

# GPD extraction from experimental data

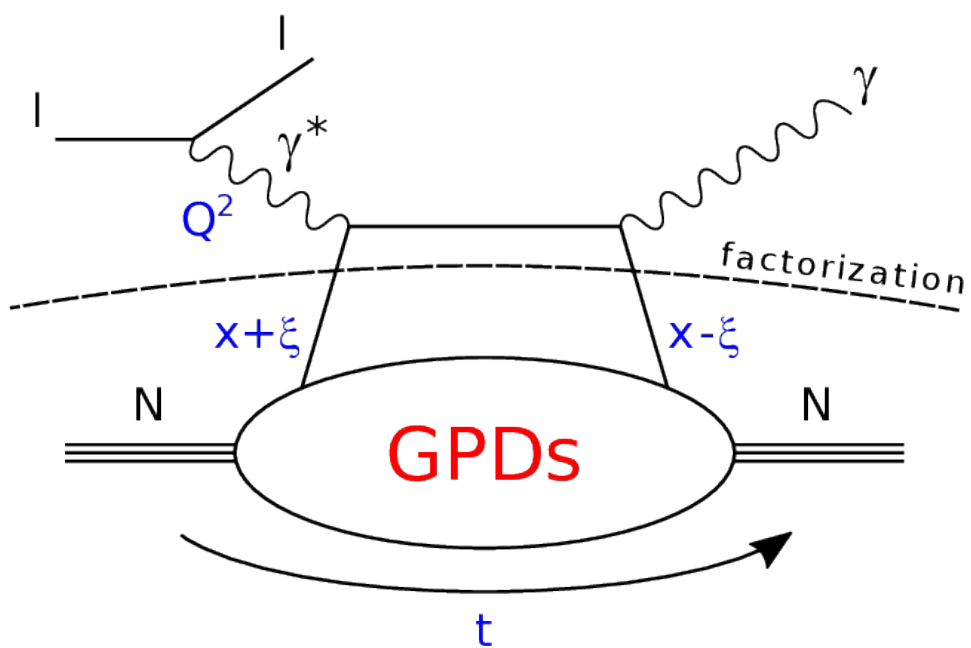
Paweł Sznajder  
National Centre for Nuclear Research, Poland



Polarisation measurements in ee, ep and pp and heavy-ions,  
IJCLab, December 18th, 2020

- Introduction
- Experimental campaign
- Recent progress
- Summary

Deeply Virtual Compton Scattering (DVCS)



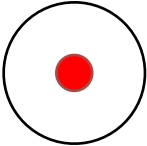
factorisation for  $|t|/Q^2 \ll 1$

Chiral-even GPDs:  
(helicity of parton conserved)

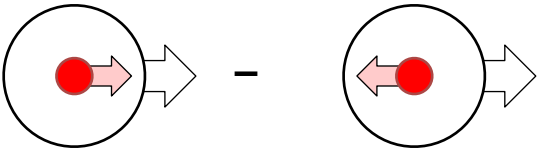
$H^{q,g}(x,\xi,t)$	$E^{q,g}(x,\xi,t)$	for sum over parton helicities
$\tilde{H}^{q,g}(x,\xi,t)$	$\tilde{E}^{q,g}(x,\xi,t)$	for difference over parton helicities
nucleon helicity conserved	nucleon helicity changed	

■ Reduction to PDFs:

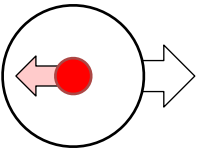
$$H^q(x, 0, 0) \equiv q(x)$$



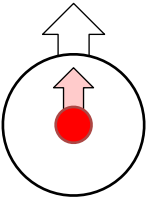
$$\tilde{H}^q(x, 0, 0) \equiv \Delta q(x)$$



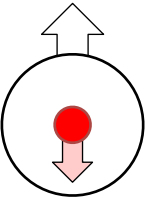
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$$H_T^q(x, 0, 0) \equiv h_1(x)$$



-



*no corresponding relations exist for other GPDs*

■ Reduction to Elastic Form Factors (EFFs):

$$\int_{-1}^1 dx H^q(x, \xi, t) \equiv F_1^q(t)$$

$$\int_{-1}^1 dx E^q(x, \xi, t) \equiv F_2^q(t)$$

$$\int_{-1}^1 dx \tilde{H}^q(x, \xi, t) \equiv g_A^q(t)$$

$$\int_{-1}^1 dx \tilde{E}^q(x, \xi, t) \equiv g_P^q(t)$$

**Polynomiality - non-trivial consequence of Lorentz invariance:**

$$\int_{-1}^1 dx \, x^n H^q(x, \xi, t) = h_0^{q,n}(t) + \xi^2 h_2^{q,n}(t) + \dots + \text{mod}(n, 2) \xi^{n+1} h_{n+1}^{q,n}(t)$$

$$\int_{-1}^1 dx \, x^n \tilde{H}^q(x, \xi, t) = \tilde{h}_0^{q,n}(t) + \xi^2 \tilde{h}_2^{q,n}(t) + \dots + \text{mod}(n+1, 2) \xi^n \tilde{h}_n^{q,n}(t)$$

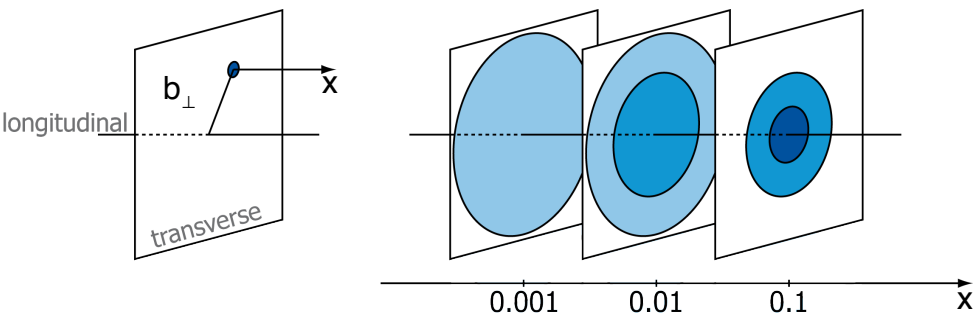
**Positivity bounds - positivity of norm in Hilbert space, e.g.:**

$$(1 - \xi^2) \left( H^q - \frac{\xi^2}{1 - \xi^2} E^q \right)^2 + \frac{t_0 - t}{4m^2} (E^q)^2 \leq q \left( \frac{x + \xi}{1 + \xi} \right) q \left( \frac{x - \xi}{1 - \xi} \right)$$

