Top-quark scale-and-scheme uncertainties in Higgs pair production at the LHC

LAPTh Thursday Seminar [remote]

June 11th, 2020

Julien Baglio

[with F. Campanario, S. Glaus, M. M. Mühlleitner, J. Ronca, M. Spira, and J. Streicher, EPJC 79 (2019) 459; JHEP 04 (2020) 181]

Outline

Introduction

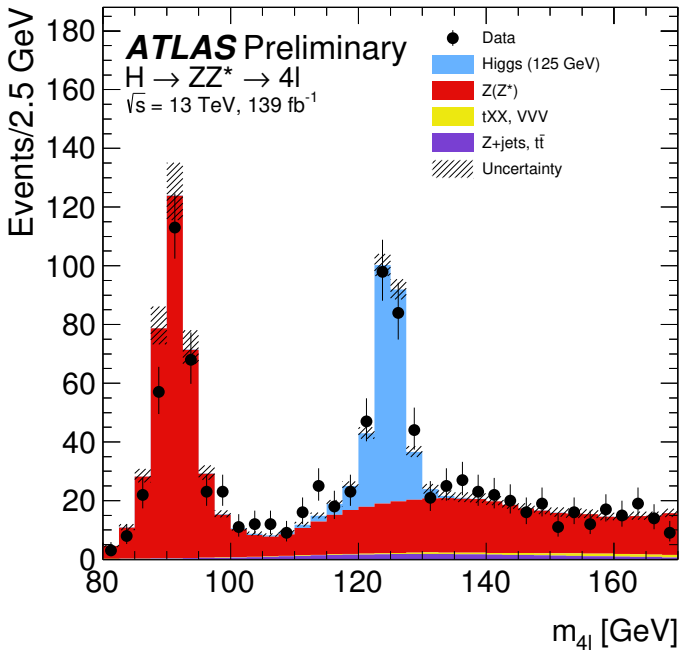
Full NLO QCD corrections to HH production

Top-quark scheme and scale uncertainties

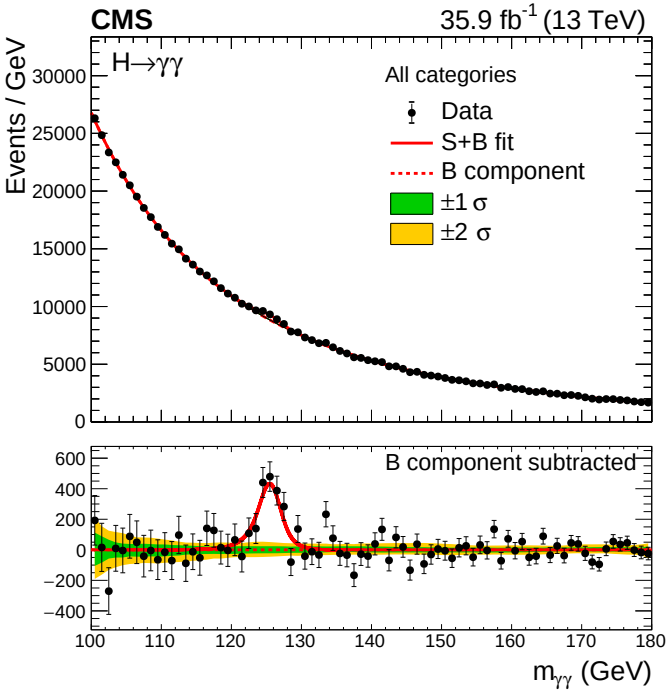
Outlook

Higgs physics facts in 2020

7/4/2012: CERN presents the discovery of a bosonic particle with Higgs-boson-like properties [ATLAS, PLB 716 (2012) 1; CMS *ibid* 30]



[ATLAS, ATLAS-CONF-2020-005]



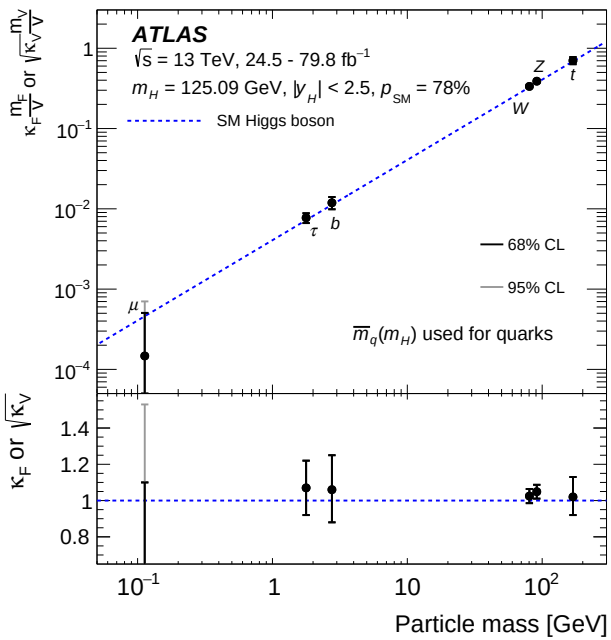
[CMS, PLB 805 (2020) 135425]

- Spin and CP-properties: Compatible with a CP-even spin 0 boson, $M_H \sim 125$ GeV: The first elementary scalar particle observed in Nature

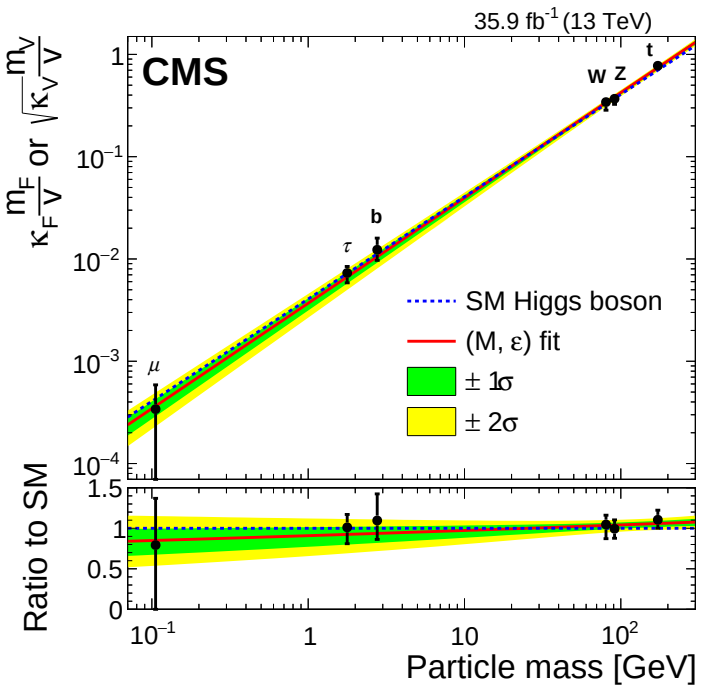
Higgs physics facts in 2020

7/4/2012: CERN presents the discovery of a bosonic particle with Higgs-boson-like properties [ATLAS, PLB 716 (2012) 1; CMS *ibid* 30]

- Couplings compatible with a Higgs boson



[ATLAS, PRD 101 (2020) 012002]



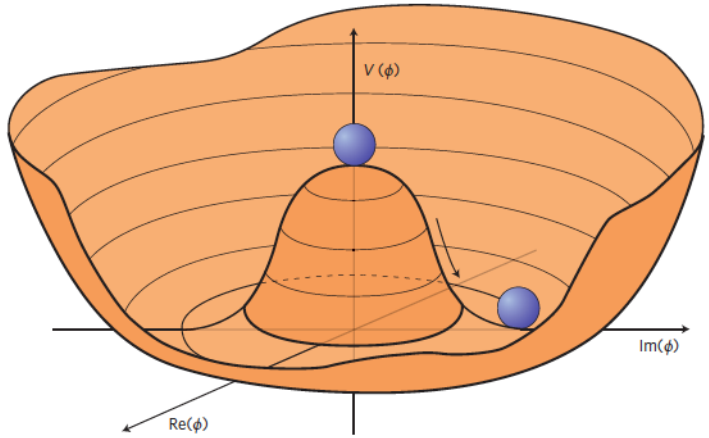
[CMS, EPJC 79 (2019) 421]

- **Still unknown: Higgs boson self-couplings**

A crucial SM test: Probing the scalar potential

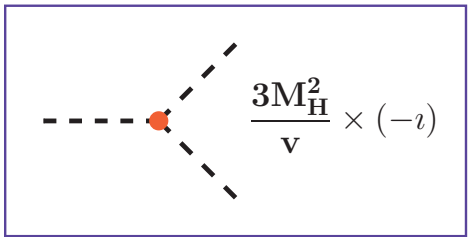
- **From the scalar potential before EWSB (ϕ as the Higgs field):**

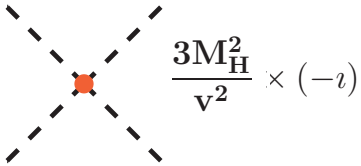
$$V(\phi) = -m^2|\phi|^2 + \lambda|\phi|^4$$



- **To $V(\phi)$ after EWSB, with $M_H^2 = 2m^2$, $v^2 = m^2/\lambda$:**

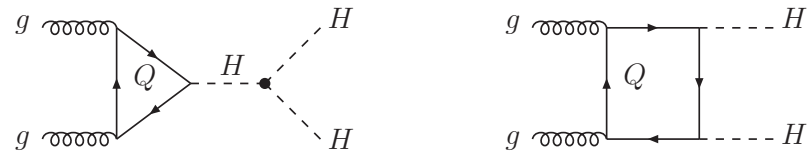
$$\phi = \begin{pmatrix} 0 \\ \dfrac{v + H(x)}{\sqrt{2}} \end{pmatrix} \Rightarrow V(H) = \frac{1}{2}M_H^2 H^2 + \frac{1}{2}\frac{M_H^2}{v}H^3 + \frac{1}{8}\frac{M_H^2}{v^2}H^4 + \text{constant}$$



