# TURNING IN THE LANDSCAPE: A NEW MECHANISM TO GENERATE PBH

Jacopo Fumagalli feat Lukas Withowski INSTITUTE D'ASTROPHYSIQUE DE PARIS

Based on ar Xiv 2004.03221 J.F., S. Renaux-Petel, J. W. Ronayne & L. Witkowski









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PLAN

- INFLATION IN NON-GEODESIC MOTION
- TRANSIENT INSTABILITY
- MECHANISM
- ANALYTICAL UNDERSTANDING
- POTENTIAL UNIQUE SIGNATURES











GENERIC/MODEL-INDEPENDENT, POSSIBLE ADVANTAGES, UNIQUE SIGNATURES?

$$\begin{split} S &= \int d^4 x \sqrt{-g} \bigg[ \frac{M_{\rm Pl}^2}{2} R - \frac{1}{2} G_{IJ} \partial^\mu \phi^I \partial_\mu \phi^J - V(\phi) \\ \\ & \text{Eon} = \mathcal{D}_t \dot{\phi}^I + 3H \dot{\phi}^I + G^{IJ} V_{,J} = 0 \\ & \downarrow \\ & \ddot{\phi}^I + \Gamma_{JK}^I \dot{\phi}^J \dot{\phi}^K \end{split}$$

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$$N^{I}(EON) \rightarrow D_{t}T^{I} = \gamma_{L}HN^{I}$$
  $\gamma_{L} = -\frac{V_{v}}{H\sigma}$ 

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$$\mathcal{T}_{\perp} = 0 \implies \text{GEODESIC MOTION}, D_{\tau}^{I} = 0$$
  
 $\mathcal{T}_{\perp} \gg 1 \implies \text{STRONGLY NON-GEODESIC}$ 



MULTI-FIELD PERTURBATIONS

$$\begin{aligned}
\delta \phi^I &= Q_s \mathcal{N}^I \\
g_{IJ} &= a^2 e^{2\zeta} \delta_{IJ}
\end{aligned}$$

$$\mathcal{L}^{(2)} = a^3 \left[ M_{\rm Pl}^2 \epsilon \left( \dot{\zeta}^2 - \frac{(\partial \zeta)^2}{a^2} \right) + 2 \dot{\sigma} \eta_\perp \dot{\zeta} Q_s \right. \\ \left. + \frac{1}{2} \left( \dot{Q_s}^2 - \frac{(\partial Q_s)^2}{a^2} - m_s^2 Q_s^2 \right) \right],$$

$$N^{I}Q_{I} \equiv Q_{s}$$

$$T^{I}Q_{I} = Q_{s}$$

 $\begin{array}{l} \hline \textit{EOM ENTROPIC FLUCTUATION} \\ \ddot{Q}_{s}+3H\dot{Q}_{s}+\left(\frac{k^{2}}{a^{2}}+m_{s}^{2}\right)Q_{s}=-\alpha\eta_{\perp}\dot{\zeta} \\ \hline & \swarrow \\ V_{;ss}+\epsilon R_{\mathrm{fs}}H^{2}M_{\mathrm{pl}}^{2}-\eta_{\perp}^{2}H^{2} \end{array}$ 





• NB.  $m_1^2 >> 1 = m_s^2 <0 ; |m_s| >> H$ 

 $m_s^2 < 0 \implies \text{UNSTABLE BKG}$ 

$$m_s^2 < 0 \not\rightarrow$$
 Unstable BKG

• SUPER-HUBBLE  $k/a \ll H$ 

$$\dot{\zeta} = (2H^2/\dot{\sigma})\eta_{\perp}Q_s \longrightarrow \mathcal{Eon}\left[Q_{\rm s}\right]$$

$$\implies \qquad \ddot{Q}_s + 3H\dot{Q}_s + \left(\frac{k^2}{a^2} + m_{\rm s,eff}^2\right)Q_s = 0$$

$$m_{\rm s,eff}^2 = V_{;ss} + \epsilon R_{\rm fs} H^2 M_{\rm pl}^2 - \eta_{\perp}^2 H^2 + 4H^2 \eta_{\perp}^2$$
$$m_{\rm s}^2 < 0$$



