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Simulation code dedicated to cosmology using the Particle-Mesh technique

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M1 Physique & Magistère de Physique Fondamentale

QMat QUANTUM SCIENCE AND NANOMATERIALS

Observatoire astronomique de Strasbourg

Plan

I. Context :
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II. Objectives :
   - Motivations

III. Methods :
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   - Theorical aspect

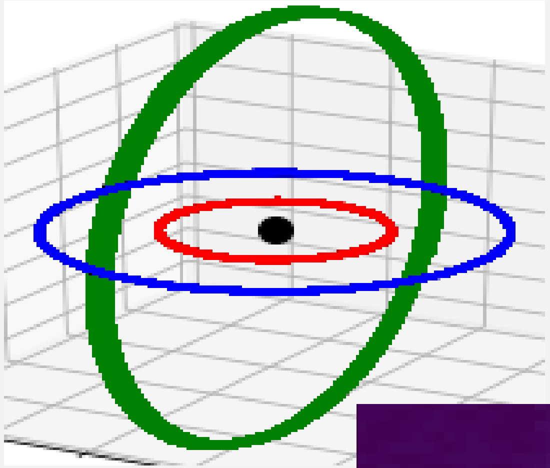
IV. Newtonian results :
   - Elliptical trajectory
   - Self-gravitational system & Jeans criterion

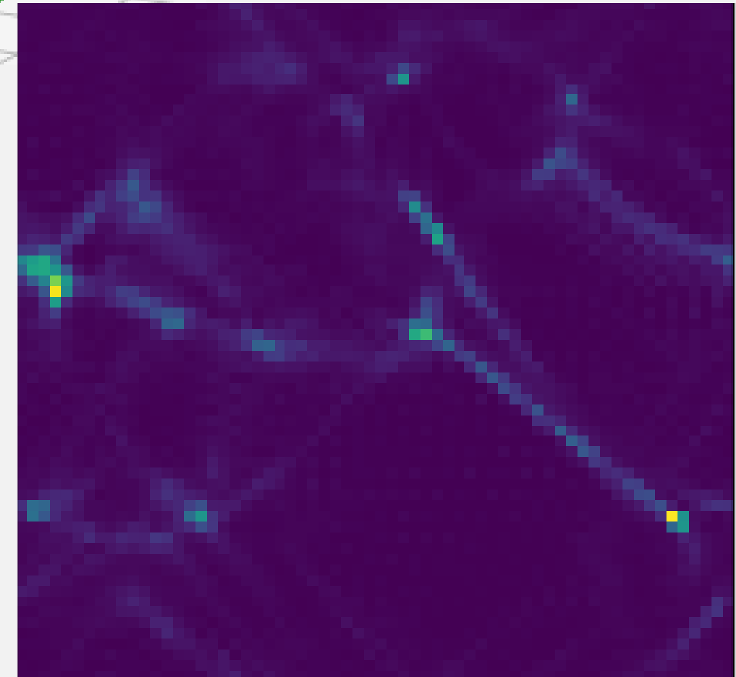
V. Cosmological case :
   - Expension of the universe
   - Gravitational dynamics in an expanding universe
   - Зельдович(Zeldovich)'s test

VI. Results
   - First result

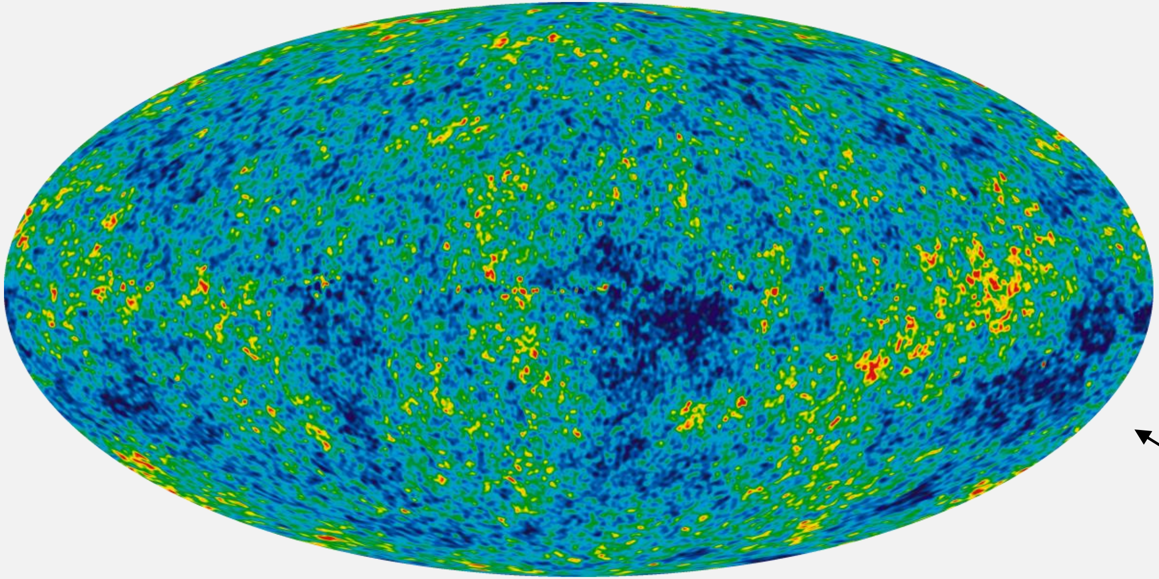
VII. Discussion
   - Secondary infall
   - Different geometry/models

VIII. Conclusion



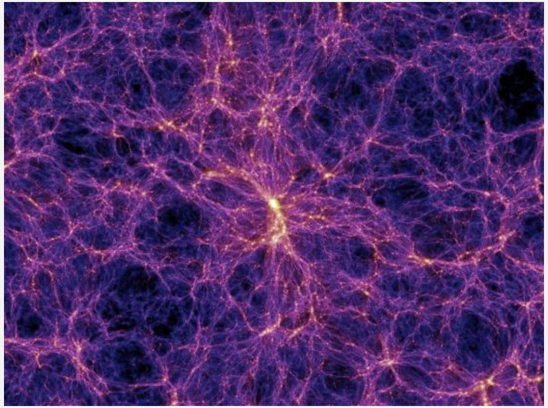


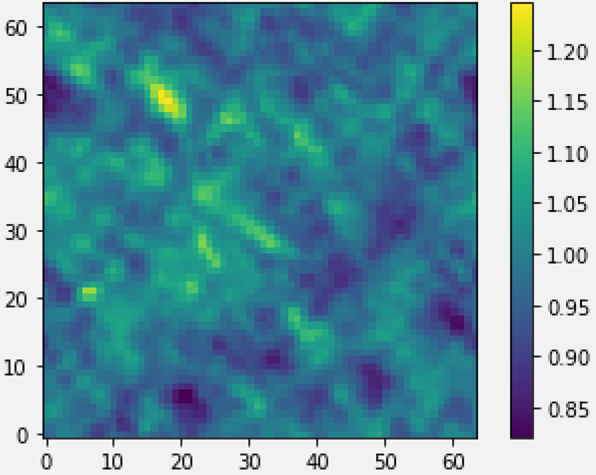
I. Context : *Cosmological simulation*



https://fr.wikipedia.org/wiki/Fond_diffus_cosmologique

https://www.gurumed.org

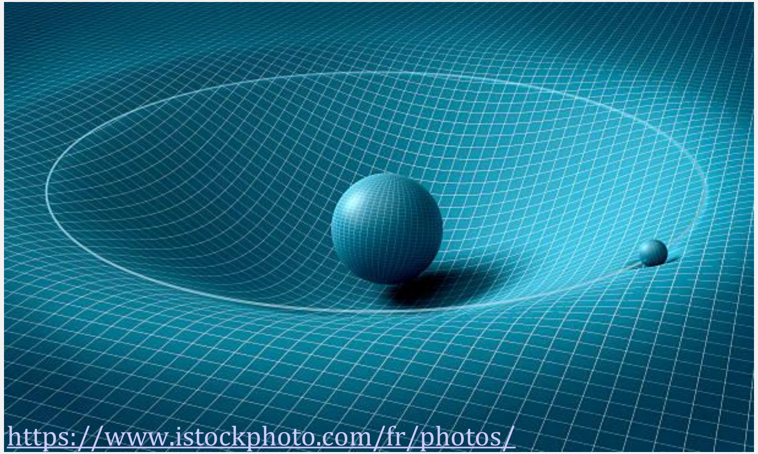




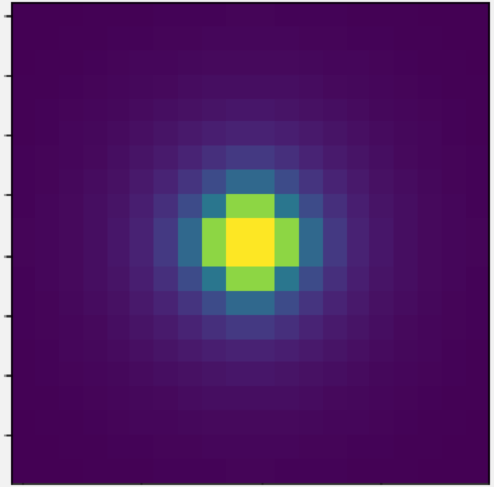
- Only one interaction : Gravitation |In our case|
- Heavy Particle ⇔Cold Dark Matter
- N bodies Interaction : need of numerical simulation
  creation of complex and non linear structures /!\

