Composite Higgs and Dark Matter

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JHEP 10 (2019) 035 and arXiv: 2007.04338

LIO international conference on Composite connections of

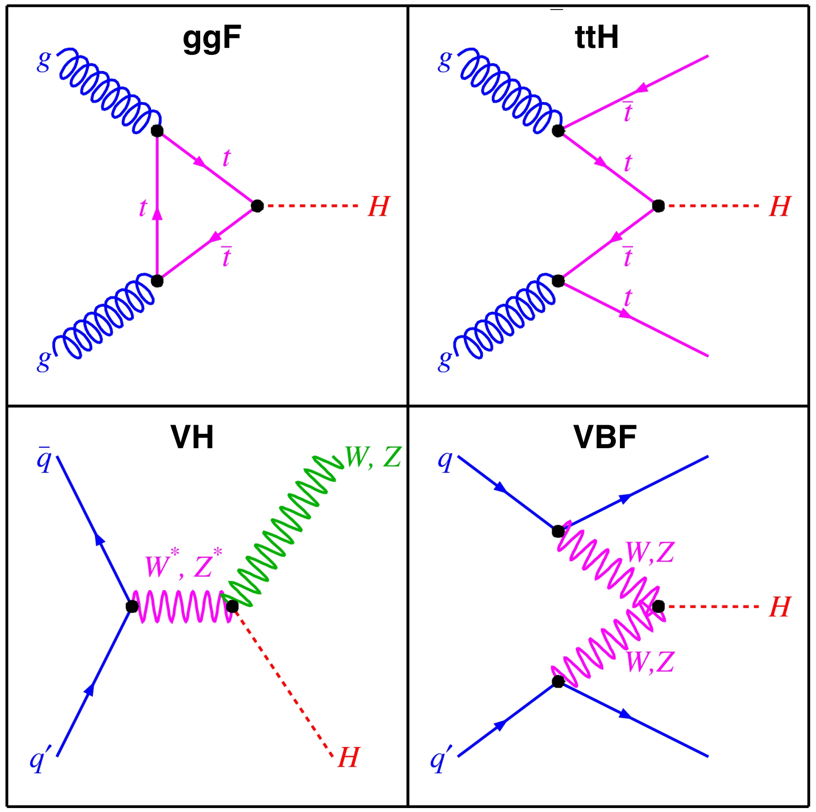
Higgs, Dark Matter and Neutrinos

September 22, 2020

Higgs boson @ LHC

In 2012, CMS and ATLAS announced the discovery of a SM-Like Higgs bosons

While new physics is still allowed …



The Higgs boson is produced in gluon-fusion, ttH, VH, and VBF channels, and decays into di-photons, WW / ZZ, quarks & leptons.

The SM Higgs boson

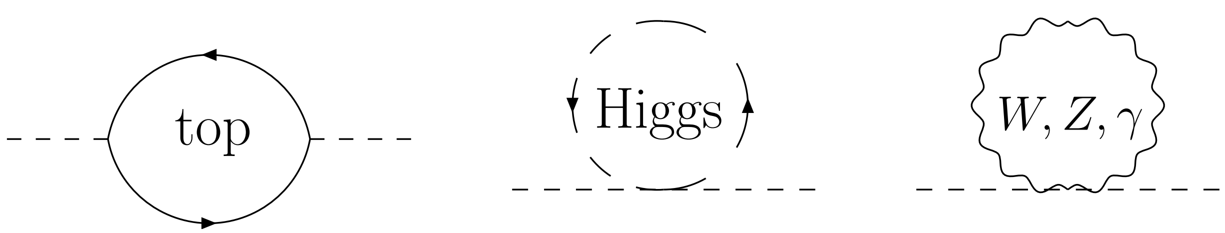
The puzzle is whether the Higgs is an elementary particle. Do we need Models Beyond the Standard Model?

$$\mathcal{L}_{\text{Higgs}} = D_\mu \phi^\dagger D^\mu \phi + \boxed{\mu^2 \phi^\dagger \phi - \lambda(\phi^\dagger \phi)^2}$$

$$\phi = e^{i\pi^i \sigma^i/2} \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v+h) \end{pmatrix} \qquad v = \mu^2/\lambda = 246 \text{ GeV}$$

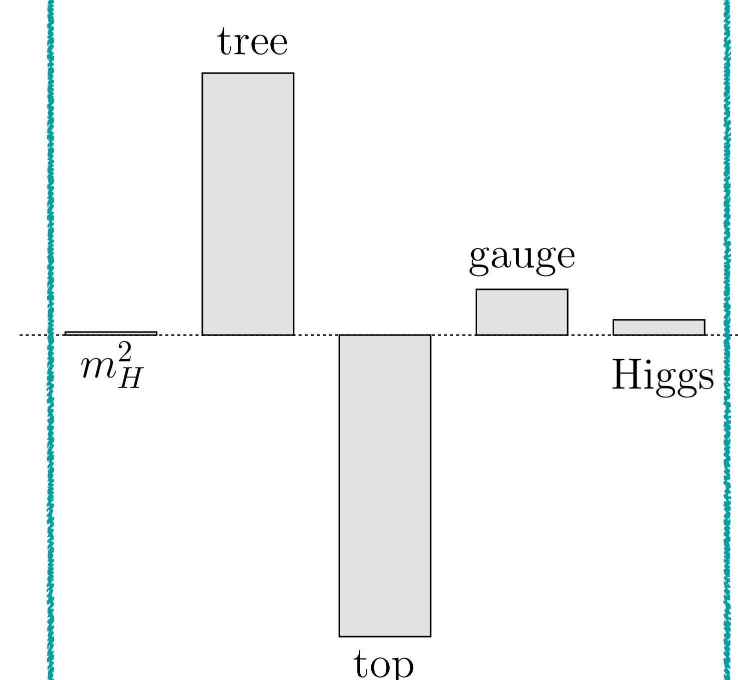
“Mexican hat” potential well describes EWSB, but no insight on Higgs property

Quantum Corrections spoil the naturalness:



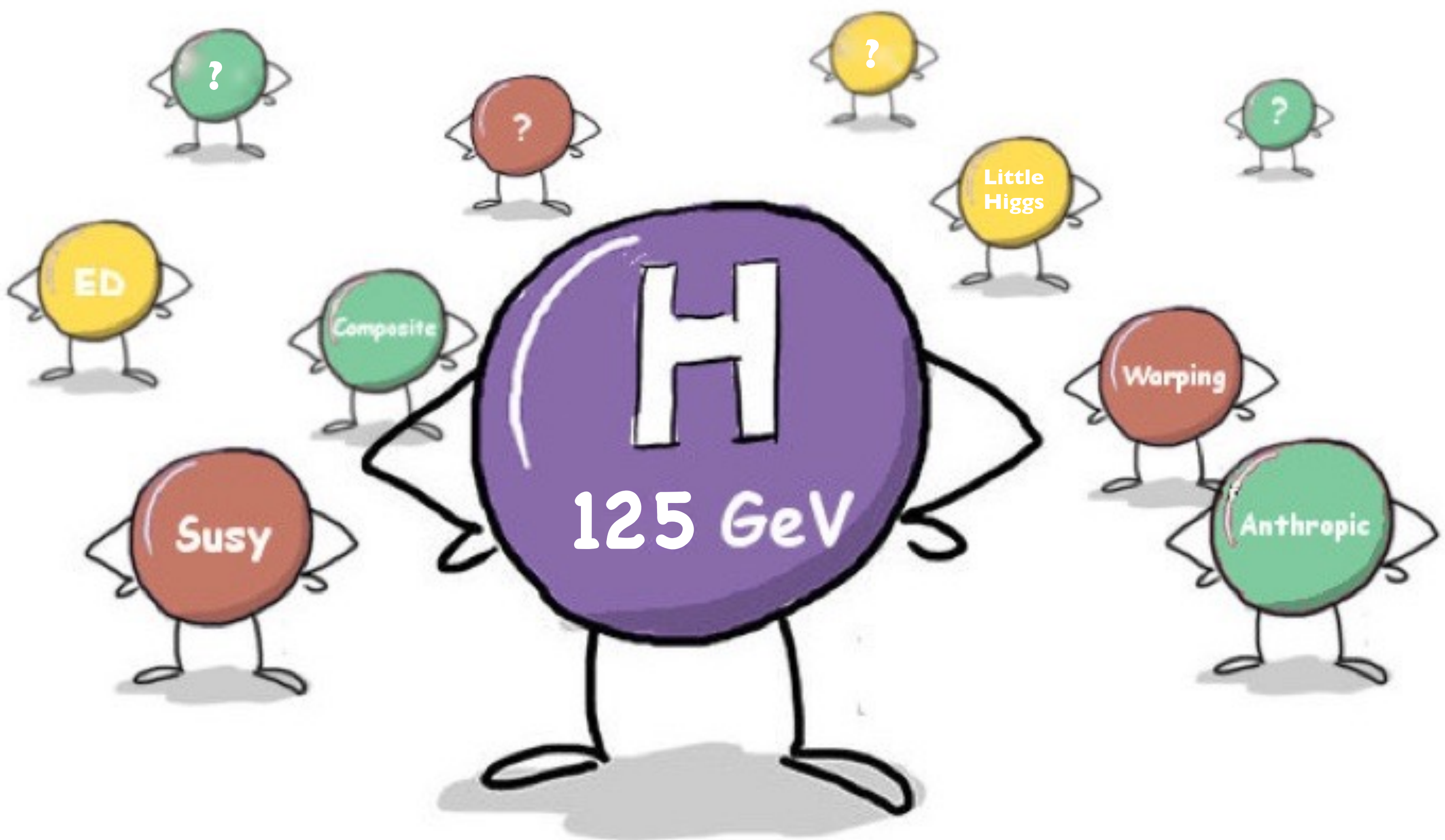
$$m_H^2 = m_{\text{tree}}^2 - \frac{3\lambda_t^2}{8\pi^2}\Lambda^2 + \frac{\lambda}{16\pi^2}\Lambda^2 + \frac{g^2}{16\pi^2}\Lambda^2$$

Fine-tuning needed for $\Lambda = 10$ TeV $\Rightarrow$

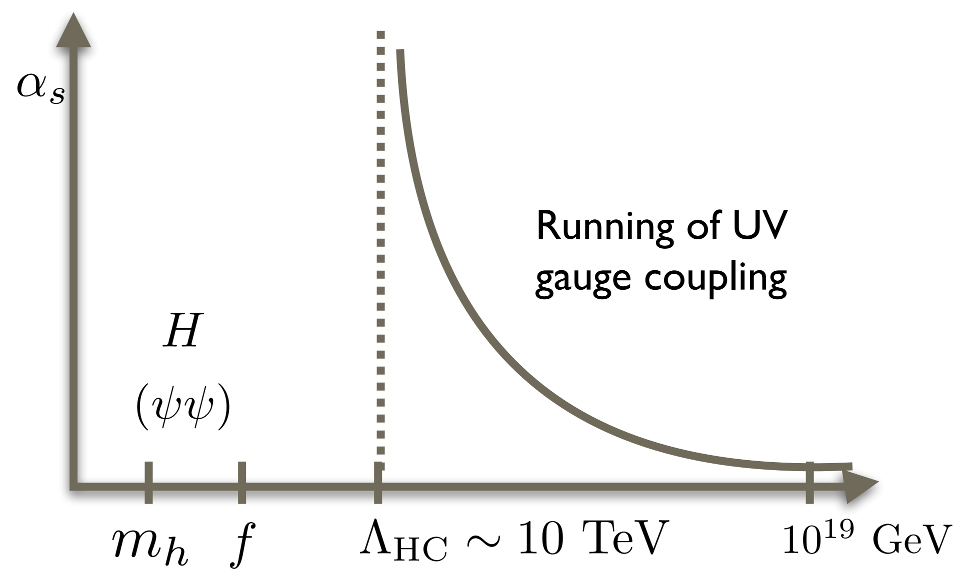


BSM might work

There can be many solutions: SUSY, Little Higgs, Extra Dimension (flat or warped), etc



