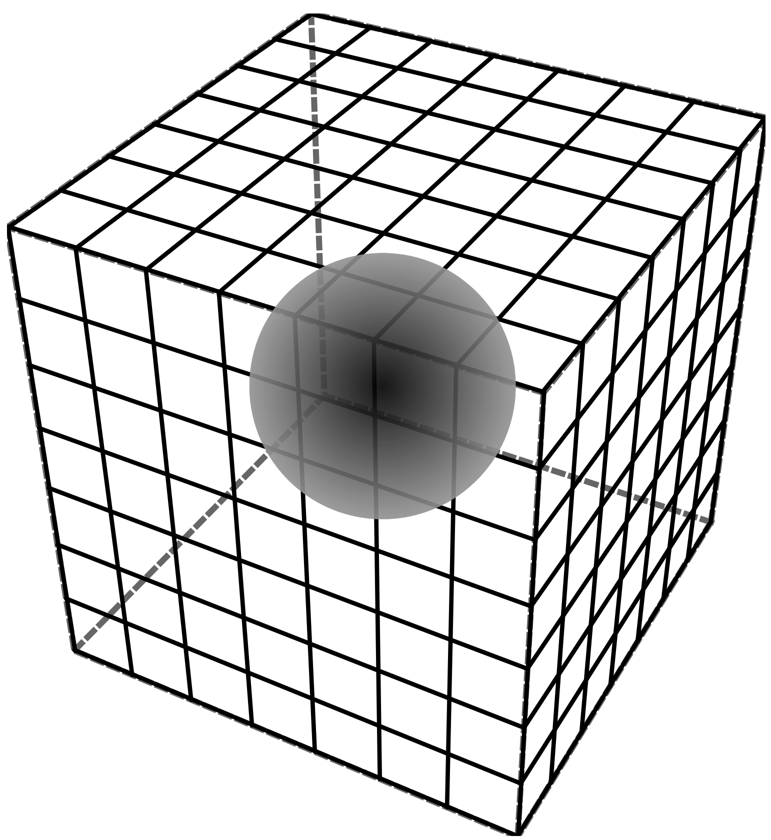
# Sp(4) gauge theories for BSM models on the lattice: towards composite Higgs and dark matter

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Workshop on composite connections @ Lyon,
Sept. 24, 2020

# Motivation - Sp(2N) gauge theory

| Coset | HC | $\psi$ | $\chi$ | $-q_\chi/q_\psi$ | Baryon | Name | Lattice |
|---|---|---|---|---|---|---|---|
| $\frac{SU(5)}{SO(5)} \times \frac{SU(6)}{Sp(6)}$ | Sp(4) | $5 \times \mathbf{A}_2$ | $6 \times \mathbf{F}$ | 5/3 | $\psi\chi\chi$ | M5 | $\surd$ |
| $\frac{SU(5)}{SO(5)} \times \frac{SU(3)^2}{SU(3)}$ | SU(4) | $5 \times \mathbf{A}_2$ | $3 \times (\mathbf{F}, \overline{\mathbf{F}})$ | 5/3 | $\psi\chi\chi$ | M6 | $\surd$ |
| | SO(10) | $5 \times \mathbf{F}$ | $3 \times (\mathbf{Sp}, \overline{\mathbf{Sp}})$ | 5/12 | | M7 | |
| $\frac{SU(4)}{Sp(4)} \times \frac{SU(6)}{SO(6)}$ | Sp(4) | $4 \times \mathbf{F}$ | $6 \times \mathbf{A}_2$ | 1/3 | $\psi\psi\chi$ | M8 | $\surd$ |
| | SO(11) | $4 \times \mathbf{Sp}$ | $6 \times \mathbf{F}$ | 8/3 | | M9 | |
| $\frac{SU(4)^2}{SU(4)} \times \frac{SU(6)}{SO(6)}$ | SO(10) | $4 \times (\mathbf{Sp}, \overline{\mathbf{Sp}})$ | $6 \times \mathbf{F}$ | 8/3 | $\psi\psi\chi$ | M10 | |
| | SU(4) | $4 \times (\mathbf{F}, \overline{\mathbf{F}})$ | $6 \times \mathbf{A}_2$ | 2/3 | | M11 | $\surd$ |

*Cacciapaglia, Ferretti, Flacke & Serodio (2019)*

modern composite Higgs

\+ composite dark matter

\+ universalities in Yang-Mills

\+ Very little is known in lattice community

*Holland, Pepe & Wiese (2003)*

- Previous work on the lattice studies of Sp(2N) gauge theories

- ☑ Glueballs & quenched meson spectrum of Sp(4) *1712.04220*
- ☑ Casimir scaling in pure SU(N), SO(N) & Sp(2N) gauge theories *1705.00286*
- ☑ Meson spectrum of Sp(4) with Nf=2 dynamical fundamental fermions *1909.12662*
- ☑ Quenched meson spectrum of Sp(4) with fundamental & antisymmetric fermions *1912.06505*
- ☑ Scalar and tensor glueballs in Sp(2N) gauge theories *2004.11063*

# Sp(4) composite Higgs & Top partial compositeness

- UV realization of `SO(6)/SO(5)~SU(4)/Sp(4)` CH model from `Sp(2N)` gauge theory

*Barnard, Gherghetta & Ray (2014)*
*Ferretti & Karateev (2013)*

**Global symmetry**

2 Dirac flavors in fund. rep.

3 Dirac flavors in anti-sym. rep.

$$SU(4)/Sp(4) \times SU(6)/SO(6)$$
$$\sim SO(6)/SO(5)$$

*SM EW*

*SM Strong*

$SU(2)_L \times U(1)_Y \subset Sp(4)$

$SU(3)_c \times U(1)_Y \subset SO(6)$

*4 of 5 PNGBs: Higgs doublets*

*Top partner = Chimera baryon*

*e.g.* $\hat{\Psi}^{a\alpha b} \equiv \left(\psi^a \chi^\alpha \psi^b\right)$ *carry color charge*

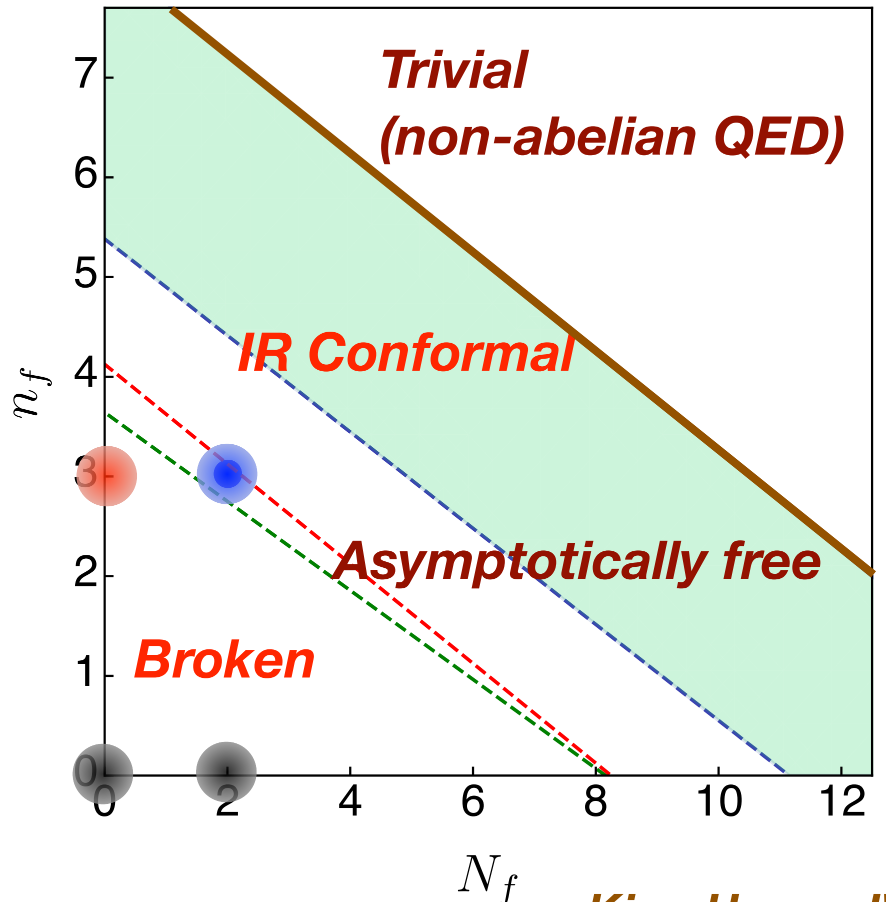
- N=2 to make it economic & QCD-like (or near-conformal)
- Easily accommodated with the lattice Monte Carlo simulation

✓ *Confirm the IR dynamics: confinement, chiral symmetry breaking, …*

✓ *Predictions: mass spectra, low-energy constants, …*

# Some features expected from analytical studies

Conformal window from conformal expansion



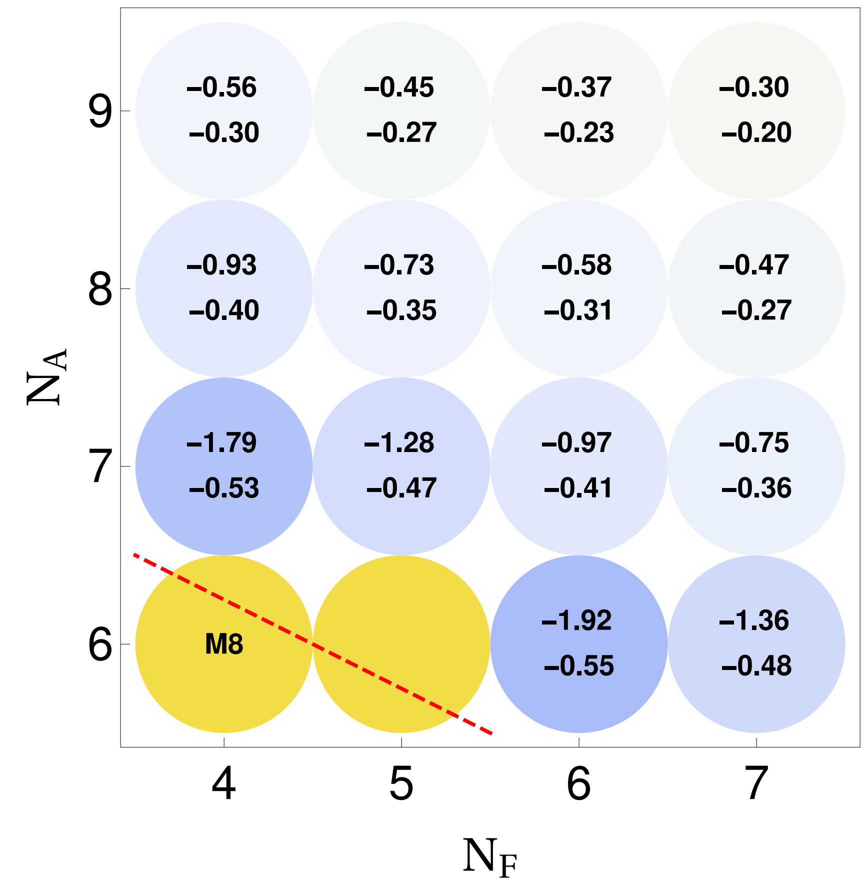

Kim, Hong, JWL (2020)

$$\mathcal{L}_{mix} = \lambda_{L,R}\Psi_{L,R}\mathcal{O}_{\Psi}$$

$$m_f \sim \lambda_L \lambda_R v$$

$$\lambda_{L,R} \sim \left(\frac{\Lambda}{\Lambda_{UV}}\right)^{\dim\,\mathcal{O}_{L,R}-\frac{5}{2}}$$

Anomalous dimension of top-partner




Franzosi & Ferretti (2019)

- The Sp(4) model is expected to be confined in the IR, but could be near-conformal.

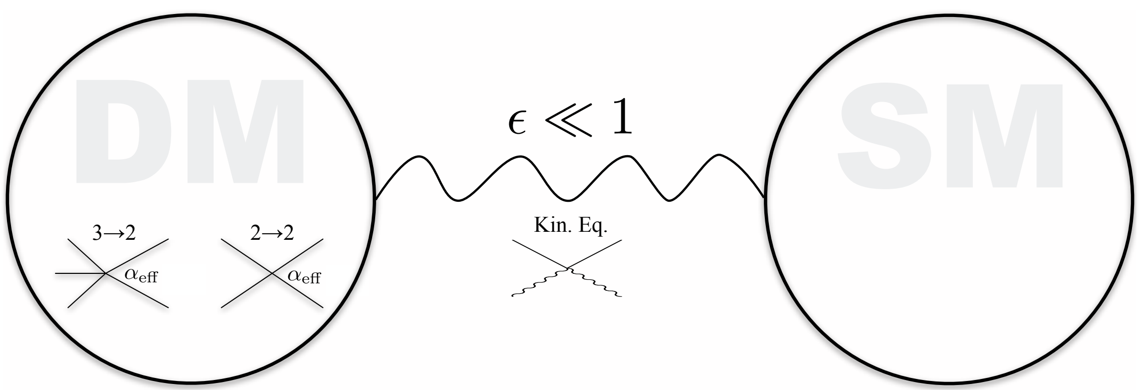
- To have the top Yukawa coupling, $y_t \approx 1$, it is preferred to have a large anomalous dimension for the top-partner, $\Delta = 9/2 + \gamma^*$ with $\gamma^* \approx -2$ . The 1-loop result finds somewhat smaller values of $\gamma^*$, but could be largely changed by higher order terms.