II. Objectives : *Motivations*

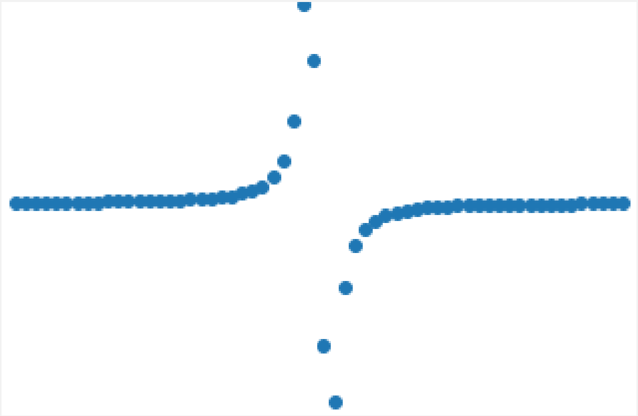
- Python version of the PM method
- Study properties of the gravitational field strength
- Study a numerical method for N body interaction
- Get a self-gravitational system



Modulus of the Force - dimensionless

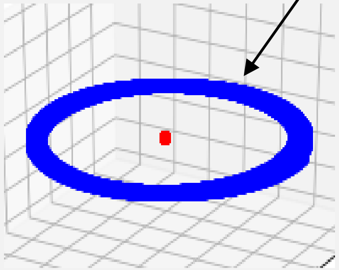


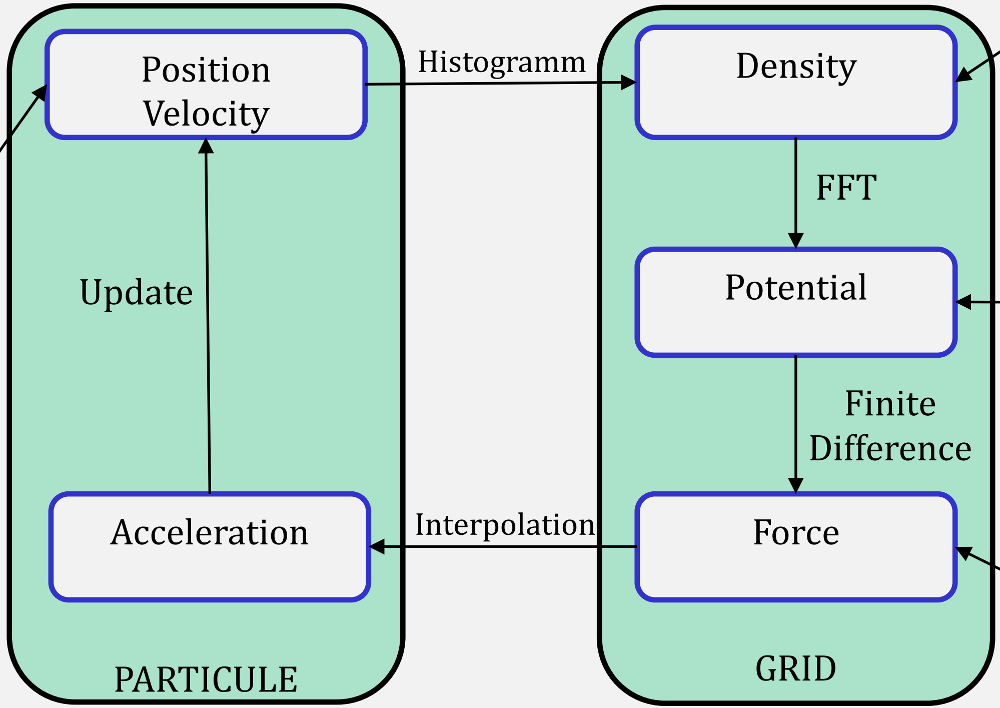
Fx in respect with x - dimensionless

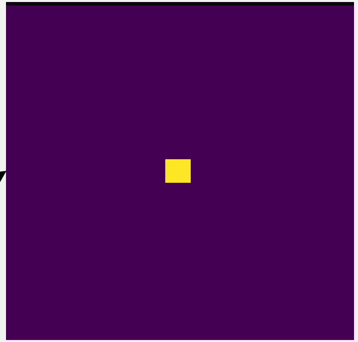


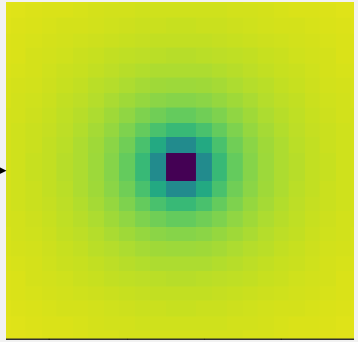
III. Methods : *Particle Mesh*

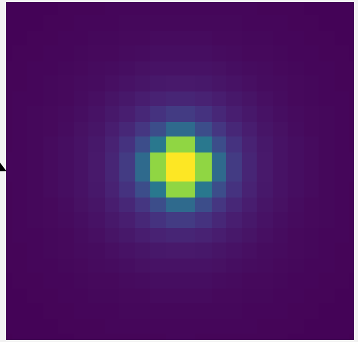
Particle	=	Matter's distribution
Mesh	=	grid ⇔ cell











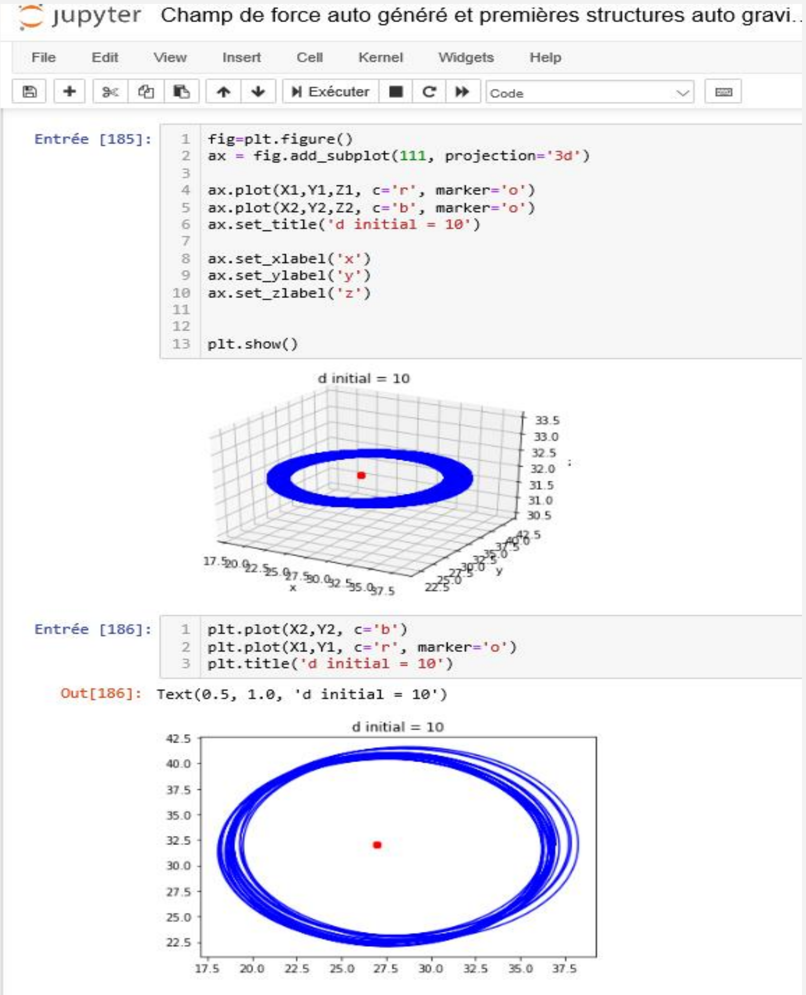
III. Methods : *Python*

Why Python :