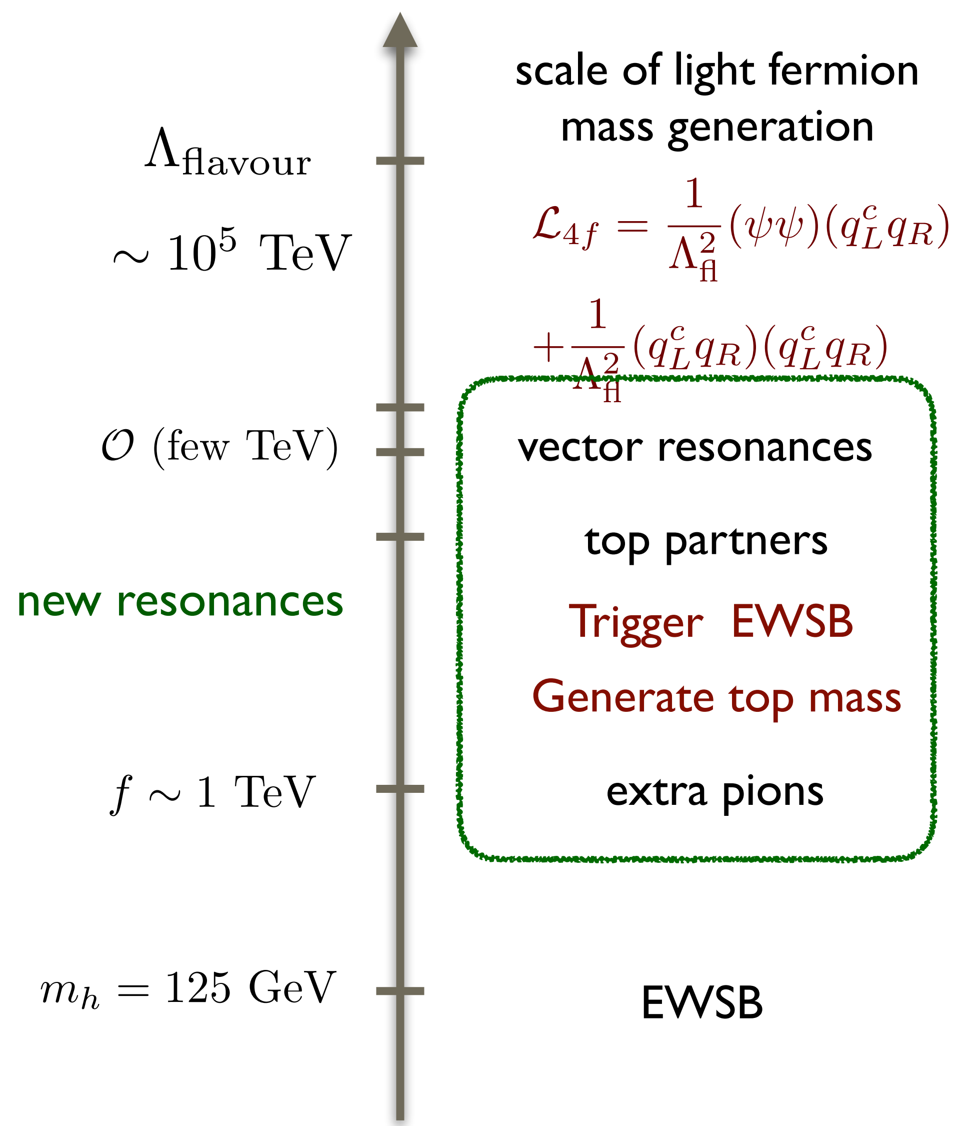
The compositeness paradigm



Compositeness is an alternative solution to " Naturalness "

- The asymptotic free UV gauge group confines at ~10 TeV, condensation states are generated.
- In EFT, Composite Higgs boson is realised as pseudo Nambu Goldstone Boson, with its lightness protected by global symmetry.

Georgi, Kaplan (1984)



Cacciapaglia, HC, Flacke, Lee, et al, JHEP 1506 (2015) 085

In this paradigm, flavour constraint can be satisfied.

Fundamental Composite Dynamics

The models we discuss are based on a confining gauge group with two species of fermions, predicting rich phenomenology in the low energy effective theory.

- Containing at least one Higgs doublet as pNGBs in custodial symmetry.
- The Hyper-Color fermionic singlets can be used as top partners.

Phenomenology Refs:

Cacciapaglia, HC, et al. JHEP 1511, 201;

Belyaev, HC, et al. PRD 94, no.1, 015004 (2016);

Buarque, HC, et al. JHEP 1611, 076

Cacciapaglia, HC, et al. JHEP 1910 (2019) 035

	$\mathcal{G}_{\rm TC}$	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
$\psi_{1/2}$	R_1	$\mathbf{1}$	$\mathbf{2}$	$1/2$
$\psi_{-1/2}$		$\mathbf{1}$	$\mathbf{2}$	$-1/2$
ψ_0		$\mathbf{1}$	$\mathbf{1}$	0
$\tilde{\psi}_0$		$\mathbf{1}$	$\mathbf{1}$	0
χ_t	R_2	$\mathbf{3}$	$\mathbf{1}$	$2/3$
$\tilde{\chi}_t$	$\overline{R}_2$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$-2/3$

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a,\mu\nu} + \bar{\Psi}(i\sigma^\mu) D_\mu \Psi - \Psi M \Psi + h.c.$$

This UV Lagrangian corresponds to SU(6)/SO(6) CHM in the IR limit. $\mathcal{G}_{TC}$: $SO(7)$ or $SO(9)$

The Sigma in SU(6)/SO(6)

The low energy dynamics of CHM can be described by $\Sigma(x)$ in EFT framework:

$$\Sigma(x) = exp\left(\frac{i2\sqrt{2}}{f}\Pi\right).\Sigma_B$$

This mapping is before EWSB

where Π are parametrized in terms of Goldstone fields as:

$$2\,\Pi = \begin{pmatrix} \varphi + \frac{\eta_1}{\sqrt{3}}1_2 & \Lambda & \sqrt{2}H_1 & \sqrt{2}H_2 \\ \Lambda^\dagger & -\varphi + \frac{\eta_1}{\sqrt{3}}1_2 & -\sqrt{2}\widetilde{H_1} & -\sqrt{2}\widetilde{H_2} \\ \sqrt{2}H_1^\dagger & -\sqrt{2}\widetilde{H_1}^\dagger & 2\left(\frac{\eta_3}{\sqrt{2}} - \frac{\eta_1}{\sqrt{3}}\right) & \sqrt{2}\eta_2 \\ \sqrt{2}H_2^\dagger & -\sqrt{2}\widetilde{H_2}^\dagger & \sqrt{2}\eta_2 & -2\left(\frac{\eta_3}{\sqrt{2}} + \frac{\eta_1}{\sqrt{3}}\right) \end{pmatrix}$$

2 Higgs doublets

$$H_1 = \begin{pmatrix} G_+ \\ \frac{h+iG_0}{\sqrt{2}} \end{pmatrix}, \qquad H_2 = \begin{pmatrix} H_+ \\ \frac{H+iA_0}{\sqrt{2}} \end{pmatrix}, \qquad \widetilde{H}_{1,2} = i\sigma_2 H_{1,2}^*$$

with (ϕ, Λ) forming a bi-triplet $(3,3)$:

Similar to Georgi Machacek model

$$\varphi = \sigma^a\varphi^a \equiv \begin{pmatrix} \varphi^0 & \sqrt{2}\varphi^+ \\ \sqrt{2}\varphi^- & -\varphi^0 \end{pmatrix}, \qquad \Lambda = \sigma^a\Lambda^a \equiv \begin{pmatrix} \sqrt{2}\Lambda^+ & 2\Lambda^{++} \\ 2\Lambda_0 & -\sqrt{2}\Lambda^+ \end{pmatrix}$$

This theory delivers 20 NGBs transforming under $SU(2)_L \times U(1)_Y$ gauge group as $3_{\pm1} + 3_0 + 2_{\pm1/2} + 2_{\pm1/2} + 3 \times 1_0$.

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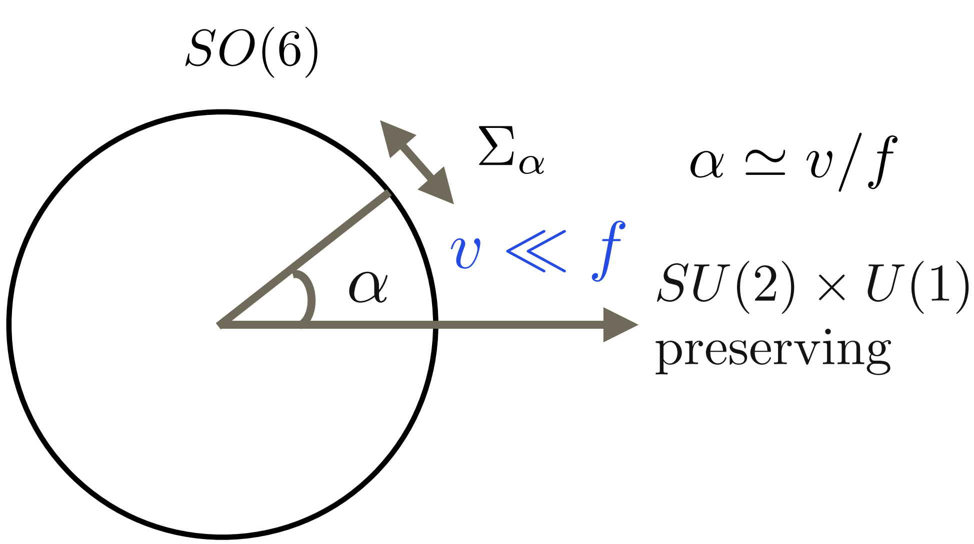
Vacuum Misalignment

The most general vacuum structure is defined by 3 angles (2 along doublets and 1 along triplet). Consider giving VEV to any one of two Higgs doublets, this can be achieved by a $SU(6)$ rotation:

$$U(\alpha,\beta) = U(\beta)U(\alpha)U(\beta)^\dagger = e^{i\sqrt{2}(\alpha_1\ X_h + \alpha_2\ X_{H_0})}$$



$$U(\beta) = \left(\begin{array}{cc|cc} 1_2 & 0 & 0 & 0 \\ 0 & 1_2 & 0 & 0 \\ \hline 0 & 0 & \cos\beta & -\sin\beta \\ 0 & 0 & \sin\beta & \cos\beta \end{array}\right)$$

$\alpha^2 = (\alpha_1^2 + \alpha_2^2)$ rotates vacuum to deviate $SU(2)_L \times U(1)_Y$ conserving direction, and tigger EWSB; The other $\tan(\beta) = \alpha_2/\alpha_1$ distributes VEV among $h,\ H_0$

Now the Sigma field after EWSB is rotated by the transformation:

$$\Sigma(x)_{\alpha,\beta} = U(\alpha,\beta)\cdot\Sigma(x)\cdot U(\alpha,\beta)^T.$$

Since $U(\beta) = e^{-i\sqrt{2}\,\beta\,\mathbf{S_{15}}}$ is defined in terms of a generator unbroken by Σ_B, it can either be absorbed into spurion (breaking symmetry) or PNGB redefinition.