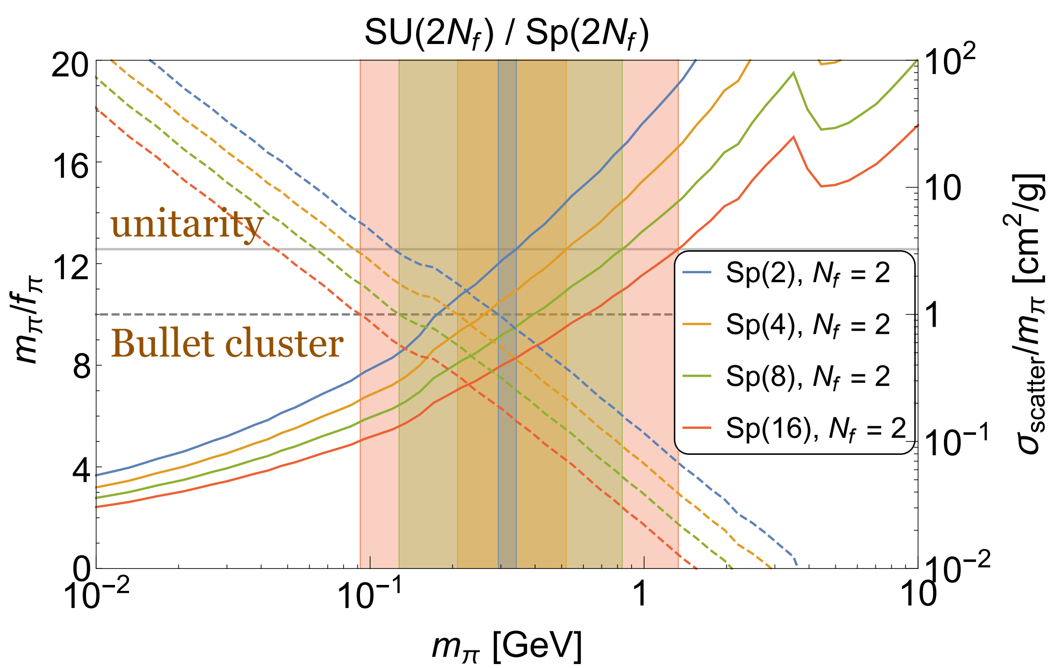
# Strongly interacting dark matter

- UV realization of strongly interacting dark matter

**Strongly interacting massive particles (SIMPs)**

*Hochberg, Kuflik, Volansky, Wacker (2014)*

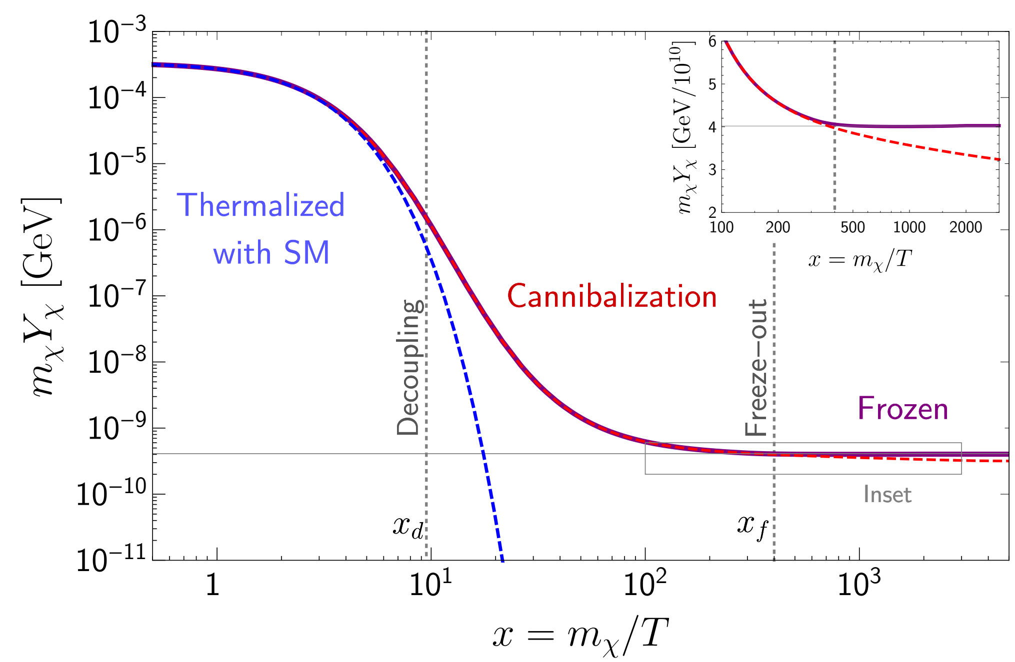


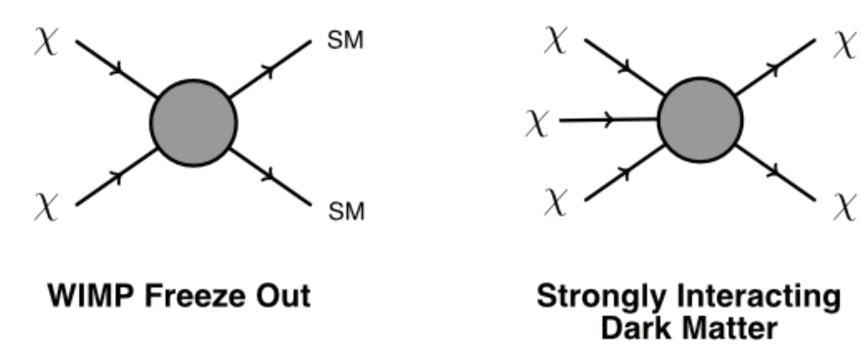




*Hochberg, Kuflik, Murayama, Volansky, Wacker (2014)*

**Elastically Decoupling Relics (ELDERs)**

*Kuflik, Pelestein, Lorier, Tsai (2015)*



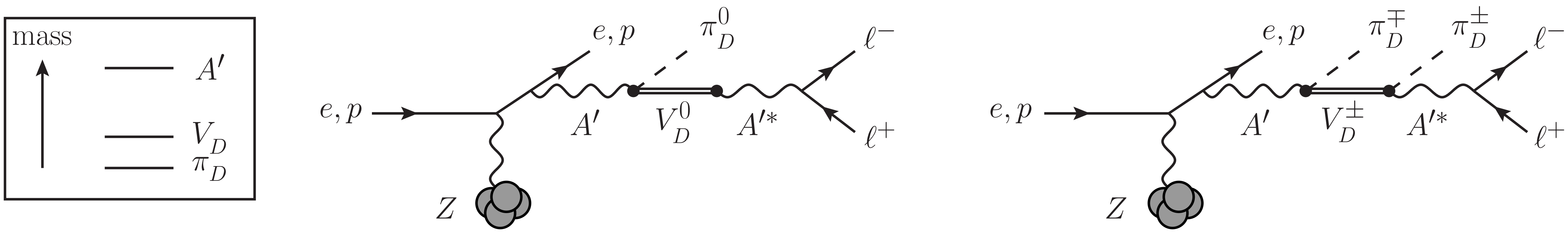




$$\mathcal{L}_{\rm WZW} = \frac{2N_c}{15\pi^2 f_\pi^5}\epsilon^{\mu\nu\rho\sigma}{\rm Tr}\left[\pi\partial_\mu\pi\partial_\nu\pi\partial_\rho\pi\partial_\sigma\pi\right]$$

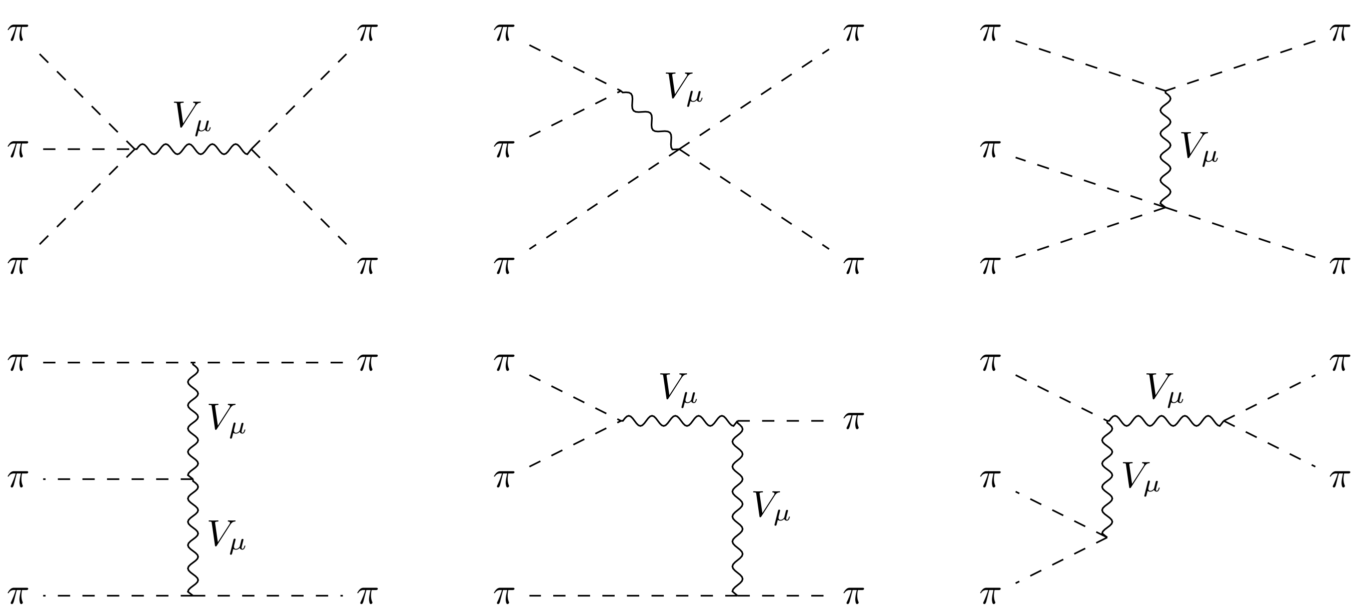
# Strongly interacting dark matter

- UV realization of strongly interacting dark matter

***Berlin, Blinov, Gori, Schuster & Toro (2018)***




***Choi, Lee, Ko & Natale (2018)***




Beyond the SIMPlest scenario, vector meson plays a crucial role to extend the viable parameter range producing the correct relic density without violating Bullet cluster bound.

$$m_\pi \sim \frac{m_V}{3}, \ \frac{m_V}{2}, \ \text{or even heavier.}$$

*Directly accessible via lattice simulations*

# Lattice action and simulation details

- Lattice formulation with the standard Wilson gauge & fermion actions

$$S \equiv \beta \sum_x \sum_{\mu<\nu} \left(1 - \frac{1}{4} \mathrm{Re}\,\mathrm{Tr}\, U_\mu(x) U_\nu(x+\hat{\mu}) U_\mu^\dagger(x+\hat{\nu}) U_\nu^\dagger(x)\right) + a^4 \sum_x \overline{Q}_j(x) D^F Q_j(x), \quad U_\mu(x) \in Sp(4) \text{ with } j = 1, 2,$$

$$D^F Q_j(x) \equiv (4/a + m_0) Q(x) - \frac{1}{2a} \sum_\mu \left\{ (1-\gamma_\mu) U_\mu^F(x) Q_j(x+\hat{\mu}) + +(1+\gamma_\mu) U_\mu^F(x-\hat{\mu}) Q_j(x-\hat{\mu}) \right\}$$

For anti-symmetric fermions $Q_j$, $U_\mu^F$ & $D^F$ are replaced by $\Psi_j$, $U^{AS}$ & $D^{AS}$ with $j = 1, 2, 3$.

$\beta = 8/g^2$ is the lattice coupling and $m_0$ is the bare fermion mass.

- HiRep code with appropriate modifications (e.g. resymplectisation)

***Del Debbio, Patella & Pica (2008)***

- Heat-bath for quenched ensembles & HMC(RHMC) for dynamical ensembles

~200 configurations for each ensemble

- Scale setting - Luscher's gradient flow method (hatted notation)

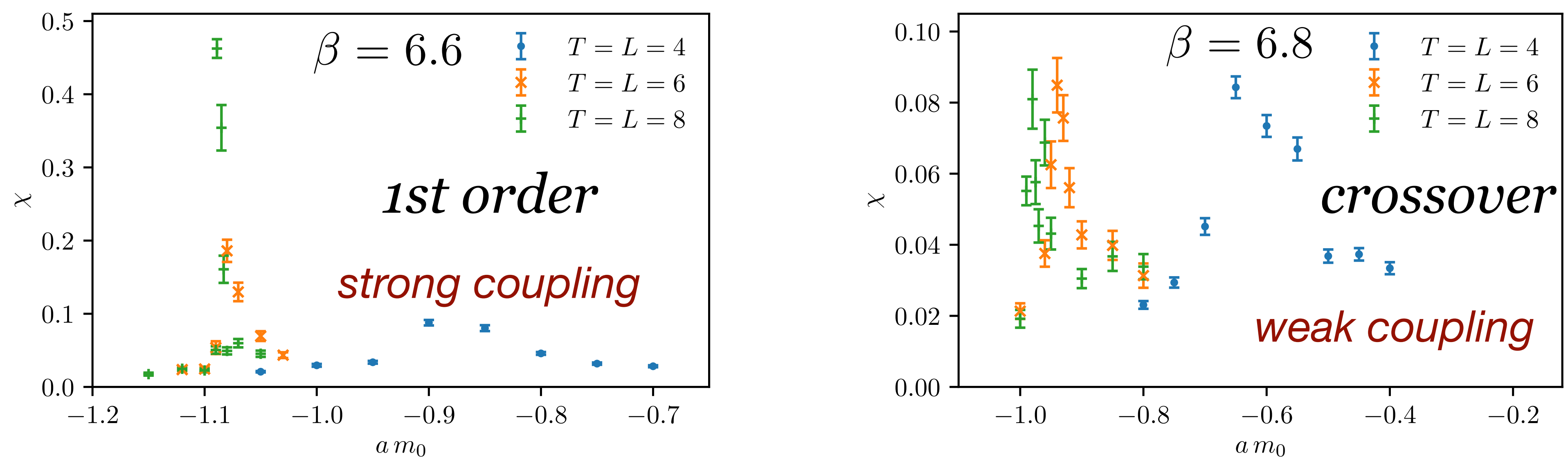
***Luscher (2010)*** ***Luscher & Wiese (2011)***

$$\mathcal{W}|_{t=w_0^2} \equiv \mathcal{W}_0 \qquad \mathcal{W}(t) \equiv \frac{\mathrm{d}}{\mathrm{d}\ln t} t^2 \langle E(t) \rangle$$