*strong constraint on GPD parameterisations!*

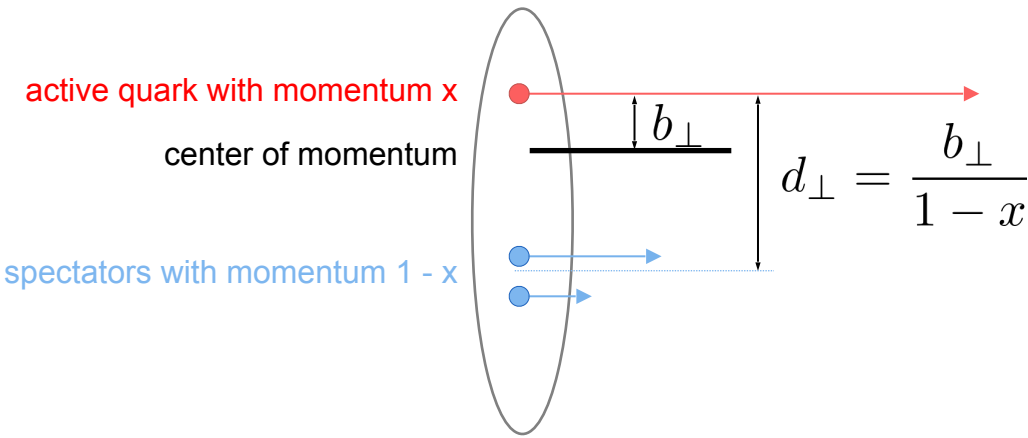
Nucleon tomography

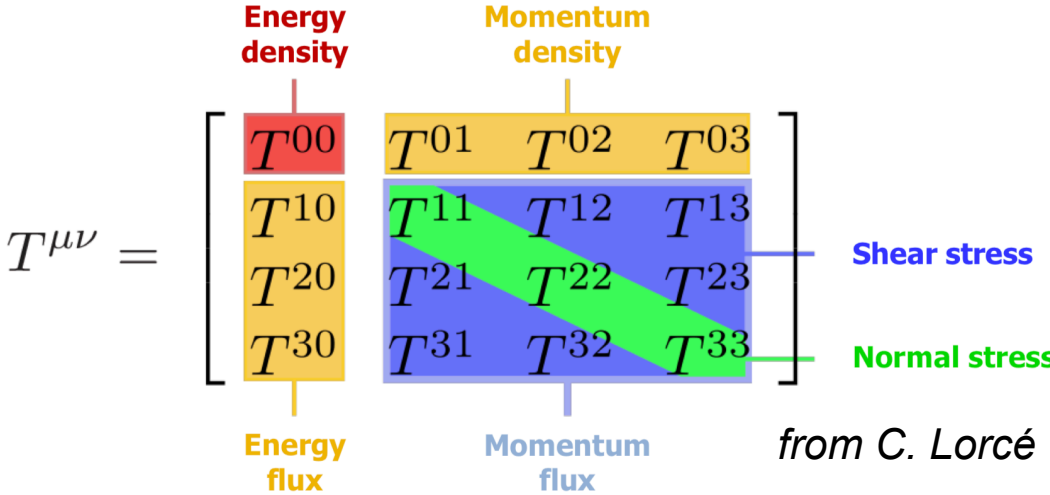
$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\Delta}{4\pi^2} e^{-i\mathbf{b}_\perp \cdot \Delta} H^q(x, 0, t = -\Delta^2)$$



Study of long. polarization with GPD  $\tilde{H}$   
Study of distortion in transv. polarized nucleon with GPD  $E$

Impact parameter  $\mathbf{b}_\perp$  defined w.r.t. center of momentum, such as  $\sum x \mathbf{b}_\perp = 0$





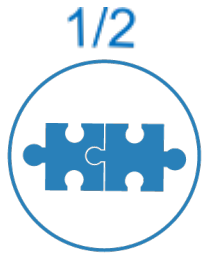
Energy momentum tensor in terms of form factors:

$$\langle p', s' | \hat{T}^{\mu\nu} | p, s \rangle = \bar{u}(p', s') \left[ \frac{P^\mu P^\nu}{M} A(t) + \frac{\Delta^\mu \Delta^\nu - \eta^{\mu\nu} \Delta^2}{M} C(t) + M \eta^{\mu\nu} \bar{C}(t) + \right. \\ \left. \frac{P^\mu i \sigma^{\nu\lambda} \Delta_\lambda}{4M} [A(t) + B(t) + D(t)] + \frac{P^\nu i \sigma^{\mu\lambda} \Delta_\lambda}{4M} [A(t) + B(t) - D(t)] \right] u(p, s)$$

Total angular momentum

$$A^q(0) + B^q(0) = \int_{-1}^1 x [H^q(x, \xi, 0) + E^q(x, \xi, 0)] = 2J^q$$

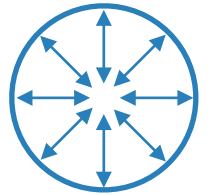
Ji's sum rule



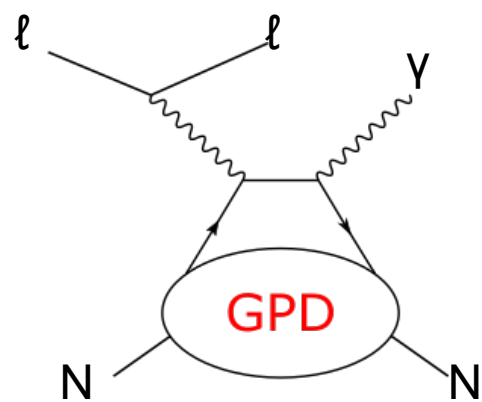


“Mechanical” forces acting on quarks, e.g. pressure in nucleon center

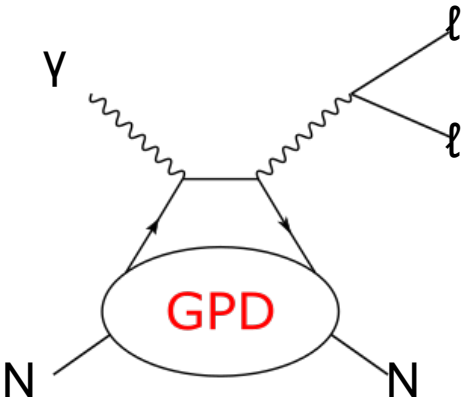
$$p(0) = \frac{1}{6\pi^2 M} \int_{-\infty}^0 dt \sqrt{-tt} C(t)$$



