Composite Connections
Lyon 2020

What the lattice can say about models for **composite Higgs and top**

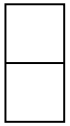
Benjamin Svetitsky
Tel Aviv University

TACO Collaboration: T. DeGrand, M. Golterman, D. Hackett, W. Jay, E. Neil, Y. Shamir, BS

- List of models based on AF gauge theories w/o elementary scalars: Ferretti & Karateev (1312.5330)
- Most economical model **M6** discussed by Ferretti (1404.7137,1604.06467)
  - $\mathrm{SU}(4)$ gauge theory, quartet fermions and sextet fermions (TACO since 2015)

COMPOSITE HIGGS

(Georgi & Kaplan 1984)

- $\mathrm{SU}(4)$ *hypercolor* gauge theory: new strong sector with scale $\Lambda_{\mathrm{HC}} \gg v$ ($5\ \mathrm{TeV}$ *vs* $246\ \mathrm{GeV}$)
- 5 Majorana fermions Q in sextet  rep $\Longrightarrow \mathrm{SU}(5)$ global (chiral) symmetry
- Hypercolor theory breaks $\mathrm{SU}(5) \to \mathrm{SO}(5)$ (Peskin 1980)
  Let Higgs multiplet $\subset$ Goldstone bosons, so $m_h = 0$ and in fact $V(h) = 0$: *Goldstone Higgs* protected from Λ_{HC} [also $\mathrm{SO}(5) \supset \mathrm{SU}(2)_L \times \mathrm{U}(1)$ and $\mathrm{SU}(2)$ custodial symmetry]
- Couple to gauge bosons/fermions of SM, generate

$$V_{\mathrm{eff}}(h) = -\alpha \cos^2(h/f) - \beta \sin^2(2h/f)$$

- If $4\beta > \alpha$ then $h = 0$ is unstable, $v = \sqrt{2}\,\langle h \rangle$ given by

$$\cos(\sqrt{2}v/f) = \alpha/(4\beta) \qquad \Longrightarrow \qquad (v/f)^2 \approx 1 - \alpha/(4\beta)$$

note $v \ll f$ demands α close to 4β — not tuning but a (calculable?) test!

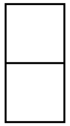
1st term ($+$ curvature): $\alpha = \frac{1}{2}(3g^2 + g'^2)C_{LR} +$ top loops*

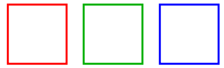
2nd term ($-$ curvature): $\beta =$ top loops*

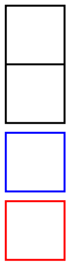
NEEDED: LECs in the HC theory

HYPERBARYONS for the TOP QUARK PARTNER

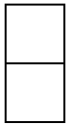
(Kaplan 1991)

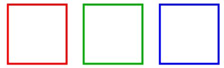
All HC singlets made of Q's  in SU(4) will be bosons (mesons, *diquarks*, etc.)

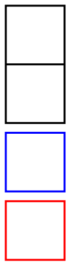
So add fund rep fermions q to the hypercolor theory. Give them 3 colors: 

Then Qqq  is a colored, fermionic chimera, ready to mix with the top quark.

HYPERBARYONS for the TOP QUARK PARTNER (Kaplan 1991)

All HC singlets made of Q's  in SU(4) will be bosons (mesons, *diquarks*, etc.)

So add fund rep fermions q to the hypercolor theory. Give them 3 colors: 

Then Qqq  is a colored, fermionic chimera, ready to mix with the top quark.

MIXING with the top quark:

- Assume extended gauge sector at $\Lambda_{\mathrm{EHC}} \gg \Lambda_{\mathrm{HC}}$ to generate 4-fermion interaction $tQqq$

  — mix t quark with chimera, generate t mass via seesaw.

