INTERNSHIP OF MASTER 1: EXPERIMENTAL INVESTIGATION OF A MINIMAL SURFACE: THE CATENOID

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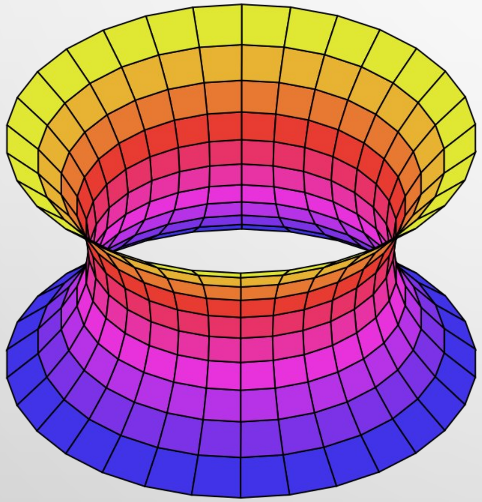
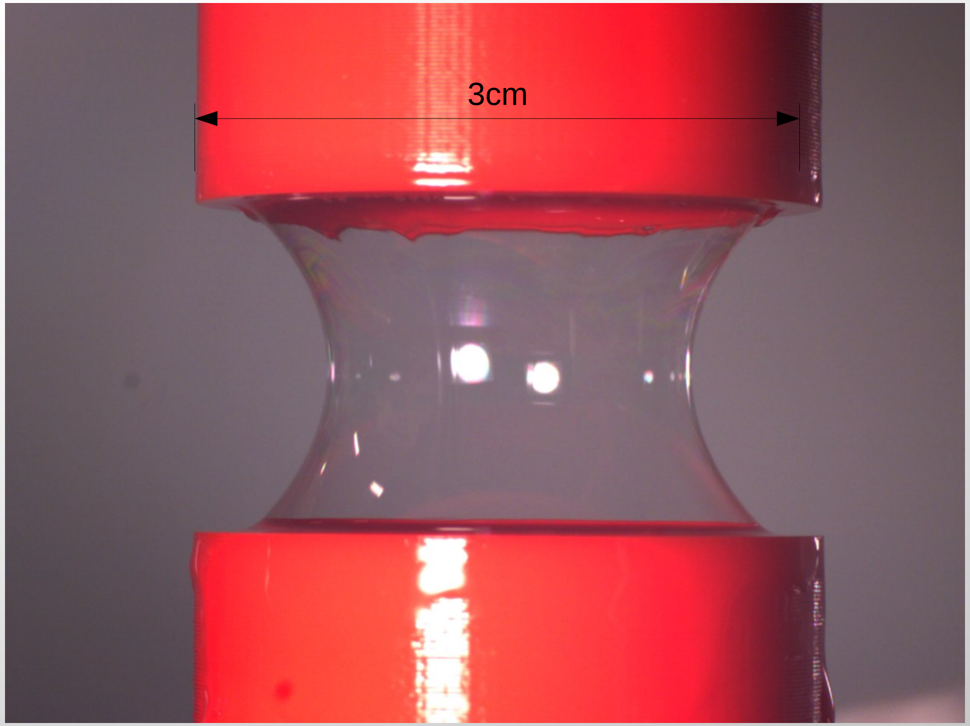
SUPERVISED BY THIERRY CHARITAT AND WIEBKE DRENCKHAN

AIM OF THE INTERNSHIP



from wikipedia.*org*



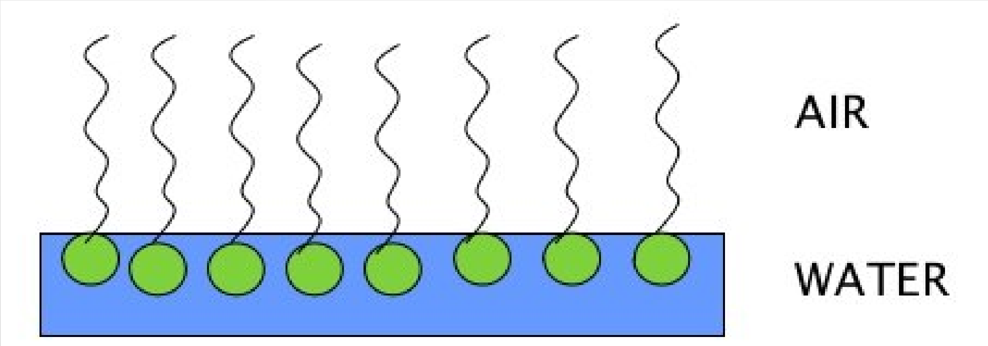
DEFINITION OF MINIMAL SURFACES

- SURFACE COST ENERGY: E=ɣ.S

from *sciencesource.com*

- DEFINITION OF MINIMAL SURFACES

- SURFACTANTS ON THE SURFACE OF WATER



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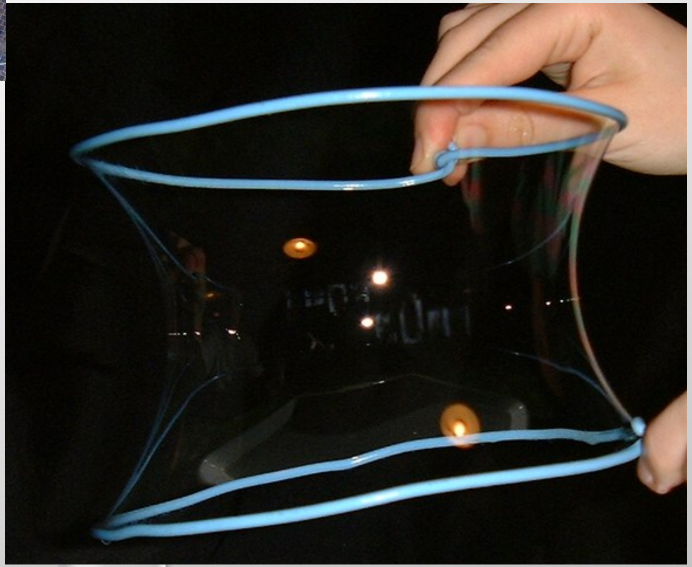
APPLICATIONS OF MINIMAL SURFACES

- ARCHITECTURE

from *wikipedia.org*

- OUR SUBJECT OF STUDY: THE CATENOID



from *wikipedia.org*

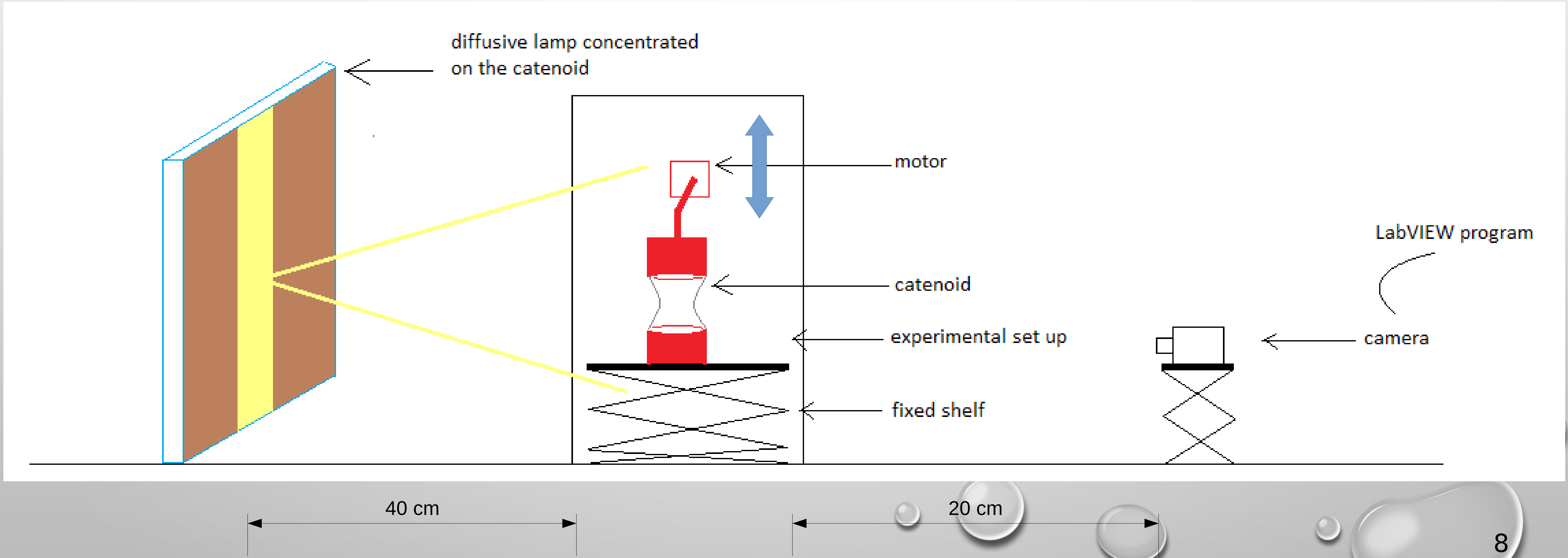
WHY THIS SUBJECT ?

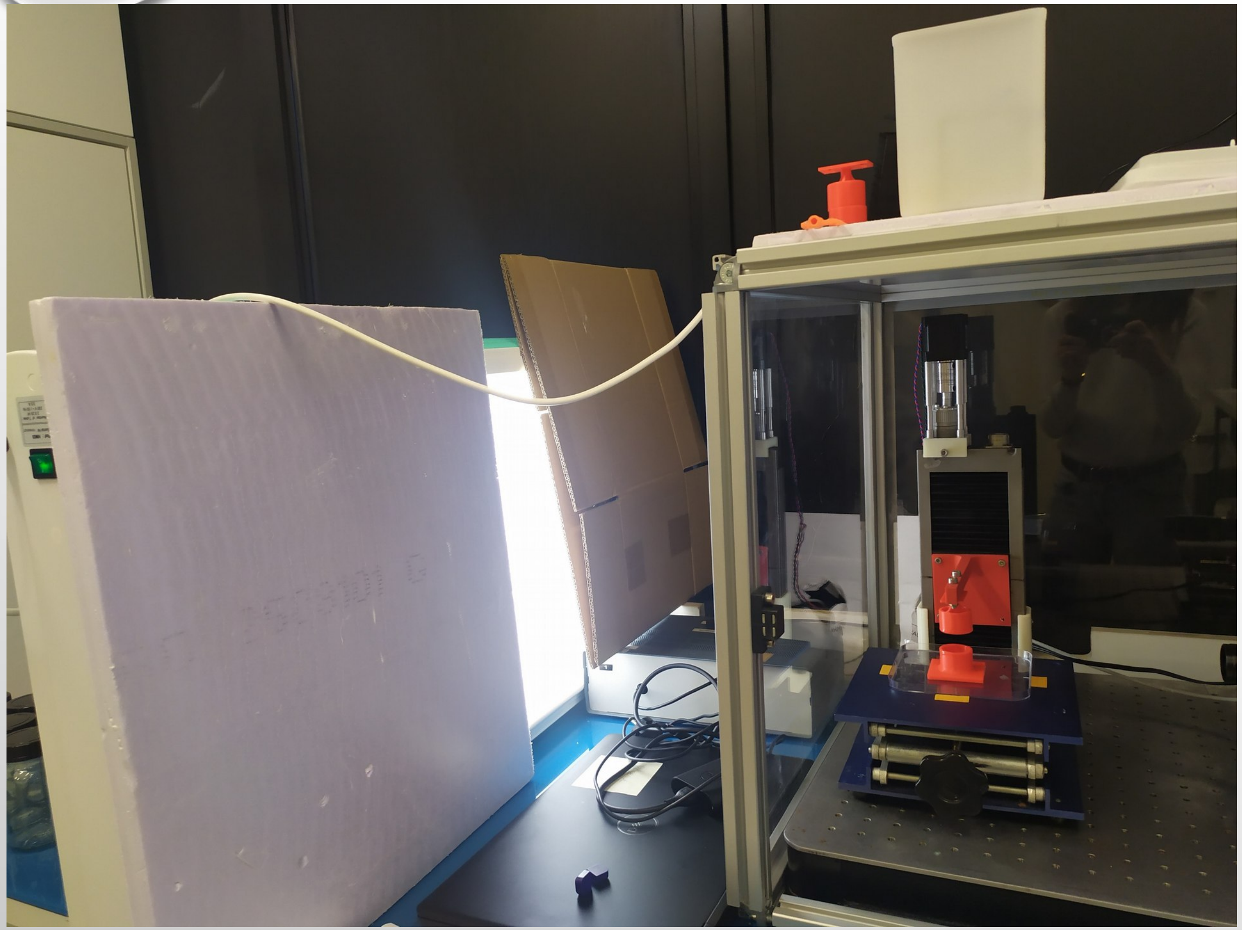
CONTENTS

I-EXPERIMENTAL PROCEDURE

- EXPERIMENTS
  - SETUP
  - LABVIEW
- IMAGE ANALYSIS
  - IMAGE J
- PROGRAMMING
  - C++ AND PYTHON OR RSTUDIO

EXPERIMENTAL SETUP





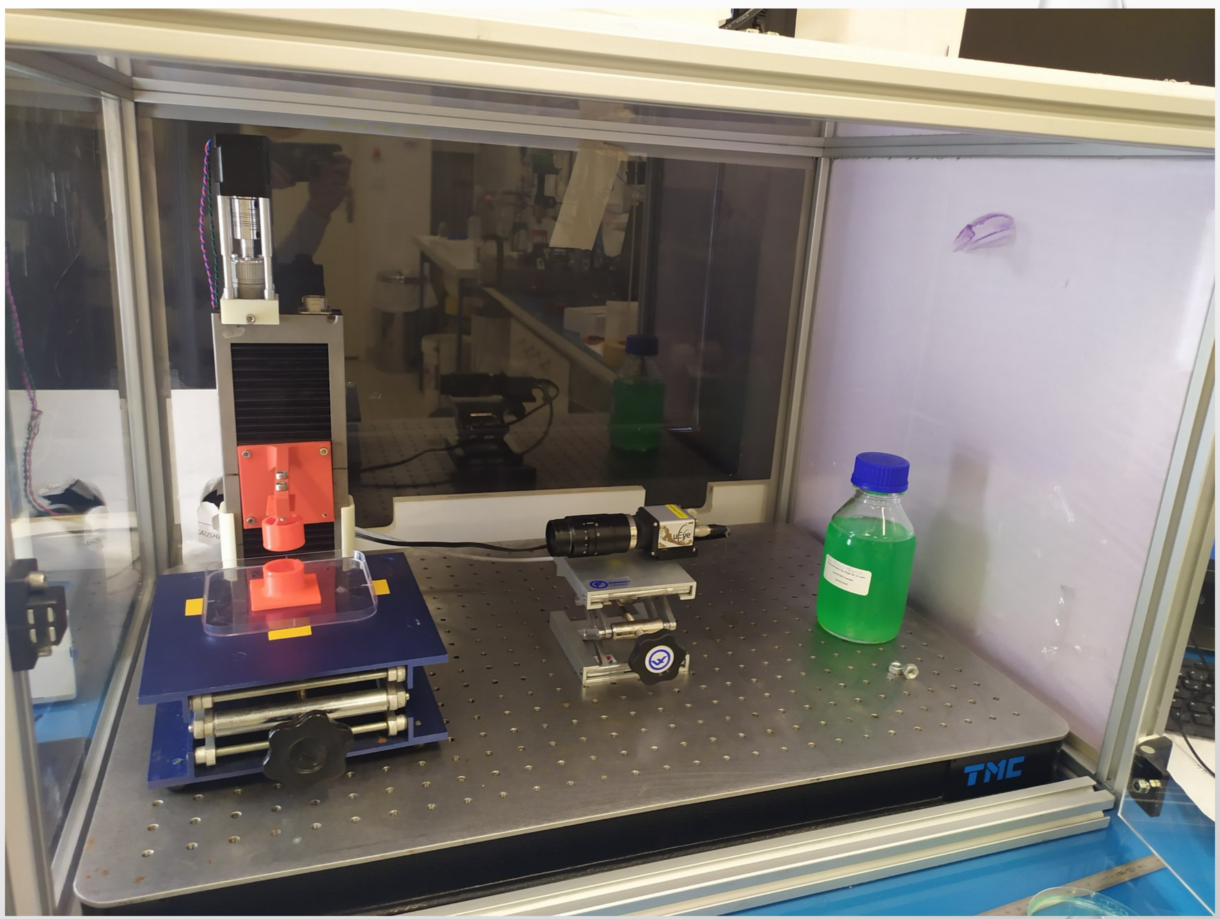
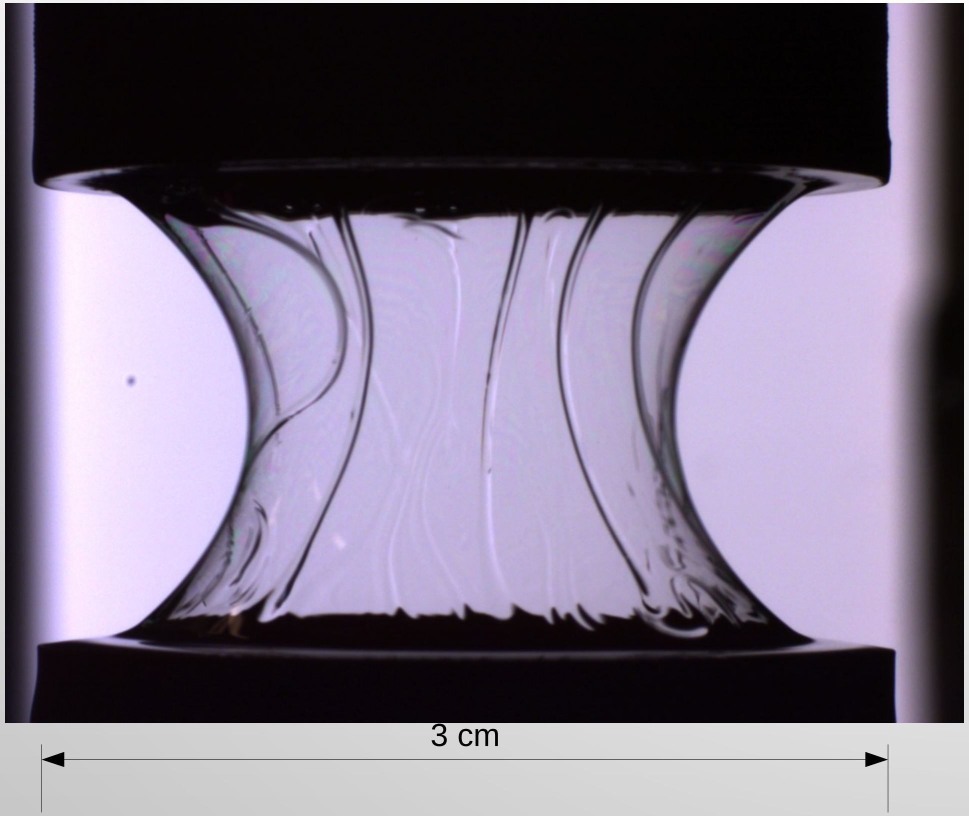
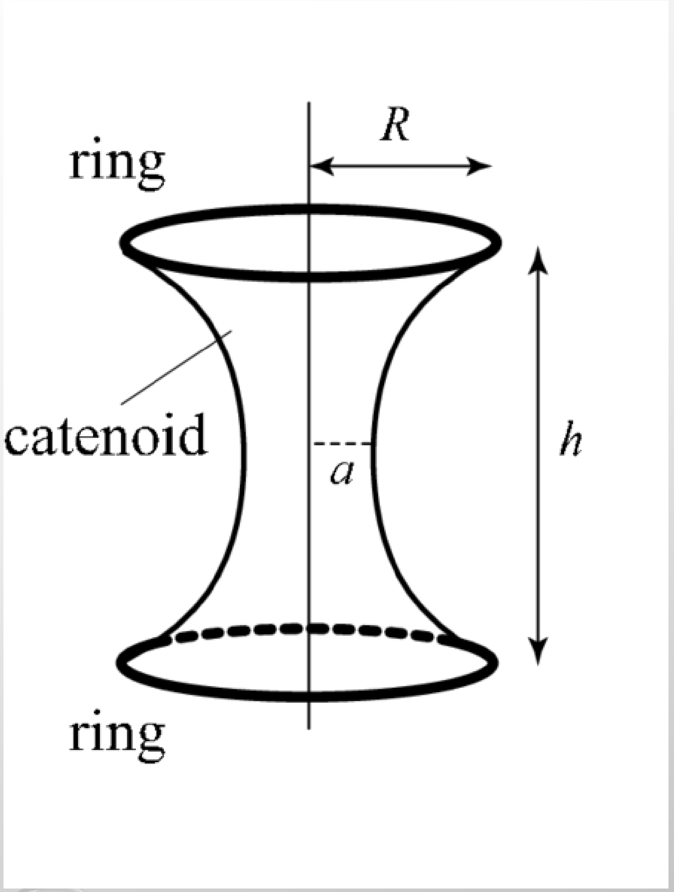

