Mössbauer experiments with neutrinos and neutrino oscillations

Based on: EA, J. Kopp & M. Lindner, JHEP 0805:005,2008 [arXiv:0802.2513] and arXiv:0803.1424

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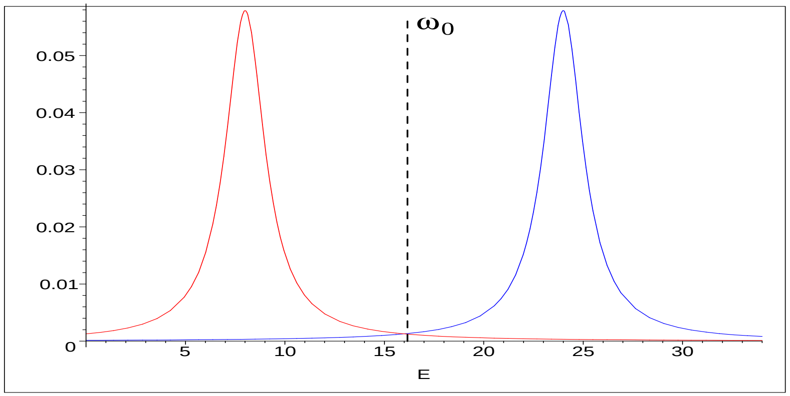
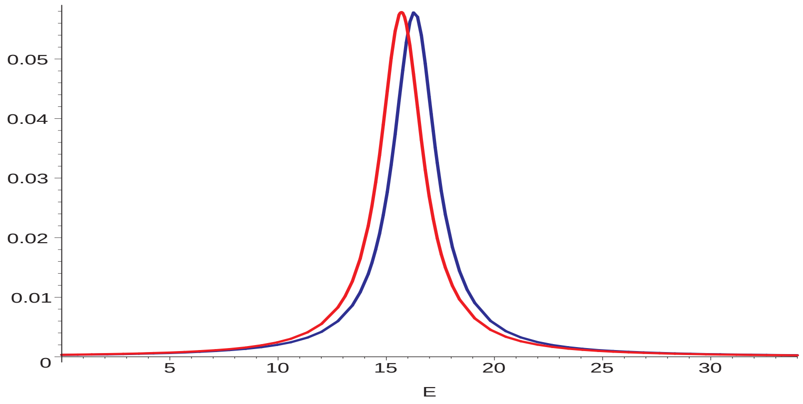
Mössbauer effect

Conventional Mössbauer effect – Res. absorption of γ quanta:

$$A^* \to A + \gamma; \qquad A + \gamma \to A^*$$





Nuclear exc. energy: ω_0.

Recoil energy: $R = \frac{\omega_0^2}{2M}$

$$E_e = \omega_0 - \frac{\omega_0^2}{2M}$$

$$E_a = \omega_0 + \frac{\omega_0^2}{2M}$$

Recoilless emission and absorption (Mössb. eff.):

$$E_e \simeq E_a \simeq \omega_0$$

Strong enhancement of absorption

Mössbauer effect with neutrinos?

Beta decay with 2 - body final state:

$$A(N,Z) \to A(N-1,Z+1) \;+ e_B^- \;+\; \bar{\nu}_e$$

Inverse process:

$$\bar{\nu}_e \;+\; e_B^- \;+\; A(N-1,Z+1) \to A(N,Z)$$

If the neuclei are embedded in solid state lattice, recoilless emission and absorption in principle possible.

Possibility of Mössbauer effect with neutrinos:

Visscher, 1959; Kells & Schiffer, 1983; Raghavan, 2005, 2006

Relevant processes considered:

Bahcall, 1961 – bound state β decay;

Mikaelyan, Tsinoev & Borovoi, 1967 – inverse process (stimulated K-electron capture)

Mössbauer effect with neutrinos?

Mössbauer effect with neutrinos on ^{3}H – ^{3}He system:

$$^3\mathrm{H} \;\rightarrow\; (^3\mathrm{He} \,+\, e^-_B) + \bar{\nu}_e\,; \qquad \bar{\nu}_e \,+\, (^3\mathrm{He} \,+\, e^-_B) \;\rightarrow\; ^3\mathrm{H}$$

Energy release: $Q = 18.6$ keV. Mean lifetime of ^{3}H is 17.8 yr $\Rightarrow$
Nat. linewidth $\Gamma_{^3\mathrm{H}} = 1.17 \times 10^{-24}$ eV – extremely small: $\Delta E/E \sim 10^{-28}$!