 $m_{s, \epsilon FF}^{2} = V_{ss} + \epsilon R_{sf} H^{2} R_{Pe}^{2} > 0$ 



 $m_{s, \epsilon FF}^{2} = V_{ss} + \epsilon R_{sf} H^{2} \pi_{Re}^{2} < O$ 



SIDETRACKED
S. Garcia Saenz, S. Renaux-Petel &
J. Ronayne '18
HYPERINFLATION
A.Brown '17
D.Marsh & T.Bjorkmo '19
ANGULAR INFLATION
P. Christadoulidis, D. Roest, E. Sfakimakis

....

 $m_{s, eff}^{2} = V_{ss} + E R_{sf} H^{2} R_{Pe}^{2} - H^{2} R_{L}^{2} + 4 H^{2} R_{L}^{2} > 0$  $m_{s}^{2} < 0$ 

#### INTERMEZZO 2

ANGULAR INFLATION ..... TOP-DOWN NEW S. Garcia Saenz, S. Renaux-Petel & J. ATTRACTORS Ronayne '18 , A. Brown '17, D. Marsh & NEGATIVE CURVATURE T.Bjorkmo '19. P.Christodaldis, D.Raest, E.Sfakimakis IN FIELD SPACE, ..... e.g. d-attractors REQUIRE MULTI-FIELD INFLATION SWAMPLAND UN EMBEDDINGS & WITH LARGE BENDING CONJECTURES INFLATTON. A. Achacarro & G. Palma '18 - PROBLEM REQUIRE INFLATING ON STEEP POTENTIALS

SIDETRACKED, HYPERINFLATION,

TRANSIENT INSTABILITY



SINGLE FIELD EFFECTIVE FIELD THEORY



• 
$$\frac{1}{c_s^2} = 1 + \frac{4H^2\eta_{\perp}^2}{m_s^2}$$

A. Achucarro et al. '11

SINGLE FIELD EFFECTIVE FIELD THEORY

• 
$$\frac{1}{c_s^2} = 1 + \frac{4H^2\eta_\perp^2}{m_s^2} = \frac{m_{s,\text{eff}}^2}{m_s^2} \stackrel{\longleftarrow}{=} \frac{\text{Stable Background}}{m_s^2}$$

A. Achacarro et al. '11



EFT WITH IMAGINARY SPEED OF SOUND S. Garcia Saenz, S. Renaux-Petel '18 J.F., S. Garcia Saenz, L. Pinol, , S. Renaux-Petel , J. Ronayne '19

HYPER NON-GAUSSIANITIES

J.F., S. Garcia Saenz, L. Pinol, , S. Renaux-Petel, J. Ronayne PRL '19



- ENHANCEMENT FOR PARTICULAR FLATTEN CONFIGURATIONS e.g.  $k_1 + k_2 - k_3 \rightarrow 0$
- MATCHING / EFT THAT ALLOWS to SHOW
  - PERTURBATIVE CONTROL
  - HIERARCHICAL ENHANCMENT OF THE N-POINT CORRELATION FUNCTIONS

See also: T.Bjorkmo, D. Marsh, R. Ferreira '19 R.Ferreria '20





MODEL-INDEPENDENT TREATMENT  $\eta_{\perp}(N), \quad m_s(N), \quad H(N)$ 



#### PBH & STABLE BACKGROUND



NO NEED FOR A WATERFALL PHASE J. Garcia-Bellido, A. D. Linde, and D. Wands '96 S. Clesse, J. Garcia-Bellido '15