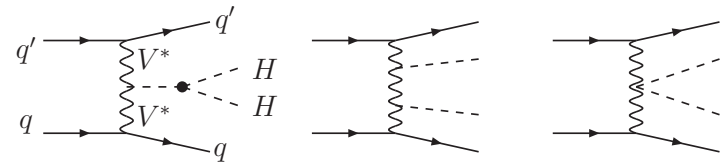


Overview of *HH* production channels

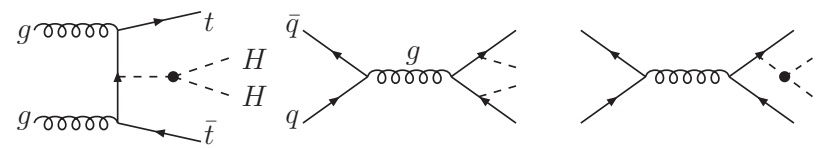
- **Gluon fusion: focus of the talk**
  Leading order (LO) QCD already loop-induced



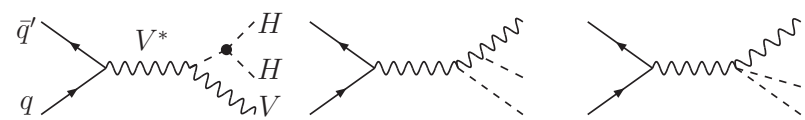
- **Vector boson fusion**
  NLO QCD [1,2], NNLO QCD [3], N^{3}LO QCD [4], NLO EW [5]



- **$t\bar{t}HH$ production** NLO QCD [2]

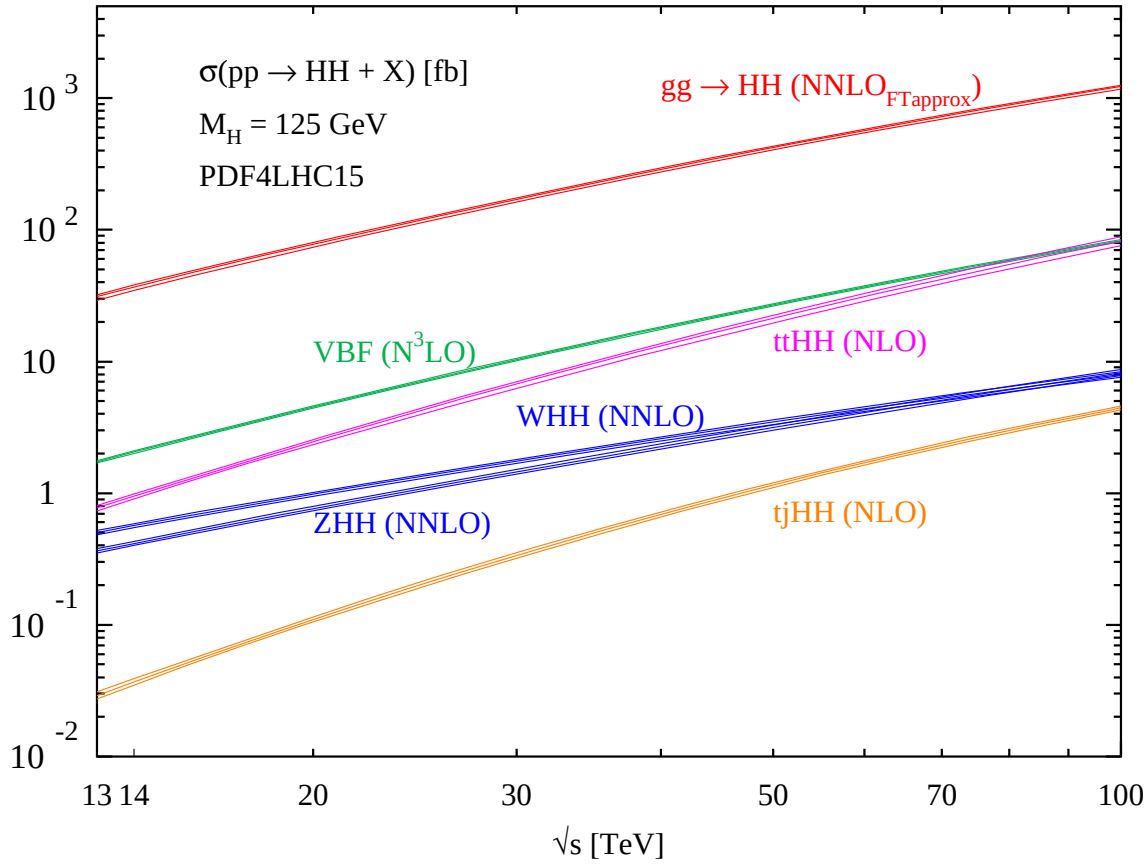


- **Double Higgs-strahlung**
  NLO QCD [1,2], NNLO QCD [1,6]



[1] **J.B.**, Djouadi, Gröber, Mühlleitner, Quevillon, Spira, JHEP 04 (2013) 151
[2] Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Torrielli, Vryonidou, Zaro, PLB 732 (2014) 142
[3] Ling, Zhang, Ma, Guo, W.H. Li, X.Z. Li, PRD 89 (2014) 073001
[4] Dreyer, Karlberg, PRD 98 (2018) 114016
[5] Dreyer, Karlberg, Lang, Pellen, arXiv:2005.13341
[6] H.T. Li, Wang, PLB 765 (2017) 265; H.T. Li, C.S. Li, Wang, PRD 97 (2018) 074026

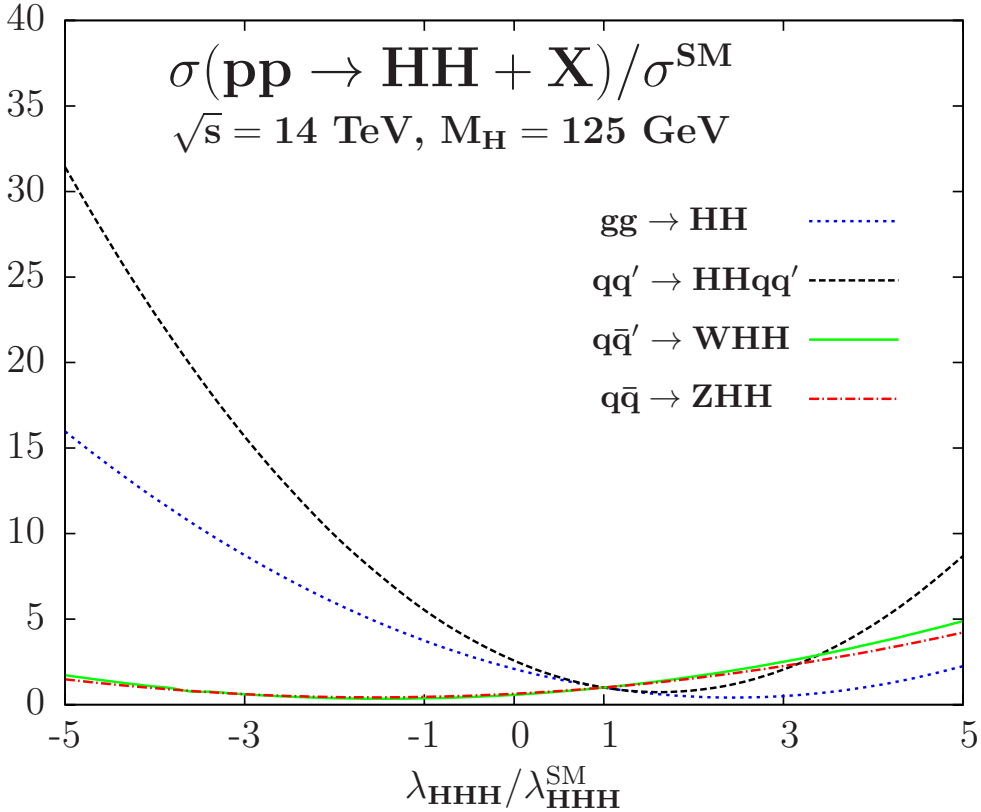
Overview *HH* production channels



[from Di Micco *et al*, arXiv:1910.00012]

Extracting the triple Higgs coupling

Assuming the SM is valid:



[**J.B.**, Djouadi, Gröber, Mühlleitner, Quevillon, Spira, JHEP 04 (2013) 151]

$$\frac{\Delta\sigma}{\sigma} \sim \frac{\Delta\lambda_{HHH}}{\lambda_{HHH}}$$

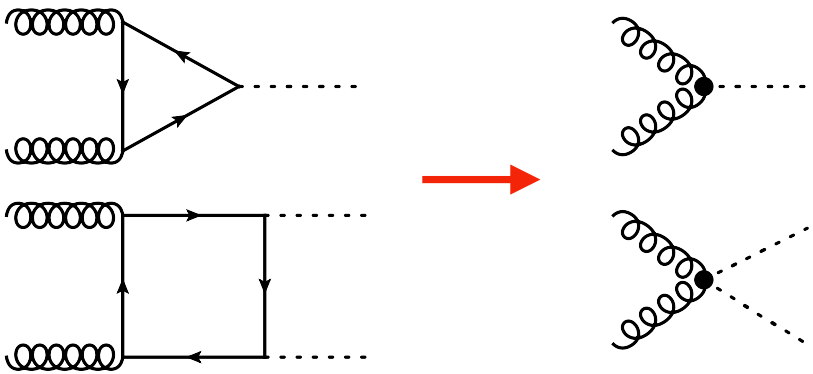
$\Rightarrow$ **Compulsory to get the cross section to high accuracy!**

Heavy top-quark limit (HTL) calculation

Gluon fusion: main production channel, top-quark loops dominant [Eboli, Marques, Novaes, Natale, PLB 197 (1987) 269; Glover, van der Bij, NPB 309 (1988) 282; Dicus, Kao, Willenbrock, PLB 203 (1988) 457]

$$\text{HTL} \equiv m_t \to +\infty$$

→ Effective tree-level *ggH* and *ggHH* couplings
→ Reduce the number of loop by one at each perturbative order



- HTL valid for $\hat{s} \ll 4m_t^2$, but *HH* production threshold $4M_H^2 \leq \hat{s}$
  ⇒ narrow energy range for which HTL is valid!
- **Born-improved NLO QCD HTL:** improve HTL result with

$$d\sigma_{\rm NLO} \simeq d\sigma_{\rm NLO}^{\rm HTL} \times \frac{d\sigma_{\rm LO}^{\rm full}}{d\sigma_{\rm LO}^{\rm HTL}}$$ [Dawson, Dittmaier, Spira, PRD 58 (1998) 115012]