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Number of ^{3}H atoms produced in the target can be counted by detecting their decay or using mass spectroscopy.

Very serious technical difficulties exist, but apparently realization of a Mössbauer experiment with neutrinos is not impossible (Raghavan, Potzel).
If realized: for $\Gamma \sim 10^{-11}$ eV, $\sigma \sim 10^{-33}\ cm^2$!

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Detection cross section: $\sigma \sim 10^{-42}\ cm^2$ – of the same order as the cross section of $\bar{\nu}_e p \longrightarrow n e^+$ for reactor $\bar{\nu}_e$'s even though the energy is $\sim$ 150 times smaller. At $L = 10$ m: counting rate $3 \times 10^{-2}/day$ for 100 MCi ^{3}H source and 1 kg ^{3}H target – still too small.

(1Ci = 3.7×10^{10} decays/sec.)

Mössbauer effect with neutrinos?

With Mössbauer effect and for the emission and absorption linewidths $\Gamma \sim 10^{-11}$ eV: $\sigma \sim 10^{-33}\ cm^2$! With recoil-free fraction $f \simeq 0.076$:

L	^{3}H	^{3}He	$\bar{\nu}_e$ capt./d	Rβ (T=65d)
5 cm	1kCi	100 mg	$\sim 4{\cdot}10^4$	$\sim$ 40
10 m	1Mci	1 g	$\sim 10^3$	$\sim$ 10

(1 kCi $\simeq$ 100 mg of ^{3}H; Rβ – reverse β activity; T – activation time)

W. Potzel. Phys. Scripta T127 (2006) 85:
Raghavan's estimates may be too optimistic because they do not include effects of inhomogeneous line broadening, but experiments on Mössbauer effect with neutrinos may still be possible

W. Potzel. arXiv:0810.2170:
Additional difficulties because of variations of binding energies of ^{3}H and ^{3}He atoms in the lattice

Mössbauer effect with neutrinos?

If a Mössbauer neutrino experiment is realized $\Rightarrow$ a unique source of extremely monochromatic low energy neutrinos. Would open up possibilities

- to detect for the first time keV neutrinos
- to detect neutrinos with g or 100 g scale (rather than t or kt scale) detectors
- to observe gravitational redshift of neutrinos
- to study neutrino oscillations at distances ~ 10 m rather than km or hundreds/thousands of km
- to search for the effects of yet unmeasured mixing angle θ_{13} and possibly measure it
- to discriminate between the normal and inverted neutrino mass hierarchies without using matter effects
- to study possible oscillations into sterile neutrino states

Will Mössbauer neutrinos oscillate?

Arguments in the literature (Bilenky et al.):

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Is that true?

The purposes of our study:

Assuming that a Mössbauer experiment with ν's can be realized

- ♢ To find out if Mössbauer neutrinos will oscillate
- ♢ To re-calculate the rate of the overall production – propagation – detection process

Will Mössbauer neutrinos oscillate?

Neutrino oscillations require some intrinsic uncertainty of energy and momentum of the emitted and detected neutrino states !

If E and p were known precisely, from $E^2 = p^2 + m_i^2$ one would determine which mass eihenstate has been emitted $\Rightarrow$ neutrinos of different mass would not be emitted coherently.

For Mössbauer effect with neutrinos in ^{3}H – ^{3}He system:

$$\diamondsuit \quad \frac{\Delta m^2}{2E} = \frac{2.5 \times 10^{-3}\,\mathrm{eV}^2}{2 \cdot 18.6\,\mathrm{keV}} \simeq 6.7 \cdot 10^{-8}\,\mathrm{eV} \gg \Gamma \sim 10^{-11}\,\mathrm{eV}\,!$$

Can neutrinos of different mass be accommodated within such a small energy unbcertainty?

Will neutrinos with such small energy uncertainty oscillate ?

Two "standard" approaches to ν oscillations

The oscillation phase: $\phi = p_\mu x^\mu = E \cdot t - p \cdot x \quad \Rightarrow$

$$\Delta\phi \;=\; \Delta E \cdot t \,-\, \Delta p \cdot L$$

I. Same momentum approach $(\Delta p \;=\; 0)$. The oscillation phase

$$\Delta\phi \;=\; \Delta E \cdot t \;-\; \Delta p \cdot L \;\;\Rightarrow\;\; \Delta E \cdot t$$

– evolution in time; needs to use $L \;\simeq\; t \quad (L \;\simeq\; vt)$.