- Python makes operation on array efficient

- Python decrease drastically developpement time

- Jupyter makes the programmation step more interactive



III. Methods : *Theorical aspect*

- **First step** : Get the density ρ over the grid ⇔ Count the number of particles per cells

- **Second step** : Resolve the Poisson equation : $\Delta\Phi = 4\pi G\rho$ ⇔ Fast Fourier Transform (FFT)

  As we know : $FT[\frac{\partial M}{\partial x}] = -ik\,\tilde{M}$ ⇔ M is here a simple fonction

  We can apply it twice to ΔΦ : $-k^2\tilde{\Phi} = 4\pi G\tilde{\rho}$

  To finaly get : $-\Phi = 4\pi G * FT^{-1}\,[\tilde{\rho}/k^2]$

Notations

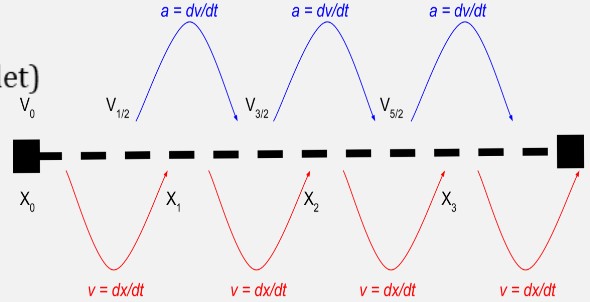
FT[M]=M̃ ⇔ Fourier transform of F

FT^{-1} inverse Fourier transform

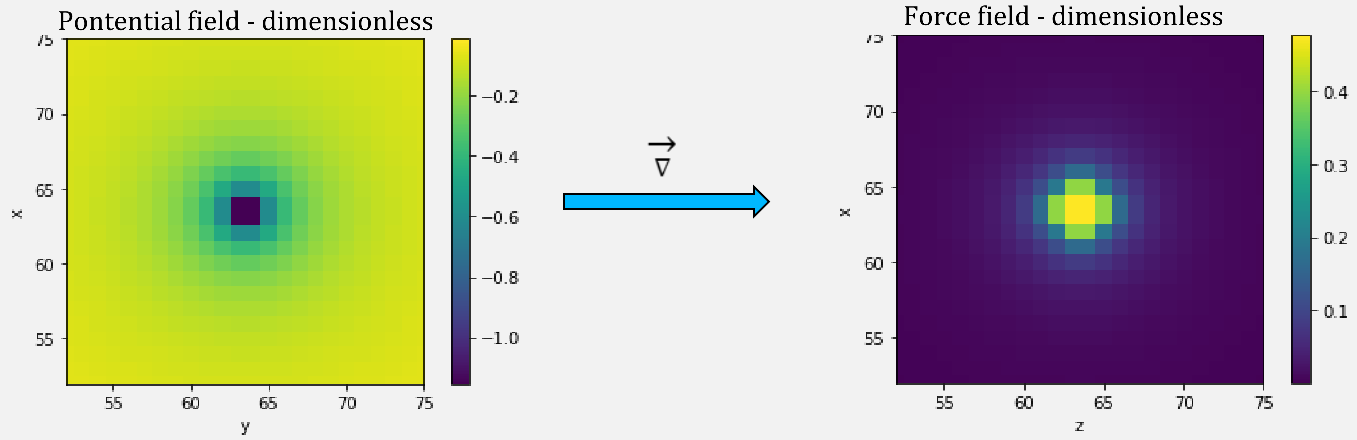
- **Third step** : Take the difference of the potential Φ to get the acceleration :

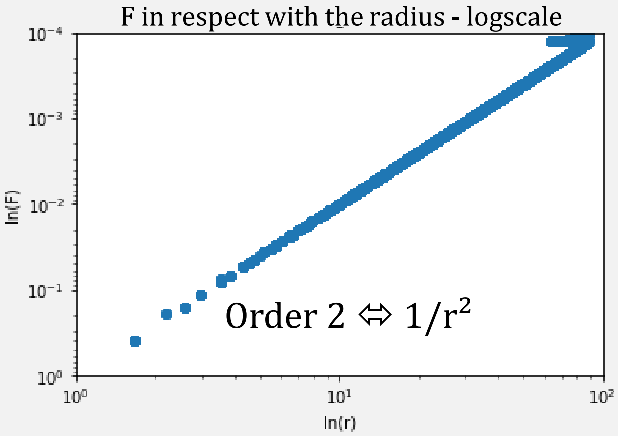
$$F_i = \frac{\Phi_{i+1} - \Phi_{i-1}}{2dx}$$

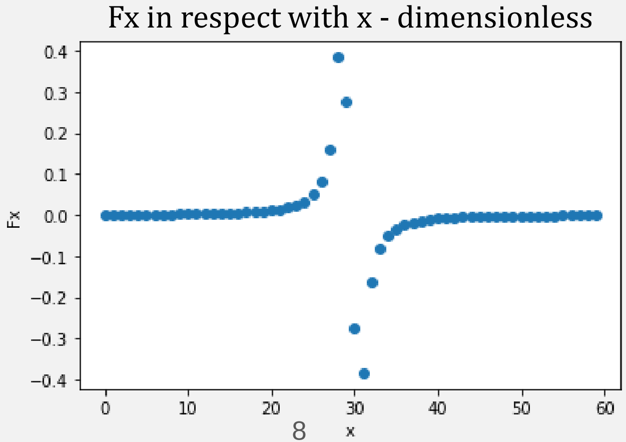
- **Last step** : Update positions and velocities with an integrator : Leap Frog (Verlet)



IV. Newtonian results : *Elliptical trajectory ➔ Validation*



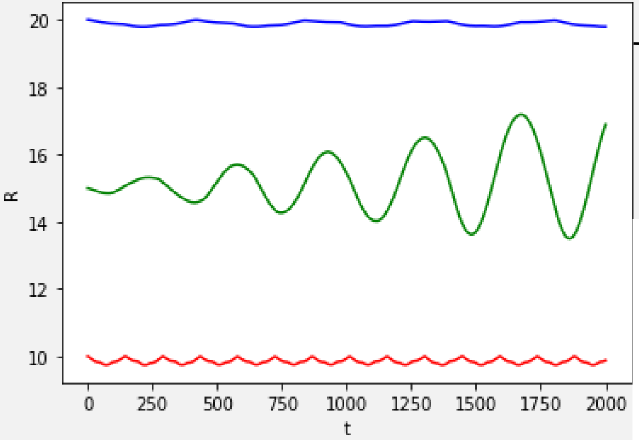




- Singularity of the gravitational force at r = 0
- Dipolar Force ⇔ at r = 0

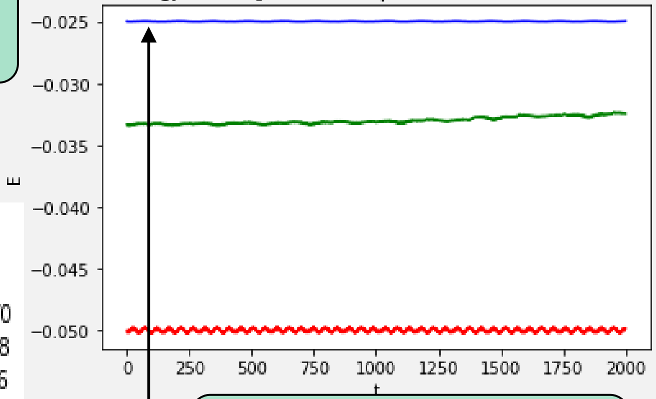
IV. Newtonian results : *Elliptical trajectory* ➔ *Results*