Gluon fusion in the HTL: Status in 2020

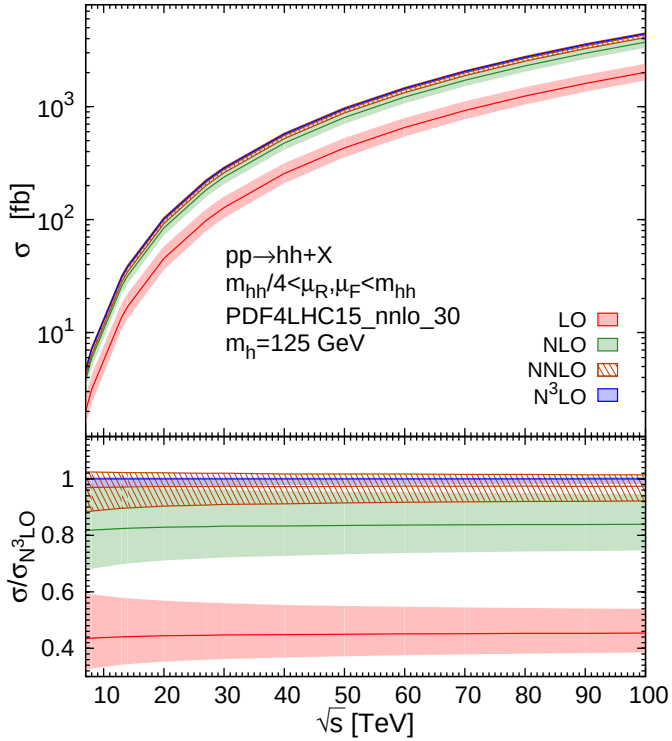
- **NLO QCD HTL (1-loop):** +93% correction

  [Dawson, Dittmaier, Spira, PRD 58 (1998) 115012]

- **NNLO QCD HTL (2-loop):** +20% on total cross section

  [De Florian, Mazzitelli, PLB 724 (2013) 306; PRL 111 (2013) 201801]

- **N^{3}LO QCD (3-loop):** +3% on total cross section



[Chen, Li, Shao, Wang, PLB 803 (2020) 135292]

Going beyond the HTL

In the quest for precision, going beyond the HTL is necessary

- **Toward full NLO QCD (2-loop):**
  $\rightarrow$ NLOFT$_{\text{approx}}$, m_t-effects in real radiation: -10%

  [Frederix *et al*, PLB 732 (2014) 079, Maltoni, Vryonidou, Zaro, JHEP 11 (2014) 079]

  $\rightarrow \mathcal{O}(1/m_t^{12})$ terms in virtual amplitudes: $\pm 10\%$

  [see e.g. Grigo, Hoff, Steinhauser, NPB 900 (2015) 412]
- **Full NLO QCD:** two independent results on the market, **see later in the talk!**
- **NNLO FT$_{\text{approx}}$:** full NLO QCD + NNLO HTL + NNLO exact reals $\Rightarrow$ $+10\%$ to $+20\%$ in distributions

  [Grazzini, Heinrich, Jones, Kallweit, Kerner, Lindert, Mazzitelli, JHEP 05 (2018) 059]
- **N^{3}LO beyond HTL:** full NLO QCD convoluted with N^{3}LO HTL $\Rightarrow$ $+3\%$ on top of the NNLO FT$_{\text{approx}}$ [Chen, Li, Shao, Wang, JHEP 03 (2020) 072]
- **Various analytical approximations:** Padé approximants, p_T^2-expansion, high-energy expansion [Gröber, Maier, Rauh JHEP 03 (2018) 020;

  Bonciani, Degrassi, Giardino, Gröber, PRL 121 (2018) 162003; Davies, Mishima, Steinhauser, Wellmann, JHEP 01 (2019) 176]

Full NLO QCD corrections to $gg \to HH$

The methods on the market

Two independent calculations on the market:

- Reduction to master integrals, sector decomposition, contour deformation [Borowka *et al*, PRL 117 (2016) 012001; JHEP 10 (2016) 107]
  - → Large mass effects in the tail up to $\sim -30\%$ w.r.t. HTL
  - → Born-improved HTL outside full NLO scale variation for $m_{HH} > 400$ GeV
  - → No quantitative statement on the top-quark scheme uncertainty

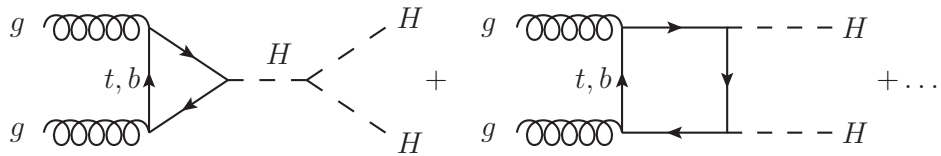
- Direct integration of the tensor integrals, integration-by-part, Richardson extrapolation [**J.B.**, Campanario, Glaus, Mühlleitner, Spira, Streicher, EPJC 79 (2019) 459; **J.B.**, Campanario, Glaus, Mühlleitner, Ronca, Spira, Streicher, JHEP 04 (2020) 181]
  - → **Same findings for the total cross section and invariant mass distributions**
  - → **Detailed study of the top-mass scale-and-scheme uncertainty**

Focus on the second method

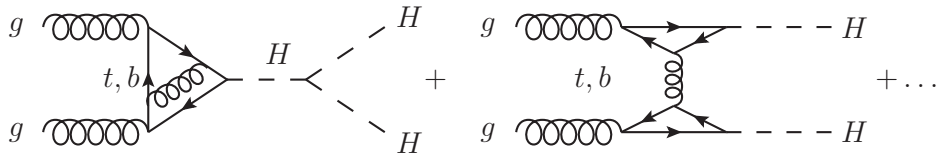
Overview of the calculation

$$\sigma_{\rm NLO}(pp \to HH + X) = \sigma_{\rm LO} + \Delta\sigma^{(1)}_{\rm virt} + \Delta\sigma^{(2)}_{\rm virt} + \Delta\sigma_{gg} + \Delta\sigma_{gq} + \Delta\sigma_{q\bar{q}}$$

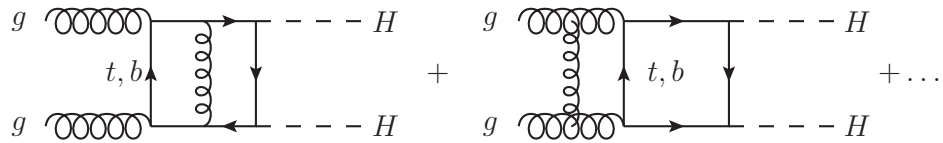
- **1-loop LO $\sigma_{\rm LO}$:**



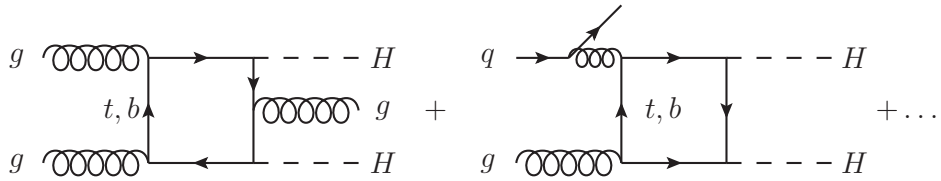
- **2-loop triangle + 1-particle reducible $\Delta\sigma^{(1)}_{\rm virt}$:**



- **2-loop box $\Delta\sigma^{(2)}_{\rm virt}$:**



- **1-loop reals $\Delta\sigma_{ij}$:**



Technical setup for the virtual corrections

- Triangle from single Higgs, 1-particle reducible analytically calculated

  [see also Degrassi, Giardino, Gröber, EPJC 76 (2016) 411]

- Classification of the **47 2-loop tensor box diagrams** into 6 topologies (+ corresponding fermion-flow reversed diagrams)

- With dimensional regularization $D = 4 - 2\epsilon$:
  calculate the **matrix-element form factors F_1 and F_2** using `FORM`/`Reduce`/`Mathematica` for $g(k_1)g(k_2) \to H(k_3)H(k_4)$,

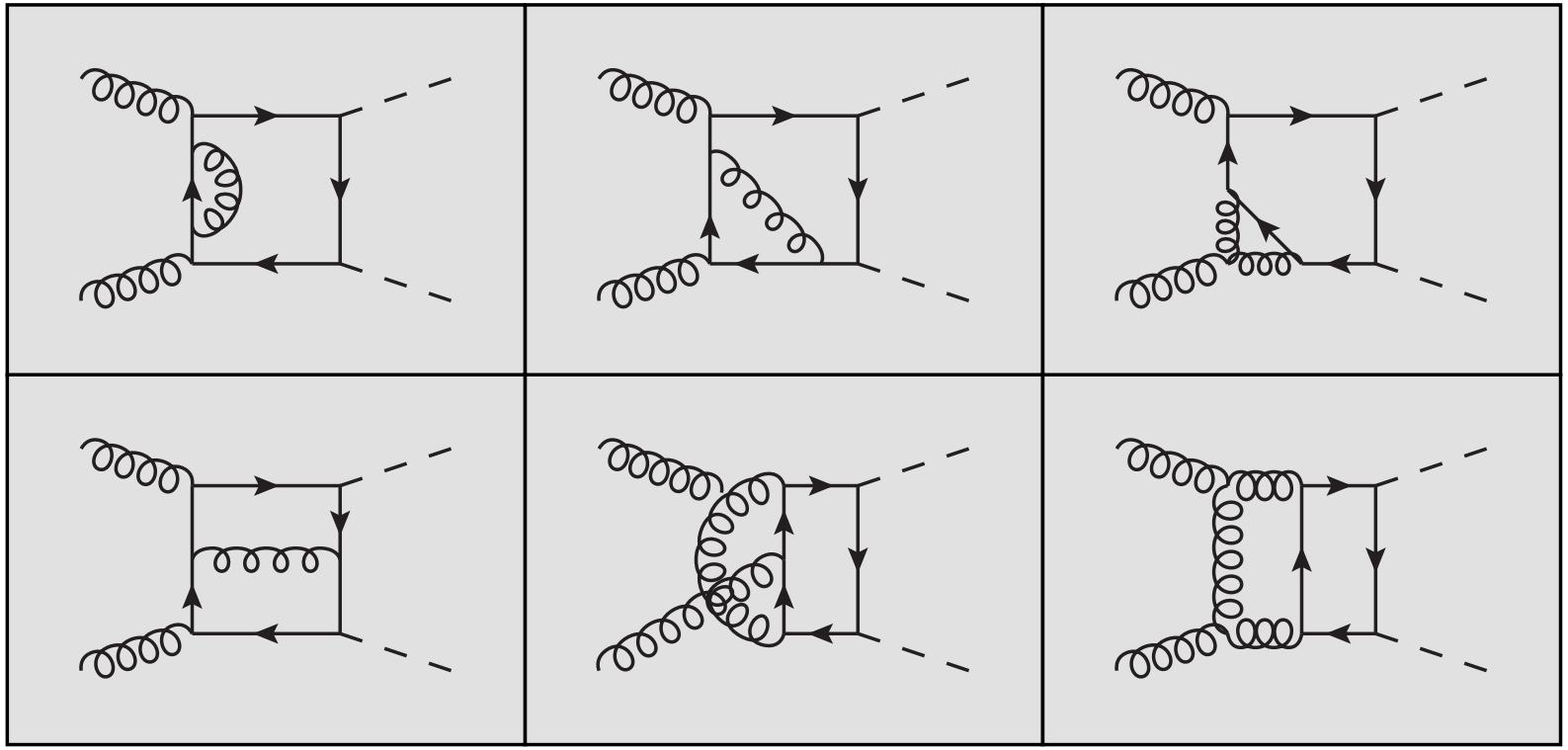
$$\mathcal{M} = \varepsilon^*_\mu(k_1)\varepsilon^*_\nu(k_2)\left(F_1\mathbf{T}_1^{\mu\nu} + F_2\mathbf{T}_2^{\mu\nu}\right),$$

