U.S. DEPARTMENT OF ENERGY

Office of Science

Quantum tomography for collider physics

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40th Polarization measurements in e+e-, pp and heavy-ion collisions

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Executive summary

We bypass 75 years of field theoretic formalism and particle physics superstructure to describe systems **model-independently in terms of basic quantum mechanics**

Schoolbooks talk about wave functions!

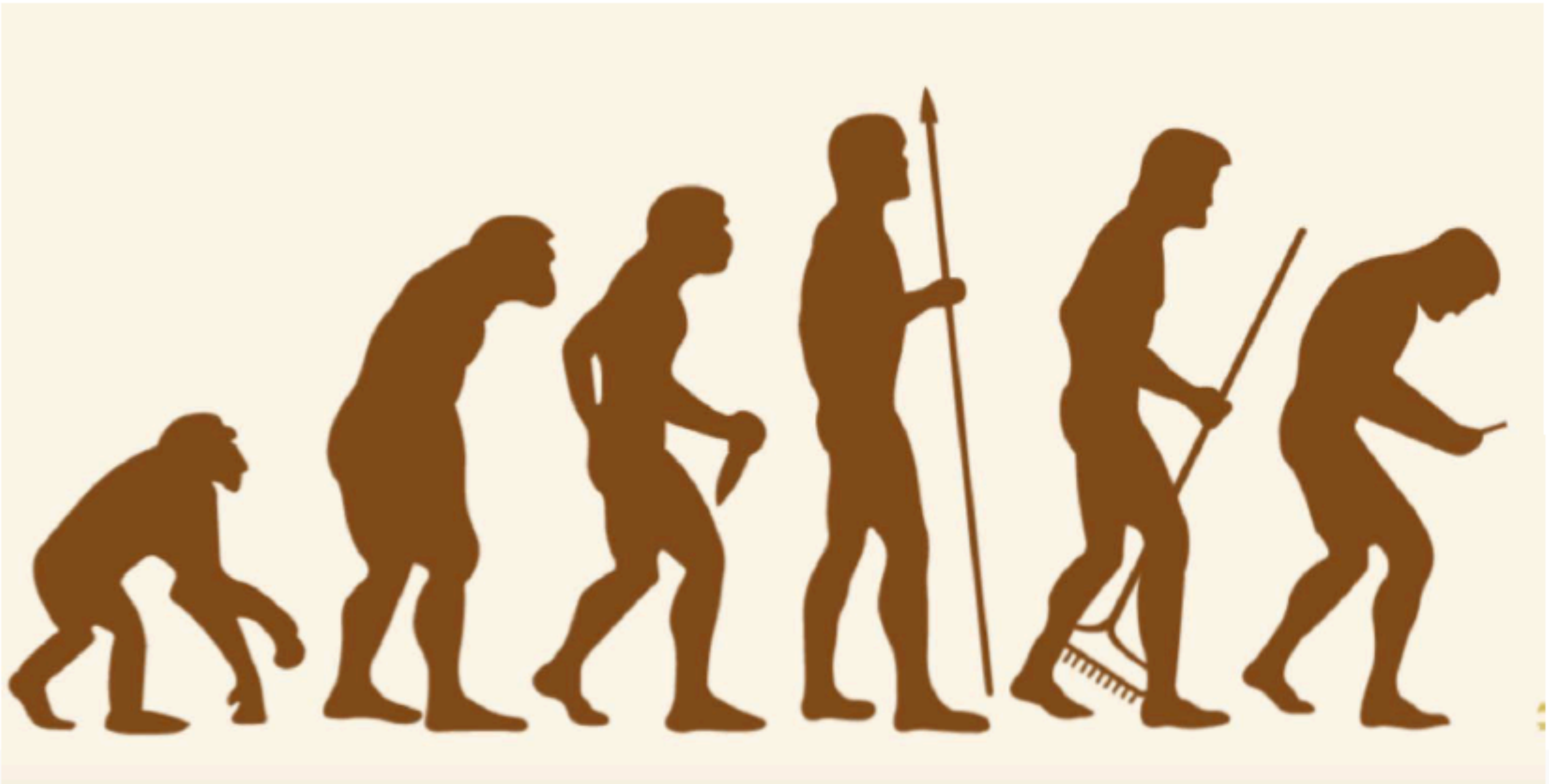
Inclusive experiments measure ***density matrices*** *traced down from larger density matrices*

