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LPSC, Grenoble

Fixed field alternating gradient accelerators

A bit of various aspects, all together :

history / theory and methods / status

Contents

1 Introduction

Heard at ICFA-HB2004 : "one of the most active fields in accelerator physics and technology".

- Several FFAG machines have been operated :

- 3 electron model rings by MURA Lab., 50's
- 2 radial scaling proof-of-principle proton rings, KEK, 2000 & 2003
- 3-ring chain, proton, Kyoto University, 2008
- 1 emittance recovery internal target ring, proton, Kyoto University, 2007
- 1 large acceptance momentum-damping ring, in view of muon pysics, Osaka University, 2007
- several compact high power electron rings by Japan industrials

- Neutrino factory R&D triggered strong activity,

– was cause of revival of 1950s' "scaling" FFAG

– gave rise to new FFAG optics concepts, "non-scaling"

- Various prototyping projects launched to study

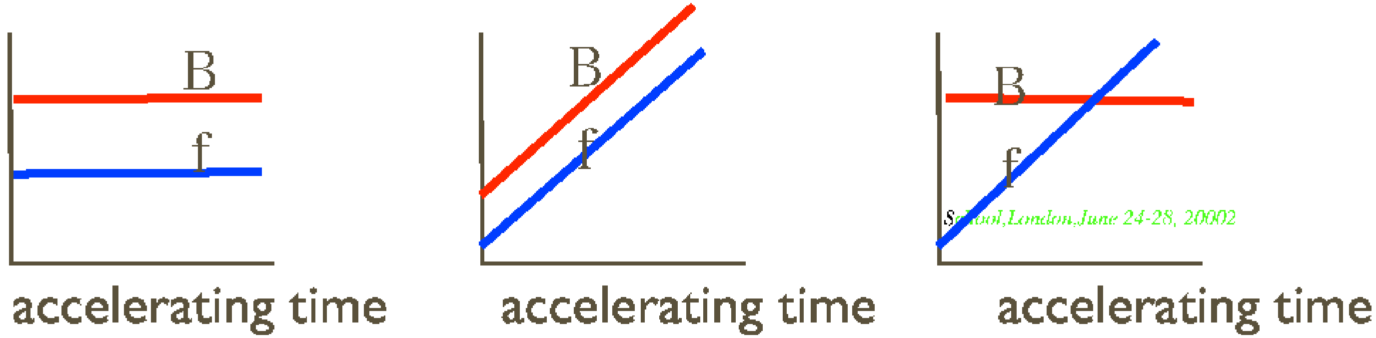
• medical application, proton driver, • fast acceleration ($\rightarrow$ high $< I >$, unstable beams), • neutron production, etc.

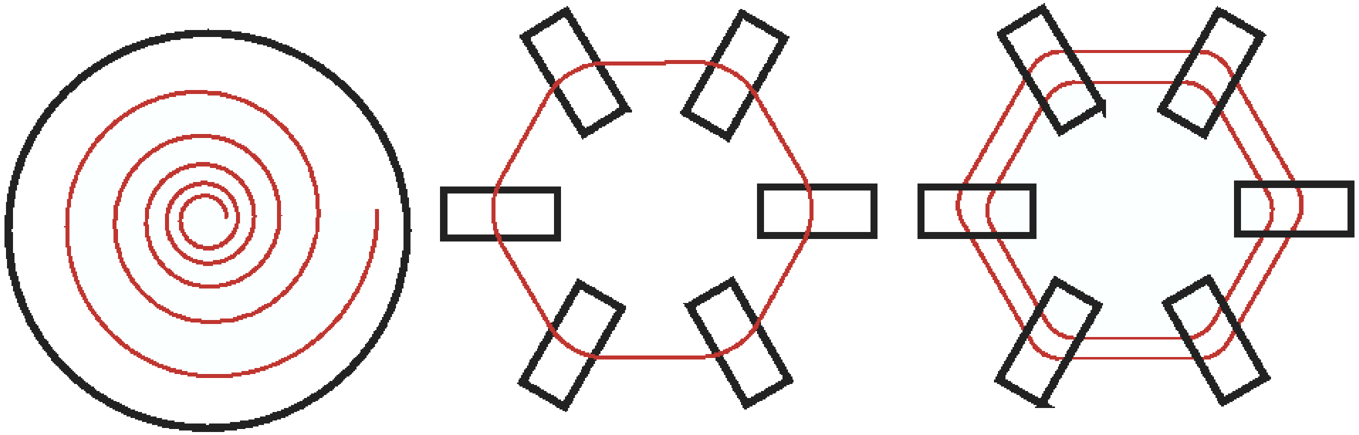
– including ANR RACCAM in France, EMMA and PAMELA in UK, involvement of several industrial companies in the US.

* close to 2 workshops per year *

• 1st	Dec. 1999	KEK
• 2nd	July 2000	CERN
• 3rd	Oct. 2000	KEK
• 4th	Feb. 2002	KEK
• 5th	Sept. 2002	LBL
• 6th	July 2003	KEK
• 7th	Sept. 2003	BNL
• 8th	Mar. 2004	TRIUMF
• 9th	Oct. 2004	KEK
• 11th	Apr. 2005	FNAL
• 11th	Dec. 2005	KURRI
• 12th	Apr. 2006	BNL
• 13th	Nov. 2006	KURRI
• 14th	Apr. 2007	LPSC
• 15th	Nov. 2007	KURRI
• 16th	Sept. 2008	Manchester
• 17th	Nov. 2008	KURRI
• 18th	Nov. 2009	FNAL

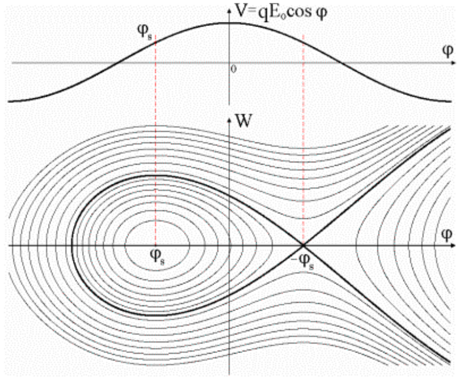
Principle





CW (High $\overline{I}$)
Limited max. Energy
Invented by
Ernest O. Lawrence,
1930

Slow acceleration (Low $\overline{I}$)
Reduced 6D acceptance
Principle of “phase stability”
Mc Millan, Veksler, 1945



Fast acceleration (High $\overline{I}$)
Huge 6D acceptance
Versatility / beam
manipulations
An invention by
Symon/Okawa/Kolomensky
1954

Large parts of the physics underpinning of contemporary particle accelerators, were first done by people associated with the invention of the FFAG and the Midwestern Universities Research Association group (MURA) in the 1950's and 1960's.

This includes :

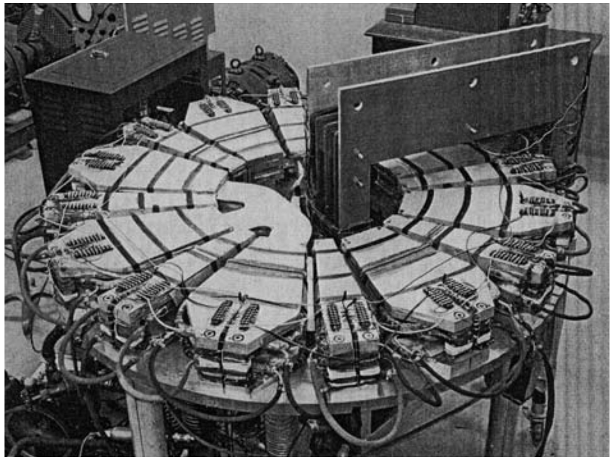
(i) beam stacking,
(ii) Hamiltonian theory of longitudinal motion,
(iv) storage rings (independently invented by O'Neill),
(vi) lattices with zero-dispersion and low-β sections for colliding beams,
(vii) multiturn injection into a strong-focusing lattice,
(viii) first calculations of the effects of nonlinear forces in accelerators,
(ix) first space-charge calculations including effects of the beam surroundings,
(x) first experimental measurement of space-charge effects,
(xi) theory of negative-mass and other collective instabilities and correction systems,
(xii) the use of digital computation in design of orbits, magnets, and RF structures,
(xiii) proof of the existence of chaos in digital computation,
(xiv) synchrotron-radiation rings.

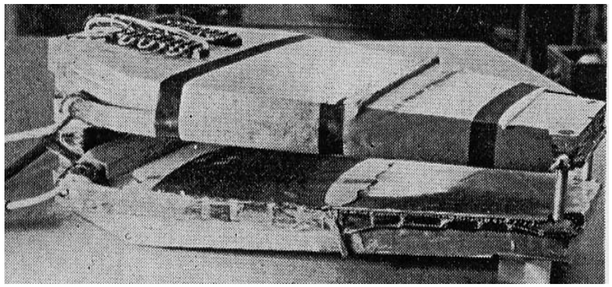
2 MURA "scaling" FFAG models

The first model, radial sector FFAG, Mark II

The objectives of this prototyping : confirm theoretical predictions ; study FFAG properties : optics, injection, test RF programs ; effects of misalignments ; effects of resonances.

First operation March 1956, U of Michigan.



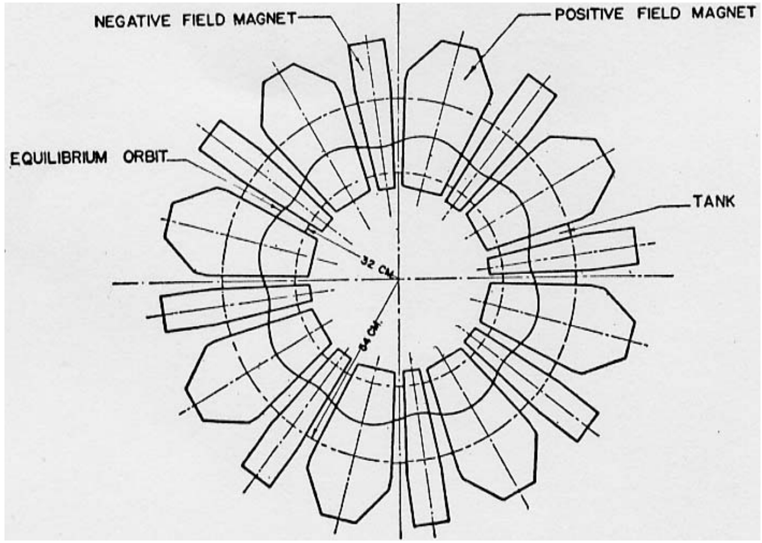


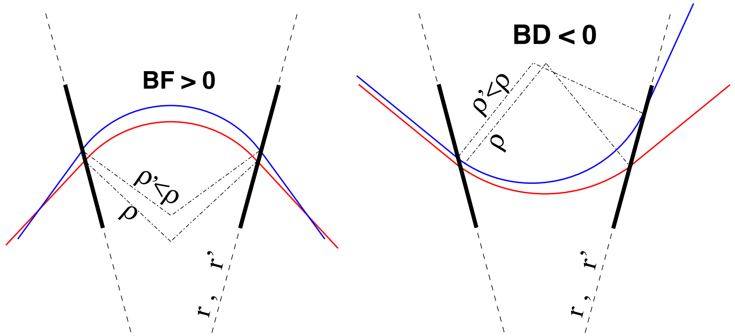
F magnet, positive field, radially focusing.

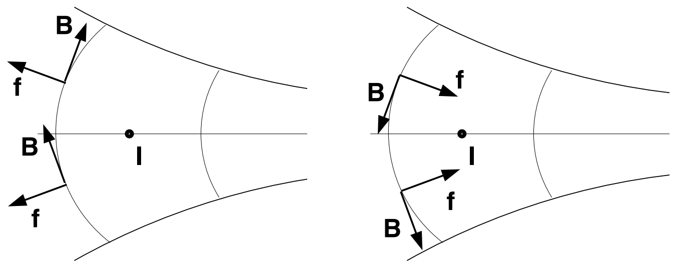
		Machine parameters	criteria / comments
$E_{inj} - E_{max}$	*keV*	25 - 400	*small size, easy to build* *field not too low, ms lifetime*
orbit radius ($\mathcal{C}/2\pi$)	*m*	0.34 - 0.50	**SPIRALING ORBIT**
Optics		**STRONG FOCUSING, SCALING** $\rightarrow \xi = 0$	
lattice		$\frac{D}{2}F\frac{D}{2}$	
number of cells		8	*16 magnets, 4.41 deg. drifts*
field index K		3.36	g/r *=Cst & coil windings*
ν_r / ν_z		2.2-3 / 1-3	*varying* K*, resp.* B_F/B_D *varies mostly* ν_r*, resp.* ν_z
γ_t		≈ 2	$\sqrt{1+K}$
Magnet		radial sector	$B = B_0(r/r_0)^K F(\theta)$
$\theta_F,\ \theta_D$	*deg*	25.74, 10.44	*sector angles*
$r_{F,D}/\rho$		2.85, 2.59	*at center of F, D magnets*
gap	*cm*	6 - 4	g/r *=Cst*
Injection		continuous or pulsed	
Acceleration		betatron first, then RF gap	*for simplicity*
swing	*Gauss*	40 - 150	
rep. rate	*Hz*	a few 10's	
		... completed with RF acc., next	*split tank*
freq. swing	*MHz*	10 in [35, 75] MHz	*for RF stacking expts*
gap voltage	*V*	50	

Radial scaling FFAG : how it works

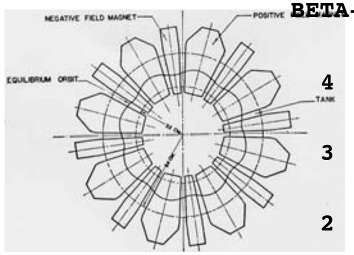
- Magnetic field fixed in time, $\boxed{B = \pm B_0(\frac{r}{r_0})^K}$
  - from lower intensity on inner orbit
  - to largest intensity on outer orbit
- Transverse motion stability is insured by **strong, AG focusing**
  - as in pulsed synchrotrons, hence small beta functions
  - AG is obtained by alternance of
    - positive curvature field sectors, hence focusing, $\frac{\rho(s)}{B(s)}\frac{dB}{d\rho} > 0$
    - negative curvature field sectors, hence defocusing, $\frac{\rho(s)}{B(s)}\frac{dB}{d\rho} < 0$
    - with ratio $|\int B_D\,ds| \approx \frac{2}{3} \times \int B_F\,ds$, to insure axial focusing
- The radial dependence $B = B_0(r/r_0)^K$ yields zero chromaticity and the *scaling* property :
  - orbits are similar wrt. geometrical center
  - tunes are independent of orbit
- Corollaries
  - large *circumference factor* $\mathcal{C}/2\pi\rho$ due to alternating curvature
  - drift length is not free
  - $\alpha = 1/(1+K) \rightarrow$ larger K insures smaller $r_{max} - r_{min}$
  - transition energy $E_{tr} = E_0/\sqrt{\alpha} \approx E_0\sqrt{1+K}$ easily beyond E_{max}
- Longitudinal motion : regular synchrotron motion. In addition
  - arbitrary RF programs are possible : ω_{RF} does not track B
  - extremely high accelerating gradients are possible : B is constant

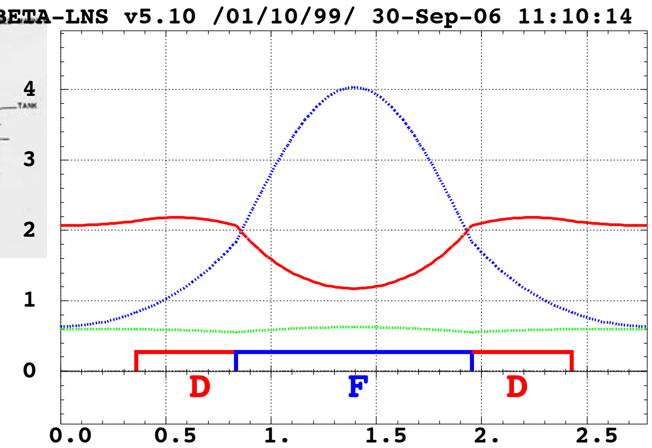






Linear optics : allows preliminary design steps based on regular "TRANSPORT" codes





First : find [a] closed orbit $\longleftarrow$ from the FFAG parameters.