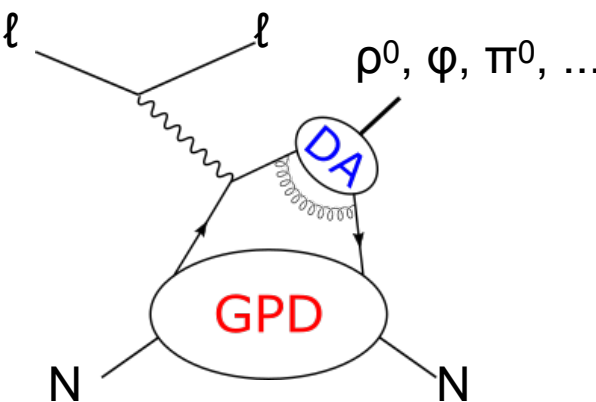
**GPDs accessible in various production channels and observables**  
→ **experimental filters**



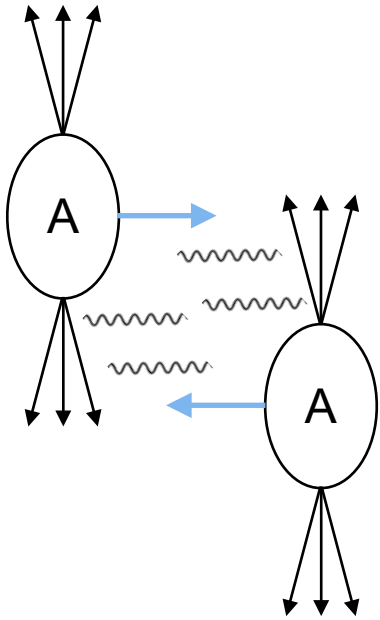
**DVCS**  
*Deeply Virtual Compton Scattering*



**TCS**  
*Timelike Compton Scattering*



**HEMP**  
*Hard Exclusive Meson Production*



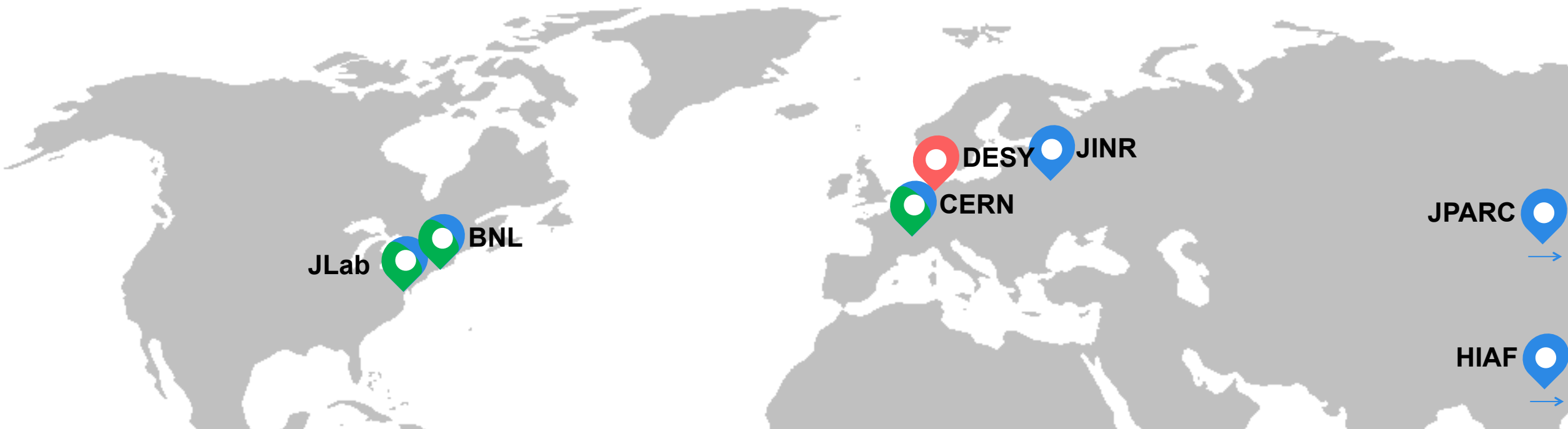
**UPC**  
*Ultra Peripheral Collisions*

*more production channels sensitive to GPDs exist!*

GPDs studied in various laboratories  
→ need to cover a broad kinematic range

experiments

closed   active   planned

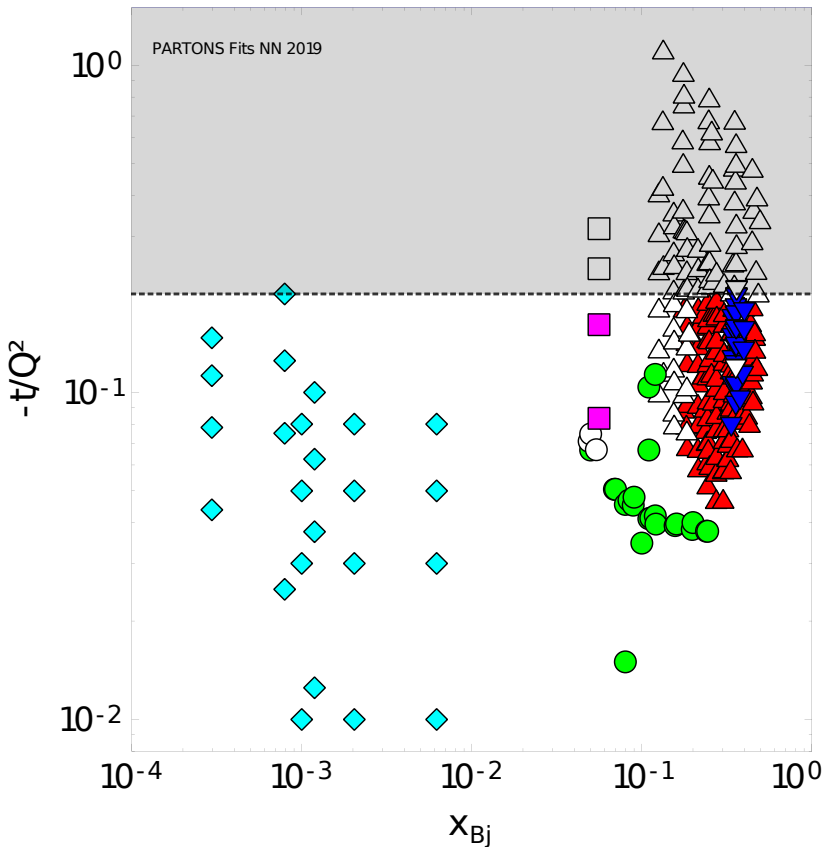
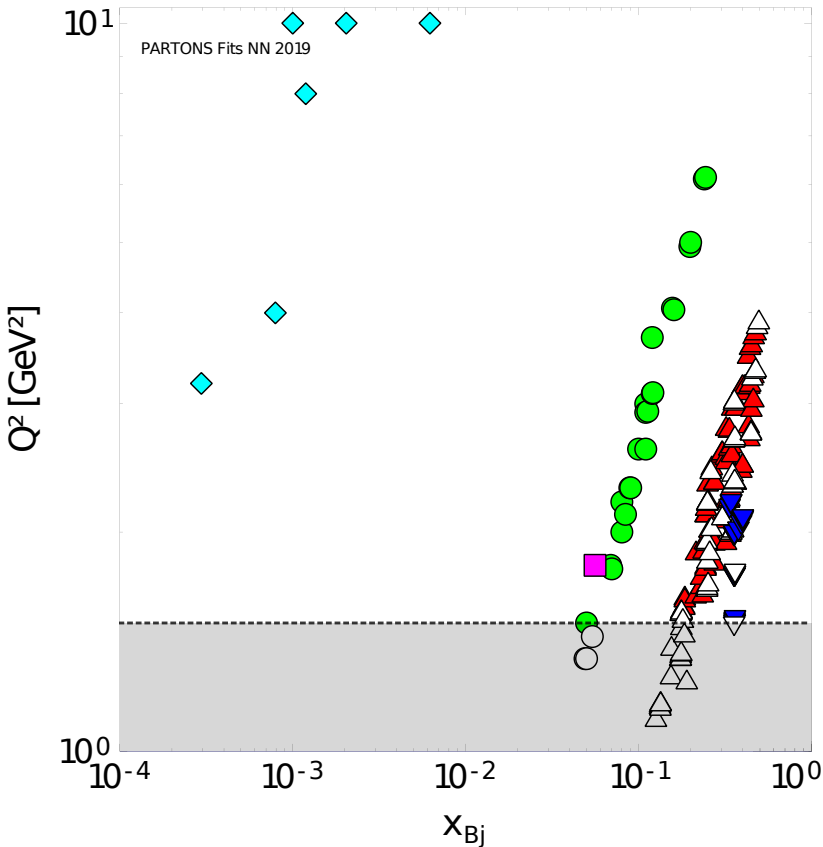