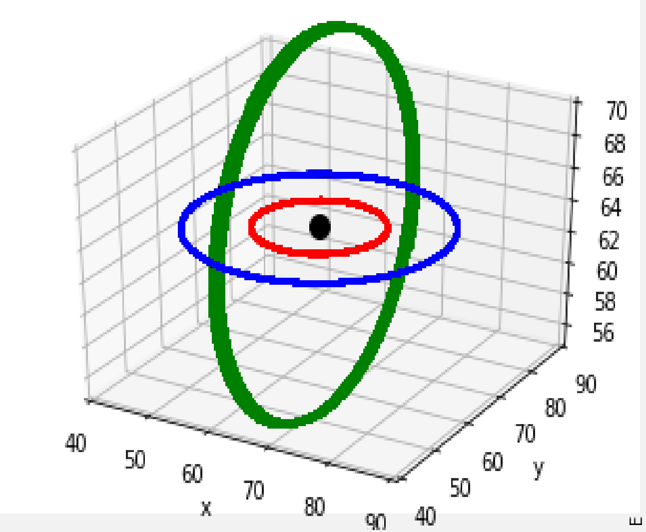
Radius - no dimension



Radius conservation for circular orbit. Errors and fluctuations due to cells more present when 3D tilted trajectory

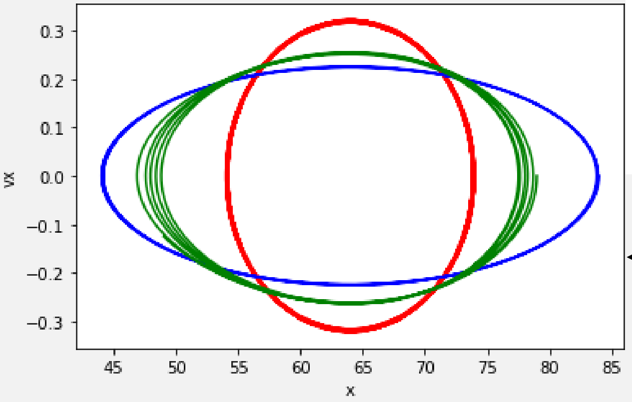
Energy in respect with the time - dimensionless





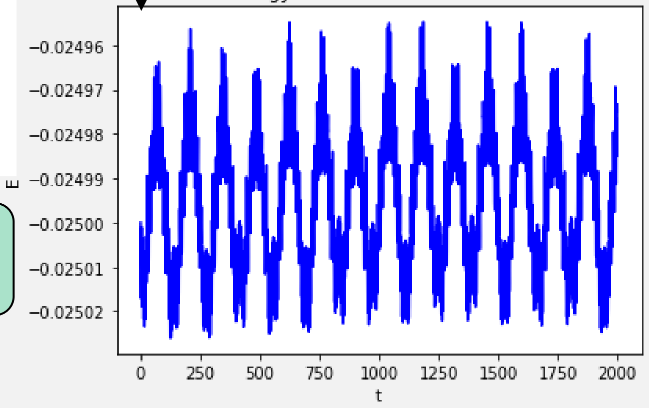
Energy conservation : Further the singularity, better is the precision

Phase space : Vx in respect with X - no dimension



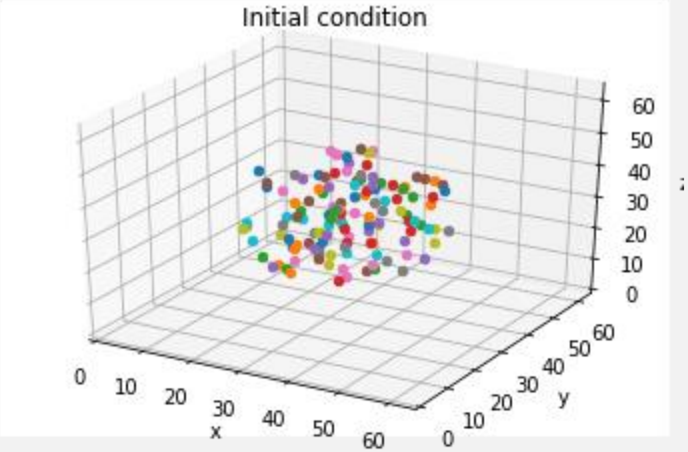
Phase space as expected for such a trajectory. Trajectories confined in a plan are more precise

Energy - no dimension - dt = 0.1



IV. Newtonian results : *self gravitational system & Jeans Criterion*

Initialisation :

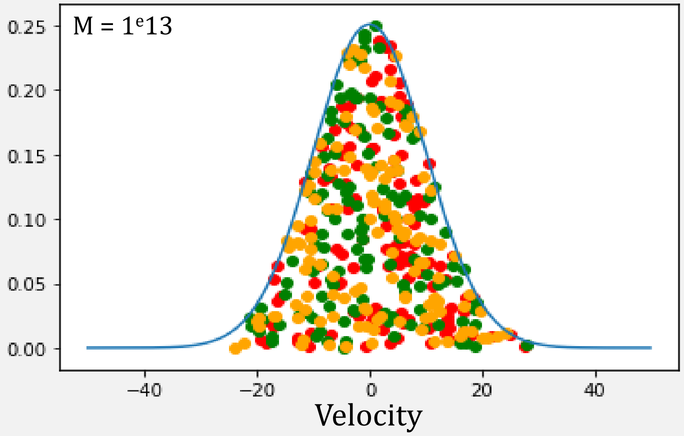


- Random distribution of matter in a sphere of radius Rs
- Each particle has a mass M.

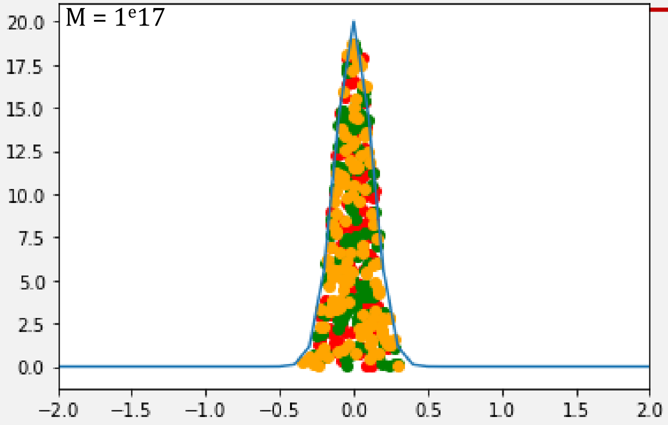
- Gaussian distribution of the velocities such that :

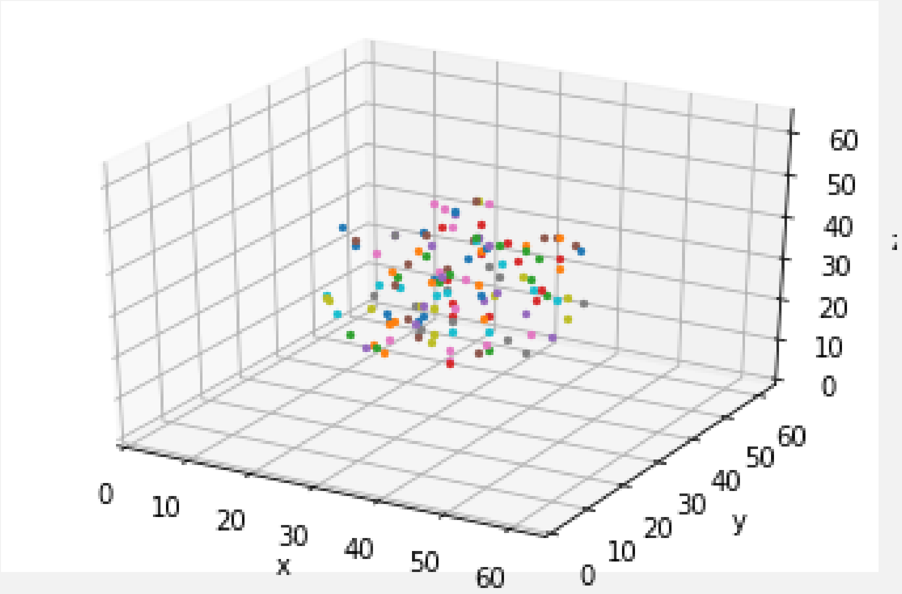
$$P = \frac{\exp\left(\frac{-x^2}{2V^2}\right)}{2V\pi}$$