  — t mass, Yukawa coupling $\Leftrightarrow$ LEC

THE simplified LATTICE THEORY: [LECs fairly insensitive to N_f — cf. QCD]

- SU(4) gauge theory
  $2\times$ sextet fermions Q (not 2.5)
  $2\times$ quartet fermions q (not 3)

producing

- $\bar{Q}Q$ mesons, including Goldstone bosons for “Higgs field”
- $\bar{q}q$ mesons, including (colored) Goldstone bosons
- U(1) (pseudo-)Goldstone boson
- q^4, Q^6 baryons
- Qqq chimera baryons for mixing with “top quark”

Inputs to each simulation:

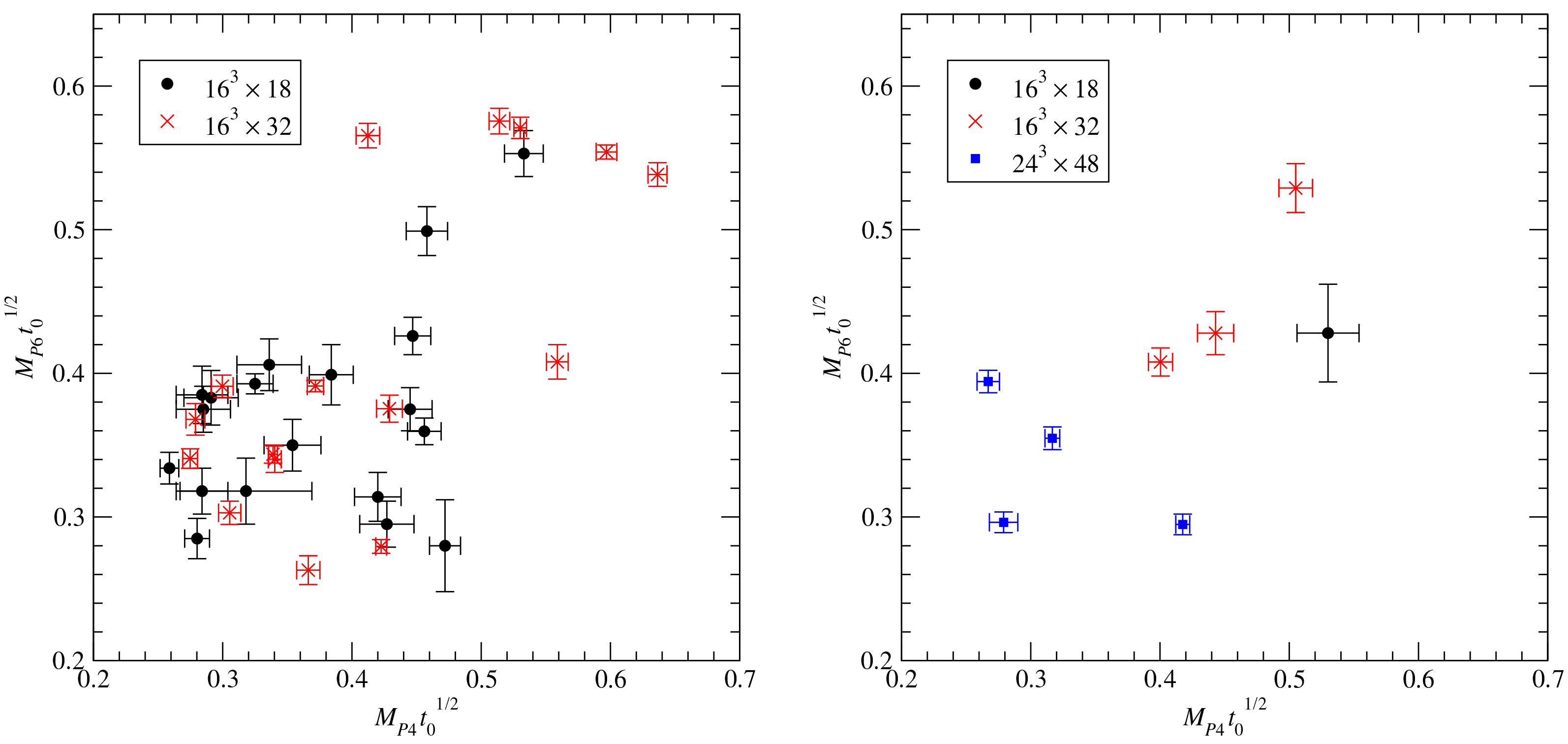
- gauge coupling β, fermion masses κ_4, κ_6

giving

- scale t_0 from gradient flow $\Longrightarrow$ lattice spacing in physical units
- meson/baryon masses (in physical units), AWI quark masses (ditto)$\Longrightarrow$ Chiral PT
- current matrix elements $\Longrightarrow$ LECs

Results: MAP OF $J^P = 0^-$ MESON MASSES

coarse lattice: $1/a \simeq 1.4\ \mathrm{GeV}$ fine lattice: $1/a \simeq 2.1\ \mathrm{GeV}$