II. Same energy approach $(\Delta E \;=\; 0)$:

$$\Delta\phi \;=\; -\,\Delta p \cdot L$$

– evolution in space.

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Our point of view: in general, there is no reason to believe that ν_i have either same energy or same momentum. No need to perform Mössbauer ν experiment to decide which approach is correct – it is sufficent to carefully examine the validity of the approximations used.

Kinematic constraints

Same momentum and same energy assumptions: contradict kinematics!

Pion decay at rest $(\pi^+ \to \mu^+ + \nu_\mu, \quad \pi^- \to \mu^- + \bar{\nu}_\mu)$:
For decay with emission of a massive neutrino of mass m_i:

$$E_i^2 \;=\; \frac{m_\pi^2}{4}\left(1-\frac{m_\mu^2}{m_\pi^2}\right)^2 + \frac{m_i^2}{2}\left(1-\frac{m_\mu^2}{m_\pi^2}\right) + \frac{m_i^4}{4m_\pi^2}$$

$$p_i^2 \;=\; \frac{m_\pi^2}{4}\left(1-\frac{m_\mu^2}{m_\pi^2}\right)^2 - \frac{m_i^2}{2}\left(1+\frac{m_\mu^2}{m_\pi^2}\right) + \frac{m_i^4}{4m_\pi^2}$$

For massless neutrinos: $E_i = p_i = E \equiv \frac{m_\pi}{2}\left(1-\frac{m_\mu^2}{m_\pi^2}\right) \simeq 30\ \ \mathrm{MeV}$

To first order in m_i^2:

$$E_i \;\simeq\; E + \xi\frac{m_i^2}{2E}\,, \qquad p_i \;\simeq\; E-(1-\xi)\frac{m_i^2}{2E}\,, \qquad \xi \;=\; \frac{1}{2}\left(1-\frac{m_\mu^2}{m_\pi^2}\right) \;\approx\; 0.2$$

Same E and same p assumptions:

No physical grounds for assuming that neutrinos have same energy or same momentum !

Also: would violate Lorentz invariance of the oscillation probability

How can wrong assumptions lead to the correct oscillation formula ? $\Rightarrow$ answer in the wave packet formalism

Coherence at neutrino production

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$\Rightarrow$ Localization condition violated $\Rightarrow$ oscillations washed out (Kayser, 1981)

How about Mössbauer neutrinos?

Very small effective linewidth Γ $\Rightarrow$ small energy uncertainty of the emitted neutrino state. Can different neutrino mass eigenstates be emitted coherently?

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– not the case. σ_x is certainly smaller than the size of the crystal (a few cm). In reality it is of the order of interatomic distances (one can destroy the crystal and find out which tritium atom turned into helium).

$\Rightarrow \quad \sigma_p \sim 10\ \mathrm{keV}$, i.e. $\quad \sigma_m^2 \;\simeq\; 2p\sigma_p \;\sim\; 4\times 10^8\ \mathrm{eV}^2 \gg \Delta m^2$

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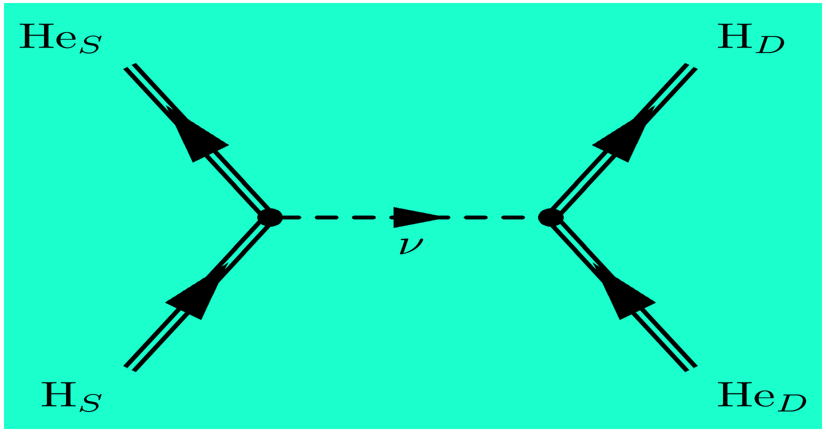
$\Rightarrow$ Oscillations must occur !