$$\mathbf{T}_1^{\mu\nu} = g^{\mu\nu} - \frac{k_2^\mu k_1^\nu}{k_1\cdot k_2}, \qquad p_T^2 = 2\frac{(k_2\cdot k_3)(k_1\cdot k_3)}{k_1\cdot k_2} - k_3^2,$$

$$\mathbf{T}_2^{\mu\nu} = g^{\mu\nu} + \frac{k_2^\mu k_1^\nu}{(k_1\cdot k_2)p_T^2}k_3^2 - \frac{2}{p_T^2}\left[\frac{k_2\cdot k_3}{k_1\cdot k_2}k_3^\mu k_1^\nu + \frac{k_1\cdot k_3}{k_1\cdot k_2}k_2^\mu k_3^\nu - k_3^\mu k_3^\nu\right]$$

- **Perform Feynman parametrization**
  $\to$ 6-dimensional integrals to be (numerically) evaluated

2-loop virtual box corrections



2-loop virtual box corrections

- **Extraction of ultraviolet (UV) divergences:**
  Endpoint subtraction of the Feynman integrals

$$\int_0^1 dx \, \frac{f(x)}{(1-x)^{1-\epsilon}} = \frac{f(1)}{\epsilon} + \int_0^1 dx \, \frac{f(x)-f(1)}{1-x} + \mathcal{O}(\epsilon)$$

- **Infrared (IR) divergences in the middle of the range**
  $\Rightarrow$ Subtraction of the integrand and analytical integration
  - Generic denominator $N = ar^2 + br + c$, $N_0 = br + c$
  - Singular infrared behavior in the limit $r \to 0$
  - $a, c = \mathcal{O}\left(1/m_t^2\right), \quad b = 1 + \mathcal{O}\left(1/m_t^2\right)$

$$\int_0^1 dxdr \, \frac{rH(x,r)}{N^{3+2\epsilon}} = \int_0^1 dxdr \left[\left(\frac{rH(x,r)}{N^{3+2\epsilon}} - \frac{rH(x,0)}{N_0^{3+2\epsilon}} \right) + \frac{rH(x,0)}{N_0^{3+2\epsilon}} \right]$$

Handling threshold instabilities

- **Threshold at $\hat{s} = m_{HH}^2 = 4m_t^2$:**
  $\Rightarrow$ Analytical continuation in the complex plane with

$$m_t^2 \to m_t^2\,(1 - i\tilde{\epsilon})\,, \quad \tilde{\epsilon} \ll 1$$

- **Enhance stability above threshold with integration by parts** Example with $N = a + bx$:

$$\int_0^1 dx\,\frac{2b\,f(x)}{N^3} = \frac{f(0)}{a^2} - \frac{f(1)}{(a+b)^2} + \int_0^1 dx\,\frac{f'(x)}{N^2}$$

- For b-quark loop, same game but with more integration by parts (b-quark loop left for future work)

UV renormalization and IR subtraction

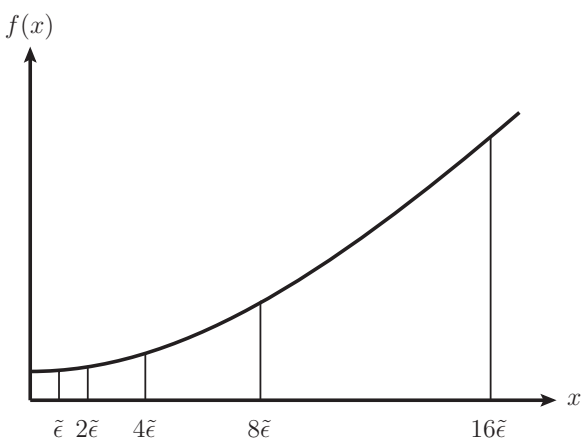
- **α_s and m_t input parameters to renormalize**
  - $\rightarrow$ $\overline{\text{MS}}$ renormalization for α_s with 5 active flavors, δ_{α_s}
  - $\rightarrow$ Top-quark contribution to the external gluon self-energies, δ_g
  - $\rightarrow$ On-shell renormalization for m_t, δ_{m_t}

- **IR subtraction, δ_{IR}:**

  Subtraction of Born-improved HTL virtual corrections to box diagrams $\Rightarrow$ IR-safe virtual mass-effects

Richardson extrapolation

- **Goal: From $m_t^2\,(1 - i\tilde{\epsilon})$, obtain the limit $\tilde{\epsilon} \to 0$**
- **Solution: Richardson extrapolation of the result!**
  Assuming $f(\tilde{\epsilon}) - f(0)$ polynomial for small $\tilde{\epsilon}$, method to accelerate the convergence of $f(\tilde{\epsilon})$ to $f(0)$



$$\mathrm{RiEx}_{2,\tilde{\epsilon}} = 2f(\tilde{\epsilon}) - f(2\tilde{\epsilon}) = f(0) + \mathcal{O}(\tilde{\epsilon}^2)$$

$$\mathrm{RiEx}_{3,\tilde{\epsilon}} = \frac{1}{3}\Big[8f(\tilde{\epsilon}) - 6f(2\tilde{\epsilon}) + f(4\tilde{\epsilon})\Big] = f(0) + \mathcal{O}(\tilde{\epsilon}^3)$$

$$\begin{aligned}\mathrm{RiEx}_{4,\tilde{\epsilon}} &= \frac{1}{21}\Big[64f(\tilde{\epsilon}) - 56f(2\tilde{\epsilon}) + 14f(4\tilde{\epsilon}) - f(8\tilde{\epsilon})\Big] \\ &= f(0) + \mathcal{O}(\tilde{\epsilon}^4)\end{aligned}$$

$$\begin{aligned}\mathrm{RiEx}_{5,\tilde{\epsilon}} &= \frac{1}{315}\Big[1024f(\tilde{\epsilon}) - 960f(2\tilde{\epsilon}) + 280f(4\tilde{\epsilon}) \\ &\qquad - 30f(8\tilde{\epsilon}) + f(16\tilde{\epsilon})\Big] \\ &= f(0) + \mathcal{O}(\tilde{\epsilon}^5)\end{aligned}$$

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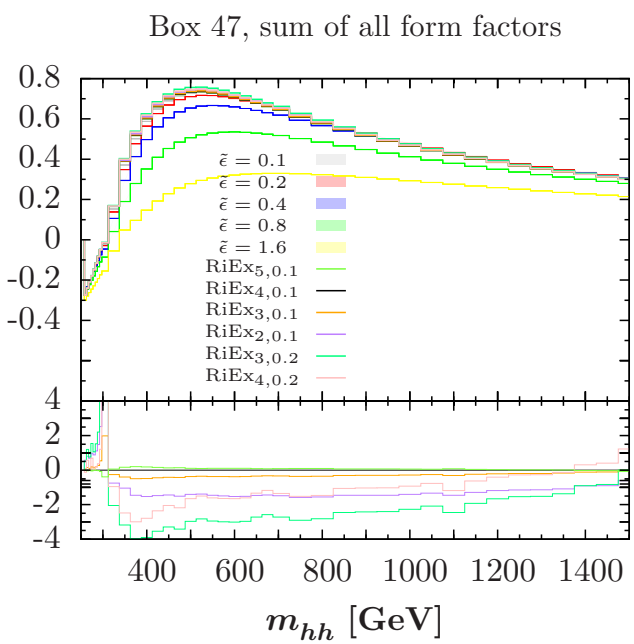

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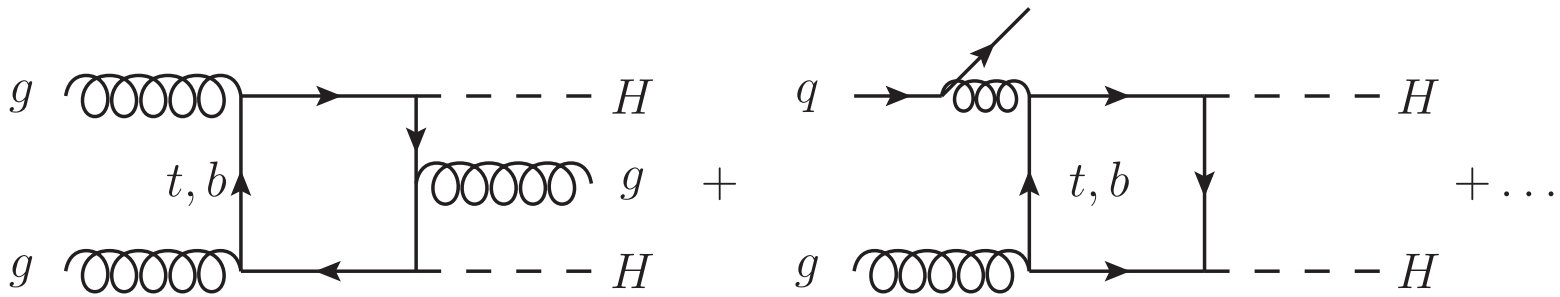
Calculation of the real corrections