Scale: if this were QCD $t_0^{-1/2} = 1.4\ \mathrm{GeV}$

CHIRAL PERTURBATION THEORY — for two inequivalent chiral fields. E.g.,

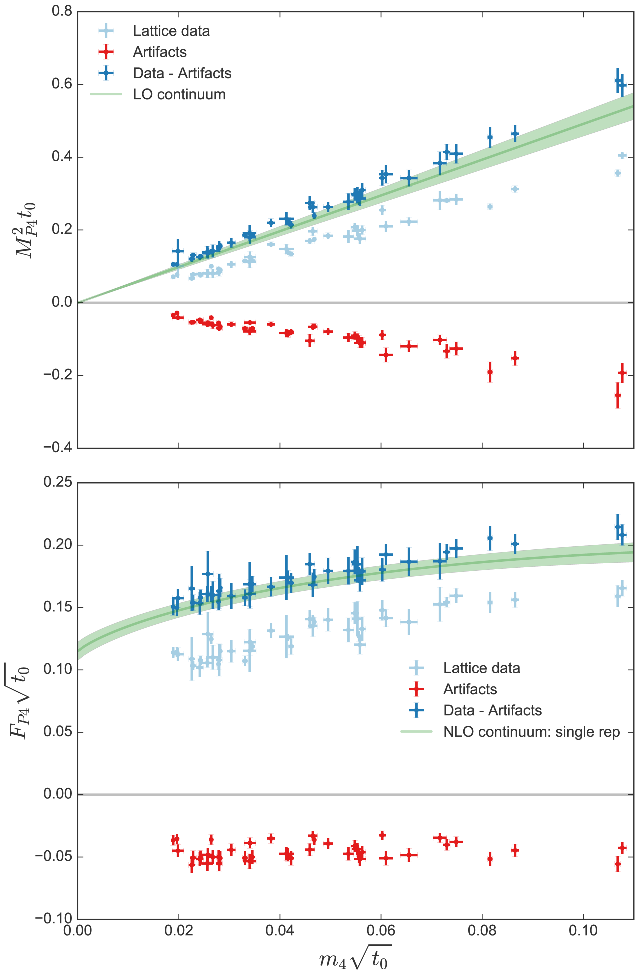
$$(\hat{M}_{P4}^2)^{\rm NLO} = 2\hat{m}_4 \mathring{B}_4 \left(1 + L_{44}^M \hat{m}_4 + L_{46}^M \hat{m}_6 + \frac{1}{2}\Delta_4 - \frac{4}{5}\Delta_\zeta\right) + \mathring{W}_{44}^M \hat{a}\hat{m}_4 + \mathring{W}_{46}^M \hat{a}\hat{m}_6 + \mathring{W}_4^M \hat{a}^2 \ ,$$

$$(\hat{M}_{P6}^2)^{\rm NLO} = 2\hat{m}_6 \mathring{B}_6 \left(1 + L_{66}^M \hat{m}_6 + L_{64}^M \hat{m}_4 - \frac{1}{4}\Delta_6 - \frac{1}{5}\Delta_\zeta\right) + \mathring{W}_{66}^M \hat{a}\hat{m}_6 + \mathring{W}_{64}^M \hat{a}\hat{m}_4 + \mathring{W}_6^M \hat{a}^2 \ ,$$