***Borsanyi et al (2012)***

## Bulk phase transition & weak coupling regime

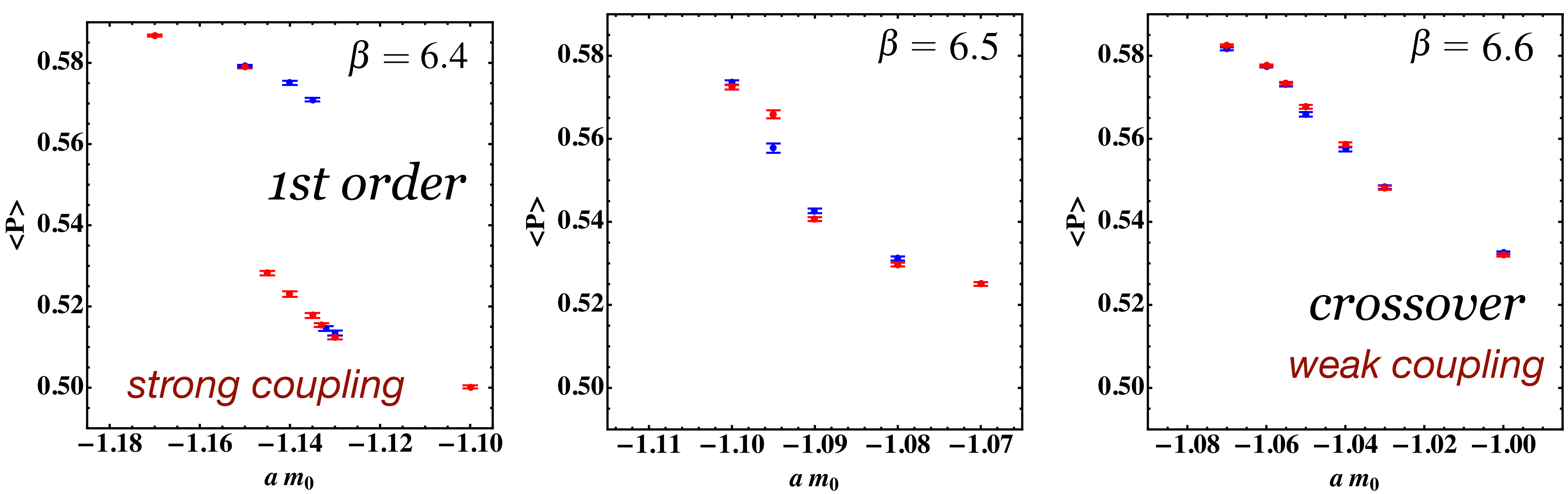
- We should stay in the weak coupling regime to be safely connected to continuum.




*$N_f$=2 fund. Sp(4)*

$\beta \gtrsim 6.8$

See how the susceptibility of $\langle P \rangle$ scales with the size of the system.




*$N_f$=3 anti-sym. Sp(4)*

$\beta \gtrsim 6.6$

Existence of Strong Hysteresis of $\langle P \rangle$ obtained from cold and hot configurations.

# Observables: spin-0 & 1lightest mesons

- Global symmetry breaking: $SU(4)/Sp(4) \times SU(6)/SO(6)$
- Gauge invariant, flavor non-singlet, i.e. $i \neq j$ or $k \neq m$

| Label $M$ | Interpolating operator $\mathcal{O}_M$ | Meson in QCD | $J^P$ | $Sp(4)$ | $SO(6)$ |
|---|---|---|---|---|---|
| PS | $\overline{Q^i}\gamma_5 Q^j$ | $\pi$ | $0^-$ | $5(+1)$ | $1$ |
| S | $\overline{Q^i} Q^j$ | $a_0$ | $0^+$ | $5(+1)$ | $1$ |
| V | $\overline{Q^i}\gamma_\mu Q^j$ | $\rho$ | $1^-$ | $10$ | $1$ |
| T | $\overline{Q^i}\gamma_0\gamma_\mu Q^j$ | $\rho$ | $1^-$ | $10(+5+1)$ | $1$ |
| AV | $\overline{Q^i}\gamma_5\gamma_\mu Q^j$ | $a_1$ | $1^+$ | $5(+1)$ | $1$ |
| AT | $\overline{Q^i}\gamma_5\gamma_0\gamma_\mu Q^j$ | $b_1$ | $1^+$ | $10(+5+1)$ | $1$ |
| ps | $\overline{\Psi^k}\gamma_5 \Psi^m$ | $\pi$ | $0^-$ | $1$ | $20'(+1)$ |
| s | $\overline{\Psi^k} \Psi^m$ | $a_0$ | $0^+$ | $1$ | $20'(+1)$ |
| v | $\overline{\Psi^k}\gamma_\mu \Psi^m$ | $\rho$ | $1^-$ | $1$ | $15$ |
| t | $\overline{\Psi^k}\gamma_0\gamma_\mu \Psi^m$ | $\rho$ | $1^-$ | $1$ | $15(+20'+1)$ |
| av | $\overline{\Psi^k}\gamma_5\gamma_\mu \Psi^m$ | $a_1$ | $1^+$ | $1$ | $20'(+1)$ |
| at | $\overline{\Psi^k}\gamma_5\gamma_0\gamma_\mu \Psi^m$ | $b_1$ | $1^+$ | $1$ | $15(+20'+1)$ |

# Meson spectroscopy

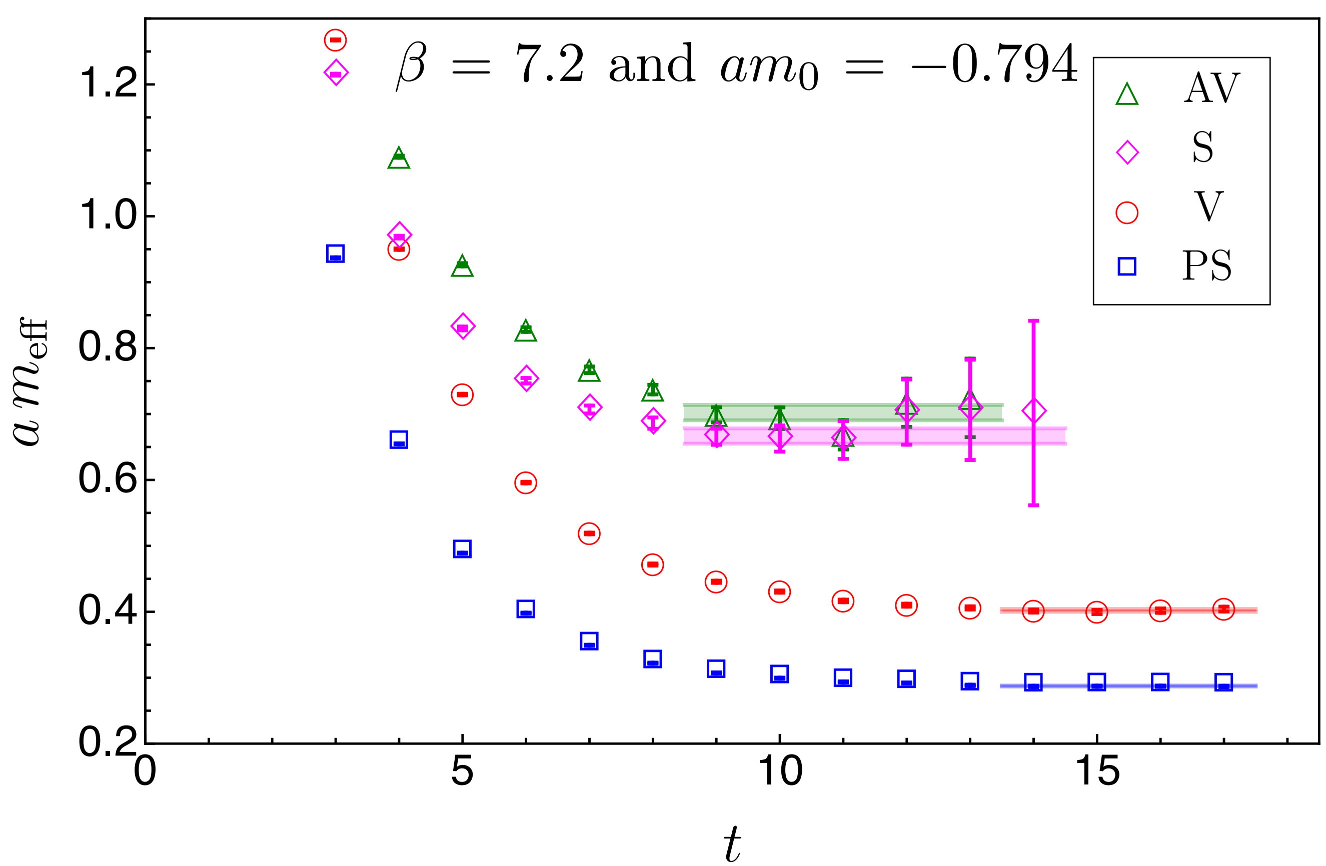
- Euclidean 2-point correlation functions at zero momentum

$$C_{\mathcal{O}_M}(t) \equiv \sum_{\vec{x}} \langle 0|\mathcal{O}_M(\vec{x},t)\mathcal{O}_M^{\dagger}(\vec{0},0)|0\rangle$$

- Masses and decay constants are extracted from their large-time behaviors

$$\langle 0|\mathcal{O}_M|M\rangle\langle 0|\mathcal{O}_M|M\rangle^* \frac{1}{2m_M}\left[e^{-m_M t}+e^{-m_M(T-t)}\right]$$

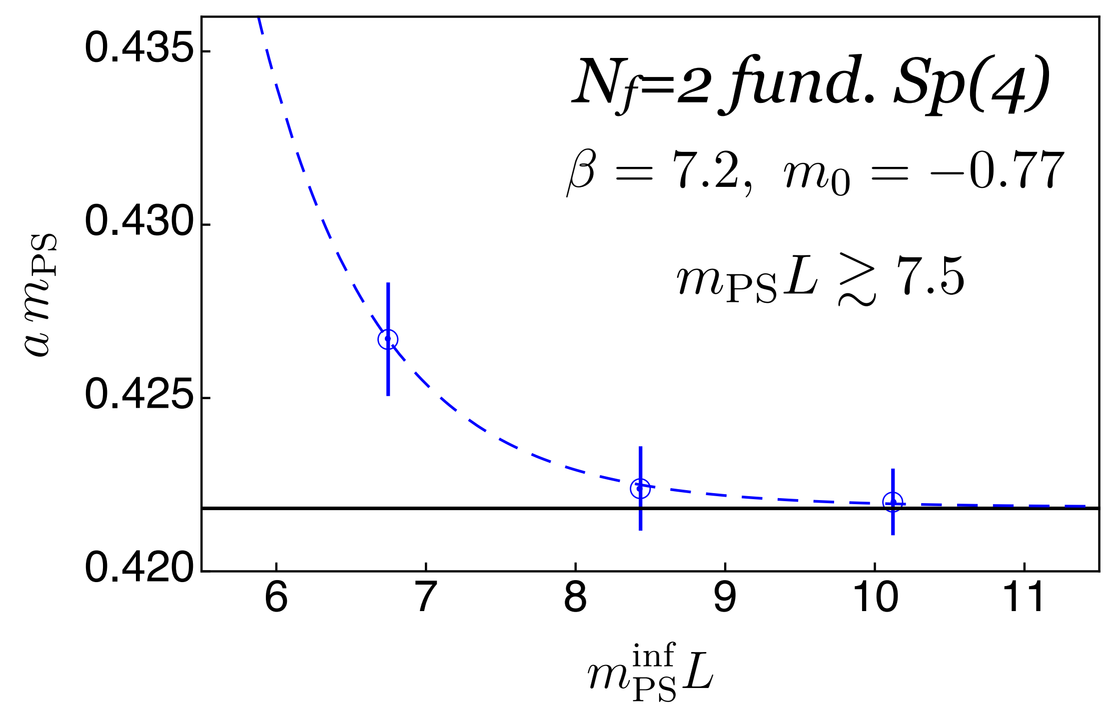
with $\langle 0|\overline{\Psi_1}\gamma_5\gamma_\mu\Psi_2|\mathrm{ps}\rangle \equiv f_{\mathrm{ps}}\, p_\mu,\ \langle 0|\overline{\Psi_1}\gamma_\mu\Psi_2|\mathrm{v}\rangle \equiv f_{\mathrm{v}}\, m_{\mathrm{v}}\,\epsilon_\mu,\ \langle 0|\overline{\Psi_1}\gamma_5\gamma_\mu\Psi_2|\mathrm{av}\rangle \equiv f_{\mathrm{av}}\, m_{\mathrm{av}}\,\epsilon_\mu$