Where : $V = Rs / \sqrt{\frac{G*M*N}{\frac{4}{3}\pi Rs^3}}$



IV. Newtonian results : *self gravitational system & Jeans Criterion*



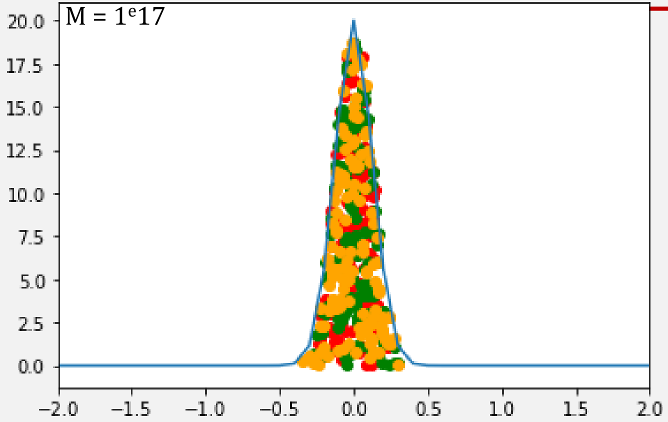


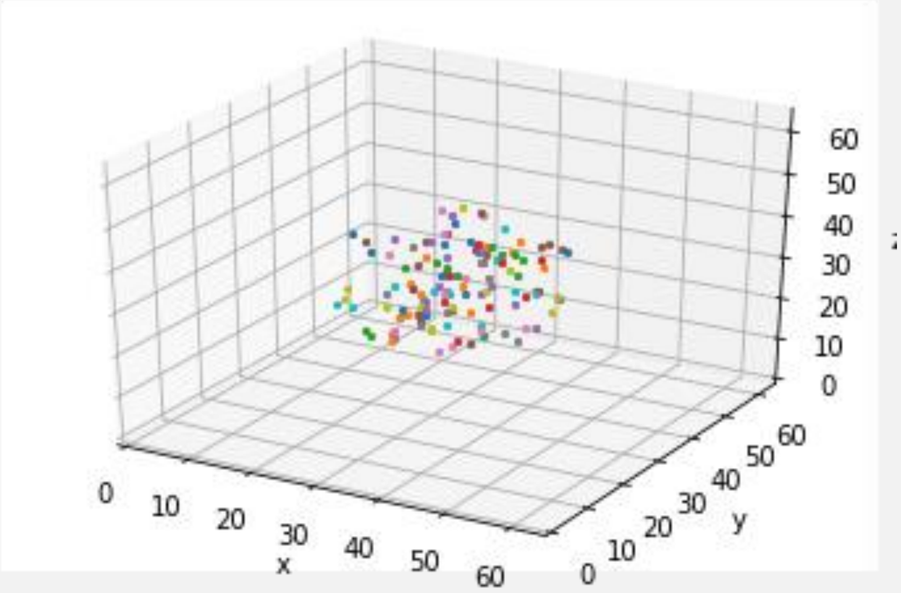
Significant mass with large amount of particles :

We expect this system to collapse /!\

Because : $R_s > \lambda_j = c_s(\frac{\pi}{G\rho})^{1/2}$

IV. Newtonian results : *self gravitational system & Jeans Criterion*





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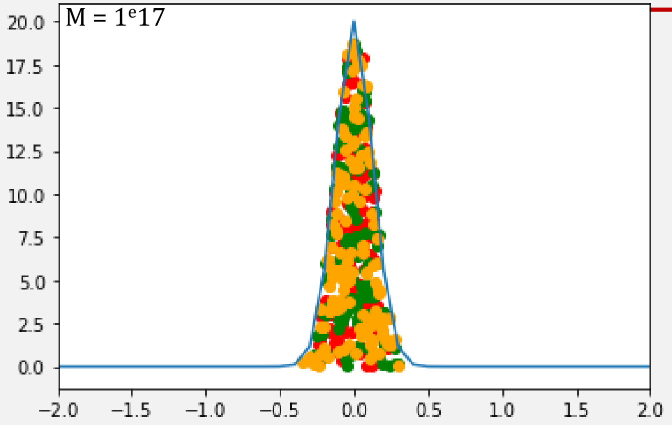

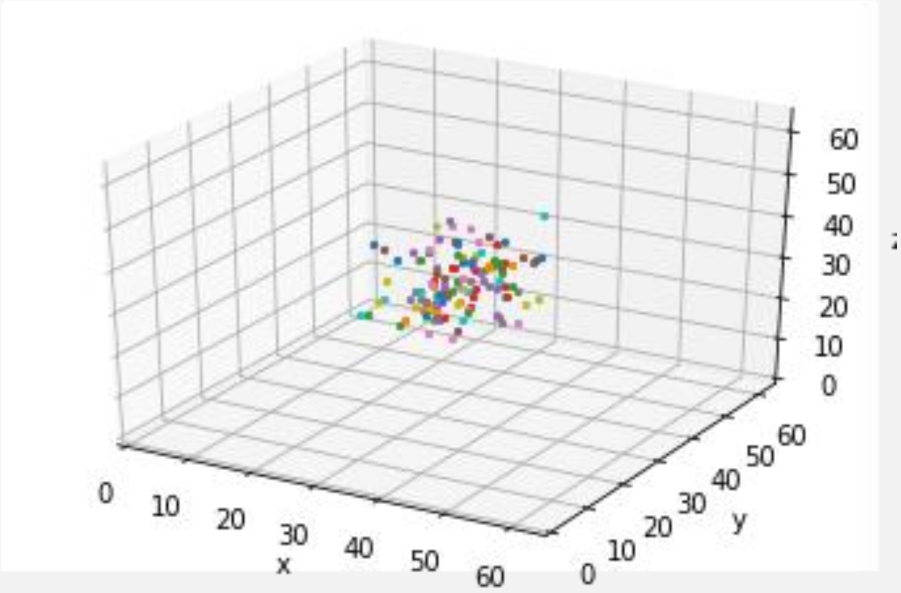

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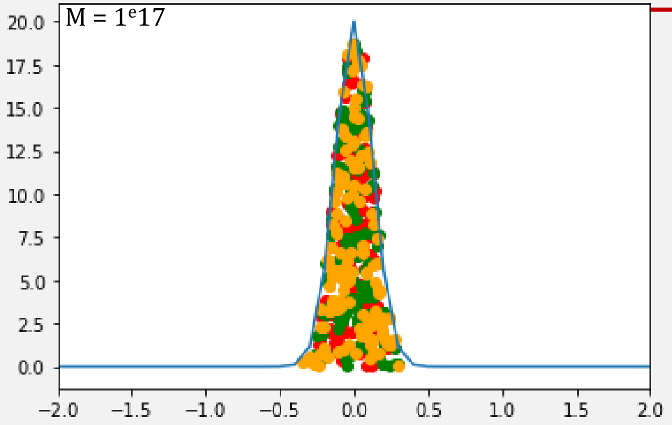
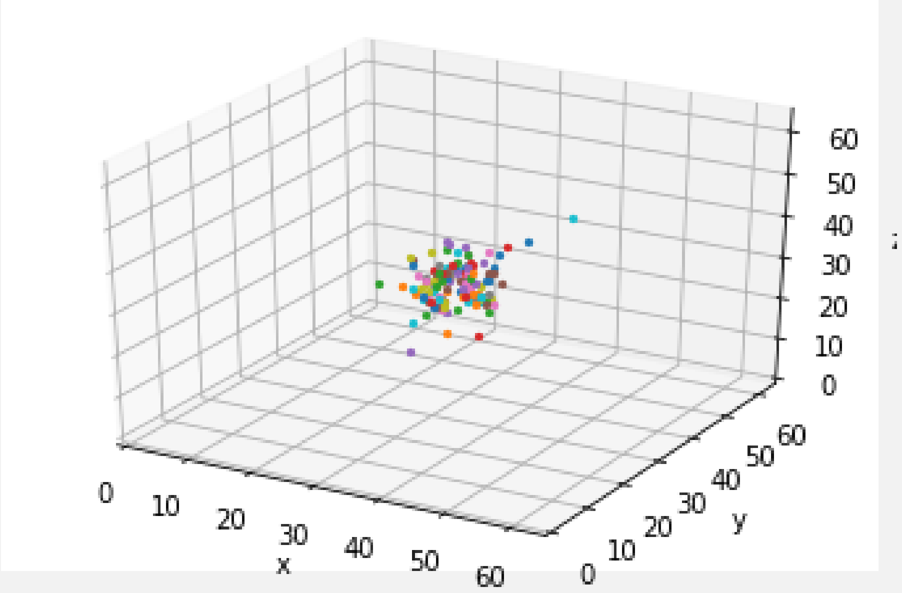




