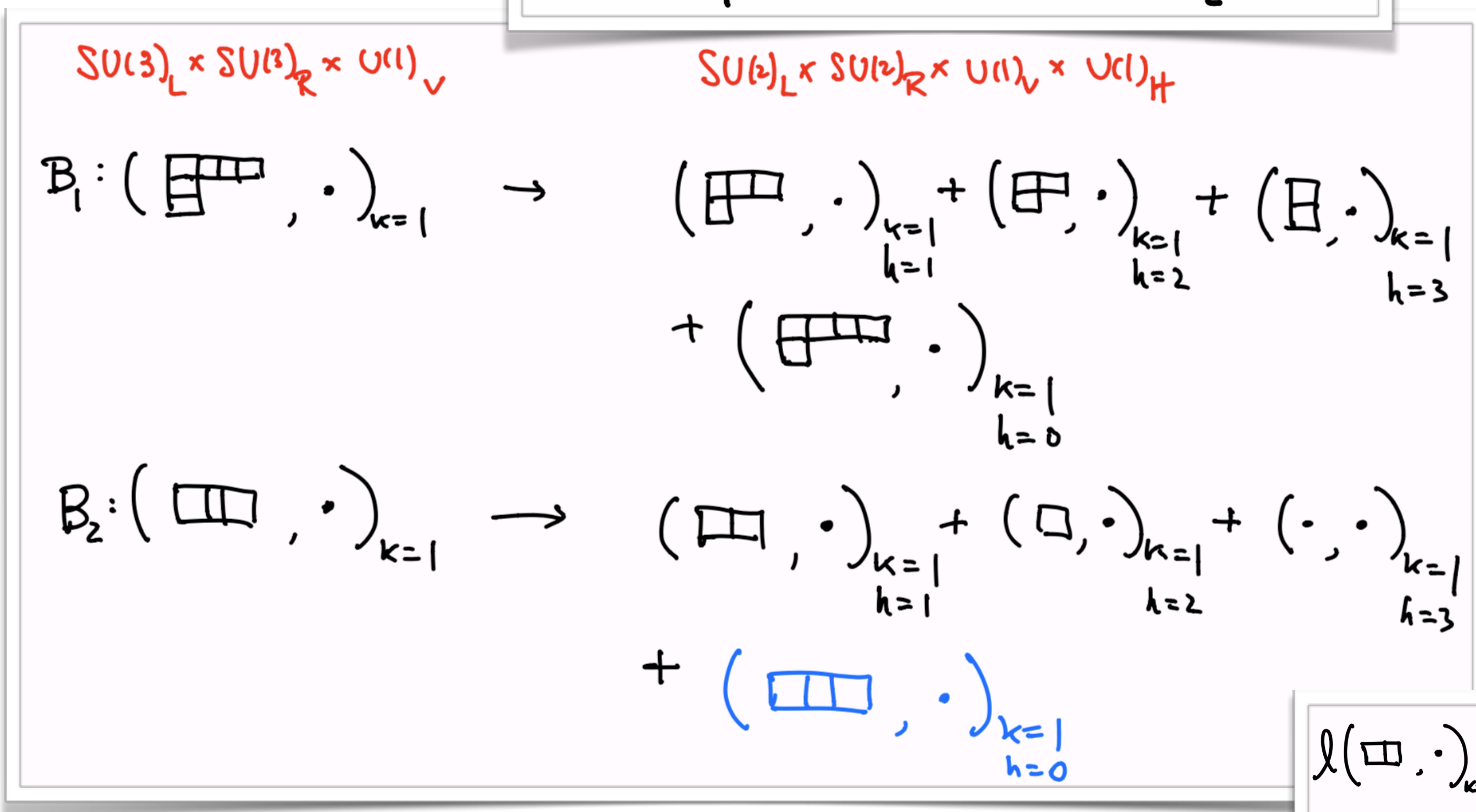

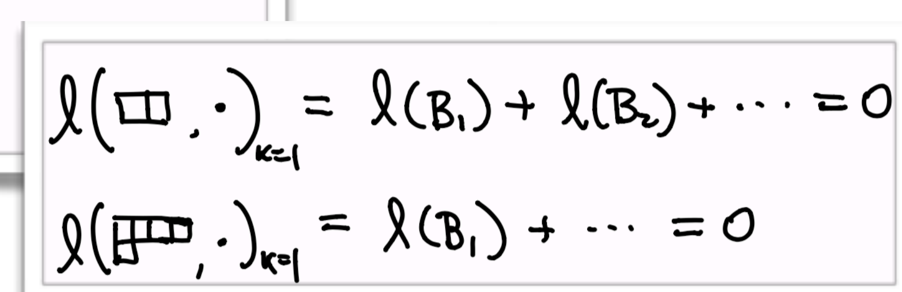

Persistent mass condition provides information on microscopic constituents, i.e. it offers a **"high energy probe"** sensitive to inner structure of bound states.

Summary & Motivation

AM[nf]: $$\sum_B l(B)A(B) = \sum_q l(q)A(q) \longrightarrow a_p n_f^p + a_{p-1} n_f^{p-1} + \cdots + a_2 n_f^2 + a_1 n_f + a_0 = 0$$

PMC[nf]: $$l'(W_L, W_R, H, V) = \sum_{Y_L, Y_R} \kappa_{Y_L}^{W_L} \kappa_{Y_R}^{W_R} \, l(Y_L, Y_R, V) = 0$$

Notice there are many massive states appearing in the decomposition of Y.

In order to prove chiral symmetry breaking, one needs to show no integral solution exists.

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Notice there are many massive states appearing in the decomposition of Y.

In order to prove chiral symmetry breaking, one needs to show no integral solution exists.

This system is infamous for its ugliness (as 't Hooft pointed out), such that it's difficult (almost impossible) to show there is no solution in general.

The property called "nf independence" (originally guessed by 't Hooft): the same set of multiplicities solves {AM[nf], PMC[nf]}, {AM[nf+1], PMC[nf+1]}, {AM[nf+2], PMC[nf+2]} and so on.

(For the moment, let's assume it's true and see what it implies.)

1) considering a solution {l(B)} of AM[nf] equation for physically confining nf:

$$a_p n_f^p + a_{p-1} n_f^{p-1} + \cdots + a_2 n_f^2 + a_1 n_f + a_0 = 0$$

2) assuming nf independence is true:

$$a_p (n_f^*)^p + a_{p-1} (n_f^*)^{p-1} + \cdots + a_2 (n_f^*)^2 + a_1 (n_f^*) + a_0 = 0$$

where $n_f{}^* = n_f + 1,\ n_f + 2,\ n_f + 3,\ n_f + 4,\ \cdots$

➡ Number of roots is larger than p.

➡ $a_i = 0,\ i = 0,1,2,\cdots,p$ which are functions of multiplicities.

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3)


ANOMALY NON-MATCHING IN COMPOSITE MODELS

Glennys R. FARRAR [1]

Rutgers University, Piscataway, NJ, USA
Institute for Advanced Study, Princeton, NJ, USA
and CERN, Geneva, Switzerland

We use Farrar's result:
the equation a0=0
only admits non-integral
solutions!

Chiral symmetry breaking arising from nf independence

Overview of literature

Original Idea (anomaly matching+decoupling condition+ requiring nf independence)

’t Hooft (1979 Cargese lecture)

nf independence if restricting to elbow-shape Young tableaux

Frishman, Schwimmer, Banks, Yankielowicz (1981)

Assuming nf independence for all nf and studying the limit nf=**0**, the only paper including exotics

Farrar (1980)

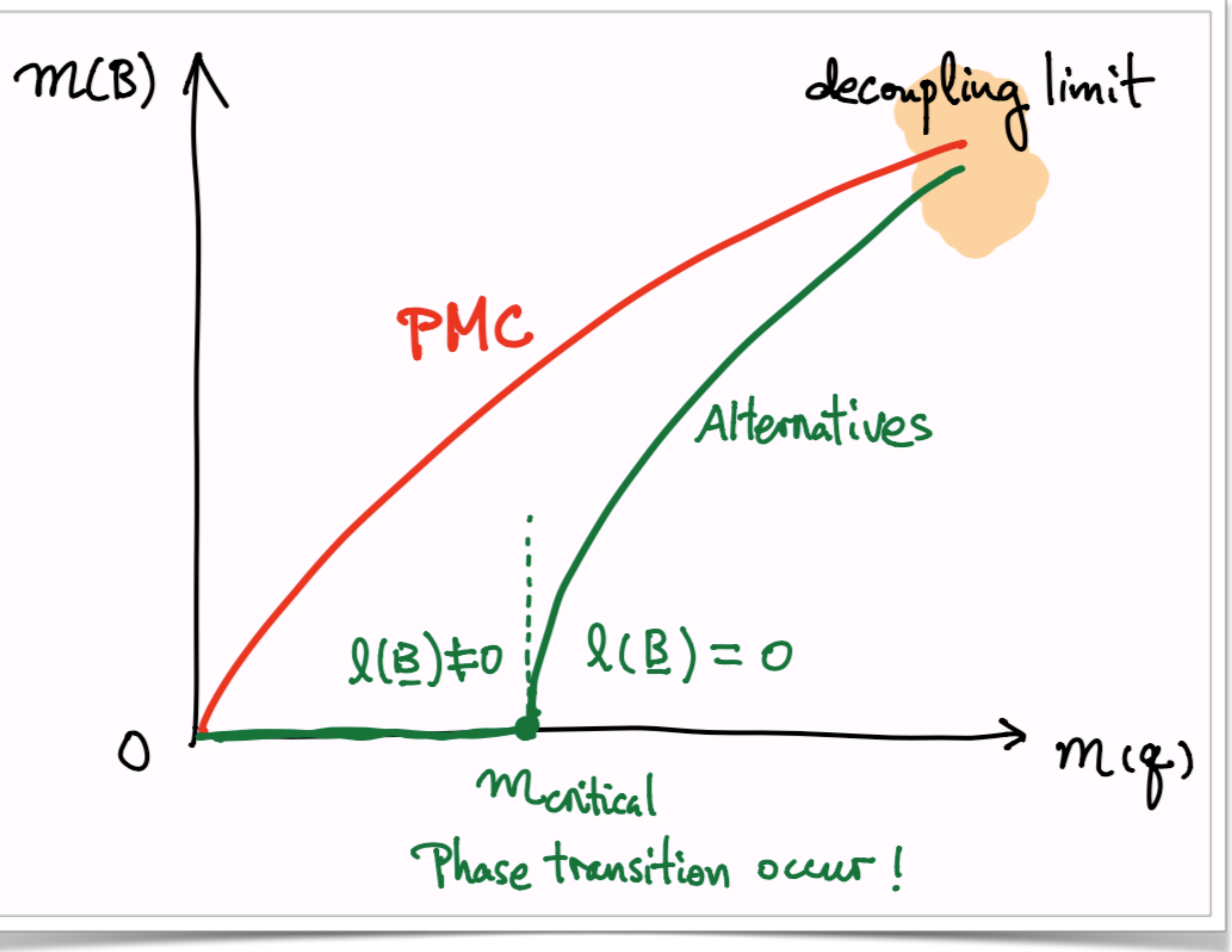
Starting quantitative formulation of nf independence in a “naive” manner, however it’s a big step forward