Partonic sub-processes $gg \to HHg$, $gq/\bar{q} \to HHq/\bar{q}$, $q\bar{q} \to HHg$



- Full matrix elements generated with `FeynArts/FormCalc` [see Hahn, PoS ACAT2010 (2010) 078], evaluated with 1-loop library `COLLIER` [Denner, Dittmaier, Hofer, CPC 212 (2017) 220]
- Then subtracted with Born-improved HTL matrix-element squared calculated analytically
  $\Rightarrow$ **IR safe mass effects in the reals**

Putting everything together

- Numerical integration performed with VEGAS on a cluster, $\hat{t}$-integration
- **Final hadronic result:**

$$\Delta\hat{\sigma}_{\rm virt} = \int d\Phi_{2\to 2}\left[(\delta_{\alpha_s} + \delta_g + \delta_{m_t} + \delta_{\rm IR} + \mathcal{M}^{\square}_{\rm virt})(\mathcal{M}_{\rm LO})^*\right] + \Delta\hat{\sigma}^{\triangle}_{\rm virt} + \Delta\hat{\sigma}^{\rm 1PR}_{\rm virt}$$

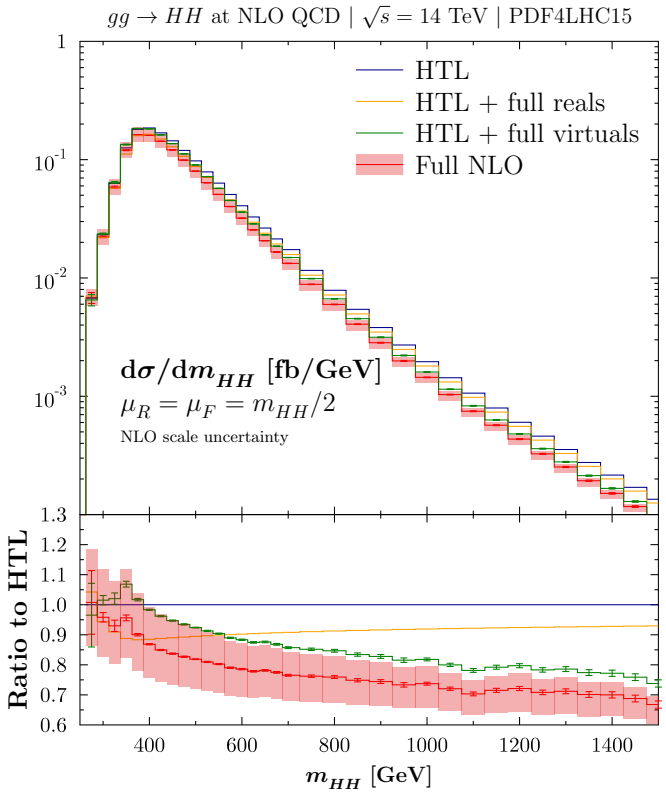
With $Q^2 = m^2_{HH}$: $$Q^2\frac{d\Delta\sigma_{\rm virt}}{dQ^2} = \tau\frac{d\mathcal{L}^{gg}}{d\tau}\Delta\hat{\sigma}_{\rm virt}(Q^2)\Big|_{\tau=\frac{Q^2}{s}}$$ $\frac{d\mathcal{L}^{gg}}{d\tau} \equiv$ gluon parton density

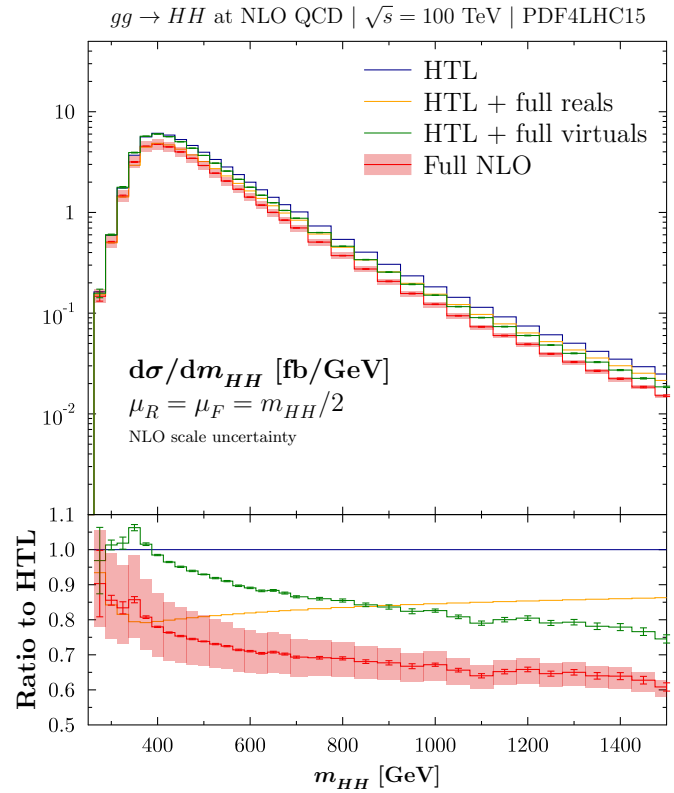
$$Q^2\frac{d\sigma_{\rm NLO}}{dQ^2} = Q^2\frac{d\sigma_{\rm HPAIR}}{dQ^2} + Q^2\frac{d\Delta\sigma_{\rm virt}}{dQ^2} + Q^2\frac{d\Delta\sigma_{\rm reals}}{dQ^2}$$

HTL hadronic result calculated with HPAIR [Spira, 1996]

- **Input parameters: can be freely chosen!** PDG values for M_W and M_Z, $M_H = 125$ GeV, $m_t = 172.5$ GeV, $G_F = 1.1663787 \times 10^{-5}$ GeV^{-2}, $\sqrt{s} = 14$ TeV

Differential cross section

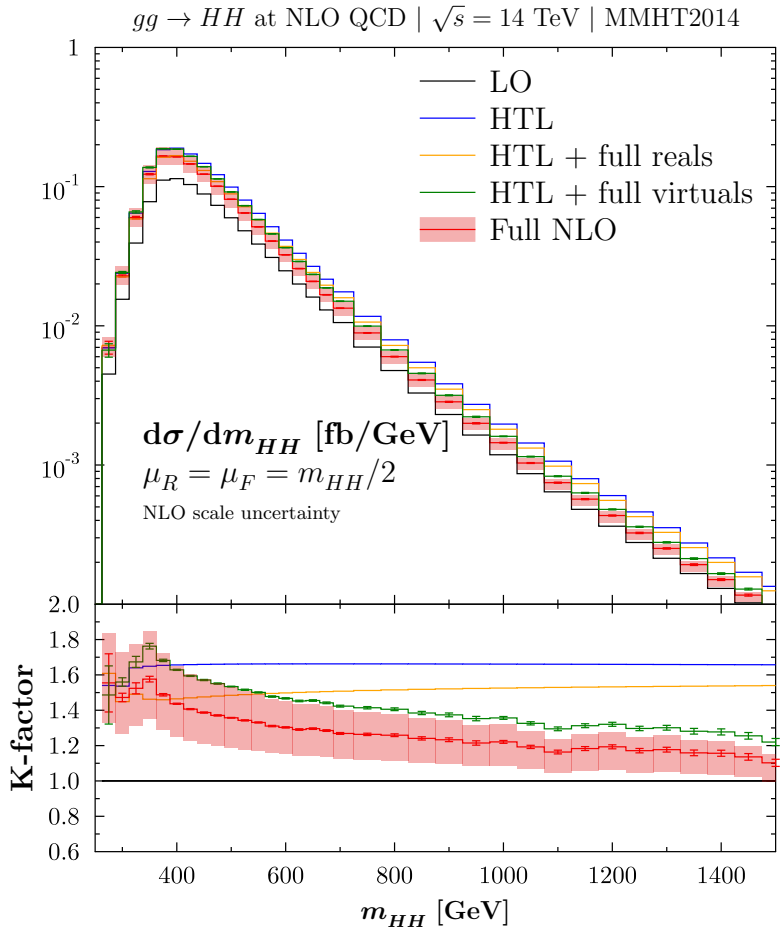




- Mass effects in the real corrections $\sim -10\%$ as in [Maltoni, Vryonidou, Zaro, JHEP 11 (2014) 079]
- Mass effects in the virtual corrections $\sim -25\%$ at $m_{HH} = 1$ TeV
- HTL results outside the scale variation band (in red) of the full results

Total cross section with scale uncertainty

Differential xs with K-factors



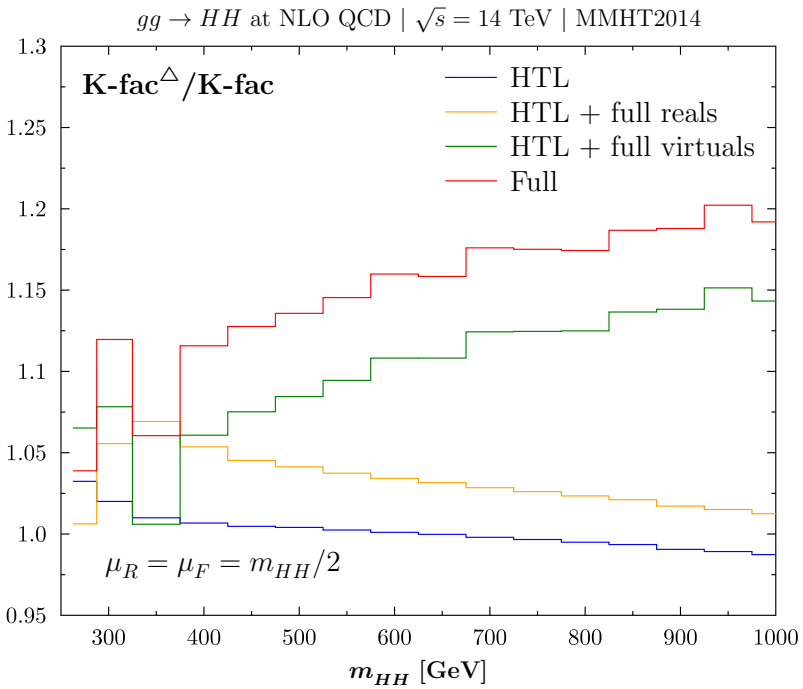
$$K = \frac{\sigma^{\text{NLO}}}{\sigma^{\text{LO}}}$$