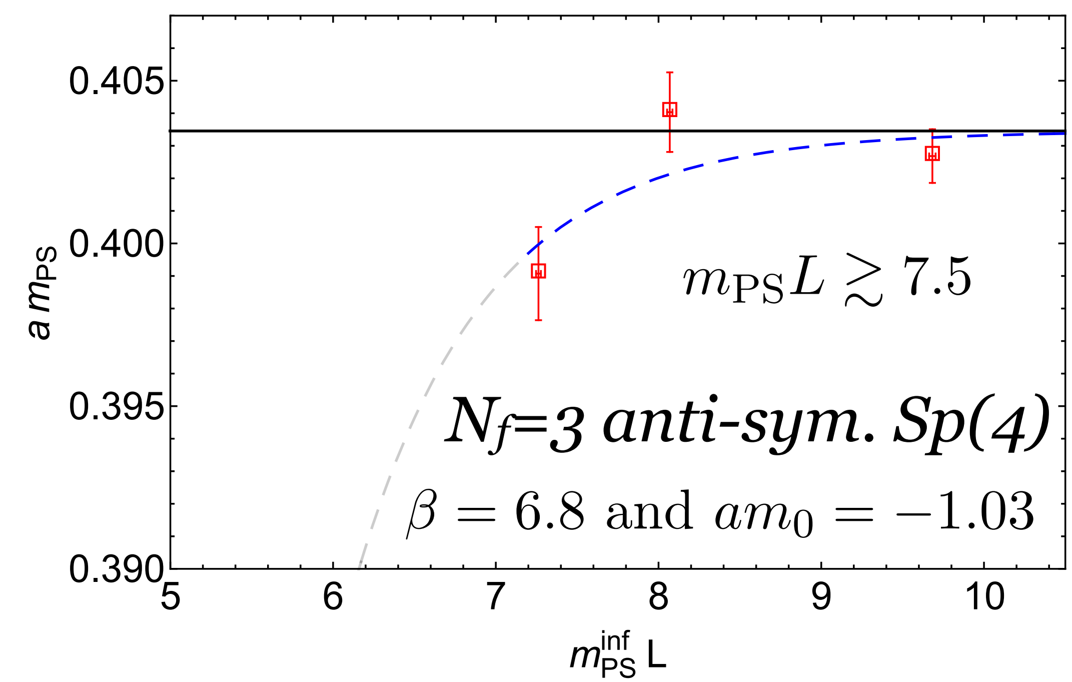



$$m_{\mathrm{eff}}(t) = \cosh^{-1}\left(\frac{C(t+1)+C(t-1)}{2C(t)}\right)$$

*$N_f$=2 fund. Sp(4)*

# Systematics 1: Finite volume effects




- Finite volume effects are exponentially suppressed, which can be understood from the low-energy chiral perturbation theory.

$$m_{\rm PS}^2 = M^2 \left(1 + a_M \frac{A(M) + A_{\rm FV}(M)}{F^2} + b_M(\mu)\frac{M^2}{F^2} + \mathcal{O}(M^4)\right)$$

**Bijnens & Lu (2009)**

$$A(M) = -\frac{M^2}{16\pi^2}\log\frac{M^2}{\mu^2} \qquad A_{\rm FV}(M) \xrightarrow{ML\gg 1} -\frac{3}{4\pi^2}\left(\frac{M\pi}{2L^3}\right)^{1/2}\exp[-ML]$$

$SU(2N_f) \to Sp(2N_f)$

$$a_M = -\frac{1}{2} - \frac{1}{N_f}$$

$SU(2N_f) \to SO(2N_f)$

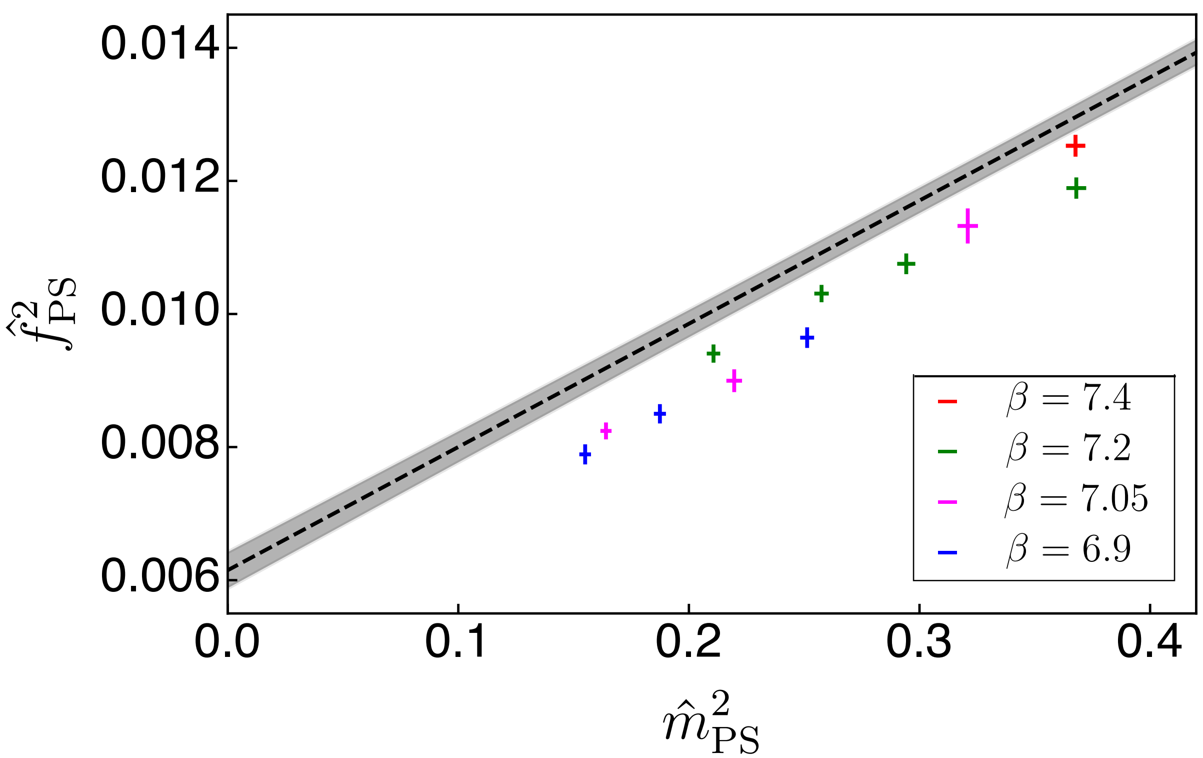
$$a_M = \frac{1}{2} - \frac{1}{2N_f}$$

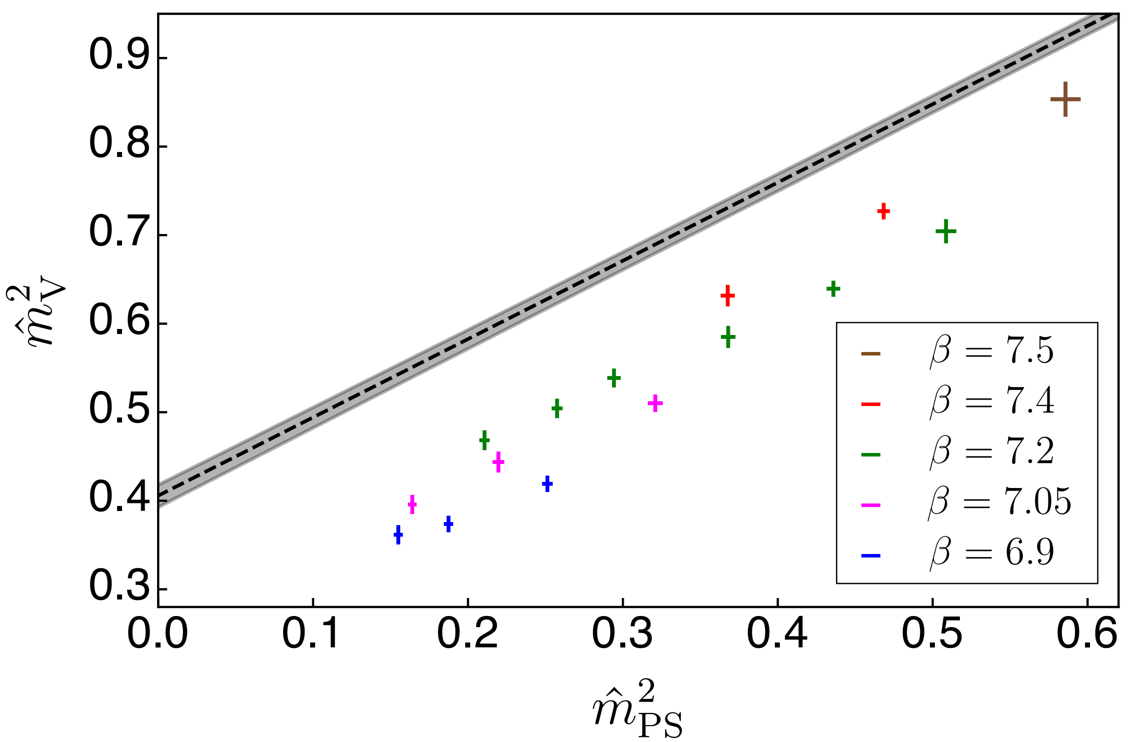
$SU(N_f) \times SU(N_f) \to SU(N_f)$

$$a_M = -\frac{1}{N_f}$$

# Systematics 2: Discretization effects

- Lattice results of Sp(4) gauge theories with 2 fund. Dirac flavors.




- Take the continuum & massless limits using the linear ansatzs,

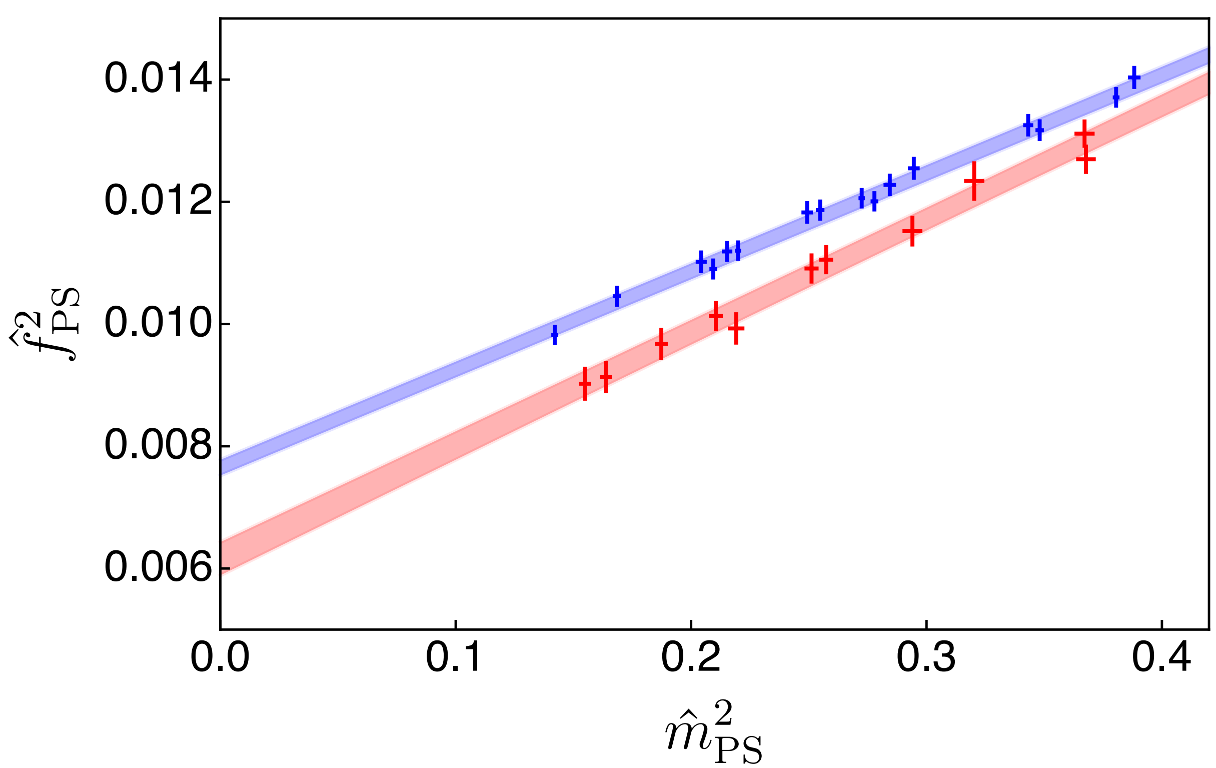
$$\hat{m}_M^{2,\rm NLO} \equiv \hat{m}_M^{2,\chi}\left(1 + L^0_{m,M}\hat{m}^2_{\rm PS}\right) + W^0_{m,M}\hat{a}\,, \quad \text{with} \quad \frac{m^2_{\rm PS}}{\Lambda^2_\chi} \sim a\Lambda_\chi < 1$$