- ➢ System collapse quickly
- ➢ Can't go further because the simulation explodes
- ➢ Relevant collapsing parameter ?
  - Mass ⇔ Density

Other phenomenon : Secondary Infall

V. Cosmological Case

Expansion of the universe

❖The space in the cosmos is expanding at large scales

❖Comobile distance & scale factor :

$$x(t) = a(t)x_0$$

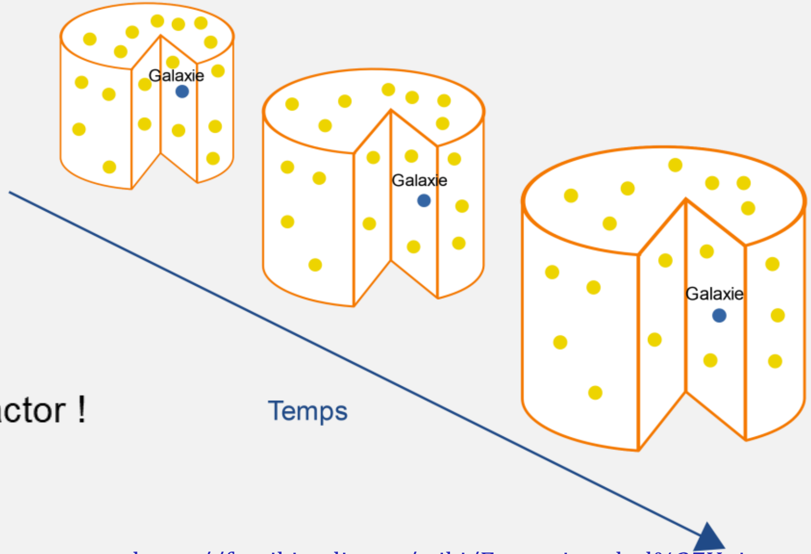
❖The integration step is not the time anymore, but the scale factor !

❖With the associated redshift z(= measure of time) :

$$z(t) = \frac{a(t_0) - a(t)}{a(t)}$$

❖a(t) is given by the content of the cosmos !

- The mass content is given by the density parameters :
  - Dark matter : $\Omega_{DM} = 0{,}3175$
  - Dark energy : $\Omega_{\Lambda} = 0{,}6825$



https://fr.wikipedia.org/wiki/Expansion_de_l%27Univers

V. Cosmological Case

Gravitationnal dynamics in an expanding universe

- Instead of the density, we work with the density contrast :

$$\delta = \frac{\rho - \bar{\rho}}{\bar{\rho}}$$

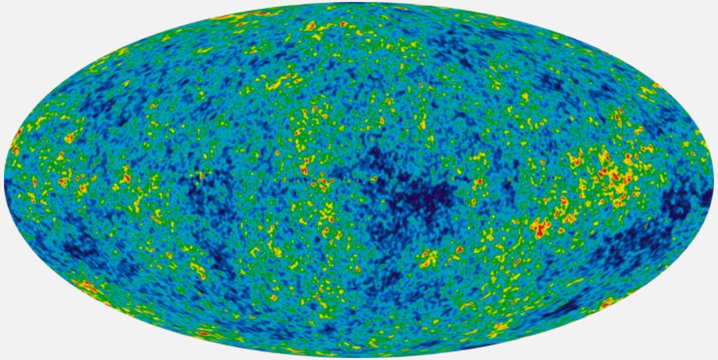
- Also, the new Poisson equation for adimensionned potential :

$$\Delta\phi = \frac{3\Omega_{DM}}{2a}\delta$$

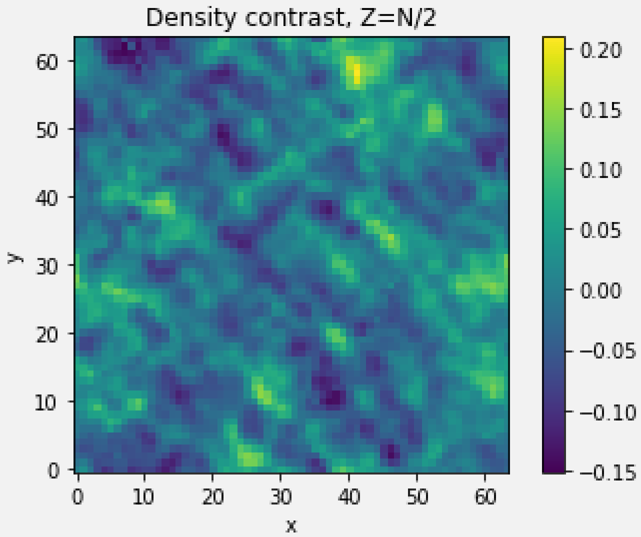
V. Cosmological Case

How it is resolved

❖Initial conditions : density fluctuations close to the cmb



https://fr.wikipedia.org/wiki/Fond_diffus_cosmologique



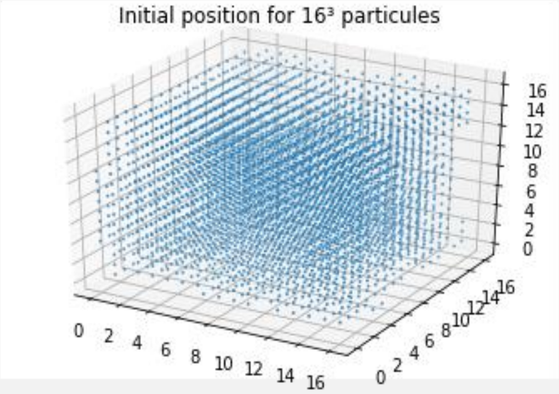
❖Validation of the code : Collapse of 1D sinus (Zeldovich test)

❖Condition to stop : a=1 (today)

a = 0.011
z = 89.9

VI. Results

Size of the box : N = 16 ⇔ 4096 Particles & Cells



z = 89,9

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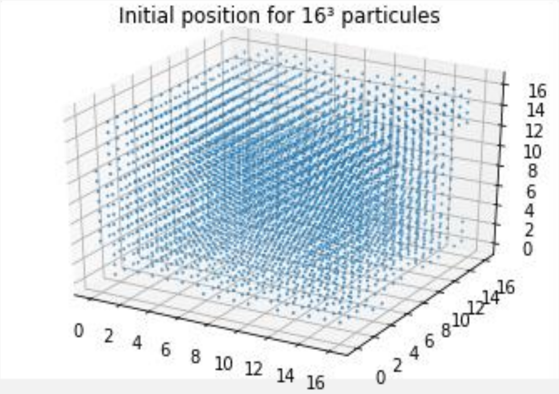
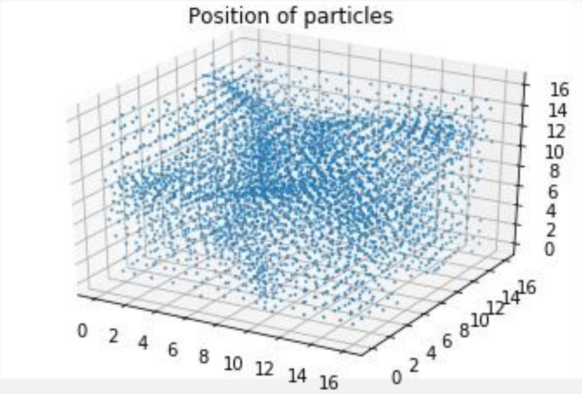