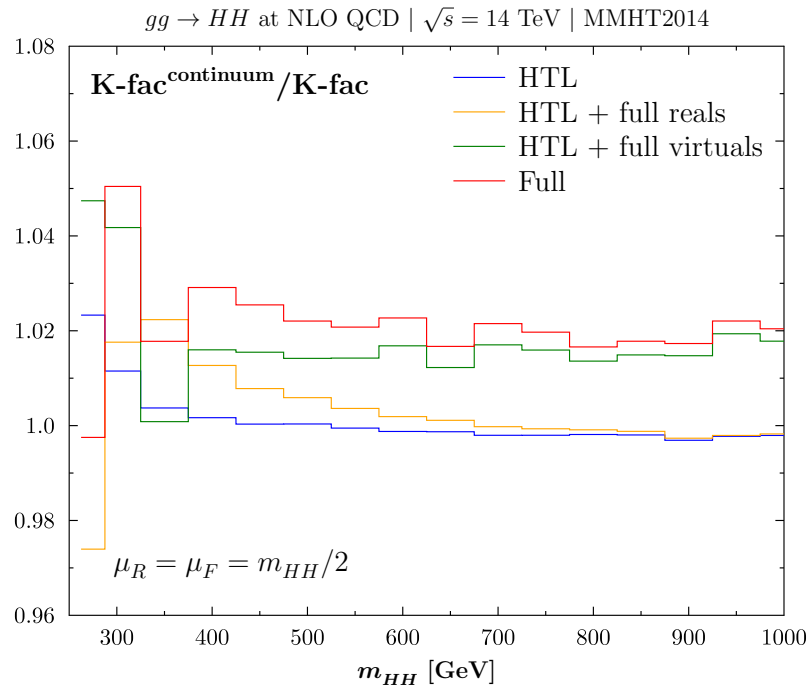
Total hadronic xs

Energy	m_t = 172.5 GeV
13 TeV	$27.73(7)^{+13.8\%}_{-12.8\%}$ fb
14 TeV	$32.81(7)^{+13.5\%}_{-12.5\%}$ fb
27 TeV	$127.0(2)^{+11.7\%}_{-10.7\%}$ fb
100 TeV	$1140(2)^{+10.7\%}_{-10.0\%}$ fb

(using PDF4LHC PDFs, central scale $\mu_R = \mu_F = m_{HH}/2$)

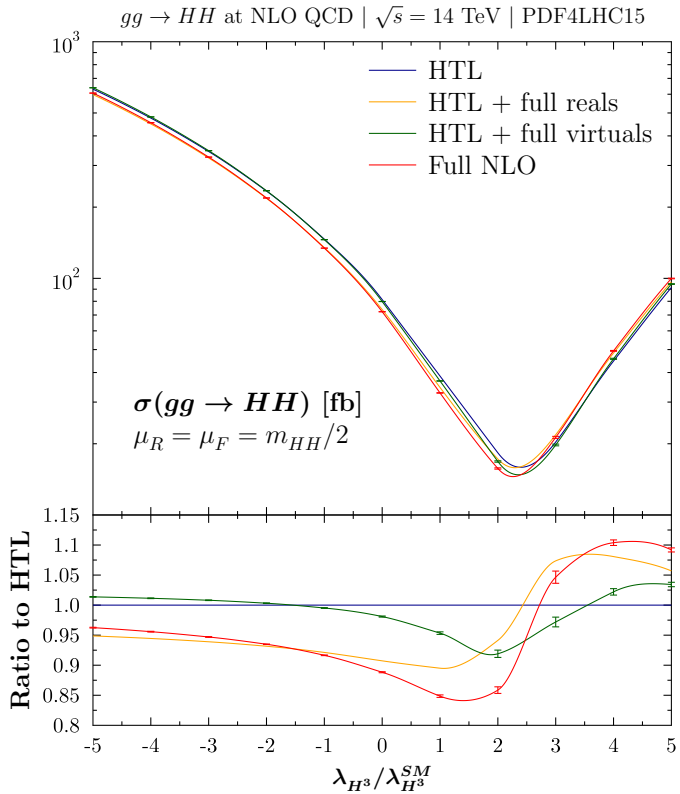
Structure of the corrections

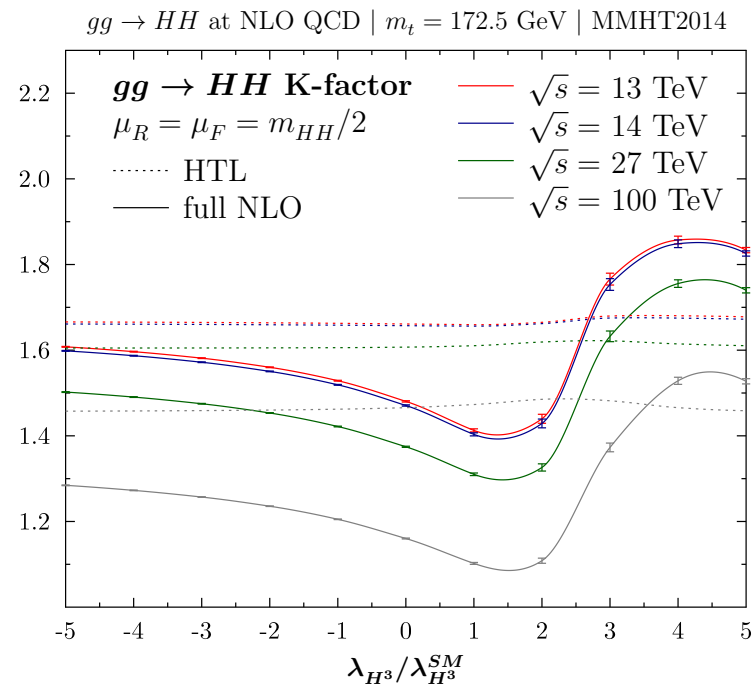




- Continuum diagrams ($\equiv$ all but with λ_{HHH}) play a dominant role at large m_{HH}
- No universal NLO top-mass effects (common in the triangle and box diagrams)
  $\Rightarrow$ not possible to approximate full NLO by single Higgs K-factors

Variation of the triple Higgs coupling





- Minimum of the cross section shifted from $\lambda/\lambda_{\text{SM}} = 2.4$ to 2.3 due to mass effects in the real corrections
- K-factors vary a lot over the $\lambda/\lambda_{\text{SM}}$ range $\Rightarrow$ **mass effects have significant impact on the extraction of λ_{HHH}**

m_t scale-and-scheme uncertainties

Introduction to the issue

- Top-quark mass can be renormalized in the on-shell (OS) scheme or in the $\overline{\text{MS}}$ scheme
- In the $\overline{\text{MS}}$ scheme: What scale choice for $\overline{m}_t(\mu_t)$?

$$\neq \text{choices} \Rightarrow \neq \text{results!}$$

Envelop of the $\neq$ results $\equiv$ top-quark scale-and-scheme uncertainty

$\rightarrow$ At LO QCD: only parametric dependence on m_t

$\rightarrow$ At NLO QCD and beyond: logarithmic dependence on m_t in the virtual (and virtual-reals, etc) corrections

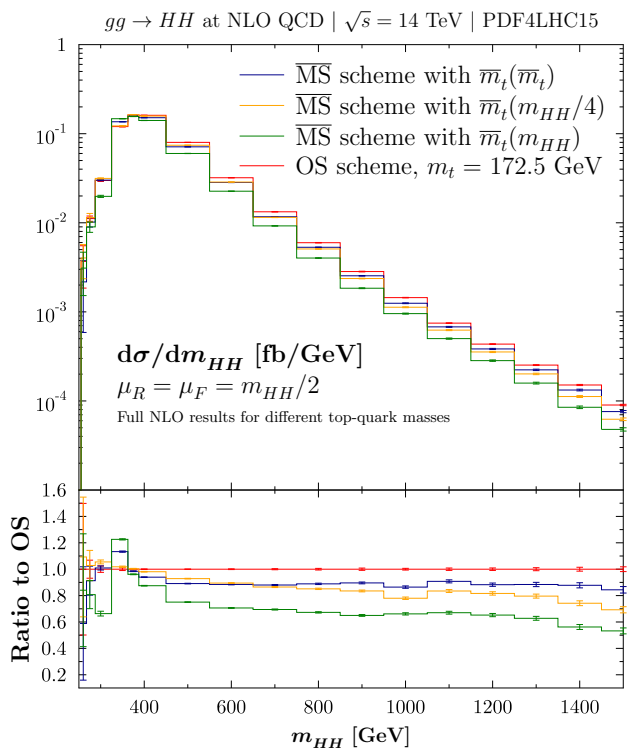
$\rightarrow$ How to cancel this dependence, and reduce the uncertainties?