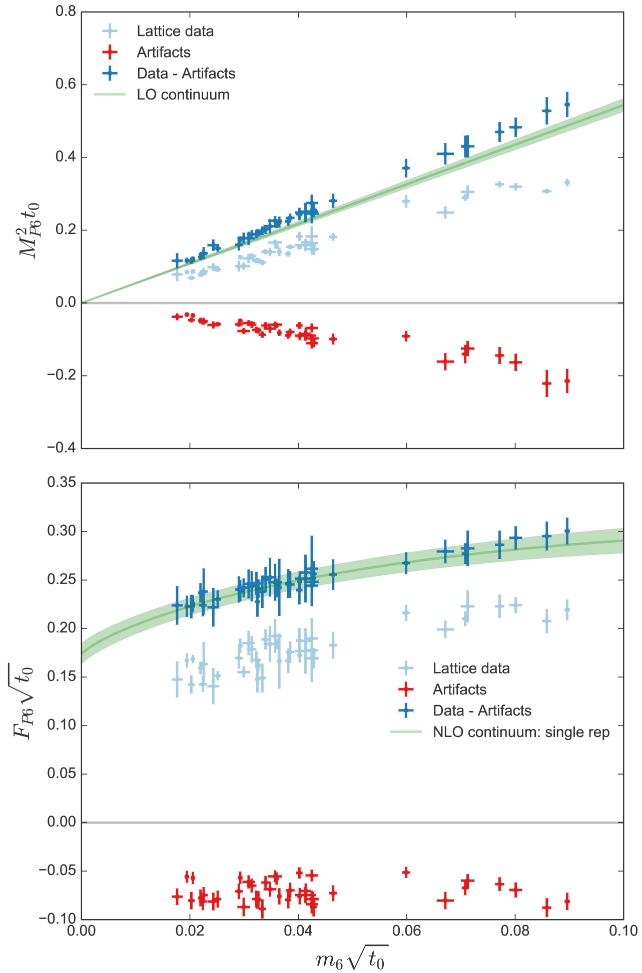
- $\bar{q}q \equiv 4$, $\bar{Q}Q \equiv 6$
- m_4, m_6 – quark masses, extracted from axial Ward identities
  $\Longrightarrow$ we want $m_6 \to 0$ [Higgs $=$ Goldstone boson]

- Cross terms! L_{46}, L_{64} in NLO
- Δ's are the usual chiral logs. Note Δ_ζ from U(1) GB.
- W's are discretization effects — subtract them from data
  $\Longrightarrow$ i.e., $a \to 0$