Takeshita, Komatsu, Kakuto, Inoue (1981)

nf independence being not true in small nf

Cohen, Frishman (1982)

Overview of literature



Pointing out the difference between PMC and decoupling condition

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PMC being proved in QCD-like theories

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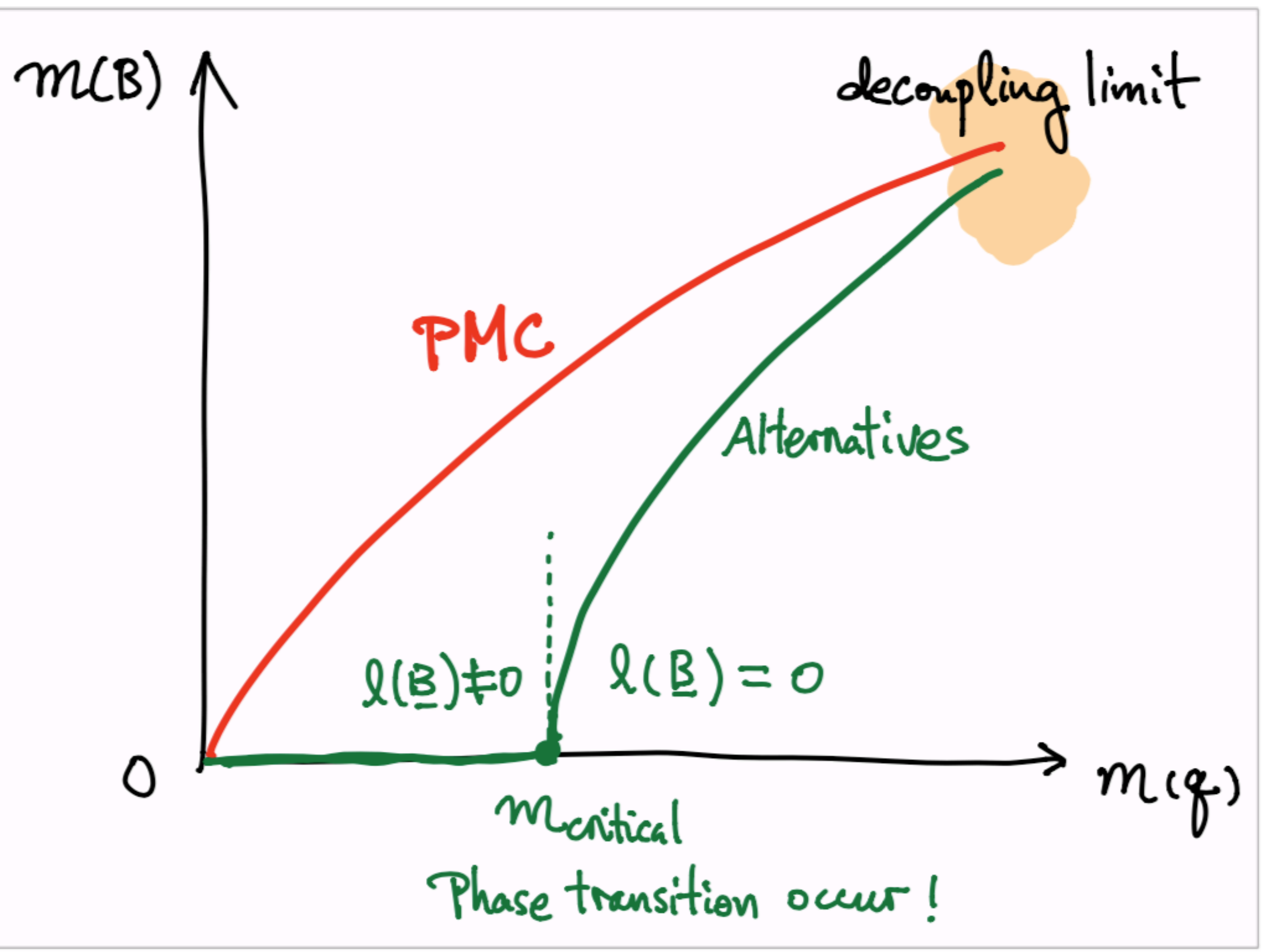

Pointing out the difference between PMC and decoupling condition

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➡ We prove rigorously nf independence in large nf with

1) general numbers of color and flavor;

2) most general spectrum, i.e. baryons & exotics.

➡ Chiral symmetry breaking will follow.

nf independence

The same set of $\{\ l(\mathbf{YT}_i)\ \}$ solves

$$\{\mathrm{AM}[n_f], \mathrm{PMC}[n_f]\} \leftrightarrow \{\mathrm{AM}[n_f+1], \mathrm{PMC}[n_f+1]\}$$

uplifting: $\longrightarrow$

downlifting: $\longleftarrow$

then one can recursively extrapolate to larger nf.

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<u>Uplifting: step 1</u>

$\mathrm{PMC}[n_f]$

$$l'(w_L, w_R, H, v) = \sum_{Y_L, Y_R} K^{w_L}_{Y_L} K^{w_R}_{Y_R}\, l(Y_L, Y_R, v) = 0$$

$\downarrow$

$\mathrm{PMC}[n_f+1]$

$$l'(w_L', w_R', H, v) = \sum_{\bar{Y}_L, \bar{Y}_R} K^{w_L'}_{\bar{Y}_L} K^{w_R'}_{\bar{Y}_R}\, l(\bar{Y}_L, \bar{Y}_R, v) = 0$$

It is true, if Young tableaux have one-to-one correspondence, and also their decompositions.

$$K^{w_L}_{Y_L} = K^{w_L'}_{\bar{Y}_L} \qquad K^{w_R}_{Y_R} = K^{w_R'}_{\bar{Y}_R}$$

Uplifting: step 2 $\mathrm{AM}[n_f] + \mathrm{PMC}[n_f+1] \rightarrow \mathrm{AM}[n_f+1]$

Summing over the **massive** Young tableaux

$$\sum_{\substack{w_L',\, w_R' \\ v>0}} \Bigg(\underbrace{\sum_{\bar{Y}_L, \bar{Y}_R} K_{\bar{Y}_L}^{w_L'} K_{\bar{Y}_R}^{w_R'} \, \ell(\bar{Y}_L, \bar{Y}_R, v)}_{\mathrm{PMC}[n_f+1]} \Bigg) d(w_R'; n_f) A_{2/3}(w_L', v; n_f) = 0$$

$$\sum_{\substack{\bar{Y}_L, \bar{Y}_R, \\ v>0}} \sum_{\substack{\text{All} \\ (\tilde{w}_L', \tilde{w}_R')}} K_{\bar{Y}_L}^{\tilde{w}_L'} K_{\bar{Y}_R}^{\tilde{w}_R'} \, \ell(\bar{Y}_L, \bar{Y}_R, v) \, d(\tilde{w}_R'; n_f) A_{2/3}(\tilde{w}_L', v; n_f)$$

$$- \sum_{\substack{\bar{Y}_L, \bar{Y}_R \\ v>0}} \sum_{Y_L, Y_R} K_{\bar{Y}_L}^{Y_L} K_{\bar{Y}_R}^{Y_R} \, \ell(\bar{Y}_L, \bar{Y}_R, v) \, d(Y_R; n_f) A_{2/3}(Y_L, v; n_f)$$

$$= 0$$

by adding and subtracting the **massless** Young tableaux,

which are different from the massive Young tableaux.