Quantum field theoretic calculations

Consider neutrino production, propagation and detection as a single inseparable process. The amplitude given by a “large” Feynman diagram:



External particles (tritium and helium) described by localized wave functions.

Neutrino is an internal line described by a propagator rather than a wave function – no assumptions on its properties (same energy, same momenta, etc.) necessary.

Model for bound tritium and helium nuclei: ground state of the corresponding harmonic oscillators (Einstein's model of crystal) – qood qualitative description of conventional Mössbauer effect. Consider $T = 0$.

Separate calculation (no osc.)

Wave functions:

$$\psi_{A,B,0}(\vec{x},t) = \left[\frac{m_A\omega_{A,B}}{\pi}\right]^{\frac{3}{4}} \exp\left[-\frac{1}{2}m_A\omega_{A,B}|\vec{x}-\vec{x}_B|^2\right]\cdot e^{-iE_{A,B}t},$$

$A=\{\mathrm{H},\mathrm{He}\}$, $B=\{\mathrm{S},\mathrm{D}\}$ (S=source, D=detector).

I. Separate calculation of the production rate and (no-oscillation) detection cross section

$$\Gamma_p = \Gamma_0\, X_S$$

$$\Gamma_0 = \frac{G_F^2\cos^2\theta_c}{\pi}\,|\psi_e(R)|^2\, m_e^2\,\left(|M_V|^2+g_A^2|M_A|^2\right)\left(\frac{E_{S,0}}{m_e}\right)^2\kappa_S$$

$$X_S = 8\left(\eta_S+\frac{1}{\eta_S}\right)^{-3} e^{-\frac{p^2}{\sigma_{pS}^2}} \equiv Y_S\, e^{-\frac{p^2}{\sigma_{pS}^2}}$$

$$\eta_S = \sqrt{\frac{m_H\,\omega_{H,S}}{m_{He}\,\omega_{He,S}}}\,, \qquad \sigma_{pS}^2 = m_H\,\omega_{H,S}+m_{He}\,\omega_{He,S}\,.$$

Separate calculation (no osc.)

Restructuring of wave function of spectator electron $\Rightarrow$

$$\kappa_S \;=\; \left| \int \Psi_{Z=2,S}(\vec{r})^* \, \Psi_{Z=1,S}(\vec{r}) \, d^3r \right|^2$$

(= $\frac{512}{729} \;\simeq\; 0.7$ in hydrogene-like approx.)
Energy spectrum $\rho(E)$ of the emitted Mössbauer neutrinos in the approximation of zero linewidth:

$$\rho(E) = \Gamma_0 \, X_S \, \delta(E - E_{S,0}) \,.$$

Cross section of recoilless detection:

$$\sigma(E) = B_0 \, X_D \, \delta(E - E_{D,0})$$

$$B_0 = 4\pi G_F^2 \cos^2\theta_c \, |\psi_e(R)|^2 \left(|M_V|^2 + g_A^2 |M_A|^2\right) \kappa_D$$

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The overall process rate (for zero linewidths and no oscillations):

$$\Gamma \,=\, \frac{1}{4\pi L^2}\int_0^\infty \rho(E)\sigma(E)\,dE \,=\, \frac{\Gamma_0\,B_0}{4\pi L^2}\,X_S X_D\,\delta(E_{S,0}-E_{D,0})$$

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For Lorentzian energy distributions for production and detection processes:

$$\rho(E) = \Gamma_0\,X_S\,\frac{\gamma_S/2\pi}{(E-E_{S,0})^2+\gamma_S^2/4}\,, \quad \sigma(E) = B_0\,X_D\,\frac{\gamma_D/2\pi}{(E-E_{D,0})^2+\gamma_D^2/4}$$

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Combined rate of the neutrino production, propagation and detection process:

$$\Gamma \simeq \frac{\Gamma_0\,B_0}{4\pi L^2}\,X_S X_D\,\frac{(\gamma_S+\gamma_D)/2\pi}{(E_{S,0}-E_{D,0})^2+(\gamma_S+\gamma_D)^2/4}$$

The Mössbauer resonance condition: $(E_{S,0}-E_{D,0})^2 \ll (\gamma_S+\gamma_D)^2/4$

QFT calculation

Inhomogeneous line broadening: Calculate the probability of the overall process for zero linewidths and then average the result over the energy distribution of ^{3}H and ^{3}He nuclei in the source and detector.