TMC

IMAGE ANALYSIS (IMAGE J)



USEFUL QUANTITIES

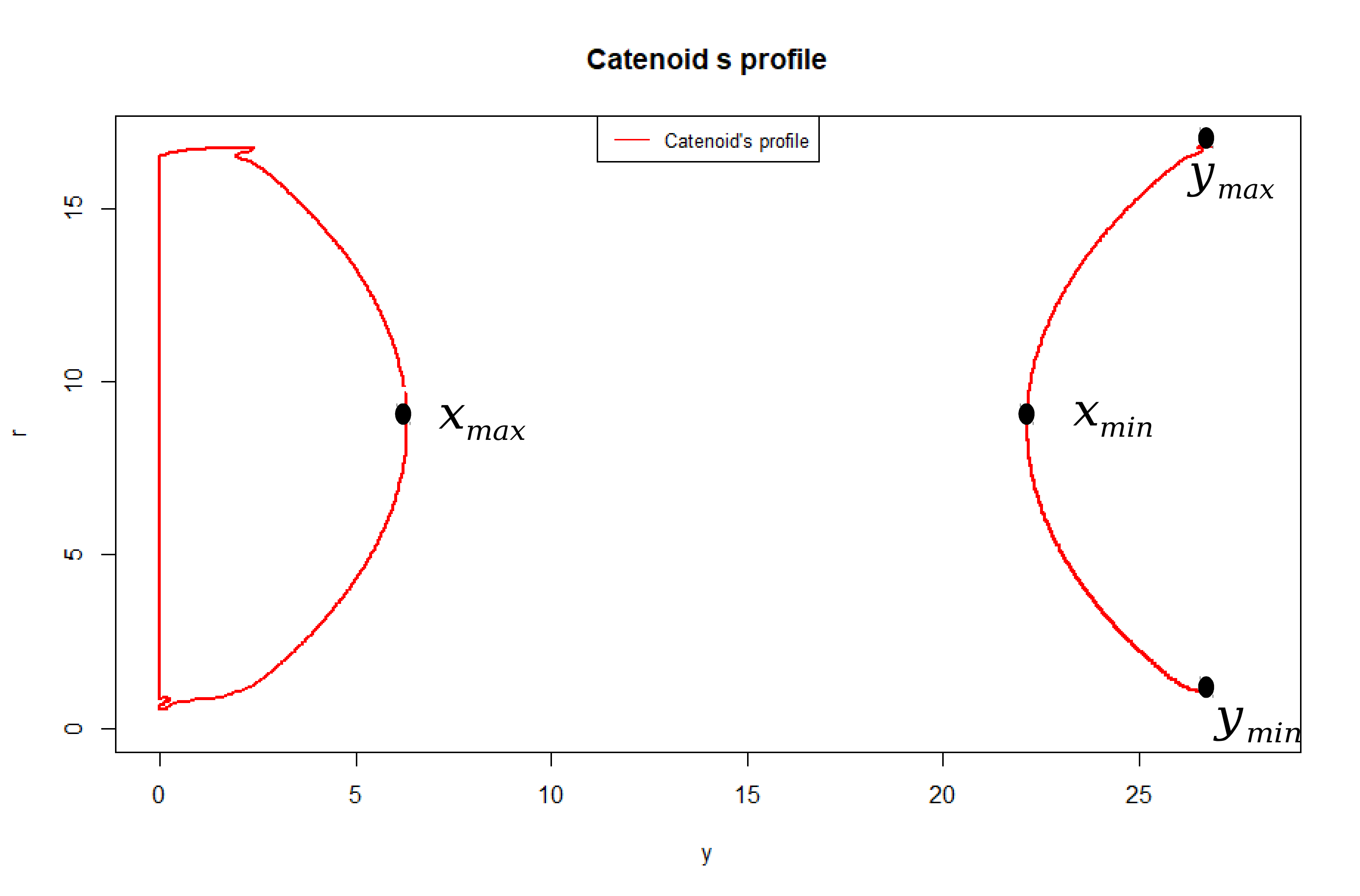
- h – height of the catenoid
- a – neck radius i.e. minimum radius of the catenoid, at h/2



from *researchgate.net*

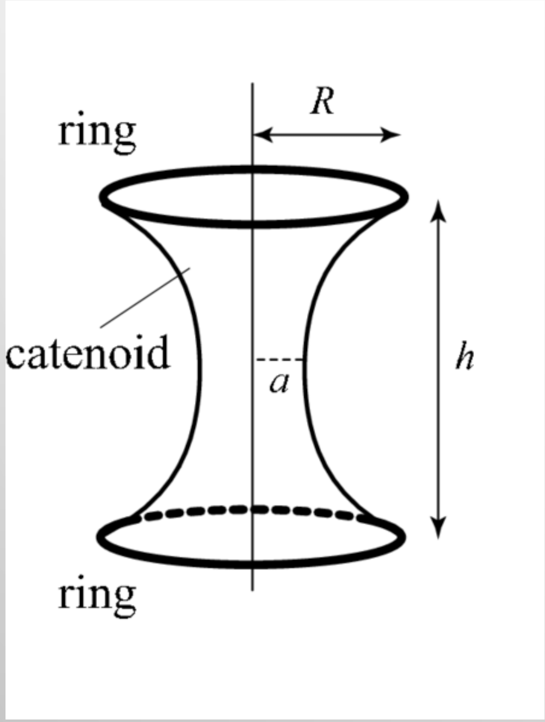
PROGRAMMING

- $a = \frac{x_{min}(right\ side) - x_{max}(left\ side)}{2}$
- $h = y_{max} - y_{min}$
- $R_{th} = 13mm$



II-EXPERIMENTS AND RESULTS

OUR FOCUS

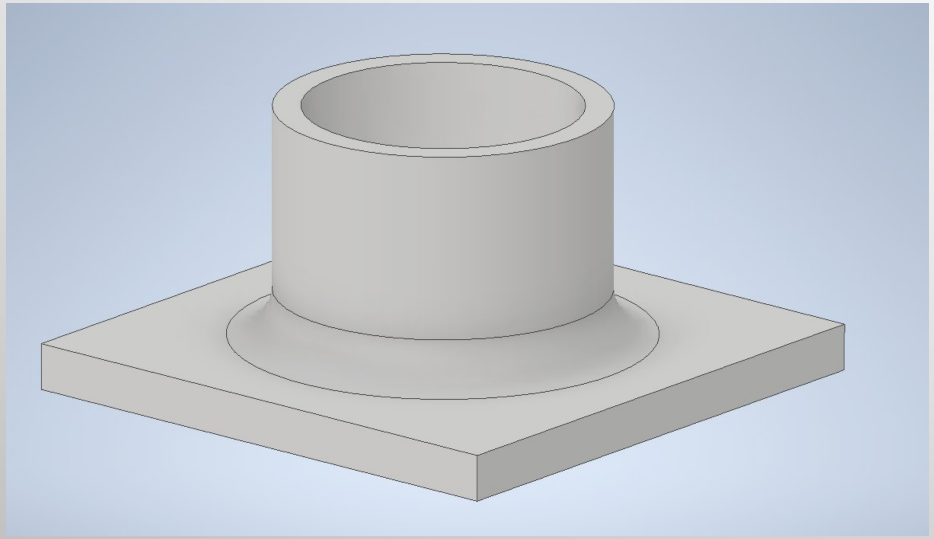


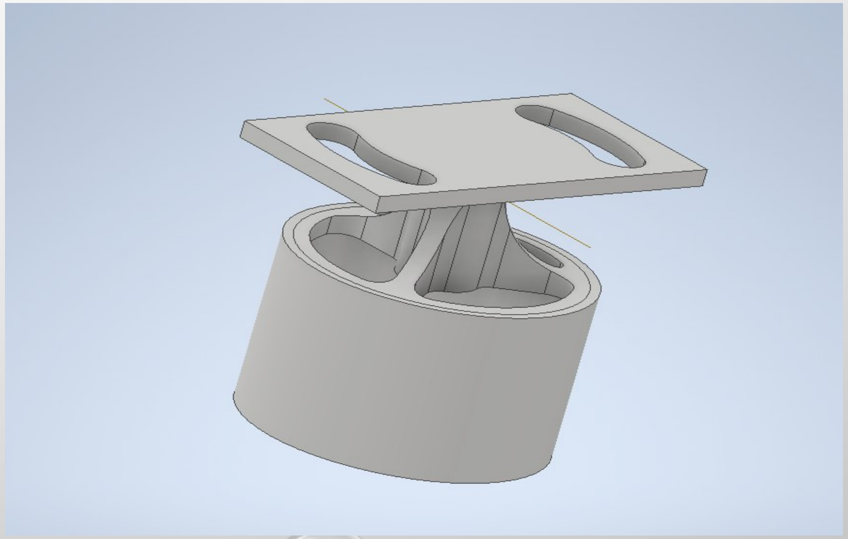
from *researchgate.net*

- EVOLUTION OF THE NECK RADIUS AS FUNCTION OF THE HEIGHT
- PARAMETERS
  - THE SPEED OF THE MOTOR
  - THE LIQUID USED
  - GEOMETRY OF THE PLASTIC PIECES

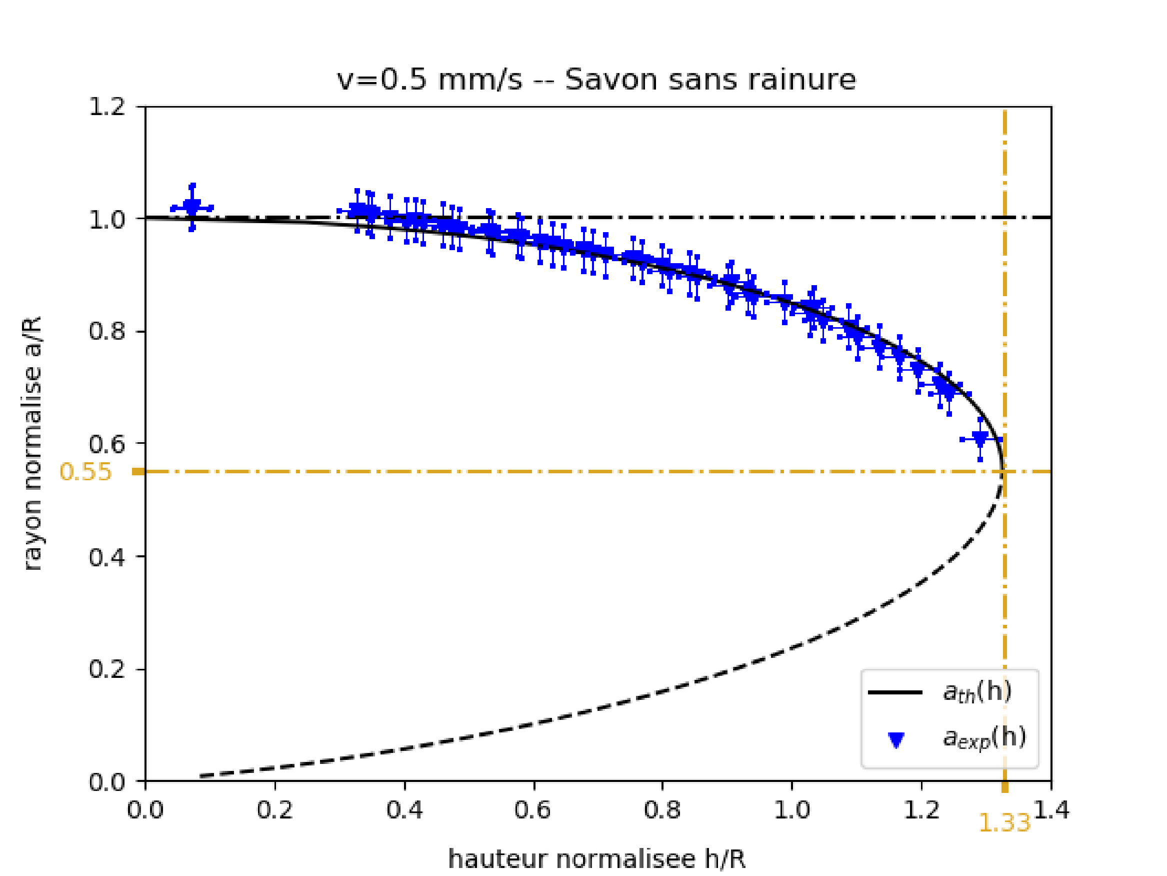
1ST EXPERIMENT

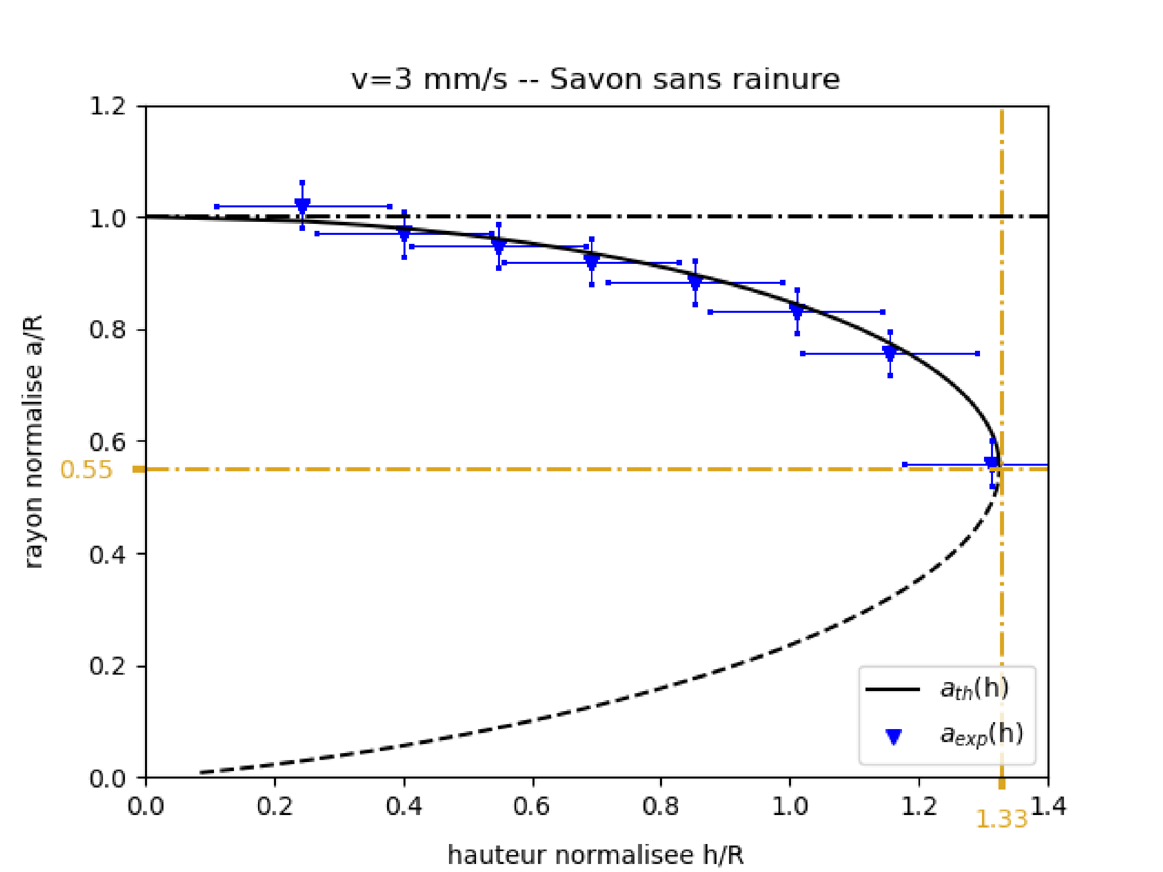
- LIQUID USED: BASIC LIQUID SOAP
- PLASTIC PIECES: CIRCLES OF EXTERNAL DIAMETER 3 CM



