Magnetic Fields and Thermal QCD

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— Polarisation measurements in ee, ep, pp and heavy-ion collisions —

To be discussed

Magnetic fields in heavy-ion collisions

- □ Rough estimates : $eB \gtrsim \Lambda^2_{\mathrm{QCD}}$

Effects on the QCD phase structures

- □ Magnetic catalysis vs Inverse magnetic catalysis

Effects on the transport / polarisation

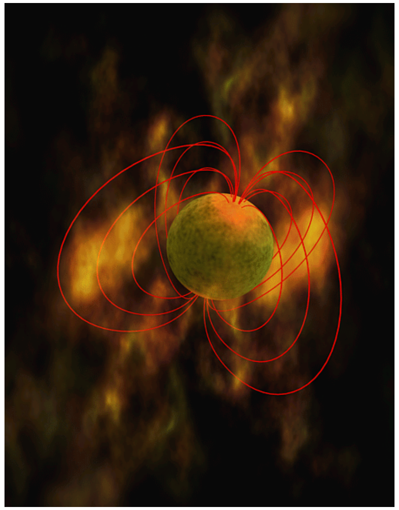
- □ Chiral Magnetic Effect **Talk by Christakoglou**
- □ Chiral Separation / Vortical Effect

Effects on the mesons / baryons

- □ Mass spectrum
- □ Deformation / polarisation of Skyrmion

Magnetic Fields in Heavy-Ion Collisions

Typical Strengths (before 2007)



Surface of the neutron star

$$\lesssim 10^{12}\,\mathrm{gauss} \sim 10^{-2}\,\mathrm{MeV}^2$$

Surface of the magnetar

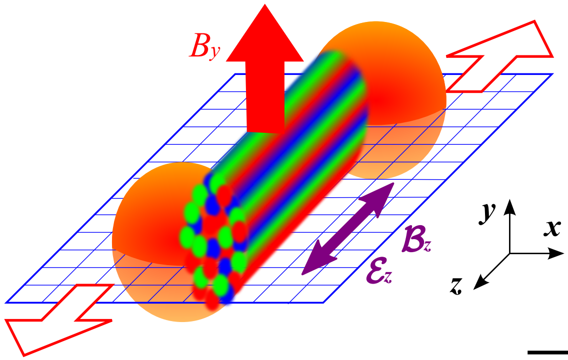
$$\lesssim 10^{15}\,\mathrm{gauss} \sim 10\,\mathrm{MeV}^2$$

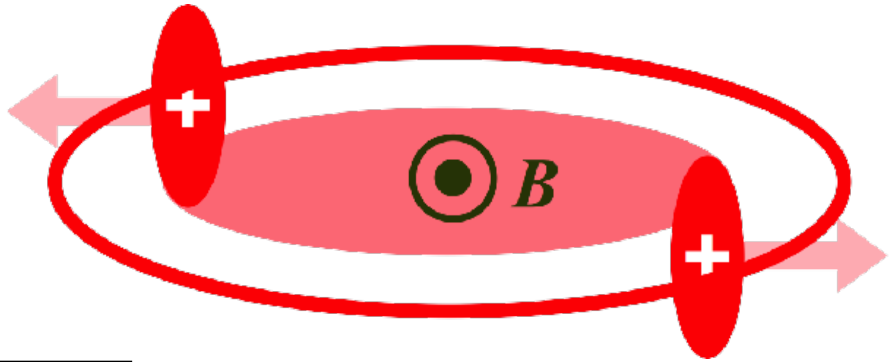
Interior of the magnetar

$$\lesssim 10^{18}\,\mathrm{gauss} \sim 10^{4}\,\mathrm{MeV}^2 \sim m_\pi^2$$

Not significant as compared to the QCD scale…

Typical Strengths (after 2007)