NLO uncertainties in differential distributions

- Switch to $\overline{\text{MS}}$ scheme $\rightarrow$ modification of the mass counterterm

$$\frac{\delta m_t}{m_t} = \frac{\alpha_s}{\pi}\Gamma(1+\epsilon)\left(\frac{4\pi\mu_R^2}{m_t^2}\right)^{\epsilon}\left(\frac{1}{\epsilon} - \log\left(\frac{\mu_{R,t}^2}{m_t^2}\right)\right)$$

- Compare the predictions with OS m_t, $\overline{m}_t(\overline{m}_t)$, $\overline{m}_t(\mu_t)$ with $Q/4 \leq \mu_t \leq Q$, take the envelop $\rightarrow$ our uncertainty



$\sqrt{s} = 14$ TeV:

$$\left.\frac{d\sigma}{dQ}\right|_{Q=300\text{GeV}} = 0.02978(7)^{+6\%}_{-34\%}\ \text{fb/GeV}$$

$$\left.\frac{d\sigma}{dQ}\right|_{Q=400\text{GeV}} = 0.1609(4)^{+0\%}_{-13\%}\ \text{fb/GeV}$$

$$\left.\frac{d\sigma}{dQ}\right|_{Q=600\text{GeV}} = 0.03204(9)^{+0\%}_{-30\%}\ \text{fb/GeV}$$

$$\left.\frac{d\sigma}{dQ}\right|_{Q=1200\text{GeV}} = 0.000435(4)^{+0\%}_{-35\%}\ \text{fb/GeV}$$

NLO uncertainties in differential distributions

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$$\frac{\delta m_t}{m_t} = \frac{\alpha_s}{\pi}\Gamma(1+\epsilon)\left(\frac{4\pi\mu_R^2}{m_t^2}\right)^{\epsilon}\left(\frac{1}{\epsilon} - \log\left(\frac{\mu_{R,t}^2}{m_t^2}\right)\right)$$

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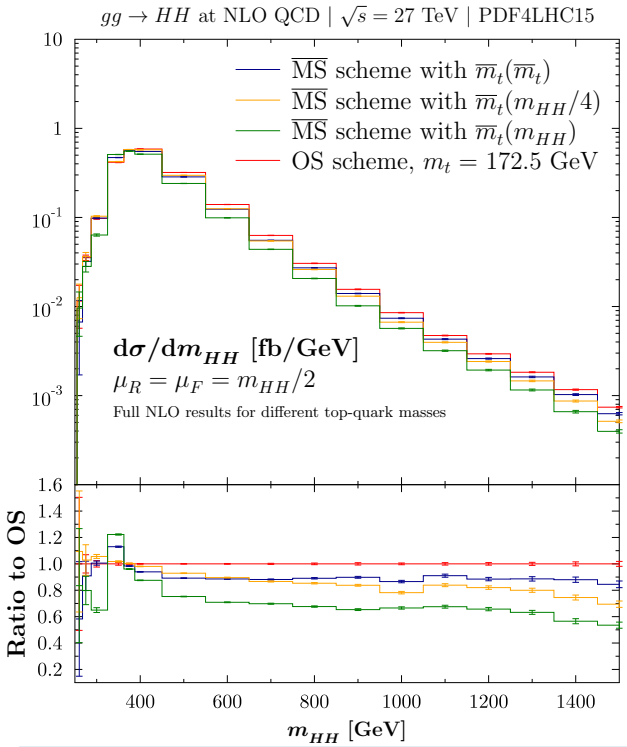

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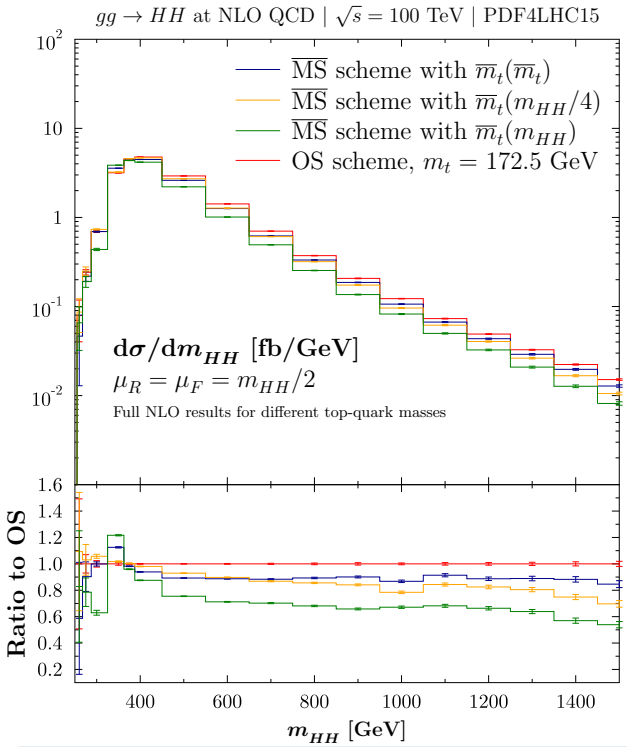

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Compare to LO results

Only parametric at LO, coming from the conversion $m_t \to \overline{m}_t(\mu_t)$
At 14 TeV:

$$\left.\frac{d\sigma}{dQ}\right|_{Q=300\text{GeV}} = 0.01656^{+62\%}_{-2.4\%}\ \text{fb/GeV}$$

$$\left.\frac{d\sigma}{dQ}\right|_{Q=400\text{GeV}} = 0.09391^{+0\%}_{-20\%}\ \text{fb/GeV}$$

$$\left.\frac{d\sigma}{dQ}\right|_{Q=600\text{GeV}} = 0.02132^{+0\%}_{-48\%}\ \text{fb/GeV}$$

$$\left.\frac{d\sigma}{dQ}\right|_{Q=1200\text{GeV}} = 0.0003223^{+0\%}_{-56\%}\ \text{fb/GeV}$$

A factor of 2 higher than at NLO

Uncertainty on the total cross section

Take for individual Q values the maximum / minimum differential cross section and integrate

$$\sigma^{\text{NLO}}_{13\text{ TeV}}(gg \to HH) = 27.73(7)^{+4\%}_{-18\%}\text{ fb}$$

$$\sigma^{\text{NLO}}_{14\text{ TeV}}(gg \to HH) = 32.81(7)^{+4\%}_{-18\%}\text{ fb}$$

$$\sigma^{\text{NLO}}_{27\text{ TeV}}(gg \to HH) = 127.0(2)^{+4\%}_{-18\%}\text{ fb}$$

$$\sigma^{\text{NLO}}_{100\text{ TeV}}(gg \to HH) = 1140(2)^{+3\%}_{-18\%}\text{ fb}$$

Sizable uncertainty comparable to the usual factorization/renormalization scale uncertainty

High-Q expansion and proper scale choice

Scale-and-scheme uncertainty from logs of μ_t $\Rightarrow$ What scale to minimize these logs?

- Low Q-values: Peak of Q-distribution around the $t\bar{t}$-threshold $\Rightarrow$ Natural choice is OS m_t, or $\overline{m}_t(\overline{m}_t)$
- High Q-values: Analytical results in the $\overline{\text{MS}}$ scheme [see also Davies, Mishima, Steinhauser; Wellmann, JHEP 01 (2019) 176]

$$F_{i,\text{LO}} \to \frac{\overline{m}_t^2(\mu_t)}{Q^2} G_i^{\text{LO}}(Q^2, \hat{t})$$

$$\Delta F_{i,mass} \to \frac{\alpha_s}{\pi} \left\{ 2F_{i,\text{LO}} \left[\log \frac{\mu_t^2}{Q^2} + \frac{4}{3} \right] + \frac{\overline{m}_t^2(\mu_t)}{Q^2} G_i(Q^2, \hat{t}) \right\}$$

G_i and G_i^{LO} do not depend on $\overline{m}_t$
$\Rightarrow$ **Natural choice at high Q is $\mu_t \propto Q$**

Off-shell Higgs production

Is this uncertainty seen in other processes?

Take a look at $\sigma(gg \to H^*)$ [Graudenz, Spira, Zerwas, PRL 70 (1993) 1372; Spira, Djouadi, Graudenz, Zerwas, NPB 453 (1995) 17; Harlander, Kant, JHEP 12 (2005) 015; Aglietti, Bonciani, Degrassi, Vicini, JHEP 01 (2007) 021; Anastasiou, Bucherer, Kunszt, JHEP 10 (2009) 068]

$$\sigma^{\rm LO}\Big|_{Q=125\rm GeV} = 18.43^{+0.8\%}_{-1.1\%}\ \text{pb}$$

$$\sigma^{\rm LO}\Big|_{Q=300\rm GeV} = 4.88^{+23.1\%}_{-1.1\%}\ \text{pb}$$

$$\sigma^{\rm LO}\Big|_{Q=600\rm GeV} = 1.13^{+0\%}_{-26.2\%}\ \text{pb}$$

$$\sigma^{\rm LO}\Big|_{Q=1200\rm GeV} = 0.0249^{+0\%}_{-41.1\%}\ \text{pb}$$

$$\sigma^{\rm NLO}\Big|_{Q=125\rm GeV} = 42.17^{+0.4\%}_{-0.5\%}\ \text{pb}$$

$$\sigma^{\rm NLO}\Big|_{Q=300\rm GeV} = 9.85^{+7.5\%}_{-0.3\%}\ \text{pb}$$

$$\sigma^{\rm NLO}\Big|_{Q=600\rm GeV} = 1.97^{+0\%}_{-15.9\%}\ \text{pb}$$

$$\sigma^{\rm NLO}\Big|_{Q=1200\rm GeV} = 0.0402^{+0\%}_{-26.0\%}\ \text{pb}$$

Similar uncertainties showing up at large Q! [Jones, Spira, in arXiv:2003.01700]

Off-shell Higgs production

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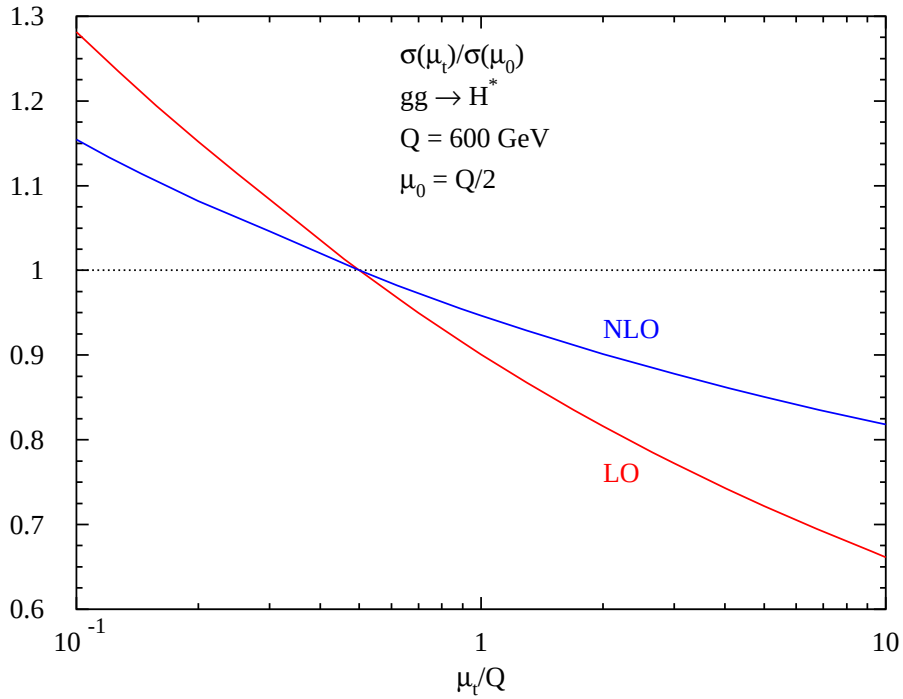
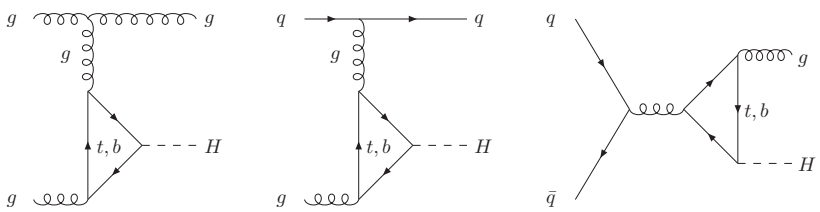