Effective Field Theory

At the leading order, the effective Lagrangian is given by the kinetic term. Since the gauge interaction is invariant under $U(\beta)$, β dependence is fully removed.

$$\mathcal{L} = \frac{1}{16} f^2 Tr[(D_\mu \Sigma(x))^\dagger \cdot D^\mu \Sigma(x)]$$

with

$$D_\mu \Sigma(x) = \partial_\mu \Sigma(x) - \left[i g_2 W_\mu^a (T^a.\Sigma(x) + \Sigma(x).T^{aT}) + i g_1 B_\mu (Y.\Sigma(x) + \Sigma(x).Y^T) \right].$$

This Lagrangian contains masses of W & Z gauge bosons, satisfying custodial symmetry:

$$m_W^2 = \frac{1}{4} g_2^2 f^2 \sin^2(\alpha)\,, \quad m_Z^2 = \frac{m_W^2}{cos^2\theta_W}\,.$$

We can define $f \sin(\alpha) = v_{\rm SM}$. The linear couplings between the Higgs boson h and the vector bosons are derived to be:

$$g_{hWW} = g_{hWW}^{SM} \cos(\alpha), \quad g_{hZZ} = g_{hZZ}^{SM} \cos(\alpha)\,.$$

This is universal in most CHMs: SO(5)/SO(4), SU(4)/Sp(4), etc.

Typical couplings

These vertices are extracted from kinetic term at the second order of pion matrix:

$$
\begin{aligned}
\mathcal{L}_{W^-W^-} &= -\frac{1}{4}g^2 W^{-\mu} W_\mu^- \Bigg(3\Lambda_+\varphi_+ \sin^2(\alpha) + 2\Lambda_+^2 \sin^4\left(\frac{\alpha}{2}\right) + 2\varphi_+^2 \cos^4\left(\frac{\alpha}{2}\right) - H_+^2 \sin^2(\alpha) \\
&+ \Lambda_{++}\left(4\left(\Lambda_0 \sin^4\left(\frac{\alpha}{2}\right) + \Lambda_0^* \cos^4\left(\frac{\alpha}{2}\right)\right) - \sin^2(\alpha)\left(\sqrt{3}\eta_1 - \sqrt{2}\eta_3 + 3\varphi_0\right)\right)\Bigg) \\
\mathcal{L}_{W^+Z} &= \frac{1}{4}g^2 \sec(\theta_w) W_\mu^+ Z^\mu \Bigg(H_-\left(-2H_0\cos(\alpha)\sin^2(\theta_w) + iA_0(\cos(2\theta_w) - \cos(2\alpha))\right) \\
&+ 4\Lambda_{--}\varphi_+ \sin^2\left(\frac{\alpha}{2}\right)\left(2\sin^2(\frac{\alpha}{2}) - 3\cos^2(\theta_w)\right) + 4\Lambda_{--}\Lambda_+ \cos^2\left(\frac{\alpha}{2}\right)\left(2\cos^2(\frac{\alpha}{2}) - 3\cos^2(\theta_w)\right) \\
&+ \Lambda_-\left(4\Lambda_0\cos^2\left(\frac{\alpha}{2}\right)\left(\cos^2(\theta_w) - 2\cos(\alpha)\right) + 2\varphi_0 \sin^2\left(\frac{\alpha}{2}\right)\left(\cos(\alpha) - \cos(2\theta_w)\right)\right) \\
&+ \varphi_-\left(4\Lambda_0^*\sin^2\left(\frac{\alpha}{2}\right)\left(\cos^2(\theta_w) + 2\cos(\alpha)\right) - 2\varphi_0 \cos^2\left(\frac{\alpha}{2}\right)\left(\cos(\alpha) + \cos(2\theta_w)\right)\right) \\
&+ (\Lambda_- + \varphi_-)\left(\sqrt{3}\eta_1 - \sqrt{2}\eta_3\right)\sin^2(\alpha)\Bigg) \\
\mathcal{L}_{W^+\gamma} &= \frac{1}{2}g^2 \sin(\theta_w) W_+^\mu \gamma_\mu \Bigg(H_-\left(H_0\cos(\alpha) + iA_0\right) - 6\sin^2(\frac{\alpha}{2})\varphi_+\Lambda_{--} - 6\cos^2(\frac{\alpha}{2})\Lambda_+\Lambda_{--} \\
&+ 2\Lambda_-\left(\Lambda_0\cos^2(\frac{\alpha}{2}) - \varphi_0\sin^2(\frac{\alpha}{2})\right) + 2\varphi_-\left(\Lambda_0^*\sin^2(\frac{\alpha}{2}) - \varphi_0\cos^2(\frac{\alpha}{2})\right)\Bigg) \\
\mathcal{L}_{Z\gamma} &= g^2 \tan(\theta_w) Z^\mu \gamma_\mu \Bigg(\left(\cos(2\theta_w) + \cos(\alpha)\right)\varphi_-\varphi_+ + \left(\cos(2\theta_w) - \cos(\alpha)\right)\Lambda_-\Lambda_+ \\
&+ \cos(2\theta_w)\left(H_-H_+ + 4\Lambda_{--}\Lambda_{++}\right)\Bigg)
\end{aligned}
$$

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There is no W, Z couplings to a single η_2 at this order, indicating protection from symmetry.

The DM Parity

Indeed our model enjoys some symmetries. First a $SO(2) \sim U(1)$ is preserved only for $\alpha = 0$, i.e. before the EW symmetry breaking. This $SO(2)$ rotation coincides with $U(\beta)$, under which:

$$
\begin{aligned}
H_1 &= \tilde{H}_1 \cos(\beta) + \tilde{H}_2 \sin(\beta), \quad H_2 = \tilde{H}_2 \cos(\beta) - \tilde{H}_1 \sin(\beta), \\
\eta_2 &= \tilde{\eta}_2 \cos(2\beta) - \tilde{\eta}_3 \sin(2\beta) \quad \eta_3 = \tilde{\eta}_2 \sin(2\beta) + \tilde{\eta}_3 \cos(2\beta)
\end{aligned}
$$

Let h obtains its VEV, $SO(2)$ will be broken, only a discrete $\mathbb{Z}_2$ remains:

$$
\Omega_{\rm DM} = \begin{pmatrix} 1_2 & & \\ & 1_2 & \\ & & \sigma_3 \end{pmatrix}, \quad \Omega_{\rm DM} \Sigma_\alpha(H_2, \eta_2) \Omega_{\rm DM} = \Sigma_\alpha(-H_2, -\eta_2)
$$

under which H_2 and η_2 are odd, serving as Dark Matter candidate.

This parity is only a good symmetry consistent with α vacuum, but broken by the $U(\beta)$ (H_2 and H_1 in opposite parity), $\beta = 0$ is required.

This indicates the second Higgs need to be inert in order to conserve DM parity.

CP Invariance (Parity)

The Lagrangian like $(D_\mu\Sigma)^\dagger(D_\mu\Sigma)$ is invariant under CP transformation. CP parity is defined from the operation which can hermitian conjugate the sigma matrix and change the sign of CP-odd particles:

$$\Omega_{\rm CP} = \begin{pmatrix} 1_2 & & \\ & 1_2 & \\ & & -1_2 \end{pmatrix}, \quad \Omega_{\rm CP}\Sigma_{(\alpha,\beta)}(\phi_{\rm odd})\Omega_{\rm CP} = \Sigma^\dagger_{(\alpha,\beta)}(-\phi_{\rm odd})$$

	h	$H_{0,\pm}$	$\mathcal{I}(\Lambda_0)$	$\mathcal{R}(\Lambda_0)$	$\Lambda_{\pm\pm,\pm}$	$\phi_{0,\pm}$	A_0	$\eta_{1,2,3}$
CP	$+$	$+$	$+$	$-$	$-$	$-$	$-$	$-$

WZW anomaly terms need be discussed in order to verify the CP property.

Wess-Zumino-Witten term

CP-Parity odd pNGBs can decay to the gauge bosons via the WZW terms involving one scalar and two gauge bosons, which can be elegantly evaluated in terms of differential forms:

$$S_{\rm WZW} = C \int Tr[(d\mathcal{A}\mathcal{A} + \mathcal{A}d\mathcal{A})d\Sigma\Sigma^\dagger + (d\mathcal{A}^T\mathcal{A}^T + \mathcal{A}^T d\mathcal{A}^T)\Sigma^\dagger d\Sigma] \\ + C \int Tr[\mathcal{A}d\Sigma\mathcal{A}^T\Sigma^\dagger - d\mathcal{A}^T d\Sigma^\dagger \mathcal{A}\Sigma]\,,$$

The differential forms $\mathcal{A}$ and $d\Sigma$ are defined to be:

$$\mathcal{A} = \mathcal{A}_\mu dx^\mu\,, \quad d\mathcal{A} = \partial_\mu \mathcal{A}_\nu dx^\mu dx^\nu\,, \quad d\Sigma = \partial_\mu \Sigma dx^\mu\,.$$

where $\mathcal{A}_\mu$ is the EW gauge fields embedded in the global $SU(6)$ symmetry, so that $\mathcal{A}_\mu = g\left(\sum_{i=1,2,3} W^i_\mu T^i_L + \tan(\theta_{\rm W}) B_\mu T^3_R\right)$.

Integrating out by parts yields anomaly interactions in the form of $\frac{\kappa_{VV'}}{f} V^{\mu\nu}\tilde{V'}_{\mu\nu}$.

In all CP-odd states, η_2 and A_0 remain anomaly free, i.e. WZW interactions conserve DM parity.