Then : linear approximation about that closed orbit

$$x'' + \frac{1-n}{\rho^2}x = 0, \quad z'' + \frac{n}{\rho^2}z = 0$$

with $\quad n(s) = -\frac{\rho(s)}{B(s)}\frac{dB}{dx} \approx -\frac{\rho}{B}\frac{dB}{dr}$ (scalloping is neglected)

Index $n(s)$ and K in $B = B_0(\frac{r}{r_0})^K$ relate as follows:

$\frac{dB}{dr} = K\frac{B_0}{r_0}(\frac{r}{r_0})^{K-1} = K\frac{B}{R}$ so that $\boxed{K/R = -n/\rho}$

The matrix representing a sector has the form $M = \begin{bmatrix} \cos(s\sqrt{k}) & \frac{1}{\sqrt{k}}\sin(s\sqrt{k}) \\ -\sqrt{k}\sin(s\sqrt{k}) & \cos(s\sqrt{k}) \end{bmatrix}$

with $k = (1-n)/\rho^2$ (radial motion) or $k = n/\rho^2$ (vertical motion)

The geometry provides the wedge angles, hence wedge matrices, M_{Fe1}, M_{Fe2}, M_{De1}, M_{2e2}

The product matrix representing a D-F sector yields the phase advance :

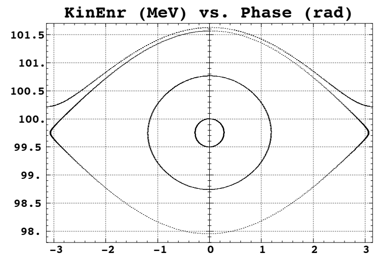
$\cos(\mu) = \frac{1}{2}Tr(M_{Fe2} \times M_F \times M_{Fe1} \times M_{De2} \times M_D \times M_{De1})/2$,...

The longitudinal motion in presence of RF satisfies, most classically

$$\Phi'' + \frac{\Omega^2}{\cos\phi_s}(\sin\phi - \sin\phi_s) = 0$$

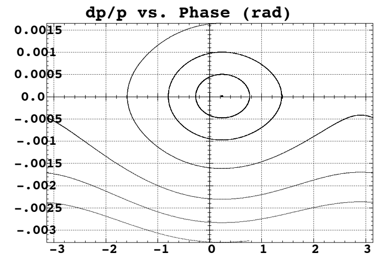
synchrotron frequency $f_s = \Omega_s/2\pi = \frac{c}{\mathcal{L}}\left(\frac{h\eta\cos\phi_s q\hat{V}}{2\pi E_s}\right)^{1/2}$,

bucket height $\pm\frac{\Delta p}{p} = \pm\frac{1}{\beta_s}\left(\frac{2q\hat{V}}{\pi h\eta E_s}\right)^{1/2}$, etc.



Stationary bucket

$\phi_s = 0$



accelerating bucket

$\phi \neq 0$

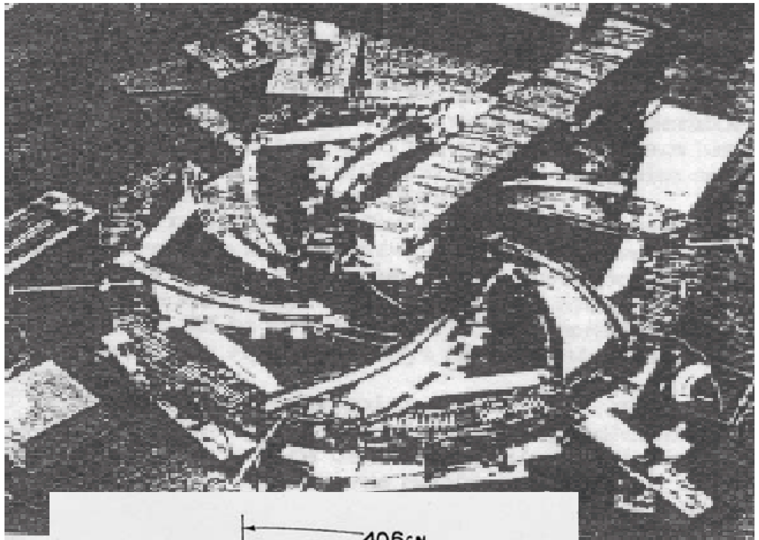
Second model, spiral sector FFAG, Mark V

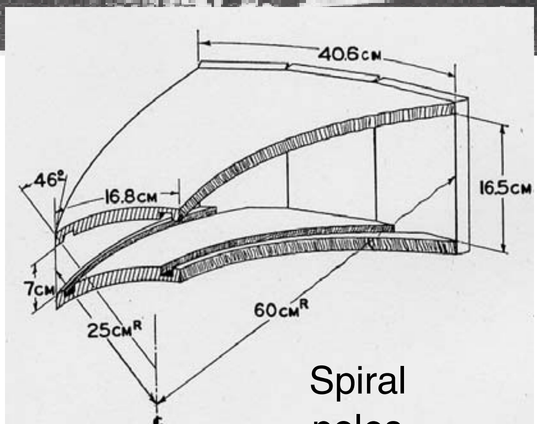
The idea in the spiral FFAG was to superpose a positive field on top of the alternating sign one of the radial sector, so as to always have the good curvature sign, hence smaller accelerator.

By doing so, the vertical focusing is strongly weakened : this is counteracted by *strong* wedge focusing by means of spiral edge.

Some objectives of this prototyping :

- confirm theoretical predictions
- first extensive use of computers to determine magnetic field and machine parameters
- long-term orbit stability ; RF acceleration methods.





Spiral poles

First operation Aug. 1957 at the MURA Lab., Madison.

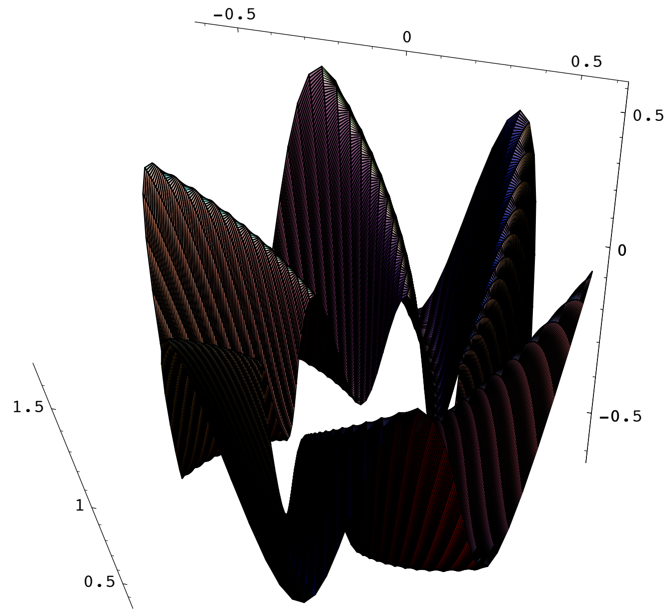
		Machine parameters	**criteria / comments**
$E_{inj} - E_{max}$	*keV*	35 - 180	*reasonable size, cost*
orbit radius	*m*	0.34 - 0.52	**SPIRALING ORBIT**
E_{tr} / r_{tr}	*keV / m*	155 / 0.49	{ *RF exprmnts* *at* $\gamma_{tr} = (1+K)^{1/2}$
Optics	**SCALING** $\rightarrow \xi = 0$		
lattice		N spiral sectors	
number of sectors		6	
field index K		0.2 – 1.16	*tunable / coil windings*
flutter F_{eff}		0.57 - 1.60	*tuning coils*
ν_r / ν_z		1.4 / 1.2	*tunable via* K, F_{eff}
β_r / β_z	*m*	0.45-1.3 / 0.6-1.4	*min-max*
Magnet		spiral sector	$B = B_0(\frac{r}{r_0})^K F(\ln\frac{r}{r_0}/w - N\theta)$
$\alpha = Arctg(Nw)$	*deg*	46	*spiral angle*
$r_{min} - r_{max}$	*m*	0.25 - 0.61	
gap	*cm*	16.5 - 7	g/r =*Cte*
Injection		cont. or pulsed	*e-gun + e-inflector*
Acceleration		betatron and RF gap	*extensive RF prog. tests*

Spiral scaling FFAG : how it works

It is not strictly AG !

Field form :

$$B(r,\theta)|_{z=0} = B_0 \left(\frac{r}{r_0}\right)^K \mathcal{F}\left(\ln\frac{r}{r_0}/w - N\theta\right)$$

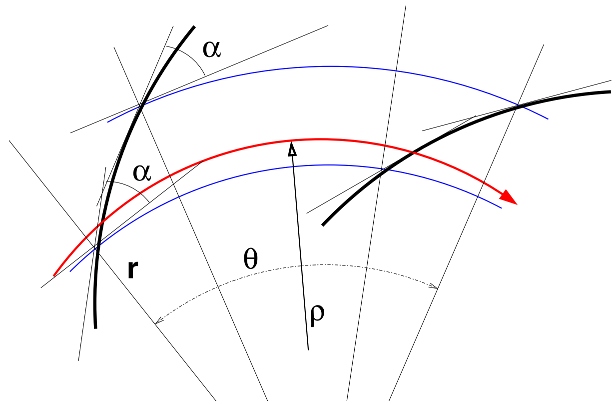


$\mathcal{F}$ is the axial modulation of the field ("flutter").
A simple, very explicit, model is sometimes used to get the bulk of the vertical focusing effect :
$\mathcal{F} = 1 + f\sin(\ln\frac{r}{r_0}/w - N\theta)$

The logarithmic spiral edge $r = r_0 \exp(Nw\theta)$ insures constant angle between spiral sector edges and closed orbits.

Expansion of the equations of motion around the scalloped orbit in the linear approximation yields the tunes

$$\nu_r \approx \sqrt{1+K}, \quad \nu_z \approx \sqrt{-K + (f/Nw)^2/2}$$

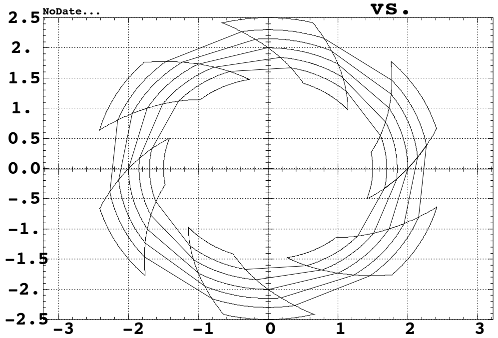


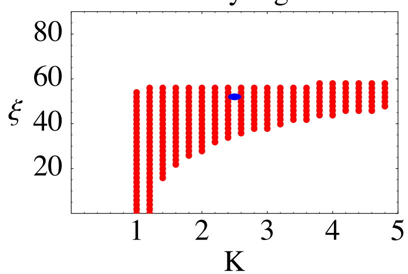
A simple, useful tool :
- matrix modeliung using hard-edge approximation and fringe field correction ;
- this is sufficient for approaching closely the bulk the first order properties, see next slide...