Uplifting: step 2 $\mathrm{AM}[n_f] + \mathrm{PMC}[n_f+1] \longrightarrow \mathrm{AM}[n_f+1]$

$$\sum_{\substack{\bar{Y}_L,\bar{Y}_R,\\ V>0}} \ \sum_{\substack{\text{All}\\ (\tilde{W}'_L,\tilde{W}'_R)}} K^{\tilde{W}'_L}_{\bar{Y}_L} K^{\tilde{W}'_R}_{\bar{Y}_R} \, \ell(\bar{Y}_L,\bar{Y}_R,V)\, d(\tilde{W}'_R; n_f)\, A_{2/3}(\tilde{W}'_L, V; n_f)$$

$$- \sum_{\substack{\bar{Y}_L,\bar{Y}_R\\ V>0}} \ \sum_{Y_L,Y_R} K^{Y_L}_{\bar{Y}_L} K^{Y_R}_{\bar{Y}_R} \, \ell(\bar{Y}_L,\bar{Y}_R,V)\, d(Y_R; n_f)\, A_{2/3}(Y_L, V; n_f)$$

$$= 0 \longrightarrow N_c - N_c$$

$$\sum_{\substack{\bar{Y}_L,\bar{Y}_R\\ V>0}} \ell(\bar{Y}_L,\bar{Y}_R,V)\, d(\bar{Y}_R; n_f+1)\, A_{2/3}(\bar{Y}_L, V; n_f+1) = N_c \longrightarrow \mathrm{AM}[n_f+1]$$

$$d(\bar{Y}_R; n_f+1) = \sum_{\substack{\text{All}\\ \tilde{W}'_R}} K^{\tilde{W}'_R}_{\bar{Y}_R}\, d(\tilde{W}'_R; n_f)$$

$$A_{2/3}(\bar{Y}_L; n_f+1) = \sum_{\substack{\text{All}\\ \tilde{W}'_L}} K^{\tilde{W}'_L}_{\bar{Y}_L}\, A_{2/3}(\tilde{W}'_L, V; n_f)$$

Uplifting: step 2 $\mathrm{AM}[n_f] + \mathrm{PMC}[n_f+1] \rightarrow \mathrm{AM}[n_f+1]$

$$\sum_{\substack{\bar{Y}_L,\bar{Y}_R,\\ v>0}} \sum_{\substack{\text{All}\\ (\tilde{W}'_L,\tilde{W}'_R)}} K_{\bar{Y}_L}^{\tilde{W}'_L} K_{\bar{Y}_R}^{\tilde{W}'_R}\, \ell(\bar{Y}_L,\bar{Y}_R,v)\, d(\tilde{W}'_R;n_f)\, A_{2/3}(\tilde{W}'_L,v;n_f)$$

$$-\sum_{\substack{\bar{Y}_L,\bar{Y}_R\\ v>0}} \sum_{Y_L,Y_R} K_{\bar{Y}_L}^{Y_L} K_{\bar{Y}_R}^{Y_R}\, \ell(\bar{Y}_L,\bar{Y}_R,v)\, d(Y_R;n_f)\, A_{2/3}(Y_L,v;n_f) = 0$$

$N_c \boxed{- N_c}$

$\mathrm{AM}[n_f]$ $$\sum_{\substack{Y_L,Y_R\\ v>0}} \ell(Y_L,Y_R,v)\, d(Y_R;n_f)\, A_{2/3}(Y_L,v;n_f) = N_c$$?

It is true, if

$$\sum_{\bar{Y}_L,\bar{Y}_R} K_{\bar{Y}_L}^{Y_L} K_{\bar{Y}_R}^{Y_R}\, \ell(\bar{Y}_L,\bar{Y}_R,v) = \ell(Y_L,Y_R,v)$$

Uplifting: step 2 $\mathrm{AM}[n_f] + \mathrm{PMC}[n_f+1] \rightarrow \mathrm{AM}[n_f+1]$

$$\sum_{\substack{\bar{Y}_L,\bar{Y}_R,\\ v>0}} \sum_{\substack{\text{All}\\ (\tilde{w}'_L,\tilde{w}'_R)}} K_{\bar{Y}_L}^{\tilde{w}'_L} K_{\bar{Y}_R}^{\tilde{w}'_R}\, \ell(\bar{Y}_L,\bar{Y}_R,v)\, d(\tilde{w}'_R; n_f)\, A_{2/3}(\tilde{w}'_L, v; n_f)$$

$$-\sum_{\substack{\bar{Y}_L,\bar{Y}_R\\ v>0}} \sum_{Y_L,Y_R} K_{\bar{Y}_L}^{Y_L} K_{\bar{Y}_R}^{Y_R}\, \ell(\bar{Y}_L,\bar{Y}_R,v)\, d(Y_R; n_f)\, A_{2/3}(Y_L, v; n_f)$$

$$= 0$$

$N_c - N_c$

$$\mathrm{AM}[n_f] \quad \sum_{\substack{Y_L,Y_R\\ v>0}} \ell(Y_L,Y_R,v)\, d(Y_R; n_f)\, A_{2/3}(Y_L, v; n_f) = N_c$$

?

It is true, if

$$\sum_{\bar{Y}_L,\bar{Y}_R} K_{\bar{Y}_L}^{Y_L} K_{\bar{Y}_R}^{Y_R}\, \ell(\bar{Y}_L,\bar{Y}_R,v) = \ell(Y_L,Y_R,v)$$

$$K_{\bar{Y}_L}^{Y_L} = \delta^{Y_L}{}_{\bar{Y}_L} \qquad K_{\bar{Y}_R}^{Y_R} = \delta^{Y_R}{}_{\bar{Y}_R}$$

$$\ell(Y_L,Y_R,v) = \ell(\bar{Y}_L,\bar{Y}_R,v)$$

Uplifting: step 2 $\mathrm{AM}[n_f] + \mathrm{PMC}[n_f+1] \Rightarrow \mathrm{AM}[n_f+1]$

$$\sum_{\substack{\bar{Y}_L,\bar{Y}_R,\\ v>0}} \sum_{\substack{\text{All}\\ (\tilde{w}'_L,\tilde{w}'_R)}} K^{\tilde{w}'_L}_{\bar{Y}_L} K^{\tilde{w}'_R}_{\bar{Y}_R}\, \ell(\bar{Y}_L,\bar{Y}_R,v)\, d(\tilde{w}'_R; n_f)\, A_{2/3}(\tilde{w}'_L, v; n_f)$$

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$$= 0$$

$$N_c - N_c$$

$$\mathrm{AM}[n_f] \quad \sum_{\substack{Y_L,Y_R\\ v>0}} \ell(Y_L,Y_R,v)\, d(Y_R; n_f)\, A_{2/3}(Y_L, v; n_f) = N_c \quad ?$$

It is true, if

$$\sum_{\bar{Y}_L,\bar{Y}_R} K^{Y_L}_{\bar{Y}_L} K^{Y_R}_{\bar{Y}_R}\, \ell(\bar{Y}_L,\bar{Y}_R,v) = \ell(Y_L,Y_R,v)$$

$$K^{Y_L}_{\bar{Y}_L} = \delta^{Y_L}{}_{\bar{Y}_L} \qquad K^{Y_R}_{\bar{Y}_R} = \delta^{Y_R}{}_{\bar{Y}_R}$$

$$\ell(Y_L,Y_R,v) = \ell(\bar{Y}_L,\bar{Y}_R,v)$$

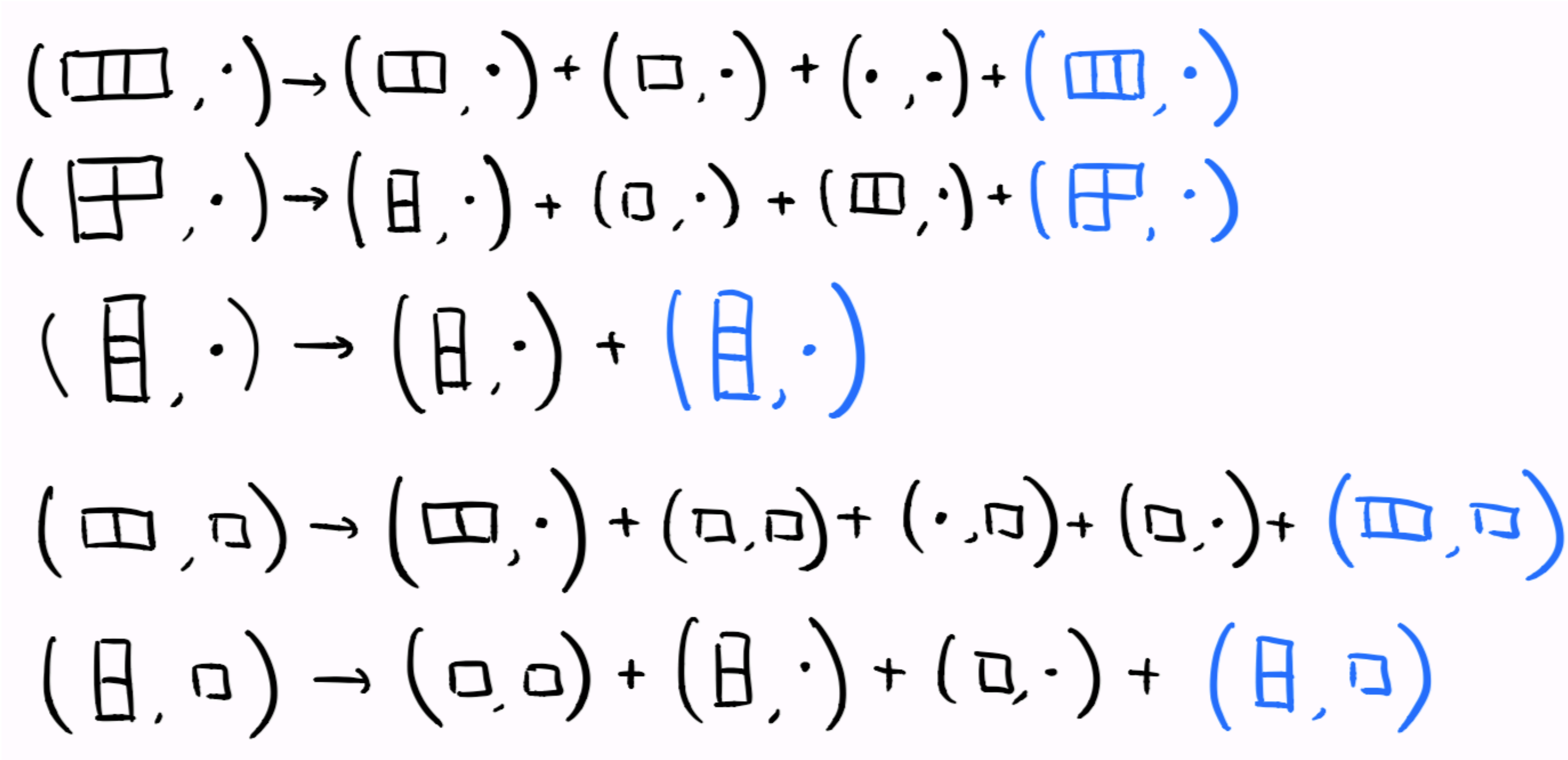
$K^{Y_L}_{\bar{Y}_L} K^{Y_R}_{\bar{Y}_R} = 0$ then $\ell(Y_L,Y_R,v) = 0$,

but $\ell(\bar{Y}_L,\bar{Y}_R,v) \neq 0$ a priori

nf independence: baryonic states as an example

$$N_c = 3,\ \kappa = 1,\ n_f > 3$$

$\bar{Y}$ $\quad$ W (massive) $\quad$ Y (massless)