Kinematic cuts  
 used in our recent analyses:

$$Q^2 > 1.5 \text{ GeV}^2$$

$$-t/Q^2 < 0.2$$

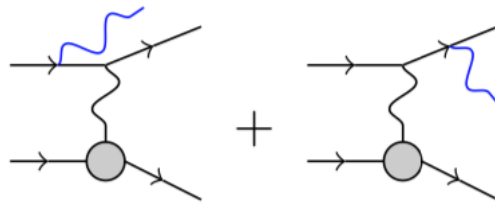
- ▼ HALLA
- ▲ CLAS
- HERMES
- COMPASS
- ◆ H1 and ZEUS



Cross-section for single photon production ( $l + N \rightarrow l + N + \gamma$ ) :

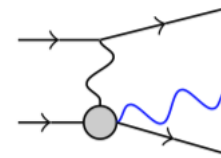
$$\sigma \propto |\mathcal{A}|^2 = |\mathcal{A}_{BH} + \mathcal{A}_{DVCS}|^2 = |\mathcal{A}_{BH}|^2 + |\mathcal{A}_{DVCS}|^2 + \mathcal{I}$$

*Bethe-Heitler process*



*calculable within QED*

*DVCS*



*parametrised by CFFs*

CFF - the most basic GPD-sensitive observables  
 - analogy with connection between structure functions and PDFs

## ■ imaginary part

$$\text{Im}\mathcal{G}(\xi, t) = \pi G^{(+)}(\xi, \xi, t) = \pi \sum_q e_q^2 G^{q(+)}(\xi, \xi, t)$$

$$G^{q(+)}(x, \xi, t) = G^q(x, \xi, t) \mp G^q(-x, \xi, t)$$

$$G^{q(+)}(\xi, \xi, t) = G^{q_{\text{val}}}(\xi, \xi, t) + 2G^{q_{\text{sea}}}(\xi, \xi, t)$$

"-" for  $G \in \{H, E\}$   
 "+" for  $G \in \{\tilde{H}, \tilde{E}\}$

## ■ real part

$$\text{Re}\mathcal{G}(\xi, t) = \text{P.V.} \int_0^1 G^{(+)}(x, \xi, t) \left( \frac{1}{\xi - x} \mp \frac{1}{\xi + x} \right) dx$$

$$\text{Re}\mathcal{G}(\xi, t) = \text{P.V.} \int_0^1 G^{(+)}(x, x, t) \left( \frac{1}{\xi - x} \mp \frac{1}{\xi + x} \right) dx + C_G(t)$$

$$C_H(t) = -C_E(t) \quad C_{\tilde{H}}(t) = C_{\tilde{E}}(t) = 0$$

connected to EMT FF

Relation between subtraction constant and D-term:

$$C_G^q(t) = 2 \int_{-1}^1 \frac{D^q(z, t)}{1 - z} dz \equiv 4D^q(t)$$

where

$$z = \frac{x}{\xi}$$

Decomposition into Gegenbauer polynomials:

$$D^q(z, t) = (1 - z^2) \sum_{i=0}^{\infty} d_i^q(t) C_{2i+1}^{3/2}(z)$$

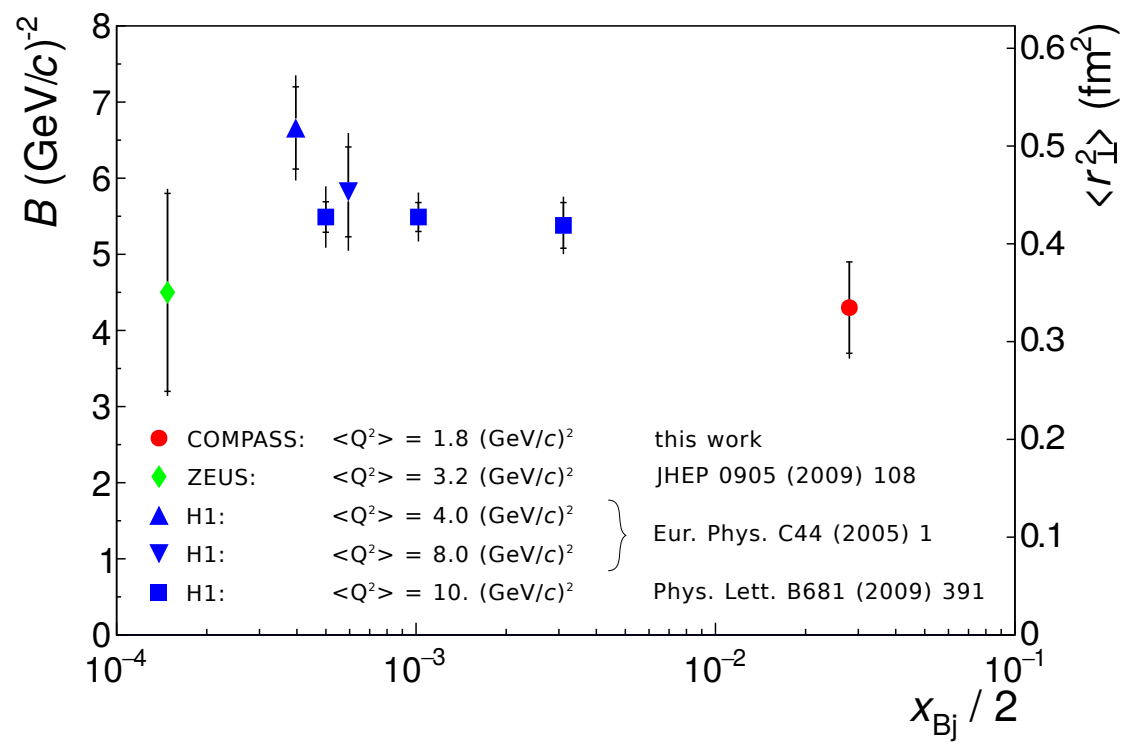
Connection to EMT FF:

$$D^q(t) = \sum_{\substack{i=1 \\ \text{odd}}}^{\infty} d_i^q(t)$$

$$d_1^q(t) = 5C^q(t)$$

■ “Direct” measurement at low-x

COMPASS Collaboration, Phys. Lett. B793 (2019) 188



Slope of t-dependance:

$$\frac{d\sigma^{\gamma^*p \rightarrow \gamma p}}{dt} \propto e^{-Bt}$$

related to transverse extension of quarks:

$$\langle r_{\perp}^2(x_{Bj}) \rangle \approx 2 \langle B(x_{Bj}) \rangle$$

under following assumptions:

- dominance of CFF Im $\mathcal{M}$
- negligible “skewness effect”  
 $H(x, x, t) \sim H(x, 0, t)$
- single-exponential dependence

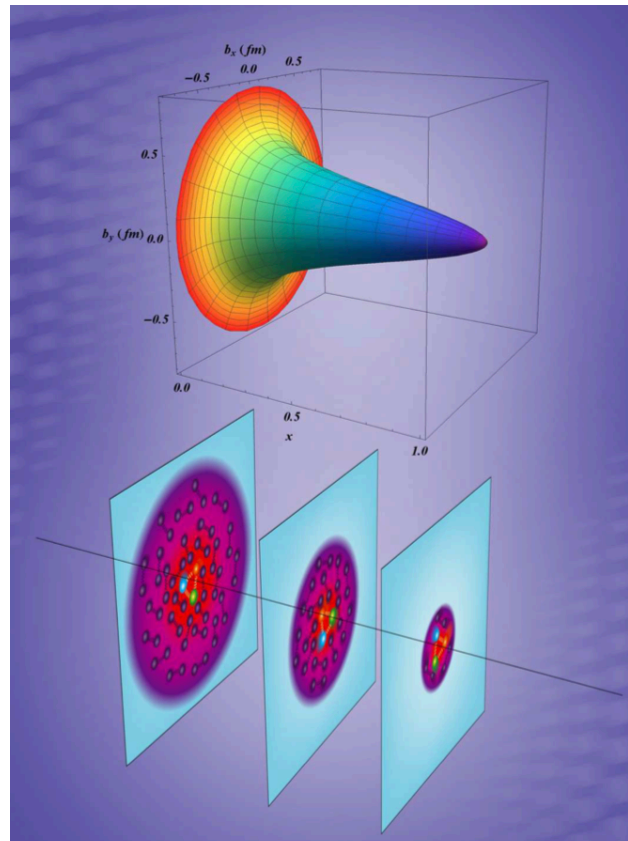
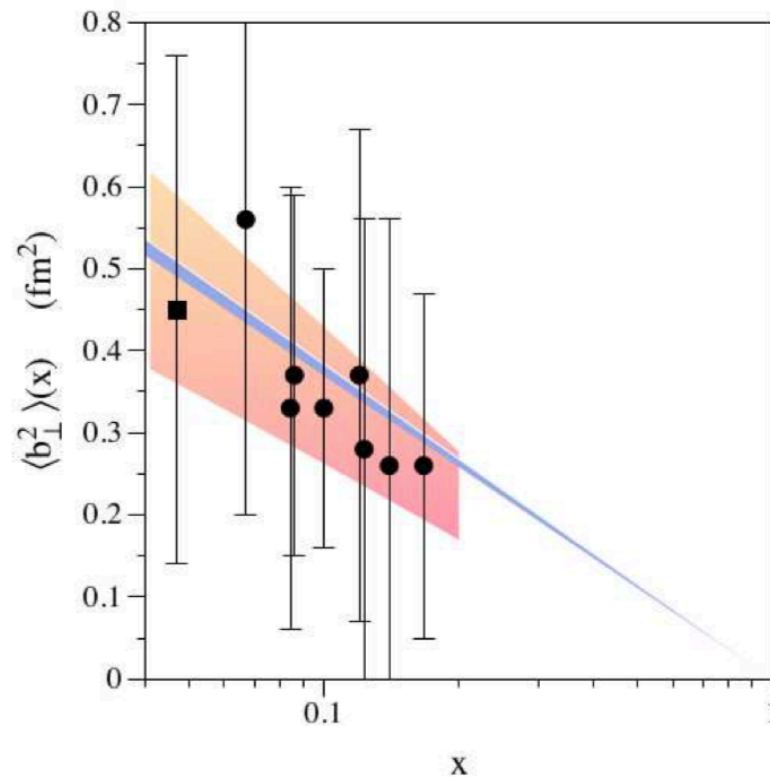
which are applicable at low-x



## Local fits and de-skewness procedure at high-x

Dupre, Guidal, Vanderhaeghen, Phys. Rev. D95 (2017) no. 1, 011501

Dupré, Guidal, Niccolai, Vanderhaeghen, Eur. Phys. J. A53 (2017) no. 8, 171



The procedure:

1. Fit CFFs separately in each  $(x_B, t, Q^2)$  bin of data
2. Fit extracted ImH values with

$$\text{Im}\mathcal{H}(\xi, t, Q^2) = A(\xi) \exp(B(\xi)t)$$

$$A(\xi) = a_A \frac{1 - \xi}{\xi}$$

$$B(\xi) = a_B \ln(1/\xi)$$

$$\xi \approx \frac{x_B}{2 - x_B}$$

- Global fits with skewness dependence encoded in Ansatz

H. Moutarde, PS, J. Wagner, Eur. Phys. J. C78 (2018) 11, 890

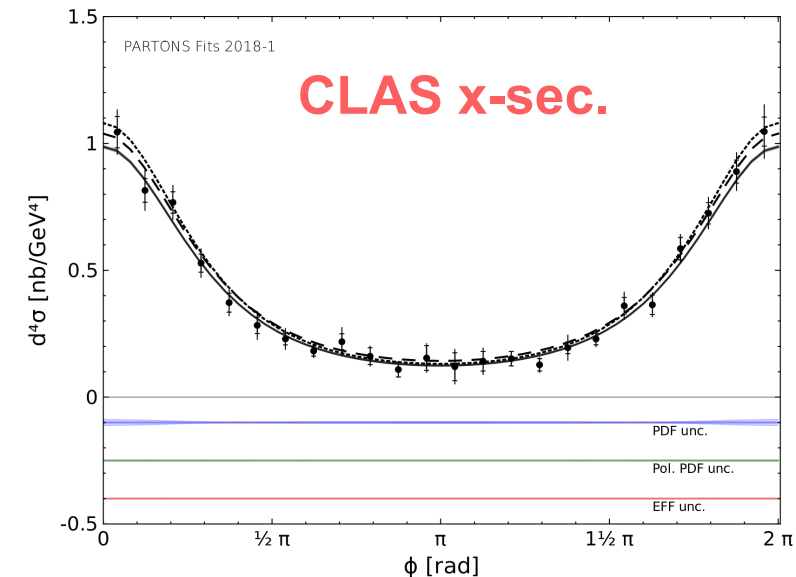
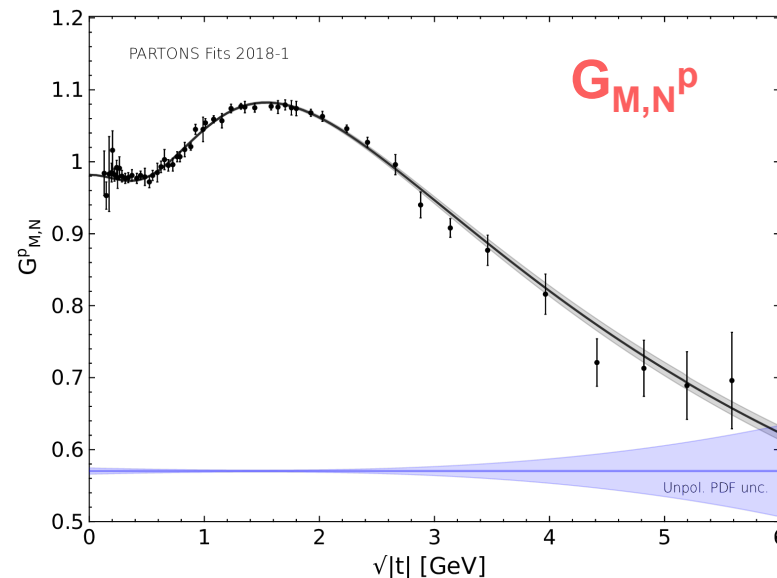
$$G^q(x, x, t) = G^q(x, 0, t) g_G^q(x, x, t)$$

$$G^q(x, 0, t) = \text{pdf}_G^q(x) \exp(f_G^q(x)t)$$

$$g_G^q(x, x, t) = \frac{a_G^q}{(1-x^2)^2} (1 + t(1-x)(b_G^q + c_G^q \log(1+x)))$$

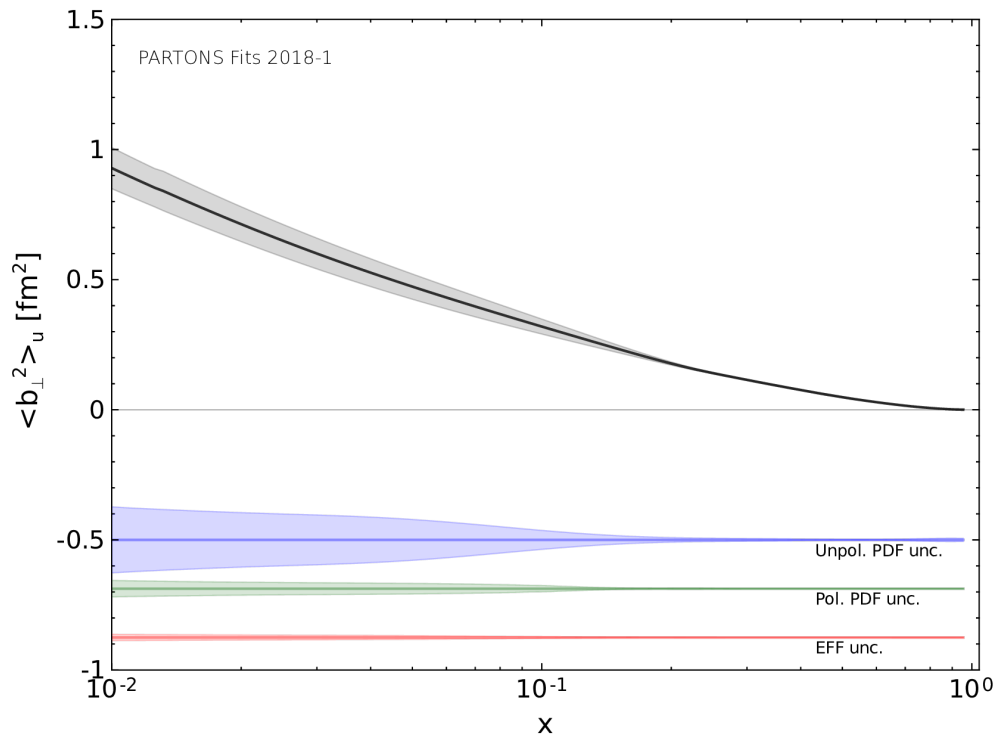
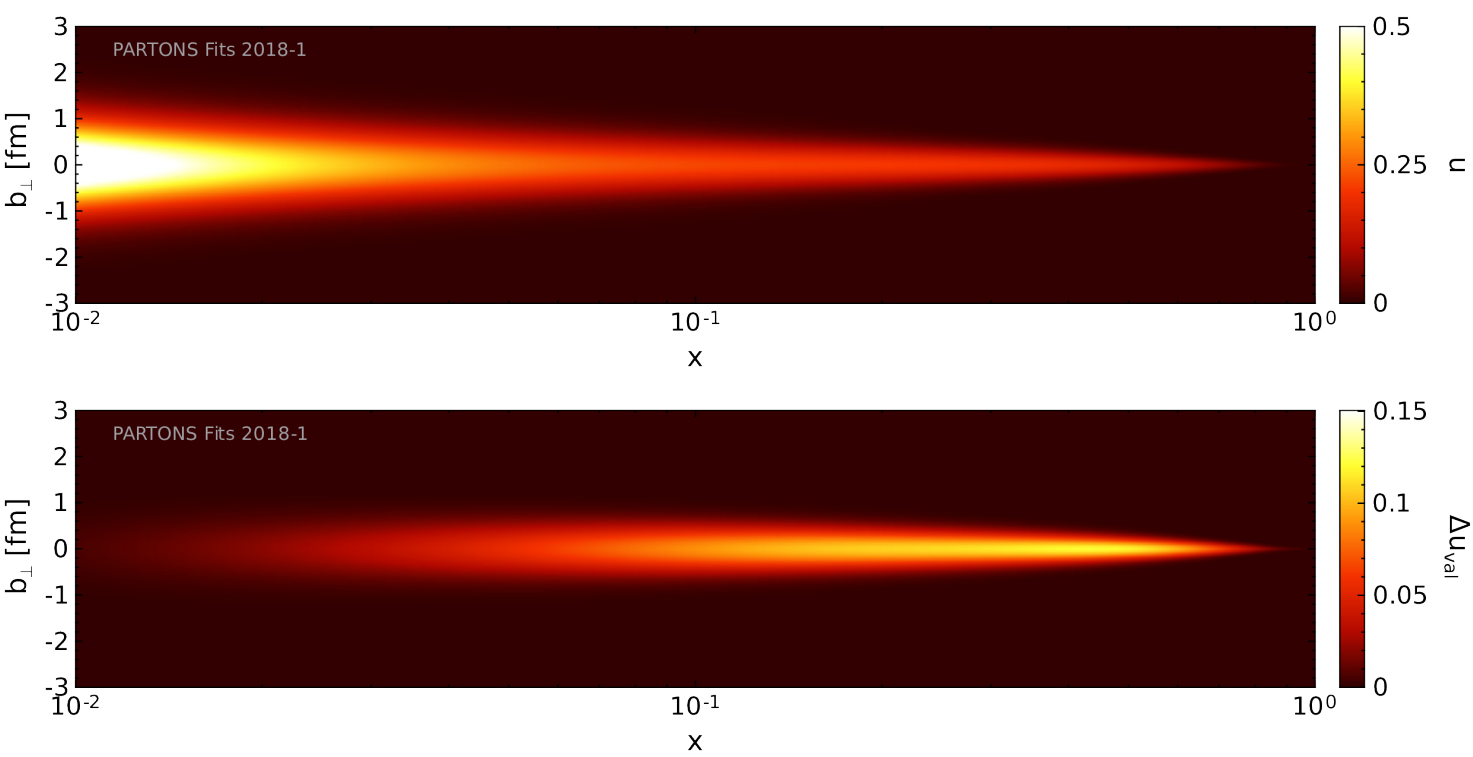
$$f_G^q(x) = A_G^q \log(1/x) + B_G^q(1-x)^2 + C_G^q(1-x)x$$