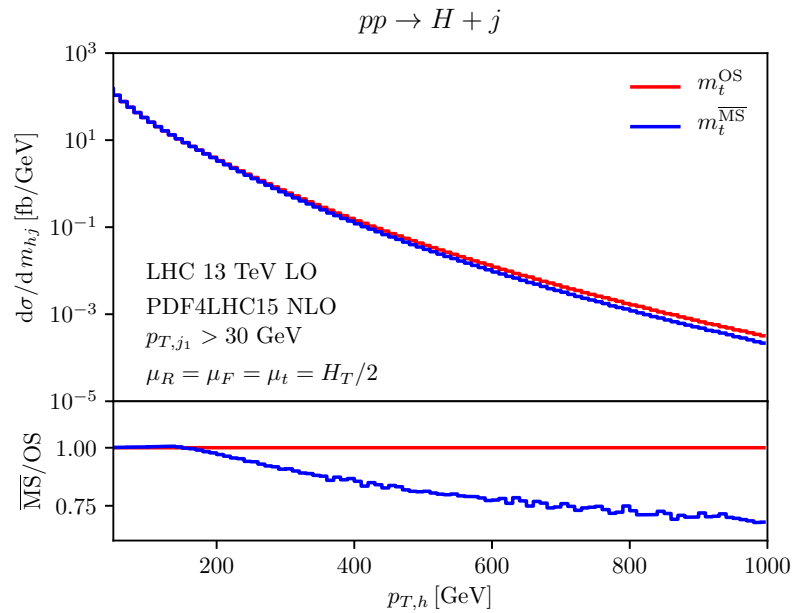
Similar uncertainties showing up at large Q! [Jones, Spira, in arXiv:2003.01700]

Higgs + jet production

Is this uncertainty seen in other processes?

Take a look at $\sigma(gg \to Hj)$, relevant for high-p_T Higgs studies [Baur, Glover, NPB 339 (1990) 38; Schmidt, PLB 413 (1997) 391; De Florian, Grazzini, Kunszt, PRL 82 (1999) 5209; Glosser, Schmidt, JHEP 12 (2002) 016; Ravindran, Smith, Van Neerven, NPB 634 (2002) 247; Jones, Kerner, Luisoni, PRL 120 (2018) 162001]





Again sizable uncertainties, showing up at large $p_{T,h}$

[Jones, Spira, in arXiv:2003.01700]

Conclusions and outlook

- **Calculation of the two-loop integrals of $gg \to HH$ with three mass scales without reduction to master integrals**

  $\to$ Results obtained in the OS scheme and in the $\overline{\text{MS}}$ scheme, scale uncertainty $\sim \pm 10 - 15\%$

  $\to$ Large NLO top-quark mass effects, $\sim -15\%$ in the total cross section

  $\to$ Extraction of λ_{HHH} sizably impacted by the top-quark mass effects

- **Sizable top-quark scale-and-scheme uncertainty:**

  $\sim 30\%$ at large Q, $\sim 22\%$ on the total cross section

$$\sigma^{\text{NLO}}_{14\ \text{TeV}}(gg \to HH) = 32.81(7)\ ^{+13.5\%}_{-12.5\%}\ (\mu_R, \mu_F)\ ^{+4\%}_{-18\%}\ (\mu_t)$$

- Top-quark scale-and-scheme uncertainty sizable in a variety of processes

  $\to$ Issue not only for HH production

  $\to$ **Full NNLO calculation required to decrease the uncertainty: Tough!**

- **Outlook: Extension to 2HDM models, bottom-quark loop**

Backup slides

Details for the renormalization

- **UV renormalization:** δ_{α_s}, δ_g, δ_{m_t}
  - $\rightarrow$ $\overline{\text{MS}}$ renormalization for α_s with 5 active flavors $N_F = 5$

$$\frac{\delta\alpha_s}{\alpha_s} = \frac{\alpha_s}{\pi}\Gamma(1+\epsilon)\,(4\pi)^\epsilon\left[-\frac{33-2(N_F+1)}{12\epsilon}+\frac{1}{6}\log\left(\frac{\mu_R^2}{m_t^2}\right)\right], \quad \delta_{\alpha_s} = \frac{\delta\alpha_s}{\alpha_s}\mathcal{M}_{\text{LO}}$$

  - $\rightarrow$ Top-quark contribution to the external gluons self-energies

$$\delta_g = \frac{\alpha_s}{\pi}\Gamma(1+\epsilon)\left(\frac{4\pi\mu_R^2}{m_t^2}\right)^\epsilon\left(-\frac{1}{6\epsilon}\right)\mathcal{M}_{\text{LO}}$$

  - $\rightarrow$ On-shell renormalization for m_t

$$\frac{\delta m_t}{m_t} = \frac{\alpha_s}{\pi}\Gamma(1+\epsilon)\left(\frac{4\pi\mu_R^2}{m_t^2}\right)^\epsilon\left(\frac{1}{\epsilon}+\frac{4}{3}\right), \quad \delta_{m_t} = -2\frac{\delta m_t}{m_t}m_t^2\frac{\partial\mathcal{M}_{\text{LO}}}{\partial m_t^2}$$

- **IR subtraction:**

$$\delta_{\text{IR}} = \frac{\alpha_s}{\pi}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left(\frac{4\pi\mu_R^2}{-m_{HH}^2}\right)^\epsilon\left[\frac{3}{2\epsilon^2}+\frac{33-2N_F}{12\epsilon}\left(\frac{\mu_R^2}{-m_{HH}^2}\right)^{-\epsilon}-\frac{11}{4}+\frac{\pi^2}{4}\right]\mathcal{M}_{\text{LO}}$$

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Analytical integration of the infrared regulator

In topology 6:

Start from the form subtracted form factors F_i,

$$\Delta F_i = \frac{\alpha_s}{\pi}\,\Gamma(1+2\epsilon)\left(\frac{4\pi\mu_0^2}{m_t^2}\right)^{2\epsilon}(G_1+G_2)\,,$$

$$G_1 = \int_0^1 d^6x\; x^{1+\epsilon}(1-x)^{\epsilon} r^{1+\epsilon} s^{-\epsilon}\left\{\frac{H_i(\vec{x})}{N^{3+2\epsilon}(\vec{x})} - \frac{H_i(\vec{x})|_{r=0}}{N_0^{3+2\epsilon}(\vec{x})}\right\}\,,$$

$$G_2 = \int_0^1 d^6x\; x^{1+\epsilon}(1-x)^{\epsilon} r^{1+\epsilon} s^{-\epsilon}\frac{H_i(\vec{x})|_{r=0}}{N_0^{3+2\epsilon}(\vec{x})}$$

with $N(\vec{x}) = ar^2 + br + c$, $N_0(\vec{x}) = br + c$, $a, c = \mathcal{O}\left(1/m_t^2\right)$, $b = 1 + \mathcal{O}\left(1/m_t^2\right)$.

Analytical integration of G_2 gives rise to hypergeometric functions

$$G_2 = \frac{1}{2+\epsilon}\int_0^1 d^5x\,\frac{x^{1+\epsilon}(1-x)^{\epsilon}s^{-\epsilon}}{c^{3+2\epsilon}}\;{}_2F_1\left(3+2\epsilon, 2+\epsilon; 3+\epsilon; -\frac{b}{c}\right) H_i(\vec{x})\big|_{r=0}$$

Calculation of the real corrections

Building the local IR counterterm:

$$d\Delta\hat{\sigma}_{ij}^{\mathrm{mass}} = d\Delta\hat{\sigma}_{ij} - d\hat{\sigma}_{\mathrm{LO}} \frac{d\Delta\hat{\sigma}_{ij}^{\mathrm{HTL}}}{d\hat{\sigma}_{\mathrm{LO}}^{\mathrm{HTL}}}$$

Local IR counterterm with a projected on-shell LO $2 \to 2$ kinematics to rescale the $2 \to 3$ HTL

$2 \to 2$ OS LO from [Catani, Seymour, NPB 485 (1997) 291] with initial-state emitter, initial-state spectator

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$\Rightarrow$ **Mass effects IR safe in the real corrections**

More on the scale-and-scheme uncertainty

- Electroweak symmetry sum rule $y_t - \sqrt{2}m_t/v = 0$
  $\Rightarrow$ no rationale behind separating the treatment of the top-quark in Yukawa couplings from the top-quark propagator masses

- Conversion from OS pole mass to $\overline{\text{MS}}$ mass at N^{3}LO [Gray, Broadhurst, Grafe, Schilcher, ZPC 48 (1990) 673; Tarasov, JINR-P2-82-900; Chetyrkin,

  PLB 404 (1997) 161]

$$\overline{m}_t(m_t) = \frac{m_t}{1 + 4/3a_s(m_t) + 10.9a_s(m_t)^2 + 107.11a_s(m_t)^3}, \quad a_s(\mu) = \frac{\alpha_s(\mu)}{\pi}$$

$$\overline{m}_t(\mu_t) = \overline{m}_t(m_t)\frac{c[a_s(\mu_t)]}{c[a_s(m_t)]}, \quad c(x) = \left(\frac{7}{2}x\right)^{\frac{4}{7}}\left(1 + 1.398x + 1.793x^2 - 0.6834x^3\right)$$

With $m_t = 172.5$ GeV, $\overline{m}_t(\overline{m}_t) = 163.01516...$ GeV