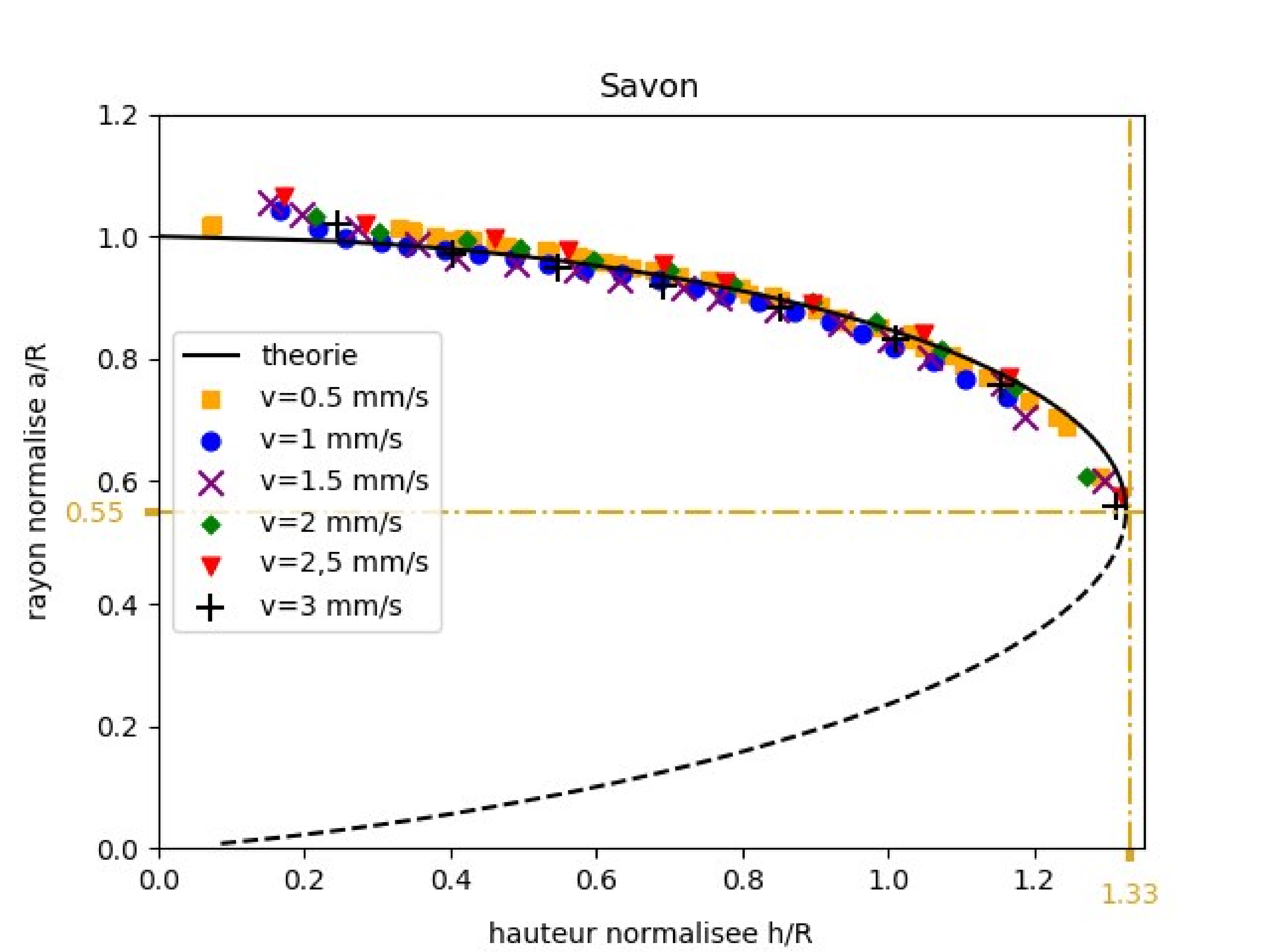


II-Experiments and Results





Critical point : Pc = (1.33 ; 0.55)

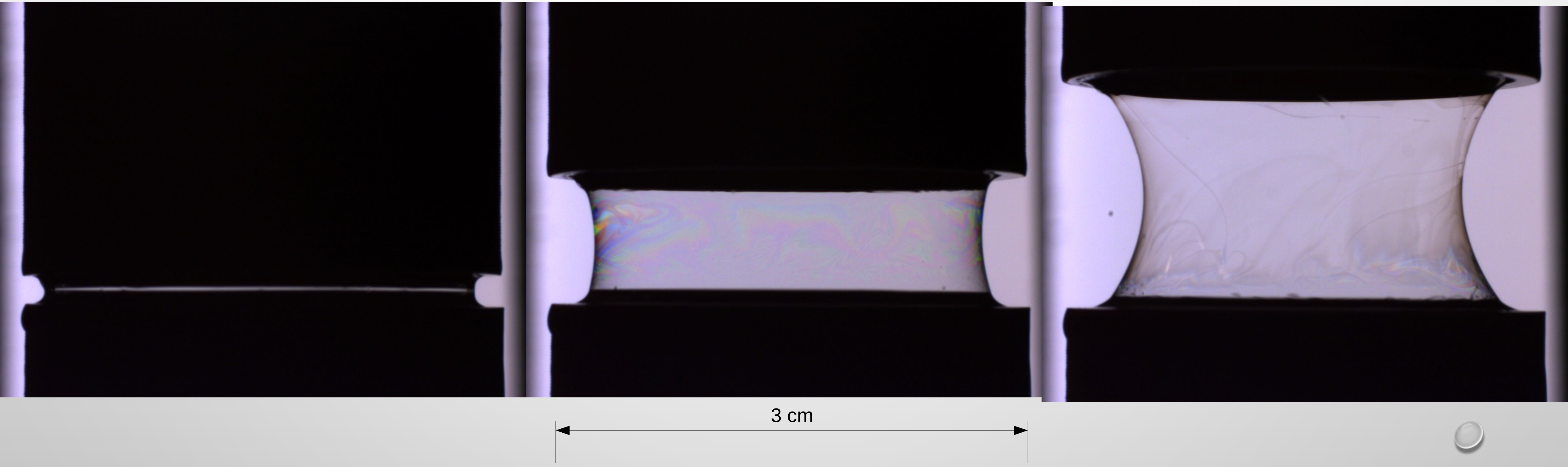


Problems :

- Catenoid not solid enough to reach the instability point
- For low h : a/R > 1

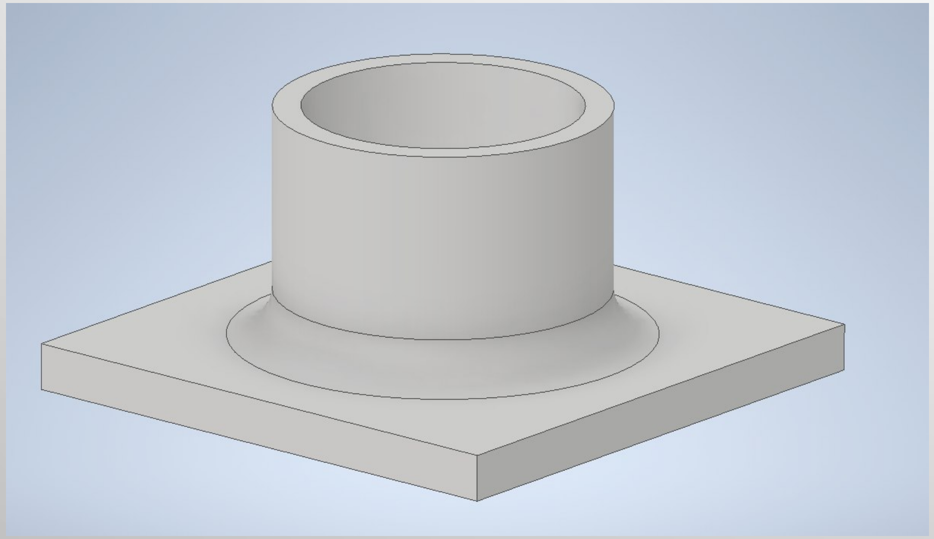
Critical point : Pc = (1.33 ; 0.55)

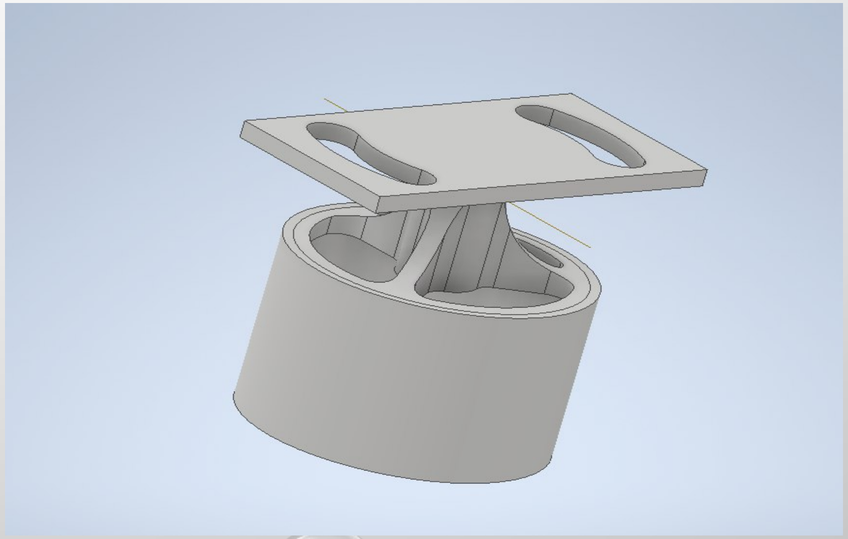
II-Experiments and Results



2ND EXPERIMENT

- LIQUID USED: J-LUBE
- PLASTIC PIECES: CIRCLES OF EXTERNAL DIAMETER 3 CM





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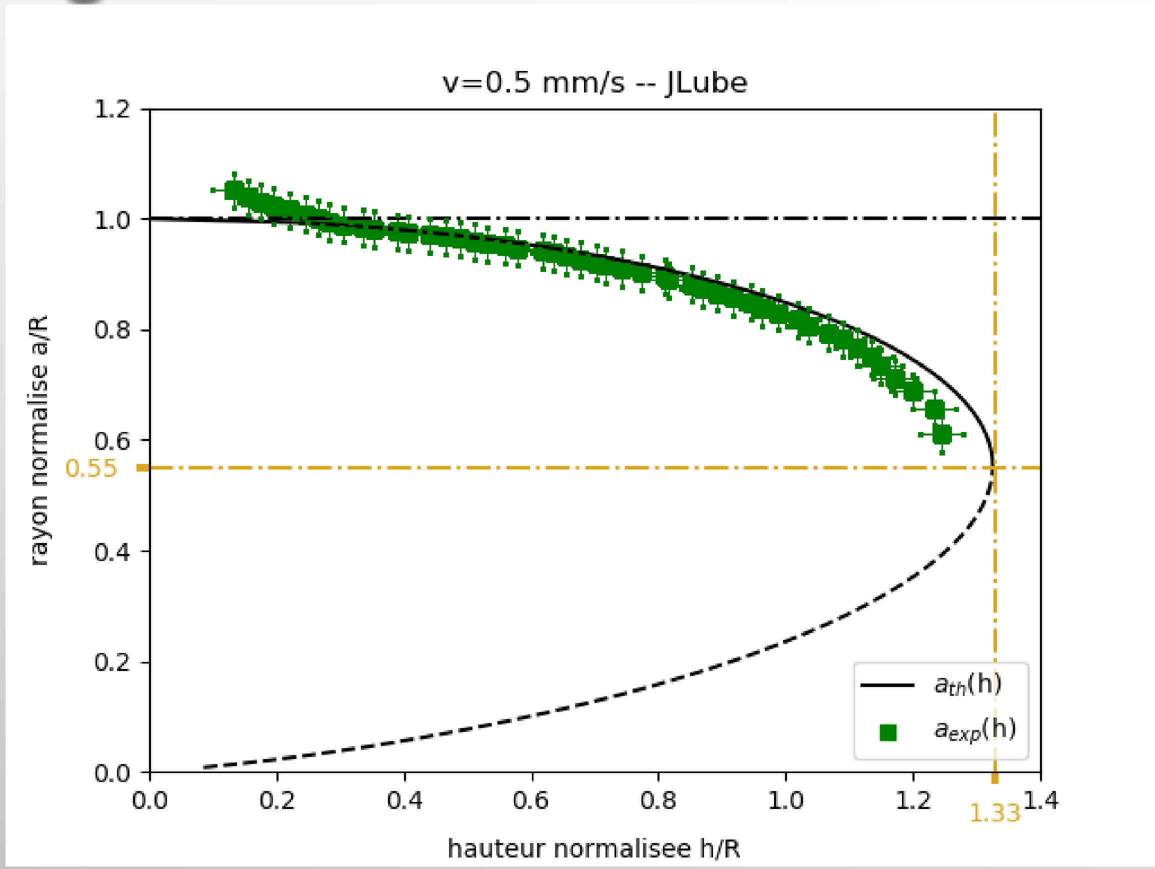