THEORY HAS EVOLVED

COPENHAGEN WENT PAST ITS PULL-DATE

WE NOW HAVE INTERNET COOKIES

INSTALLED IN YOUR BRAIN'S OPERATING SYSTEM



No assumptions on perturbative theory nor one-photon exchange needed

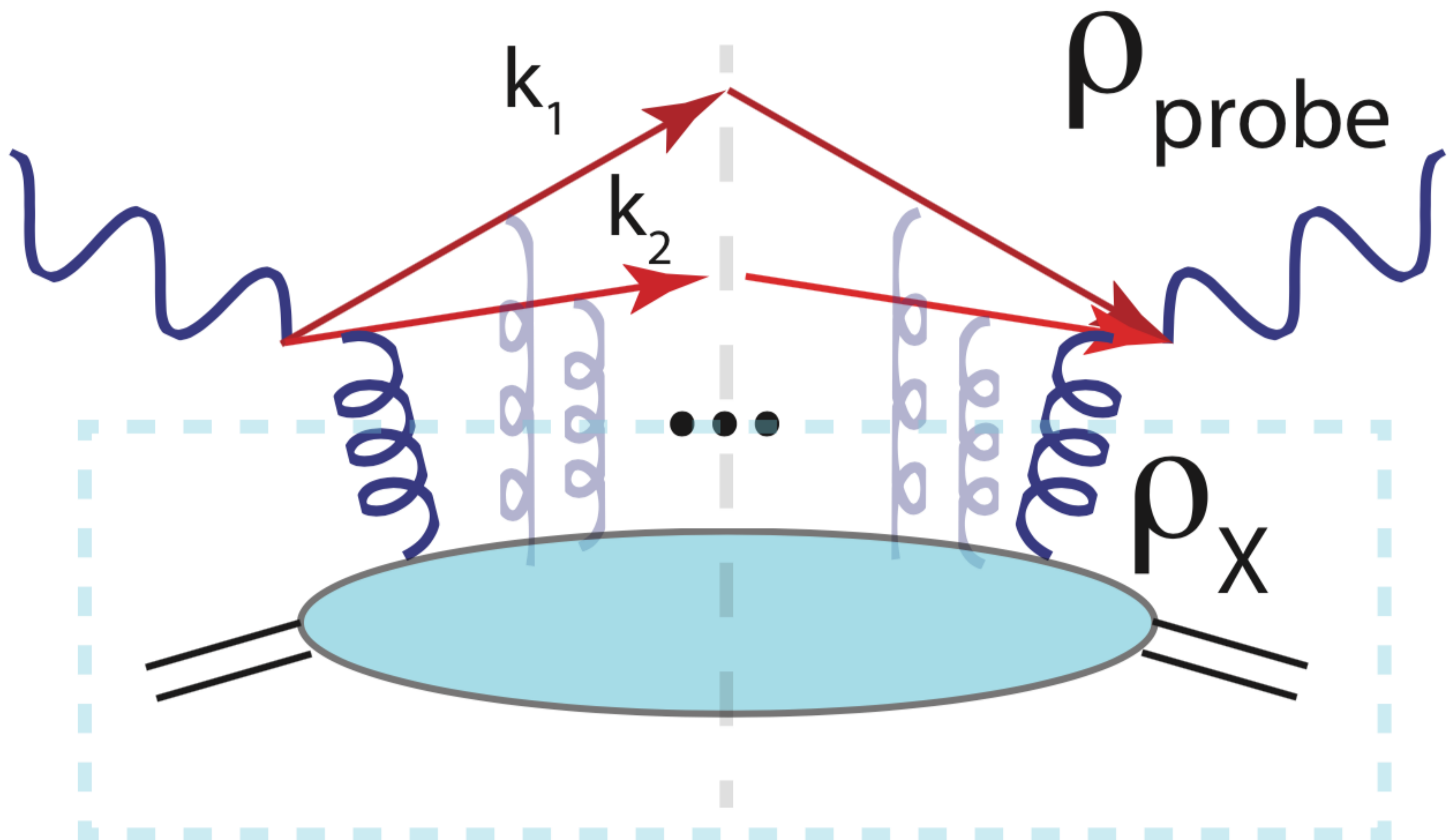


FIG. 1: By analogy with deeply inelastic scattering, a dijet probe replaces the handle of the handbag diagram with a shoulder strap (red) defining new elements of the probe density matrix ρ_{probe}. Each orthogonal element of ρ_{probe} can extract a corresponding projection of the unknown system density matrix ρ_X inside the dashed box. Unlike the deeply inelastic structure functions no assumptions of perturbation theory or one-photon exchange need be made.

Experimentally measure the density matrix

$$\frac{dN}{d\cos\theta d\phi} \sim tr(\rho_{probe}\rho_X)$$

ρ_{probe} = known density matrix

ρ_X = unknown density matrix

The notation does not look Lorentz invariant,
but the quantities are

Begin with Drell-Yan

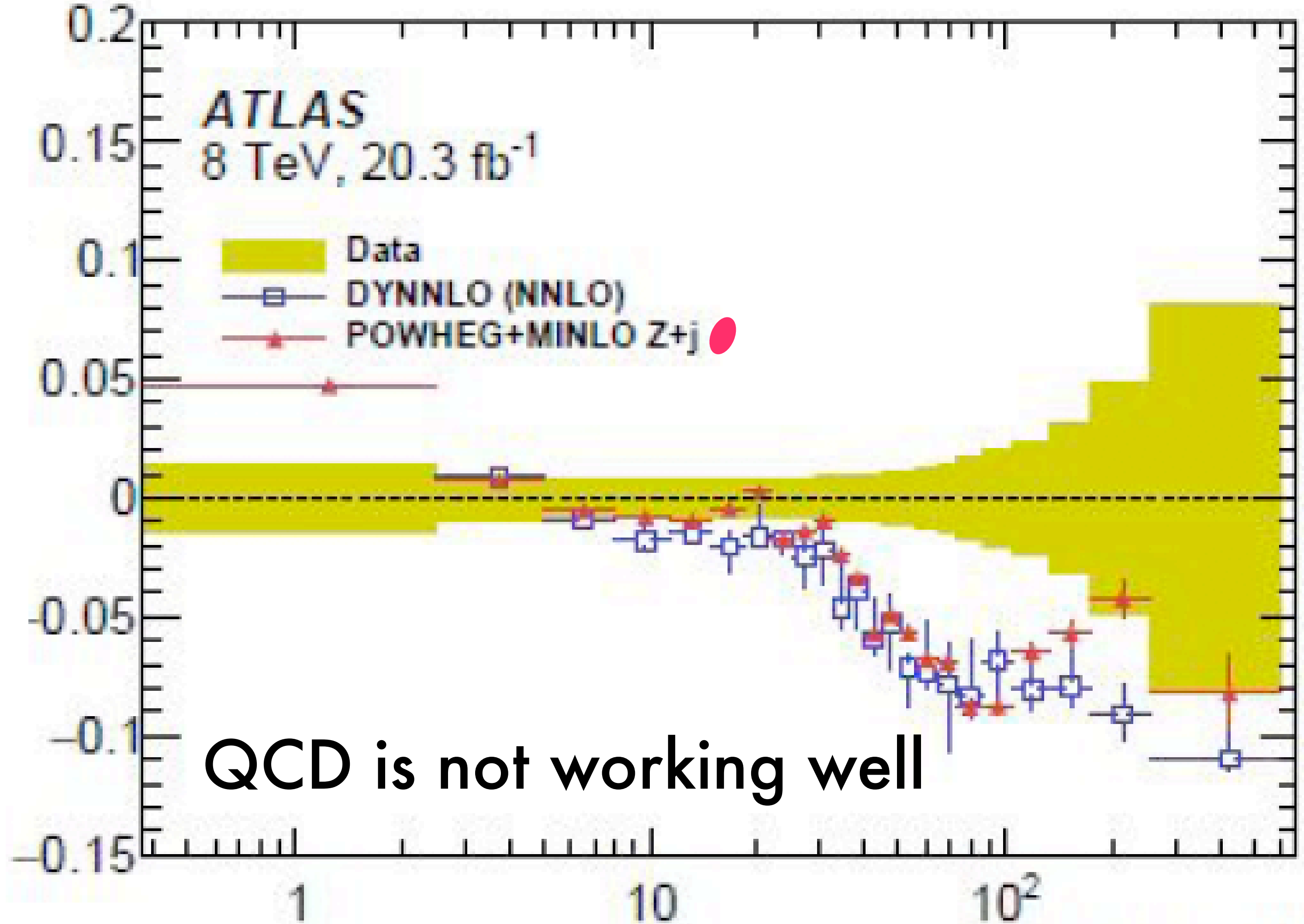
ATLAS data

JHEP 08 (2016)159

$$proton + proton \to Z + anything \to \mu^+ + \mu^- + anything$$

"lepton pairs"