Linear optics using regular "TRANSPORT" methods (i.e., matrix modelling)

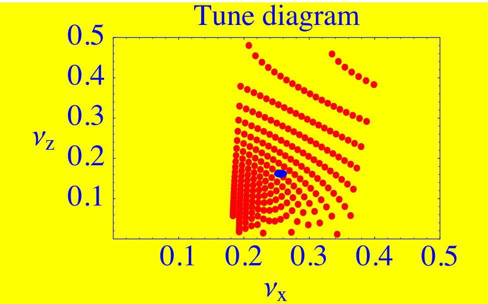
The rule still is the pseudo-harmonic motion

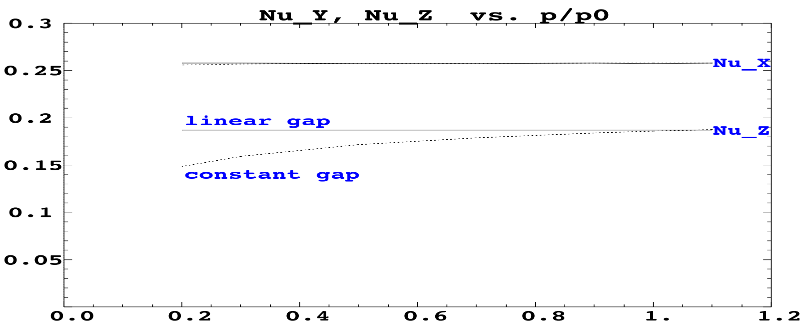
$$x'' + \frac{1-n}{\rho^2}x = 0, \quad z'' + \frac{n}{\rho^2}z = 0$$



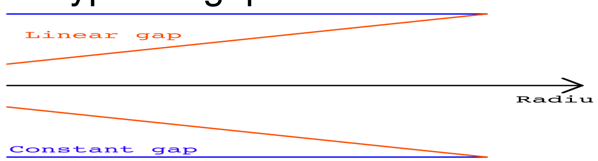


Proton, 8 cells,
3 MeV $\rightarrow$ 100 MeV
Working point :
K=2.5, ξ=52 deg
$\nu_x \,/\, \nu_z = 0.25 \,/\, 0.19$

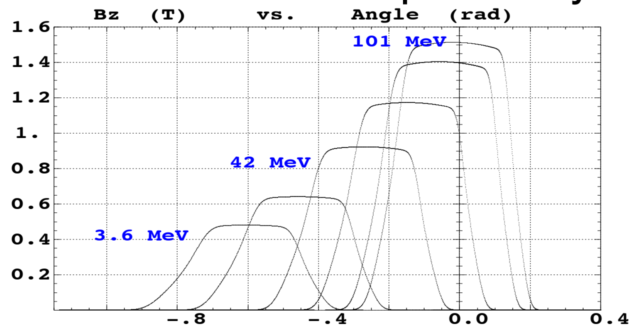




2 types of gap: constant / linear



"TRANSPORT" versus "stepwise ray-tracing" :



(MeV) Energy	linear gap Mathematica / BeamOptics ν_X	ν_Z
3.55	0.263381	0.187253
7.96	0.263381	0.187253
21.0	0.263381	0.187253
42.6	0.263381	0.187253
85.	0.263381	0.187253

	Ray-tracing ν_X	ν_Z
3.55	0.257628	0.187178
7.96	0.257625	0.187190
21.0	0.257616	0.187203
42.6	0.257616	0.187211
85.	0.257619	0.187220

Edge effect and Fringing Field Corrections :

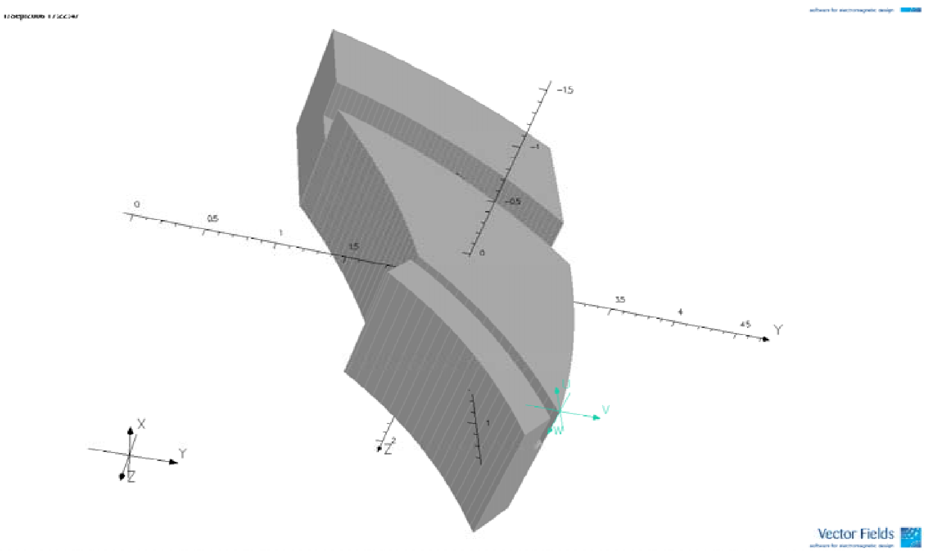
Focusing due to wedge ϵ : $\frac{1}{f}=\frac{-\tan(\epsilon)}{\rho}$.

Correction for field extent : $\epsilon \rightarrow \epsilon - \frac{g I_1 (1+\sin(\alpha)^2)}{\rho\cos(\alpha)}$,

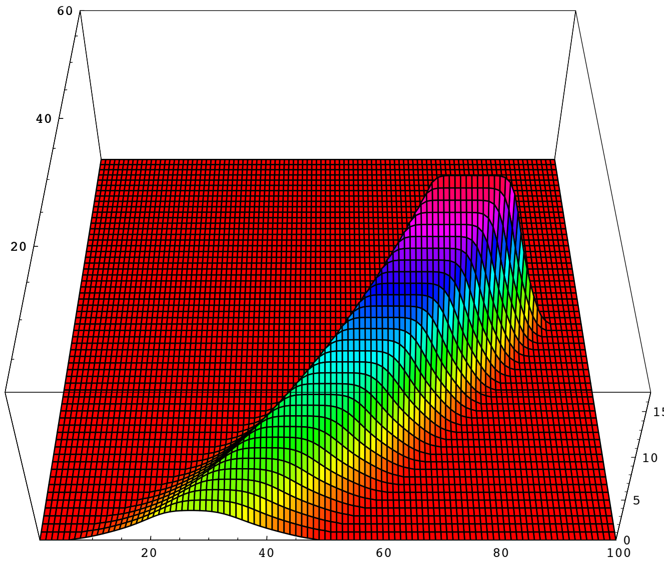
with $I_1 = \int_{-\infty}^{+\infty} \frac{B_Z(s)(B_0 - B_Z(s))}{g B_0^2} ds, \quad \alpha = \epsilon - 1.2\frac{K_1 g}{\rho}$

g = local gap,ρ = local curvature radius, B_0 = reference field.

Field maps : not a simple problem, either

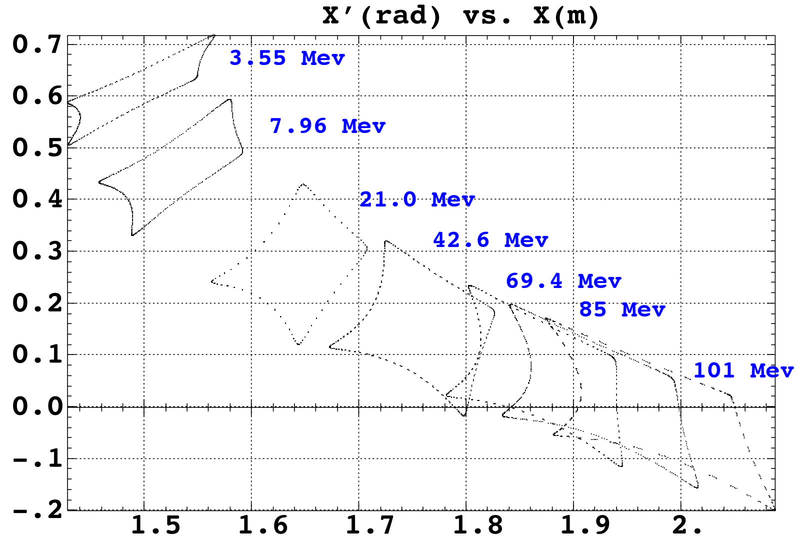


Spiral pole. TOSCA code.

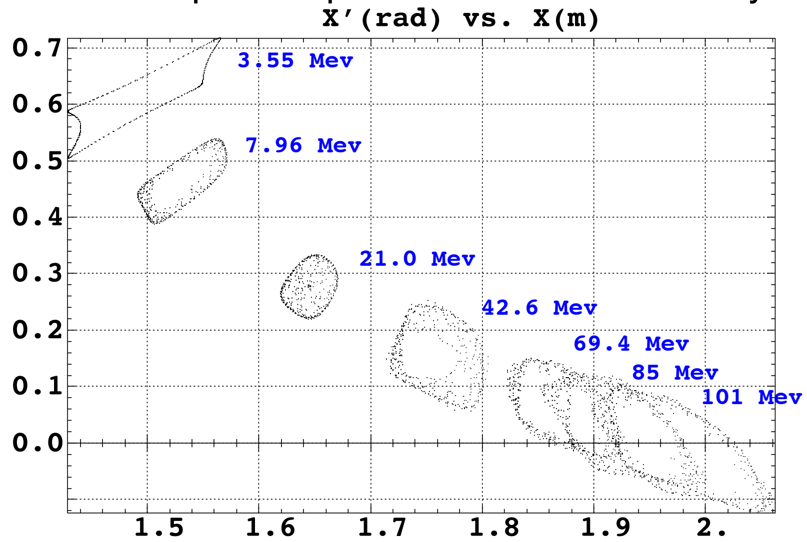


Mid plane field map.

Allows large amplitude, DA tracking :



Evolution in phase space at horizontal stability limit, z=0



Evolution in phase space at horizontal stability limit, z=ϵ

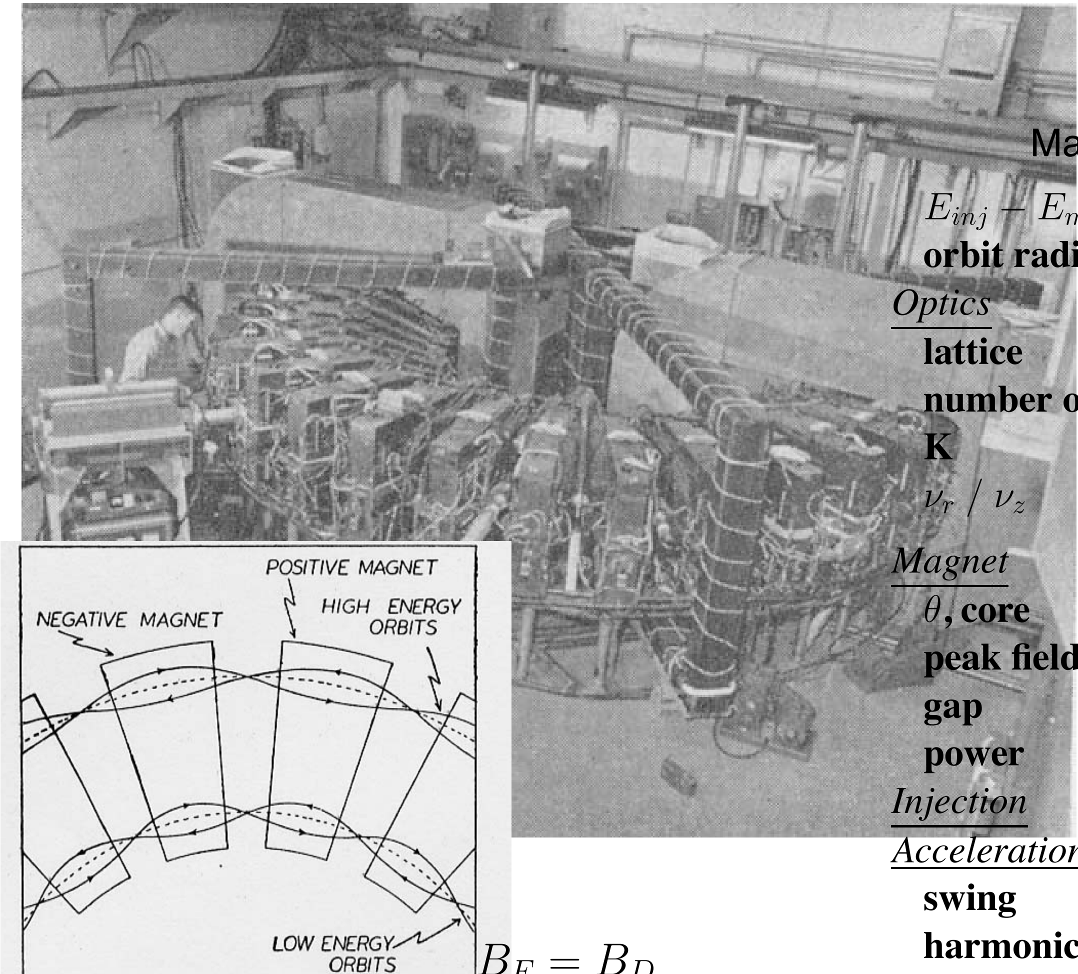
Second MURA radial sector FFAG, electron, 50 MeV, 2-way

Preliminary studies early 1957. The spiral sector e-model was not yet completed - this determined the choice of radial sector : easier to design, better understood.

Some objectives of that study : RF stacking, high circulating I, 2-way storage.

First operation Dec. 1959, 2-beam mode, 27 MeV ;

disassembled in 1960, magnets corrected ; second start Aug. 61, single beam, 50 MeV.



$B_F = B_D$

Today : $\mu + /mu^-$ acceleration in single ring !