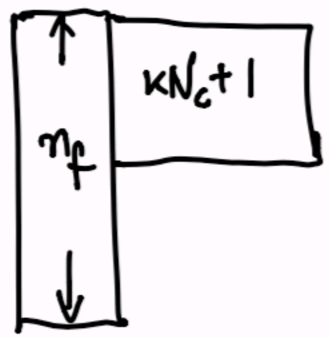


$$K^{Y_L}{}_{\bar{Y}_L} = \delta^{Y_L}{}_{\bar{Y}_L} \qquad K^{Y_R}{}_{\bar{Y}_R} = \delta^{Y_R}{}_{\bar{Y}_R}$$

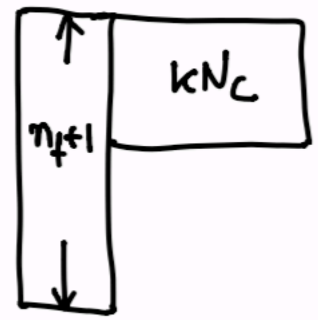
$$\ell(Y_L, Y_R, v) = \ell(\bar{Y}_L, \bar{Y}_R, v)$$

All these conditions are satisfied.

Possible caveat for exotic bound states



(a)



(a′)

(a), (a′) are $\overline{Y}$ in $AM[n_f+1]$

(a′) does not admit massless Young Tableaux.

$K^{Y_L}_{\overline{Y}_L} K^{Y_R}_{\overline{Y}_R} = 0$ then $\ell(Y_L, Y_R, v) = 0$,

but $\ell(\overline{Y}_L, \overline{Y}_R, v) \neq 0$ a priori

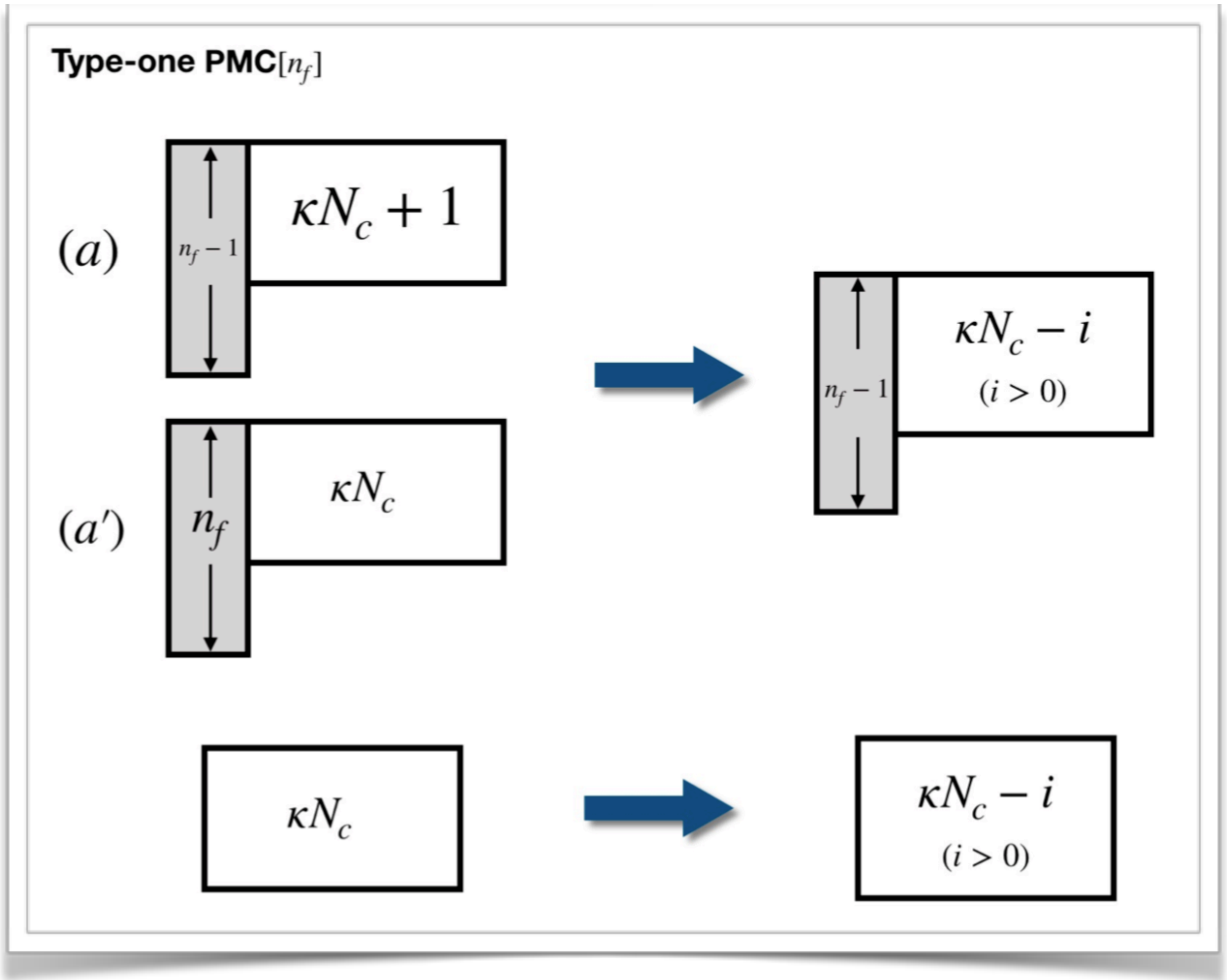
$$\sum_{\overline{Y}_L, \overline{Y}_R} K^{Y_L}_{\overline{Y}_L} K^{Y_R}_{\overline{Y}_R} \ell(\overline{Y}_L, \overline{Y}_R, v) = \ell(Y_L, Y_R, v)$$