χPT for $\bar{q}q$ MESONS



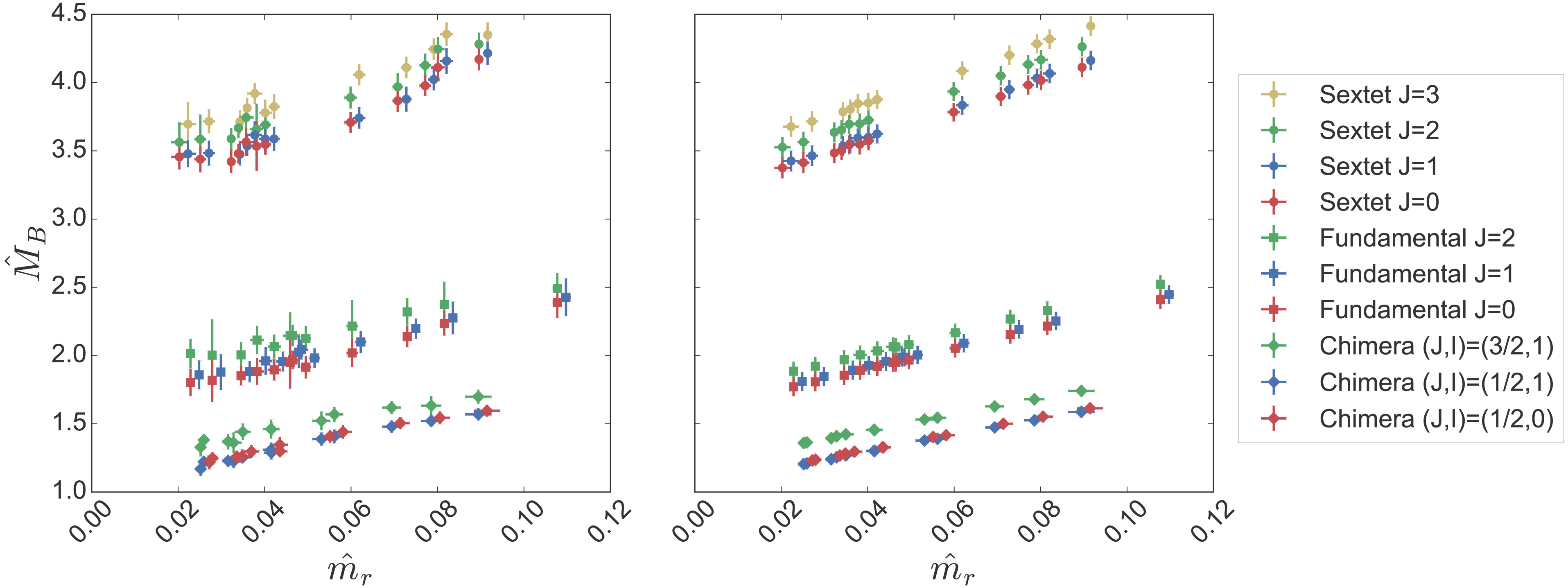
χPT for $\bar{Q}Q$ MESONS



Note decay constants F_4, F_6; also vector boson masses

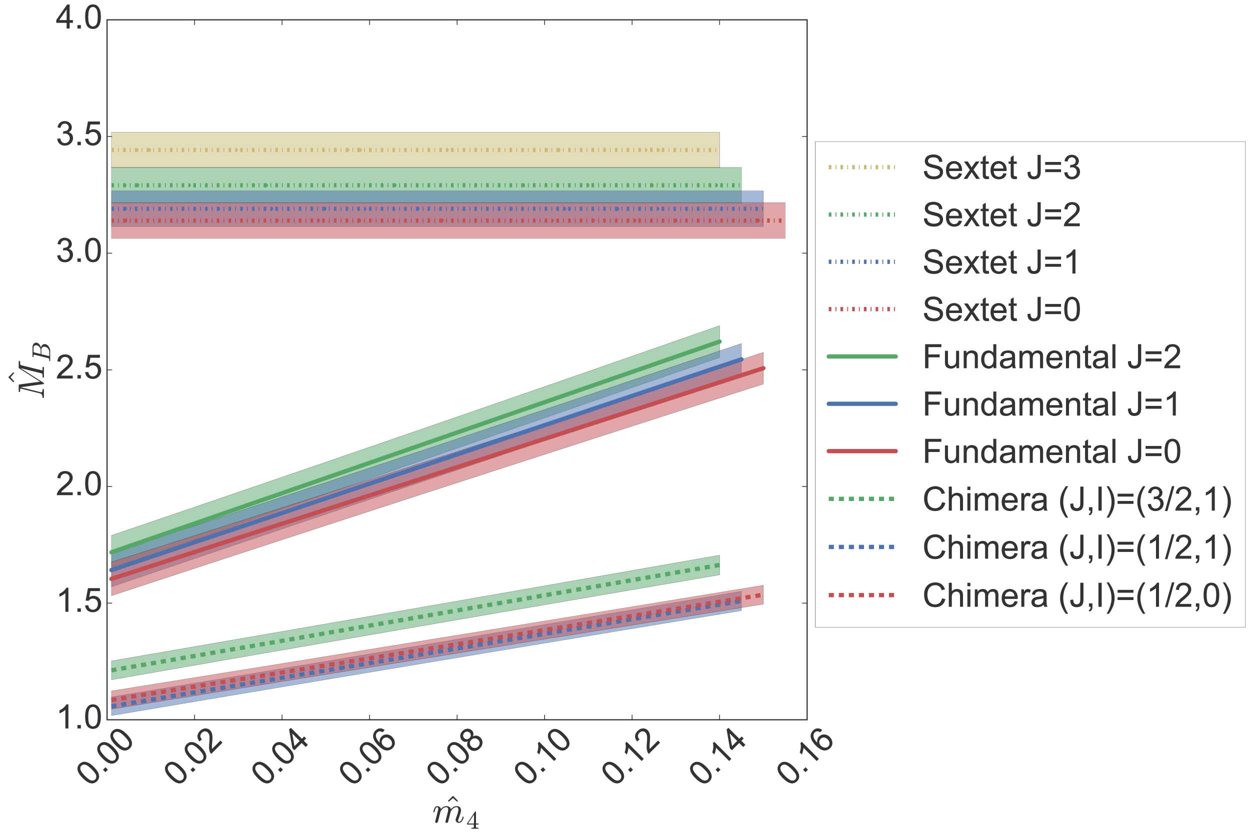
BARYON MASSES: $6Q$, $4q$, and $\ldots$ Qqq (the chimera)

Data: Linear fits:



$\hat{m}_r = \hat{m}_6,\ \hat{m}_4,\ a\hat{m}_4 + b\hat{m}_6$ resp.

Chiral sextet limit $m_6 \to 0$ ($6Q$, $4q$, Qqq)



Bottom line: $F_6 \gtrsim 1.1$ TeV $\Longrightarrow M_{\text{Chimera}} \gtrsim 6.5$ TeV, $M_V \gtrsim 5$ TeV.