The WZW Coefficients

$\eta_1 W^+_{\mu\nu}\tilde{W}^-_{\mu\nu}$	$\eta_3 W^+_{\mu\nu}\tilde{W}^-_{\mu\nu}$	$\varphi_0 W^+_{\mu\nu}\tilde{W}^-_{\mu\nu}$
$\frac{(7\cos(2\alpha)+17)}{4\sqrt{6}f\sin^2(\theta_W)}$	$\frac{\sin^2(\alpha)}{2f\sin^2(\theta_W)}$	$\frac{\sin^2(\alpha)}{2\sqrt{2}f\sin^2(\theta_W)}$
$\frac{(\Lambda_0+\Lambda_0^*)}{\sqrt{2}} W^+_{\mu\nu}\tilde{W}^-_{\mu\nu}$	$\Lambda_+ W^-_{\mu\nu}\tilde{Z}_{\mu\nu}$	$\varphi_+ W^-_{\mu\nu}\tilde{Z}_{\mu\nu}$
$\frac{3\sin^2(\alpha)}{2f\sin^2(\theta_W)}$	$-\frac{\sin^2\left(\frac{\alpha}{2}\right)(1-2\cos(\alpha)-3\cos(2\theta_W))}{\sqrt{2}f\sin^2(\theta_W)\cos(\theta_W)}$	$-\frac{\cos^2\left(\frac{\alpha}{2}\right)(1+2\cos(\alpha)-3\cos(2\theta_W))}{\sqrt{2}f\sin^2(\theta_W)\cos(\theta_W)}$
$\Lambda_{++} W^-_{\mu\nu}\tilde{W}^-_{\mu\nu}$	$\Lambda_+ W^-_{\mu\nu}\tilde{A}_{\mu\nu}$	$\varphi_+ W^-_{\mu\nu}\tilde{A}_{\mu\nu}$
$\frac{\sin^2(\alpha)}{\sqrt{2}f\sin^2(\theta_W)}$	$\frac{3\sqrt{2}\sin^2\left(\frac{\alpha}{2}\right)}{f\sin(\theta_W)}$	$\frac{3\sqrt{2}\cos^2\left(\frac{\alpha}{2}\right)}{f\sin(\theta_W)}$
$\eta_1 A_{\mu\nu}\tilde{A}_{\mu\nu}$	$\eta_3 \quad \frac{(\Lambda_0+\Lambda_0^*)}{\sqrt{2}} \quad A_{\mu\nu}\tilde{A}_{\mu\nu}$	$\varphi_0 A_{\mu\nu}\tilde{A}_{\mu\nu}$
$\frac{\sqrt{6}}{f}$	0	$\frac{3\sqrt{2}}{f}$
$\eta_1 Z_{\mu\nu}\tilde{A}_{\mu\nu}$	$\eta_3 \quad \frac{(\Lambda_0+\Lambda_0^*)}{\sqrt{2}} \quad Z_{\mu\nu}\tilde{A}_{\mu\nu}$	$\varphi_0 Z_{\mu\nu}\tilde{A}_{\mu\nu}$
$\frac{2\sqrt{6}}{f\tan(2\theta_W)}$	0	$\frac{6\sqrt{2}}{f\tan(2\theta_W)}$
$\eta_1 Z_{\mu\nu}\tilde{Z}_{\mu\nu}$	$\eta_3 \quad \frac{(\Lambda_0+\Lambda_0^*)}{\sqrt{2}} \quad Z_{\mu\nu}\tilde{Z}_{\mu\nu}$	$\varphi_0 Z_{\mu\nu}\tilde{Z}_{\mu\nu}$
$\frac{\left(2+7\cos^2(\alpha)+3\cos(4\theta_W)\right)}{\sqrt{6}f\sin^2(2\theta_W)}$	$\frac{\sin^2(\alpha)}{f\sin^2(2\theta_W)}$	$\frac{\left(2-5\cos^2(\alpha)+3\cos(4\theta_W)\right)}{\sqrt{2}f\sin^2(2\theta_W)}$

Coefficients omit a prefactor $\frac{e^2 d_\psi}{48\pi^2}$; compared to SU(5)/SO(5), major difference arises from $\eta_{1,3}$, $\frac{\Lambda_0+\Lambda_0^*}{\sqrt{2}}$, φ_0, because of a more complicated singlet sector in SU(6)/SO(6) CHM.

pNGB potential

pNGB potentials $\Rightarrow$ vacuum misalignment and mass spectrum

The pNGB potential is generated by (a) gauge loops, (b) techni-fermion mass, (c) top couplings. Taylor expanding in pNGBs, it reads:

$$\mathcal{V}(\phi_i) = \mathcal{V}_0 + \frac{\partial \mathcal{V}}{\partial \phi_i}\phi_i + m^2_{\phi_i}\phi_i^2 + \cdots$$

Trigger EWSB

$\frac{\partial \mathcal{V}}{\partial \phi_i}|_{min} = 0$
(no tadpole)

$m^2_{\phi_i} > 0$
(no tachyons)

Assuming the first Higgs h develops VEV (just α angle):

$$\mathcal{V}_0(\alpha) = f^4/4\,(C_2\cos(2\alpha) + C_4\cos(4\alpha))$$ quartic term

we need tuning $C_2 \sim -4\,C_4$ to achieve $\alpha \sim v/f \ll 1$.

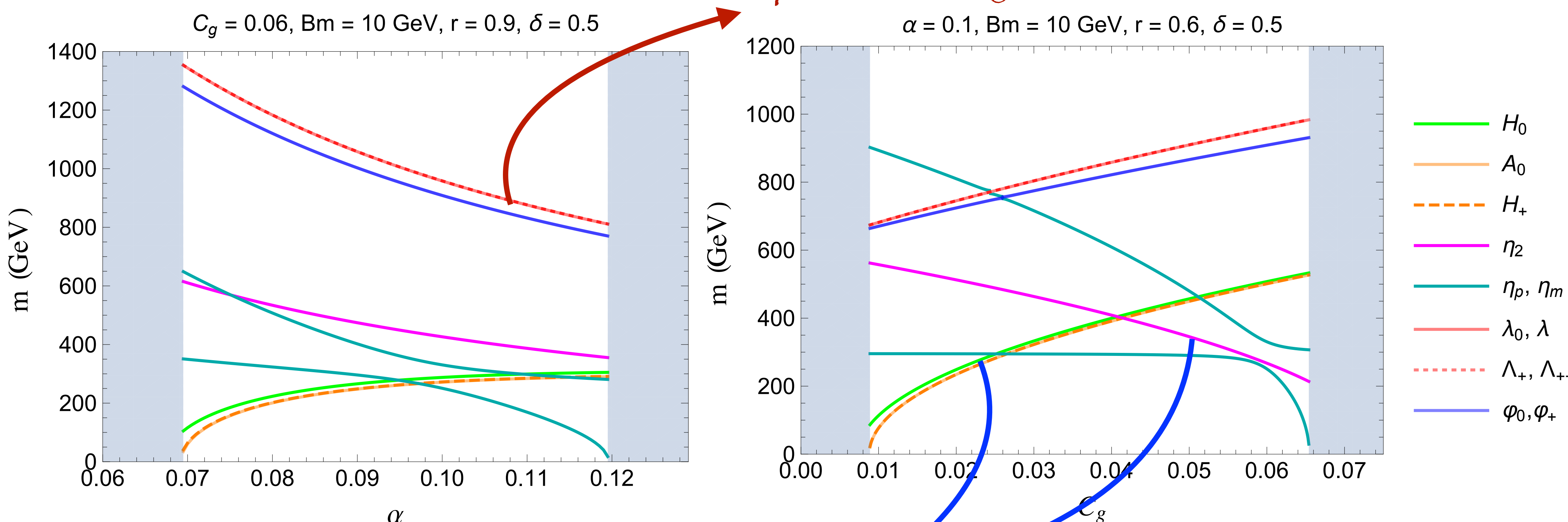
5 free parameters remain after imposing m_h^2 and minimum condition:

$$C_g\,, \quad Bm = B\frac{m_1+m_2}{2}\,, \quad \delta = \frac{m_2+m_1}{m_2+m_1}\,, \quad \alpha\,, \quad \text{and} \quad r = R'_S/R_S$$

Mass spectrum

Unmixed states: η_2, $H_{0,\pm}$, A_0, $\lambda_0 = \mathcal{I}(\Lambda_0)$ and Λ_{++}; Two mixed charged states: Λ_+, φ_+; Four mixed neutral states: $\lambda = \mathcal{R}(\Lambda_0)$, φ_0 and $\eta_{p,m}$ (from $\eta_{1,3}$).

Triplets normally heaviest



DM and CP-odd singlets are light states

The light blue area indicates the presence of tachyon (either A_0 or a singlet mostly aligned with η_m), thus not accessible.

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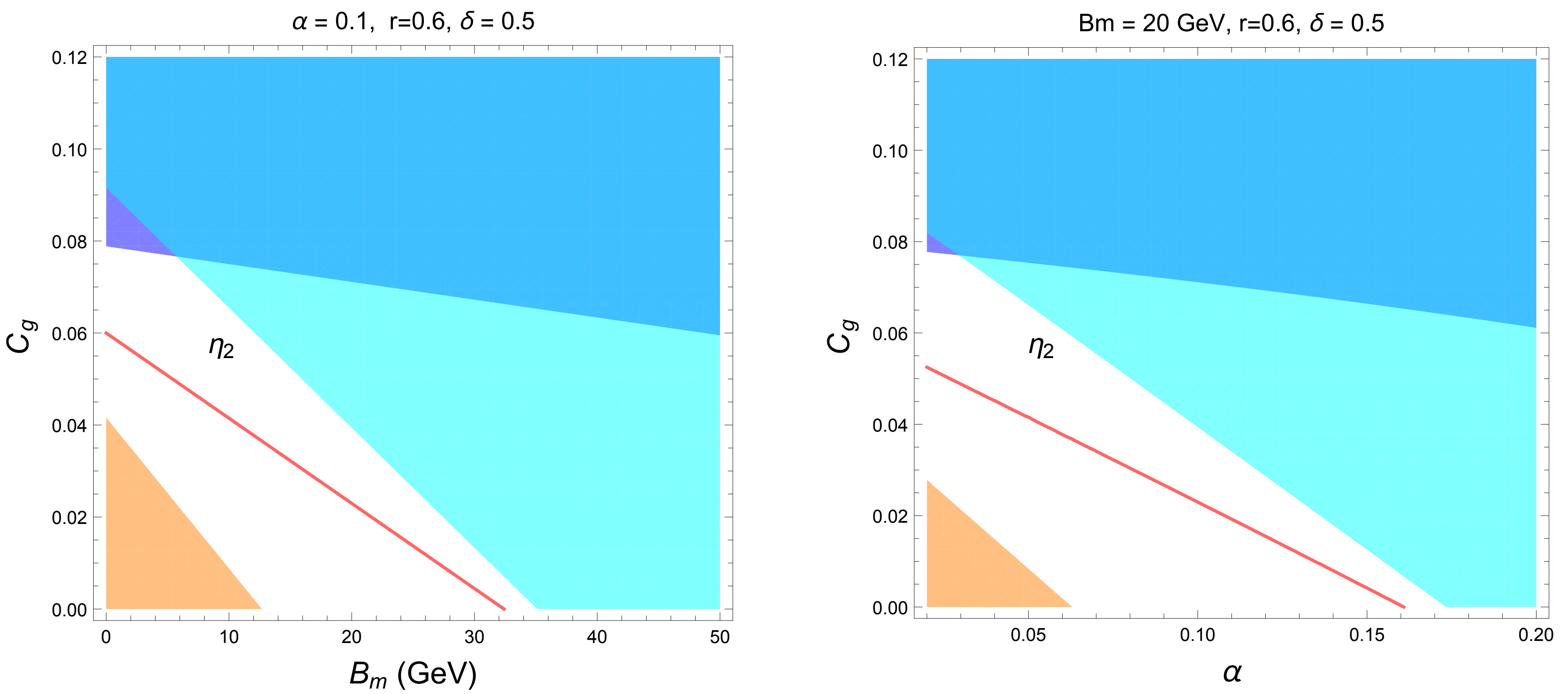
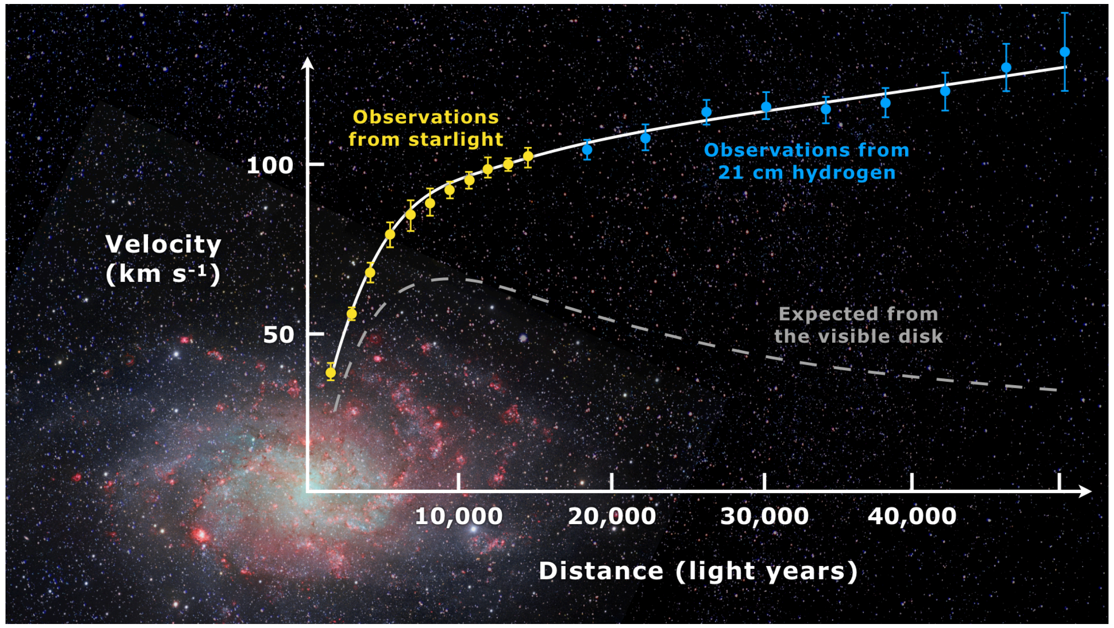
Allowed region