$(A_0 - A_2)$



one of many terms in an old traditional expansion of a certain

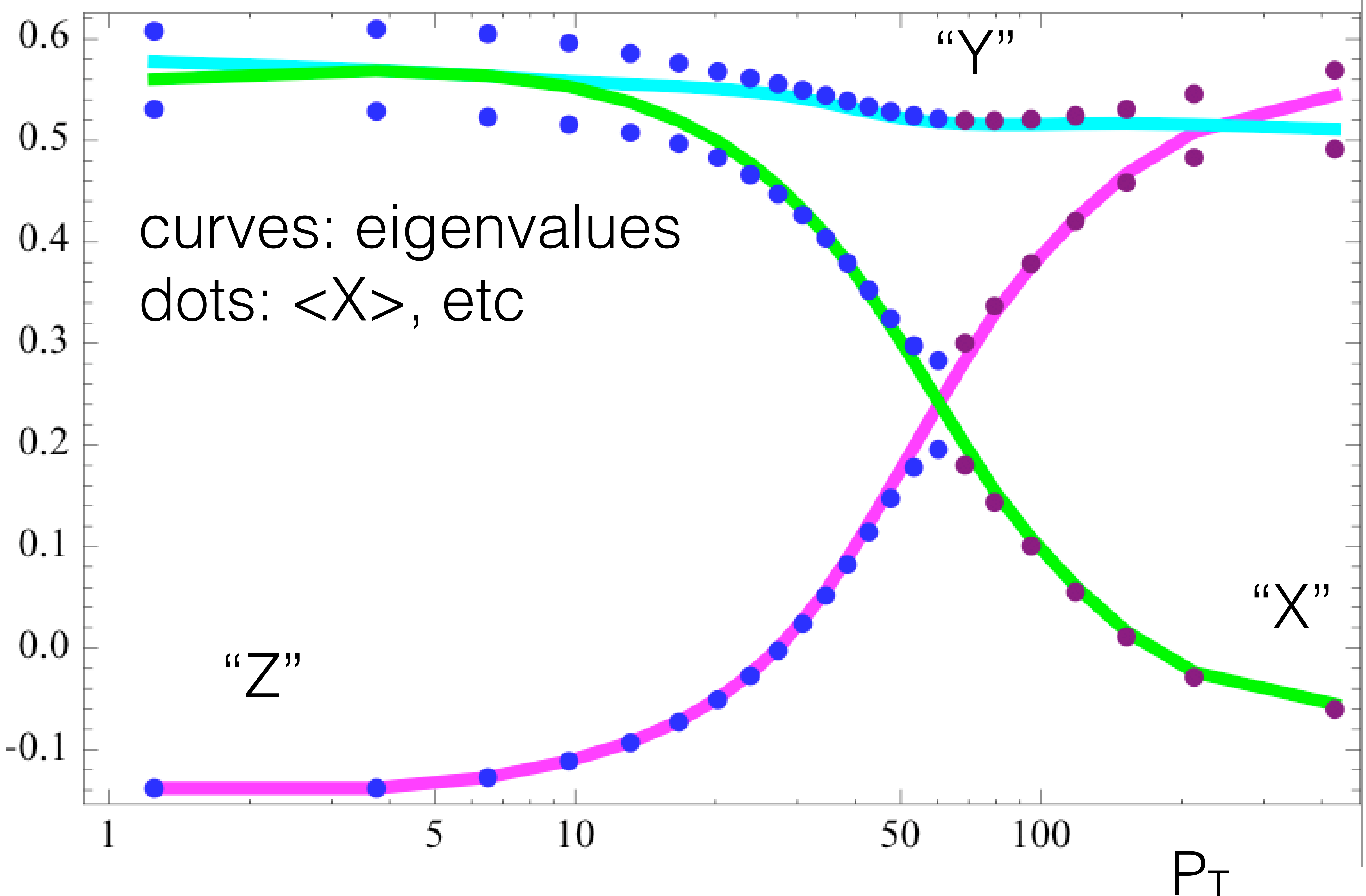
QCD is not working well

"Lam-Tung fails"

P_T of Z = our q_T p_T [GeV]

Avoided level crossing; eigenvectors swap

true QM expectation values <X>, <Y>, <Z> v PT

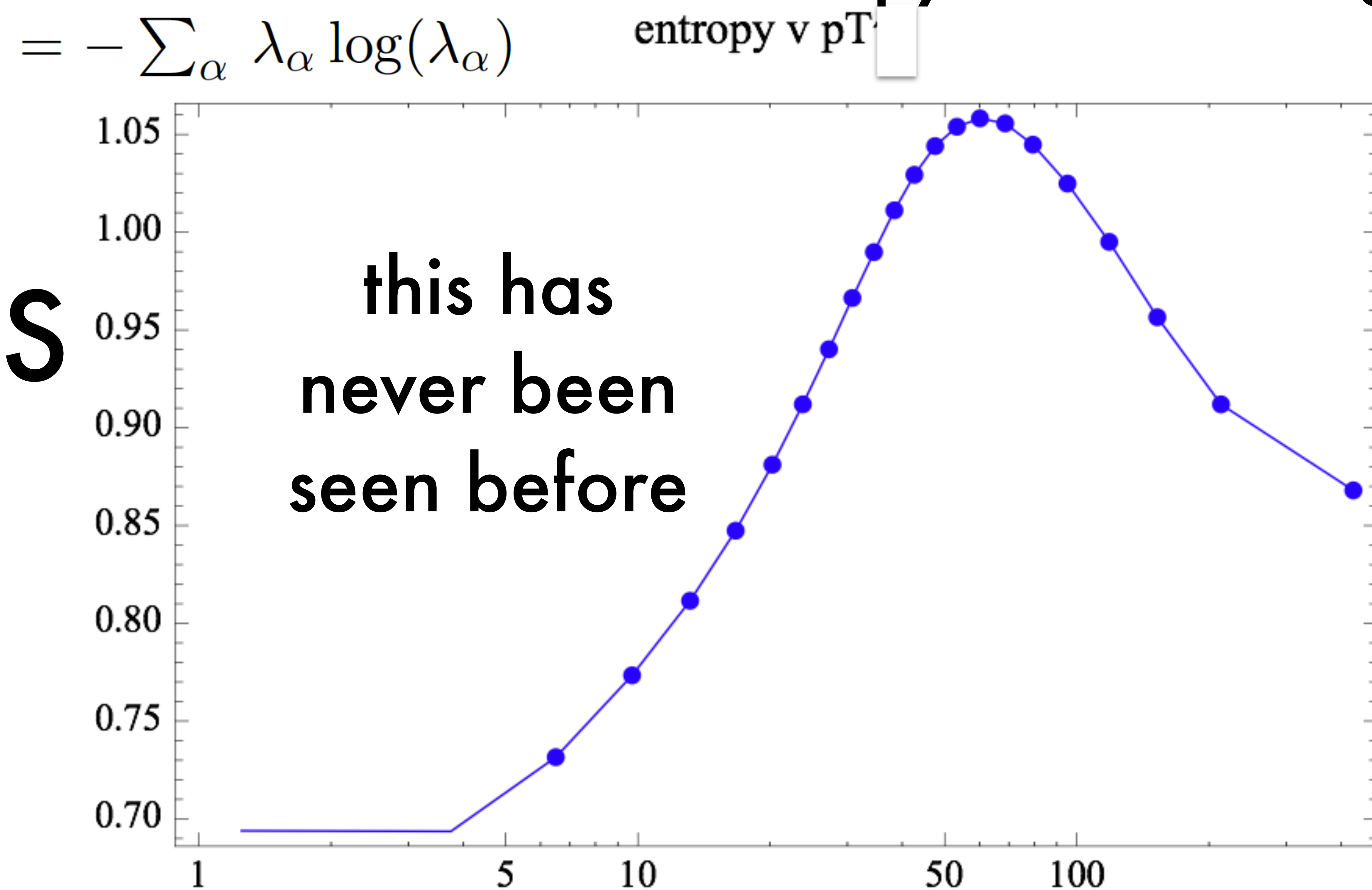


new: no precedent

P_T of Z = our q_T

The entanglement entropy is interesting

$S = -\sum_{\alpha} \lambda_{\alpha} \log(\lambda_{\alpha})$



this has never been seen before

p_T (same as q_T) of pair in GeV units

Tomography builds higher dimensional structure from lower dimensional projections