LOW-ENERGY CONSTANT C_{LR}: the gauge contribution to the COMPOSITE HIGGS POTENTIAL

$$C_{LR} = \int_0^\infty dq^2 q^2 \, \Pi_{LR}(q^2)$$

where Π_{LR} is the transverse part of

$$\Pi_{\mu\nu}(q) = -\int d^4x \, e^{iqx} \, \langle J^L_\mu(x) J^R_\nu(0) \rangle \qquad \text{(currents of } \mathbf{6} \text{ fermions)}$$

Chiral & continuum limit: m_6, $a \to 0$ [no discernible dependence on m_4]

RESULT:

$$\frac{C_{LR}}{F_6^4} = 29(8)(5)$$

Cf. QCD:

$$m^2_{\pi^\pm} - m^2_{\pi^0} = \frac{3\alpha}{4\pi} \frac{C_{LR}}{f_\pi^2} \quad \Longrightarrow \quad \frac{C_{LR}}{f_\pi^4} \approx 42$$

(Das, Guralnik, Mathur, Low, and Young 1967)

— not too different.

MIXING WITH THE TOP QUARK — 4-fermion interaction in the HC theory

$$V_{\rm top}^{\rm HC} = G_R \bar{t}_L B_R + G_L \bar{t}_R B_L + {\rm h.c.} \quad — \quad t \equiv {\rm top\ quark}, B \equiv Qqq$$

where $G_{L,R} \sim g_{\rm EHC}^2/\Lambda_{\rm EHC}^2$. Then $t \leftrightarrow B$ mixing $\Longrightarrow$

$$m_t \approx G_L G_R \frac{Z_L Z_R}{M_B} \frac{v}{F_6} \qquad (= y_t v)$$

where $Z_{L,R}$ are matrix elements (*cf. proton decay in QCD*)

$$\left\langle 0 \left| (Qqq)^\alpha_{L,R} \right| {\rm Chimera} \right\rangle = Z_{L,R}\, u^\alpha$$

Result: $Z_L \simeq Z_R \equiv Z$ and

$Z/F_6^3 \simeq 0.35(8)$ Cf. QCD: $Z/f_\pi^3 \simeq 7$ — a factor of 20!

Rearrange:

$$y_t \approx \left(\frac{g_{\rm EHC} F_6}{\Lambda_{\rm EHC}}\right)^4 \left(\frac{Z}{F_6^3}\right)^2 \frac{F_6}{M_B} \simeq 0.01 \left(\frac{g_{\rm EHC} F_6}{\Lambda_{\rm EHC}}\right)^4$$

But $y_t \simeq 1$ and certainly $g_{\rm EHC} < 1$, so

$$\frac{F_6}{\Lambda_{\rm EHC}} \gtrsim 3$$

contradicting $\Lambda_{\rm EHC} \gg F_6$. BAD. ($\Longrightarrow$ large 4-fermi interactions at Λ_{HC}, etc.)

*** A *general problem*: Even if Z/F_6^3 comes out large, it's hard to push $\Lambda_{\rm EHC}$ very far.

Required: an enhancement (that recalls WALKING TECHNICOLOR).

Again

$$y_t \approx G_L G_R \frac{Z_L Z_R}{M_B} \frac{1}{F_6} \quad ,$$

$$G_{L,R} \sim g^2_{\rm EHC}/\Lambda^2_{\rm EHC} \quad .$$

and we need a large $G_{L,R}$ — but at the HC scale where Z is measured:

$$G(\Lambda_{\rm HC}) = G(\Lambda_{\rm EHC}) \exp\left(- \int_{\Lambda_{\rm HC}}^{\Lambda_{\rm EHC}} \gamma_B(g_{\rm HC}(\mu)) \frac{d\mu}{\mu} \right)$$

where γ_B is the anomalous dimension of Qqq. This can be much bigger if γ_B is large (and < 0).

Is it?

Not in this theory — which resembles QCD (dynamically speaking).

WHAT HAPPENS IN OTHER MODELS?