$$\frac{eB_0}{[1+(t/t_0)^2]^{3/2}}$$

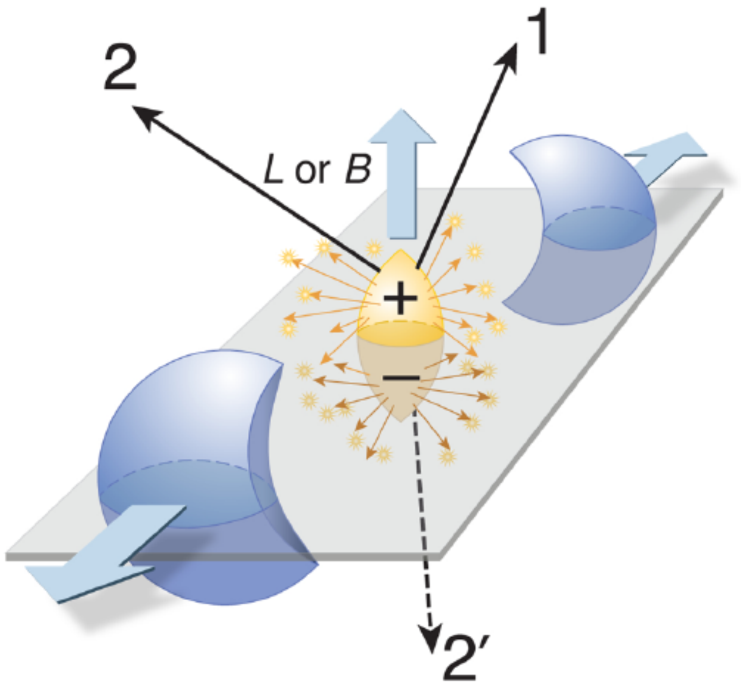
$$eB_0 = (47.6\ \mathrm{MeV})^2 \left(\frac{1\ \mathrm{fm}}{b}\right)^2 Z \sinh Y$$