Homogeneous line broadening: modify the amplitude of the process and apply a proper averaging procedure to take into account the stochastic nature of the processes leading to homog. broadening. $\Rightarrow$ Results in both cases are

formally very similar. Mössbauer res. condition:

$$|E_S - E_D| \ll \gamma_S + \gamma_D$$

If it is satisfied $\Rightarrow$ neutrino detection cross section enhanced by a factor

$$\sim (\alpha Z m_e)^3/[p_e E_e(\gamma_S + \gamma_D)] \sim 10^{12}$$

compared to non-resonance $\sigma(\bar{\nu}_e + A \to A' + e^+)$ for neutrinos of same energy (assuming recoil-free fraction ~ 1).

QFT calculation – contd.

The amplitude for zero linewidths:

$$i\mathcal{A} = \int d^3x_1\, dt_1 \int d^3x_2\, dt_2\, \Psi^*_{He,S}(\vec{x}_1) e^{+iE_{He,S}\, t_1}\; \Psi_{H,S}(\vec{x}_1) e^{-iE_{H,S}\, t_1}$$

$$\cdot\, \Psi^*_{H,D}(\vec{x}_2) e^{+iE_{H,D}\, t_2}\; \Psi_{He,S}(\vec{x}_2) e^{-iE_{He,D}\, t_2}$$

$$\cdot \sum_j \mathcal{M}^\mu_S \mathcal{M}^{\nu *}_D |U_{ej}|^2 \int \frac{d^4p}{(2\pi)^4} e^{-ip_0(t_2-t_1)+i\vec{p}(\vec{x}_2-\vec{x}_1)}$$

$$\cdot\, \bar{u}_{e,S}\gamma_\mu(1-\gamma_5)\, \frac{i(\not{p}+m_j)}{p_0^2-\vec{p}^2-m_j^2+i\epsilon}\, \gamma_\nu(1-\gamma_5) u_{e,D}$$

Here

$$\mathcal{M}^\mu_{S,D} = \frac{G_F \cos\theta_c}{\sqrt{2}}\, \psi_e(R)\, \bar{u}_{He}\, (M_V\, \delta^\mu_0 - g_A M_A \sigma_i\, \delta^\mu_i/\sqrt{3})\, u_H\, \kappa^{1/2}_{S,D}$$

QFT calculation – contd.

The overall process rate:

$$\Gamma = \frac{\Gamma_0 \, B_0}{4\pi L^2} \, Y_S Y_D \int_0^\infty dE_{H,S} \, dE_{He,S} \, dE_{He,D} \, dE_{H,D}$$

$$\cdot \, \delta(E_S - E_D) \rho_{H,S}(E_{H,S}) \, \rho_{He,D}(E_{He,D}) \, \rho_{He,S}(E_{He,S}) \, \rho_{H,D}(E_{H,D})$$

$$\cdot \sum_{j,k} |U_{ej}|^2 |U_{ek}|^2 \, \exp\left[-\frac{2E_S^2 - m_j^2 - m_k^2}{2\sigma_p^2}\right] e^{i\left(\sqrt{E_S^2 - m_j^2} - \sqrt{E_S^2 - m_k^2}\right)L}$$

σ_p – effective momentum uncertainty of the emission/absorption processes:

$$\frac{1}{\sigma_p^2} = \frac{1}{m_H \omega_{H,S} + m_{He} \omega_{He,S}} + \frac{1}{m_H \omega_{H,D} + m_{He} \omega_{He,D}} \, ,$$

An analogue of the Debye - Waller (Lamb - Mössbauer) factor:

$\diamondsuit$ $$\exp[-(2E_S^2 - m_j^2 - m_k^2)/2\sigma_p^2] \;=\; \exp[-(p_j^2 + p_k^2)/2\sigma_p^2]$$

QFT calculation – contd.

Generalized Lamb – Mössbauer (Debye – Waller) factor

$$\exp\left[-\frac{p_j^2+p_k^2}{2\sigma_p^2}\right] \;=\; \exp\left[-\frac{(p_{jk}^{\min})^2}{\sigma_p^2}\right]\exp\left[-\frac{|\Delta m_{jk}^2|}{2\sigma_p^2}\right]$$

First factor $\Rightarrow$ suppression of emission and absorption, i.e. a generalized Lamb-Mössbauer factor, second factor $\Rightarrow$ suppression of oscillations.