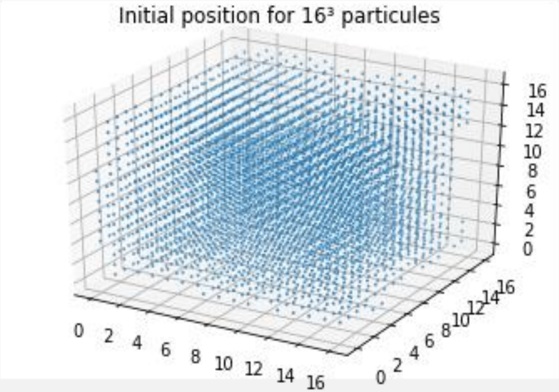


z = 89,9

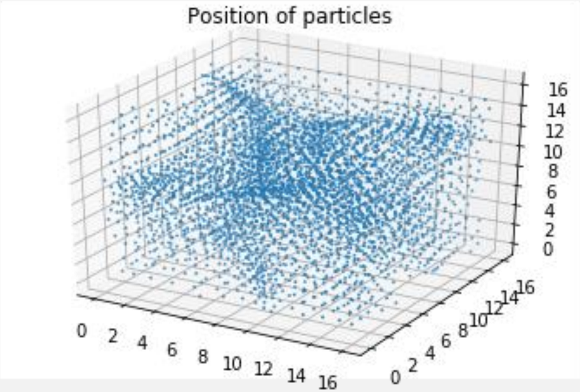
z = 8,62

VI. Results

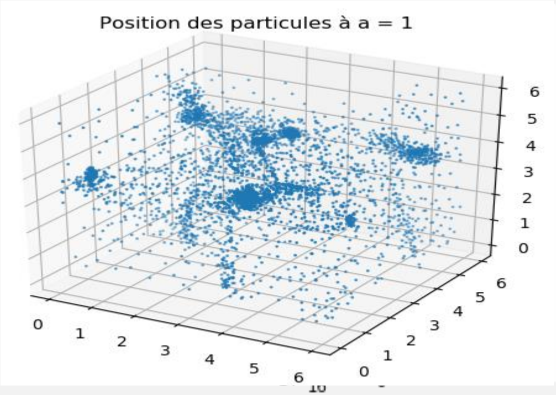
Size of the box : N = 16 ⇔ 4096 Particles & Cells



z = 89,9



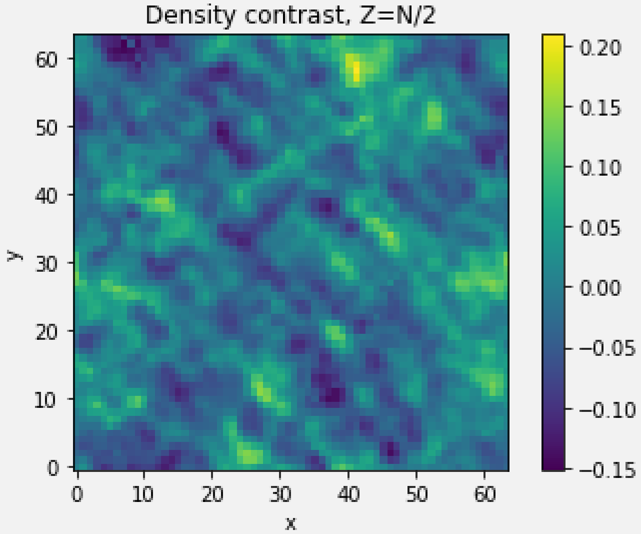
z = 8,62



z = 0

VI. Results

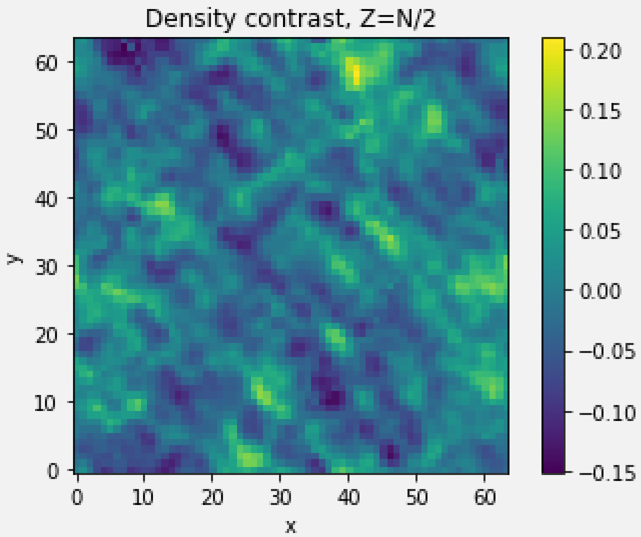
Size of the box : N = 64 ⇔ 262 144 Particles & Cells



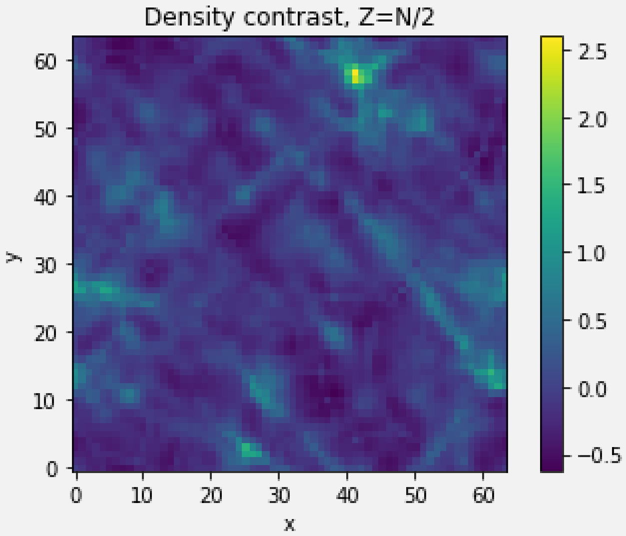
z = 89,9

VI. Results

Size of the box : N = 64 ⇔ 262 144 Particles & Cells



z = 89,9



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VI. Results

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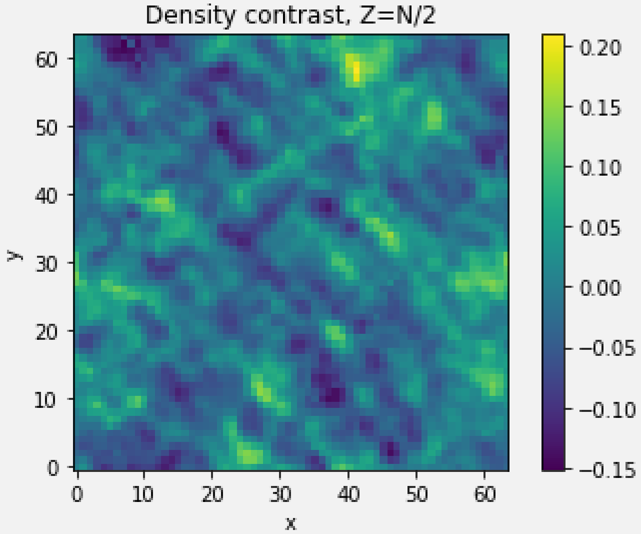
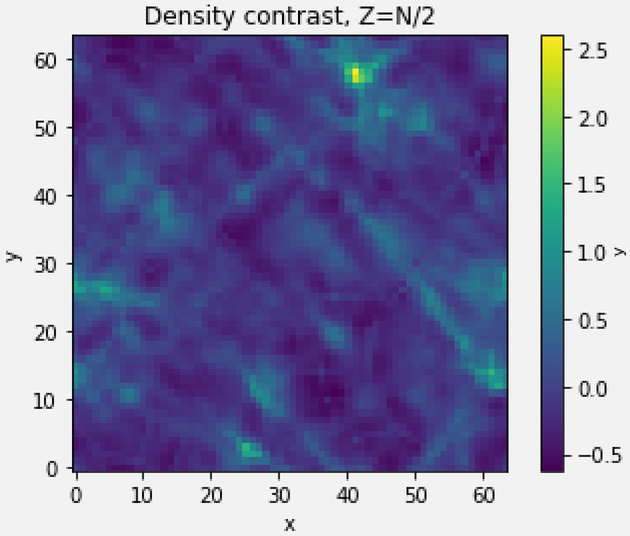
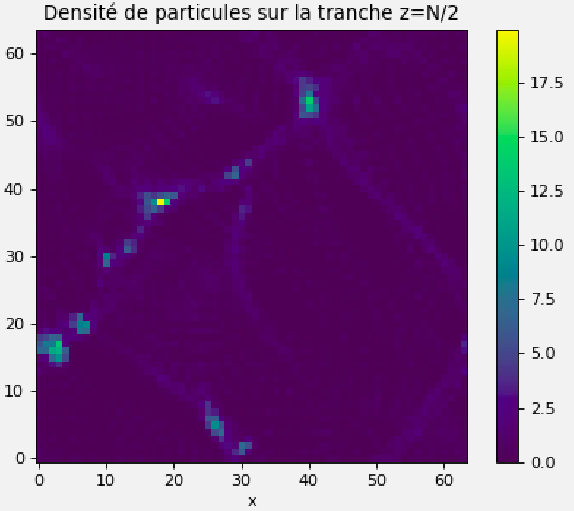