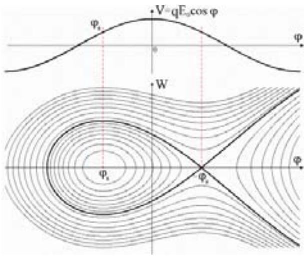
[Typical] data

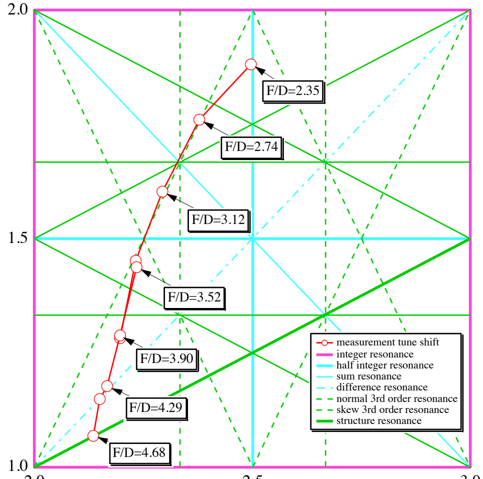
Machine parameters			criteria / comments
$E_{inj} - E_{max}$	*MeV*	**0.1 - 50**	*reasonable size & beam life-time*
orbit radius	*m*	**1.20 - 2.00**	
Optics			
lattice		**FODO**	$B \approx B_0(r/r_0)^K \cos(16\,\theta)$
number of cells		**16**	*32 magnets, 3.15 deg. drifts*
K		**9.25**	
ν_r / ν_z		**4.42 / 2.75**	
Magnet		**radial sector**	
θ**, core**	*deg*	**6.3**	
peak field	*T*	**0.52**	r_{max}
gap	*cm*	**8.6**	
power	*kW*	**100**	
Injection			*e-gun + e-inflector*
Acceleration			
swing	*MHz*	**20 - 23**	
harmonic		**1**	
voltage p-to-p	*kV*	**1.3 - 3**	
cycle rep. rate	*Hz*	**60**	

3 Prototyping and construction in Japan

First proton FFAGs, KEK

- **POP - Proof of principle. First accelerated beam 2000.**

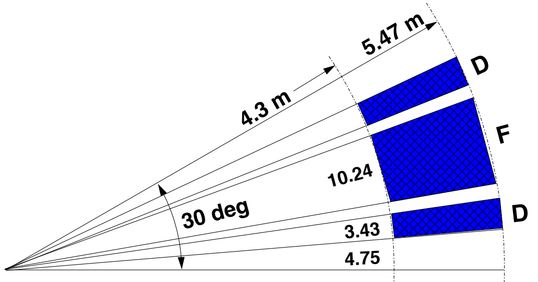


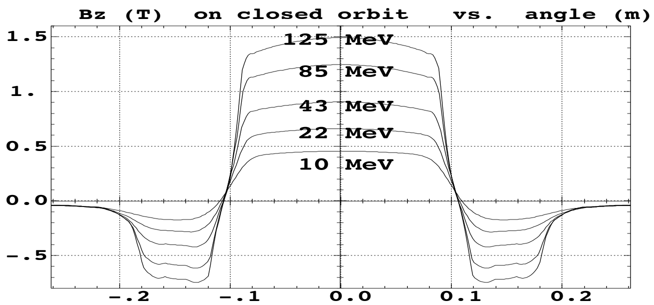


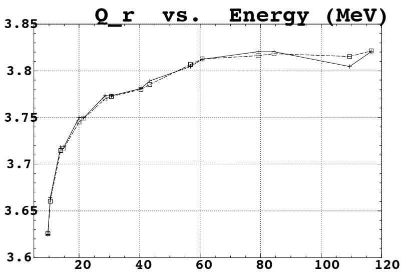
[Typical] data

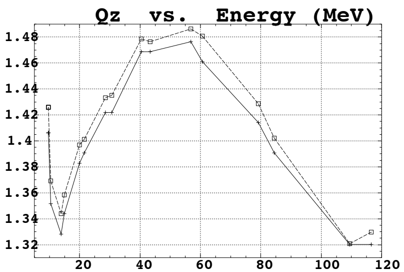
$E_{inj} - E_{max}$	keV	50 - 500	
orbit radius	m	0.8 - 1.14	
Optics			
lattice		DFD	
number of cells		8	
K		2.5	$(B = B_0(r/r_0)^K F(\theta))$
$\beta_r,\ \beta_z$ max.	m	0.7	
$\nu_r\ /\ \nu_z$		2.2 / 1.25	*tunable via B_F/B_D ratio*
Magnet	**high field, non-linear gradient**		*sector triplet*
$\theta_D\ /\ \theta_F$, core	deg	2.8 / 14	
$B_D\ /\ B_F$	T	0.04-0.13 / 0.14-0.32	$r_{inj} \to r_{max}$
gap	cm	30-9	$gap = g_0(r_0/r)^K$
Injection		multi- or single-turn	{ *electrostatic inflector + 2 bumpers*
Extraction	**massless septum exprmnt**		
Acceleration			55 kW amp.
MA alloy RF core		**high $\vec{E}$, low Q, broad band, RF**	**; Ex. : 2-beam accel.**
swing	MHz	0.6 - 1.4	
harmonic		1	
voltage p-to-p	kV	1.3 - 3	
rep. rate cycle time	**ms**	**1**	*fast acceleration*
rep. rate	**kHz**	**1**	*high average current*
equiv. $\dot{B}$	T/s	180	

"return yoke free" magnet









• 150 MeV protype FFAG : medical beams, ADS-reactor, NuFact muon accelerators

Start up 2003. Full acceleration cycle, 9-100 MeV mode, spring 2005.

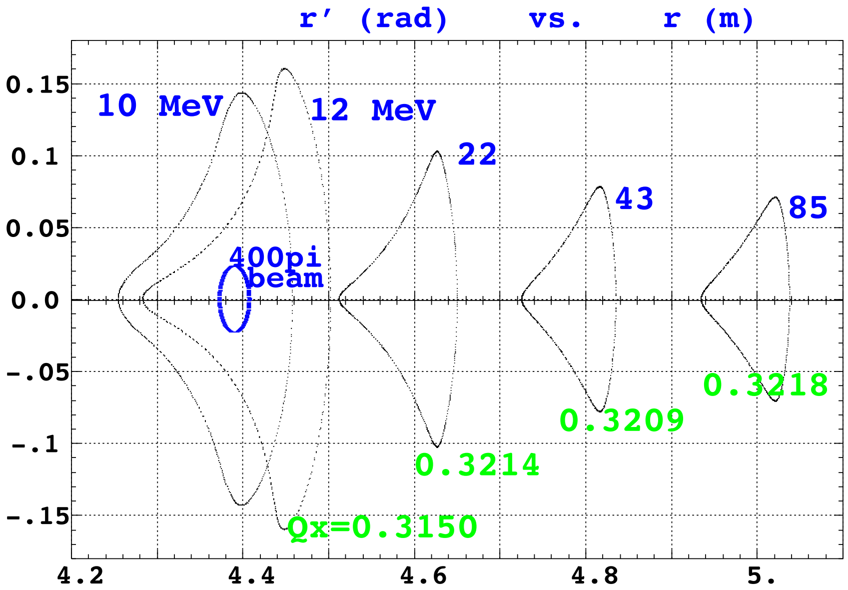
[Typical] data

$E_{inj} - E_{max}$	MeV	12 - 150	
orbit radius	m	4.47 - 5.20	
Optics			
lattice		DFD/12 cells	*9.5 deg. drift*
K		7.6	$(B = B_0(r/r_0)^K F(\theta))$
β_r / β_z max.	m	2.5 / 4.5	
ν_r / ν_z		3.7 / 1.3	*tunable via* B_F/B_D *ratio*
α, γ_{tr}		0.13, 2.95	$1/(1+K)$, $(1+K)^{1/2}$
Magnet	**Return yoke free magnet**		
θ_D / θ_F	deg	3.43 / 10.24	
B_D / B_F	T	0.2-0.78 / 0.5-1.63	$r_{inj} \longrightarrow r_{max}$
gap	cm	23.2 - 4.2	*at* r_{inj} - r_{max} $(gap = g_0 \left(\frac{r_0}{r}\right)^K)$
Injection		multi-turn	$\left\{\right.$ *B-septum + E-septum + 2 bumpers*
Extraction		single-turn	**fast kicker (1kG, 150 ns)**
Acceleration	*Amorphous MA*		
swing	MHz	1.5 - 4.5	
harmonic		1	
voltage p-to-p	kV	2	
ϕ_s	deg	20	
ν_s		0.01 - 0.0026	
rep. rate	**Hz**	**250**	*high average current*
equiv. $\dot{B}$	**T/s**	**300**	*fast acceleration*

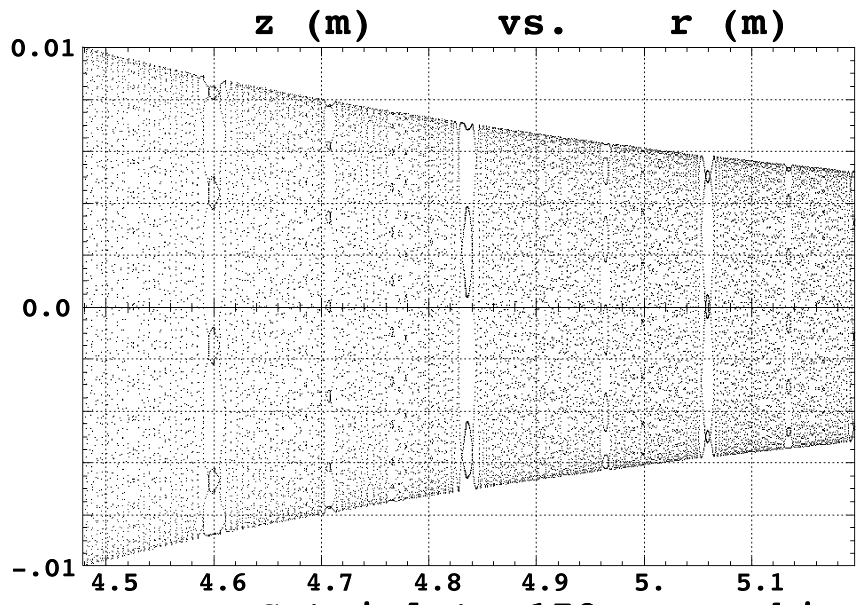
A strong concern in the design of the Neutrino Factory :

The design of such large acceptance, non-linear machine *must* resort to precision tracking - a statement by the MURAs', 1950s, they were using Runge-Kutta type of methods...

Regarding tunes (field inhomogeneities), amplitude detuning, motion stability limits (DA), 6-D transmission, etc.,



Motion stability limits ("DA") about 1πcm.

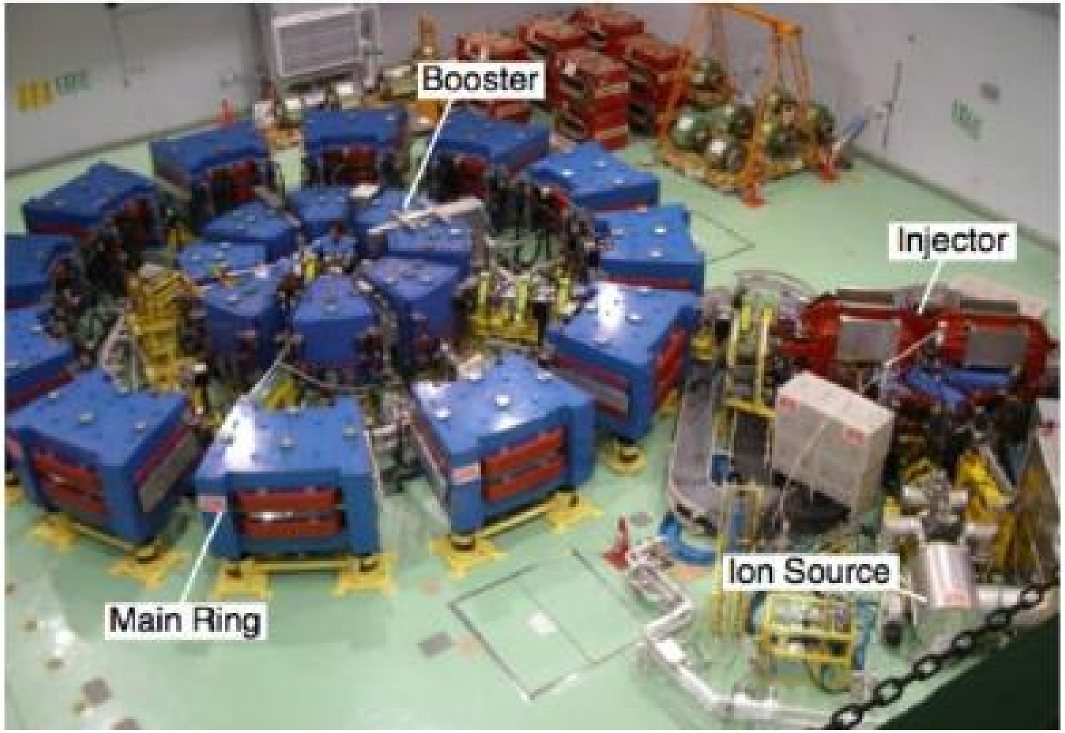


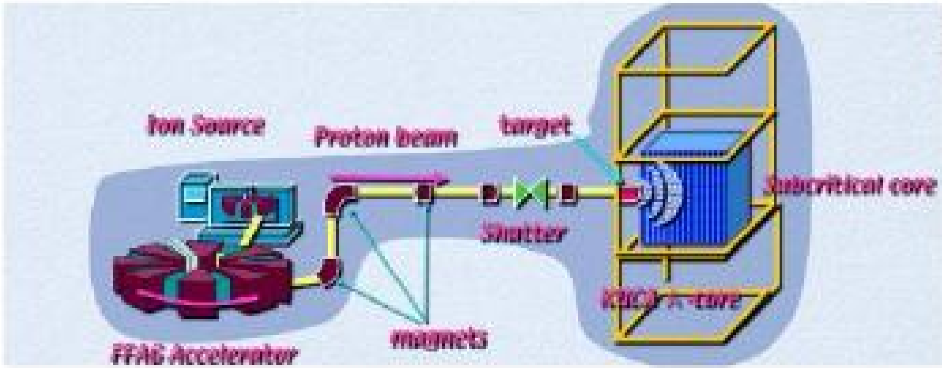
6-D acceleration, 12 to 150 MeV - adiabatic damping of vertical motion. The motion spans $\Delta R \approx 0.5$ m !.

It means that, end-to-end simulations in the neutrino factory require dedicated developement of stepwise raytracing based computation methods.

ADS/reactor prototyping, Kyoto University Research Reactor Institute

ADS/Reactor *and* accelerator prototyping, output power $\sim$10 W
Beam power needed from FFAG < 0.1 W (typically 100 MeV, $<$1 nA)
Further (longer term) objectives from accelerator installation : 25-150 MeV p beam, up to 1 μA (120 Hz)
Now operated at 100 MeV, 0.1 nA, beam extracted towards 5W reactor core





B ~ r^K

ERIT : 10 MeV neutron production for BNCT

A compact proton storage ring for BNCT application.