Allows for a global fit of both elastic FF and DVCS data



- Global fits with skewness dependence encoded in Ansatz

H. Moutarde, PS, J. Wagner, Eur. Phys. J. C78 (2018) 11, 890



$Q^2 = 2 \text{ GeV}^2$

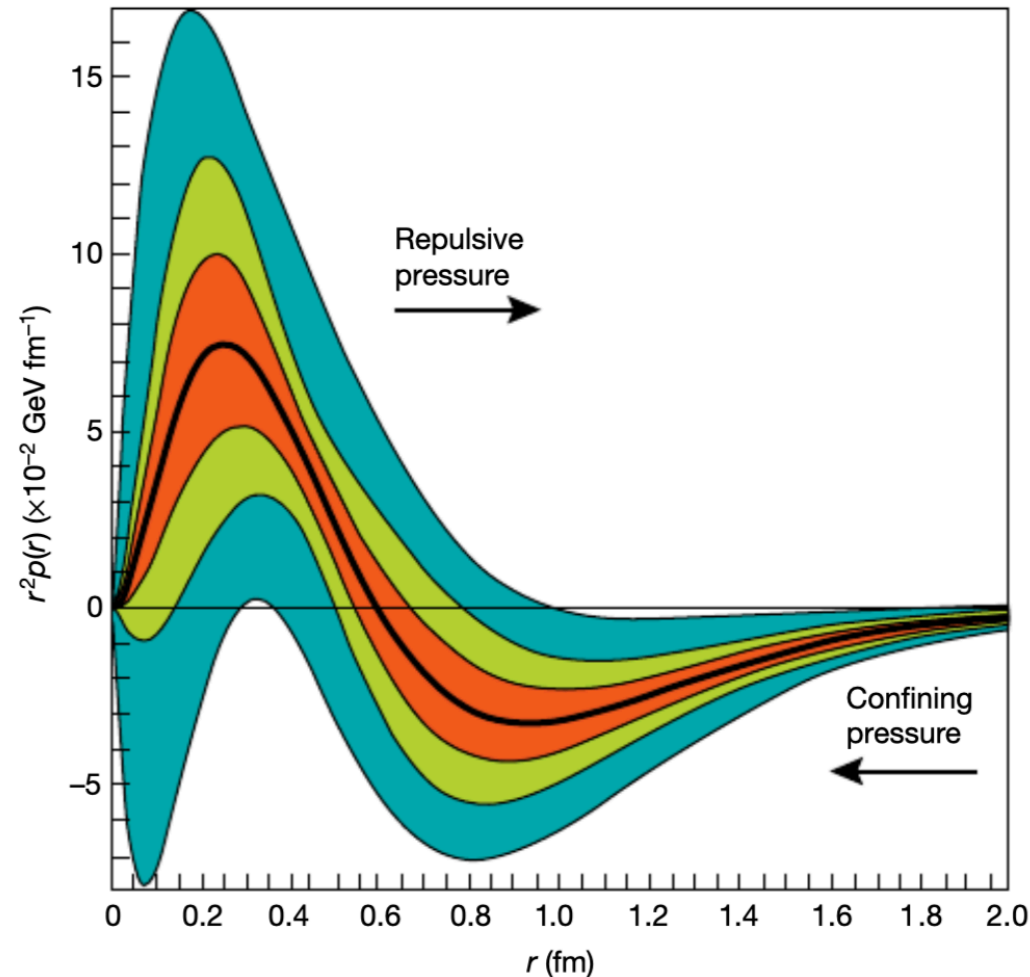
No recent progress here :-(

Need for more neutron data or data taken with transversely polarized targets

Need for modern GPD models to access  $H/E(x, \text{const } \xi, t=0)$  from exclusive data

Elastic data provide important constraints on GPDs  $E$  (however, only for valence contribution), see for instance [Eur. Phys. J. C73 \(2013\) no. 4, 2397](#)