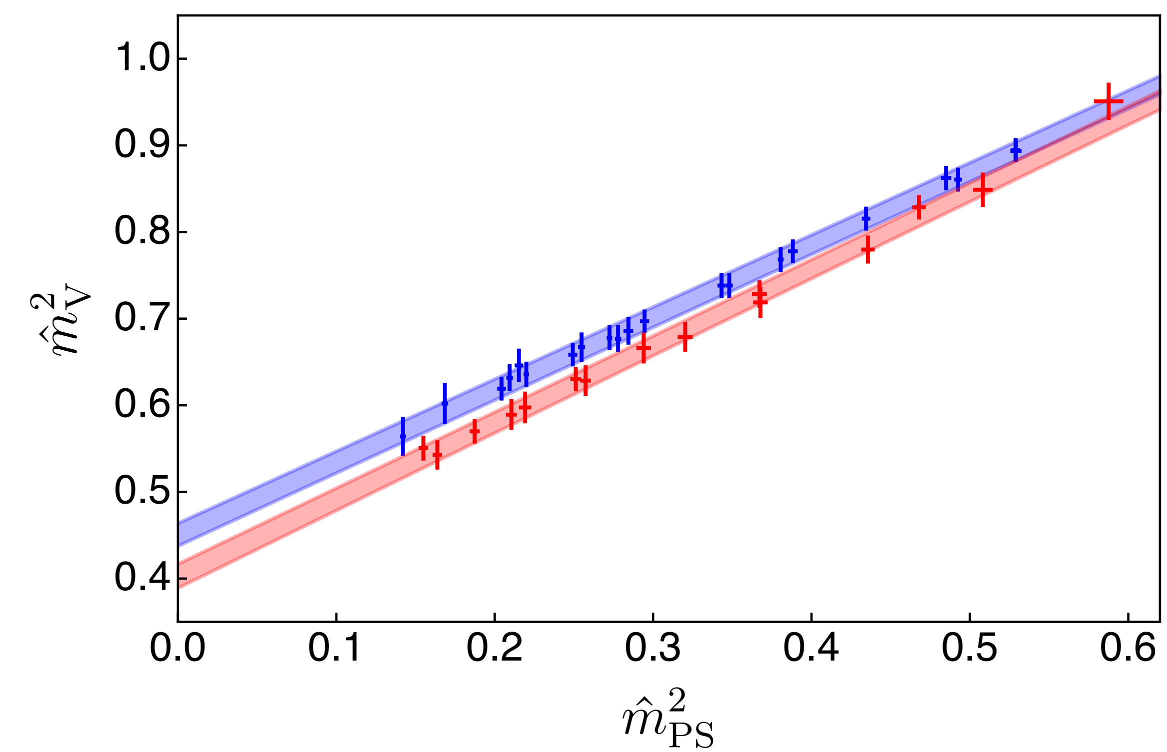
$$\hat{f}_M^{2,\rm NLO} \equiv \hat{f}_M^{2,\chi}\left(1 + L^0_{f,M}\hat{m}^2_{\rm PS}\right) + W^0_{f,M}\hat{a}\,, \quad \text{with} \quad \frac{m^2_{\rm PS}}{\Lambda^2_\chi} \sim a\Lambda_\chi < 1$$

# Systematics 3: Quenching effects

**Blue: Quenched fundamental fermions**

**Red: $N_f$=2 dynamical fundamental fermions**



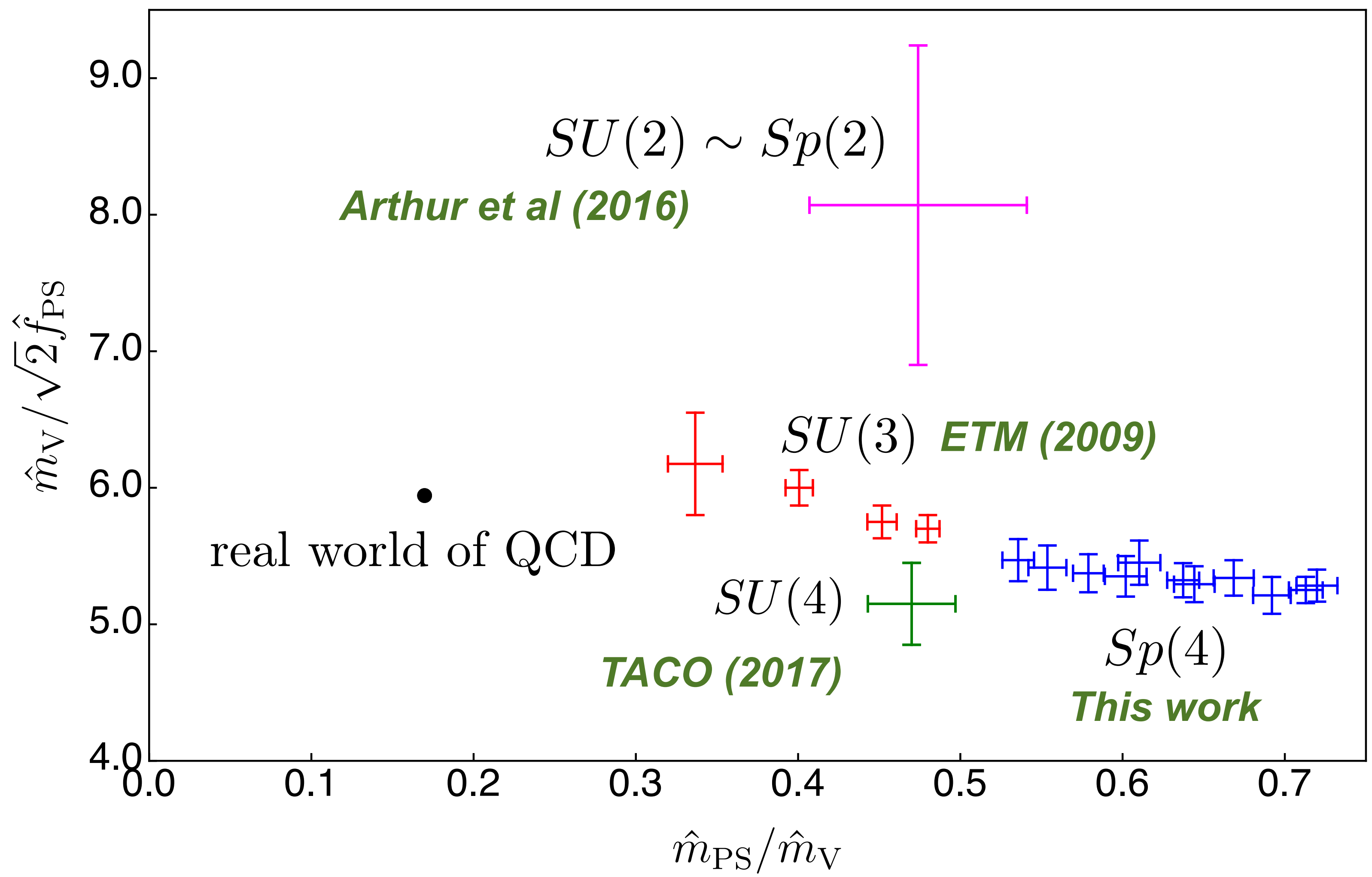




- Quenching effects are getting larger as the fermion mass gets smaller.

- Quenching effects in the massless limit are $\delta_{\hat{f}^2_{\rm PS}}/\hat{f}^2_{\rm PS} \sim 20\%$ and $\delta_{\hat{m}^2_V}/m_{\hat{m}^2_V} \sim 10\%$ .

# Vector meson mass

- Collected lattice results for gauge theories with 2 fund. Dirac flavors.
- The hypothesis vector meson dominance leads to the KSRF relation




*Kowarabayashi & Suzuki (1966)*
*Riazuddin & Fayyazuddin (1966)*

$$g_{VPP} = \frac{m_{\rm V}}{\sqrt{2} f_{\rm PS}}$$

HLS EFT fit results:
in the massless limit

$$g^{\chi}_{\rm VPP} = 6.0(4)(2)$$

Large Nc argument:

$$f_{\rm PS} \sim \sqrt{N_c}$$

$$m_{\rm V}/\sqrt{2} f_{\rm PS} \times \sqrt{N_c/3} \sim 6$$

# Vector meson mass

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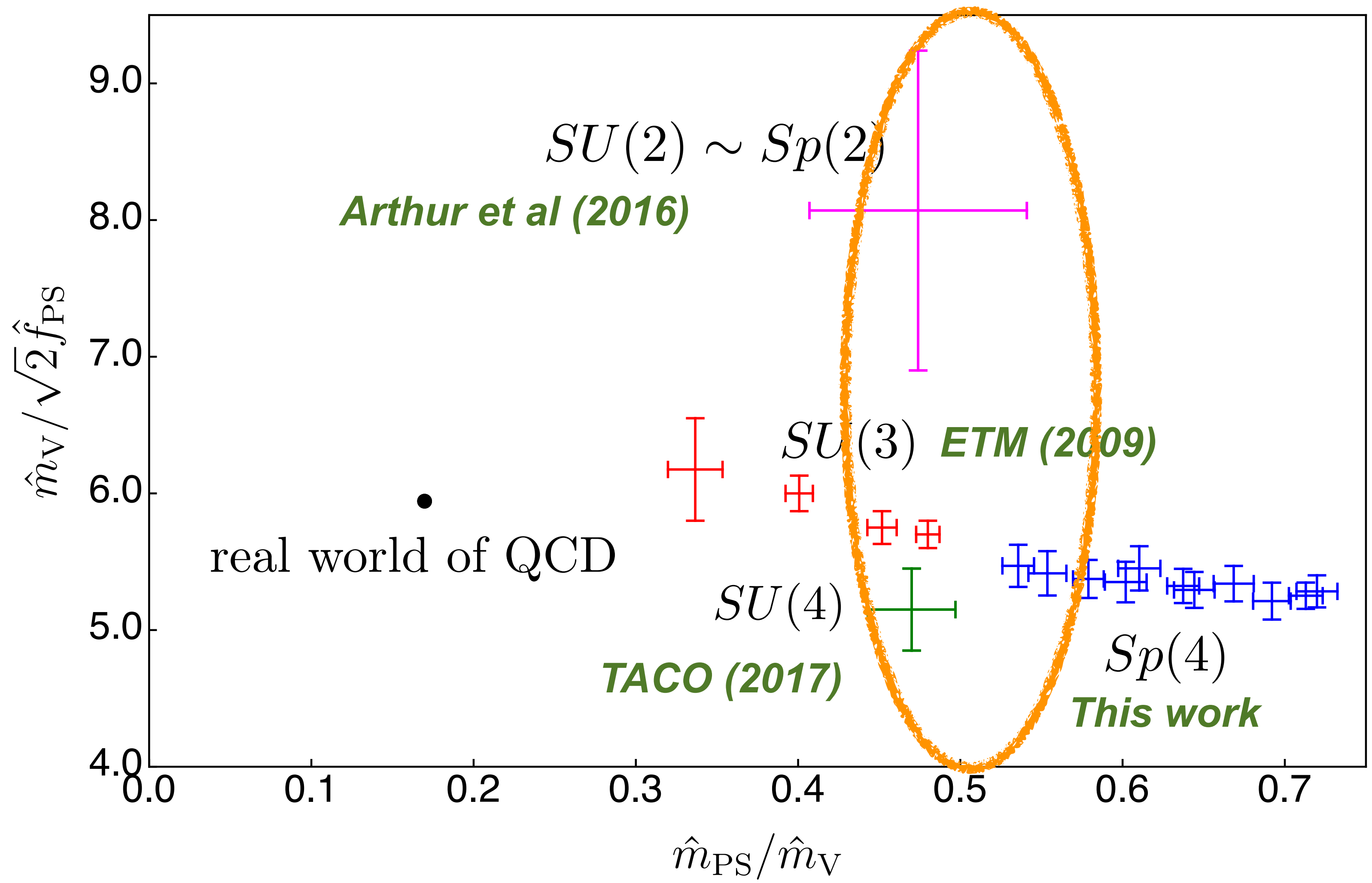