z = 89,9



z = 8,62

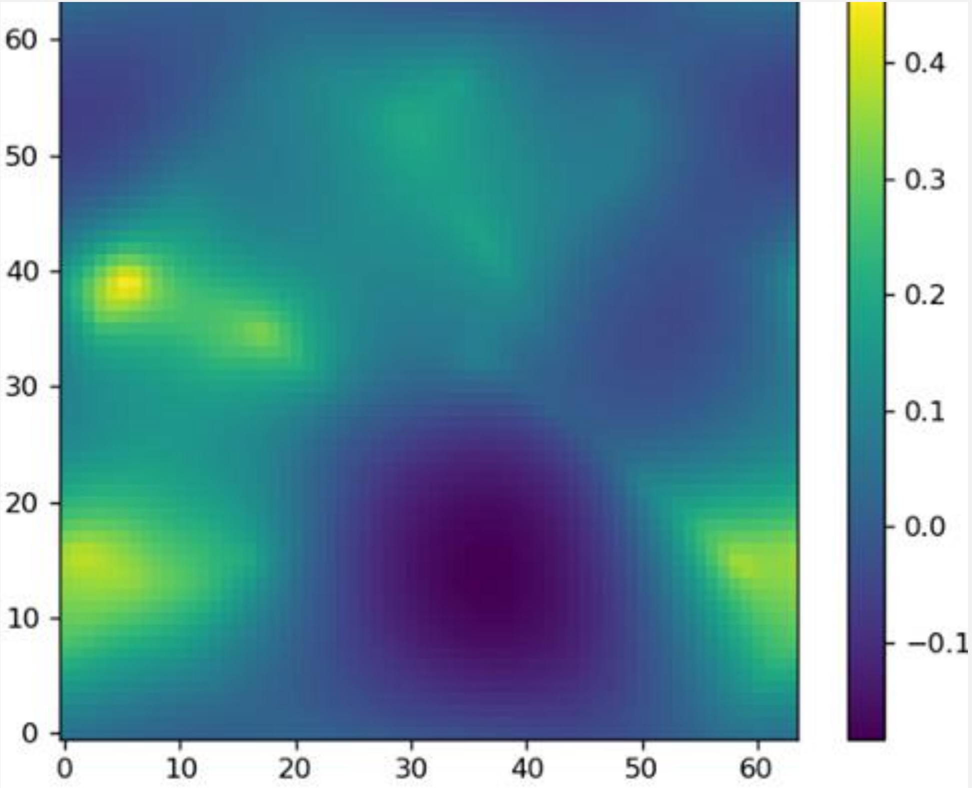


z = 0

VI. Results

Size of the box : N = 64 ⇔ 262 144 Particles & Cells

Potential at N = 52



- ❖ Greater contrast between dense region of the box and region that is empty
- ❖ Structure are clearly visible
- ❖ Slight filaments linking the dense regions of the box

VII. Discussion & perspective

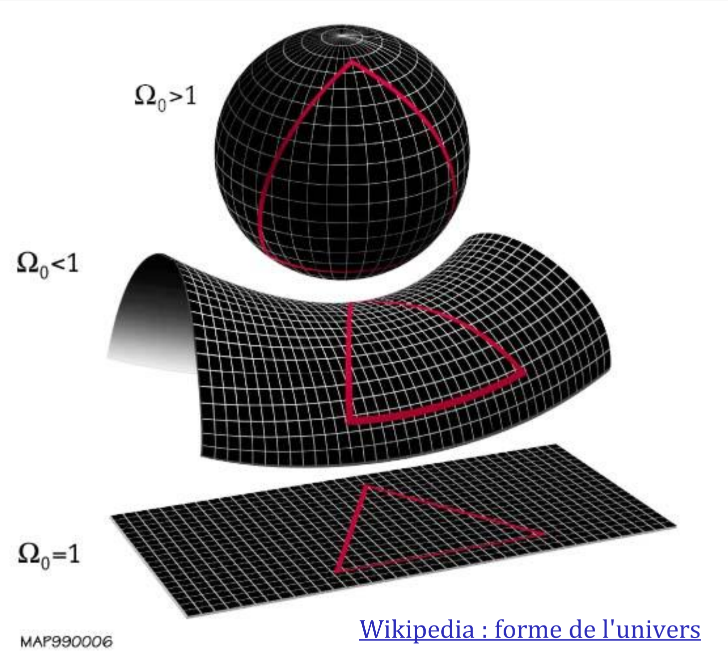
❖Use of python : useful

❖Performances

❖Use of known models to validate the code (Keplerian and Zeldovich test)

To go further :

- ❖ Different values for density parameters
- ❖ To put a geometry (not a flat universe)
- ❖ Secondary infall



Wikipedia : forme de l'univers

VIII. Conclusion

❖Main purpose of the internship done ⇔ Python version of a PM algorithm

❖Particle-Mesh : effective for gravitation and mean-field problems (Poisson)

❖Execution time versus precision

❖Other disciplines :

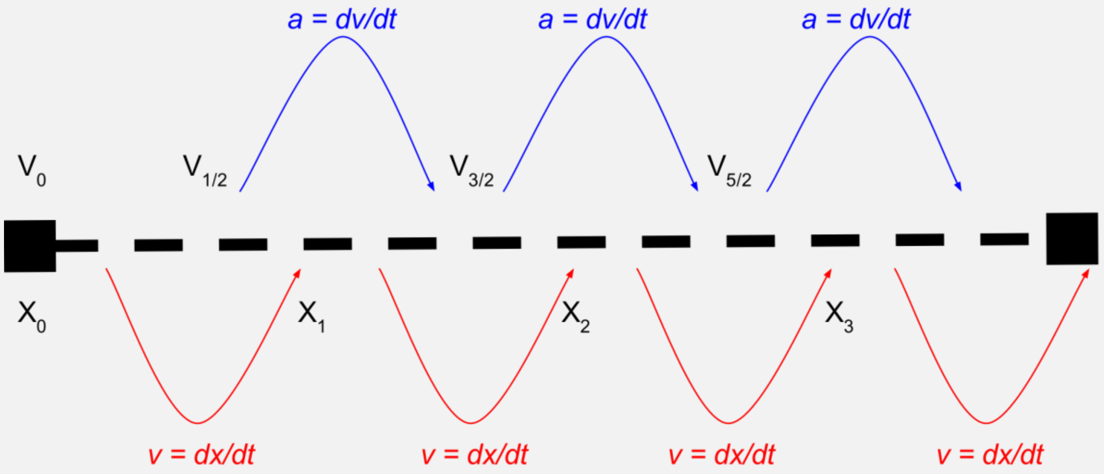
- Molecular physics
- Plasma Physics

Appendix: *Leap Frog scheme*

Idea : to implement the position using speed at a intermediar steps

$$v_{n+\frac{1}{2}} = v_{n-\frac{1}{2}} - \nabla\Phi\delta t$$

$$x_{n+1} = x_n + v_{n+\frac{1}{2}}\delta t$$



Wikipédia : leap frog

Leap Frog scheme : Cosmological case

❖ All variables (velocity, potential, position) are adimensionned

❖ The integration step is not the time anymore, but the scale factor

❖ New integration scheme :

- Momentum : $p_{n+\frac{1}{2}} = p_{n-\frac{1}{2}} - f(a_n)\nabla\tilde{\Phi}\delta a$
- Position : $x_{n+1} = x_n + f\left(a_{n+\frac{1}{2}}\right)\frac{p_{n+\frac{1}{2}}}{a^2_{n+\frac{1}{2}}}\delta a$

With : $f(a) = \sqrt{\frac{a}{\Omega_{DM}+\Omega_{\Lambda}a^3}}$

❖ Condition to stop : a=1 ⇔ present day

Appendix : Зельдович (Zeldovitch) test

Known solution : Validation of the code

Initial condition similar to a sinus then it has to collapse such as shown on the right plot.