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Oscillations would be suppressed only if $|\Delta m_{jk}^2| \gtrsim 2\sigma_p^2$.

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In reality: $|\Delta m_{jk}^2|_{\max} \simeq 2.5\cdot 10^{-3}\ \mathrm{eV}^2$; $\sigma_p^2 \sim (10\ \mathrm{keV})^2$ $\Rightarrow$

oscillations will not be suppressed.

QFT calculation – contd.

For Lorentzian energy distributions of external particles:

$$\rho_{A,B}(E_{A,B}) = \frac{\gamma_{A,B}/2\pi}{(E_{A,B}-E_{A,B,0})^2+\gamma_{A,B}^2/4}$$

$(A=\{H,He\},\ B=\{S,D\},\ E_{A,B,0}=m_A+\frac{1}{2}\omega_{A,B})\quad\Rightarrow$

$$\Gamma \simeq \frac{\Gamma_0\, B_0}{4\pi L^2}\, Y_S Y_D \sum_{j,k} |U_{ej}|^2 |U_{ek}|^2 \exp\left[-\frac{(p_{jk}^{\min})^2}{\sigma_p^2}\right] \exp\left[-\frac{|\Delta m_{jk}^2|}{2\sigma_p^2}\right]$$

$$\cdot\,\frac{1}{2}\left(e^{-L/L_{jk,S}^{\rm coh}}+e^{-L/L_{jk,D}^{\rm coh}}\right)\exp\left[-i\frac{\Delta m_{jk}^2}{2\bar{E}}\,L\right]\frac{(\gamma_S+\gamma_D)/2\pi}{(E_{S,0}-E_{D,0})^2+\frac{(\gamma_S+\gamma_D)^2}{4}}$$

$L_{jk,B}^{\rm coh}$ – coherence lengths:

$$L_{jk,B}^{\rm coh} = \frac{4\bar{E}^2}{\gamma_B|\Delta m_{jk}^2|} = \frac{\sigma_x}{\Delta v_g}\,, \qquad \sigma_x = \frac{2}{\gamma_B} \qquad (B=S,D)$$

QFT calculation – contd.

For realistic values of parameters – just the expected result: the rate of no-oscillation production-detection process times the standard oscillation probability (probability of $\bar{\nu}_e$ survival). Decoherence and delocalization can be neglected.

A theoretically interesting case:

Natural linewidth dominance

Time dependence of the wave functions of unstable ^{3}H in the source and detector have to be modified:

$$\Psi_{H,S}(\vec{x}_1)e^{-iE_{H,S}\,t_1} \;\longrightarrow\; \Psi_{H,S}(\vec{x}_1)e^{-iE_{H,S}\,t_1-\frac{1}{2}\gamma t_1}$$

$$\Psi_{H,D}(\vec{x}_2)e^{-iE_{H,D}\,t_2} \;\longrightarrow\; \Psi_{H,D}(\vec{x}_2)e^{-iE_{H,D}\,t_2-\frac{1}{2}\gamma(T-t_2)}$$

($\gamma \;=\; (17.8\;\mathrm{yr})^{-1} = 1.17\times 10^{-24}\;\mathrm{eV}$ – natural linewidth of tritium).

Time integrations in the interval $[0,\,T]$ rather than $(-\infty,\,\infty)$.

QFT calculation – contd.

The result:

$$\mathcal{P} = \frac{\Gamma_0\, B_0}{4\pi L^2}\, Y_S Y_D\, \frac{2}{\pi} \sum_{j,k} |U_{ej}|^2 |U_{ek}|^2 \exp\left[-\frac{(p_{jk}^{\min})^2}{\sigma_p^2}\right] \exp\left[-\frac{|\Delta m_{jk}^2|}{2\sigma_p^2}\right]$$

$$\cdot\, e^{-L/L_{jk}^{\rm coh}}\, e^{-\gamma T}\, e^{-i\frac{\Delta m_{jk}^2}{2\bar{E}}L}\, \frac{\sin^2\left[\frac{1}{2}(E_{S,0}-E_{D,0})T\right]}{(E_{S,0}-E_{D,0})^2}$$