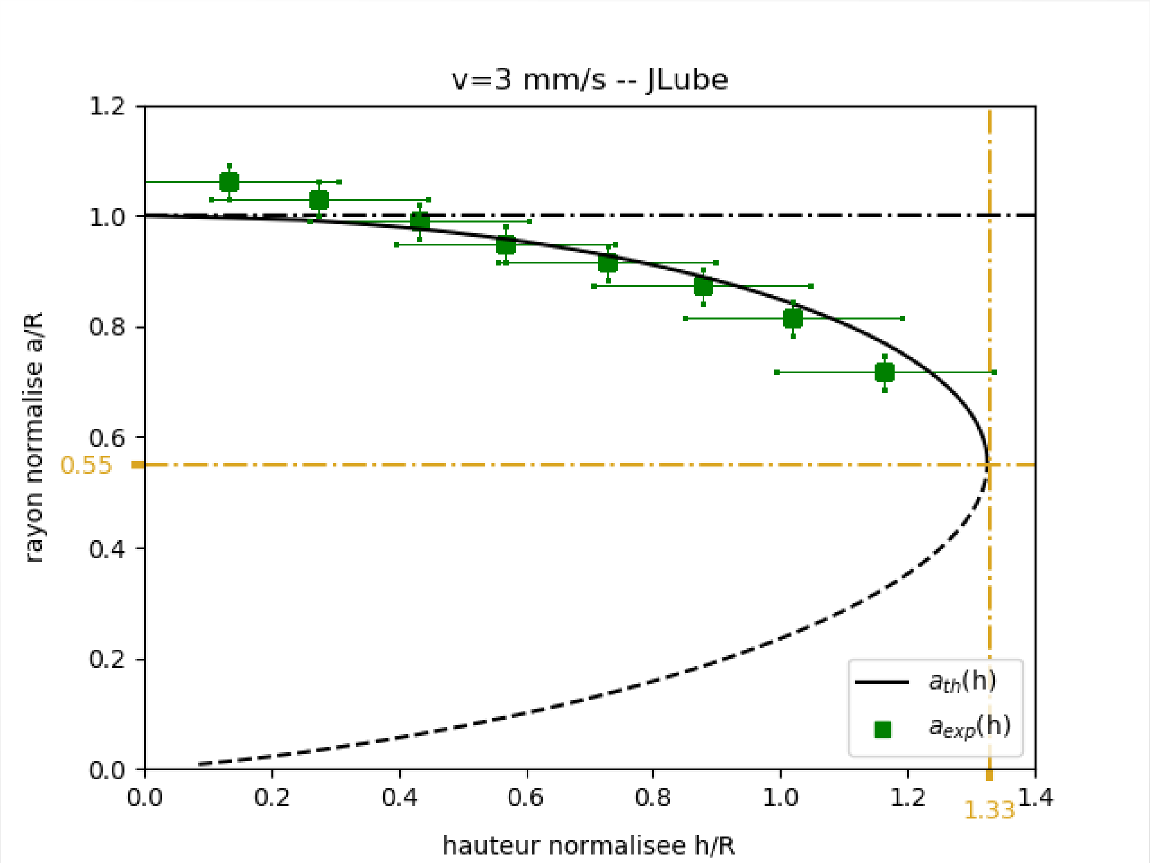

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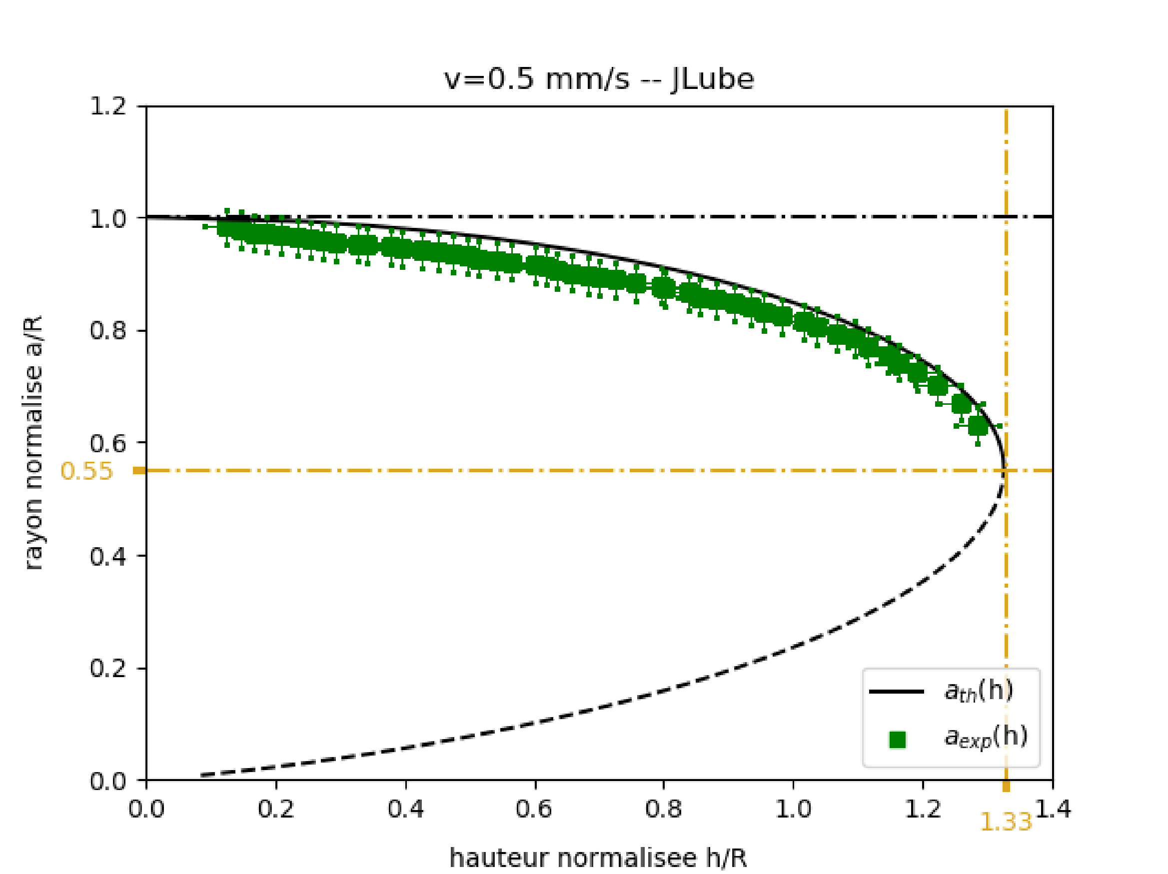

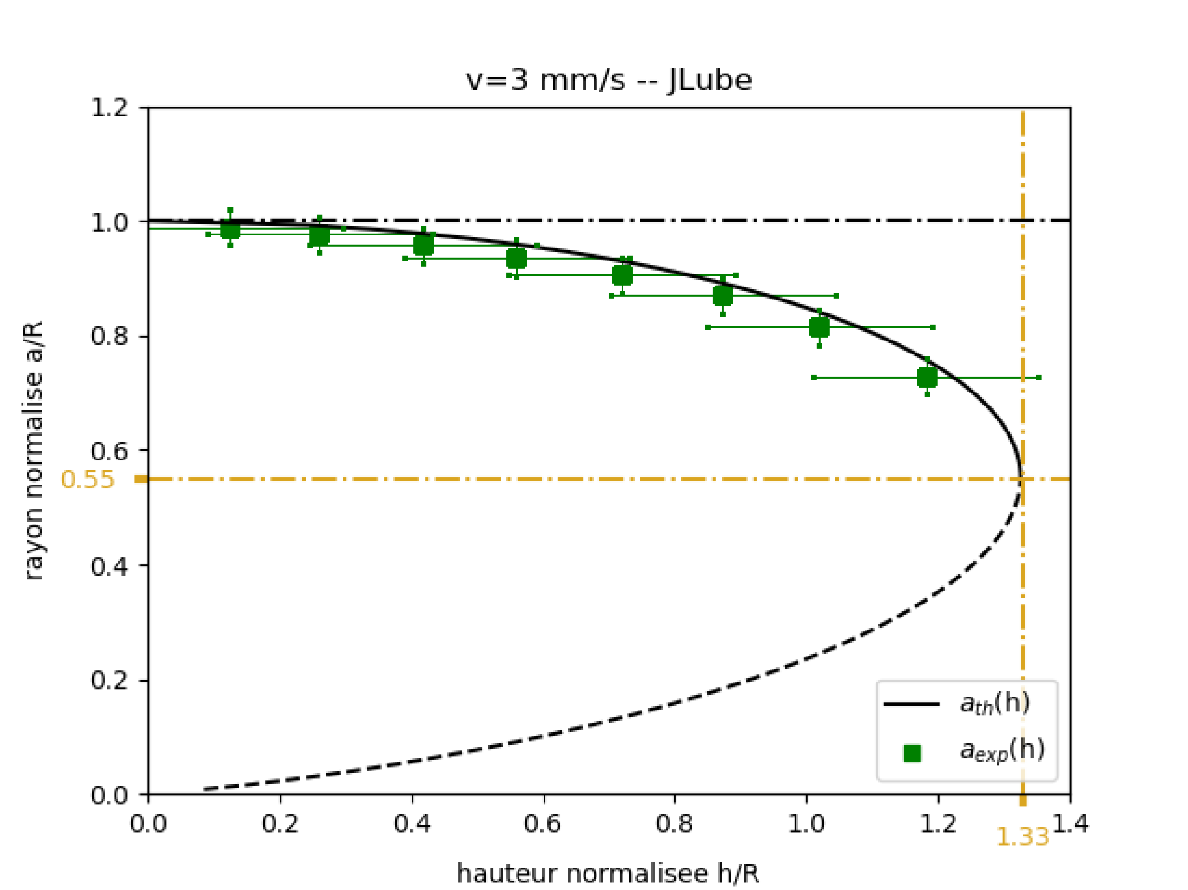

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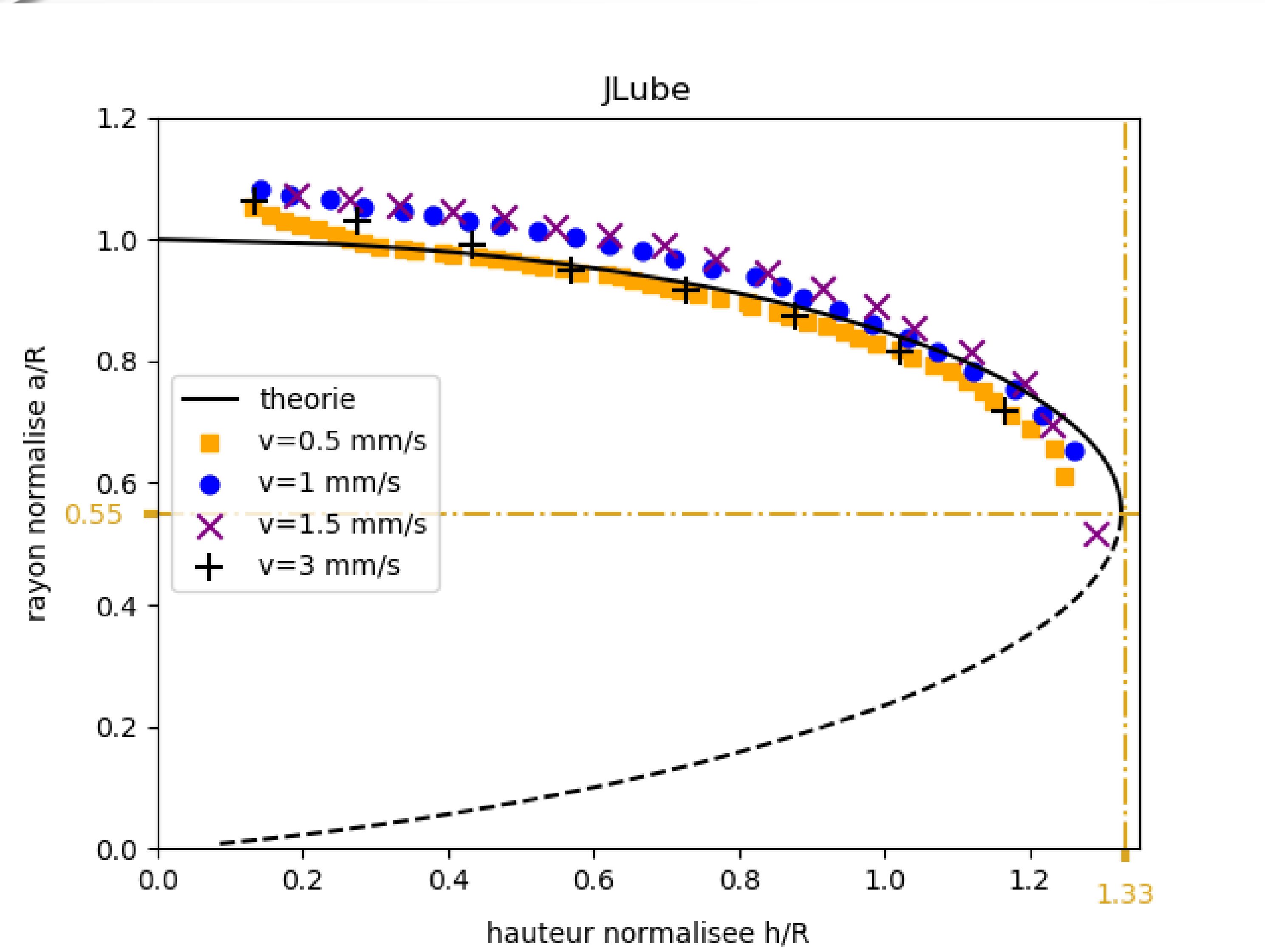


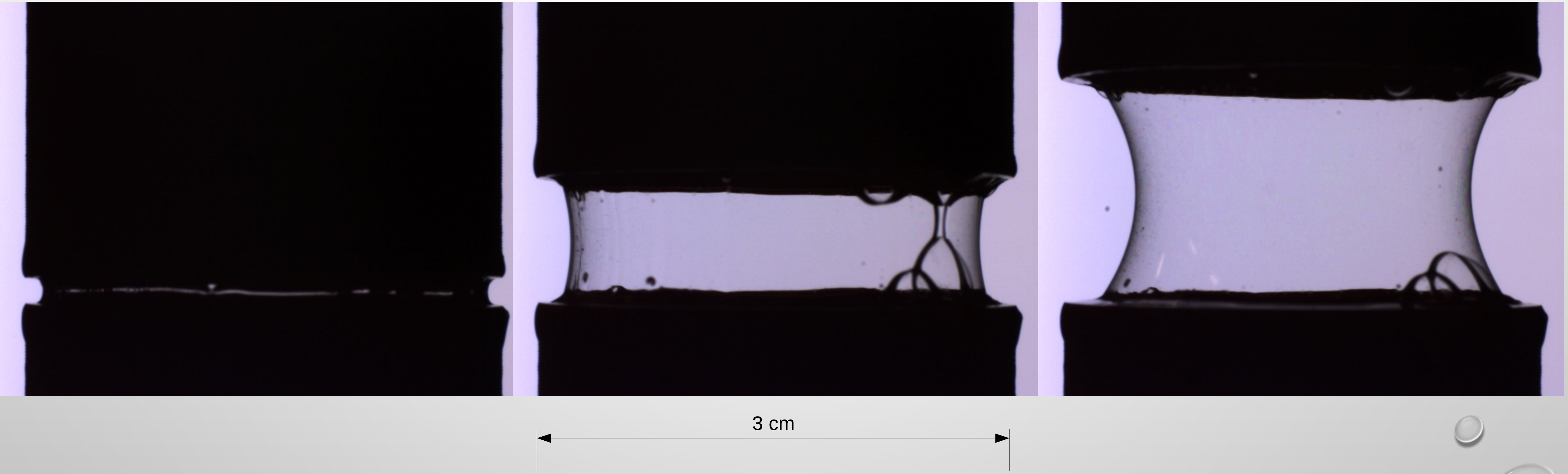
Catenoid more stable

Problem :

- For low h : a/R > 1 stronger

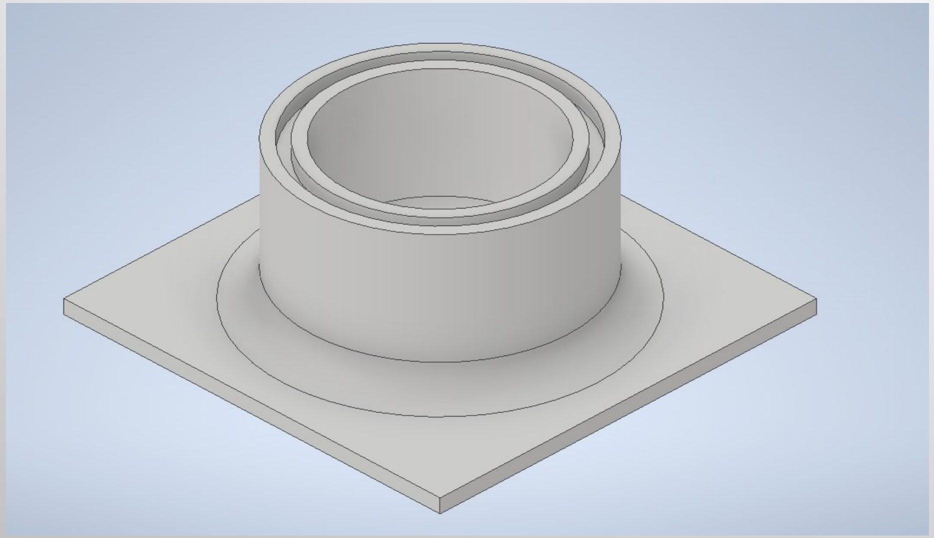
Critical point : Pc = (1.33 ; 0.55)