probe operators G_ℓ

$$tr(G_\ell G_k) = \delta_{\ell k} \quad \text{orthonormal matrices}$$

observable:

$$< G_\ell >= tr(G_\ell \rho_X)$$

$$\rho_X = \text{unknown system}$$

reconstruction:

$$\rho_X = \sum_\ell < G_\ell > G_\ell$$

Completeness? *It's complete for what it spans*

The density matrix is observable

If and when rank=1,

$$\rho|\psi> = |\psi>$$

defines $|\psi>$

Wave functions are observable, up to the undetermined phase of eigenstates

Bring us data: We'll give you a density matrix

Example : events with 2 particles, or 2 jets plus anything else

4-momenta $k,\ k'$

total pair momentum $Q = k + k'$

$$l^{\mu} = k^{\mu} - k^{'\mu} = \sqrt{Q^2}(0,\ \hat{\ell});$$

$$\hat{\ell} = (\sin\theta\cos\phi,\ \sin\theta\sin\phi,\ \cos\theta).$$

pair rest frame $Q^{\mu} = (\sqrt{Q^2},\ \vec{0})$

$$P(Q,\ \ell \big| \, init) = P(\ell \big| Q,\ init) P(Q \big| init).$$

Martens, Ralston, Tapia Takaki Eur. Phys. J. C78, 5, 2018

Experimentally measure the density matrix

$$P(Q,\, \ell \big|\, init) = P(\ell \big| Q,\, init) P(Q \big| init).$$

$$\downarrow$$

$$\frac{dN}{d\Omega} = \frac{1}{\sigma}\frac{d\sigma}{d\Omega} = \frac{3}{4\pi} tr\left(\rho(\ell)\rho(X)\right),$$

$\rho(\ell)$ = known density matrix $= \sum_{\ell} c_\ell G_\ell$

$\rho(X)$ = unknown density matrix

reconstruction: $$\rho_X = \sum_{\ell} < G_\ell > G_\ell$$

IF probe is two "massless" fermions
1/2 x 1/2 x 1/2 x 1/2

$$\rho_{ij}(\ell) = \frac{1+a}{3}\delta_{ij} - a\hat{\ell}_i\hat{\ell}_j - \imath b\epsilon_{ijk}\hat{\ell}_k$$

from symmetry

Standard Model + shelf of books

predicts nothing more than two numbers

$$a = 1/2; \quad b = \sin^2\theta_W$$

One could get a, b tomographically

from another experiment. Indeed we did.

We don't need a theory. Sometimes less theory is better theory.

For tomography in general,

expression above is not exact - only for DY

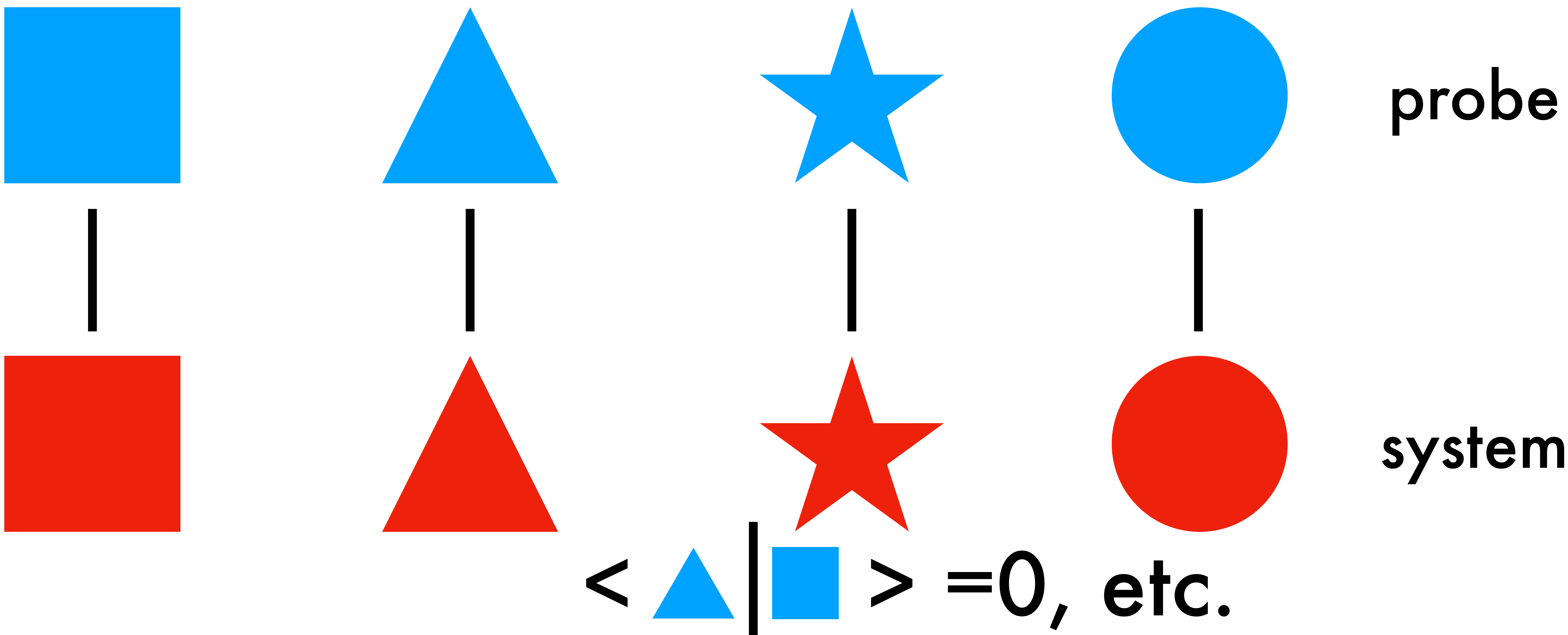
The Mirror trick

3 spin 1 tensors

5 spin 2 tensors

$$\text{Probe:}\quad \rho_{ij}(\ell) = \frac{1}{3}\delta_{ij} + b\hat{\ell}\cdot\vec{J}_{ij} + aU_{ij}(\hat{\ell});\quad \text{where}\quad U_{ij}(\hat{\ell}) = \frac{\delta_{ij}}{3} - \hat{\ell}_i\hat{\ell}_j = U_{ji}(\ell);\ tr(U(\ell)) = 0; \tag{1}$$

$$\text{System:}\quad \rho_{ij}(X) = \frac{1}{3}\delta_{ij} + \frac{1}{2}\vec{S}\cdot\vec{J}_{ij} + U_{ij}(X);\quad \text{where}\quad U(X) = U^T(X);\quad tr(U(X)) = 0.$$



Everything is Lorentz Invariant and easy !