Coherence length:

$$\frac{1}{L_{jk}^{\rm coh}} = \gamma\left|\frac{1}{v_j} - \frac{1}{v_k}\right| \simeq \gamma\, \frac{|\Delta m_{jk}^2|}{2\bar{E}^2}$$

Allowed detuning of $E_{S,0}$ and $E_{D,0}$ is determined not by γ but by the overall observation time T through the relation

$$|E_{S,0} - E_{D,0}| \lesssim 1/T$$

Quant. optics analogy – subnat. spectroscopy

Consider a system with two excited states, $|\psi_A\rangle$ and $|\psi_B\rangle$, width γ_A and γ_B.

At time $t = 0$ state $|\psi_B\rangle$ is instantaneously excited from the ground state by a short laser pulse. Also: the atom is continuously exposed to electromagn. radiation of a frequency ω. Probability of transition into state $|\psi_A\rangle$

$$\mathcal{P} \propto \frac{1}{(E_a - E_b - \omega)^2 + (\gamma_A - \gamma_B)^2/4}$$

$\Rightarrow$ depends on $(\gamma_A - \gamma_B)$ rather than $(\gamma_A + \gamma_B)$. In our case $\gamma_A = \gamma_B = \gamma$.

Time – energy uncertainty relation

Bilenky, v. Feilitzsch & Potzel, arXiv:0803.0527: neutrino oscillations in Mössbauer neutrino experiments contradict time-energy uncertainty relation.

Mandelstam - Tamm time-energy uncertainty relation:

$$\Delta E \Delta O \,\geq\, \frac{1}{2}\left|\frac{d}{dt}\overline{O}(t)\right|$$

$O(t)$ – an arbitrary operator in the Heisenberg representation,
$\overline{O}(t) = \langle\psi_H|O(t)|\psi_H\rangle$. Choose $O \equiv |\nu_l\rangle\langle\nu_l|$ $\Rightarrow$ $\overline{O} = |\langle\nu_l|\Psi(t)|^2 = P_{ll}$.

$$\Delta E \,\geq\, \frac{1}{2}\frac{|\frac{d}{dt}P_{ll}(t)|}{\sqrt{P_{ll}(t) - P_{ll}^2(t)}}$$

Bilenky et al.: set

$$x \simeq t$$

and integrate the inequality for ΔE from 0 to $t_{1\mathrm{min}} \equiv 2\pi E/\Delta m^2$ ("time it takes the neutrino to reach the first oscillation maximum").

$\Rightarrow$ Integration gives

$$\Delta E \geq \frac{1}{\pi}\frac{\Delta m^2}{2E} \qquad\qquad (\mathrm{Note:\ their}\ \Delta E\ \mathrm{is\ our}\,\sigma_E)$$

Mössbauer neutrinos: $\Delta E \sim 10^{-11}\ \mathrm{eV}$, $\frac{\Delta m^2}{2E} \sim 10^{-7}\,\mathrm{eV}$ $\Rightarrow$ oscillations contradict time-energy uncertainty relation?

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satisfied for point-like neutrinos (more generally, when the size of the neutrino wave packet σ_x is small compared to the distance x they have traveled). In reality: baselines of interest $x \sim 10\ m$, but $\sigma_x \sim 1/\gamma \sim 10\ km$!

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⇒ Time of arrival is not well defined ⇒ integration over time is pointless. Without time integration, using $P_{ll}(x,t) = 1 - \sin^2 2\theta \sin^2 \phi(x,t)$,

$$\Delta E \geq |E_1 - E_2|$$

– satisfied for Mössbauer neutrinos: r.h.s. can vanish because $\sigma_p \sim 10$ keV.

Mössbauer neutrinos as a theoretical lab

Mössbauer neutrinos – a very interesting theoretical laboratory for neutrino oscillations. Oscillations under extreme conditions (highly monochromatic ν's, $\sigma_E \to 0$).

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An interesting example of coherence restoration at production – restoration of coherence in the momentum space. At production: $\sigma_E \sim 10^{-11}$ eV, $\sigma_p \sim 10^4$ eV. For neutrinos on the mass shell: $p\sigma_p = E\sigma_E \quad \Rightarrow$ $\sigma_p \sim \sigma_E \sim 10^{-11}$ eV – both are tiny! W. packets of different mass eigenstate widely separated in p-space ($\Delta p \simeq \Delta m^2/2p \simeq 10^{-8}$ eV). But: momentum uncertainty at detection $\sigma_p \sim 10^4$ eV restores coherence and allows coherent detection of different mass eigenstates (EA, Smirnov, 2009).