Tachyonic region is colored, blue from η_2, cyan from mixing neutral states, orange from the inert doublet. No constraint comes from mixing charged states.

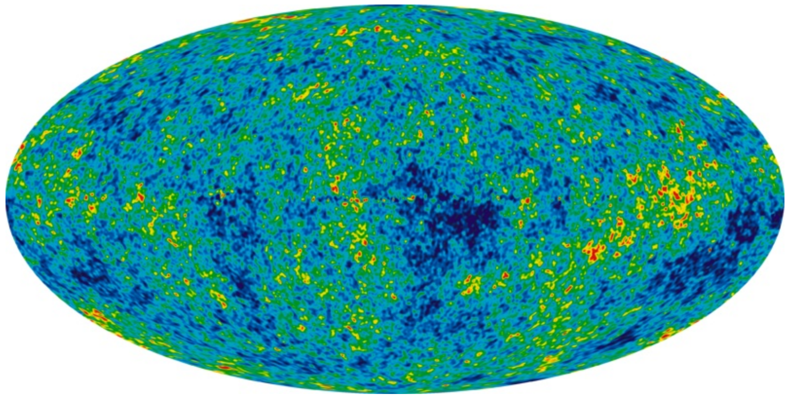


The red line signals the boundary where the DM candidate switches. The white region is permitted. Note that the part tagged with η_2 indicates that η_2 is DM in that parameter space.

Evidence of Dark Matter

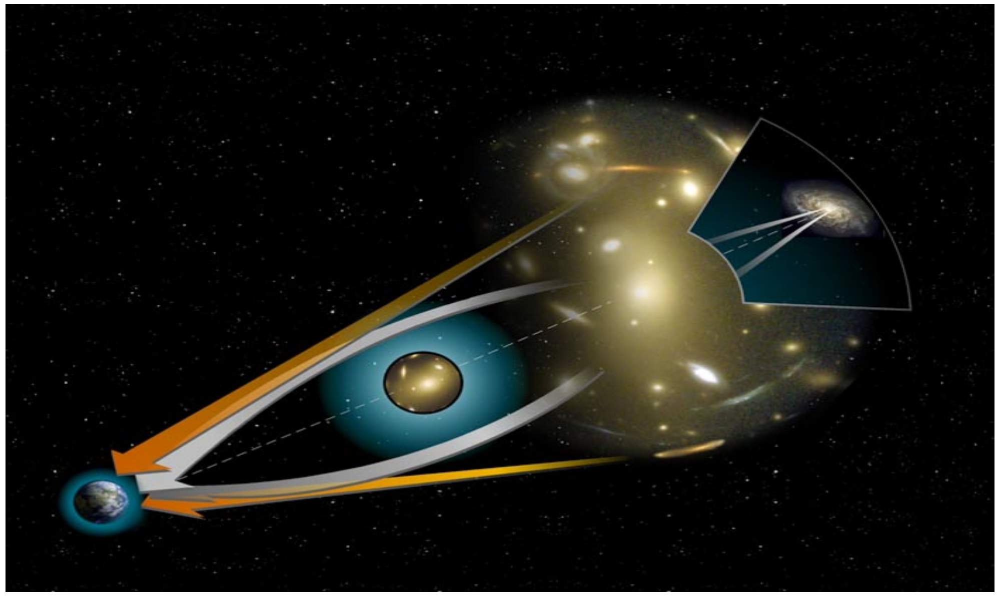


Galaxy rotation Curves



Comic microwave background

$$\Omega_{\rm DM} \sim 5\Omega_b$$



Gravitational Lensing

Gravitational Lensing

"The Bullet Cluster " originated from the collision of two sub-clusters, shows the spatial separation of dark matter from X-ray emitting plasma.

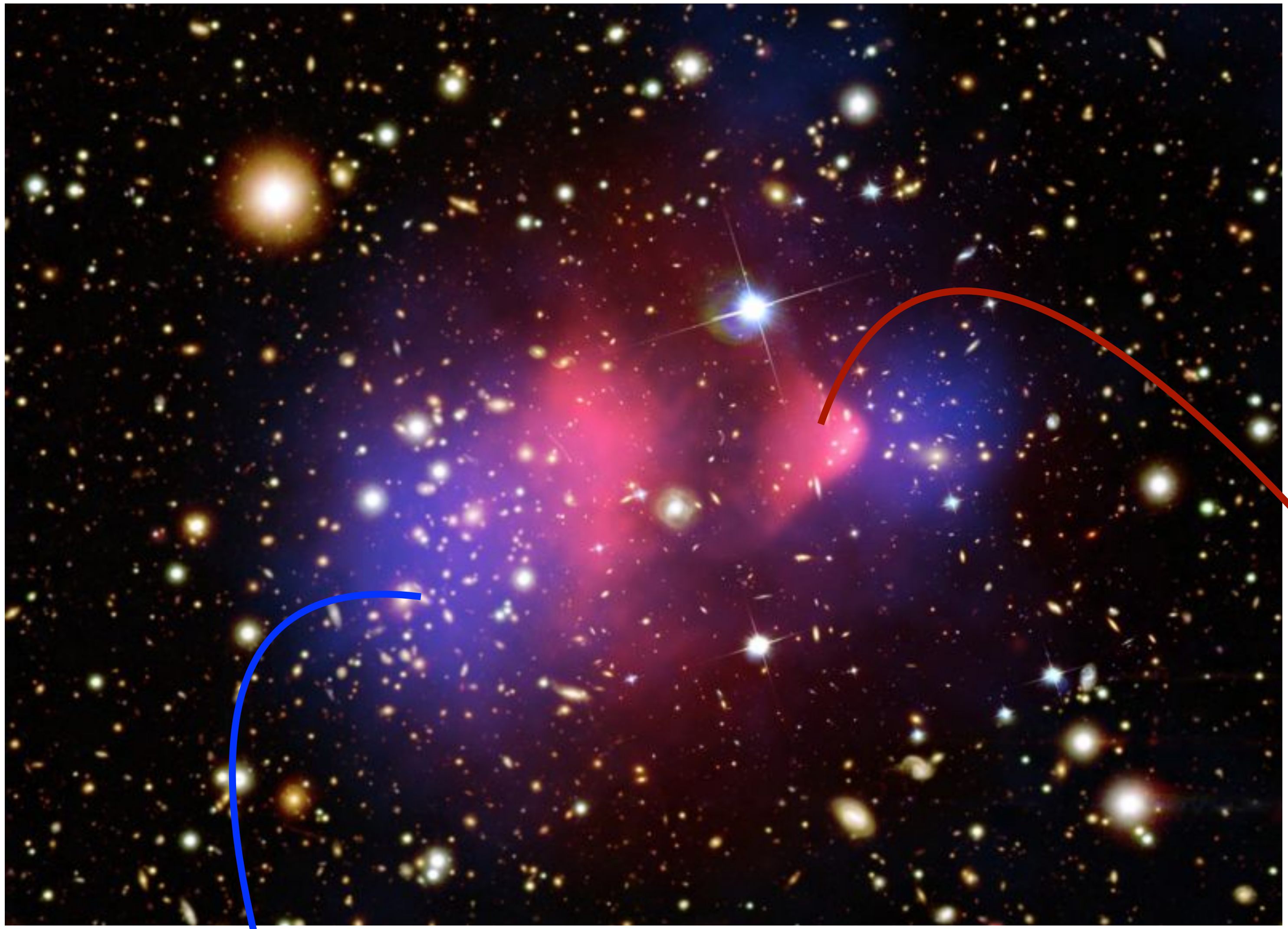


Image of plasma with Electromagnetic interactions

Image of DM via Gravitational Lensing

Clowe, Bradac, et.al
Astro-ph/0608407

Freeze out

Boltzmann Equation:

$$\frac{dn_X}{dt} + 3Hn_X = -\langle \sigma_{\rm eff} v \rangle [n_X^2 - n_{\rm eq}^2]$$

Hubble expansion   Collision rate

Freeze out condition:

$$\langle \sigma_{\rm eff} v \rangle_{x_f} n_{eq} = H(x_f), \quad x_f = m_X/T_f$$