$\hat{m}_V = 2\,\hat{m}_{PS}$

$$g_{VPP} = \frac{m_{\rm V}}{\sqrt{2} f_{\rm PS}}$$

HLS EFT fit results:
in the massless limit

$$g^{\chi}_{\rm VPP} = 6.0(4)(2)$$

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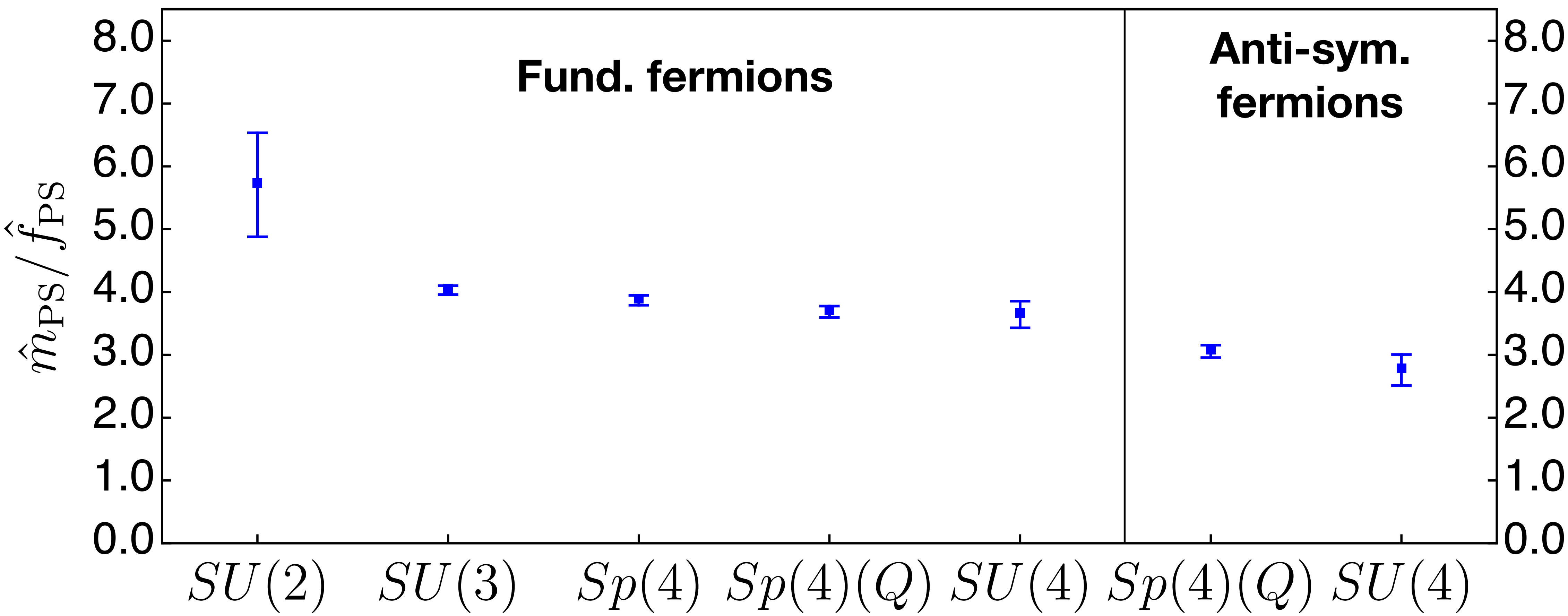
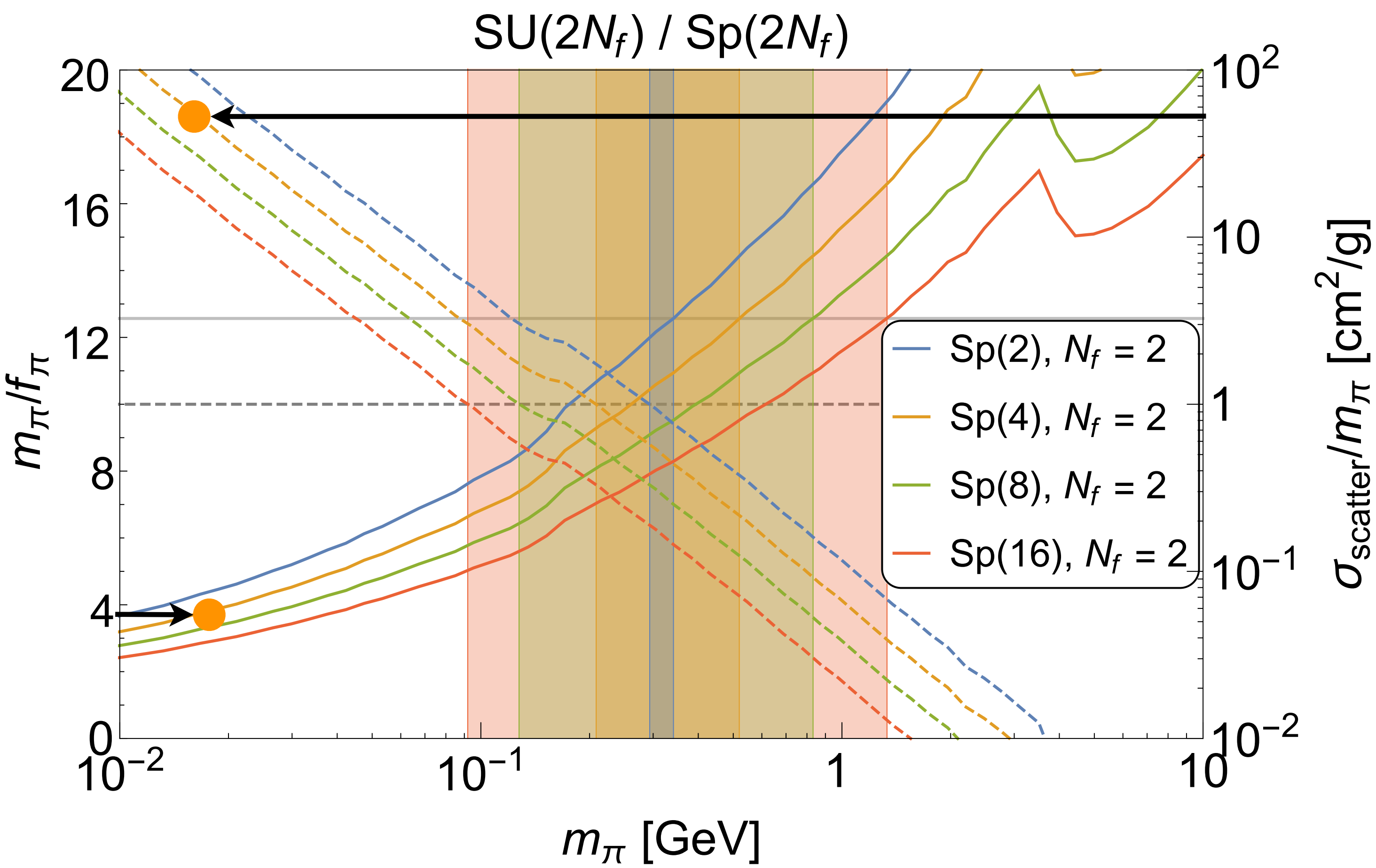
# Pseudoscalar meson masses at the threshold




- For SU(2), SU(3) & Sp(4), simulations were performed including two dynamical fundamental Dirac flavors near the V-PS-PS threshold.
- (Q) denotes the “quenched calculation”, i.e. no dynamical fermions in the sea.
- In *SU(4)* results, both two fundamental and two anti-symmetric Dirac fermions are in the sea.

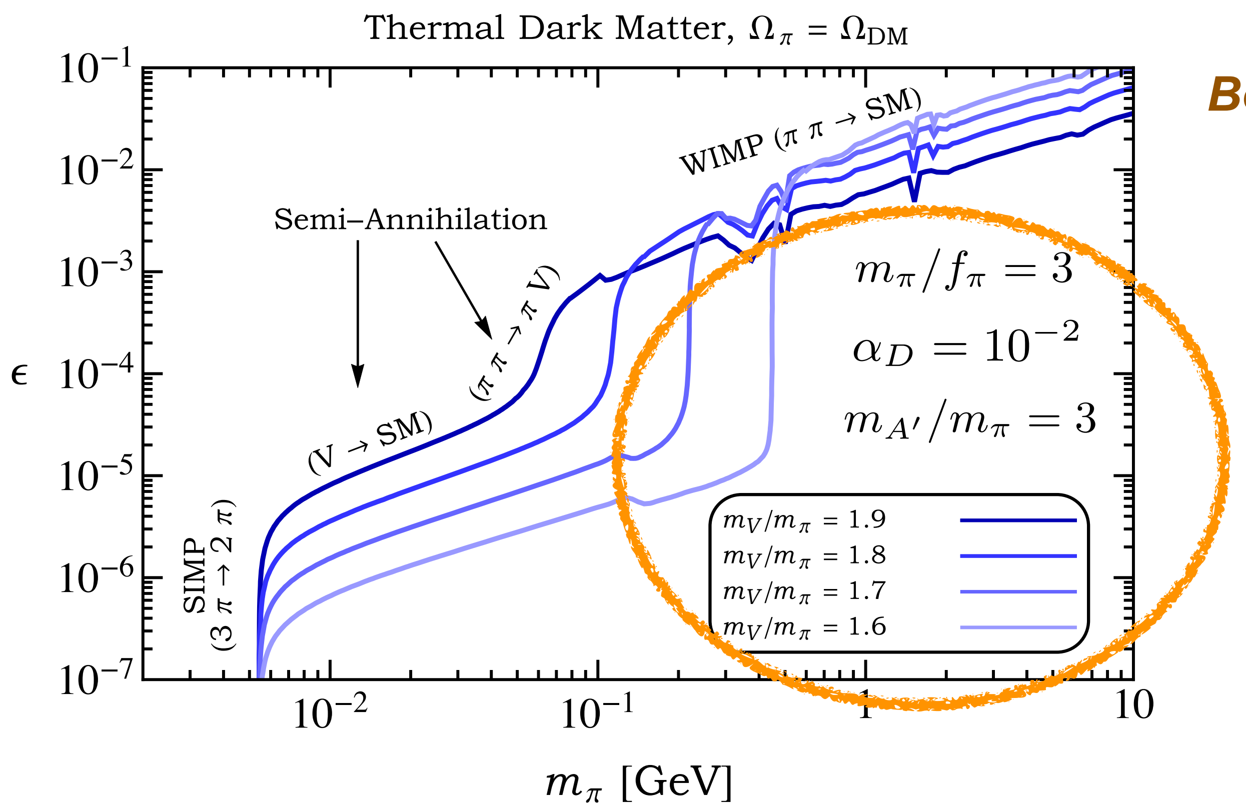
# Strongly interacting massive particles




***Hochberg, Kuflik, Murayama, Volansky, Wacker (2014)***

- The SIMPest scenario severely violates the Bullet cluster bound if the UV model is QCD-like theory, like the ones studied on the lattice.

# SIMPs with vector resonances

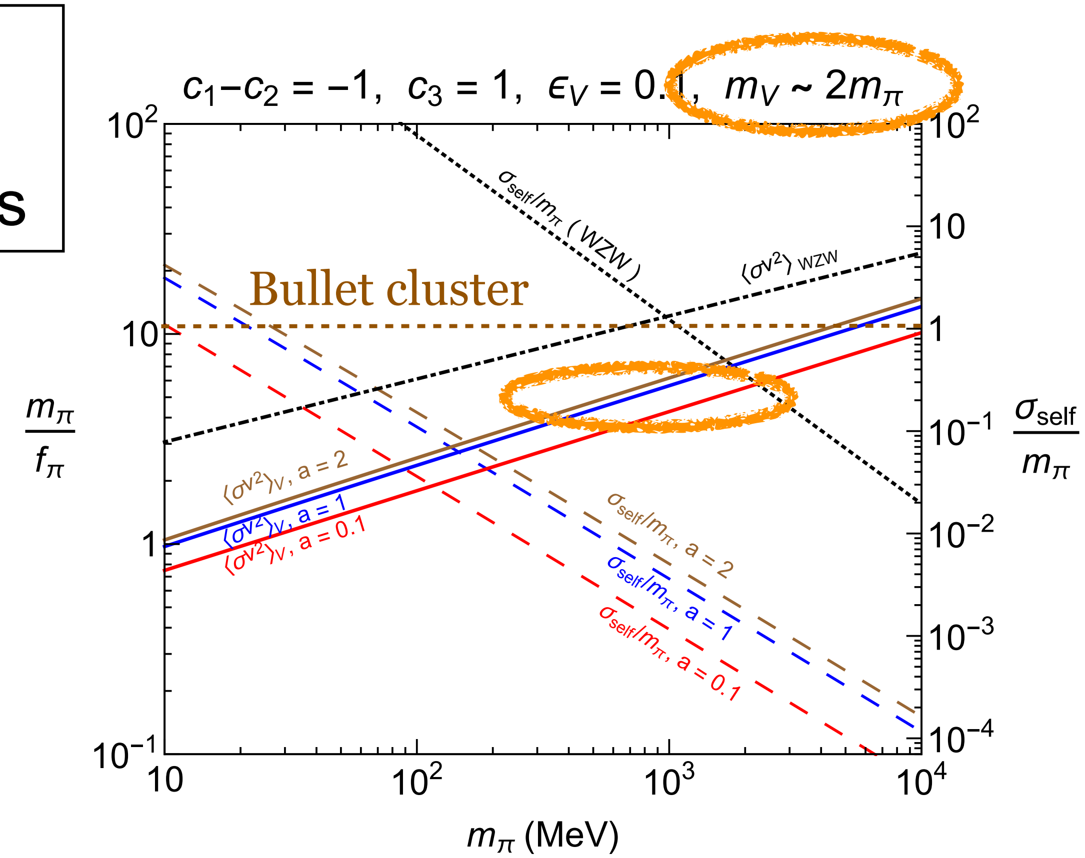


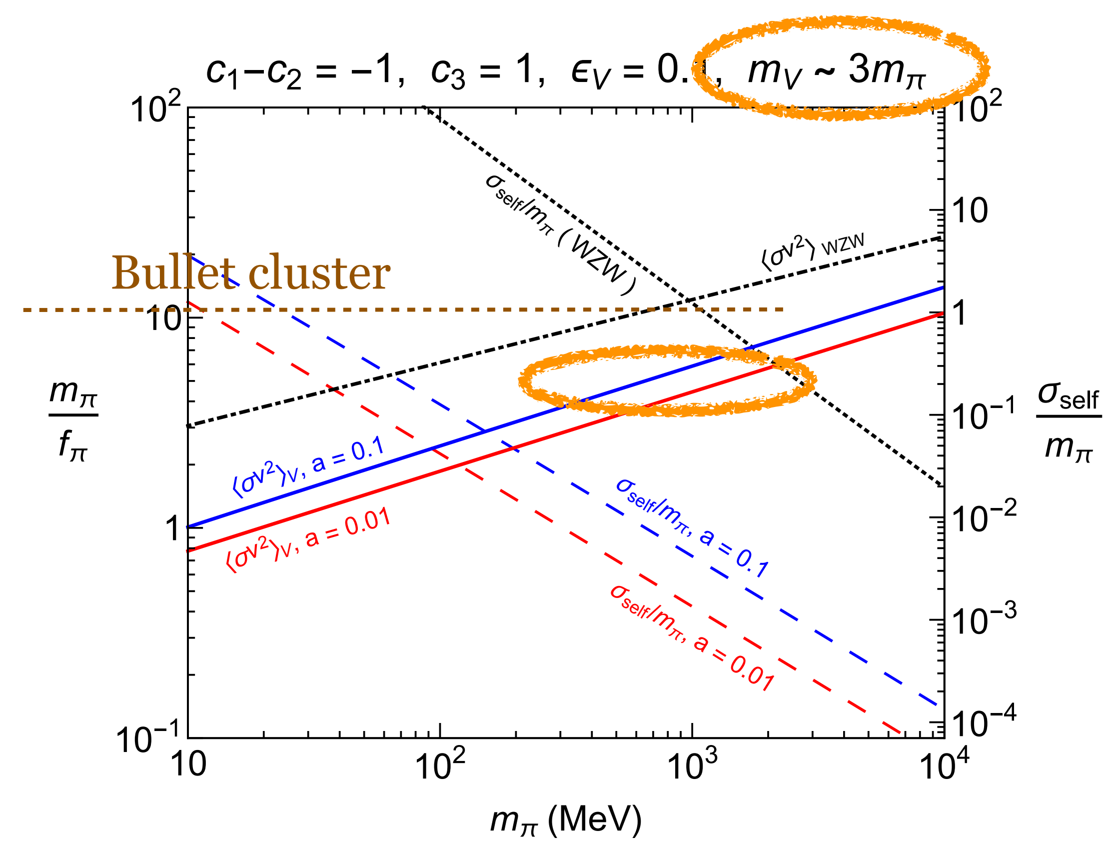

***Berlin, Blinov, Gori, Schuster & Toro (2018)***

- The vector resonances may rescue the SIMPs scenarios realized in QCD-like theories.
- Further lattice results will provide more useful information for the phenomenological inputs.

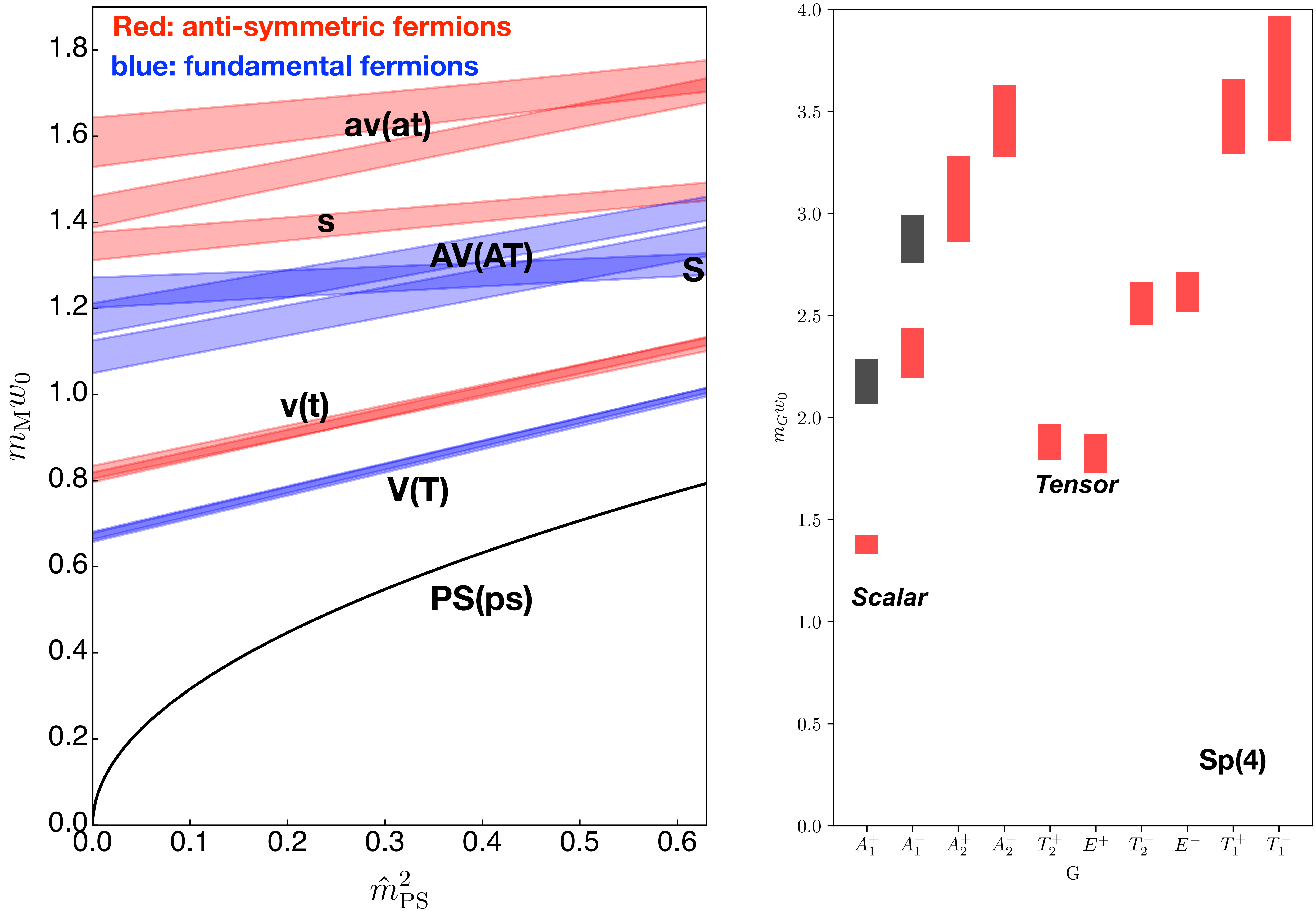
***Choi, Lee, Ko & Natale (2018)***

SU(3) with $N_f$=3 fund. Dirac flavors



- Spectra of composite states in Sp(4) (quenched mesons & glueballs)



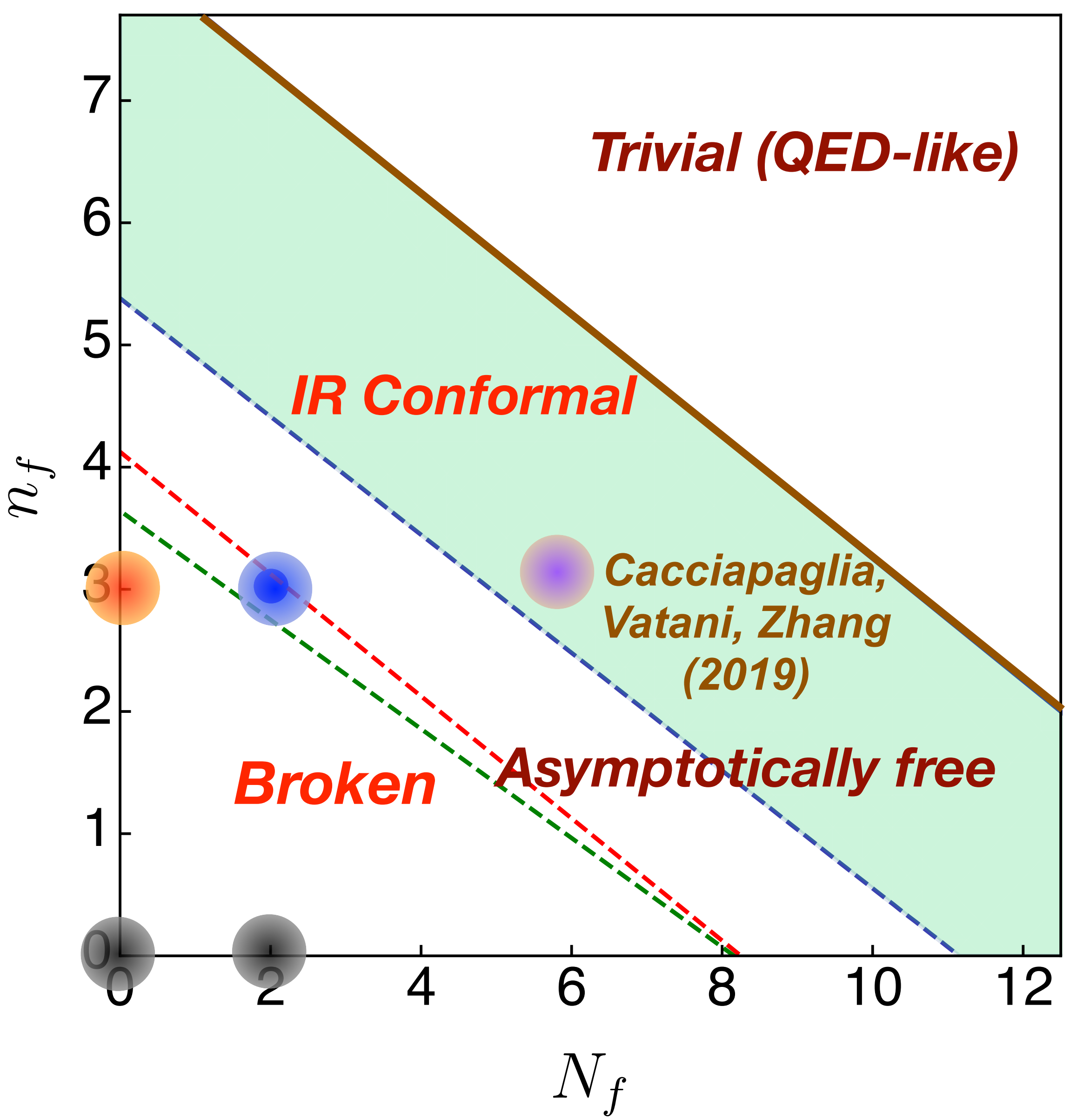
**On going work**

Large-volume study of `Sp(4)` with $N_f$=2 (dynamical) fund. Dirac fermions

`Sp(4)` gauge theory with $n_f$=3 (dynamical) anti-sym. Dirac fermions

`Sp(4), Sp(6), Sp(8)` gauge theory with quenched fund., anti-sym. & sym. Dirac fermions

Code improvement
- Fully dynamical multi-representation
- Gaussian-smeared operators to improve the signals, especially for heavy states




**Future directions**

Meson spectra of `Sp(4)` with $N_f$=2 fund. & $n_f$=3 anti-sum. dynamical Dirac fermions

Spectrum of chimera baryon (e.g. top partner)

Scattering states of mesons (e.g. ps-ps)

Four-fermi operators relevant to Higgs potential

Phase structure of Sp(4) gauge theory with/without fermion matter at finite temperature & density

Thank you for your attention!