Alternatives?

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Speed should be maintained to an accuracy $\sim 10^{-10}$!

Conclusions

If a Mössbauer neutrino experiment is realized – recoillessly emitted and absorbed neutrinos will oscillate

Backup slides

The oscillation phase

Oscillations are due to phase differences of different mass eigenstates:

$$\Delta\phi \;= \Delta E\cdot t \;-\; \Delta p\cdot L \qquad\qquad (E_i \;=\; \sqrt{p_i^2+m_i^2})$$

Consider the case $\Delta E \ll E$ (relativistic or quasi-degenerate neutrinos) $\Rightarrow$

$$\Delta E \;=\; \frac{\partial E}{\partial p}\Delta p + \frac{\partial E}{\partial m^2}\Delta m^2 \;=\; v_g\,\Delta p \;+\; \frac{1}{2E}\,\Delta m^2$$

$$\Delta\phi \;=\; (v_g\,\Delta p + \frac{1}{2E}\Delta m^2)\;t \;-\; \Delta p\cdot L$$

$$=\; -\;(L\;-\;v_g\,t)\Delta p \;+\; \frac{\Delta m^2}{2E}\,t$$

In the center of wave packet $(L\;-\;v_g\,t)=0$. In general, $|L\;-\;v_g\,t|\lesssim\sigma_x$ (the spatial size of the w. packet); if $\sigma_x \ll l_{\rm osc}$, $|L\;-\;v_g\,t|\Delta p \ll 1 \quad\Rightarrow$

The oscillation phase – contd.

$$\Delta\phi = \frac{\Delta m^2}{2E}\,t\,, \qquad\qquad L \simeq v_g t \simeq t$$

– the result of the “same momentum” approach recovered!

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♢ $$\Delta\phi = -\frac{1}{v_g}(L - v_g\, t)\Delta E + \frac{\Delta m^2}{2p}\, L \quad \Rightarrow \quad \frac{\Delta m^2}{2p}\, L$$

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- Neutrinos are highly relativistic with $\Delta E \ll E$

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The reasons why wrong assumptions give the correct result:

- Neutrinos are highly relativistic with $\Delta E \ll E$
- Neutrino production/detection processes are coherent (typically this implies $\sigma_x \ll l_{\rm osc}$)

QFT calculation – Grimus-Stockinger f-la.

Integrating over the coordinates and applying the Grimus-Stockinger theorem for $\vec{p}$ - integration of the neutrino propagator:

$$\int d^3p \, \frac{\psi(\vec{p})\, e^{i\vec{p}\vec{L}}}{A - \vec{p}^2 + i\epsilon} \xrightarrow{|\vec{L}|\to\infty} -\frac{2\pi^2}{L}\psi(\sqrt{A}\frac{\vec{L}}{L})e^{i\sqrt{A}L} + \mathcal{O}(L^{-\frac{3}{2}})$$

to leading order in $1/L$ one gets

$$i\mathcal{A} = \frac{-i}{2L}\mathcal{N}\,\delta(E_S - E_D)\sum_j \exp\left[-\frac{E_S^2 - m_j^2}{2\sigma_p^2}\right]\mathcal{M}_S^{\mu}\mathcal{M}_D^{\nu *}|U_{ej}|^2\, e^{i\sqrt{E_S^2 - m_j^2}L}$$

$$\cdot\, \bar{u}_{e,S}\gamma_\mu(1-\gamma_5)(\not{p}_j + m_j)(1+\gamma_5)\gamma_\nu u_{e,D}\,,$$

where $p_j \equiv (E_S, (E_S^2 - m_j^2)^{1/2}\,\vec{L}/L)$. Averaging procedure:

$$\mathcal{P} = \int_0^\infty dE_{H,S}\, dE_{He,S}\, dE_{He,D}\, dE_{H,D}$$

$$\cdot\, \rho_{H,S}(E_{H,S})\,\rho_{He,D}(E_{He,D})\,\rho_{He,S}(E_{He,S})\,\rho_{H,D}(E_{H,D})\,\overline{|\mathcal{A}|^2}$$