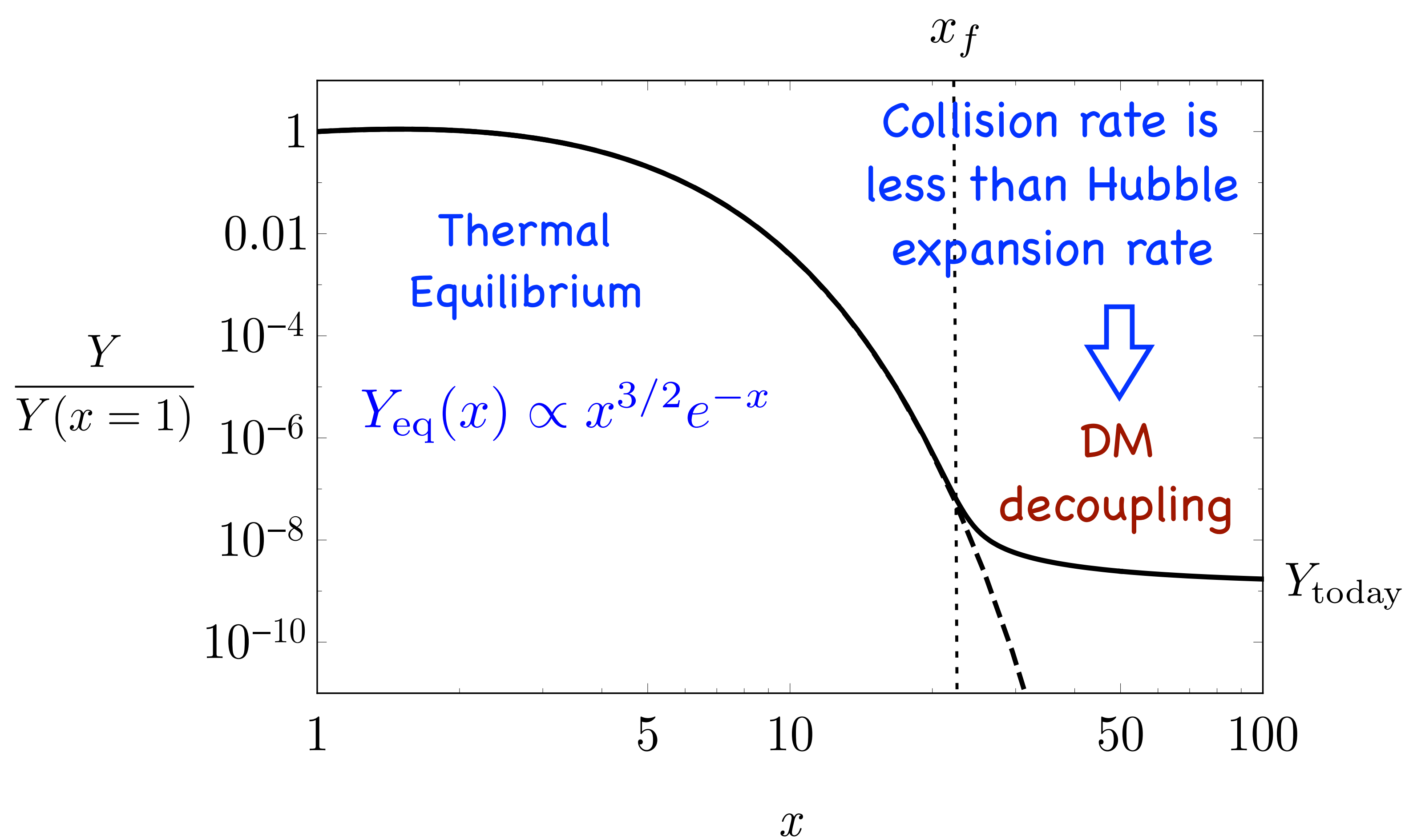
Number density

$$Y(x) = \frac{n(x)}{s(x)}$$

Entropy density

$$\Omega_X h^2 = m_X s_0 Y_0/\rho_c h^2 \propto \frac{1}{\langle \sigma_{\rm eff} v \rangle_{x_f}}$$

Relic density is inversely proportional to $\langle \sigma_{\rm eff} v \rangle_{x_f}$



Lisanti, arXiv:1603.03797

The vacuum structure

HC , Cacciapaglia
arXiv:2007.04338

Light quarks can only get mass from 4-fermion interactions. Both h & $\lambda_0 = \mathcal{I}(\Lambda_0)$ obtain VEVs.

$$\Sigma_{\alpha,\gamma}(x) = U_\gamma \cdot U_\alpha \cdot \tilde{\Sigma}(x) \cdot U_\alpha^T \cdot U_\gamma^T$$

$$\tilde{\Sigma}(x) = U_\gamma^\dagger \cdot \Sigma(x) \cdot U_\gamma^*$$

$$\begin{aligned}
\tilde{h} &= h\cos(\gamma) - \lambda_0 \sin(\gamma) \\
\tilde{\lambda}_0 &= h\sin(\gamma) + \lambda_0 \cos(\gamma) \\
\tilde{A}_0 &= A_0\cos(\gamma) - \eta_2 \sin(\gamma) \\
\tilde{\eta}_2 &= \eta_2\cos(\gamma) + A_0 \sin(\gamma) \\
\tilde{\Lambda}_+ &= \Lambda_+ \cos^2\left(\frac{\gamma}{2}\right) - \varphi_+ \sin^2\left(\frac{\gamma}{2}\right) \\
&- \frac{i}{\sqrt{2}} G_+ \sin(\gamma) \\
\tilde{\varphi}_+ &= \varphi_+ \cos^2\left(\frac{\gamma}{2}\right) - \Lambda_+ \sin^2\left(\frac{\gamma}{2}\right) \\
&- \frac{i}{\sqrt{2}} G_+ \sin(\gamma) \\
&\ldots\ldots
\end{aligned}$$

After the field redefinition, tadpole terms observe a structure:

$$\frac{1}{f}\frac{\partial V_0(\alpha,\gamma)}{\partial \alpha}\tilde{h} - \frac{1}{v}\frac{\partial V_0(\alpha,\gamma)}{\partial \gamma}\tilde{\lambda}_0$$

like Taylor expansions, tadpoles vanish affirmatively at the minimum.

There exist vertices: η_2-A_0-λ_0 and λ_0-$X_{\rm SM}$-$X_{\rm SM}$; The lighter one is DM.

The Bound of gamma

Using minimum conditions: $\frac{\partial V_0}{\partial \alpha} = \frac{\partial V_0}{\partial \gamma} = 0$

we can fix the γ in terms of α :

$$\gamma \simeq \frac{12\,\alpha\, m_b^2}{C_g m_W^2 (72 + 40 \tan^2(\theta_W)) + m_h^2}$$

For a non-zero gamma, the tree-level custodial symmetry breaking is :

$$\delta\rho_\gamma \simeq \frac{\sin^2(\gamma) + 1}{(2\cos(2\alpha) + 1)\sin^2(\gamma) + 1} - 1$$

Counting loop contributions to S & T gives:

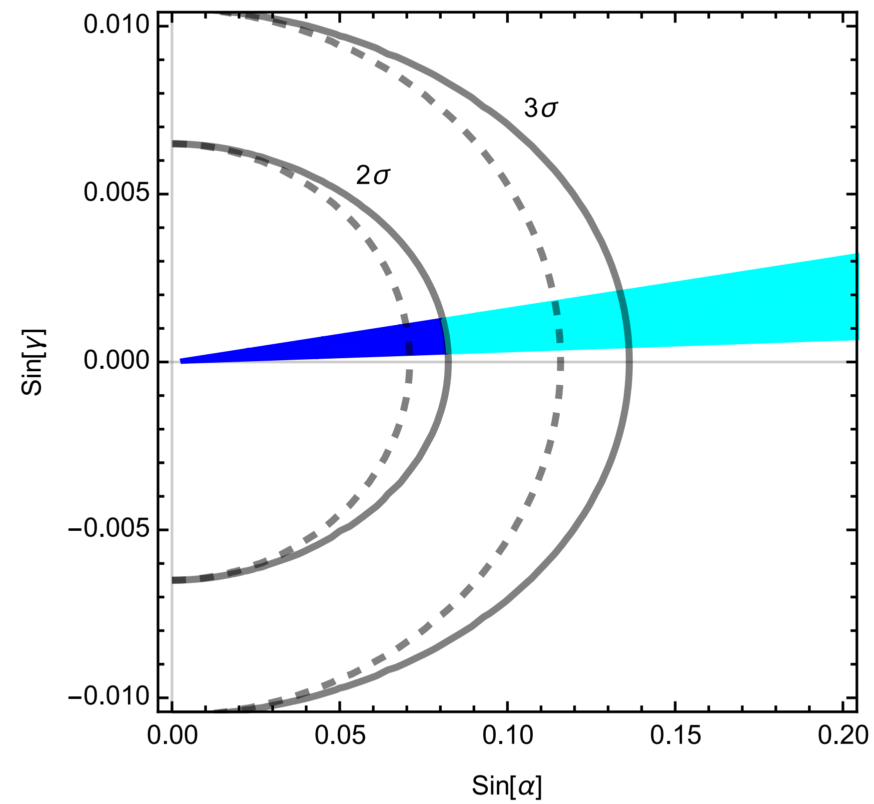
$$S = \frac{\sin^2\alpha}{6\pi}\left(\ln\frac{4\pi f}{m_h} + N_D\right)$$

Strong sector

$$T = -\frac{3\sin^2\alpha}{8\pi\cos^2\theta_W}\ln\frac{4\pi f}{m_h} + \frac{\delta\rho_\gamma}{\alpha_{em}}$$