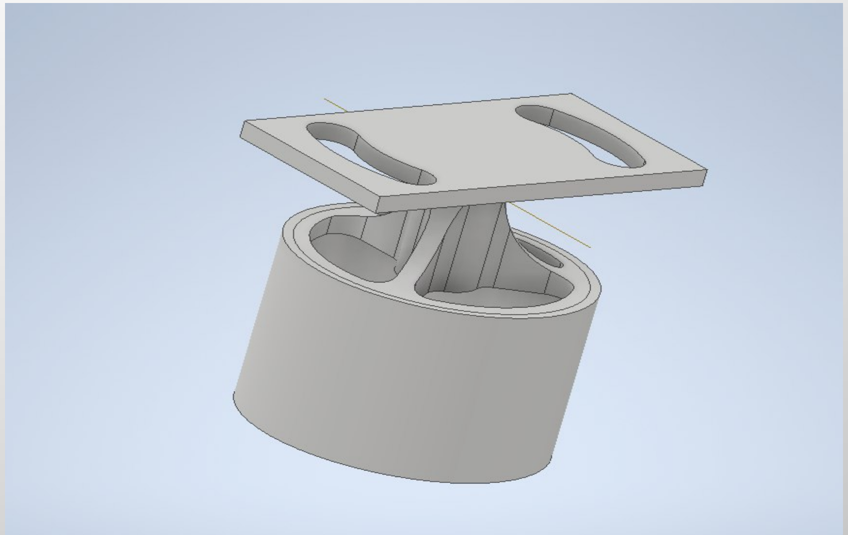
II-Experiments and Results



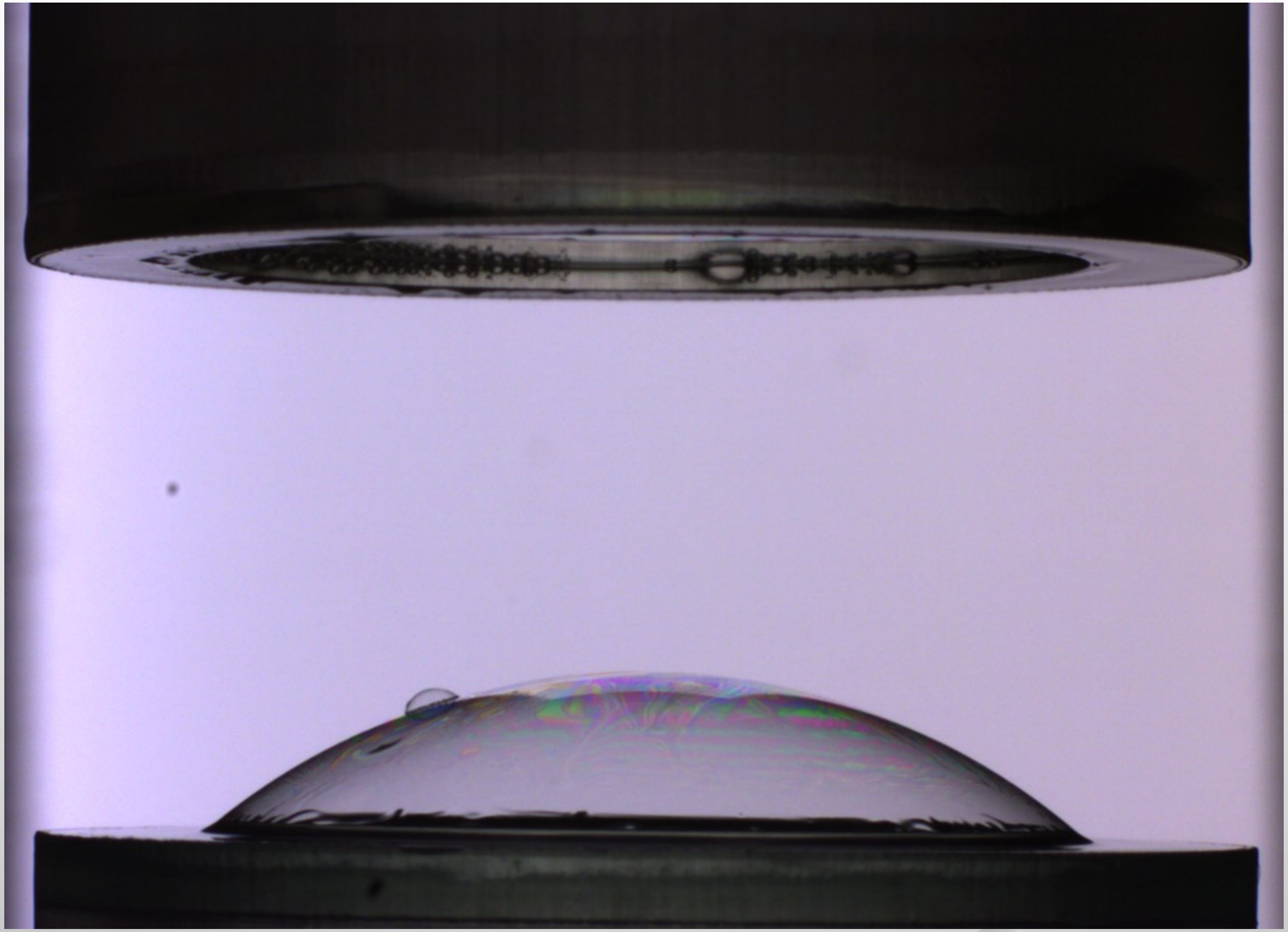
3RD EXPERIMENT

- LIQUID USED: BASIC LIQUID SOAP
- PLASTIC PIECES: CIRCLES WITH GROOVE AND EXTERNAL RADIUS 3 CM



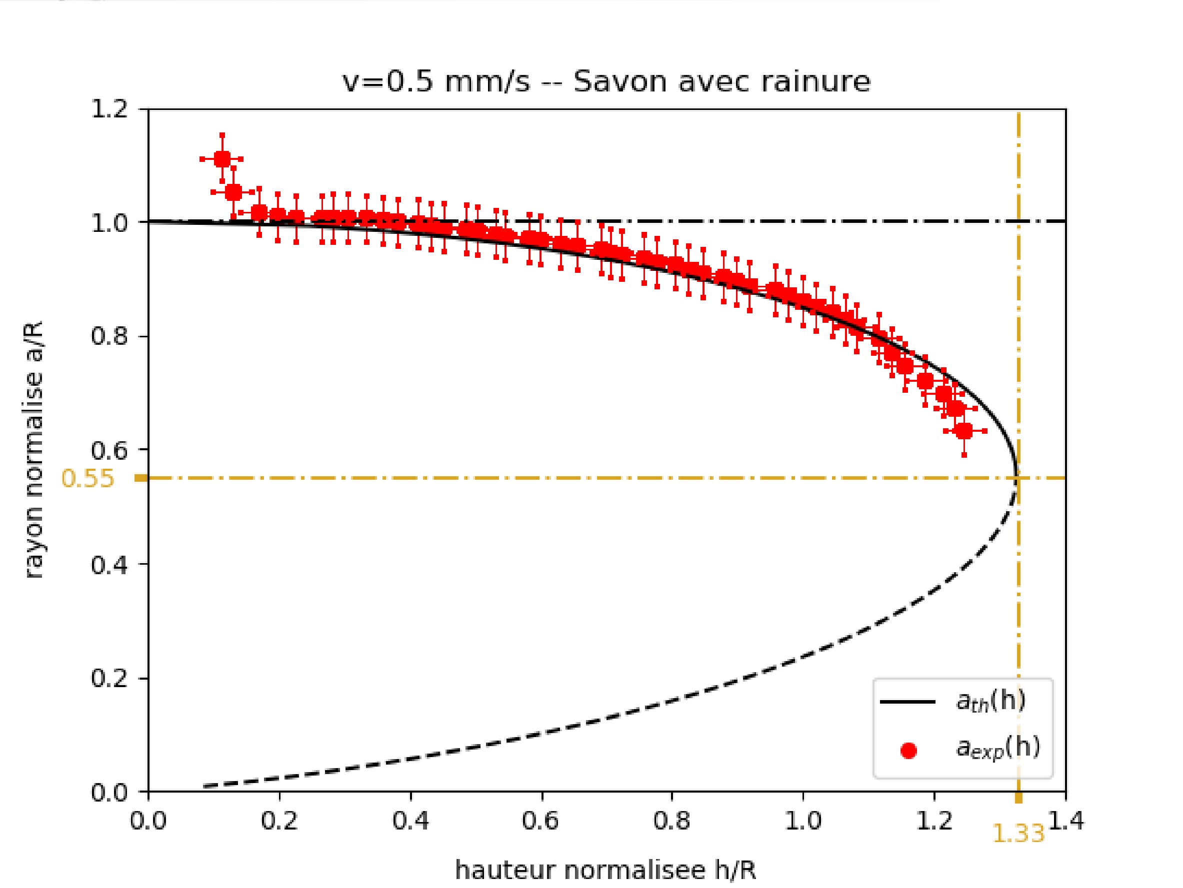


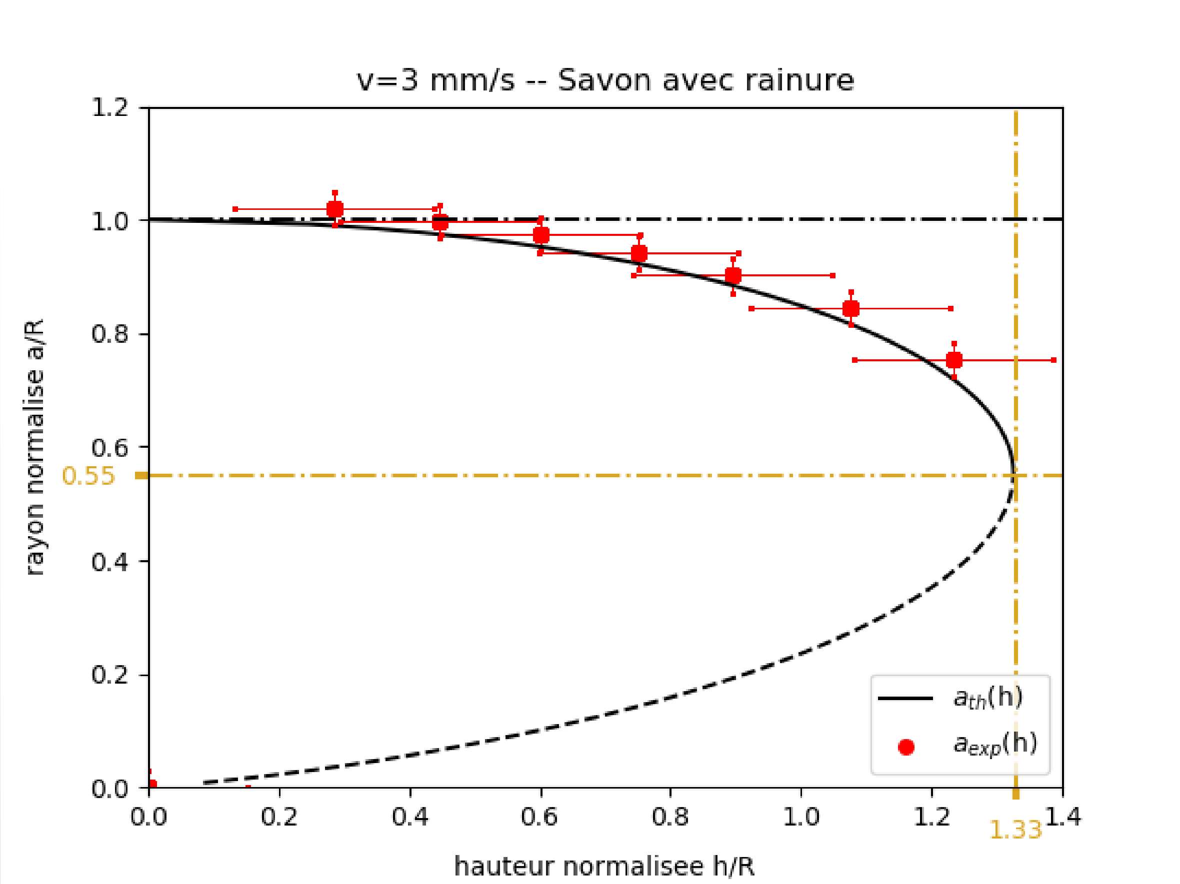
II-Experiments and Results



3 cm

II-Experiments and Results





Critical point : Pc = (1.33 ; 0.55)

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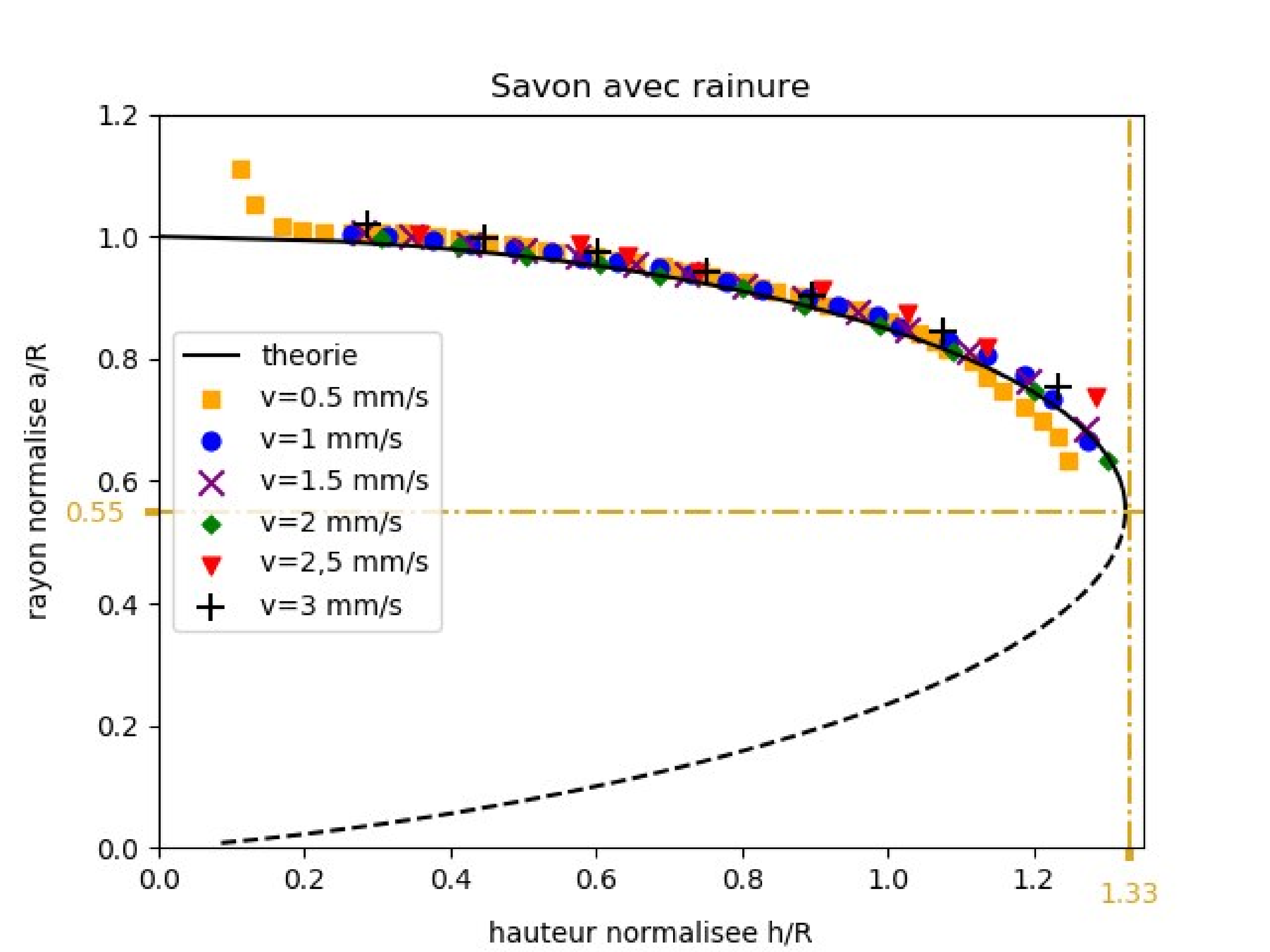

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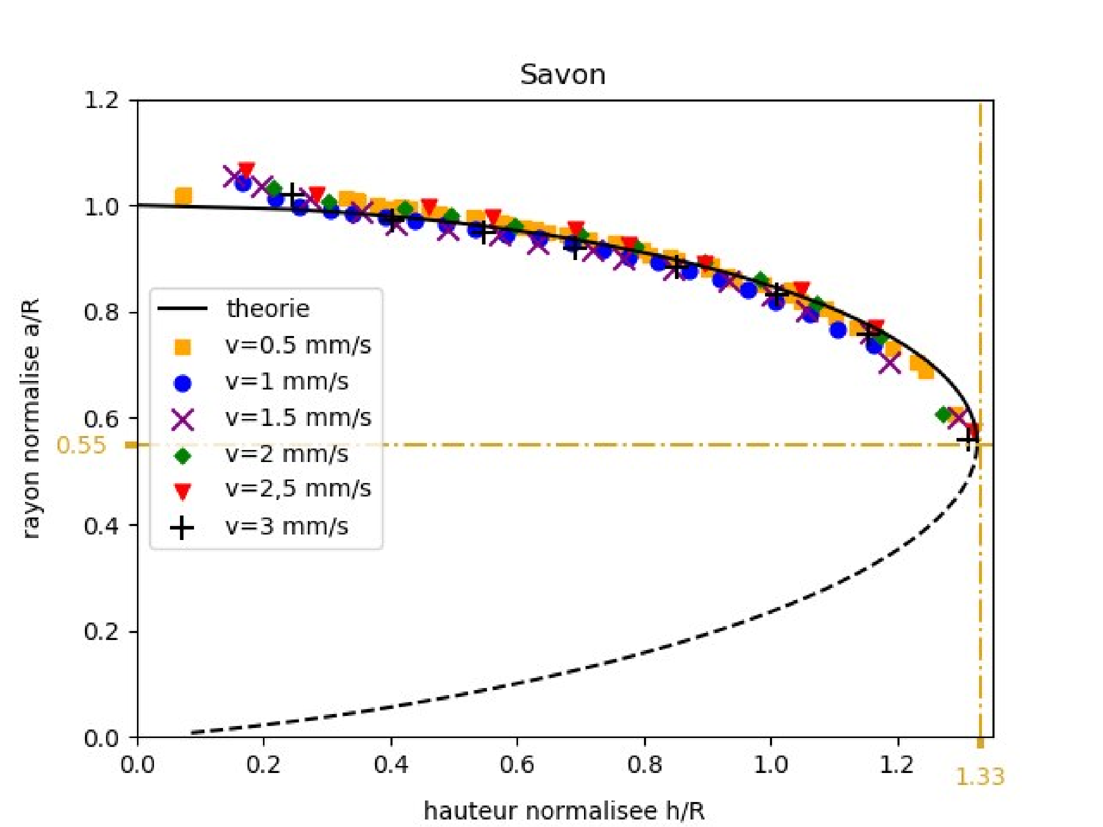
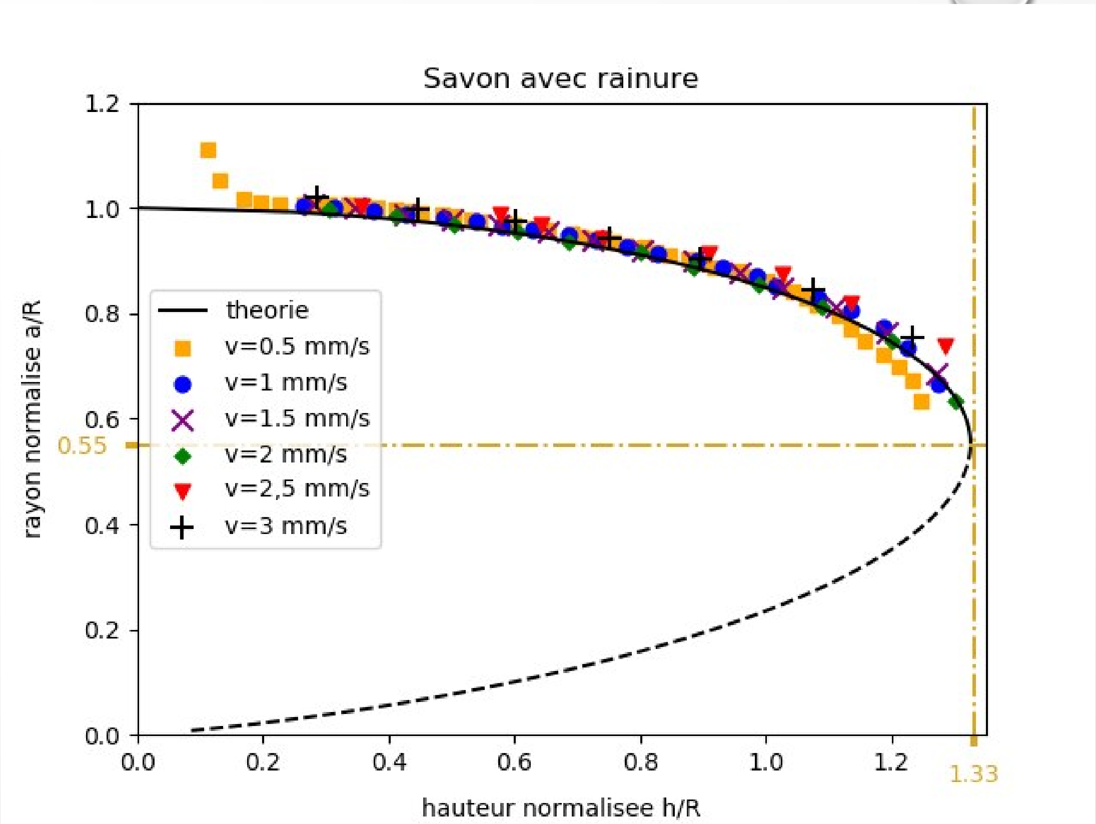