High flux is needed at patient : $\approx 2\,10^{13}$neutrons in 30 minutes for typical tumor volume

Today, a 5-10 MW reactor is used, there is needed for hospital environment compliant equipment : ERIT

Injector (425 MHz RFQ + IH-DTL)

H-, kinetic energy 11 MeV
Peak/average beam current 5 mA / $> 100\mu$A
Repetition rate 200 Hz, d.c. 2%

FFAG ring

FDF lattice, 8 cells
H- injection on internal Be target ($5 - 10\mu$m thick)
proton energy 11 MeV
circulating current 70 mA

ERIT system

Beam survival 500-1000 turns
Target lifetime $>$ 1 month
ΔE / turn 70 keV

RF cavity

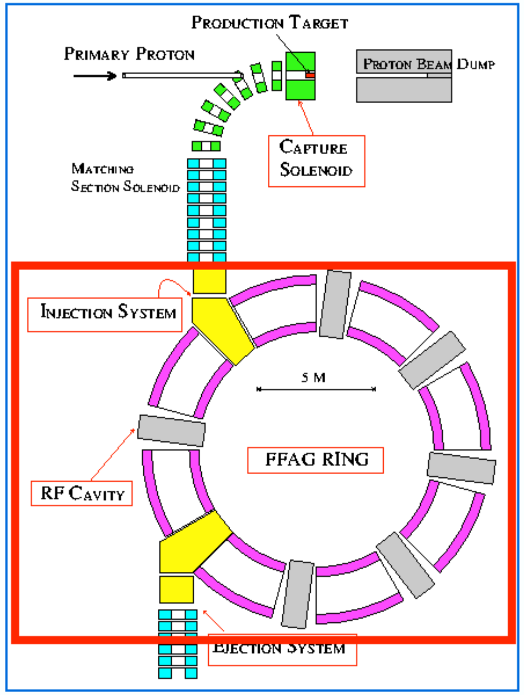
Operated CW, 100 kW input power
RF voltage / frequency 250 kV / 18.1 MHz
Harmonic number 5

PRISM. Muon bunch, phase rotator

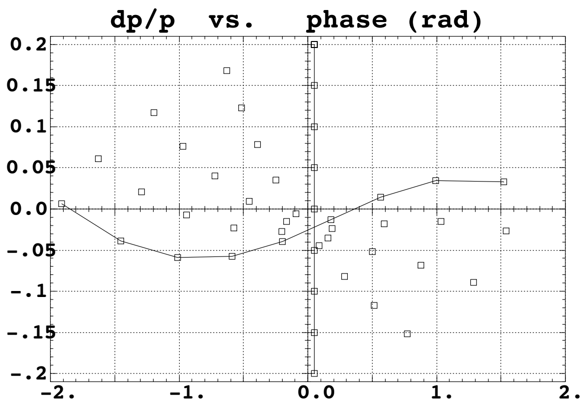
FFAG used as phase rotator, for momentum compression

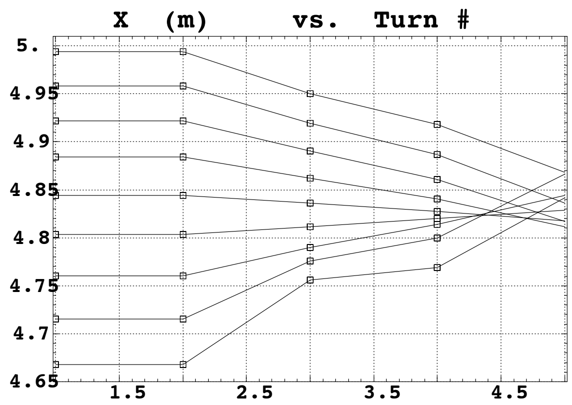
p=68MeV/c +/-20% down to +/-2% in 6 turns

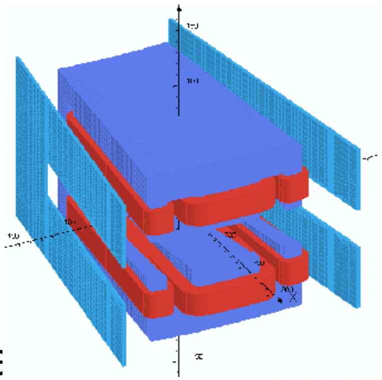
FFAG brings : large geometrical acceptance, zero chromaticity

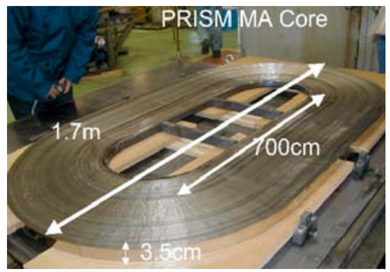


- DFD lattice 14t triplet yoke, 120 kW/triplet
- K, B_F/B_D variable $\rightarrow$ quasi-decoupled ν_x, ν_z adjustments
- H / V apertures : 1 / 0.3 m
- acceptance : 4 π cm.rad $\times$ 0.65 π cm.rad
- RF : 5-gap cavity, 33 cm gap, 150-200 kV/m, 2MV/turn









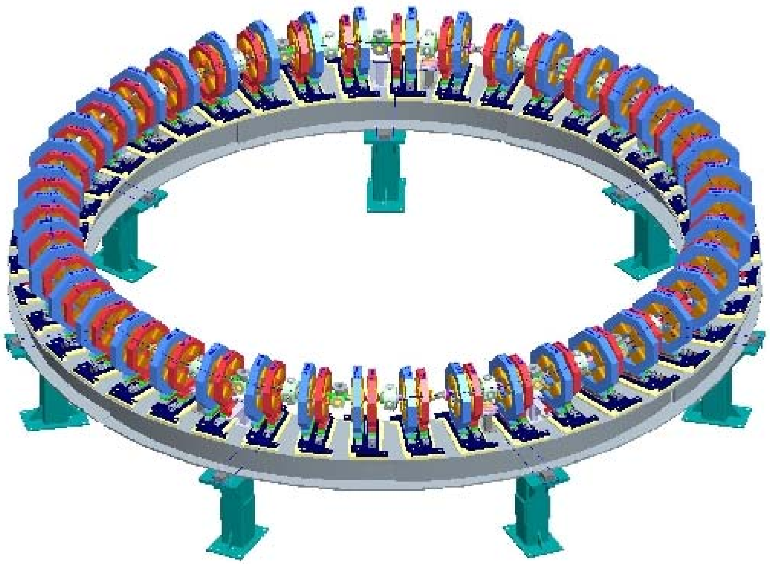
- Optics design : requested large acceptance can be achieve
- difficult task : injection & extraction
- 6 magnets produced, 6-cell ring built
- RF system : more than 156kV/m at 5MHz expected, under R&D

4 Prototyping and construction in EU

• The EMMA experiment. Experimental model of a linear FFAG

Goal : investigate the new concept of "linear FFAG" :

- linear magnets ("easy", quadrupoles) ⟶ yields huge acceptance
- with fixed field ("easy", e.g. regular SCRF) ⟶ yields fast acceleration
- fast acceleration ⟶ requires lots of RF, and fixed frequency, gutter acceleration

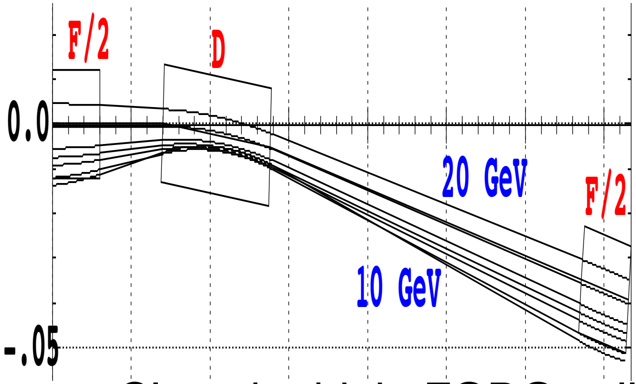


A model of Study IIa FFAG
10 to 20 MeV
42 cells, doublet
pole-tip $fields\ \approx 0.2\,T$
apertures $\approx \phi 40$ mm
37cm cell length
16m circumference
1.3GHz RF
1 cavity every other cell

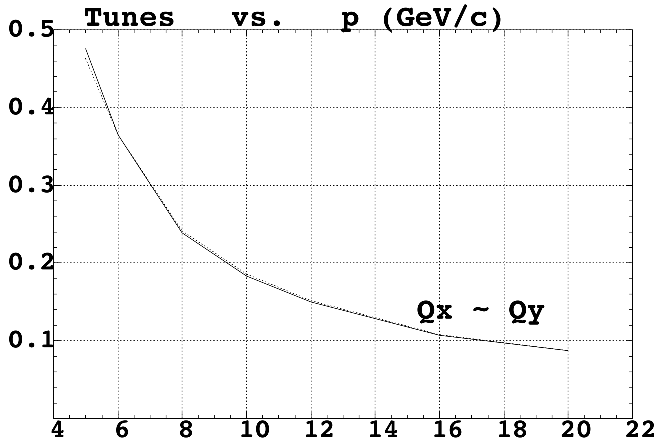
- **Launched in the frame of Neutrino Factory R&D**
- **An experimental model of muon accelerators**
- **International collaboration :**
  BNL, CERN, FNAL, LPSC, STFC, J.Adams Inst., Cockcroft Inst., TRIUMF
- **Recollection :**
  1999 : principle of linear FFAG optics, FNAL
  2001 : first e-model meeting, BNL
  2006 : project funded by "British Accelerator Science and Radiation Oncology Consortium",
  3.5 years : 04-2007 / 09-2010, £5.6M budget
- **Construction started at Daresbury, 04/2007, first beam planned summer 2009**

Beam due Autumn 2009

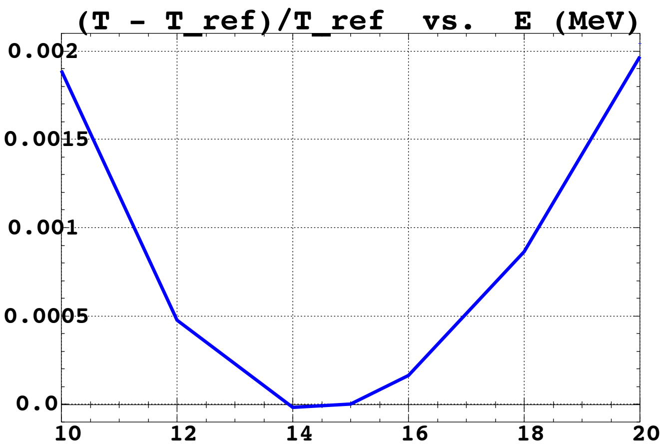
Principles of particle dynamics in a linear FFAG



Closed orbit in FODO cell

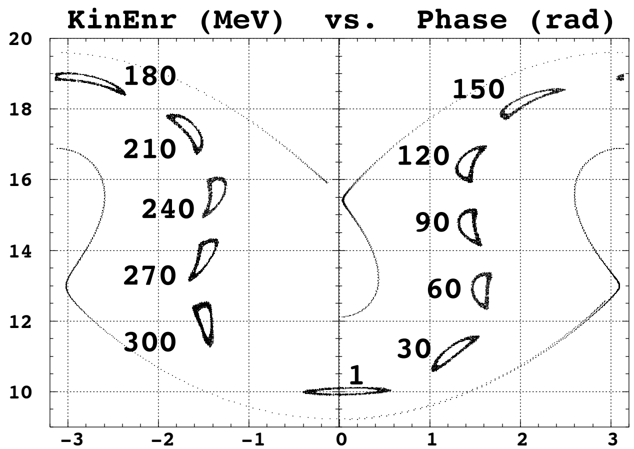


Tunes versus energy



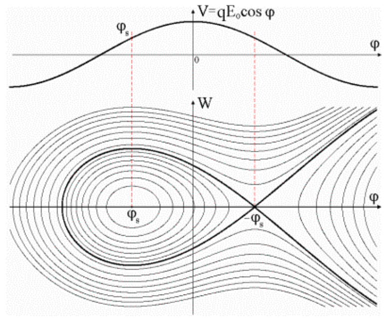
Time of flight parabola ($\gamma \approx \gamma_{tr}$)

$$\frac{\delta TOF}{TOF} \approx \left[\eta_0 \frac{\delta p}{p}\right] + \eta_1 \left(\frac{\delta p}{p}\right)^2$$



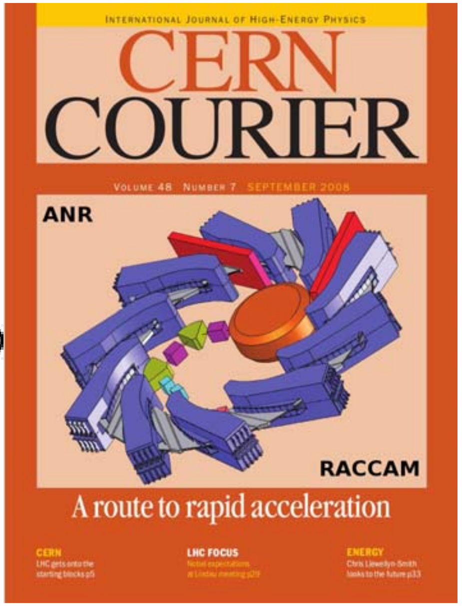
Longitudinal motion

$$H \approx \sin^2 \pi\phi + [a \left(\frac{\delta p}{p}\right)^2] + b \left(\frac{\delta p}{p}\right)^3$$



• **Medical application : the RACCAM project**

A 3 year ANR Contract, 2005-2009. 500 kEU grant, 1.3 MEu total budget.