Modified Higgs coupling

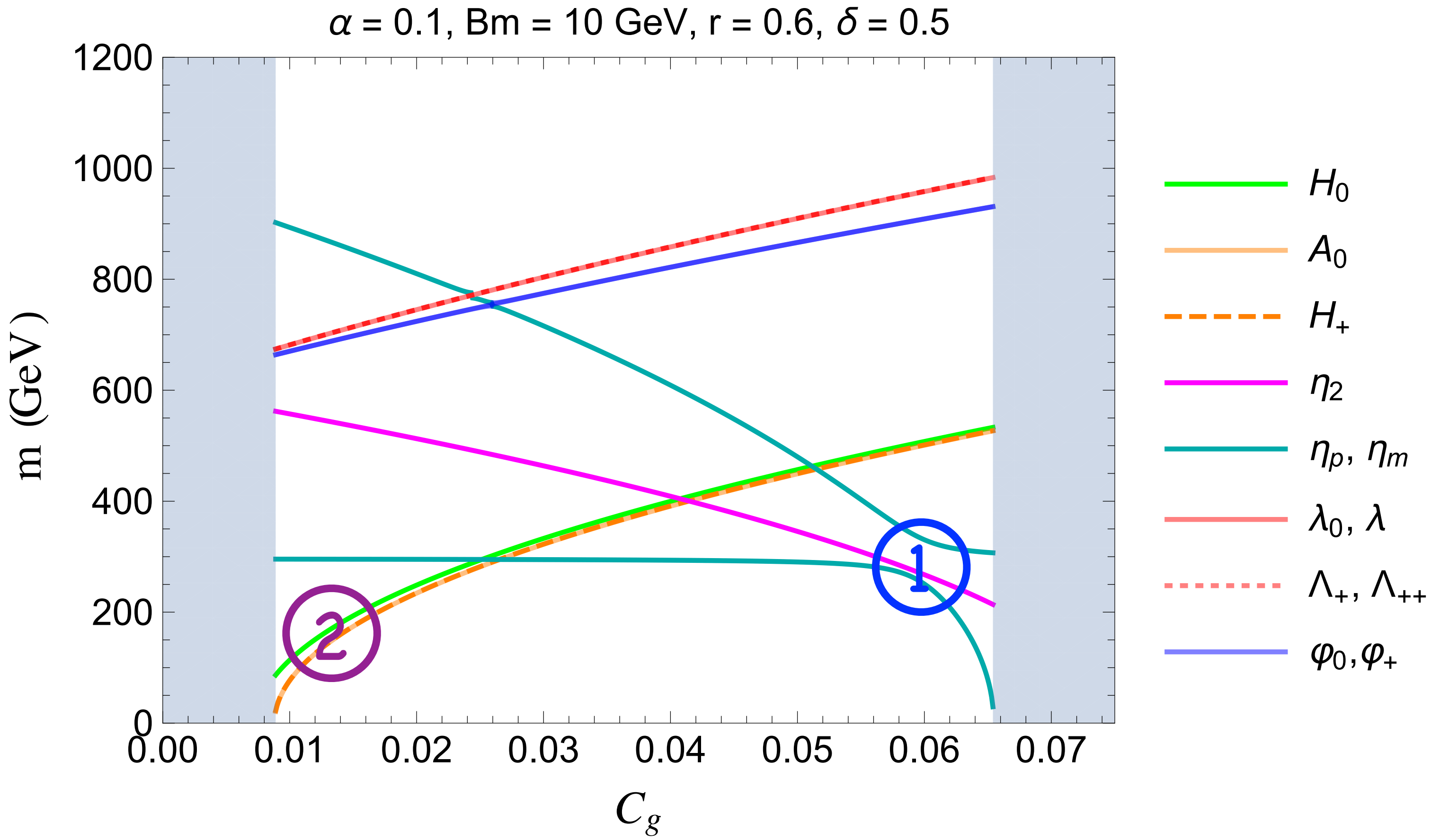
HC , Cacciapaglia
arXiv:2007.04338



At 3σ C.L., $\sin\alpha \lesssim 0.12$
$|\sin\gamma| \lesssim 0.01$

Theory gives: $\sin\gamma \lesssim 0.002$

Annihilation or Co-annihilation

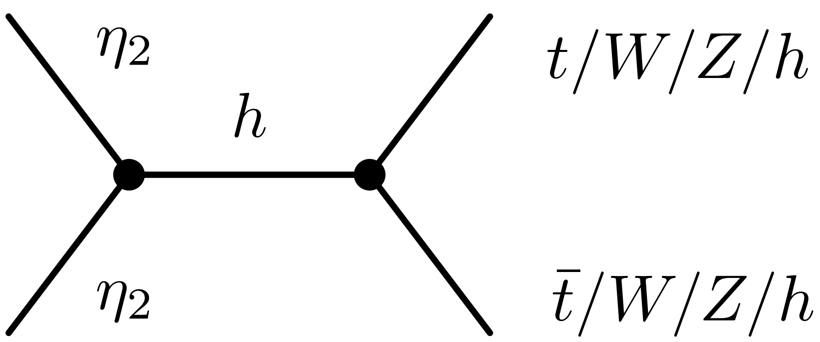


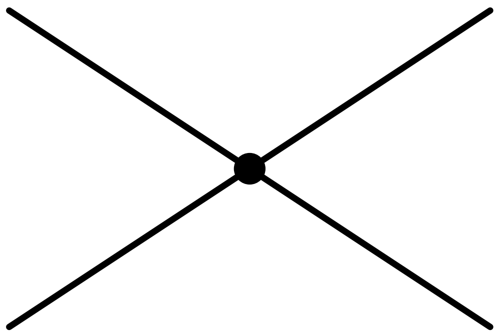
① Singlet annihilation

$m_{\eta_2} \ll m_{A_0}$

② Inert Higgs doublet co-annihilation

$m_{A_0,H_\pm} \ll m_{\eta_2}$





Nonlinear term in CHM

Direct Detection

Direct Detection experiments measure the recoil energy deposited by the scattering of DM into nucleus (A: mass number; Z: proton number).

$$\sigma_{\rm SI} = \frac{m_N^2}{\pi m_{\eta_2}^2}\left(\frac{m_N m_{\eta_2}}{m_N + m_{\eta_2}}\right)^2 \frac{(Zf_p + (A-Z)f_n)^2}{A^2} \qquad \left(\sigma^{\rm SI}_{\rm nucleus} = \frac{\mu_T^2}{\mu_N^2} A^2 \cdot \sigma_{\rm SI}\right)$$

$f_{p,n}$ characterise the interaction between DM and a nucleon; m_N : nucleon mass.

Nonlinearity

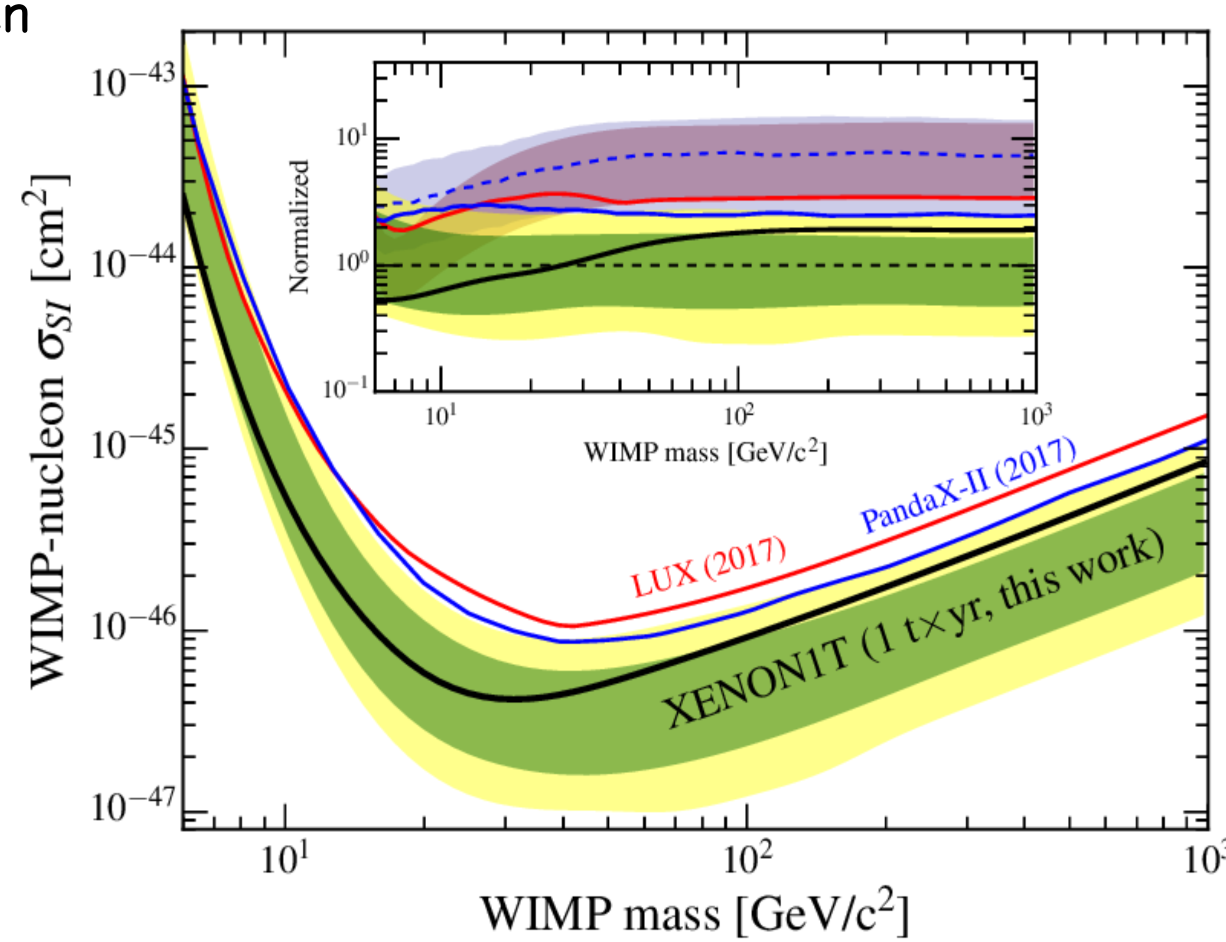
$$\mathcal{L} \supset \frac{c_q\, m_q}{f^2}\, \eta_2^2 \bar{q} q - \lambda_{h\eta_2^2}\, h\eta_2^2$$

$$f_{p,n} = \sum_q f_q^{(p,n)} \left[\frac{c_q}{f^2} + \frac{\lambda_{h\eta_2^2}}{v m_h^2}\right]$$

$\frac{m_q}{m_N}\langle N|\bar{q}q|N\rangle$ Form factor of quark content

m_{η_2}, c_q & $\lambda_{h\eta_2^2}$ are functions of (α, C_g, Bm, r).