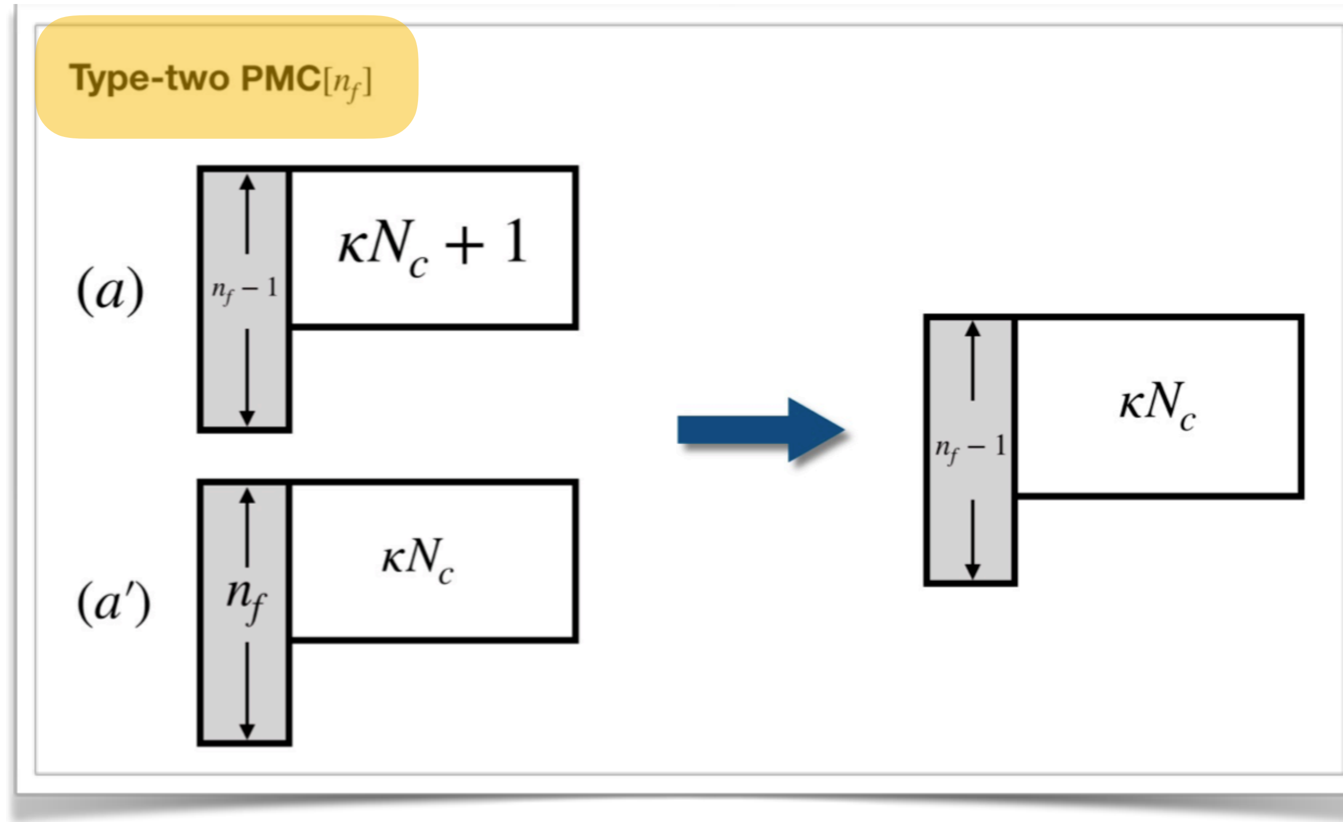
The resolution is to prove

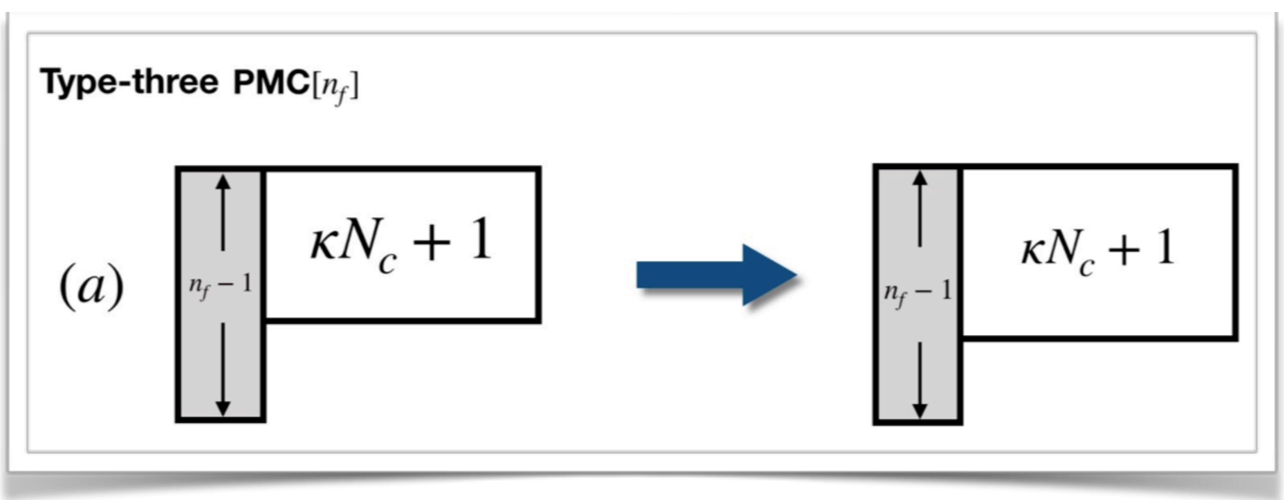
$$\ell(\overline{Y}_L, \overline{Y}_R, v) = 0$$

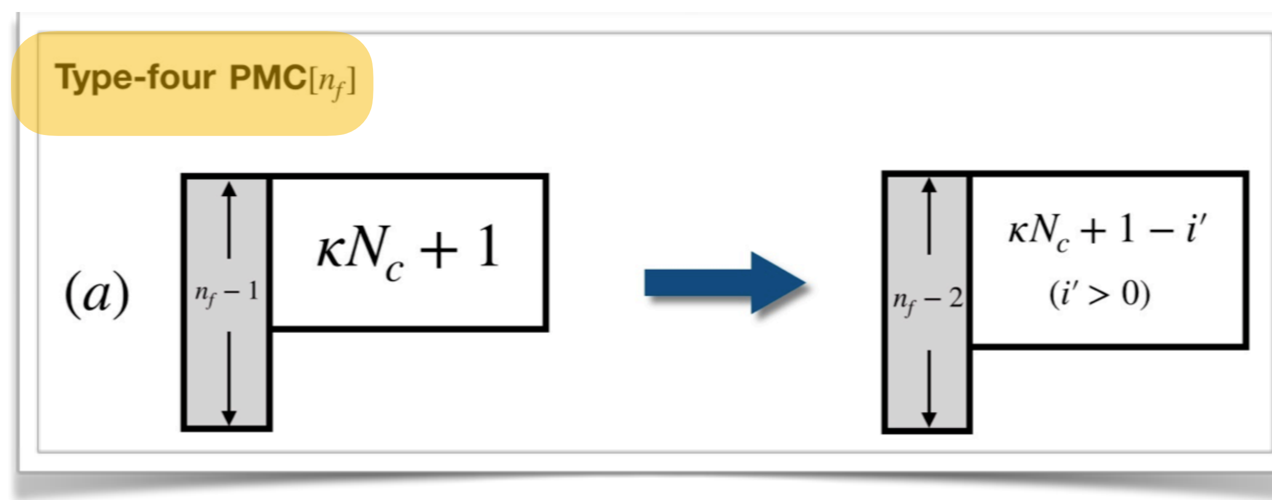
if $(\overline{Y}_L, \overline{Y}_R)$ does not admit massless YT.