$$t_0 = \frac{b}{2\sinh Y}$$

$$\boxed{\lesssim 10^{20}\ \mathrm{gauss} \sim \mathrm{GeV}^2}$$

Strongest magnetic field in the (present) Universe

Strong B and Rapid Rotation



Chirality n_5 probed by B
Vorticity ω coupled to J

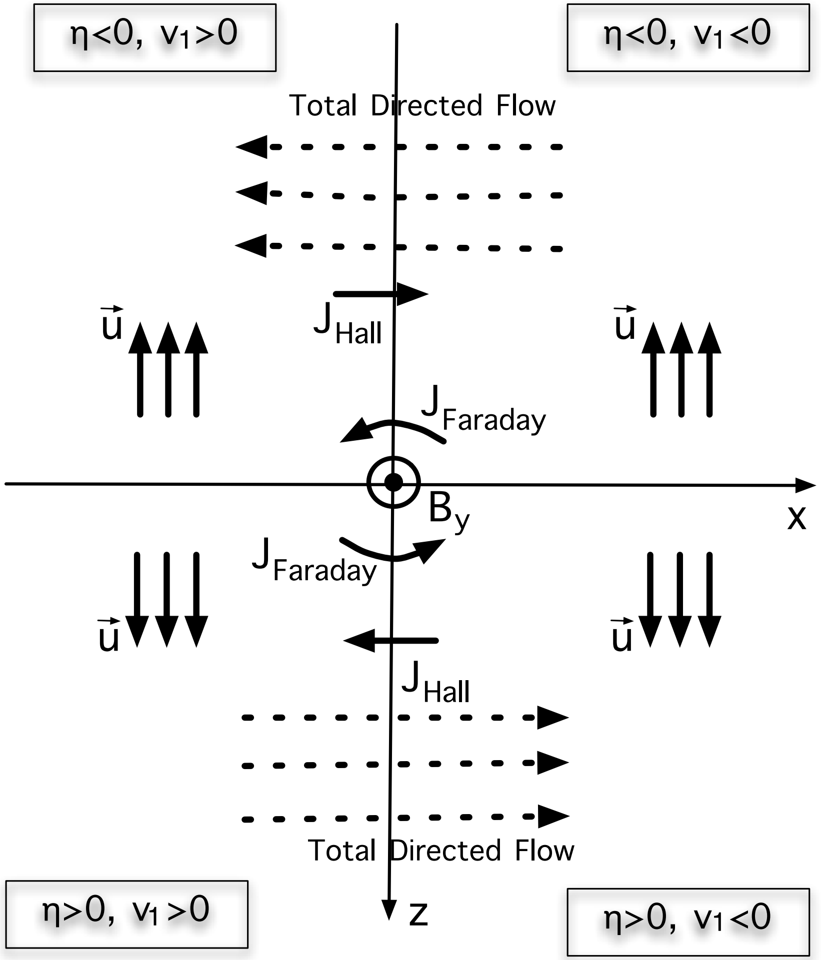
$$B \sim 10^{15}\,\mathrm{T}$$