V.D. Burkert, L. Elouadrhiri, F.X. Girod, Nature 557 (2018) no. 7705, 396



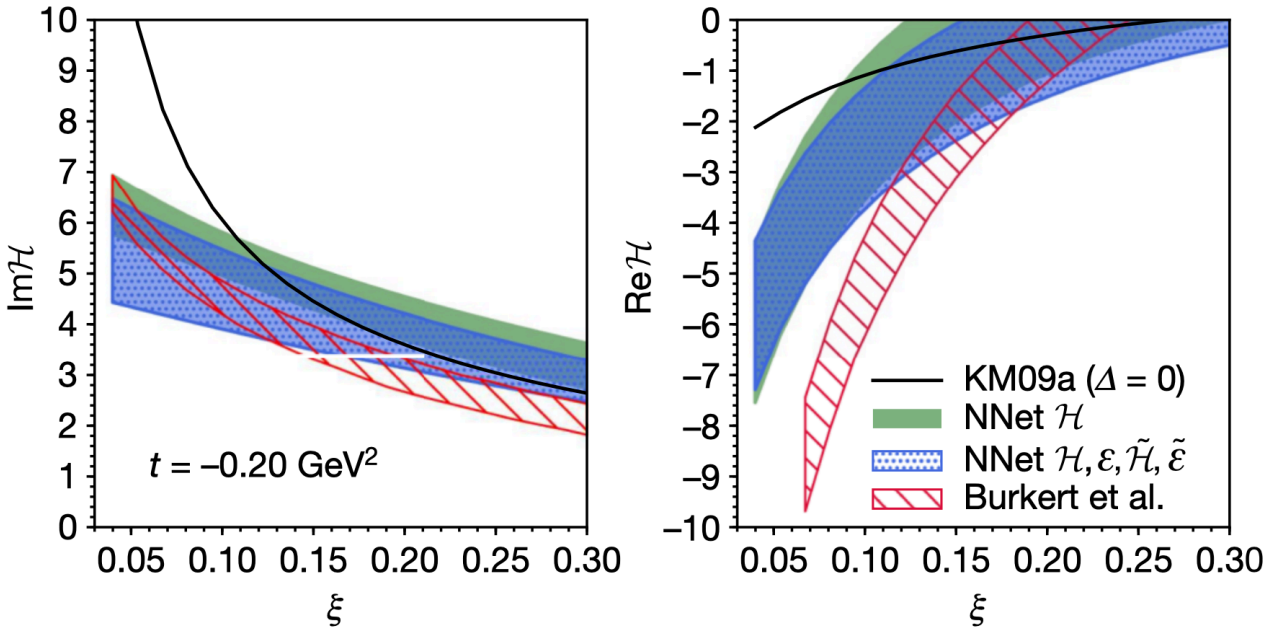
The procedure:

1. Extract subtraction constant using KM model and CLAS data
2. Fit extracted values with

$$d_1(t) = d_1(0) \left( 1 - \frac{t}{M^2} \right)^{-\alpha}$$

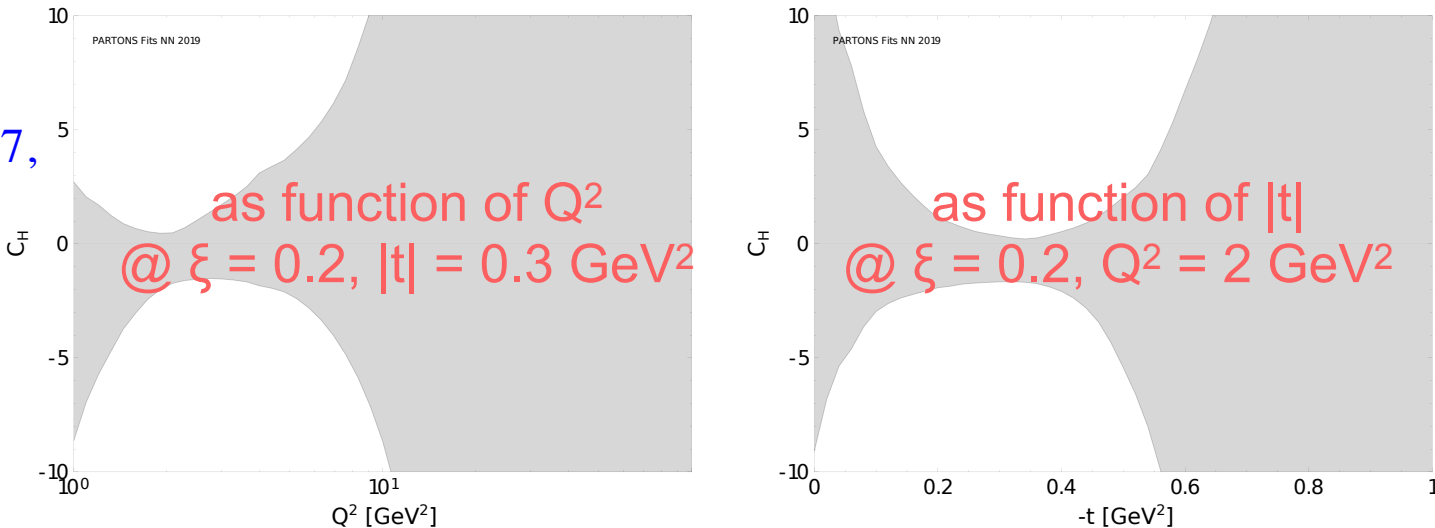
- without JLab 6 GeV data
- with JLab 6 GeV data
- with JLab 12 GeV data (projected)

K. Kumerički, Nature 570  
(2019) no.7759, E1



Result:  $C(t) = 0.78 \pm 1.5$   
almost no  $t$ -dependence

H. Moutarde, PS, J. Wagner,  
Eur. Phys. J. C79 (2019) no. 7,  
614

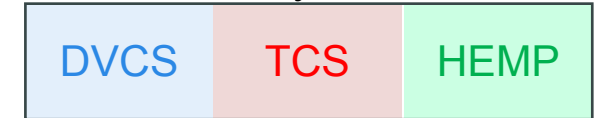


## Generalised Parton Distributions

- novel way to describe partonic structure of nucleon
- allows to study (highlights):
  - nucleon tomography
  - total angular momentum of partons
  - “mechanical” properties of parton distributions
- need for more data, better description of exclusive processes, extraction methods

- **PARTONS** - platform to study GPDs
- Come with number of available physics developments implemented
- Addition of new developments as easy as possible
- To support effort of GPD community
- Can be used by both theorists and experimentalists

## Observable Layer



## Process Layer



## CCF Layer



## GPD Layer





- **PARTONS** - platform to study GPDs
- Come with number of available physics developments implemented
- Addition of new developments as easy as possible
- To support effort of GPD community
- Can be used by both theorists and experimentalists

$H^u @ x_i = 0.2, t = -0.1 \text{ GeV}^2, \mu_F^2 = 2 \text{ GeV}^2$