Persistent mass condition of exotic states as a resolution

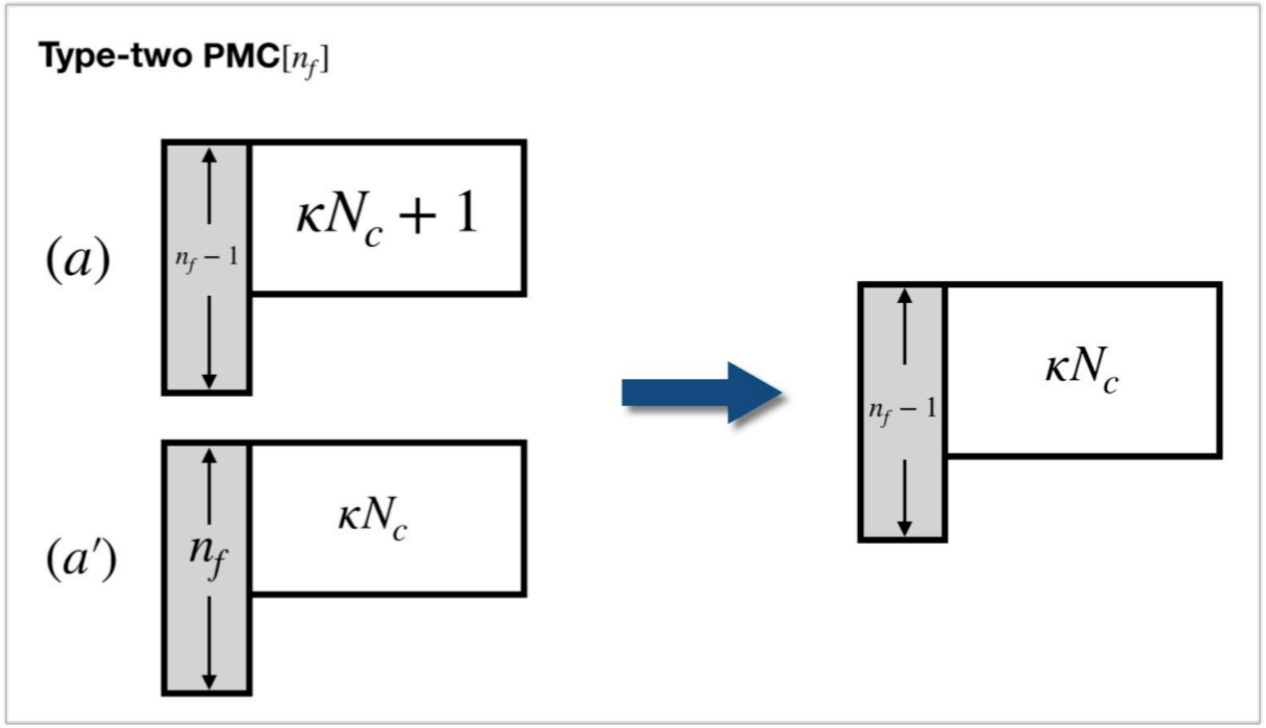




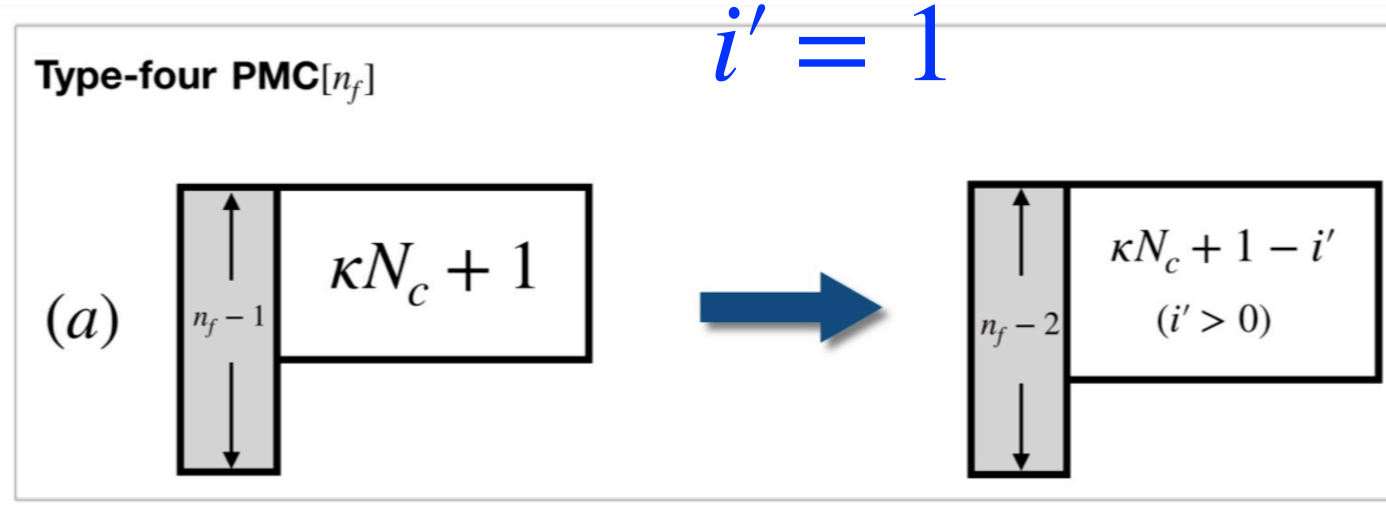




Before decomposition: $\bar{Y}$ After decomposition: W



$i'=1$



$$\sum_a \kappa(a)\, l(a) + l(a') = 0$$

$$\sum_a \kappa(a)\, l(a) = 0$$

If we subtract them,

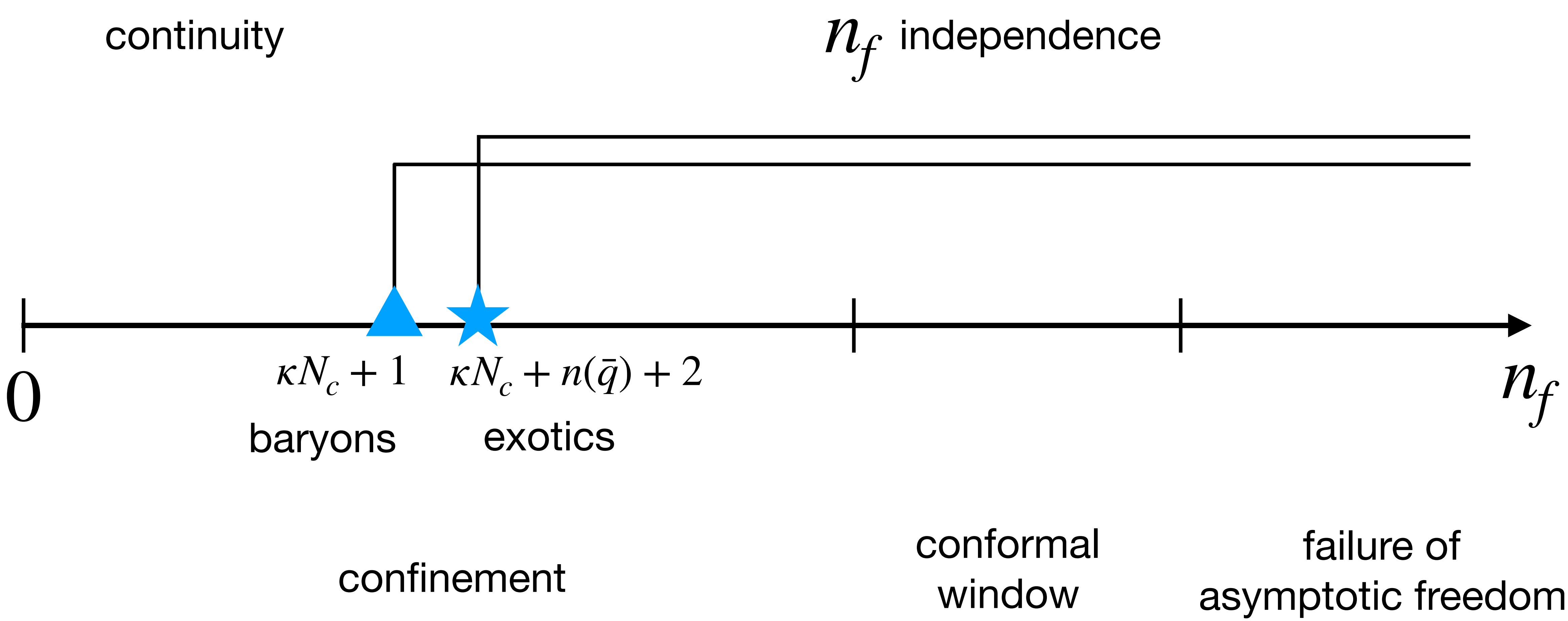
$$l(a') = 0$$

In the same spirit, one can show generic exotic states containing any number of quark-antiquark singlets are vectorial.

Remarkably, this property is forced by PMC.

Global overview

For any N_c :



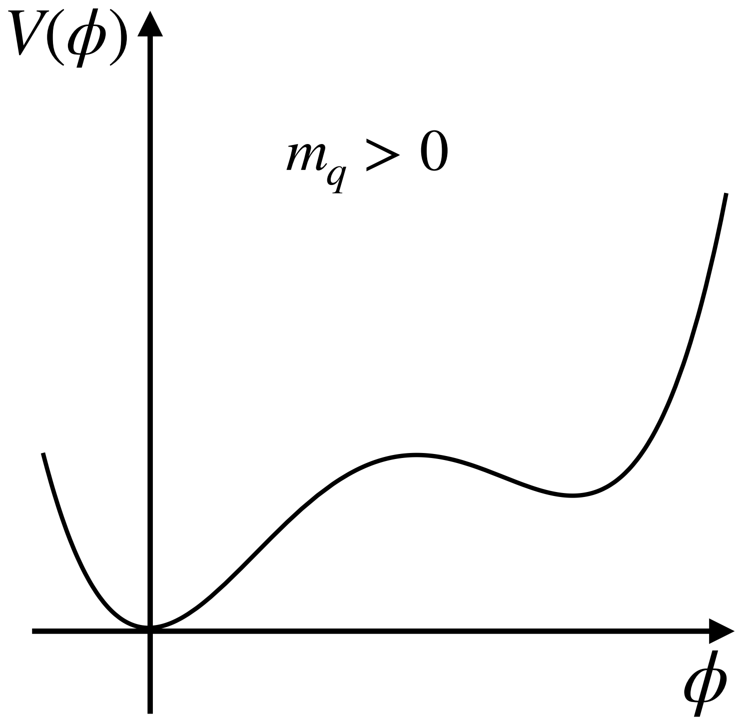
Limitations:

nf independence is not true in small nf regime.

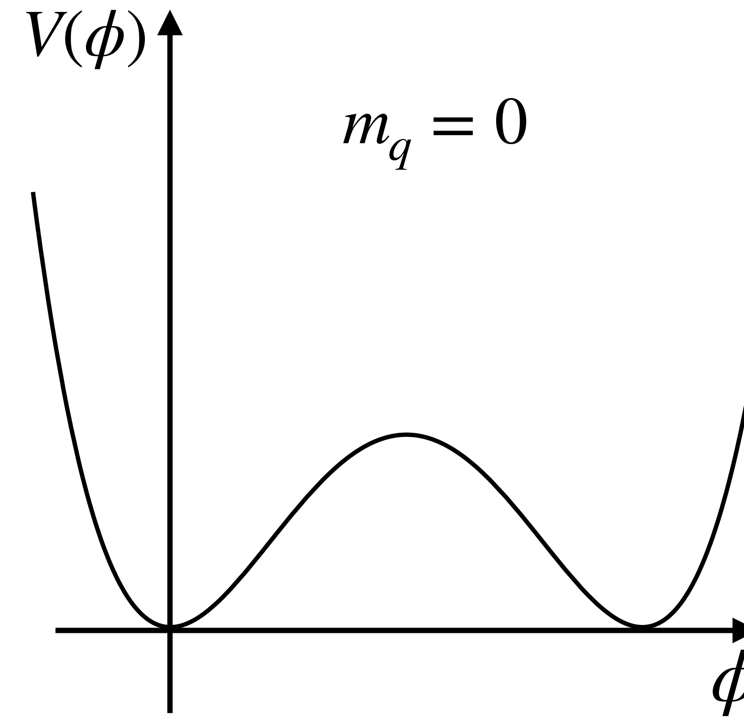
Nothing can be learnt if baryon number or antiquark number is too large, i.e. above the lower edge of conformal window.

(Notice baryon number and antiquark number should be determined by dynamics. Here they are just two parameters for us.)

Continuity in small nf



continuity



nf massless flavors
+1 massive flavor

(nf+1) massless flavors

1) Assuming the vacuum of unbroken chiral symmetry $SU(n_f)_L \times SU(n_f)_R \times U(1)_V$ remains the global minimum.

2) In (nf+1) massless flavors, vectorial $SU(n_f+1)_V$ is unbroken. (Vafa-Witten)

$SU(n_f+1)_L \times SU(n_f+1)_R \times U(1)_V$ is unbroken in (nf+1) massless flavors

3) nf massless flavors +(mf-nf) massive flavors $\xrightarrow{\text{continuity}}$ mf massless flavors

nf massless flavors
+(mf-nf) massive flavors $\xrightarrow{\text{continuity}}$ mf massless flavors

"Theory A" **"Theory B"**

Notice the continuity argument is dynamical, which however is different from 't Hooft anomaly matching.

Implications of continuity argument:

1) Continuity: chiral symmetry in "theory A" is unbroken, then chiral symmetry in "theory B" is unbroken.

2) If chiral symmetry in "theory B" is necessarily broken suggest by 't Hooft anomaly matching. By contradiction, chiral symmetry in "theory A" must be broken.

3) Assuming if there is no phase transition when massive flavors in "theory A" decouple, at this limit "theory A" is equivalent to a theory of nf massless flavors.

→ Chiral symmetry breaking occurs for two-flavor QCD.