Define spatial axes X^μ, Y^μ, Z^μ satisfying Lorentz invariant

$$Q \cdot X = Q \cdot Y = Q \cdot Z = 0. \quad (1)$$

The frame vectors being orthogonal implies

$$X \cdot Y = Y \cdot Z = X \cdot Z = 0$$

$$\tilde{Z}^\mu = P_A^\mu Q \cdot P_B - P_B^\mu Q \cdot P_A;$$
$$\tilde{X}^\mu = Q^\mu - P_A^\mu \frac{Q^2}{2Q \cdot P_A} - P_B^\mu \frac{Q^2}{2Q \cdot P_B};$$
$$\tilde{Y}^\mu = \epsilon^{\mu\nu\alpha\beta} P_{A\nu} P_{B\alpha} Q_\beta.$$

The first step in our quantum tomographic (QT) analysis expresses everything in a Lorentz-covariant fashion

To analyze data for each event labeled J:

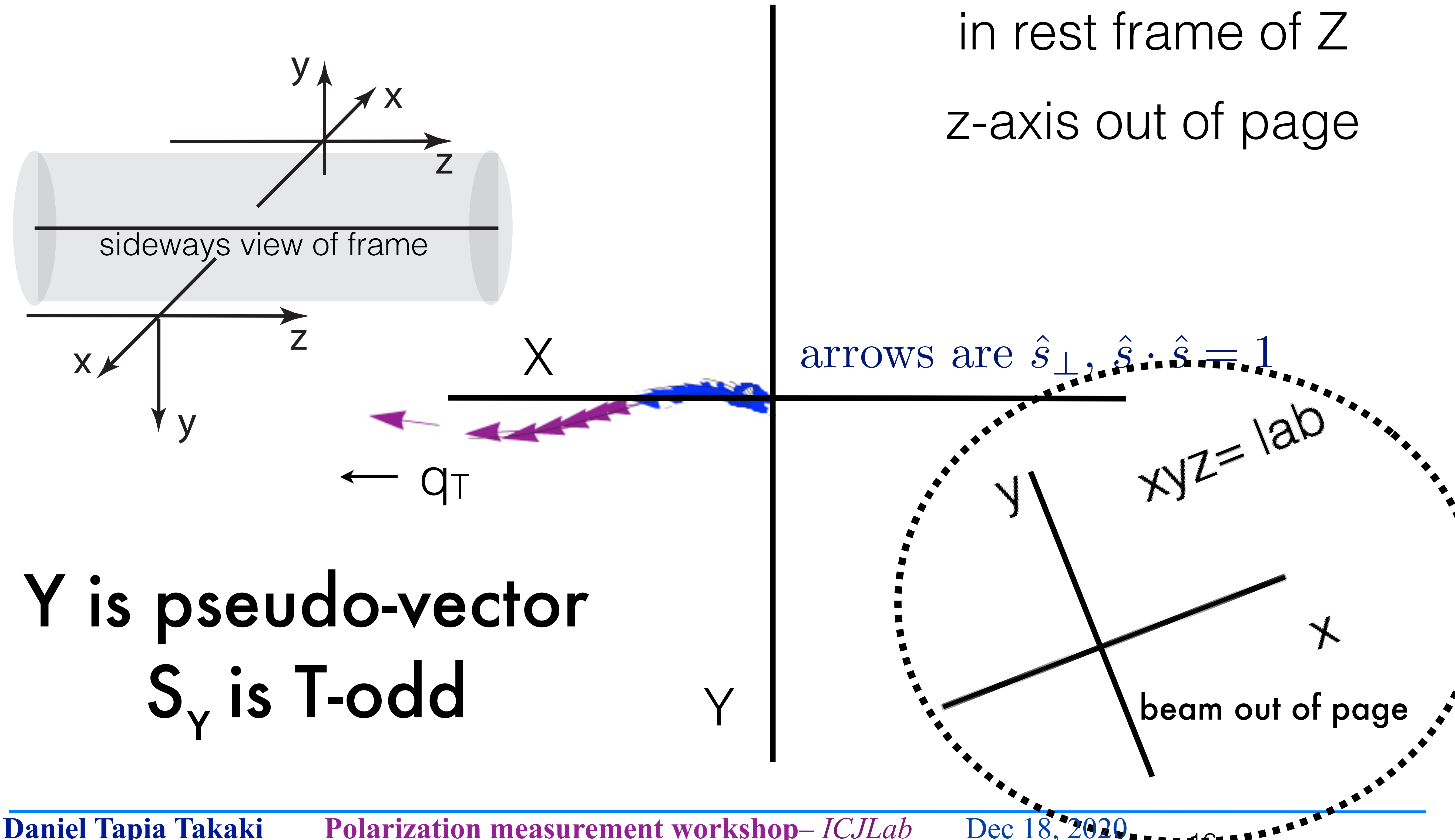
$$\text{Compute} \quad Q_{(J)} = k_J + k'_J; \quad \ell_J = k_J - k'_J; \quad (X_J^\mu, Y_J^\mu, Z_J^\mu);$$
$$\vec{\ell}_{XYZ,J} = (X_J \cdot \ell_J, Y_J \cdot \ell_J, Z_J \cdot \ell_J);$$
$$\hat{\ell}_J = \ell_{XYZ,J} / \sqrt{-\ell_{XYZ,J} \cdot \ell_{XYZ,J}}.$$