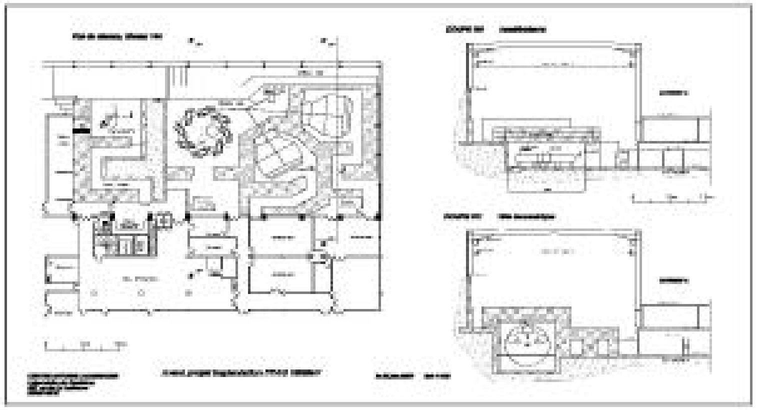
• **Motivations for a medical FFAG project :**

(i) Hadrontherapy is considered to be more effective for cancer treatment co
to photons

(ii) FFAGs appear to have various advantages in medical applications:

- Potential for variable energy operation (→ no need of degrader, ESS)
- High dose delivery, potentially >> Gy/min (←high rep rate, potentially 100s Hz)
- Potential for reasonably compact size (if needed...) and low cost
- Flexibility : synchrotron-like manipulattion of beams
  (injection, extraction, multi-particle)
- Stable and easy operation (← fixed magnetic field)
- Potential for multiple extraction ports
- Natural scanning method : bunch-to-voxel

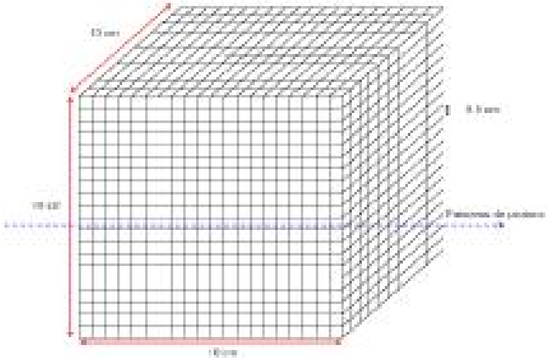
(iii) Possible implementation of a demonstration machine at the Nice anti-cancer clinic (MEDICYC)



Beam delivery hypothesis (our working hypothesis)

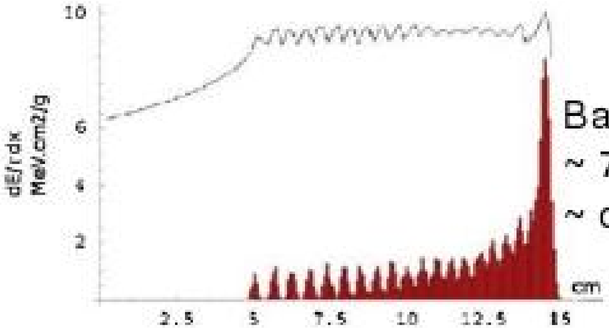
(i) Reference volume :

- 1 liter cube, 10x10x10cm^3
- Voxel size : 5x5x5 mm^3



(ii) Spread out Bragg pic (SOBP) :

$$\left|-\frac{dE}{dx}\right| = 2\pi N_a r_e^2 m_0 c^2 \rho \frac{Z}{A}\frac{z^2}{\beta^2}\left[\ln\left(\frac{2m_0\gamma^2 v^2 W_{max}}{I^2}\right) - 2\beta^2 - \delta - 2\frac{C_S}{Z}\right]$$



Basic results :
~ 7 10^{11} proton/Gy in a liter
~ only weak dependence on E

(iii) Bunch filling for bunch-to-pixel scanning :

- **400 pixels/slice**, about 20% of the dose in the distal layer, yields a maximum

  5Gy x 7.10^{11} p/Gy x 20% / 400pixel ~ **2x10^9 p/pixel in the distal layer**
- Cyclotron produces 1.5x10^7 ppb, hence need ~**130 cyclotron bunches**
- Cyclotron RF 70MHz (h=3) and FFAG 3MHz @ injection,

i.e. 70MHz/3MHz*h3 = **70 cyclotron bunches in one FFAG turn.**

Hence, for distal pixels,

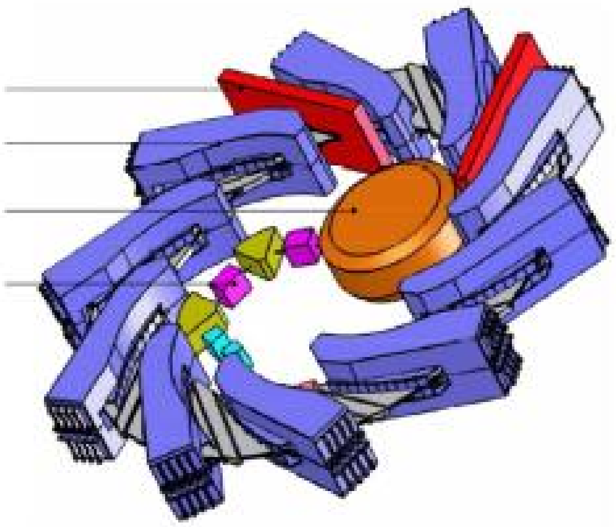
either (i) 5-turn injection at 50% efficiency

or (ii) single-turn injection and a minimum of 4 paintings

(iv) Repetition rate needed :

20x20x20 pixels in a minute, hence, ~**130Hz**

We end up with a set of optimized parameters of the RACCAM 10 cell ring and magnet :

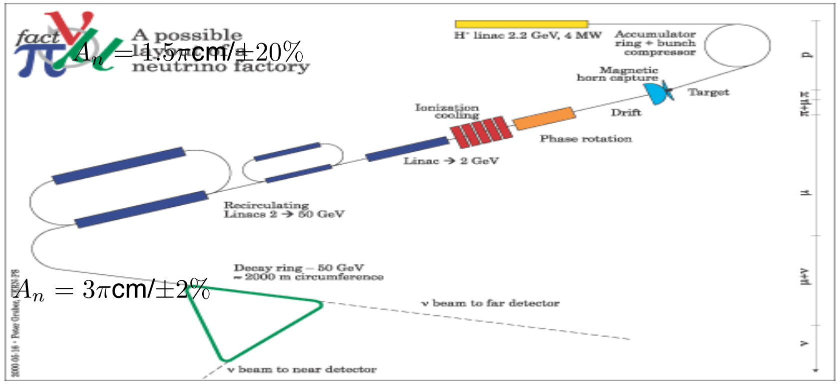


Extracion energy, variable (MEDiCYC specs.)	70 – 180 MeV
Injection energy	5.5 – 17 MeV
Nomentum ratio	3,62
Number of cells	10
Packing factor	0,34
Field index, k	5
Spiral angle	53.7 deg.
Qh / Qv	2.76 / 1.55~1.60
Radius on extraction/injection orbit : dR	3.46 m / 2.78 m / 0.67 m
Drift length, extraction/injection orbit	1.42 m / 1.15 m
Frev, 15->180 MeV	3.03 -> 7.54 MHz
Frev, 5.5->70 MeV	1.86 -> 5.07 MHz

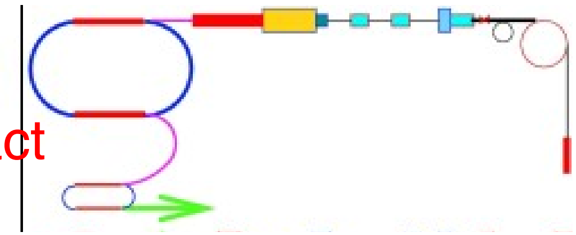
5 The Neutrino Factory

NuFact R&D has triggered strong activity in FFAG design, and lead to the development of new concepts.
Recollection :

Europe NuFact



US NuFact



Study 2 Costs ….

- Study I, II ν-Factory – feasible but too expensive
- Biggest cost item: acceleration (~600M$)

Table A.1: Construction Cost Rollup per Components for Study-II Neutrino Factory. All costs are in FY01 dollars.

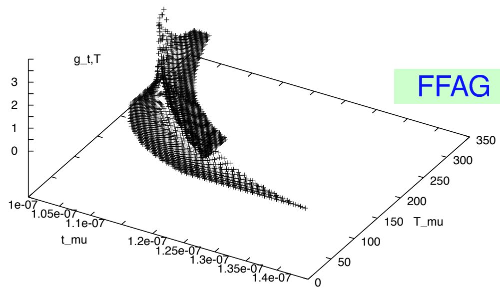
System	Magnets ($M)	RF power ($M)	RF cav. ($M)	Vac. ($M)	PS ($M)	Diagn. ($M)	Cryo ($M)	Util. ($M)	Conv. Facil. ($M)	Sum ($M)
Proton Driver	5.5	7.0	66.1	9.8	26.6	2.2	28.5		21.9	167.6
Target Systems	30.3			0.8	3.5	8.0	18.8		30.2	91.6
Decay Channel	3.1			0.2	0.1	1.0	0.2			4.6
Induction Linacs	35.0		90.3	4.4	163.3	3.0	3.6		19.5	319.1
Bunching	48.8	6.5	3.2	2.7	2.1	5.0	0.3			68.6
Cooling Channel	127.6	105.6	17.7	4.3	4.8	28.0	9.5		19.5	317.0
Pre-accel. linac	46.3	68.4	44.1	7.5	3.0	6.0	13.6			188.9
RLA	129.0	89.2	63.4	16.4	5.6	4.0	28.9		19.0	355.5
Storage Ring	38.5			4.8	2.2	29.0	4.8		28.1	107.4
Site Utilities								126.9		126.9
Totals	**464.1**	**276.7**	**284.8**	**50.9**	**211.2**	**86.2**	**108.2**	**126.9**	**138.2**	**1,747.2**

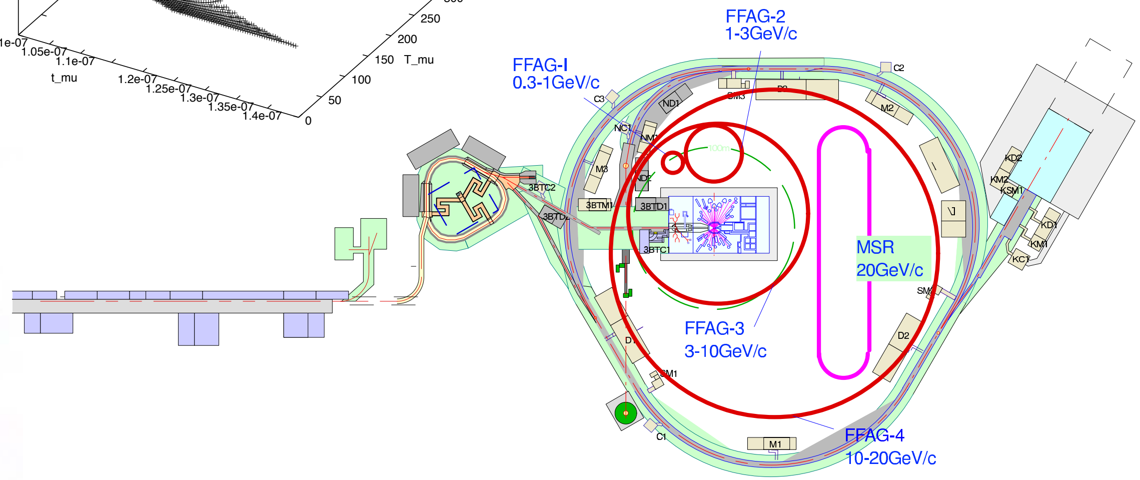
The Europe and the two US NuFact studies at first proposed to accelerate muons up to the storage energy (20 or 50 GeV) by means of one or two 4- or 5-pass RLA's. RLA's are complicated machines (spreaders, combiners), hence expensive.

The Japan NuFact

50-GeV, $3.3\,10^{14}$ ppp with 0.3 Hz (15 μA) / 0.75 MW
Four muon FFAG's : 0.2-1 GeV, 1-3, 3-10 (SC), 10-20 (SC).
No cooling, technology simpler, compact (R≈200m)

30ns/300±50% MeV bunch

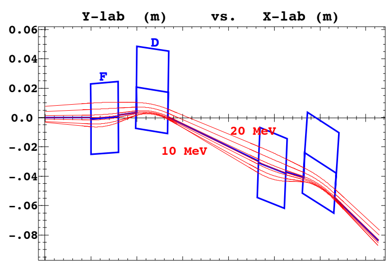


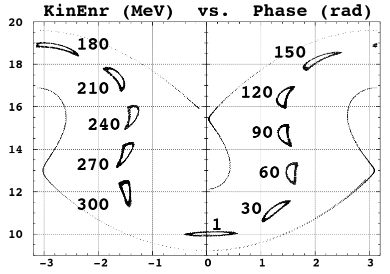


Acceleration rate is lower than RLA, requires larger distance, but, acceptance is larger both transversally (twice : DA $3\ \pi$cm norm. at $\delta p = 0$) and longitudinally (≈ 5 eV.s). Hence achieve comparable production rate : $\approx 10^{20}$ muon decays per year (1 MW p power).