Conclusions

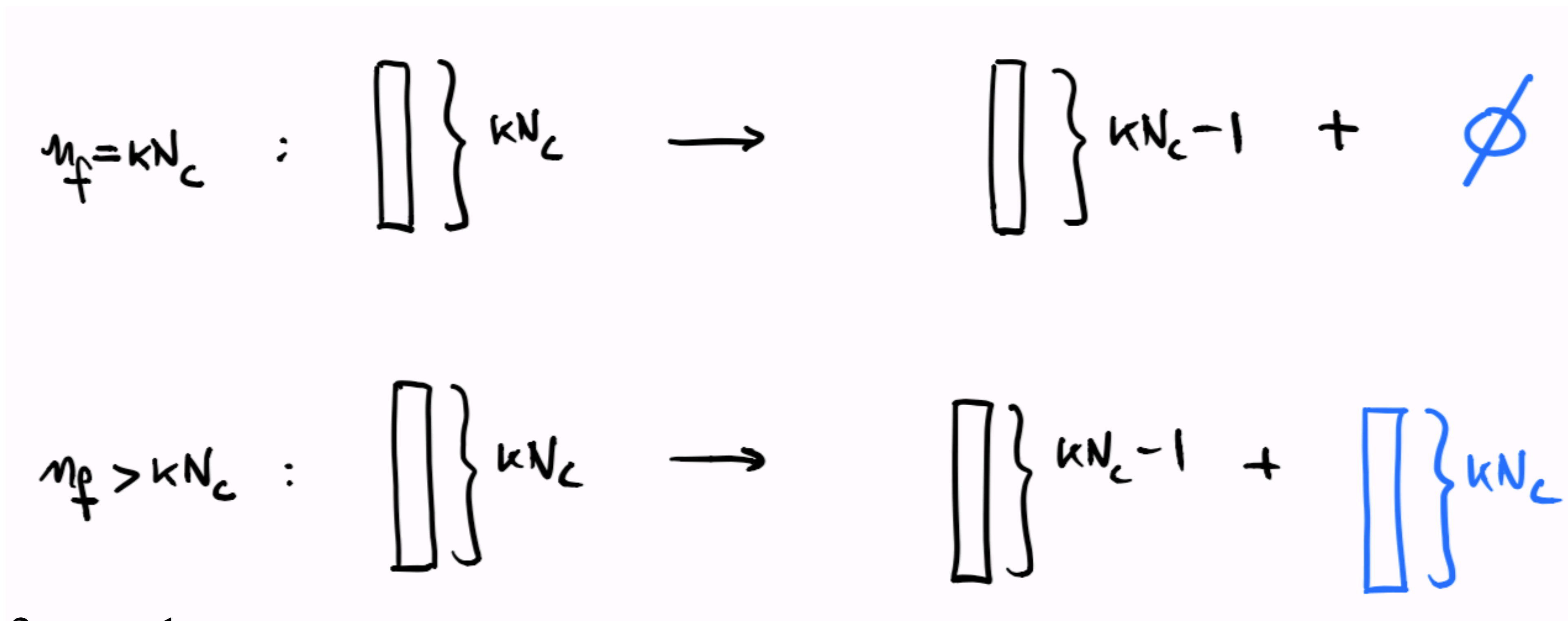
- In large nf, we show rigorously “nf independence” in the confining phase of strongly-coupled QCD-like gauge theories, with any number of color and the general spectrum of bound states. Chiral symmetry breaking follows.
- Limitations: small nf; large baryon number and antiquark number.
- Extension to proving chiral symmetry breaking in other theories seems less plausible but certainly interesting, e.g. gluequarks in adjoint QCD where strong dynamics is still vectorlike; see Michele Redi’s talk.
- Many possibilities in strongly-coupled chiral dynamics (e.g. tumbling, confining, color-flavor locking, and their complementarities).

Questions and comments are welcome!!!

Backup slides

Lower edge of nf independence

Baryons:



For example: $N_c = 3,\ \kappa = 1$

Uplifting from 4 flavors to 5 flavors works (and so on), because Young tableaux and their massive decompositions are one-to-one correspondent (PMC are the same); at the same time there is no overlap between massive and massless Young tableaux after decomposition.

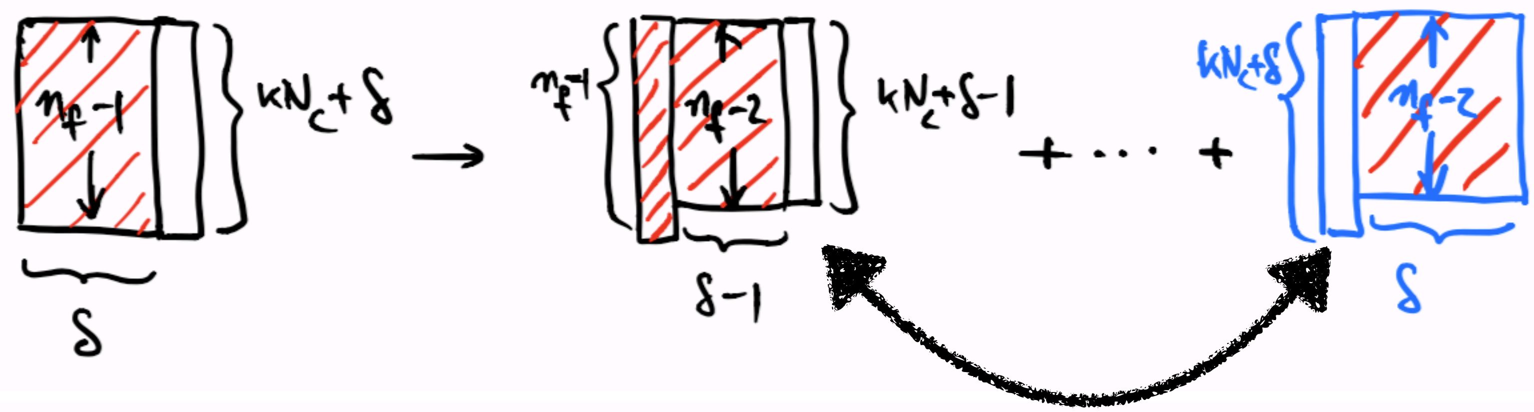
Uplifting from 2 flavors to 3 flavors doesn't work, because one-to-one correspondence is not true.

Uplifting from 3 flavors to 4 flavors doesn't work, because the column is a singlet in PMC[3] and it won't give constraints, while it's not a singlet in PMC[4].

➡ Step 1 in the proof fails.

Exotics:

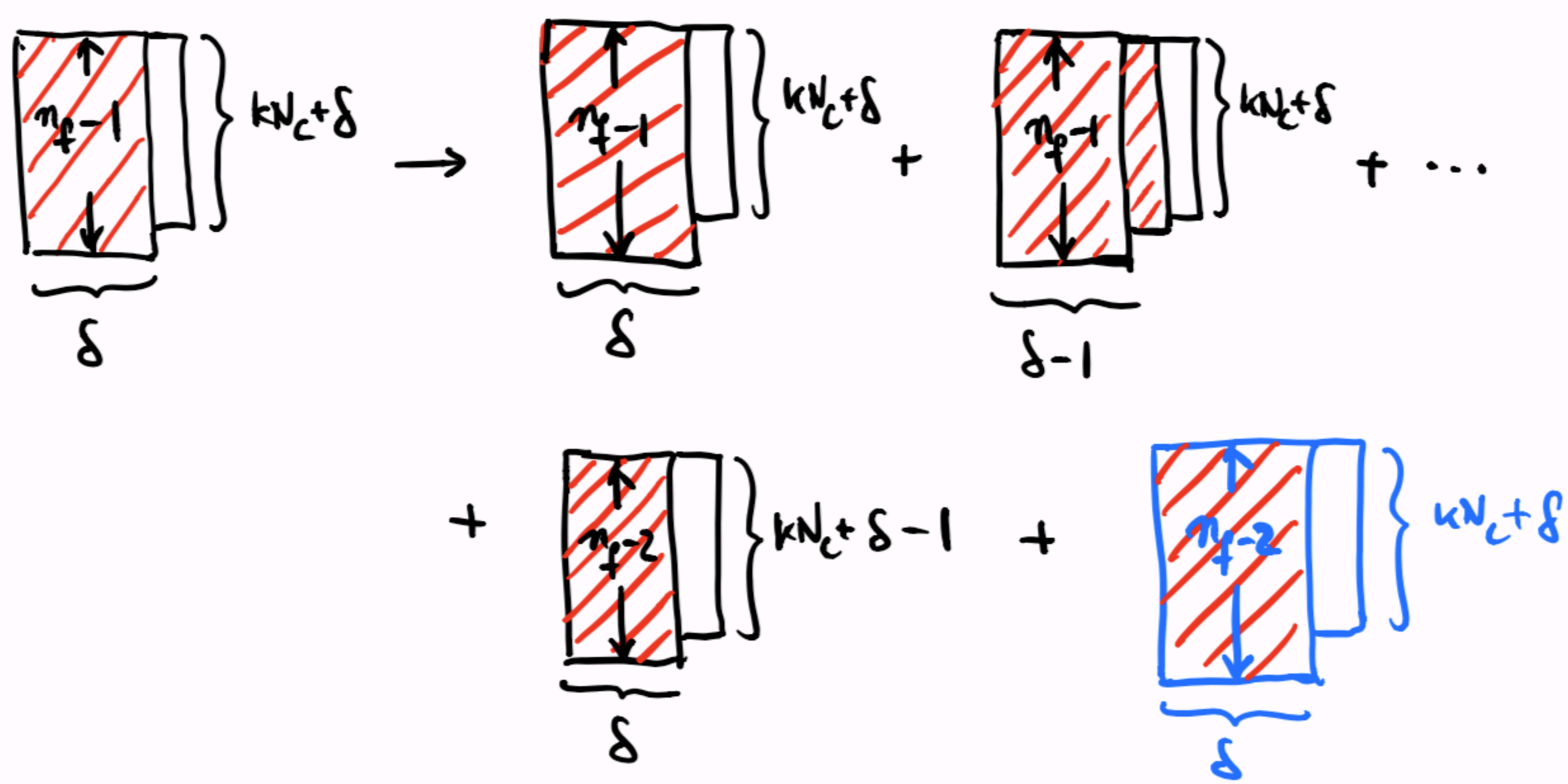
$n_f - 1 = kN_c + \delta$: (e.g. $k=1,\ N_c=3,\ \delta=1 \Rightarrow n_f = 5$)



In the example of only one antiquark, uplifting from 4 flavors to 5 flavors doesn't work, because there is overlap between massless and massive Young tableaux.

Step 2 in the proof fails.