use lab momenta to compute invariants

Unexpected discovery in spin parameters of the Z

in rest frame of Z

z-axis out of page

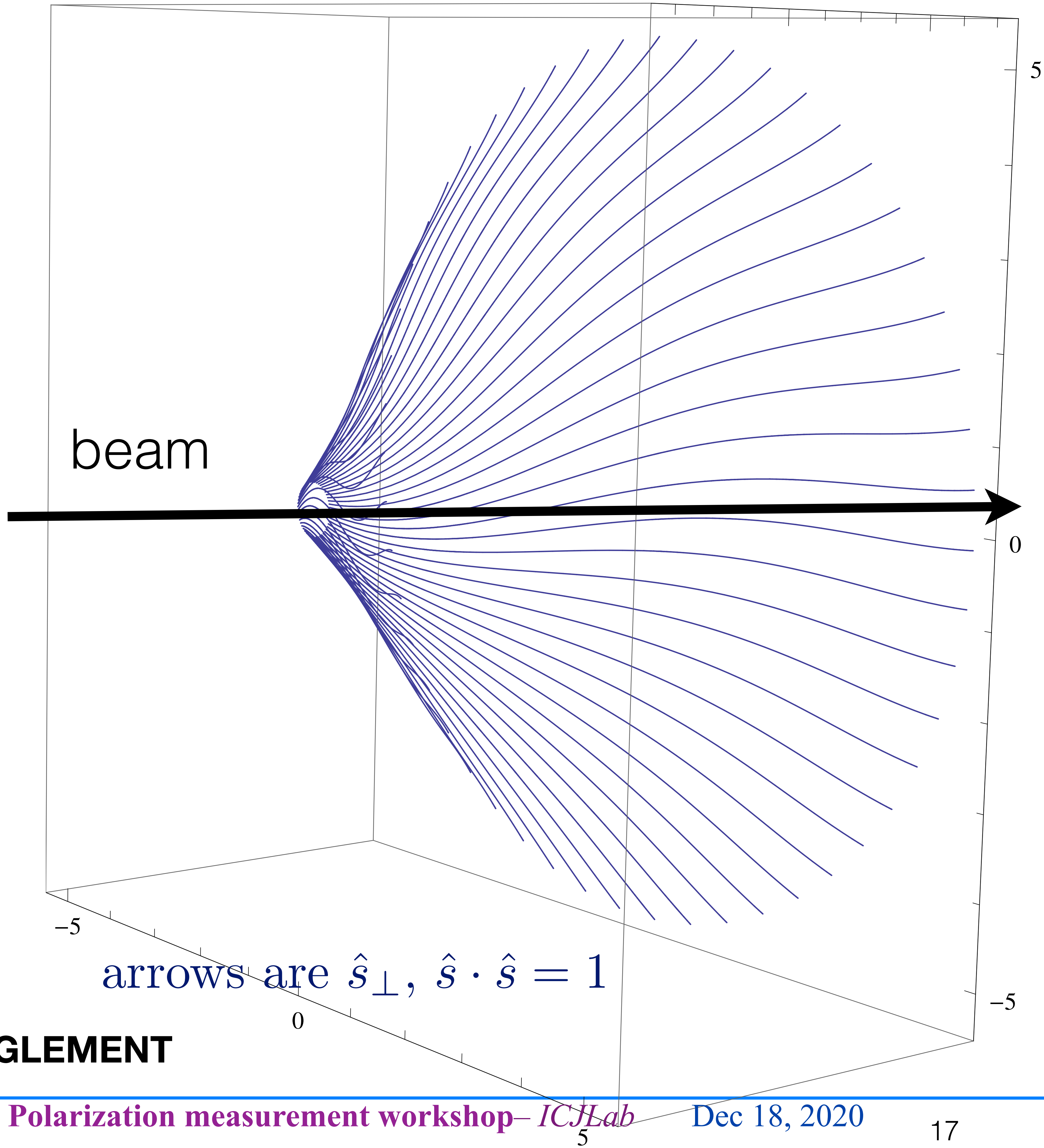


Y is pseudo-vector

S_Y is T-odd

3D holography of the Z spin, lab frame

(q_x, q_y, q_z)



arrows are $\hat{s}_\perp$, $\hat{s}\cdot\hat{s} = 1$

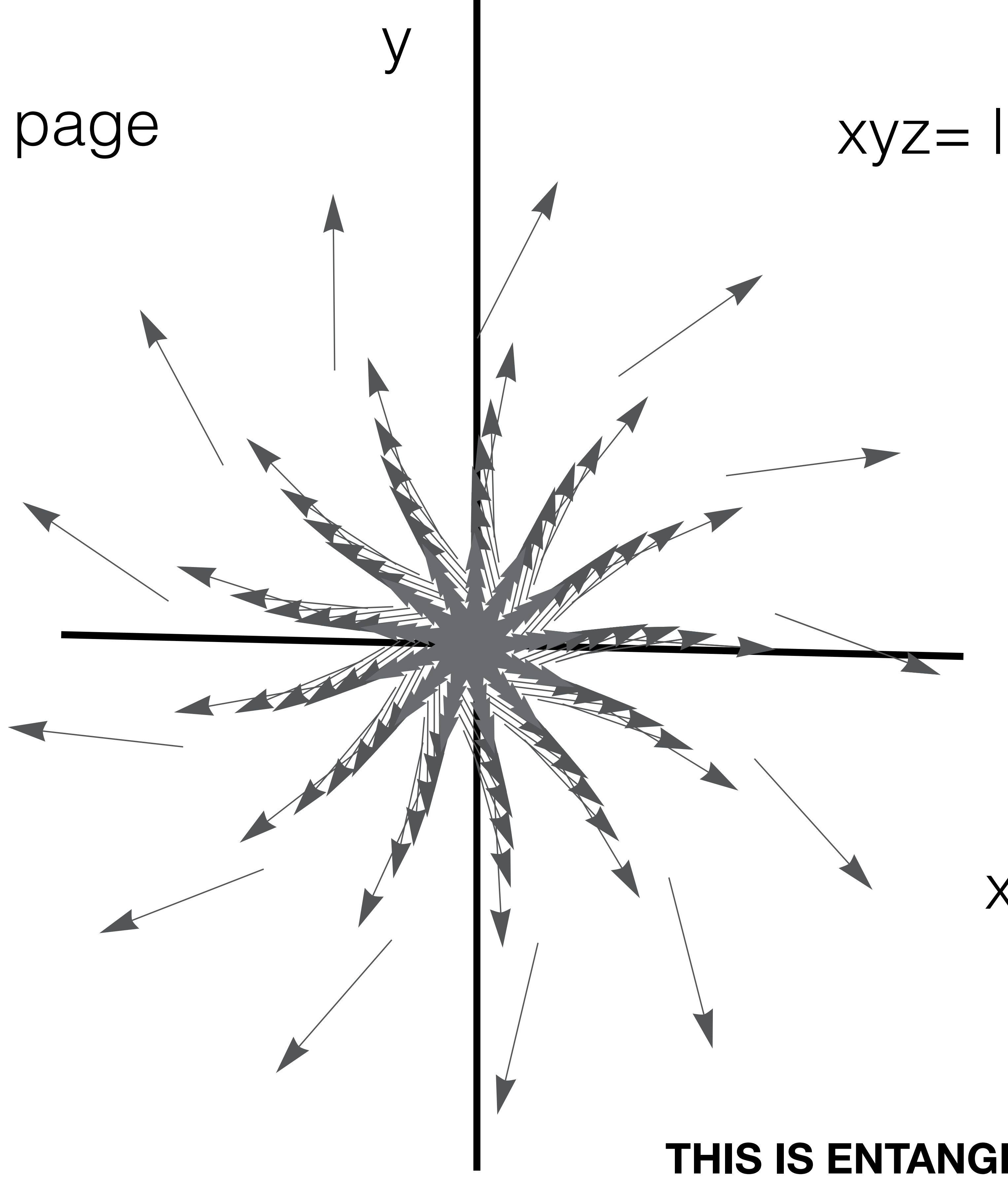
2% of Z's are polarized pure state spinning as shown