US-Study-2a : based on linear FFAG

- FFAG based on linear optical elements (quadrupoles)
- orbits no longer scale, tunes are allowed to vary with energy





This has a series of consequences :

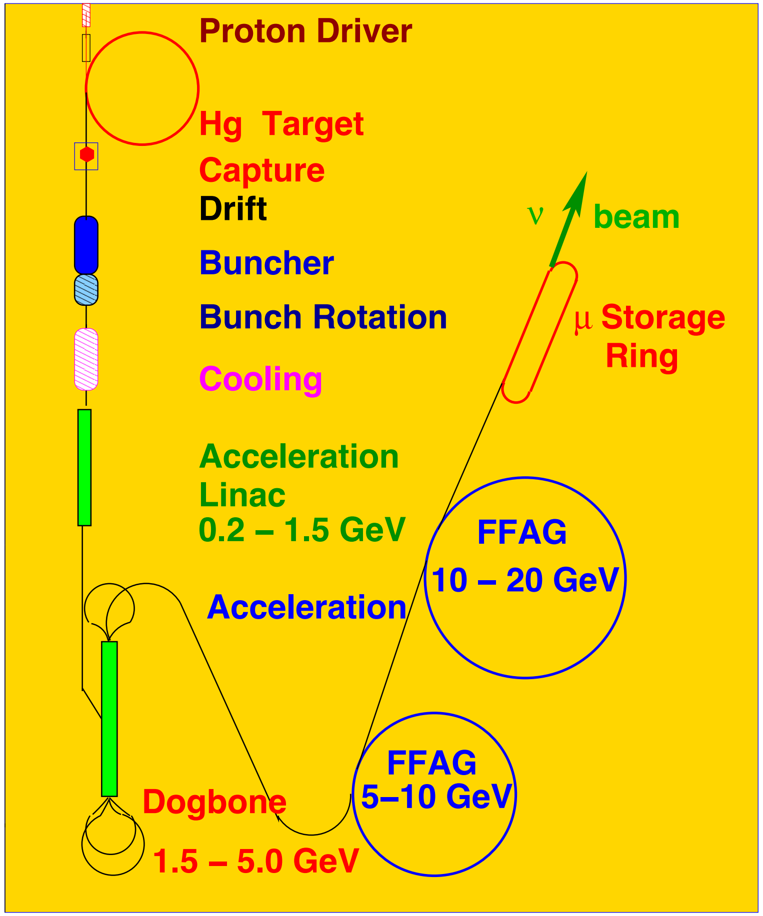
• $R/\rho < 2$ - this decreases the machine size compared to classical (scaling) FFAG

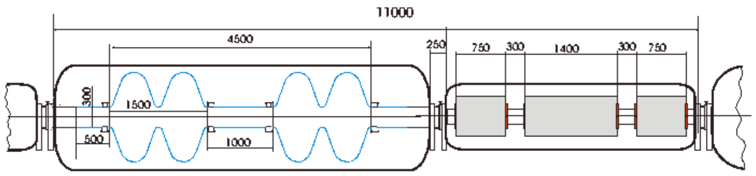
• horizontal beam excursion is reasonable (small D_x)
$\rightarrow$ **magnets apertures are much smaller**

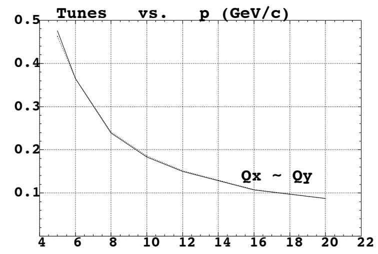
• yields **large transverse acceptance** $\leftarrow$ fields are linear (3πcm achieved)

• small δTOF over energy span, allows **fast acceleration** high gradient RF (200 MHz type SCRF cavities)

• Above 5 GeV, non-scaling linear FFAG method yields **lower cost/GeV than RLA.**





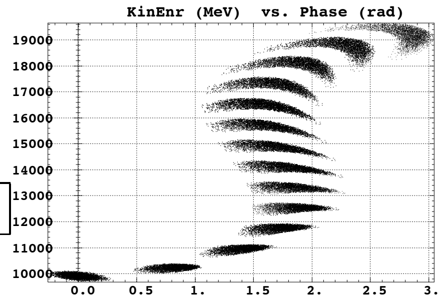


In practice, the cell tune is allowed to decrease from just below a half-integer value at injection, to just above the lower integer (negative natural chromaticity)

- hence tens of cells cause **resonance Xing**, tens of integer and $\frac{1}{2}$integer resonances.
- yet the **crossing is fast**, this should result in not too stringent tolerance on alignments and field defects.

There are other issues, as

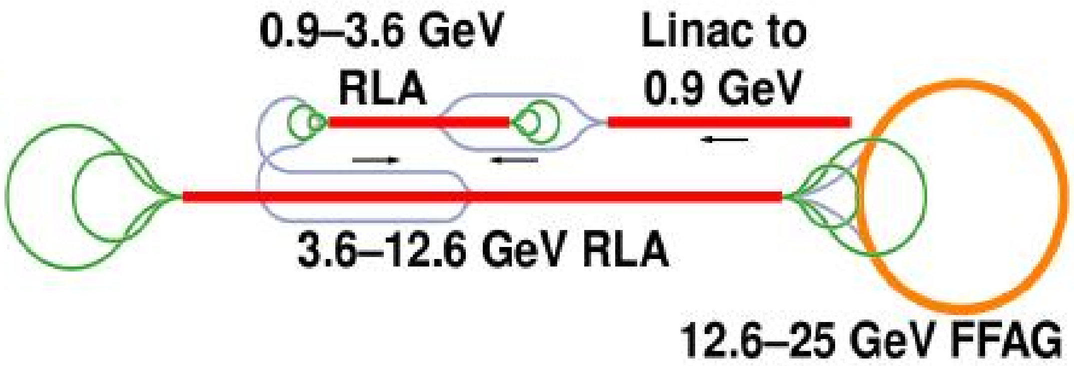
- amplitude de-timing
- difficulty of longitudinal matching from upstream muon FFAG to downstream one



• THESE QUESTION REQUIRE EXPERIMENTAL WORK → EMMA prototyping project.

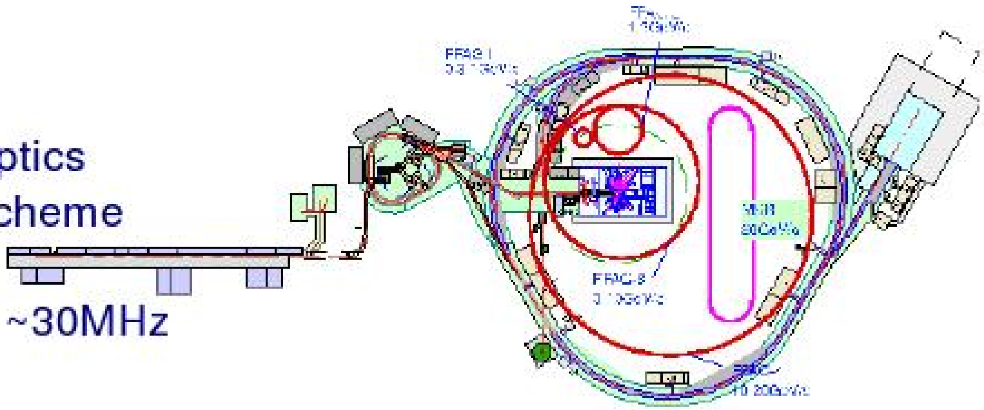
Today's, "IDS-NuFact" baseline muon ccelerators layout relies on single, top energy linear FFAG ring

Based on 200 MHz RF, ~1500MV
30 mm transverse acceptance
Circumference ~450m
Quads r / field ~ 10-20cm / 4-8T



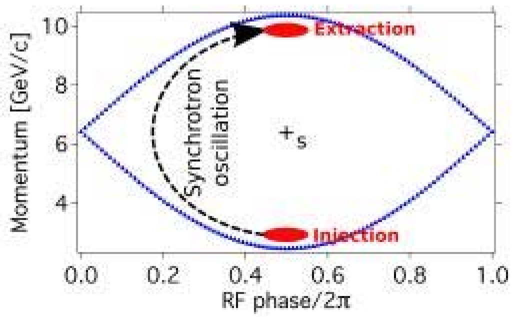
Further "scaling" non-linear FFAG method assessements

- "Scaling" (tune-invariant) optics
  The japan NuFact scheme



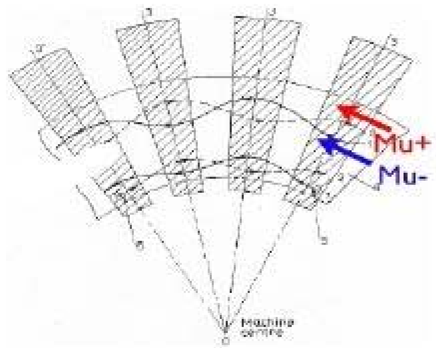
Based on low-frequency RF ~30MHz
Large 6D acceptance

Stationary bucket acceleration
f_{RF} < 100 Hz
small phase slip, convenient

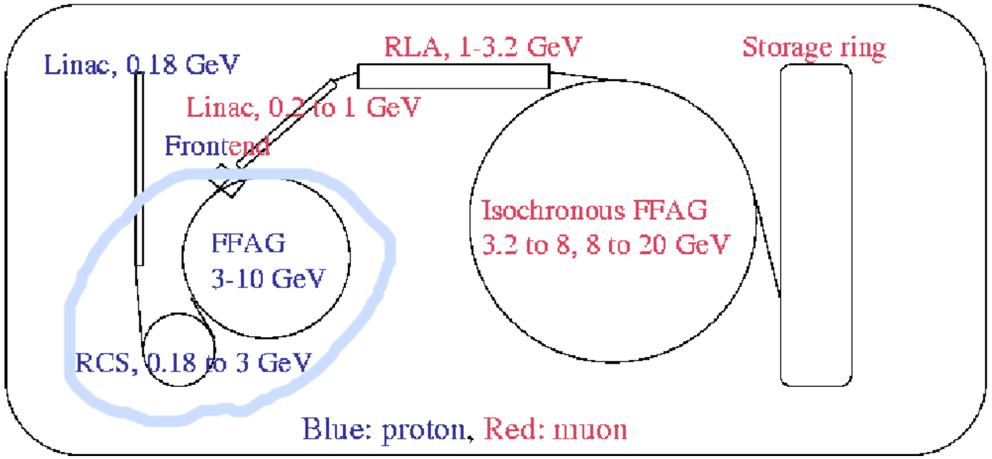


Harmonic number jump, T_n-T_{n-1} = p / f_{RF}
$f_{RF > 100\ Hz}$

compatiblity with two-beam ring ?



UK NuFact possible options : FFAGs for proton driver stage and for muon acceleration



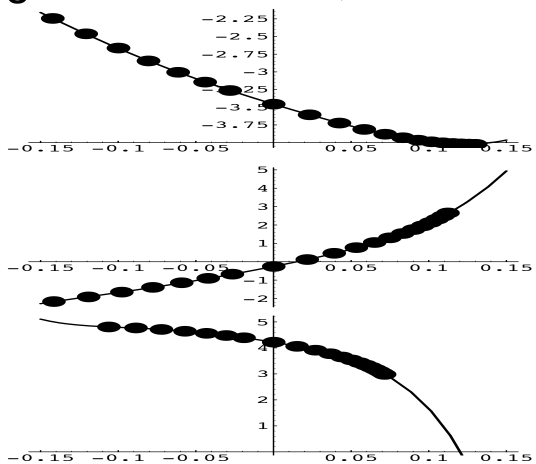
NFFAGI p-Driver		
Energy	(GeV)	3→10
Power	MW	4
Circumference	m	686
Q_x/Q_z		19.2 → 19.4 / 13.7
# N/I cells per super-p		21 / 13
# super-periods		2
# cells		70
N-cell/I-cell length	m	6.4/10.2
RF range	MHz	14-14.37 / 86.2
RF voltage	MV/turn	1
h accel./compress.		33 / 198
bunch length / compressed	ns	2 / 1
# bunches		5
ppb		10^{13}
pulse rate	Hz	50

Based on non-linear lattice that yields **possibility of insertions** , with the advantages of

1. easier injection and extraction,
2. space for beam loss collimators,

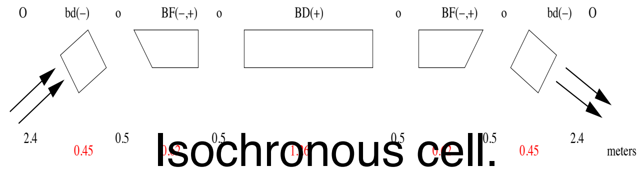
Muon lattice case : $\gamma = \gamma_{tr}$ at all energy hence $\eta \equiv 0$ (isochronism), allows on-crest acceleration

Magnetic field in bd, BF and BD.

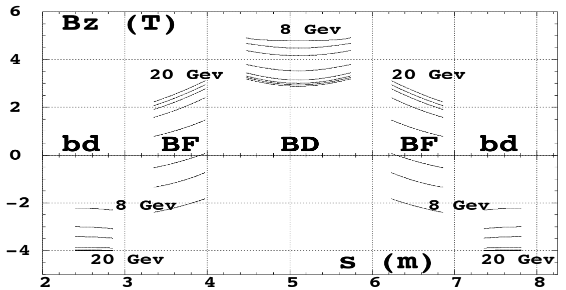


Isochronous muon FFAG lattice (cf. Thèse Franck Lemuet, CEA/DAPNIA & CERN/AB).

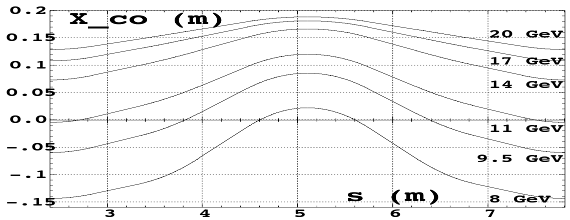
- Geometry, fields, isochronism design have first been assessed from matrix methods
- stepwise ray-tracing follows



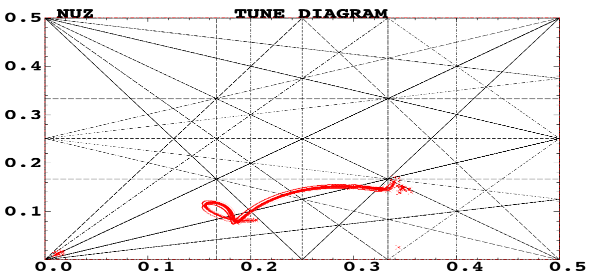
Isochronous cell.



Magnetic field on closed orbits at various energies.