Soap : curves more spread around the theoretical one

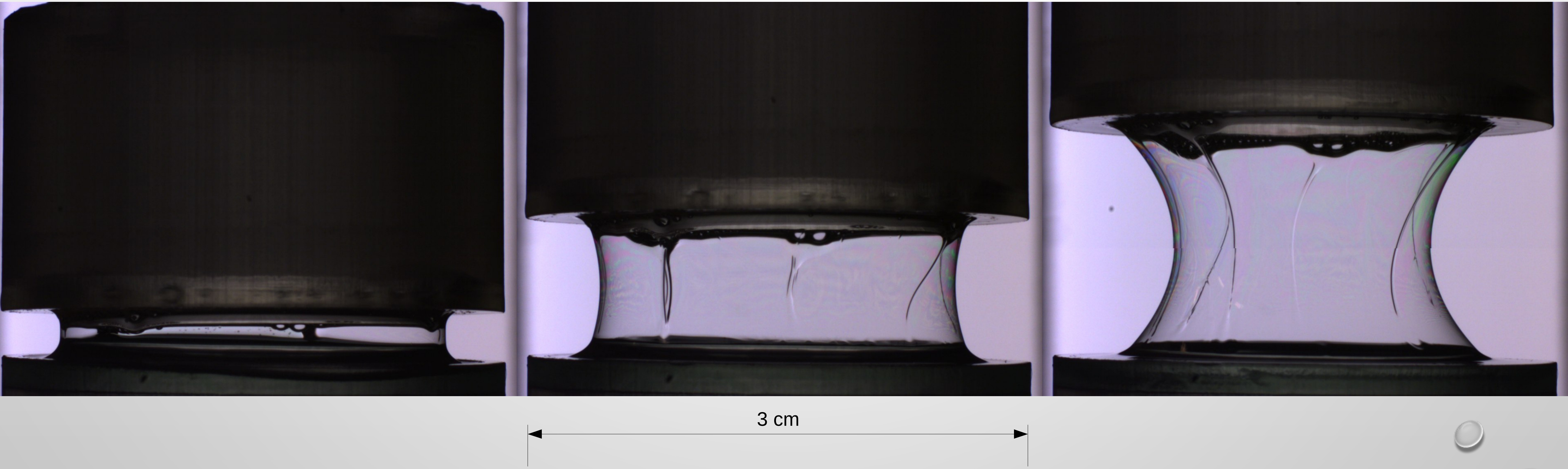
Soap+groove :

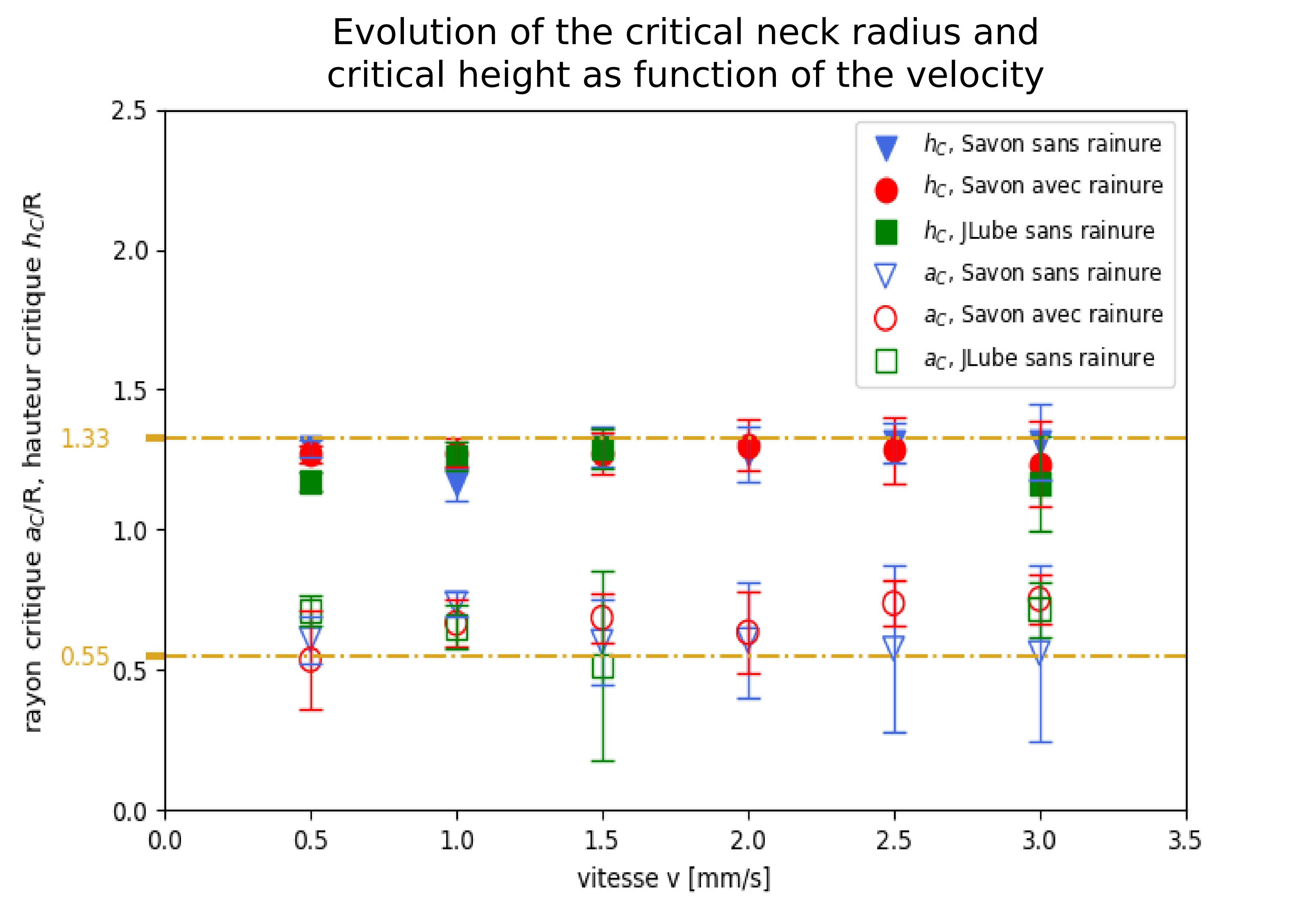
- Translation phase shorter
- Perfect overlap has h increases

BOTH : points more diffuse close to Pc : because of more instable ?

Conclusion : shape has an impact only on stability

II-Experiments and Results





- Theoretical critical neck radius ac = 0.55
- Theoretical critical point hc = 1.33

Critical point : Pc = (1.33 ; 0.55)

UNCERTAINTIES

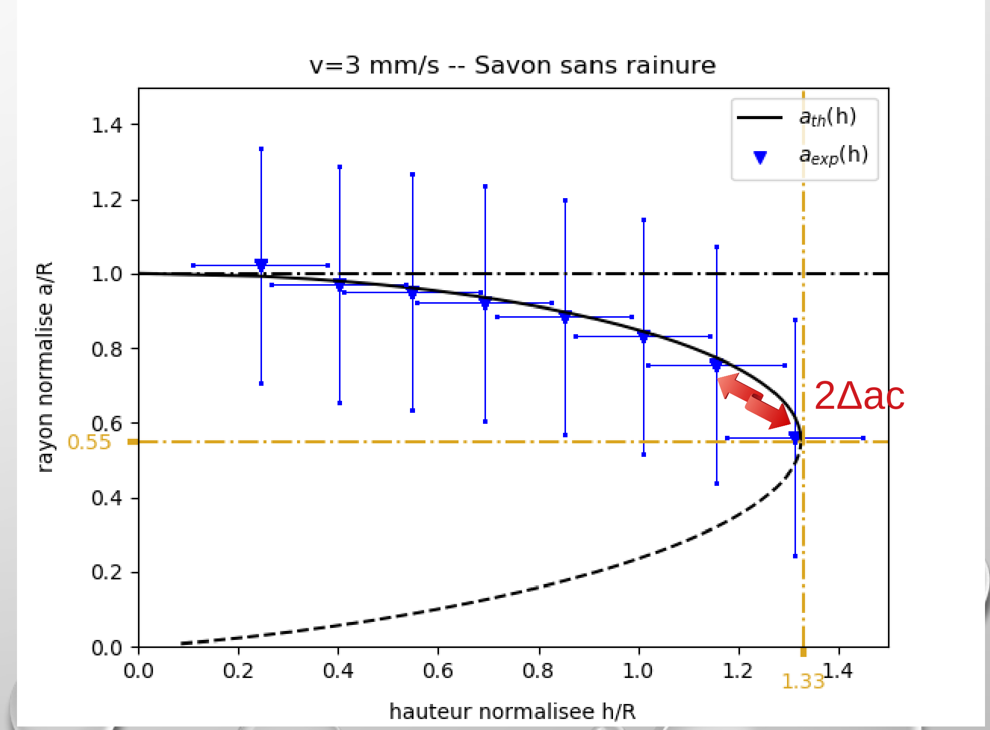
- OVER h: VELOCITY, BECAUSE ONE SECOND BETWEEN TWO IMAGES
- OVER a_c: DISTANCE BETWEEN THE TWO LAST POINTS ON THE GRAPH FOR A GIVEN SPEED

  → WE DETERMINE Δa_c

- OVER $\frac{a_c}{R}$ AND $\frac{h}{R}$:

$$\Delta\left(\frac{X}{Y}\right) = \frac{Y}{X}\sqrt{(\frac{\Delta X}{X})^2 + (\frac{\Delta Y}{Y})^2}$$

- $\Delta R = \frac{0.5 \times graduation}{\sqrt{3}} = 0.2887mm$; $R = 13mm$; a_c, h EXPERIMENTAL VALUES



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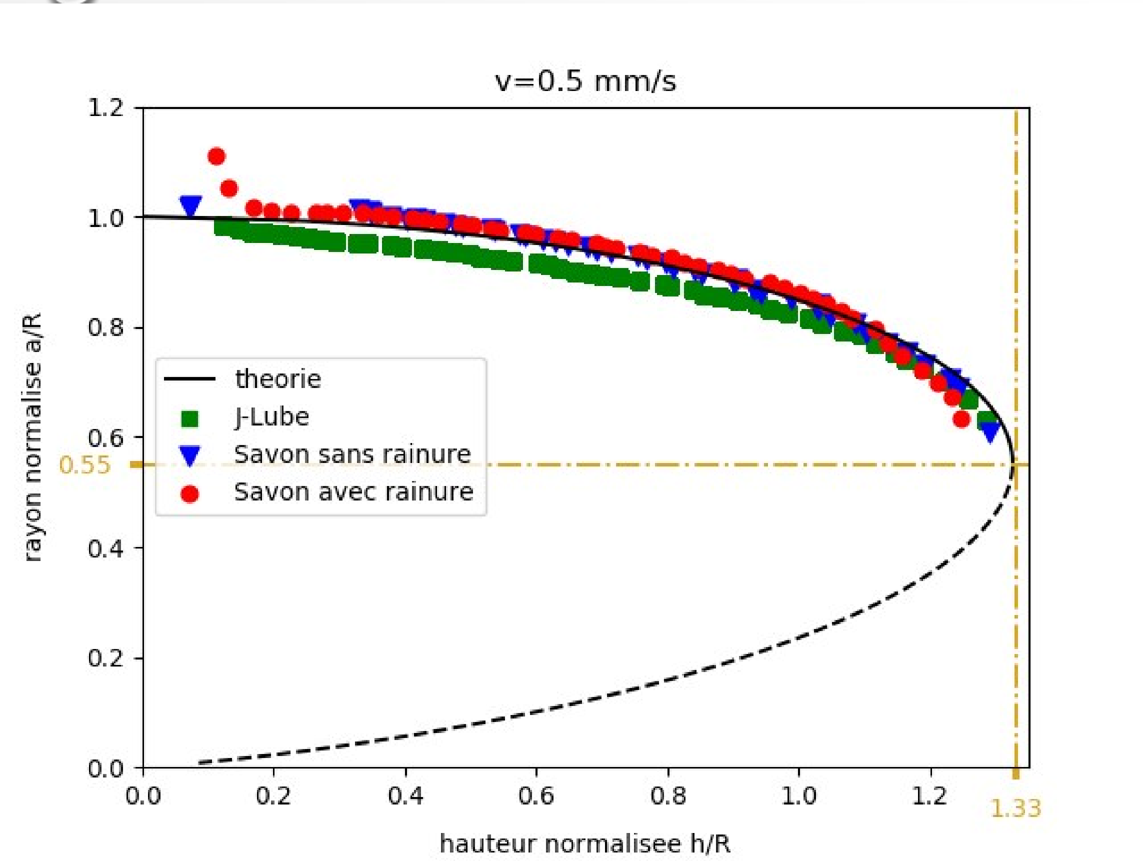
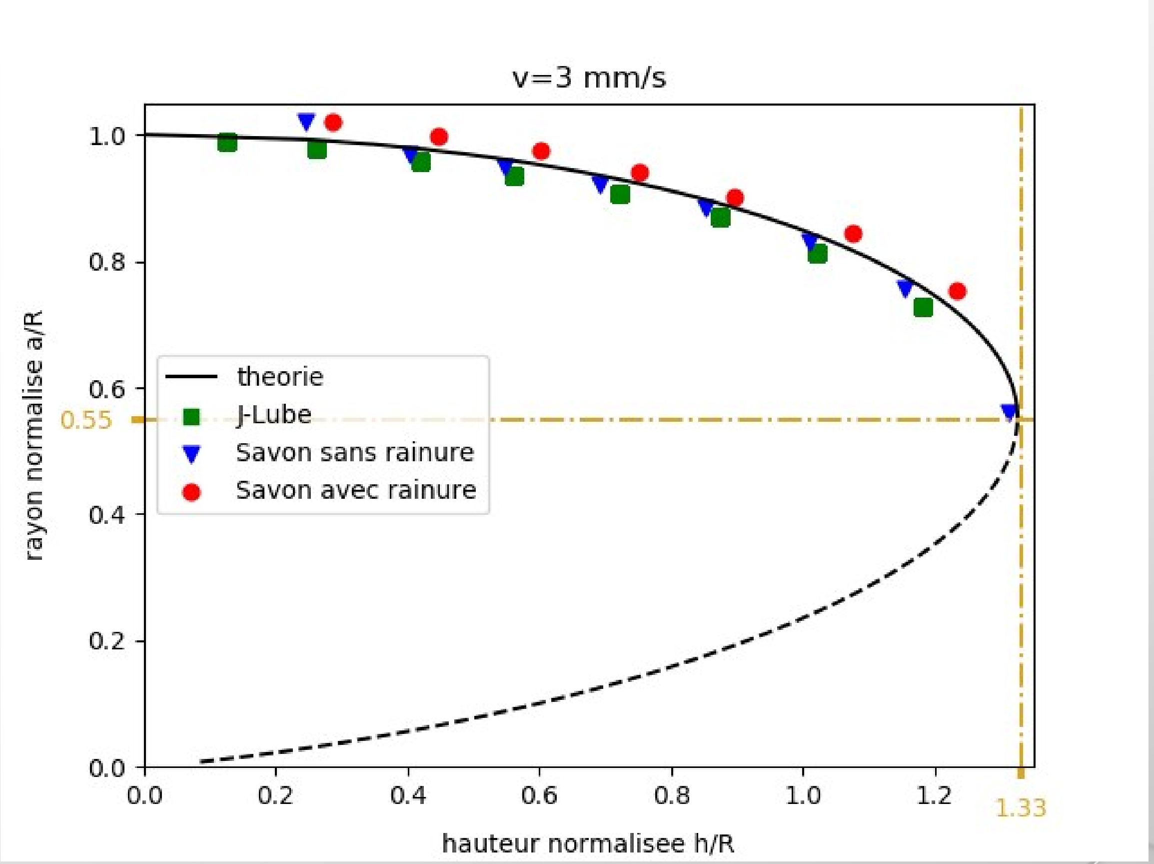