THIS IS ENTANGLEMENT

beam-axis out of page

xyz= lab

2% of Z's
are
polarized
pure state
spinning
as shown

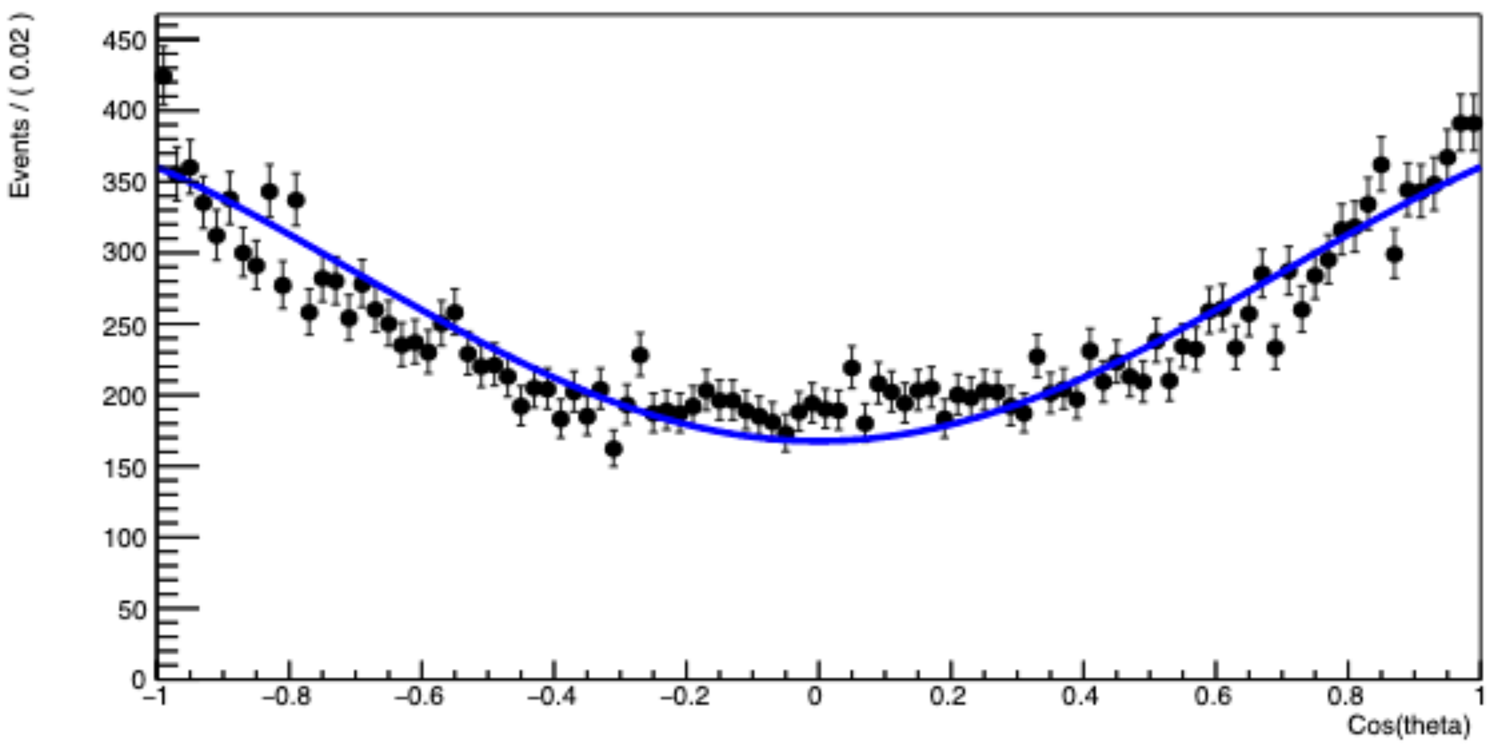


THIS IS ENTANGLEMENT

arrows are $\hat{s}_\perp$, $\hat{s} \cdot \hat{s} = 1$

Quarkonia production

Quantum tomography can also be use for polarization measurements with limited statistics



Lorentz covariant angles and direct link to density matrix

Dijet angular correlation

histograms show a Lorentz-invariant angular distribution of jet1 v jet 2

measuring a density matrix

Quantum tomography Prediction
from MC generated events of DIS (RAPGAP)

We note that the polarization and transverse momentum degrees of freedom are entangled. **No possibility to describe the system as separable. Need a more general description**