NO NEED FOR USR and/or STOCHASTIC TREATMENT Many in this Workshop.,



$$\eta_{\perp} = \eta_{\perp}^{\max} e^{-\frac{(N-N_{\rm f})^2}{2\Delta^2}}$$





• SHAPE

• GROWTH OF 
$$\mathcal{P}_{\zeta}$$

## ANALYTICAL UNDERSTANDING

S. Garcia-Saenz and S. Renaux-Petel, '18  
JF et al. '19  
T. Bjorkmo, R. Z. Ferreira, and M. C. D. Marsh '19 
$$\int$$
 WKB  
 $\mathcal{P} = \mathcal{P}_0 e^{2x}, \qquad x = \frac{\pi}{2} \left(2 - \sqrt{3 + \xi}\right) \eta_\perp \qquad m_s^2/H^2 = \xi \eta_\perp^2 - \eta_\perp^2$ 

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#### ANALYTICAL UNDERSTANDING



PEAK: HEIGHT & LOCATION



•  $N_p \simeq N_f + \ln \eta_{\perp}^{\max} = N_f + \ln(\gamma/c)$ 

SYMMETRIC  $\gamma_1^2 \implies ASYMETRIC PEAK$ 

#### MODE ENHANCED IF





#### MODE ENHANCED IF



#### GROWTH OF THE POWER SPECTRUM

$$\mathcal{P}(k) = \mathcal{P}_0(k) e^{2x} |_{\widetilde{N}_k}$$



C. T. Byrnes, P. S. Cole, and S. P. Patil' '18 P. Carrilho, K. A. Malik, and D. J. Malryne, '19





LUKAS WITKOWSKI

**Standard picture** for PBH formation in inflationary models:



**Standard picture** for PBH formation in inflationary models:



**Quantity of interest:** energy density in PBHs per logarithmic mass interval as a fraction of the DM energy density today:

Mass function 
$$f(M) \equiv \frac{1}{\Omega_{\text{CDM}}} \frac{d\Omega_{\text{PBH}}}{d \ln M}$$

**Standard picture** for PBH formation in inflationary models:



**Procedure** for computing the mass function f(M):

- Consider a moment during the PBH formation period.
- Calculate the fraction of energy density collapsing into PBHs at that time.
- Apply a redshift factor to relate this to an energy fraction today.
- Sum / integrate over the whole period of PBH formation.

**Heuristic:** PBH formation upon horizon re-entry of primordial fluctuations that are "sufficiently large".

**More precisely:** Relevant quantity for PBH formation is the **smoothed density contrast**. [S.Young et al. 1405.7023; S.Young 1905.01230]

Density contrast: 
$$\delta\equivrac{
ho-ar
ho}{ar
ho}$$
 .

This is to be smoothed over the relevant scale, i.e. Horizon scale.

PBH formation if  $\delta > \delta_c$  with  $\delta_c$  the threshold value that is typically determined from numerical studies of collapse.

An important quantity for calculating the PBH abundance is the **probability distribution function**  $P_H(\delta)$  for the density contrast at a given time (denoted by the Hubble parameter H).

The energy fraction collapsing into PBHs at a given time is:

$$\beta_H = 2 \int_{\delta_c}^{\infty} P_H(\delta) \, d\delta$$

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$$\beta_H = 2 \int_{\delta_c}^{\infty} P_H(\delta) \, d\delta \, .$$

The energy fraction in PBHs crucially depends on the **tail of the PDF.** The PBH spectrum is highly sensitive to the detailed statistical properties of the fluctuations.

Dependence of PBH abundance on the detailed statistics of the fluctuations makes PBHs an interesting observable for **primordial non-Gaussianities (NGs)**.