$n_f - 1 > kN_c + \delta$: (e.g. $k=1,\ N_c=3,\ \delta=1 \Rightarrow n_f = 6, 7, 8 \cdots$)



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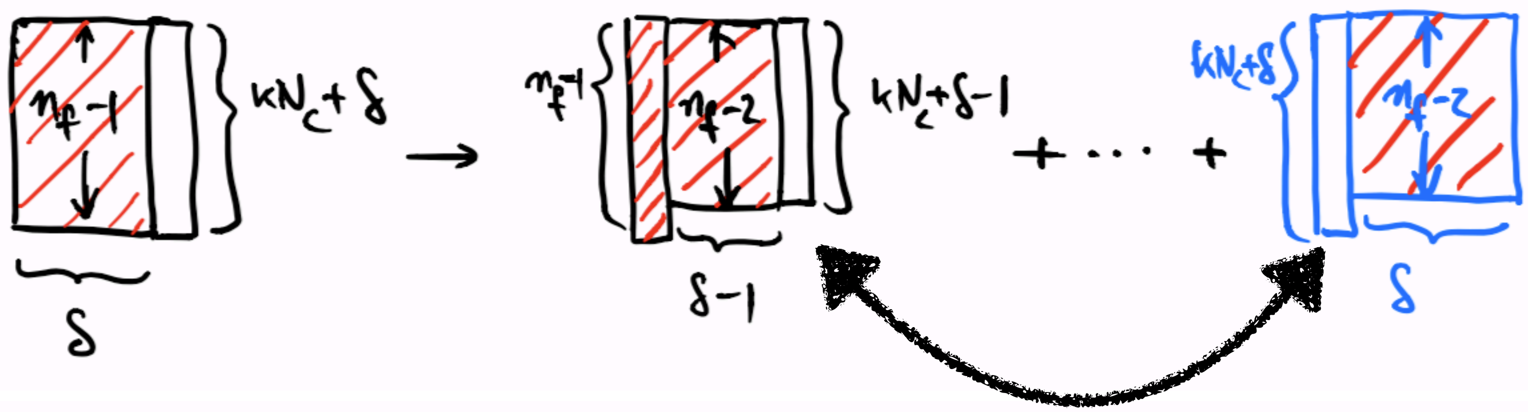

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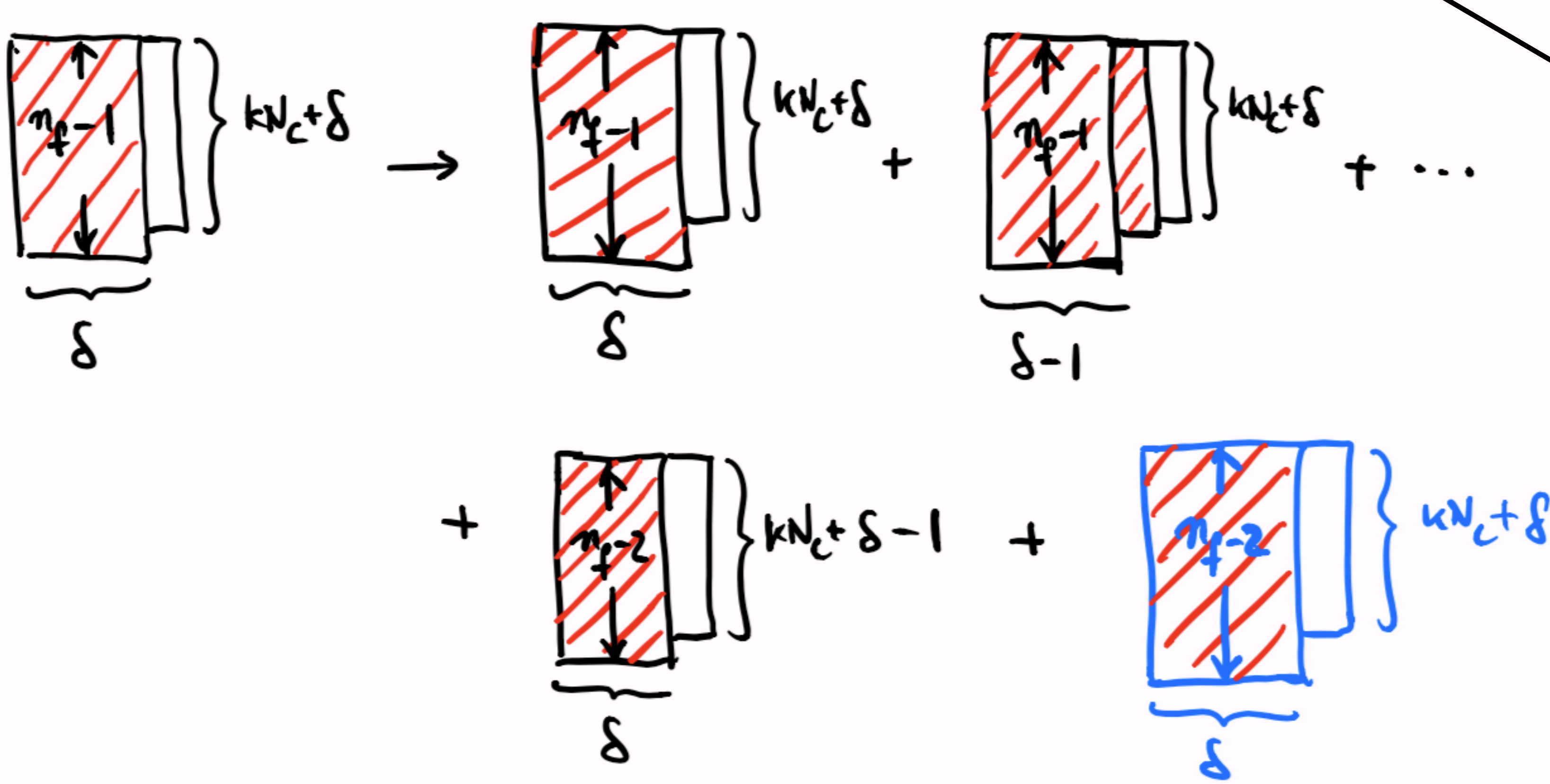


Uplifting from 6 flavors to 7 flavors works, and so on.

In general, what about uplifting from $\kappa N_c + \delta + 1$ to $\kappa N_c + \delta + 2$? (Step 2 works.)

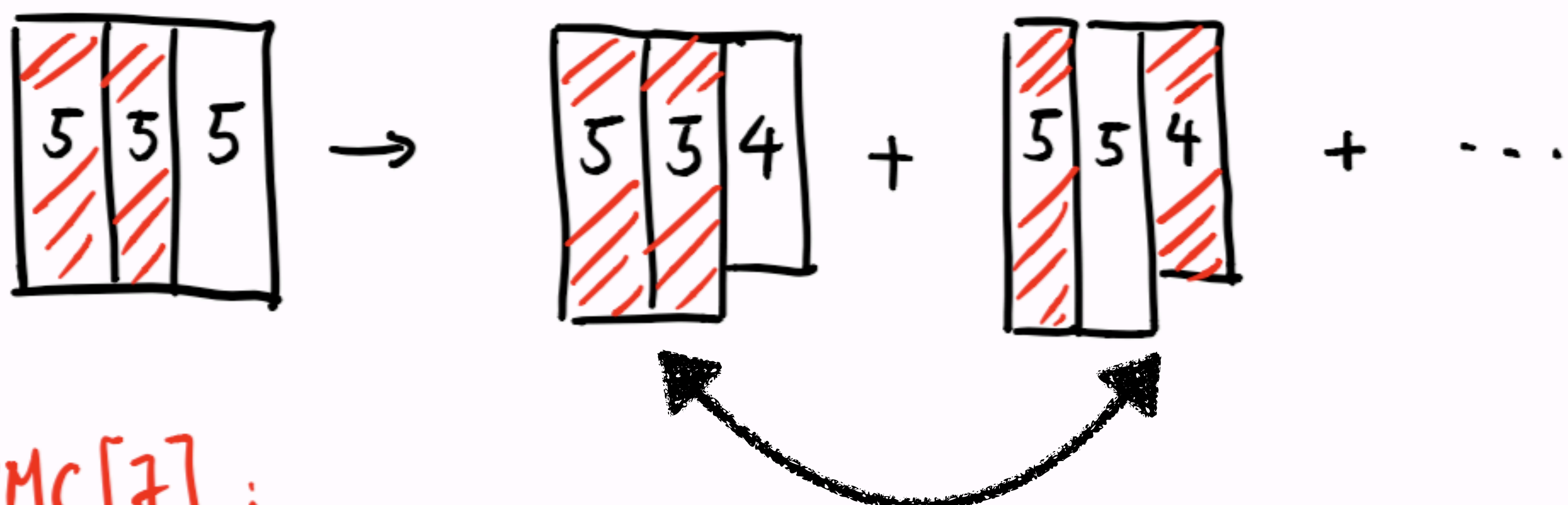
then what about step 1? **PMC**$[\kappa N_c + \delta + 1]$ $\Rightarrow$ **PMC**$[\kappa N_c + \delta + 2]$

In general it doesn't work, although it works for pentaquark case.

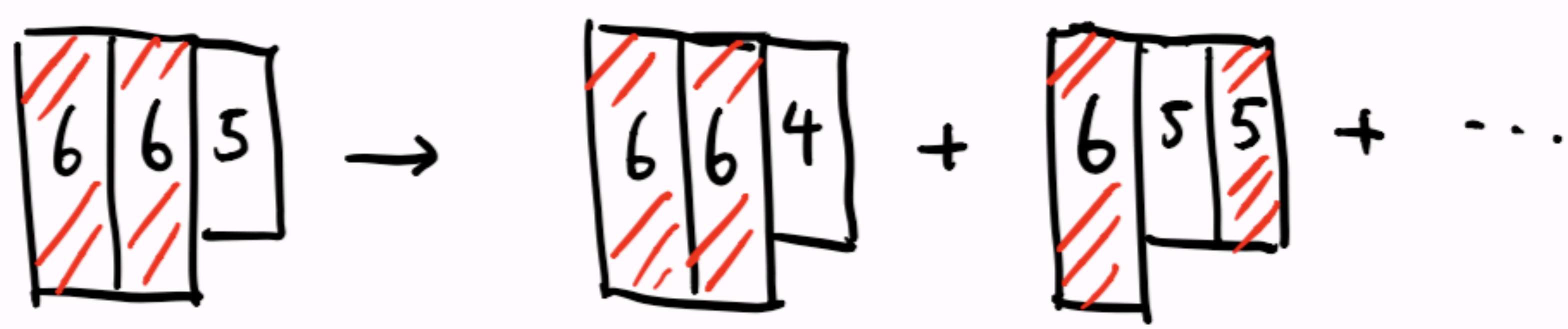
e.g. $\kappa = 1$, $N_c = 3$, $\delta = 2$

PMC[6] $\not\Rightarrow$ PMC[7]

PMC[6] :



PMC[7] :



Equations in small nf

Baryonic case: