# Sp(4) gauge theories for BSM models on the lattice:

## Strategies, opportunities and challenges

arXiv:1712.04220, 1909.12662, 1912.06505, 2004.11063

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Composite Connections of Higgs, Dark Matter and Neutrino
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# Collaboration on Sp(2N) gauge theories


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# UV completion for composite Higgs models

| Name | Gauge group | $\psi$ | $\chi$ | Baryon type |
|---|---|---|---|---|
| M1 | $SO(7)$ | $5 \times \mathbf{F}$ | $6 \times \mathbf{Spin}$ | $\psi\chi\chi$ |
| M2 | $SO(9)$ | $5 \times \mathbf{F}$ | $6 \times \mathbf{Spin}$ | $\psi\chi\chi$ |
| M3 | $SO(7)$ | $5 \times \mathbf{Spin}$ | $6 \times \mathbf{F}$ | $\psi\psi\chi$ |
| M4 | $SO(9)$ | $5 \times \mathbf{Spin}$ | $6 \times \mathbf{F}$ | $\psi\psi\chi$ |
| M5 | $Sp(4)$ | $5 \times \mathbf{A}_2$ | $6 \times \mathbf{F}$ | $\psi\chi\chi$ |
| TACO Collab. Edinburgh-Torino: M6 | $SU(4)$ | $5 \times \mathbf{A}_2$ | $3 \times (\mathbf{F}, \overline{\mathbf{F}})$ | $\psi\chi\chi$ |
| M7 | $SO(10)$ | $5 \times \mathbf{F}$ | $3 \times (\mathbf{Spin}, \overline{\mathbf{Spin}})$ | $\psi\chi\chi$ |
| Our work: M8 | $Sp(4)$ | $4 \times \mathbf{F}$ | $6 \times \mathbf{A}_2$ | $\psi\psi\chi$ |
| M9 | $SO(11)$ | $4 \times \mathbf{Spin}$ | $6 \times \mathbf{F}$ | $\psi\psi\chi$ |
| M10 | $SO(10)$ | $4 \times (\mathbf{Spin}, \overline{\mathbf{Spin}})$ | $6 \times \mathbf{F}$ | $\psi\psi\chi$ |
| M11 | $SU(4)$ | $4 \times (\mathbf{F}, \overline{\mathbf{F}})$ | $6 \times \mathbf{A}_2$ | $\psi\psi\chi$ |
| M12 | $SU(5)$ | $4 \times (\mathbf{F}, \overline{\mathbf{F}})$ | $3 \times (\mathbf{A}_2, \overline{\mathbf{A}_2})$ | $\psi\psi\chi$, $\psi\chi\chi$ |

★Before starting lattice simulations

➡ Choose a model wisely before spending years on the simulation

# Our implementation

| Field | $Sp(4)$ gauge | $SU(4)$ global | $SU(6)$ global |
|---|---|---|---|
| $A_\mu$ | 10 | 1 | 1 |
| $\psi$ | 4 | 4 | 1 |
| $\chi$ | 5 | 1 | 6 |

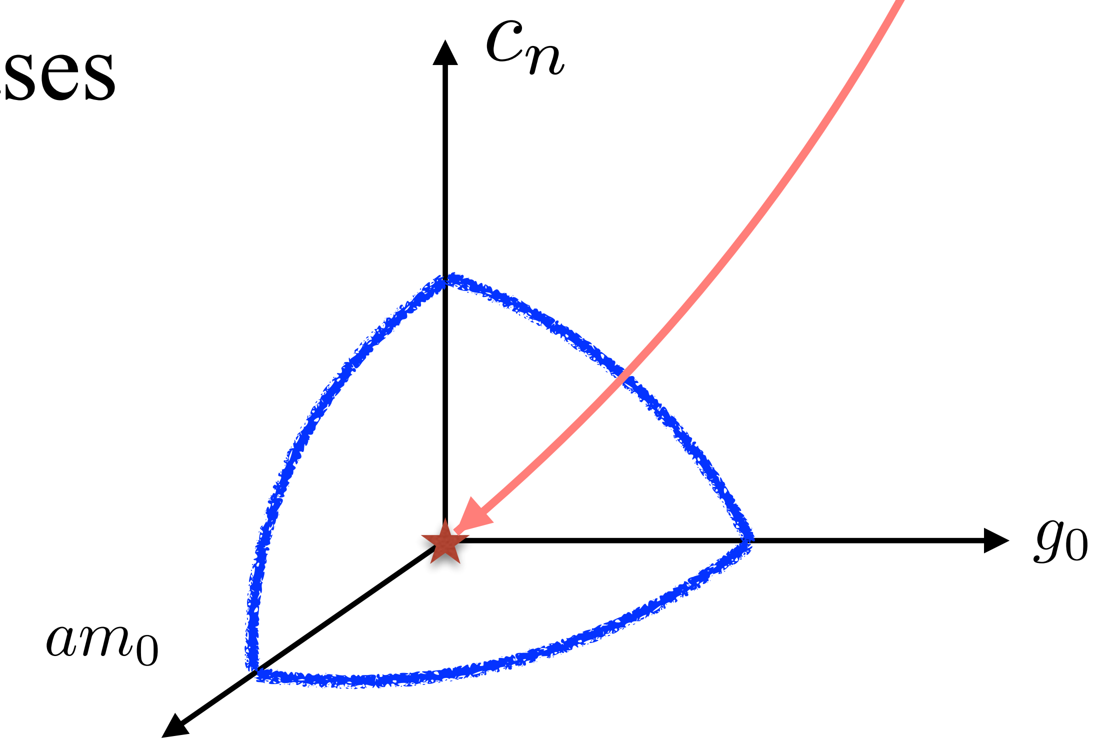
- ★ Two Dirac fermions in the fundamental repn
  pseudoreal
- ★ Three Dirac fermions in the antisymmetric repn
  real
- ★ $G_{HC} = Sp(4),\, G/H = \{SU(4) \times SU(6)\}/\{Sp(4) \times SO(6)\}$
- ★ Wilson's lattice gauge and fermion actions

# The first thing to do when simulating a new theory

Probe the non-thermal phase structure of the lattice theory

$$\mathcal{L}_{\mathrm{latt}} = \mathcal{L}_{\mathrm{cont}} + \sum_{n=1}^{\infty} c_n a^n O_{n+4}$$

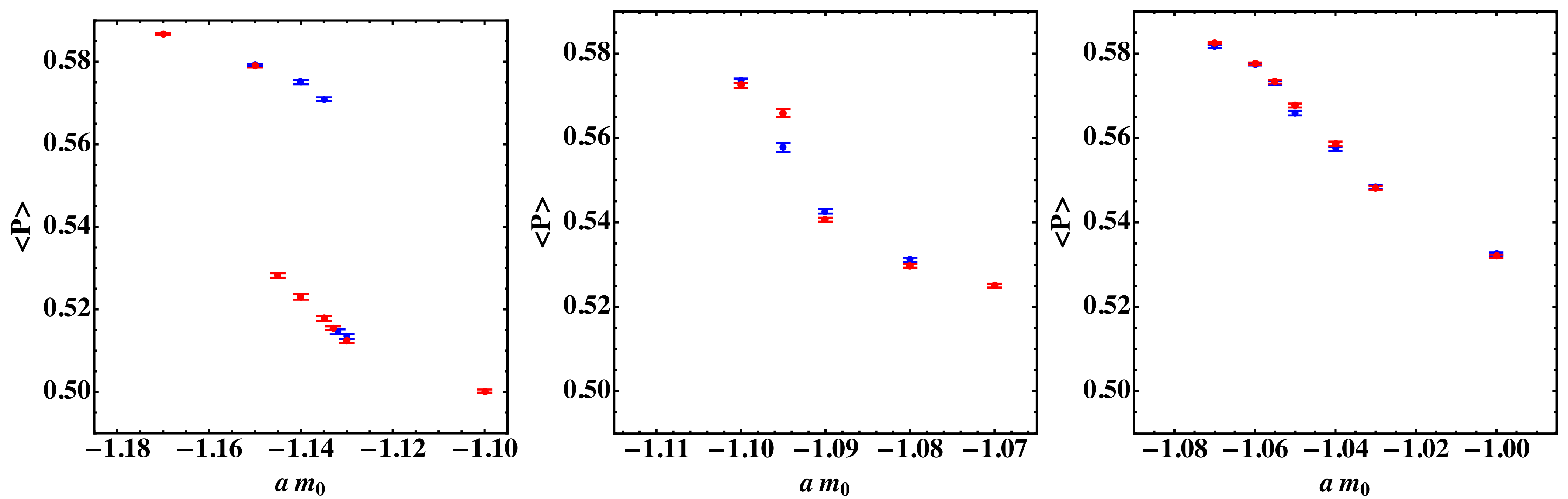
- Important for simulations and the continuum limit
- Avoid artefact phases



# Exploring the non-thermal phase structure

Sp(4) gauge theory with 3 antisymmetric-repn Wilson-Dirac flavours

$8^4$ lattices




stronger bare coupling | weaker bare coupling

coarser lattice | finer lattice

# Probing Sp(4) gauge theories step by step

$$Z = \int DU\, D\psi D\bar{\psi}\ \mathrm{e}^{-S_g[U]} \mathrm{e}^{-2\int \bar{\psi}_f (\not{D}_f[U]+m)\psi_f} \mathrm{e}^{-3\int \bar{\psi}_a (\not{D}_a[U]+m)\psi_a}$$

$$= \int DU\ \underline{\det(\not{D}_f[U]+m)^2 \det(\not{D}_a[U]+m)^3\ \mathrm{e}^{-S_g[U]}}$$

Monte-Carlo simulations

★ Computing the determinants is numerically very demanding

★ First calculations performed in the quenched approximation

$$\det(\not{D}_f[U]+m) = \det(\not{D}_a[U]+m) = 1$$

Systematics about 10% in QCD spectrum calculations

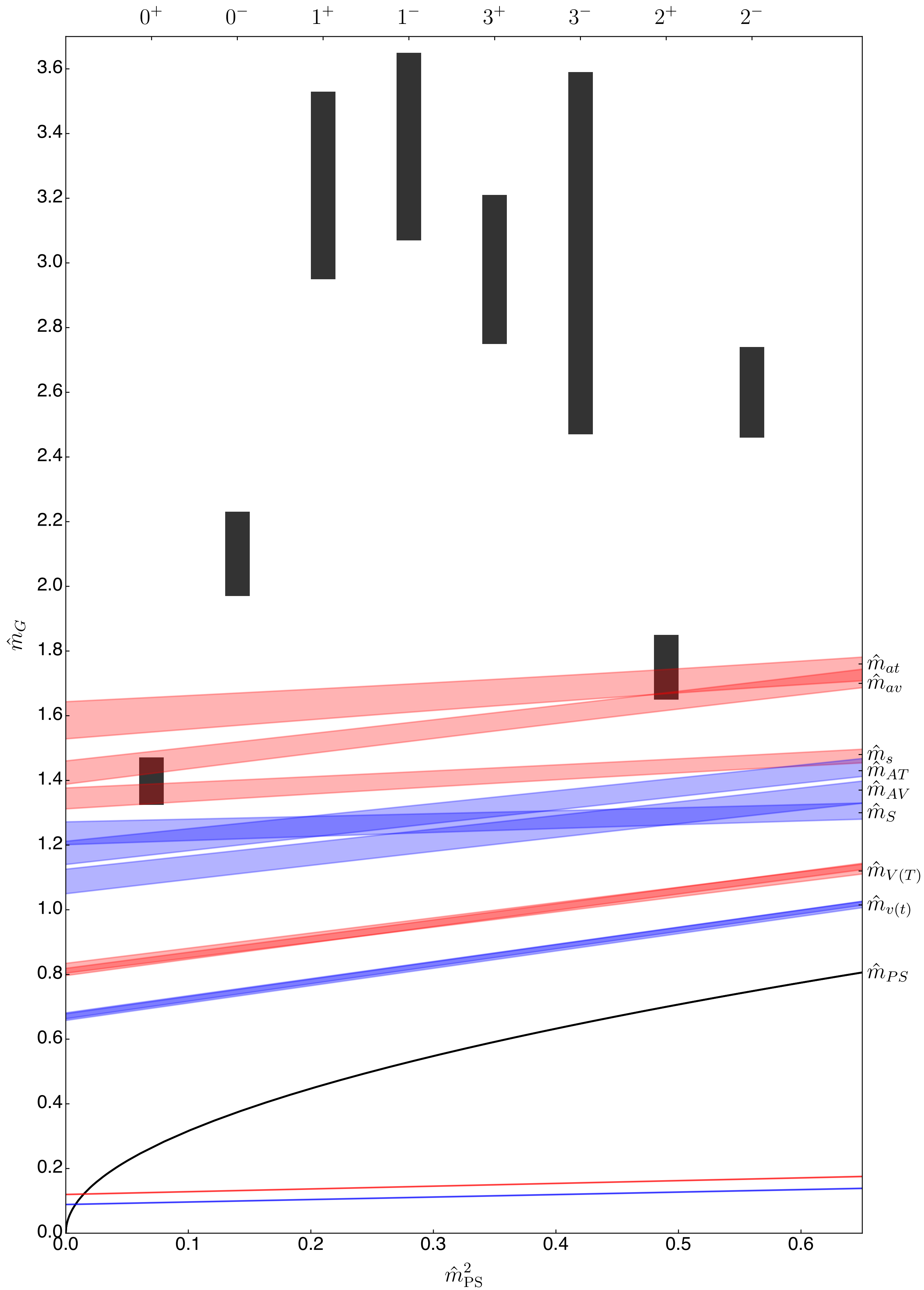
Loss of unitarity forbids computations for scattering amplitudes

★ Partial quenching as the intermediate step

$$\det(\not{D}_f[U]+m) = 1 \text{ or } \det(\not{D}_a[U]+m) = 1$$

# Quenched Sp(4) gauge theories spectrum

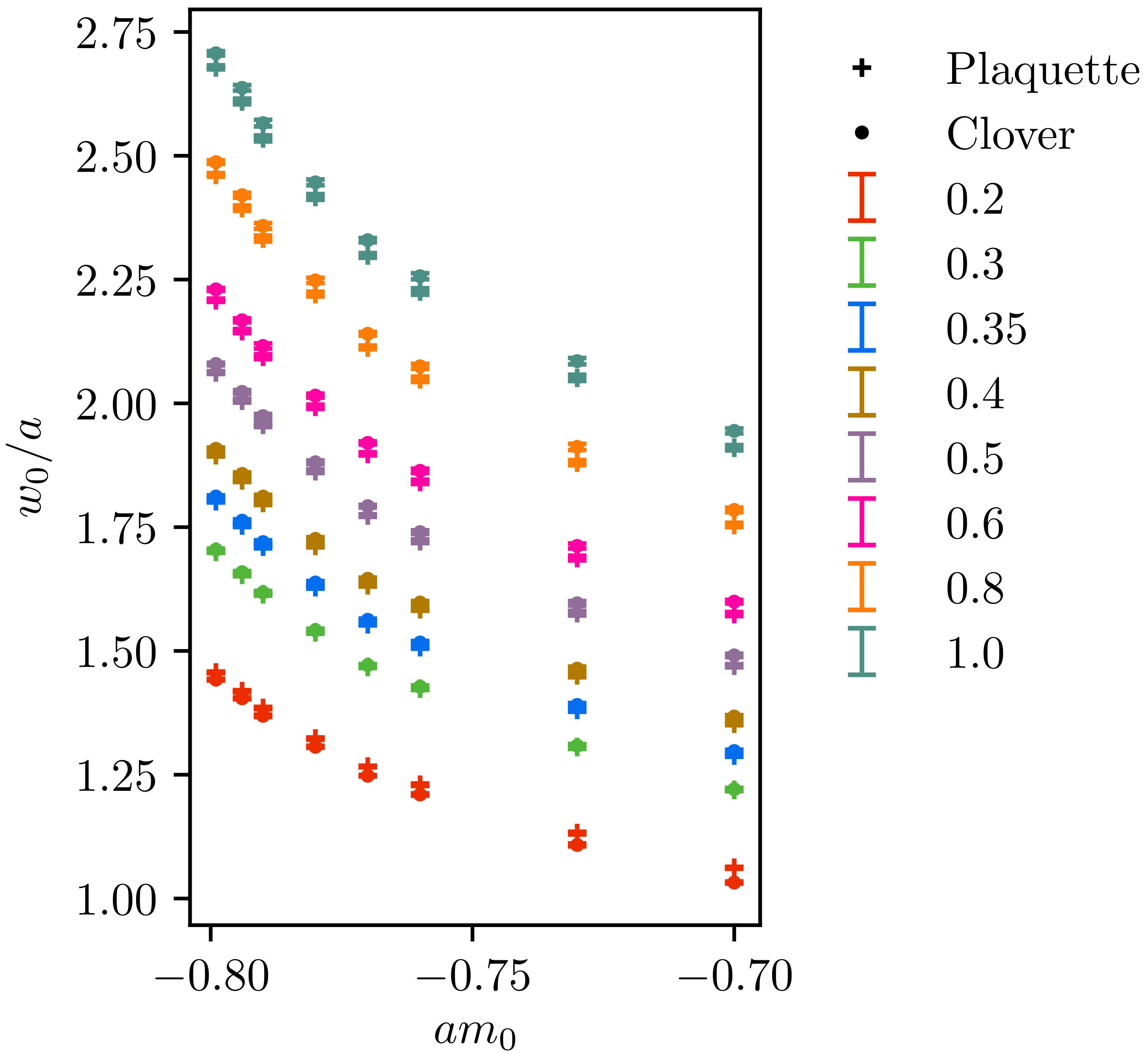
## in the continuum limit




Jong-Wan Lee's talk for more details

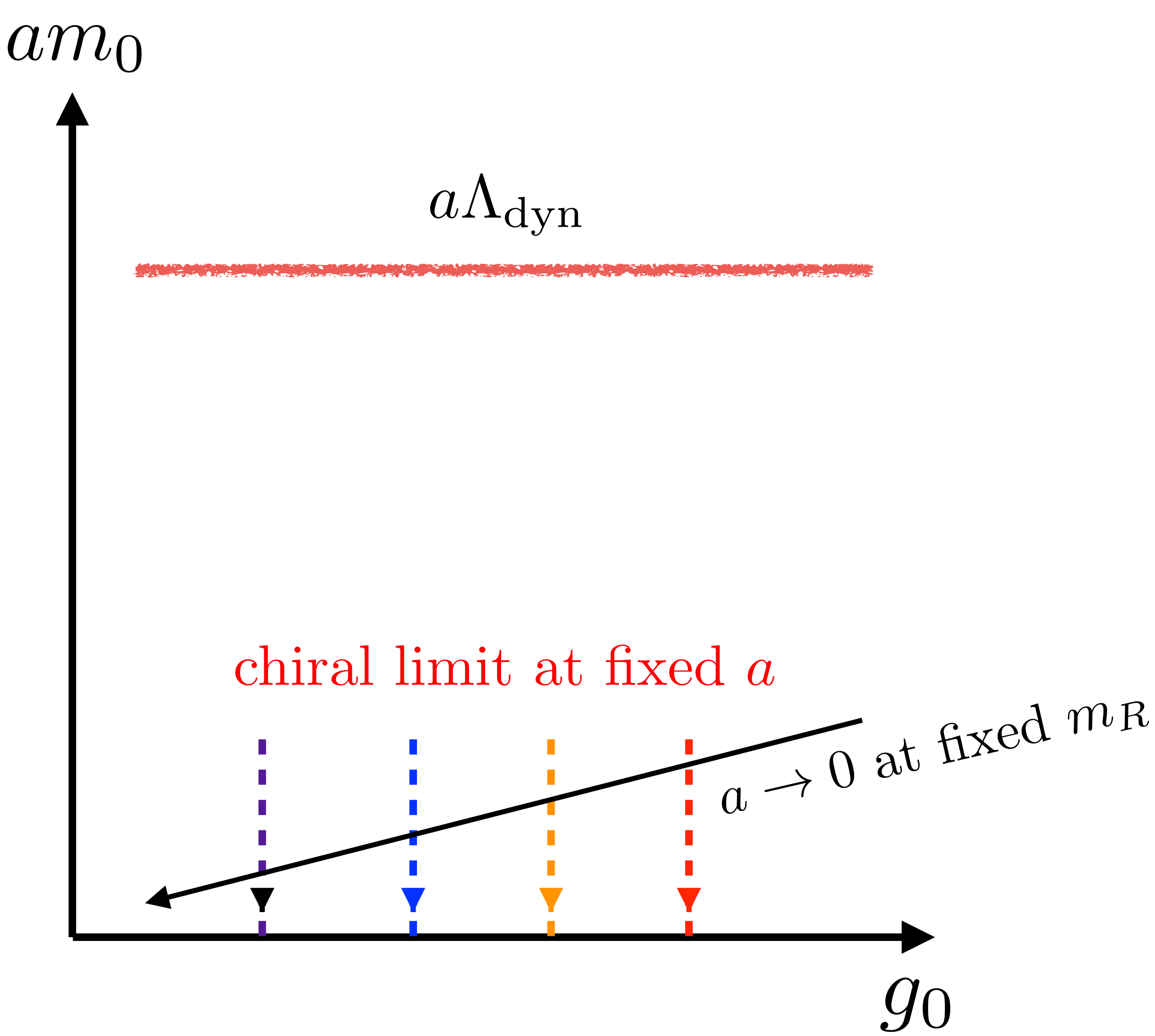
# Going dynamical

Marked difference from QCD, what we learned hitherto…

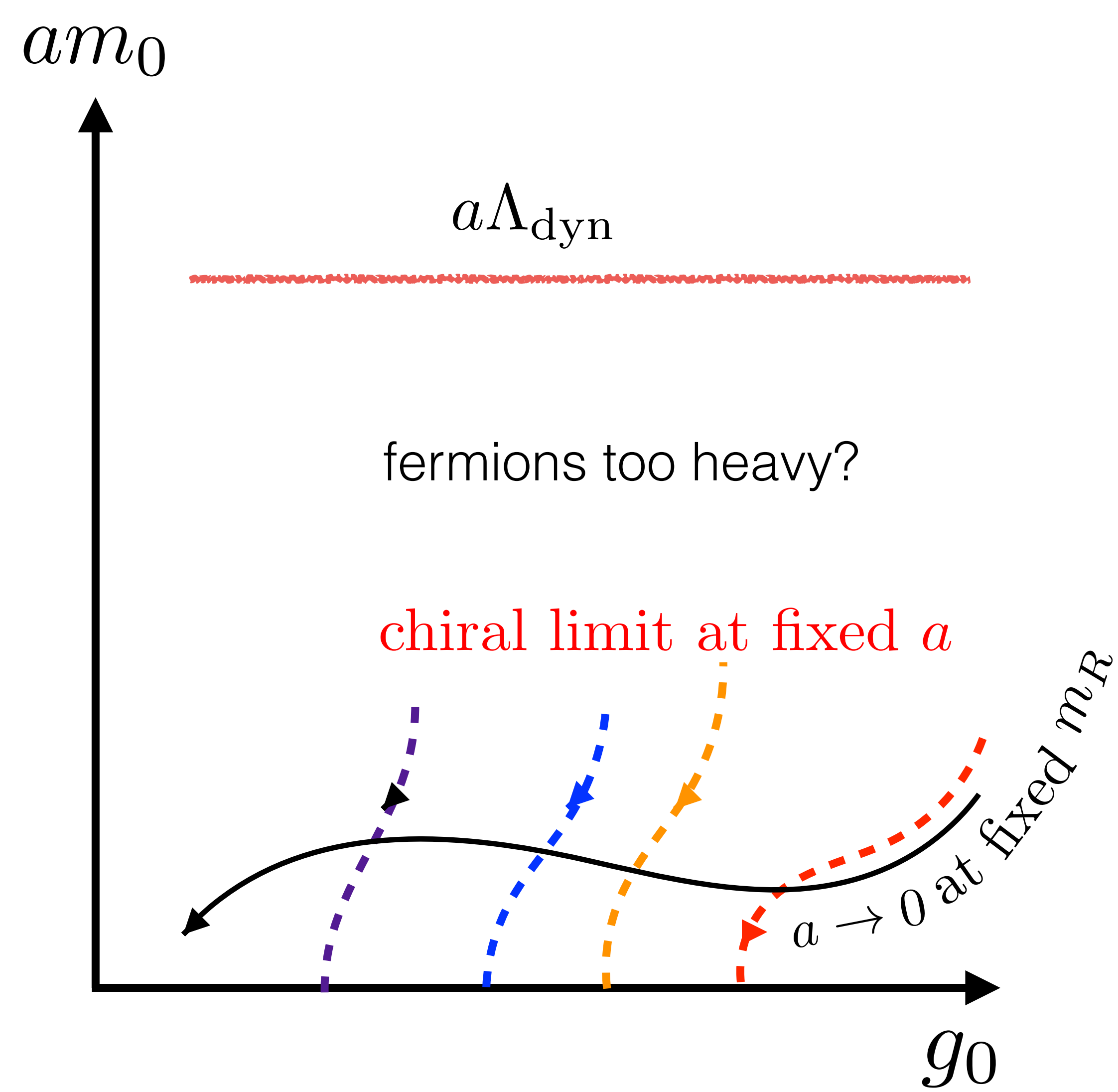


# Going dynamical

This is making the continuum extrapolation more involved…




QCD




Composite Higgs

# Chiral-continuum extrapolation

Use of EFT for these two extrapolations simultaneously

- ★ Recall the expression of a lattice theory:

$$\mathcal{L}_{\rm latt} = \mathcal{L}_{\rm cont} + \sum_{n=1}^{\infty} c_n a^n O_{n+4}$$

- ★ For Wilson fermions: $O_5 = O_{\rm SW} = \bar{\psi}\sigma_{\mu\nu}F^{\mu\nu}\psi$
- ★ $O_{\rm SW}$ and $\bar{\psi}\psi$ break chiral symmetry the same way
- ★ Spurion analysis with $\chi = 2B_0 m,\ \rho = 2W_0 a$
- ★ Simultaneous expansions in fermion mass and lattice spacing

# Status of unquenched calculations

★ Published results of low-lying meson spectrum for

$\det(\not{D}_f[U]+m)^2 = \mathrm{dynamical}$ and $\det(\not{D}_a[U]+m) = 1$

★ Ongoing Monte-Carlo simulations for

$\det(\not{D}_a[U]+m)^3 = \mathrm{dynamical}$ and $\det(\not{D}_f[U]+m) = 1$

See Jong-Wan Lee's talk for more details

# What next

★ In terms of Monte-Carlo simulations

Multi-representation dynamical simulations

Run at smaller fermion masses

★ In terms of observables to compute

Spectrum of excited-state mesons and the top partner

Four-fermion operator matrix elements of Goldstone

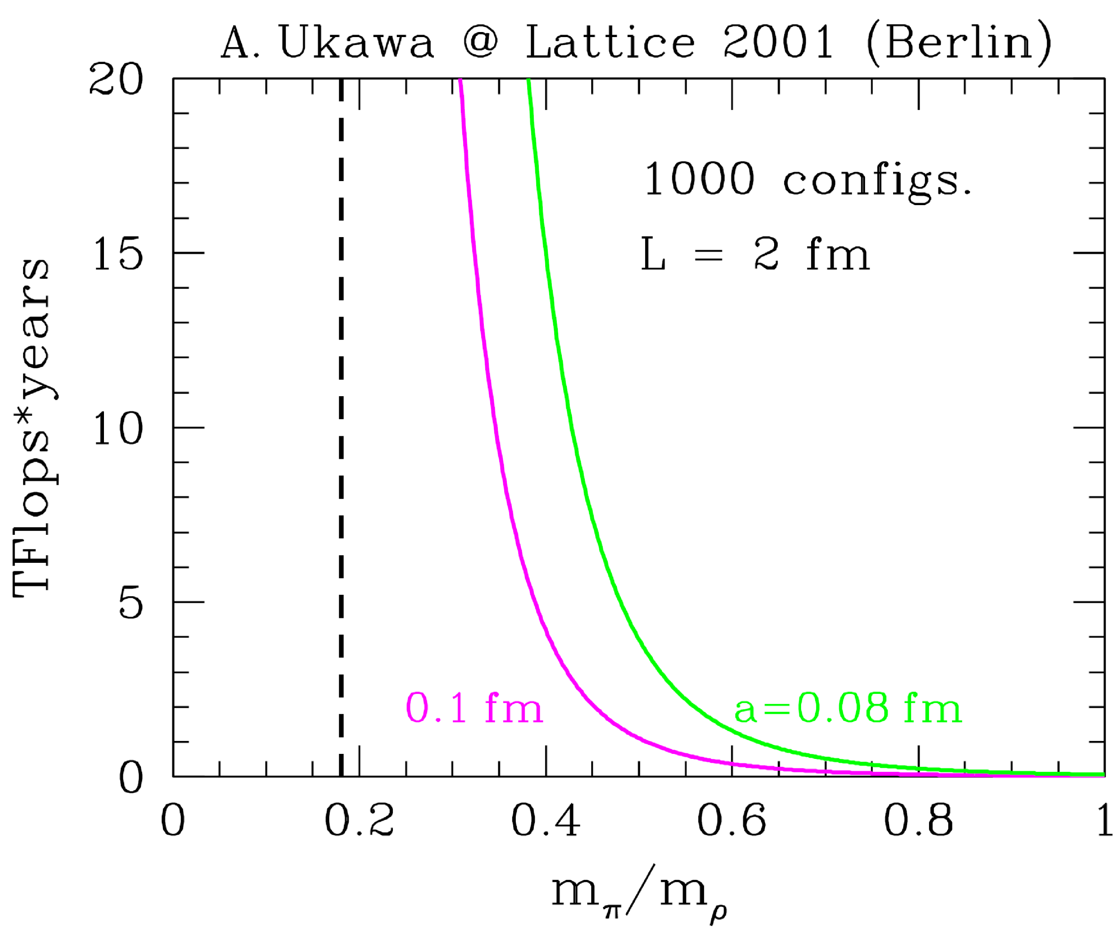
$\longrightarrow$ Inputs for the Higgs potential

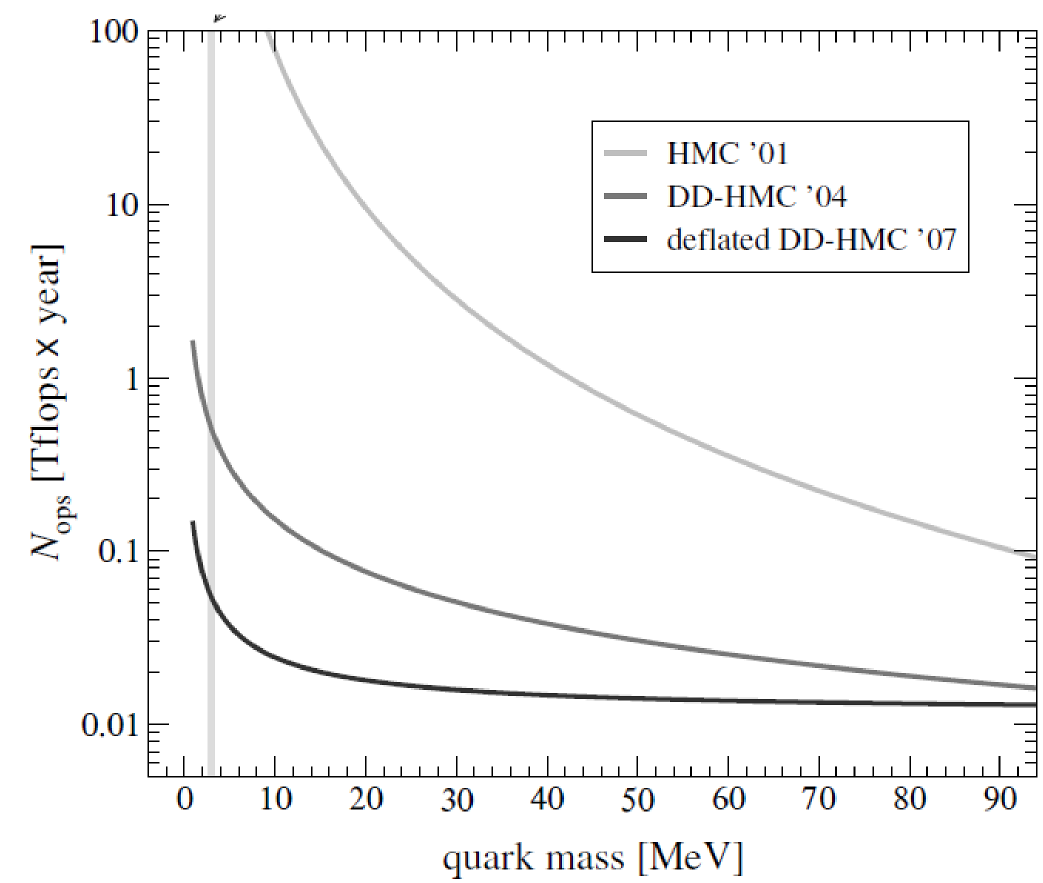
Resonance masses and decay widths

Anomalous dimension of the top partner: conformal window?

# The challenge ahead for dynamical simulations

The Berlin Wall




Improve the fermion action, use more sophisticated algorithm…

# Back-up slides

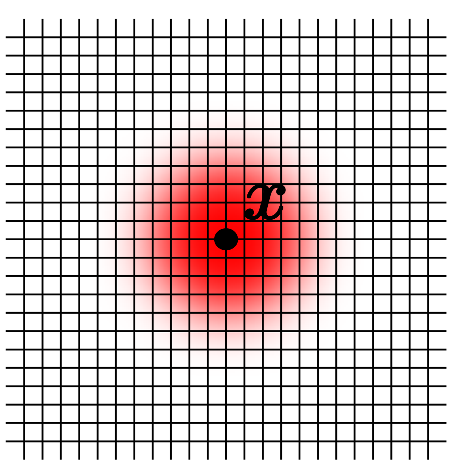
# Scale setting

## Determining the lattice spacing

- Converting from lattice to “physical” units.
- For extrapolating to the continuum limit.
- Modern popular method: the gradient flow.

bare $\longrightarrow$ renormalised

ref. scale




$\sim 2\sqrt{8t}$

$$\alpha(\mu) = k_\alpha t^2 \langle E(t) \rangle \equiv k_\alpha \mathcal{E}(t)\,,$$

$$\text{with } \mu = \frac{1}{\sqrt{8t}}, E(t) = -\frac{1}{2}\mathrm{Tr}(G_{\mu\nu}G_{\mu\nu})\,.$$

scale setting by specifying values

$$\mathcal{E}(t)|_{t=t_0} = \mathcal{E}_0$$

$$\mathcal{W}(t)|_{t=w_0^2} = \mathcal{W}_0, \; \mathcal{W}(t) \equiv t\frac{\mathrm{d}\mathcal{E}(t)}{\mathrm{d}t}\,.$$

# Looking for resonances

## Luscher's method in a nutshell

1. Elastic scattering phase shift from finite-volume Euclidean field theory

1.1. $E_{\mathrm{non-int}} = \sqrt{m_\pi^2 + |\vec{p}_1|^2} + \sqrt{m_\pi^2 + |\vec{p}_2|^2}, \;\; \vec{p}_i = \frac{2\pi}{L}\vec{n}_i, \;\; \vec{n}_i \in Z^3$

$$\vec{P} = \vec{p}_1 + \vec{p}_2$$

1.2. Measure $E_n^{\pi\pi}$ at total momentum $\vec{P}$ with interactions in finite volume

$$E_{n,\mathrm{CM}} = \sqrt{(E_n^{\pi\pi})^2 - |\vec{P}|^2} = \sqrt{s_n} = 2\sqrt{m_\pi^2 + k^2}$$

1.3. $\boxed{\phi(q)} + \boxed{\delta(k)} = n\pi$, where $q = kL/2\pi$

Infinite-volume phase shift

This function knows the finite-volume geometry

2. Fit to the Breit-Wigner form to describe the resonance

# Looking for resonances

## State of the art calculation in QCD

C. Alexandrou *et al.*, Phys. Rev. D 96 (2017) 034525