Look for models with large γ_B AND large Z —

- E.g., add quartet flavors$\Longrightarrow$ approach conformality (?), larger anomalous dimensions (?)
- Or maybe ... other models entirely (Franzosi & Ferretti 1905.08273)
  - M11 — also $\mathrm{SU}(4)$ gauge group but $4\times$ quartet fermions $+ 3\times$ (Dirac) sextet fermions
  - M5 — $\mathrm{Sp}(4)$ gauge theory $\Longrightarrow \mathrm{SU}(4)/\mathrm{Sp}(4)$ composite Higgs

(E. Bennett, *et al.* 1712.04220 *et seq.*)

(TODAY: David Lin, Jong-Wan Lee)

LOW-ENERGY CONSTANT L_{10}; and S

In the chiral Lagrangian, L_{10} is the LEC that multiplies

$$\mathcal{O}_{10} = -\mathrm{tr}\,\left((\mathcal{V}_{\mu\nu} - \mathcal{A}_{\mu\nu})\Sigma(\mathcal{V}_{\mu\nu} + \mathcal{A}_{\mu\nu})\Sigma^\dagger\right)$$

— coupling of Higgs to external field strengths. Calculate from fit to current correlator:

$$\hat{\Pi}_{LR}(q^2) = \frac{\mathcal{G}(N)}{48\pi^2}\left[\frac{1}{3} + \log\left(\frac{M_{vs}^2}{\mu^2}\right) - H(s)\right] + 8L_{10}$$

where

$$\hat{\Pi}'_{LR}(q^2) = \frac{F^2}{q^2} + \hat{\Pi}_{LR}(q^2)$$

is the longitudinal part of

$$\Pi_{\mu\nu}(q) = -\int d^4x\, e^{iqx}\,\langle J_\mu^L(x) J_\nu^R(0)\rangle$$

Result: $L_{10} = -0.0100(12)(35)$. So . . .

- contributes to $H \to gg, WW^*$, etc.
- gives $S_{HC} = 0.8(2)\xi$
  Expt: $S < 0.09$ OK if $\xi \equiv 2(v/F)^2 < 0.11(3)$, consistent with other estimates

Can we calculate ξ? Back to $V_{\mathrm{eff}}(h) \implies$ we need several LECs $\{C_i\}$ of top loops.
. . . Lattice relates these to 4-fermi couplings $\{G^a\}$ — but we have no knowledge of $\{G^a\}$!

SUMMARY

Composite Higgs/top model (almost **M6**):

$$\mathrm{SU}(4) \text{ gauge theory, } 2\times \text{ quartet fermions } + 2\times \text{ sextet fermions}$$

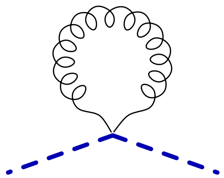
1. Masses of mesons, top partner — straightforward. Lower limits on Λ_{HC}.
2. Gauge loop $C_{LR} = \int_0^\infty dq^2 q^2\, \Pi_{LR}(q^2) \Longrightarrow$ a piece of the Higgs potential. No surprises.
3. LEC L_{10} — together with S gives bound on $\xi = 2(v/F)^2$
4. Top quark mixing: Too small! Doesn't work!

FUTURE

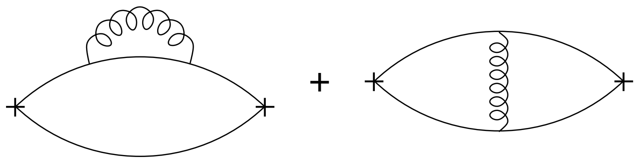
- Lots more models. Look for larger Z and/or large γ_B.
- Systematic study of LECs requires systematics of 4-fermi couplings that come from EHC.

EXTRA C_{LR}: THE GAUGE CONTRIBUTION IN THE COMPOSITE HIGGS POTENTIAL

Gauge tadpole on Higgs propagator

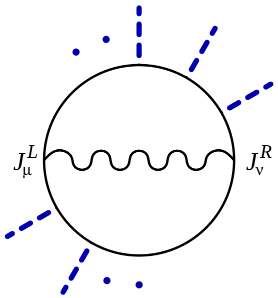


becomes in the gauge theory



where "+" is a vertex to the GB.
(Hyperglue not shown.)

In the chiral Lagrangian for the GB's, all n-GB amplitudes are related



— including the zero-GB amplitude!

SO

$$C_{LR} = \int_0^\infty dq^2 q^2\, \Pi_{LR}(q^2)$$

where

$$(q^2\delta_{\mu\nu} - q_\mu q_\nu)\,\Pi_{LR}(q^2) = -\int d^4x\, e^{iqx}\,\langle J^L_\mu(x) J^R_\nu(0)\rangle$$