Critical point : Pc = (1.33 ; 0.55)

SUMMARY

- NO IMPACT OF THE SPEED
- NO IMPACT OF THE LIQUID ON THE MINIMAL RADIUS
- IMPACT OF THE LIQUID ON THE STABILITY OF THE CATENOID
- IMPACT OF THE GEOMETRY ON THE STABILITY

III-THEORY

- EXPRESSION OF THE ENERGY

$$E = \gamma S$$

- STARTING POINT: LAGRANGIAN OF THE SYSTEM

$$\mathcal{L} = \rho(z)\sqrt{1 + \rho'^2(z)}$$

- SATISFYING THE EULER-LAGRANGE EQUATION:

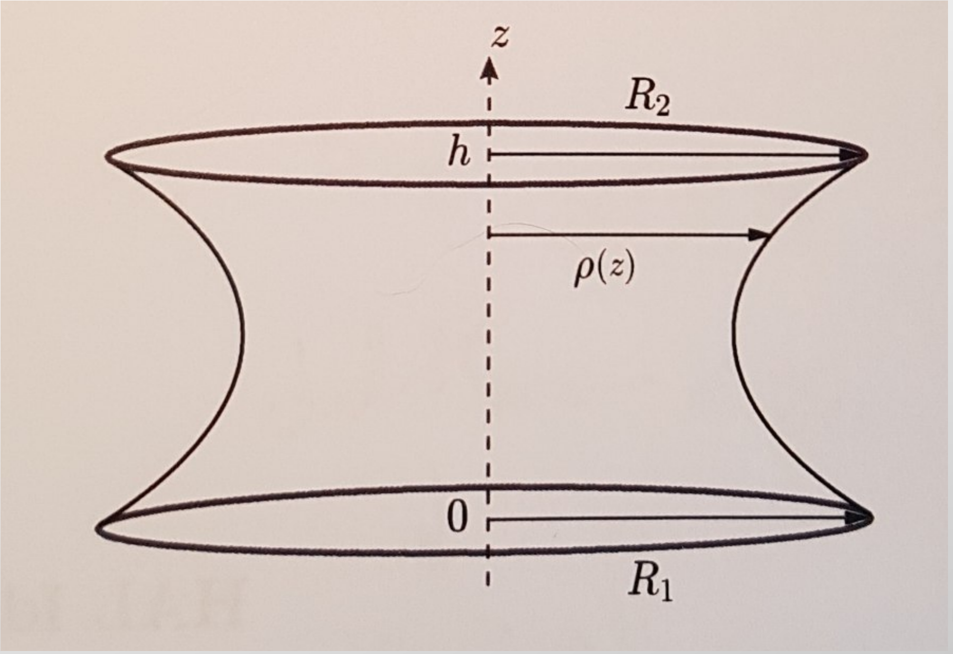
$$\frac{\partial \mathcal{L}}{\partial \rho} = \frac{d}{dz}\left(\frac{\partial \mathcal{L}}{\partial \rho'}\right)$$

- OBTAINING A 2ND ORDER EQUATION IN ρ:

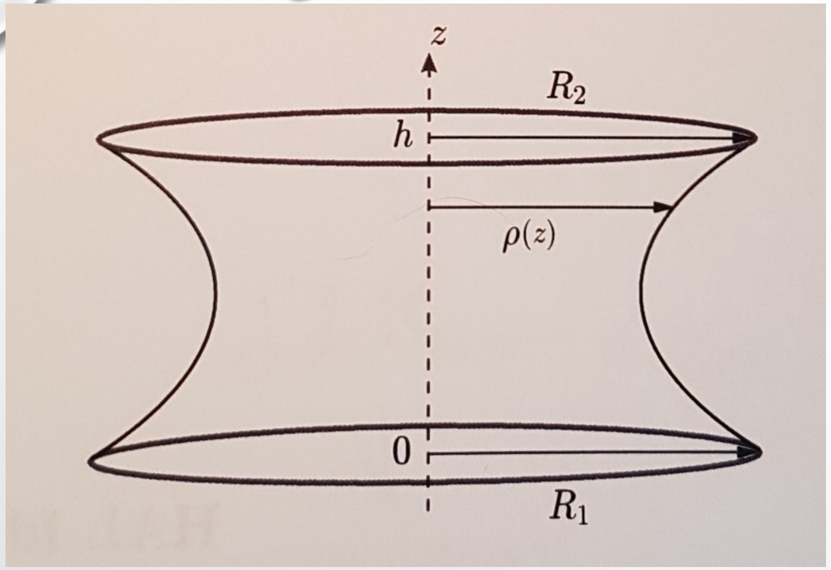
$$1 + \rho'^2(z) - \rho(z)\rho''(z) = 0$$

- SOLVING IT WE FINALLY OBTAIN:

$$\boxed{\rho(z) = a \cosh\left(\frac{z}{a} + C\right)}$$



from *Influence of boundary conditions on the existence and stability of minimal surfaces of revolution made of soap films* – Louis Salkin



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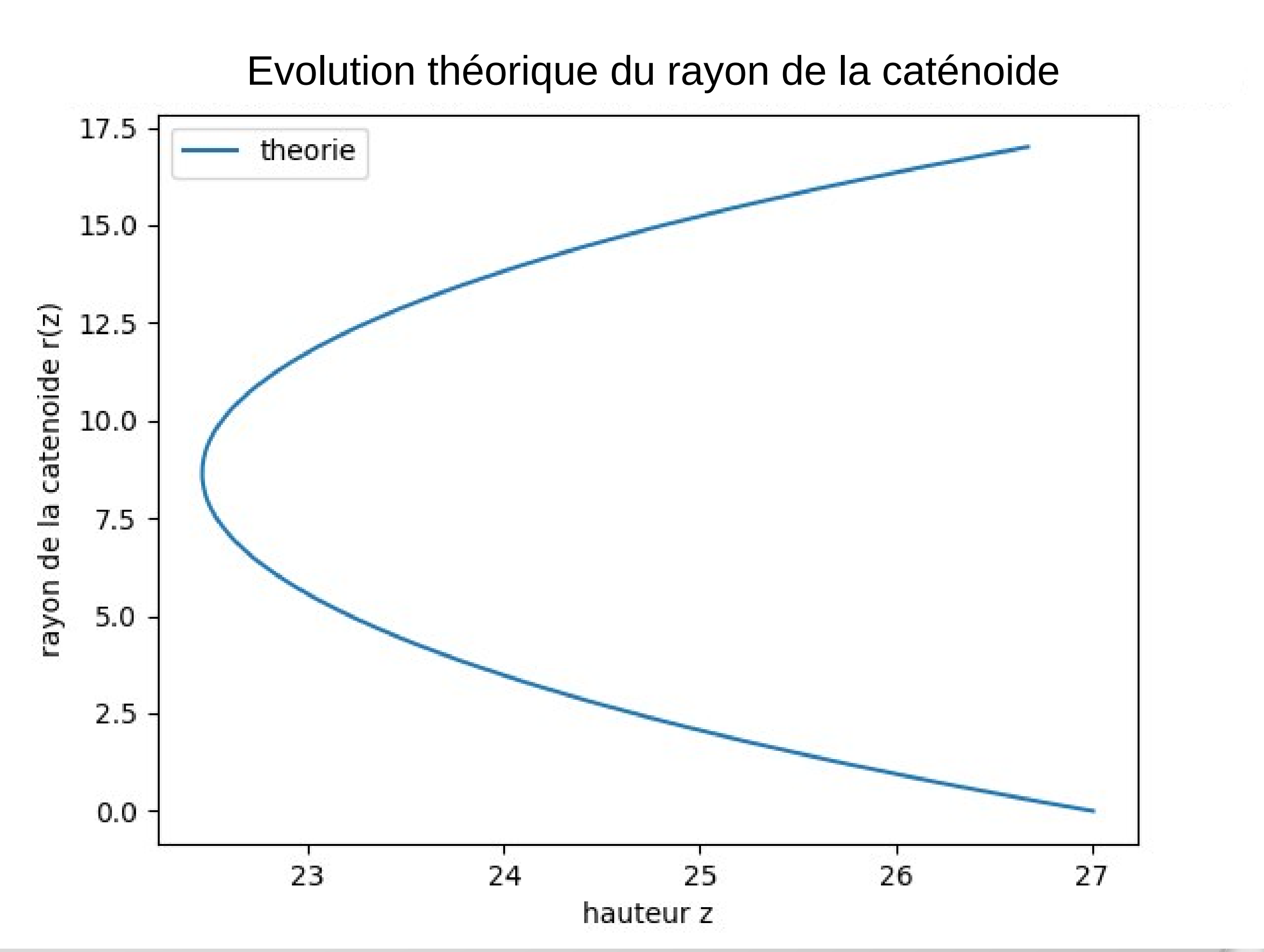
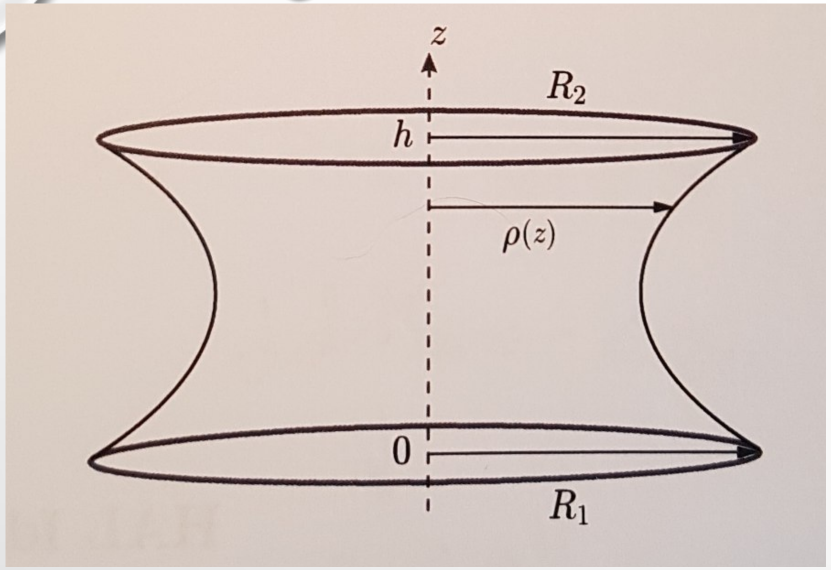
- IN OUR CASE : $R_1 = R_2 = R$

- BOUNDARY CONDITIONS:

$$\rho(0) = \rho(h) = R$$

$$\boxed{\rho(z) = a\,cosh(\frac{z}{a} - \frac{h}{2a})}$$





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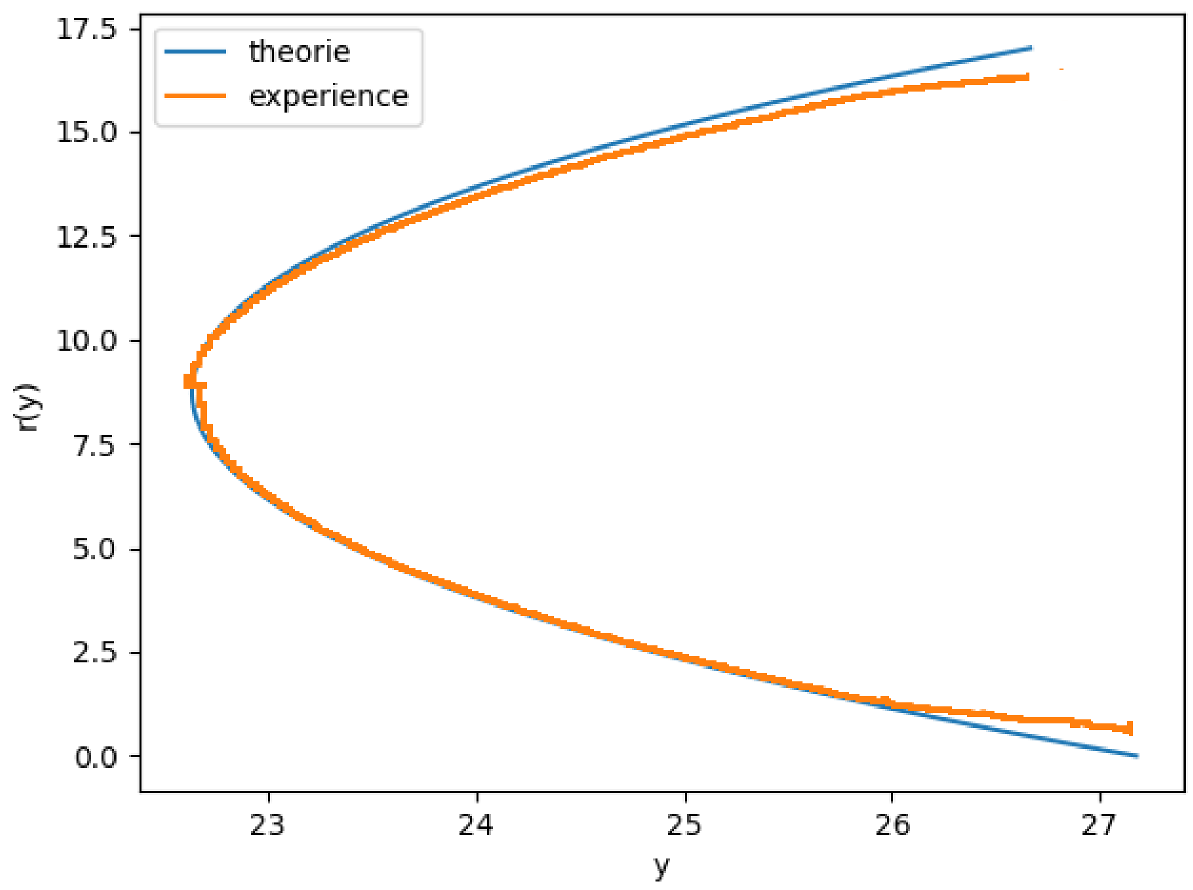
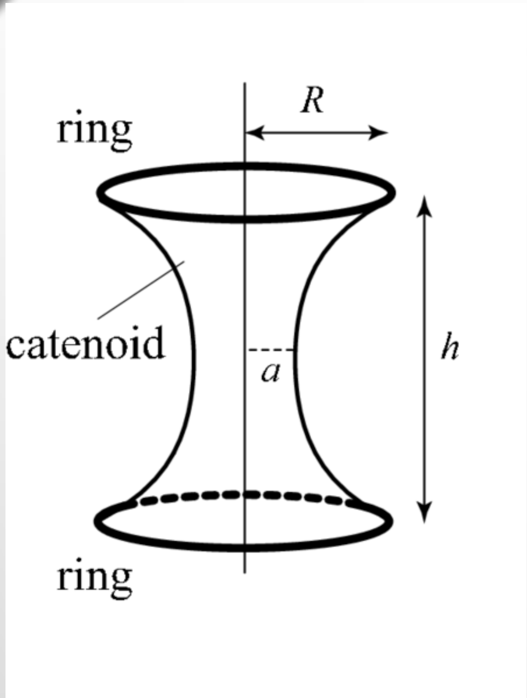
Evolution du rayon de la caténoide pour v=1mm/s, savon sans rainure