10^3 times larger than the surface magnetic field of the magnetar

$$J \sim 10^7\,\hbar$$

Largest spin states of nuclei (Yrast states) < 100 $\hbar$

Quantifying B experimentally

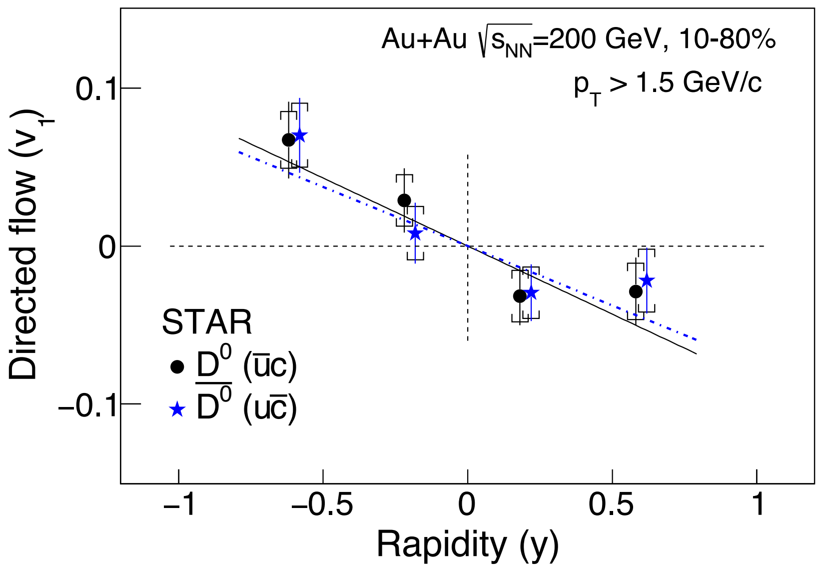


Rapidity (and charge) dependent directed flow

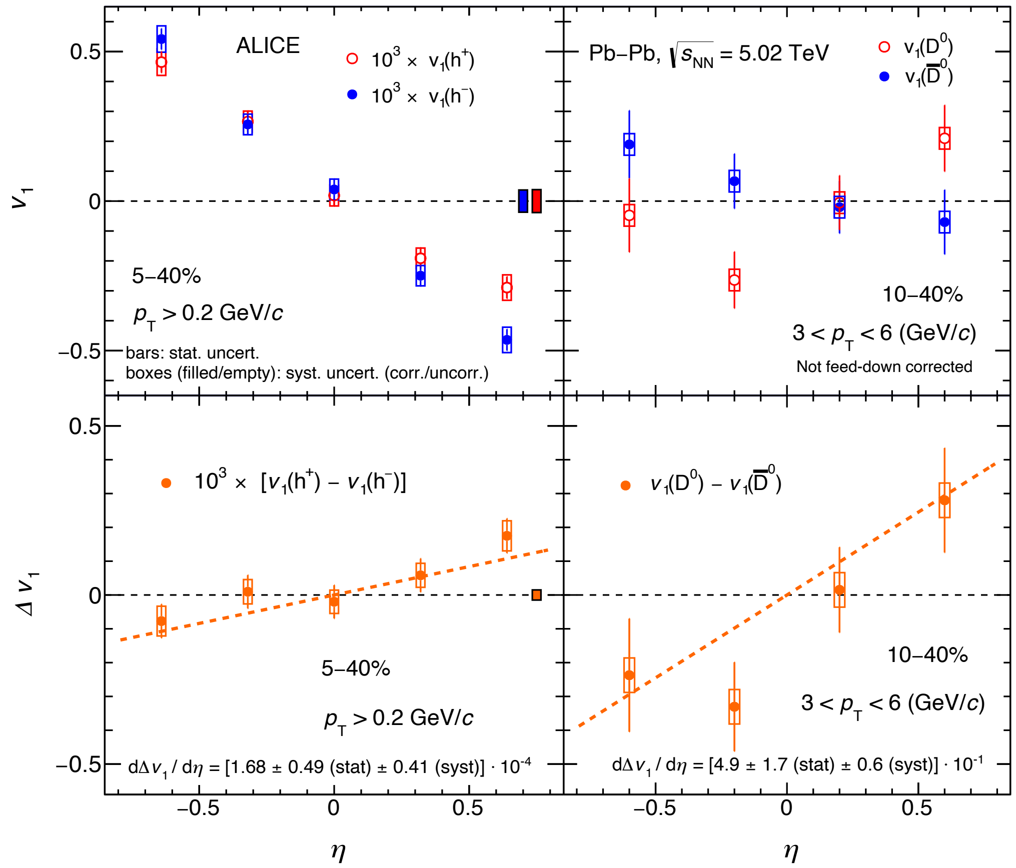
Gursoy-Kharzeev-Rajagopal (2014)