[C.T. Byrnes et al. 1206.4188; S. Young and C.T. Byrnes 1307.4995; G. Franciolini et al. 1801.09415; V. Atal, C. Germani 1811.07857; G. Panagopoulos, E. Silverstein 1906.02827; J.M. Ezquiaga et al. 1912.05399]

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**Our model:** uses strongly non-geodesic motion that was shown to exhibit a characteristic pattern of NGs, with a bispectrum & higher-order correlation functions enhanced for **flattened** 

Configurations. [S. Garcia-Saenz, S. Renaux-Petel 1805.12563; J. Fumagalli et al. 1902.03221; R. Z. Ferreira 2003.13410]



This unusual pattern of NG may lead to **characteristic features** in the **PBH mass function**.

**Caveat:** PDF of  $\delta$  is generically difficult to calculate.

- Knowledge of a number of moments generically not enough.
- Need non-perturbative computational techniques, like the stochastic  $\delta N$ -formalism. [see David Wands', Chris Pattison's and Julien Grain's talks; C. Pattison et al. 1707.00537; J.M. Ezquiaga et al. 1912.05399]

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**To proceed:** make assumption for the PDF and calculate f(M).

• For simplicity assume **Gaussian statistics**:

$$P_k(\delta) = \frac{1}{\sqrt{2\pi \sigma^2(k)}} \exp\left(-\frac{\delta^2}{2\sigma^2(k)}\right),$$
  
$$\sigma^2(k) = \left(\frac{4}{9}\right)^2 \int_0^\infty \frac{\mathrm{d}q}{q} \left(qk^{-1}\right)^4 e^{-q^2k^{-2}} \mathcal{P}_{\zeta}(q)$$
  
smoothing

• Results still instructive.

**Standard picture** for PBH formation in inflationary models:



#### **Approximations / omissions** in the computation of f(M):

- Assume Gaussian statistics for fluctuations.
- Ignore merger and accretion effects after formation of PBHs.





**Example with sharper bending**  $(\Delta^2, \eta_{\perp}^{\max}) = (0.1, 68)$ :

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**Example with sharper bending**  $(\Delta^2, \eta_{\perp}^{\max}) = (0.1, 68)$ :



Oscillations washed out in f(M) as a result of smoothing and the integration over the formation time.

Could this be an artefact of the Gaussian approximation?

## **Observational Fingerprints**

- I) Inherent non-Gaussianity with characteristic pattern may lead to unique features in PBH mass function.
- 2) A potential signal in Gravitational Waves (GWs):
  - GWs are sourced during the phase of inflation, but also through **anisotropic stresses during the re-entry** of scalar fluctuations. [S. Mollerach et al. astro-ph/0310711; K.N. Ananda et al. gr-qc/0612013; D. Baumann et al. hep-th/0703290]
  - The latter give a contribution to the **stochastic background of GWs**, potentially detectable by LISA and other upcoming GW detectors.
  - The **unusual statistical properties** of fluctuations from strongly non-geodesic motion may also leave an imprint in this GW background.

# Summary

Phases of strongly non-geodesic motion lead to an enhancement of primordial fluctuations that is sufficient for subsequently producing PBHs with the abundance to be all or a fraction of DM.

The position, amplitude and shape of the peak in  $\mathcal{P}_{\zeta}(k)$  depends on the details of the non-geodesic motion in a precise and quantifiable way.

#### Potentially unique observational features:

- The **non-Gaussianity** inherent to the mechanism may lead to characteristic **features in the PBH mass spectrum**.
- Also may find a specific pattern in the stochastic background of GWs, potentially visible in LISA and other near-future GW observatories.

#### **Extra Slides**

#### **PBH formation: mass function**



 $M_H$  = mass contained in a Hubble volume.

 $C, \gamma$ : parameters describing the collapse, typically taken from numerical simulations. Here:  $C = 3.3, \gamma = 0.36$ . [see e.g.T. Koike et al., Phys. Rev. Lett. 74, 5170 1995); J.C. Niemeyer, K. Jedamzik, Phys. Rev. Lett. 80, 5481 (1998)]

eq = matter-radiation equality