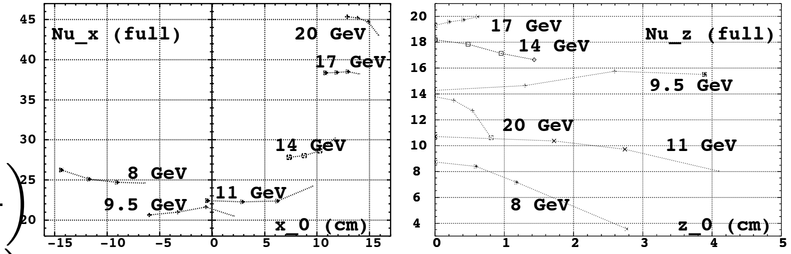


Closed orbits

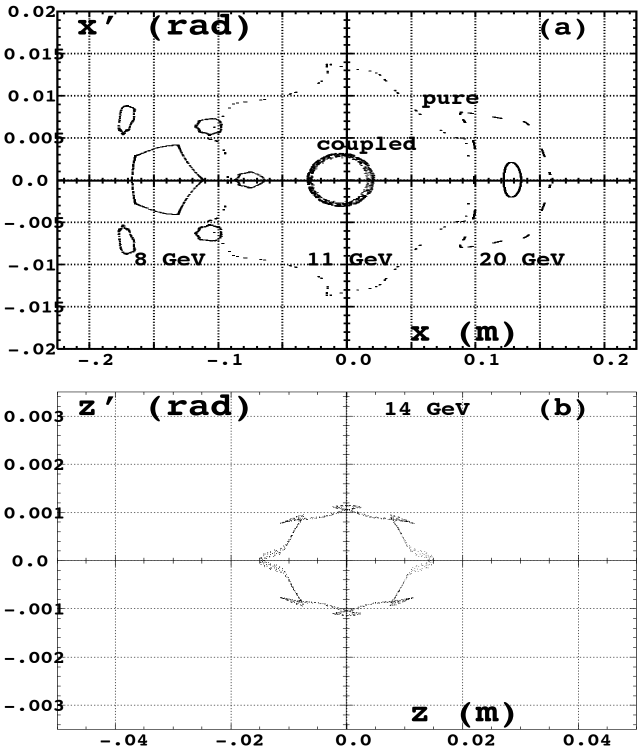


Beam trajectory in tune diagram, from 6 to 20 GeV (10 turns)

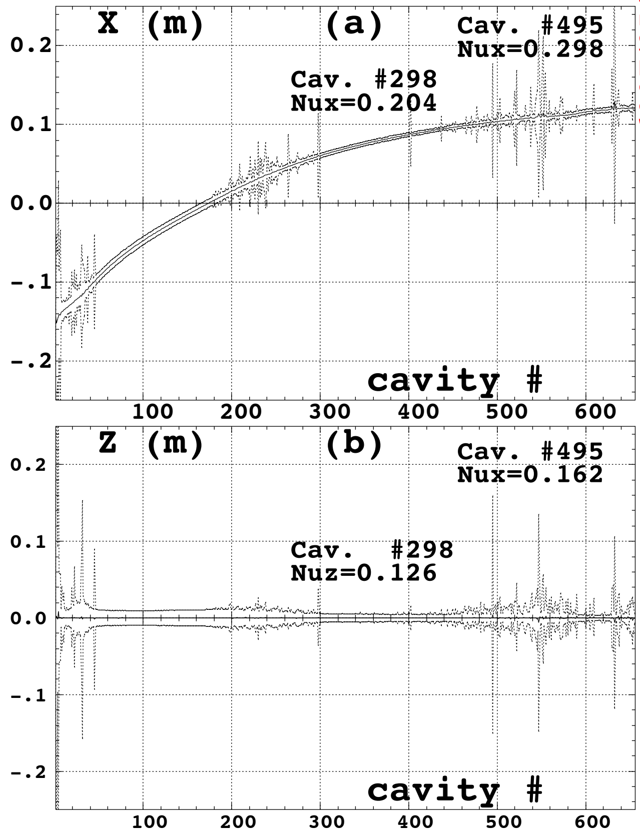
$$V_n(s,x,z) = (n!)^2 \left(\sum_{q=0}^{\infty} \frac{(-)^q\, G^{(2q)}(s)(x^2+z^2)^q}{4^q q!(n+q)!}\right) \times \left(\sum_{m=0}^{n} \frac{\sin(m\frac{\pi}{2})\, x^{n-m} z^m}{m!(n-m)!}\right)$$



Amplitude detuning and its energy dependence.



Stability limits, H and V.



Muon rate transmission

Further proton driver ideas

BNL / Linear FFAG

- **For neutrino factory,** 12 GeV design, several MW

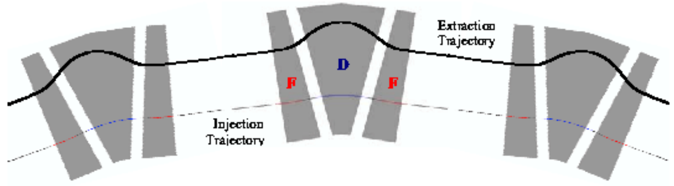
		Ring 1	Ring 2	Ring 3
Energy, Inj.	(GeV)	0.4	1.5	4.5
Extr.	(GeV)	1.5	4.5	12
# of turns		1800	3300	3600
cycle time	ms	6	9	10
Circumf.	m	807	819	831
# cells		136	136	136
cell length	(m)	5.9	6	6.1
h		136	138	140
RF freq.	MHz	36-46	46-49.7	49.7-50.4
E gain / turn	MeV	0.6	0.9	2

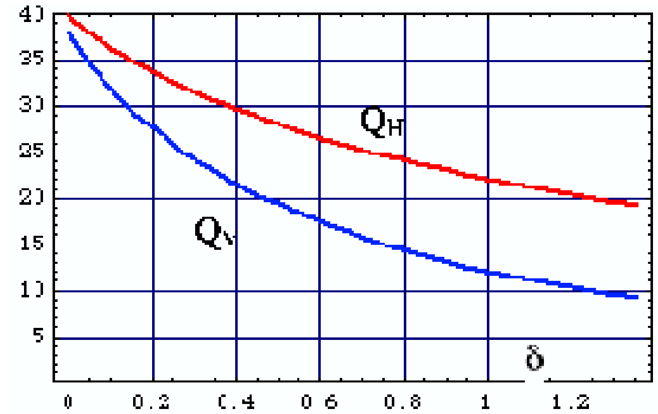
  Pulsed mode, 10^{14}ppp, rep. rate 100 Hz.

- Possibility of CW with acceleration based on “harmonic number jump”

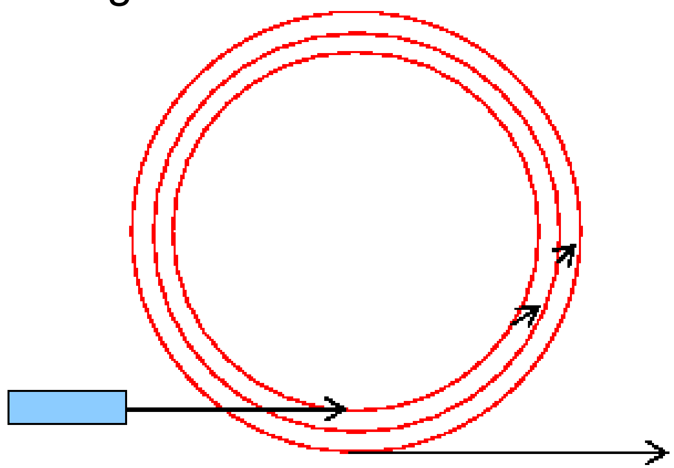
- Refs: (BNL) C-A/AP/208, C-A/AP/218, C-A/AP/219.

- FDF triplet





- 3 stages

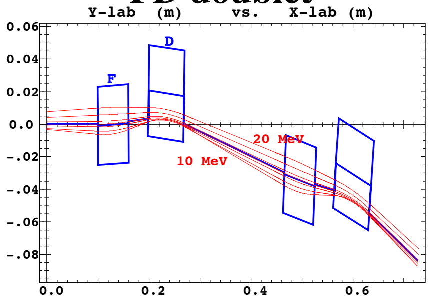


6 To summarize : FFAG lattices, DA, Chromaticity

Linear, non-scaling

(natural $\xi_{x,z}$)

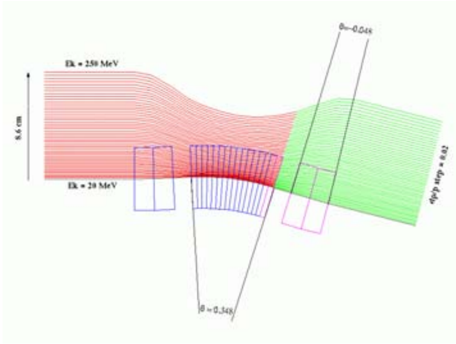
FD doublet



Concerns : muon, EMMA

Apps. : NuFact

FDF triplet



Concerns : proton, p-model

Apps. : hadrontherapy, p-driver

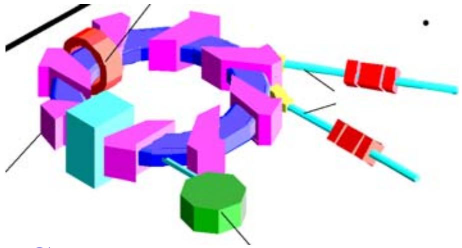
Scaling - $\frac{B}{B_0} = \left(\frac{r}{r_0}\right)^K$

(zero chromaticity, $\forall p$)

DFD triplet, doublet

SC technology

Spiral

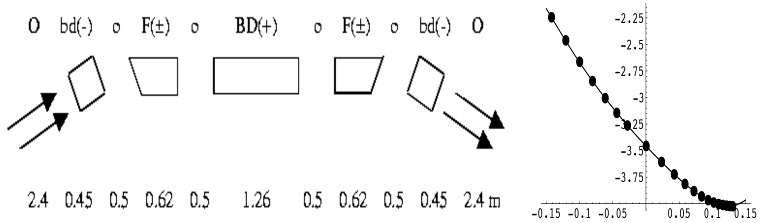


Concern : muon, e and p

Apps. : muon phys., NuFact, high power e and p, hadrontherapy, [R]Ions

Non-linear, non-scaling

Pumplet lattice

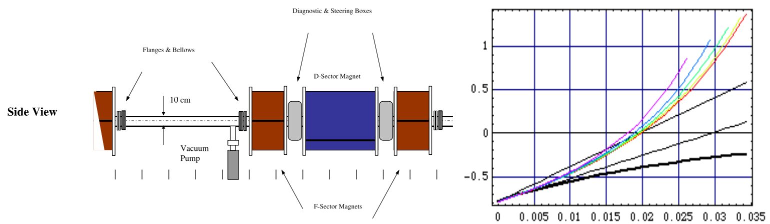


Concerns : muon and e-model (isochronous, $\xi_x > 0$, $\xi_z \rightarrow 0$) ; proton

Apps. : NuFact, p driver

Adjusted field profile ?

($\xi_{x,z} \longrightarrow$ small)



Concerns : proton

Apps. : p driver, hadrontherapy

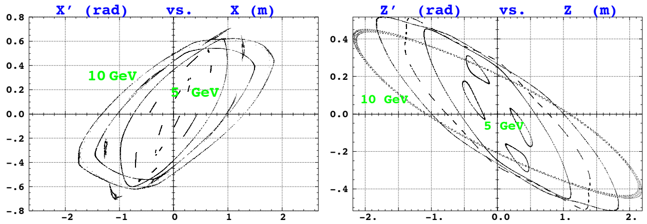
DA's

- DA's are large, possibly *very* large - a key interest of FFAGs.

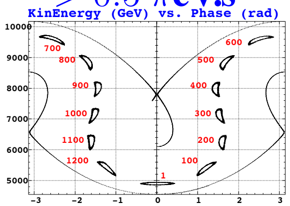
Linear, non-scaling

FD doublet

Muon :
$\gg 3\pi$**cm norm.**



$> 0.5\ \pi$**eV.s**



EMMA (electron) :
$\gg$**200-300** π**mm.mrad norm.**

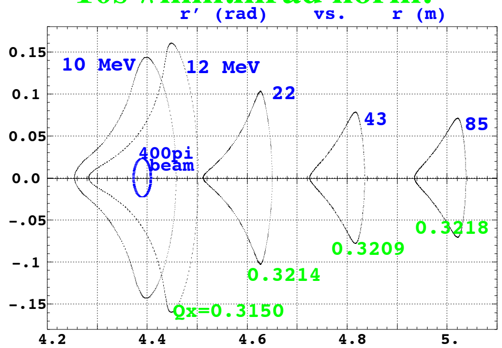
Proton :
10s π**mm.mrad norm.**

Non-linear, scaling

DFD triplet, doublet, spiral

$0.3 - 20$ **GeV muon :**
$> 3\pi$**cm norm.,** $1.5\ \pi$**eV.s**

Proton :
10s π**mm.mrad norm.**



Non-linear, non scaling

Pumplet lattice

$8 - 20$ **GeV muon**
isochronous
$\approx \pi$**cm norm.** $-0.5\ \pi$**eV.s**

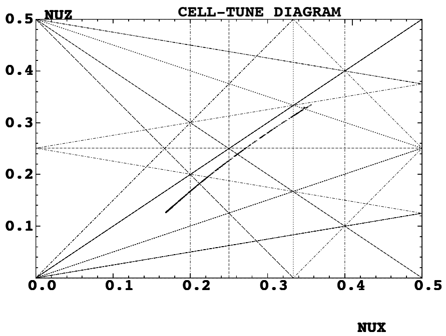
p-Driver :
10s π**mm.mrad norm.**

electron model :
100-300 π**mm.mrad norm.**

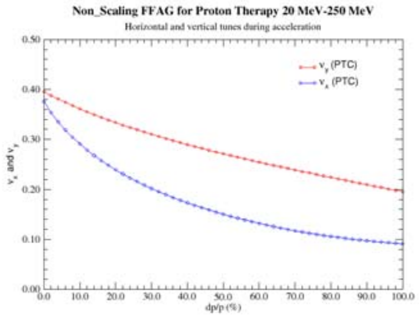
Adjusted field profile
p apps.,
10s π**mm.mrad norm.**

Chromaticity

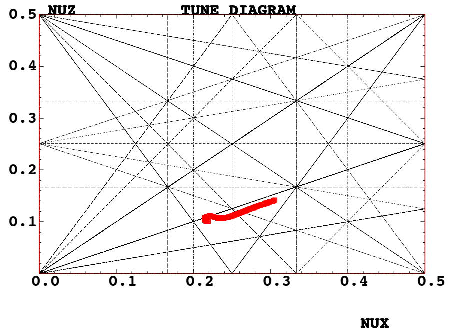
muon, EMMA,
linear, non-scaling
FD doublet



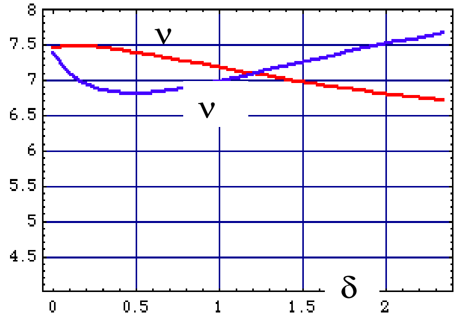
250MeV p-therapy
linear, non-scaling
FDF triplet



pumplet, e-model
non-linear
non-scaling



AFP
non-linear
non-scaling



THANK YOU