$$\rho_X(Q_T) = \sum_\alpha |\psi_\alpha > \rho_\alpha < \psi_\alpha|$$

will submit a study to arXiv soon

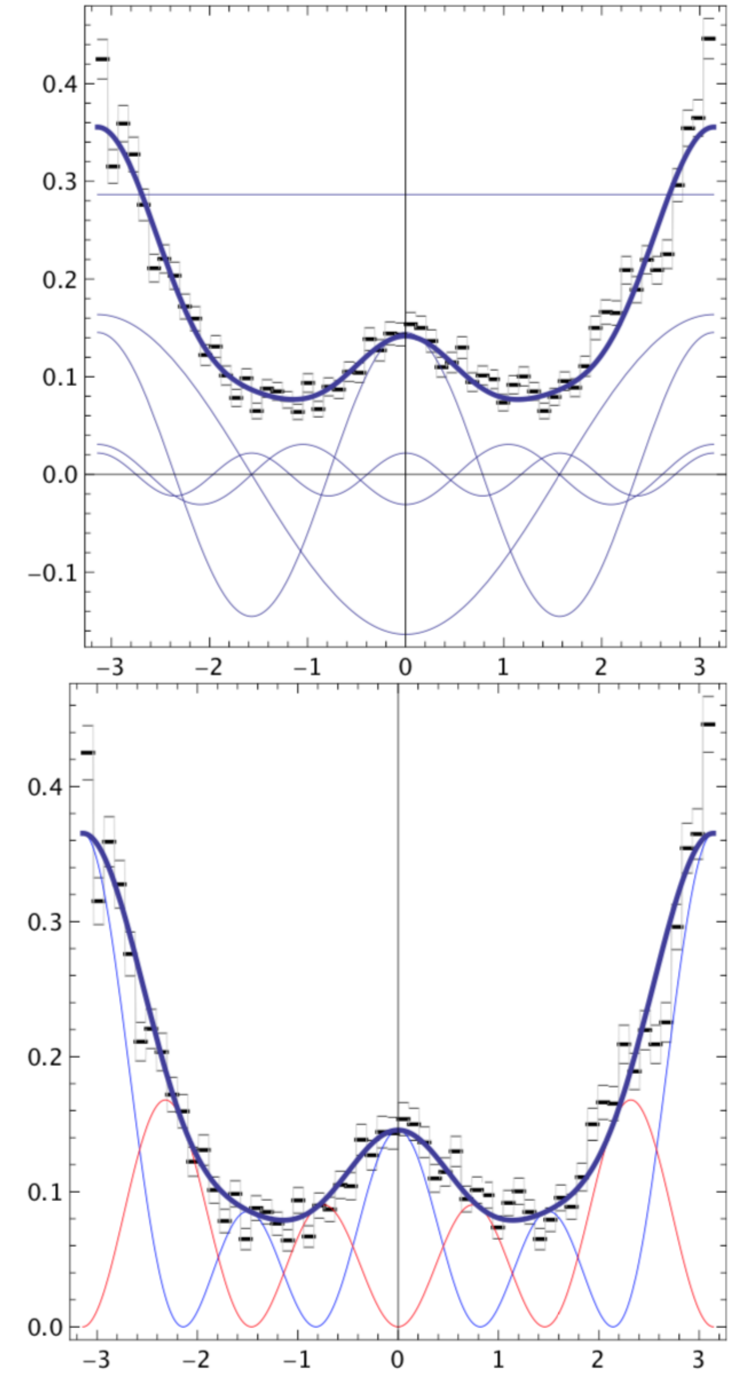


FIG. 4: Top: Maximum likelihood fit, with the contributions of $\cos m\phi$ for $m = 0 - 4$. Bottom: Two weighted distributions defined by $f_+(\phi) = Re(\psi)^2$ (blue) and $f_-(\phi) = Im(\psi)^2$ (red), coming from the eigenstates of the rank two density matrix.

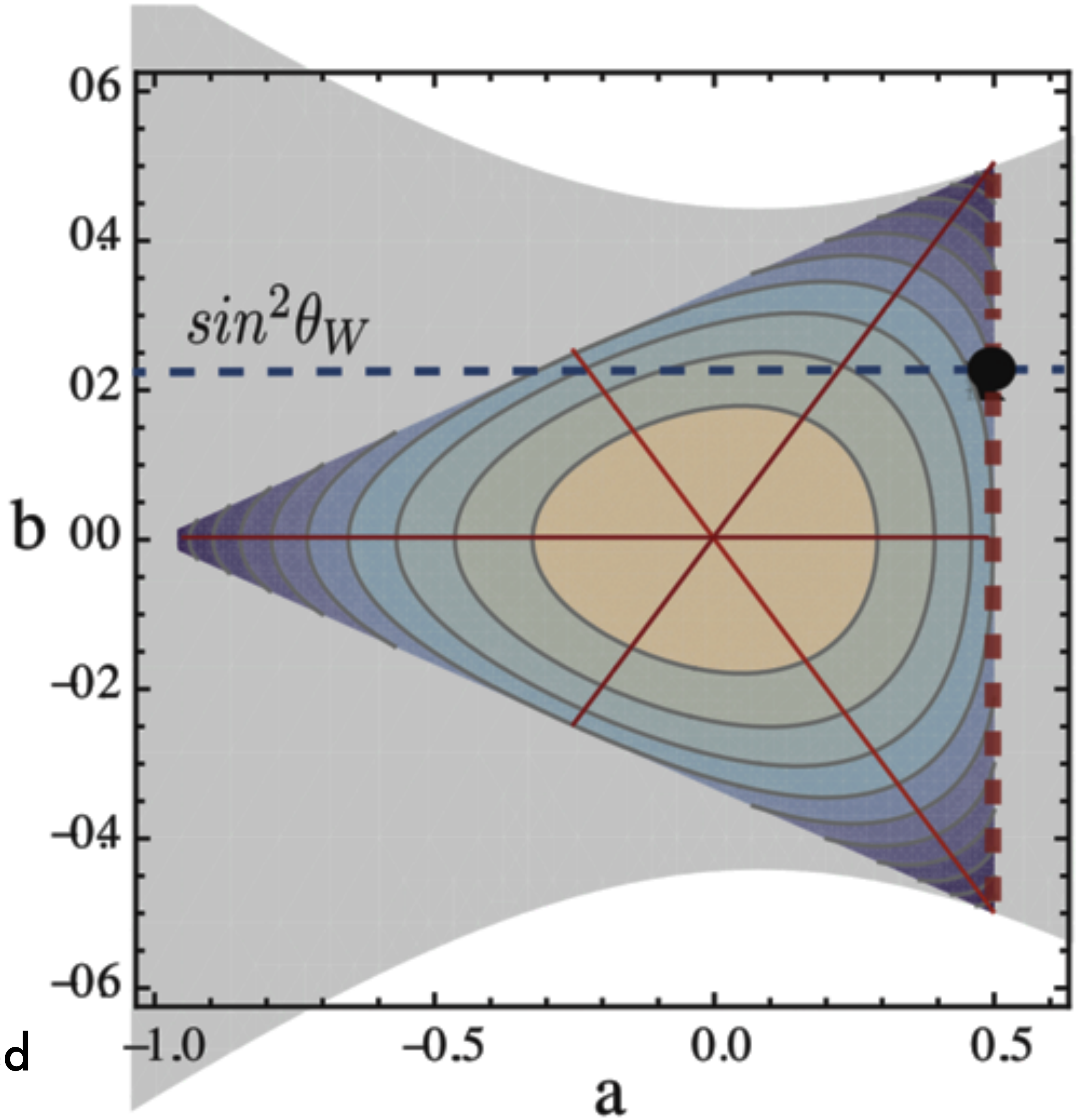
Summary

- Quantum mechanics experiencing a renaissance. Opportunity to study QCD in novel ways, in a model independent way
- Quantum tomography (QT) of dimuon and dijet distributions is feasible with data sets of the size for RHIC and LHC, and future colliders
- A Lorentz-invariant formalism exists which expedites analysis using 4-vectors in the Lab frame
- By using QT, there is much more information available than from moments of a distribution
- QT yields independent eigenvalues and eigendistributions with Born rule probabilities. Natural topic for theoretical comparison
- QT applied to ATLAS data on Drell-Yan production in pp shows quantum entanglement

Additional slides

Bonus:

gray: positivity of cross section



vertical line:on shell helicity conservation

color: positivity of eigenvalues

contours: constant entropy

both CMS and ATLAS have positivity wrong, when it's even mentioned

FIG. 2: Contours of constant entropy $\mathcal{S}$ of the lepton density matrix $\rho(\ell)$ (Eq. 3) in the plane of parameters (a, b). Contours are separated by 1/10 unit with $\mathcal{S} = 0$ at the central intersection. The horizontal dashed line shows the lowest order Standard Model prediction $b = sin^2\theta_W$. Annihilation with on-shell helicity conservation is indicated by the vertical dashed line $a = 1/2$. The left corner of the triangle is a pure state with longitudinal polarization, while the two right corners are pure states of circular polarization. The interior lines represent matrices with maximal symmetry, where two eigenvalues are equal. They cross at the unpolarized limit. The curved gray region represents the much less restrictive constraints of a positive distribution using Eq. 8 and lepton universality.