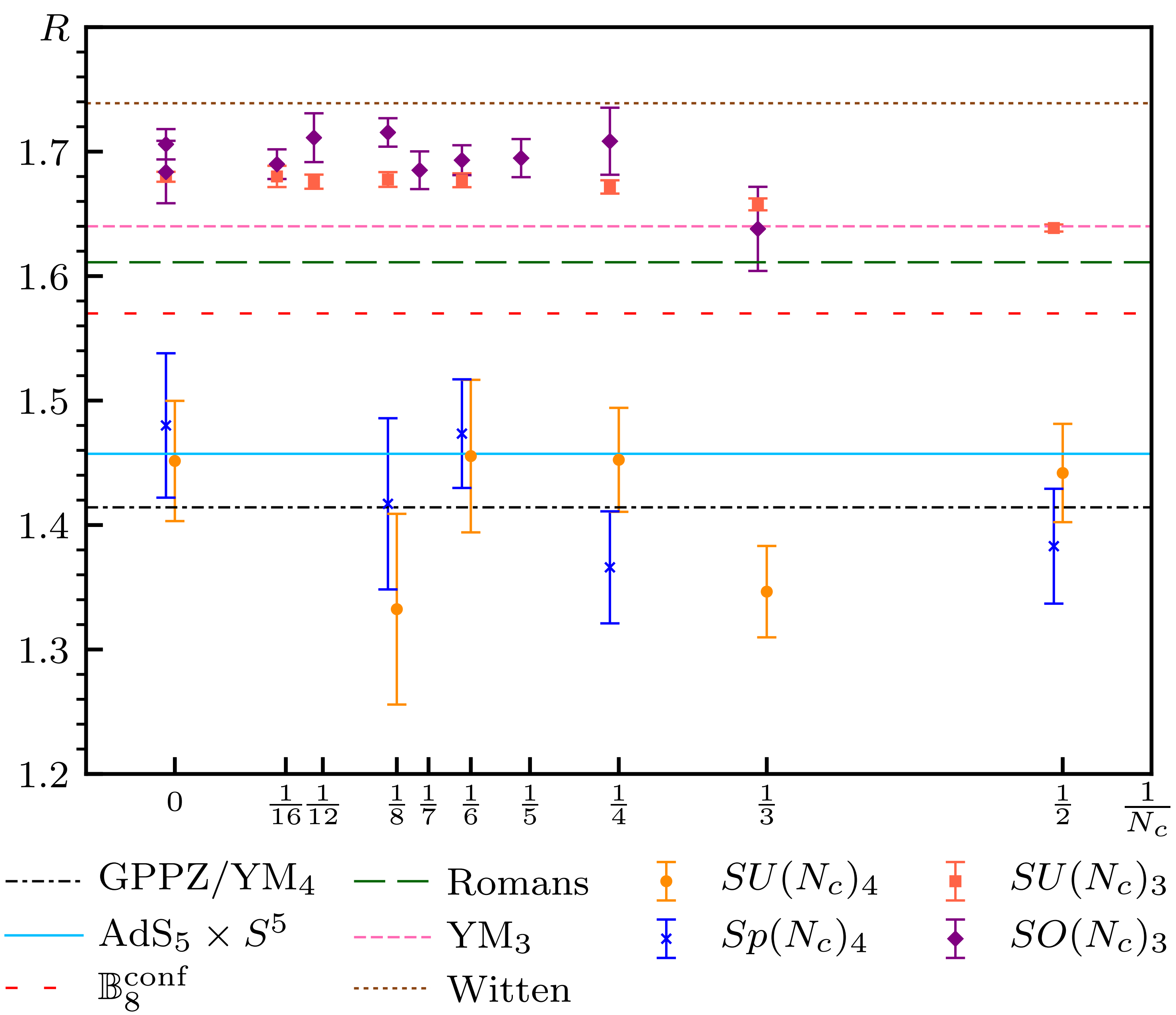
# Backup slides

# Mass ratio of scalar & tensor glueballs

- Universal arguments for infrared confining theories yield

$$m_{2^{++}}/m_{0^{++}} = \sqrt{2}$$

*A. Athenodrou et al (2015)*




$$R \equiv \frac{m_{2^{++}}}{m_{0^{++}}}$$

*PRD(R) 102 (2020) 1, 011501*

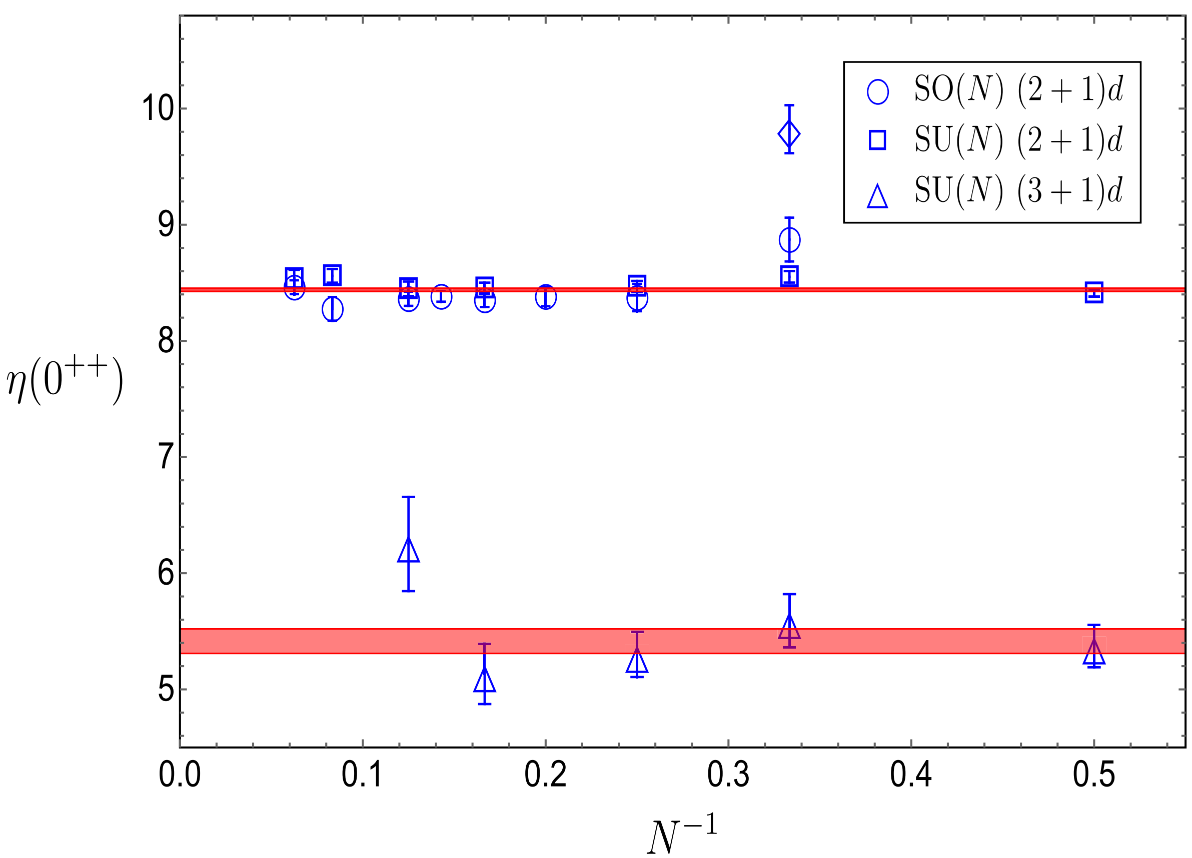
# Scalar glueball mass and string tension

- Casimir scaling: Universality in pure `SU(N), SO(N), Sp(2N)` Yang-Mills

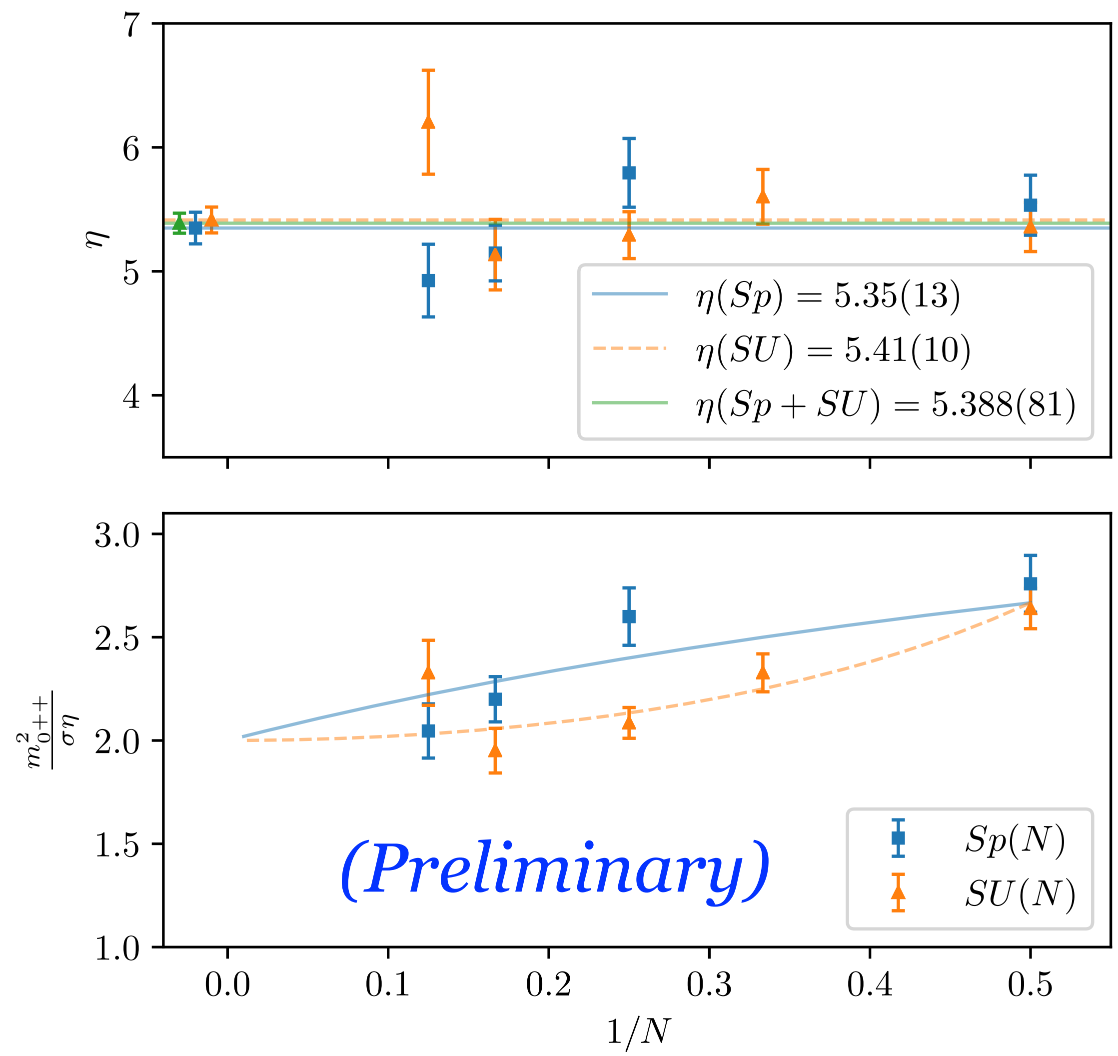
$$\frac{m_{0^{++}}^2}{\sigma} = \eta \frac{C_2(A)}{C_2(F)}$$

Conjecture proposed by
***D. Hong et. al. (2017)***

***The universal constant***, **only depends on the space-time dimension.**




***PLB 775 (2017) 89-93***




***E. Bennett et al (in preparation)***

## Results: $N_f$=2 dynamical fundamental Dirac fermions

- Considered mass range, $1.39 \lesssim \hat{m}_{\rm V}/\hat{m}_{\rm PS} \lesssim 1.87$, is applicable for SIMP studies, but requires an extrapolation to be used for composite Higgs.

- In the massless limit, we found

$$m_{\rm V}/f_{\rm PS} = 8.08(32),\ m_{\rm AV}/f_{\rm PS} = 13.4(1.5),\ \text{and}\ m_{\rm S}/f_{\rm PS} = 14.2(1.7)$$

$$f_{\rm V}/f_{\rm PS} = 2.15(8)\ \text{and}\ f_{\rm AV}/f_{\rm PS} = 2.3(4)$$

The first KSRF relation $f_{\rm V} = \sqrt{2}\, f_{\rm PS}$ is largely violated.

- V-PS-PS coupling

from the Global fit using HLS EFT compared to

$$g_{\rm VPP} = 6.0(4)(2) \qquad m_{\rm V}/(\sqrt{2}f_{\rm PS}) = 5.72(18)(13)$$

The second KSRF relation $g_{\rm VPP} = \dfrac{m_{\rm V}}{\sqrt{2}f_{\rm PS}}$ is satisfied.

# Results: quenched fund. & anti-sym. Dirac fermions

- Physical quantities relevant to composite Higgs phenomenology

in the massless limit

$$\hat{f}^2_{\rm ps}/\hat{f}^2_{\rm PS} = 1.81 \pm 0.04$$

$$\hat{m}^2_{\rm v}/\hat{m}^2_{\rm V} = 1.46 \pm 0.08$$

- V-PS-PS coupling

from the Global fit using HLS EFT

$$g^{\chi}_{\rm VPP} = 4.95(21)(8)$$

$$g^{\chi}_{\rm vpp} = 3.80(24)(16)$$

KSRF relation

$$\hat{m}^{\chi}_{\rm V}/\sqrt{2}\hat{f}^{\chi}_{\rm PS} = 5.48(9)(4)$$

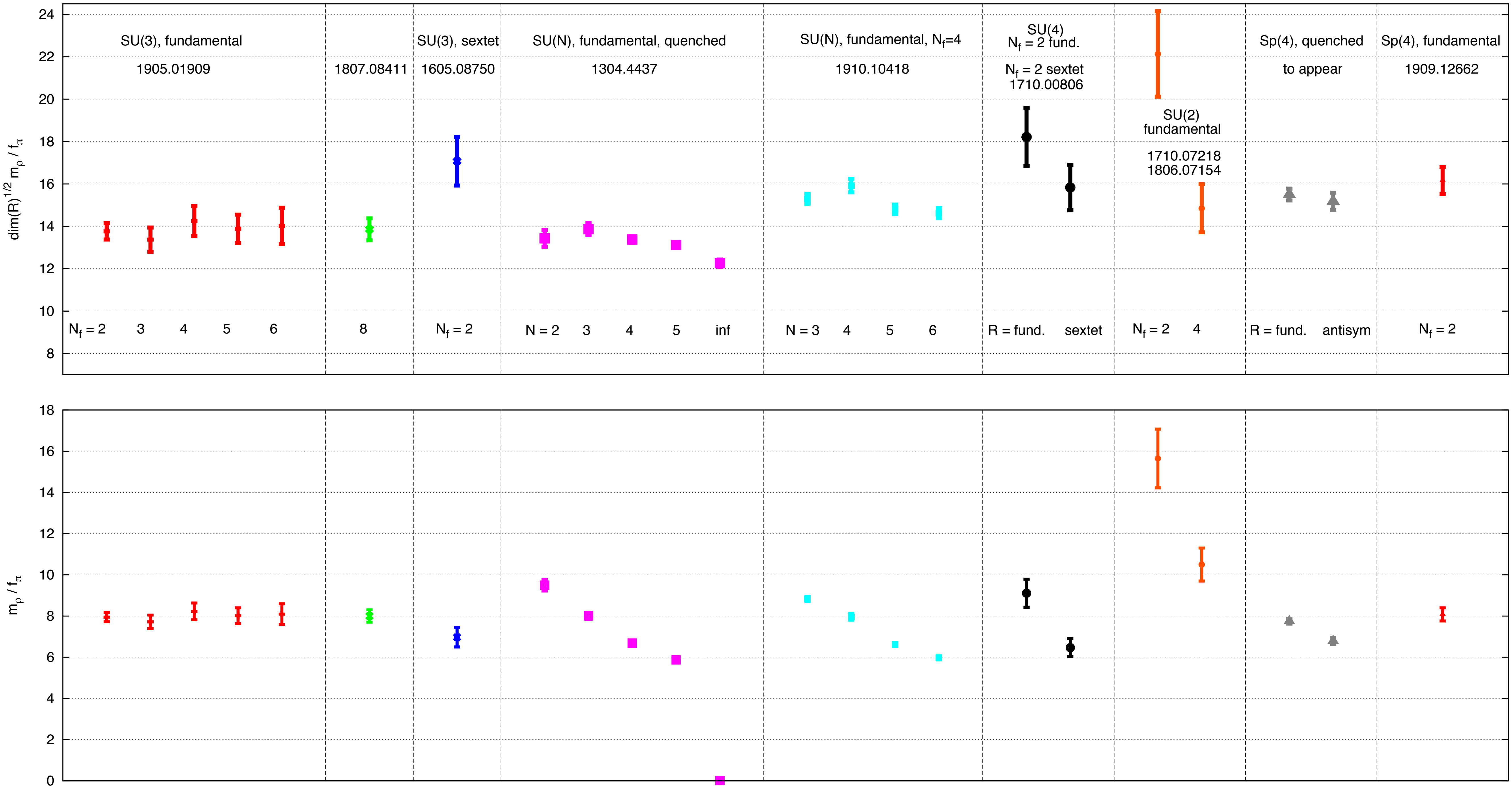
$$\hat{m}^{\chi}_{\rm v}/\sqrt{2}\hat{f}^{\chi}_{\rm ps} = 4.80(12)(4)$$

- Quenching affects in mesons interpolated by fundamental bilinear operators are 10~30% depending on the observables.

- Quenching affects in mesons interpolated by antisymmetric bilinear operators could be larger.

***Dynamical simulations are required.***

# Vector meson masses in various QCD-like theories




*Nogradi & Szikszai (2019)*