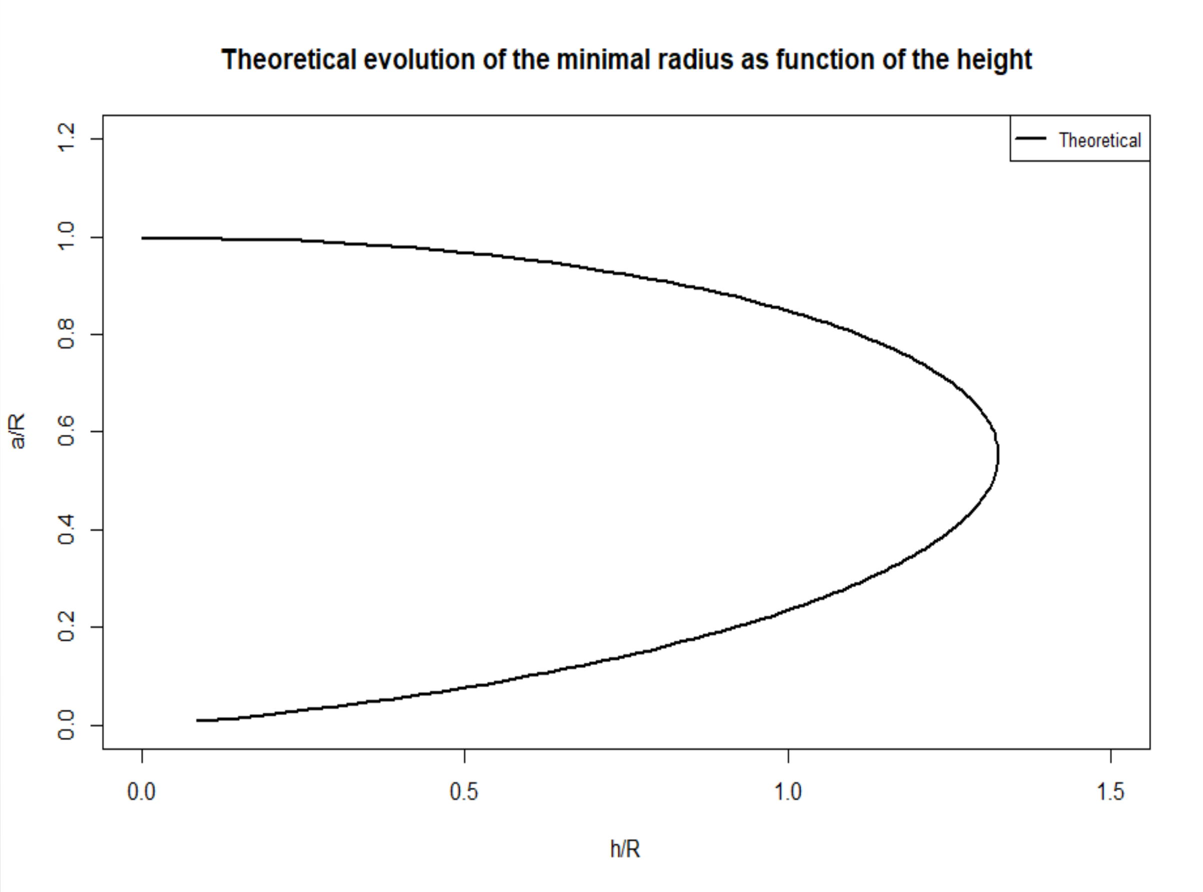
III-Theory



from *researchgate.net*

- $\rho(0) = \rho(h) = R$

$$\rightarrow \frac{h}{R} = \frac{2a}{R} argcosh(\frac{R}{a})$$



- $$cosh\left(\frac{\Delta X}{2}\right) - X = 0$$

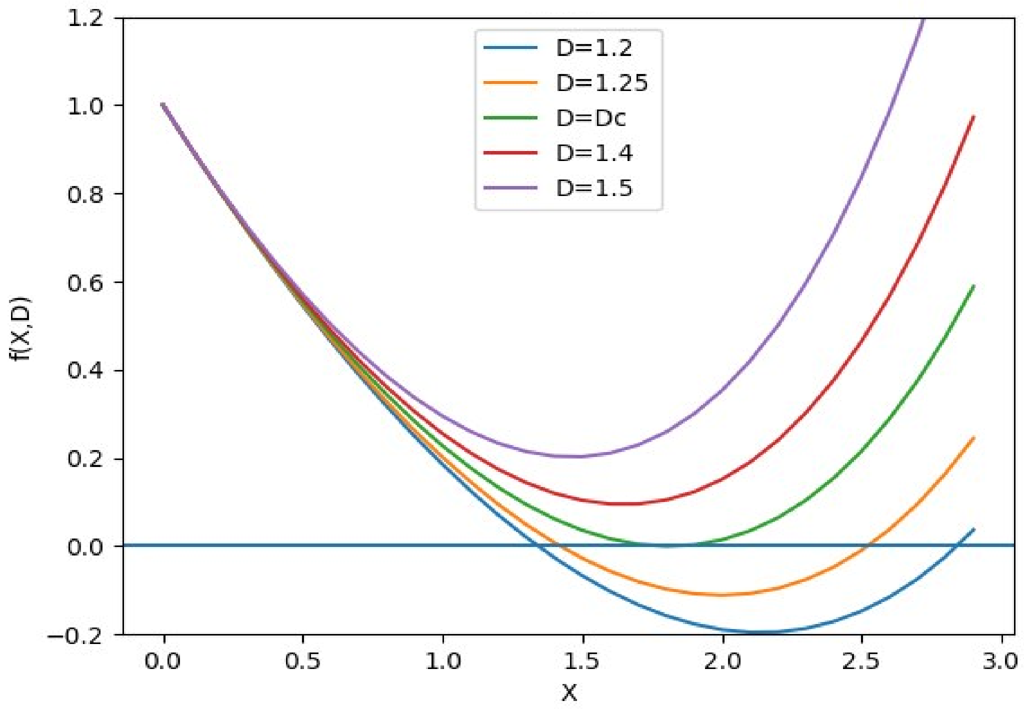
$$\Delta = \frac{h}{R}; X = \frac{R}{a}$$

- $\Delta > \Delta_c$: NO CATENOID EXISTS
- $\Delta = \Delta_c \approx 1.33$: ONLY ONE CATENOID SATISFIES THE BOUNDARY CONDITIONS $X_c \approx 1.81$

CATENOID COLLAPSES INTO TWO PLANAR FILMS SUSPENDED ON THE TWO RINGS (CASE OF THE SOAP + GROOVE)

- $\Delta < \Delta_c$: TWO CATENOIDS POSSIBLE, BUT THE LARGER NECK IS LESS STABLE
- $\frac{a_c}{R} = \frac{1}{X_c} \approx 0.55$; $\frac{h_c}{R} = \Delta_c \approx 1.33$

Graph of cosh(DX/2) – X as a function of X for different values of D



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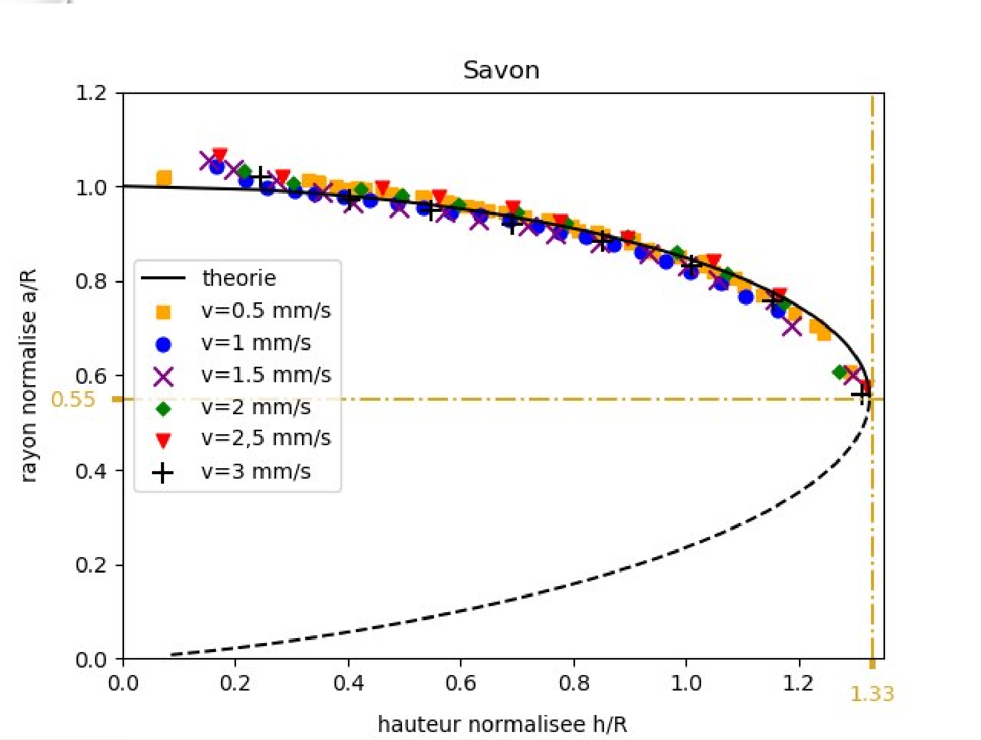
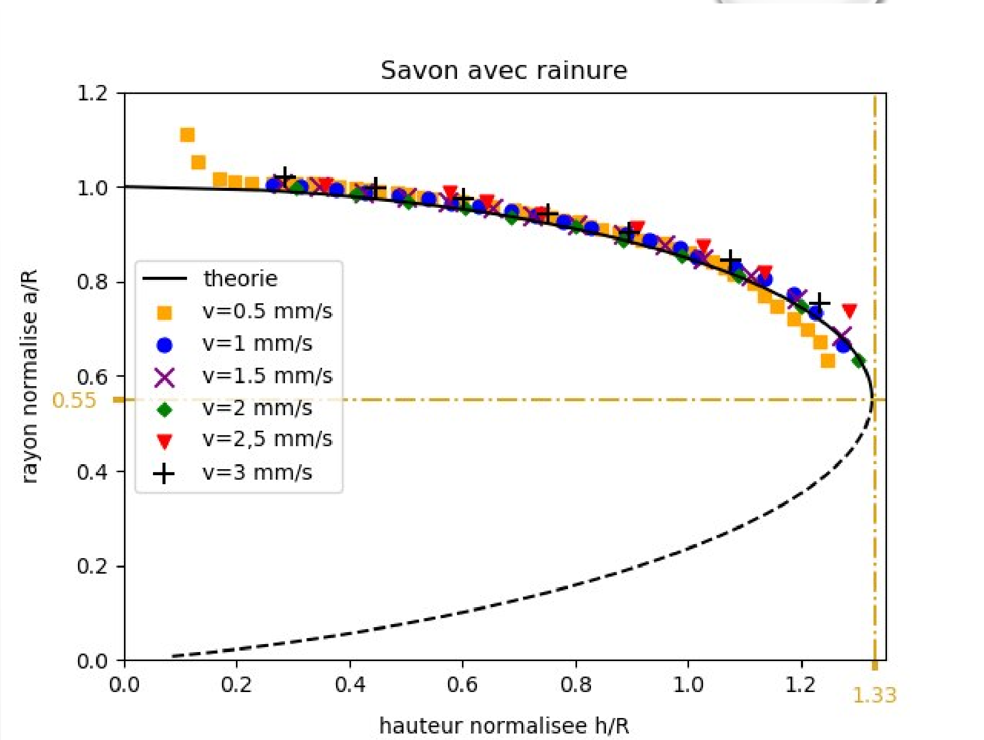
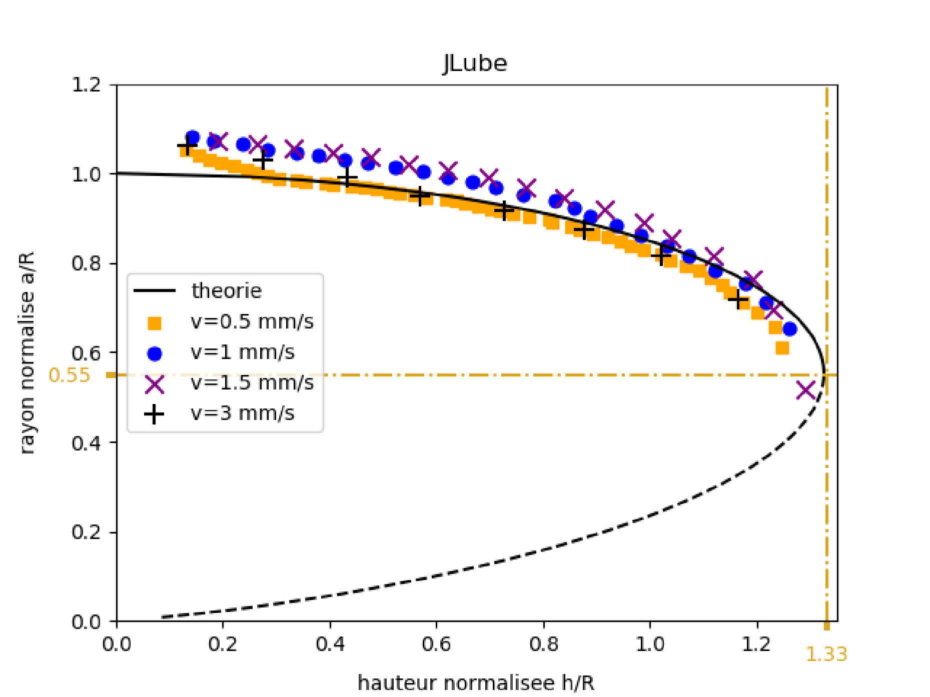




Critical point : Pc = (1.33 ; 0.55)



IV-SUMMARY AND CONCLUSION

SUMMARY OF OUR SCIENTIFIC APPROACH

- 1ST STEP: UNDERSTAND MORE THE MECHANISM
- 2ND STEP: MAKE FIRST MEASUREMENTS
- 3RD STEP: SOLVE PROBLEMS MET
- 4TH STEP: ANALYSE ALL THE DATA
- 5TH STEP: COMPARE THEM BETWEEN THEMSELVES AND WITH THE THEORY

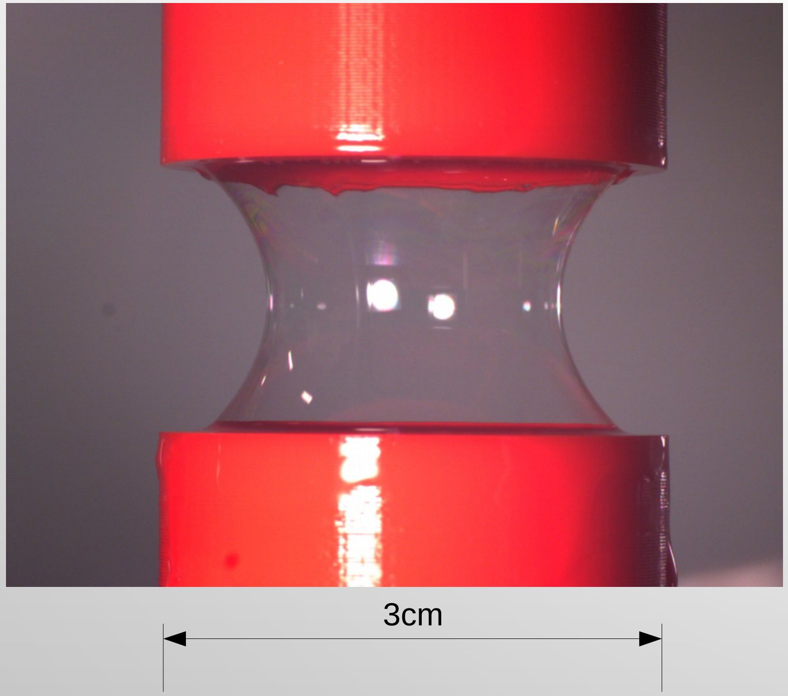
CONCLUSION

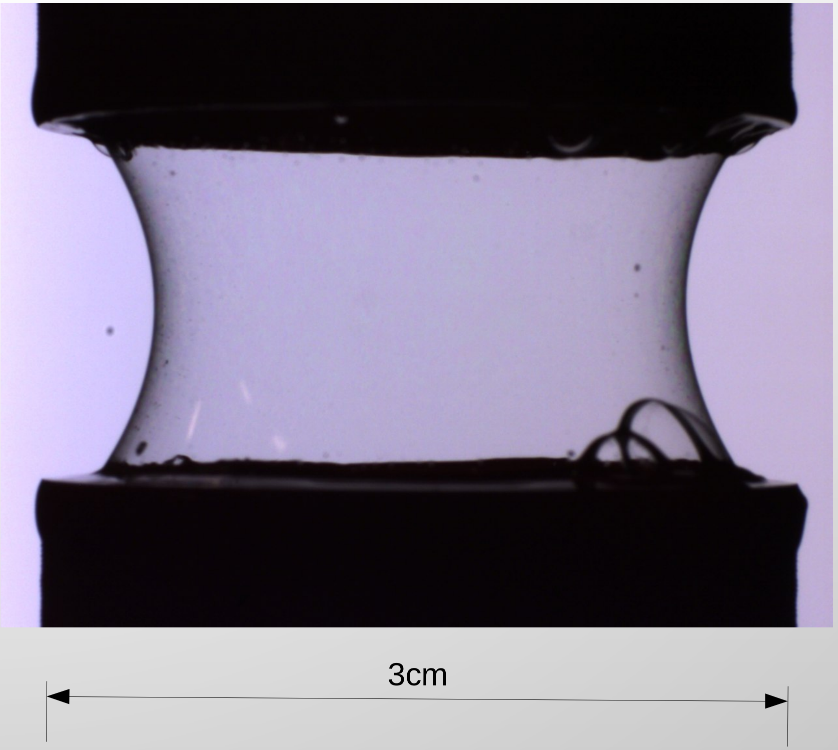
- RESULTS AGREE WITH THE THEORY. NEVER GO OVER THE CRITICAL POINT
- NO DEPENDENCE OF NECK RADIUS ON THE SPEED, LIQUID USED, PLASTIC PIECES
- IT LOOKS TO BE INDEPENDENT OF ANY PARAMETER
- STABILITY DEPENDS ON SEVERAL PARAMETERS (LIQUID, PLASTIC PIECES)

ADDITIONAL EXPERIMENTS

- CHANGING THE GEOMETRY (ELLIPSOID)
- CHANGING THE RADIUS OF THE PLASTIC PIECES (LIKE IN THE ARTICLE)
- ASYMMETRIC CATENOID ($R_1 \neq R_2$)
- ELASTIC EXPERIMENTS (GOING BACK IN THE MIDDLE OF THE EXPERIMENT)
- STATIC OBSERVATION
- FOCUS MORE ON THE STABILITY OF THE CATENOID

THANKS FOR YOUR ATTENTION !


3cm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