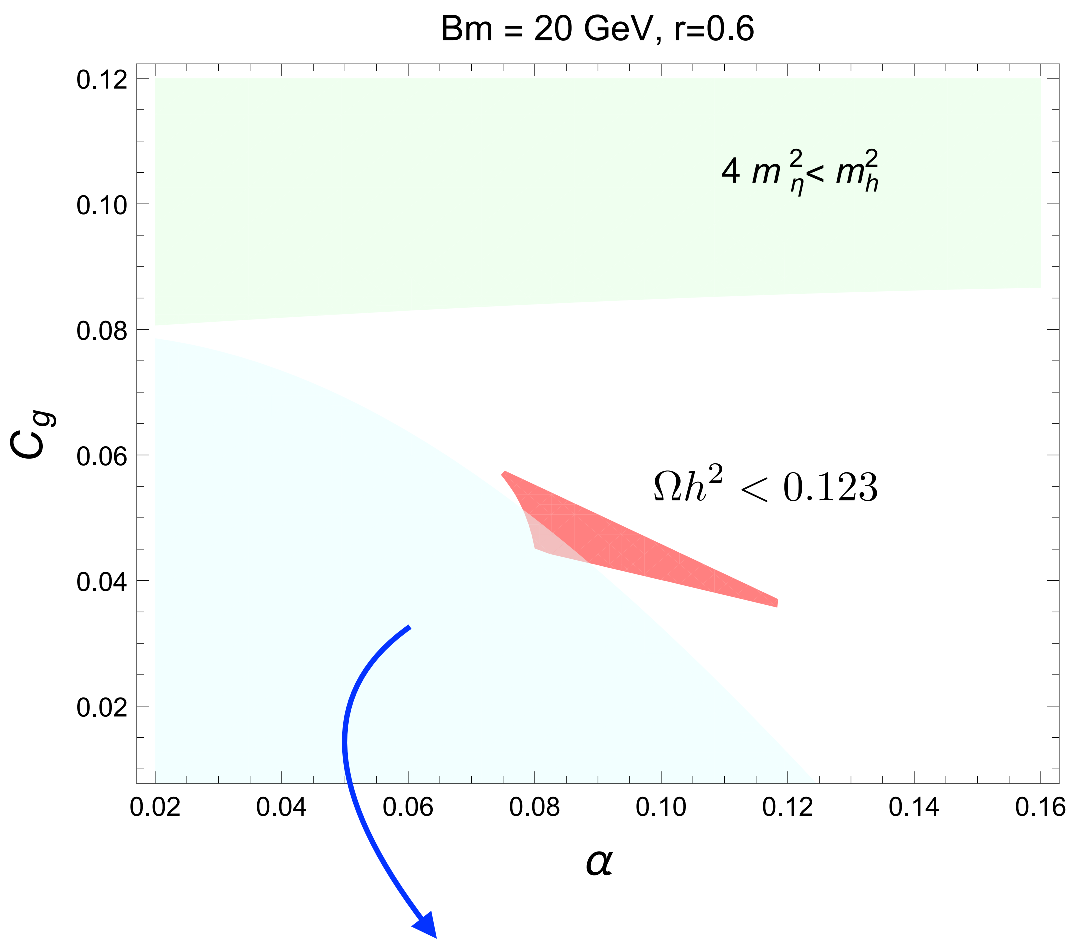
$\delta = 0$

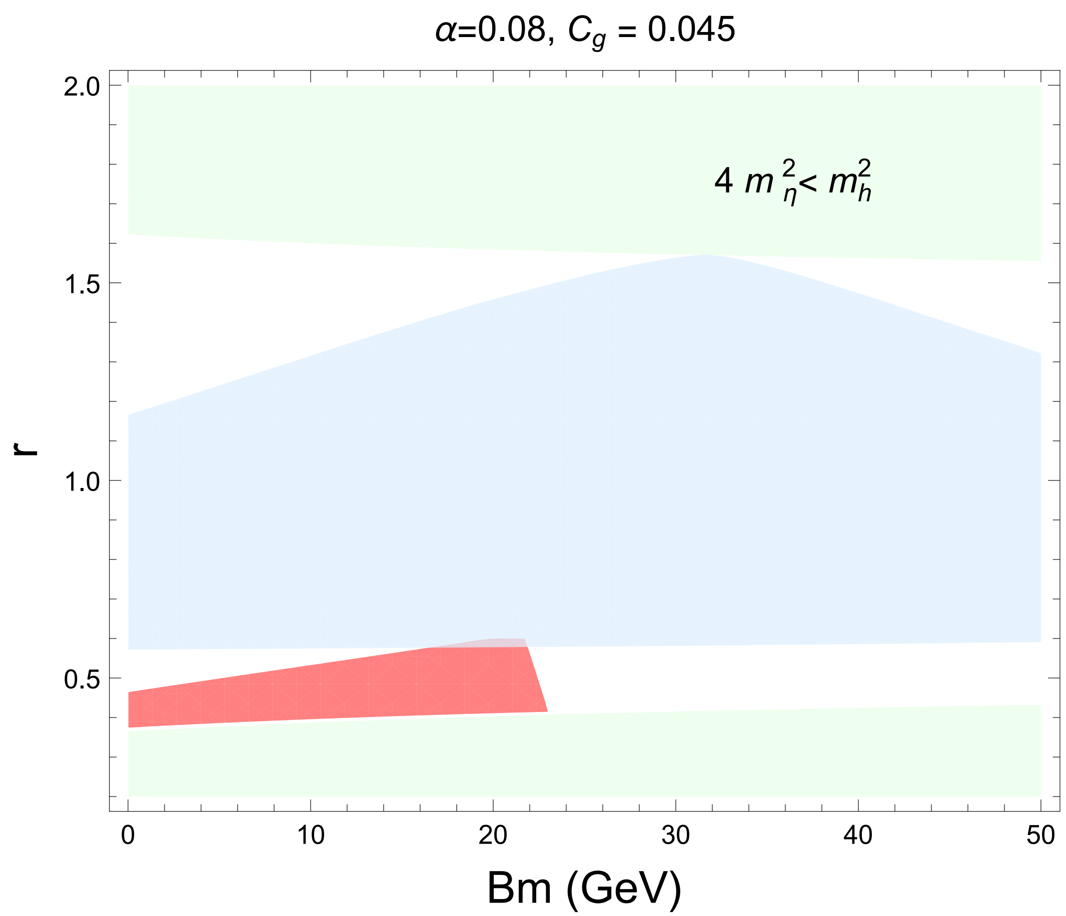


Aprile et al. (XENON) 2018

The viable DM region

HC, Cacciapaglia
arXiv:2007.04338





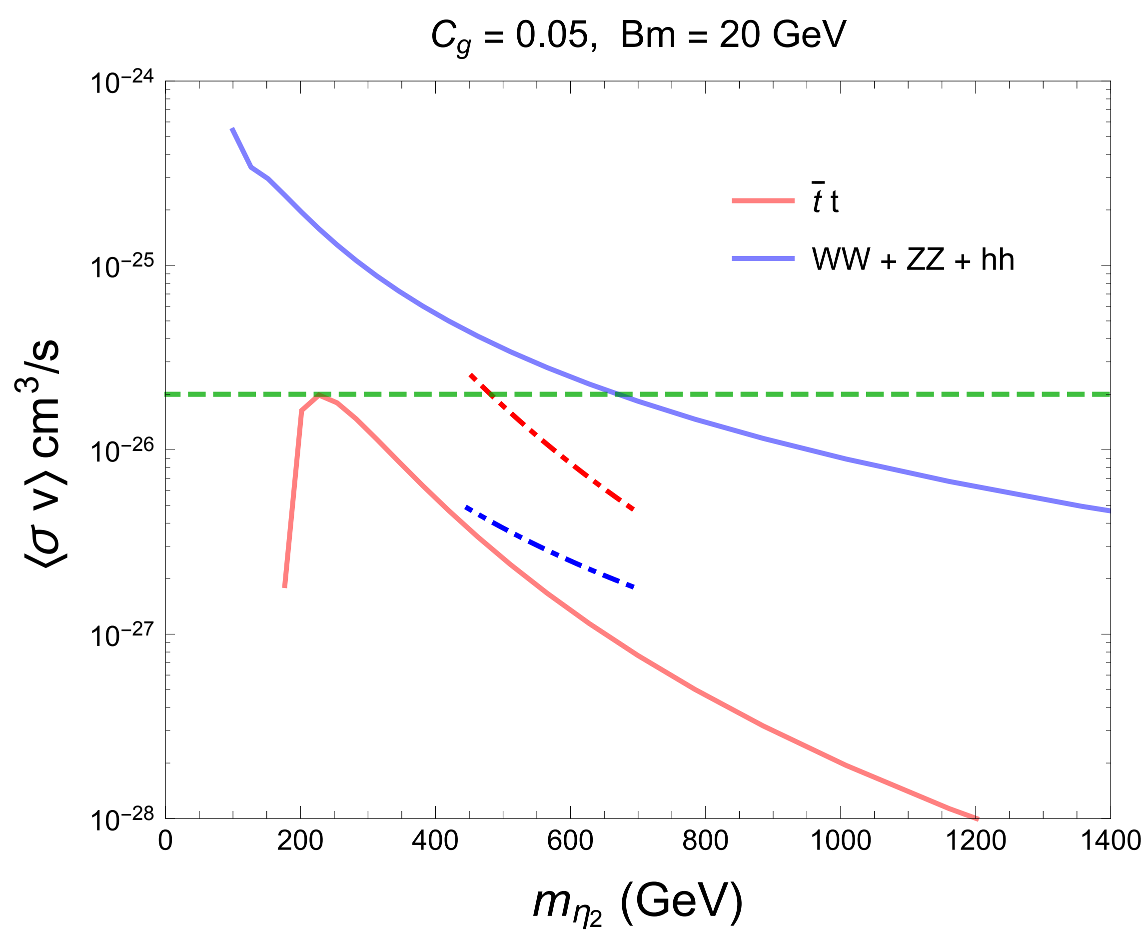
Direct Detection
allowed

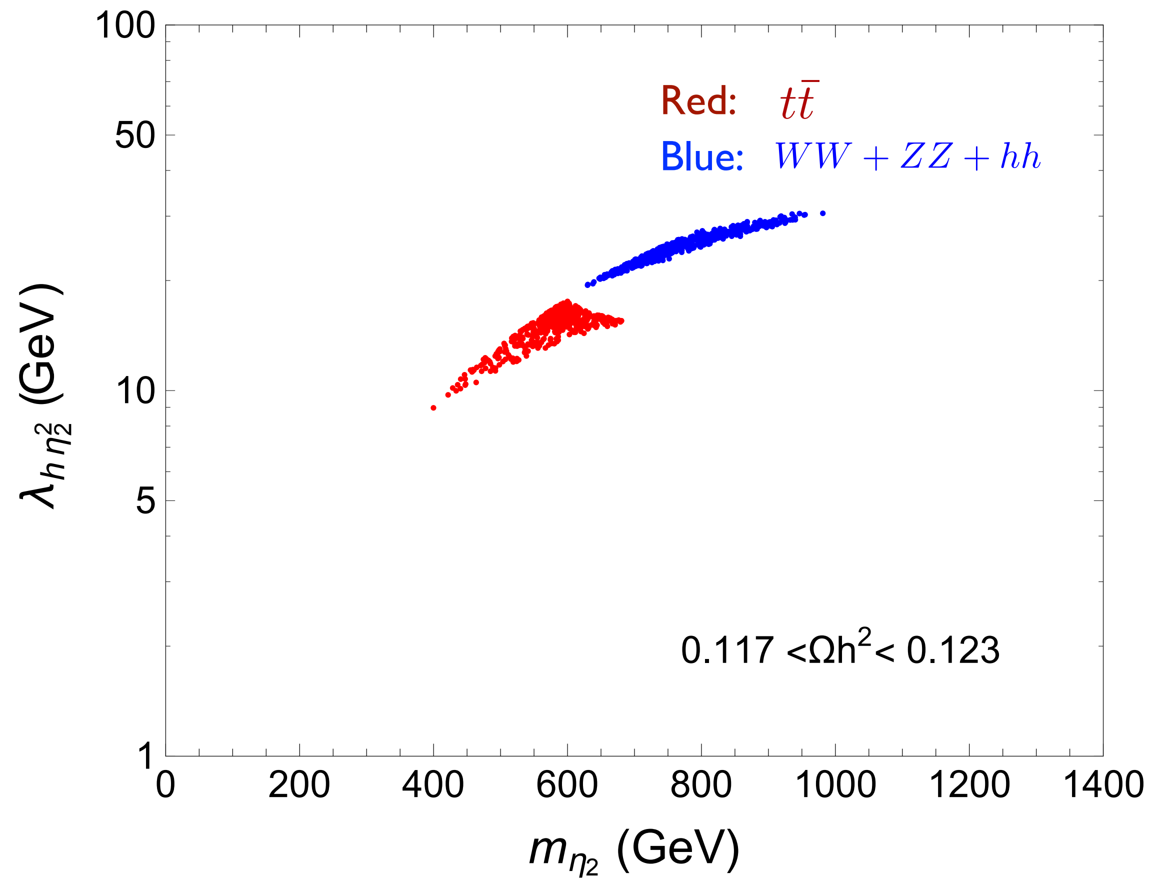
Only the overlap of red and
blue is viable, fairly limited.

The singlet DM mass

HC , Cacciapaglia
arXiv:2007.04338

Dashed line (r=0.4), Solid line (r=0.6)





$$\Omega h^2 \simeq 0.12 \Rightarrow \langle \sigma v \rangle \sim 2.0 \cdot 10^{-26} cm^3/s$$

The intersection with reference green line roughly gives DM mass.

$\lambda_{h\eta_2^2}$ is close to the upper limit of Xenon1T detection.

Conclusion

- Non-minimal CHM with DM candidates is an interesting direction for investigation (minimum CHM has no DM).

- For the SU(6)/SO(6) model, η_2 or A_0 can actually be a relic DM, as annihilation or co-annihilation depending on parameter regions.

- The gamma vacuum signaling custodial symmetry violation is proved to be in oder of : m_b^2/m_h^2 , consistent with EWPT. Light fermions can obtain masses from 4-fermion interactions.