Quantifying B experimentally

STAR (2019)



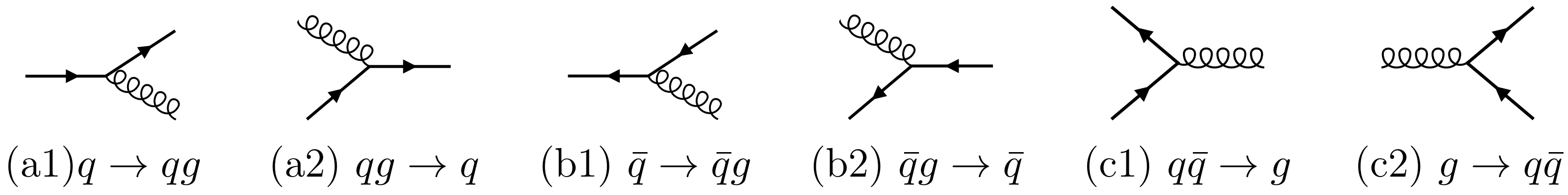
ALICE (2019)

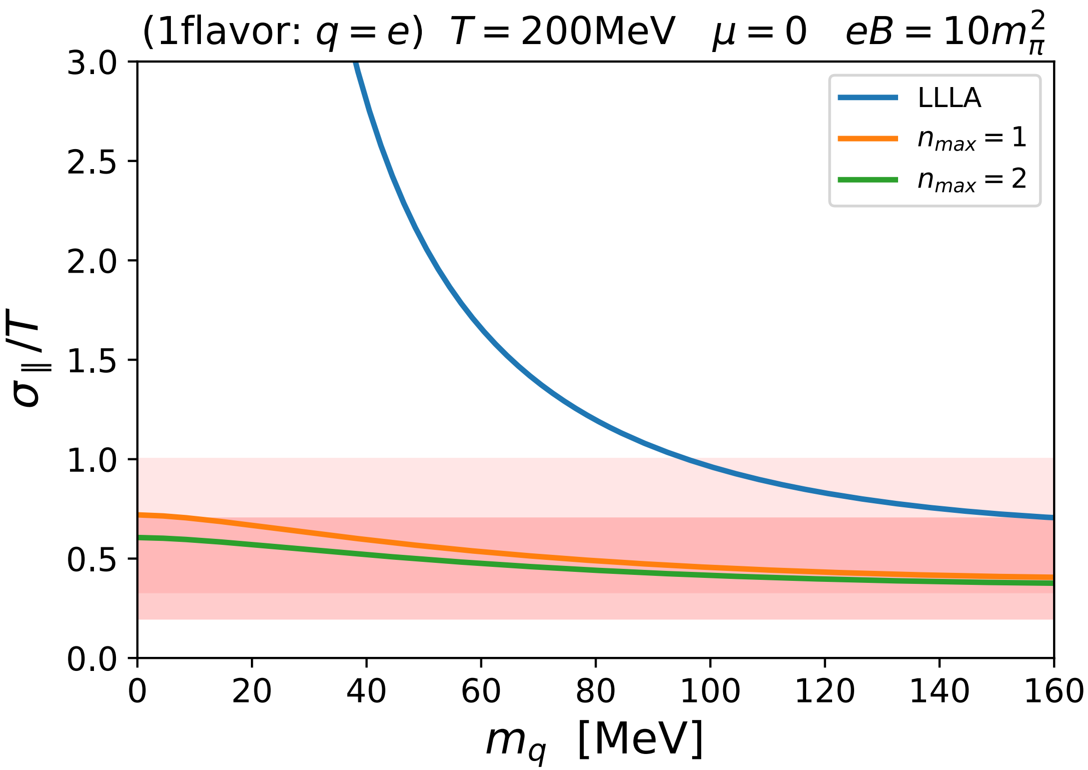


Charm quarks are more sensitive to the magnetic fields

Interpretations depend on the electric conductivity…

QCD Electric Conductivity





Fukushima-Hidaka (2017/2019)

Conductivity is not large enough to support decaying magnetic fields

Bands ~ Lattice QCD by H.-T Ding *et al.* (2011/2016)

Λ Polarisation

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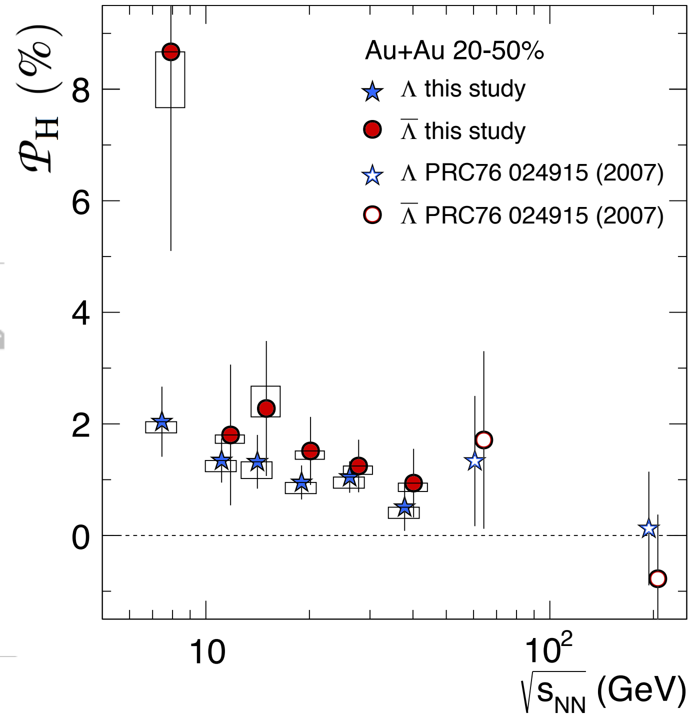
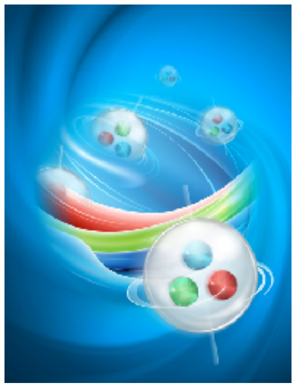
'Perfect Liquid' Quark-Gluon Plasma is the Most Vortical Fluid

Swirling soup of matter's fundamental building blocks spins ten billion trillion times faster than the most powerful tornado, setting new record for "vorticity"





$$\omega = \nabla \times v \longrightarrow \textbf{Polarization}$$

Λ Polarisation

Effects of the rotation and the magnetic field

$$H_{\text{eff}} = H - \boldsymbol{\omega} \cdot \boldsymbol{J} - \boldsymbol{\mu} \cdot \boldsymbol{B}$$

cf. Cranking Hamiltonian in nuclear physics

Thermal equilibrium $\sim e^{-H_{\text{eff}}/T}$ gives polarisation

$$P_H = \frac{N_{H\uparrow} - N_{H\downarrow}}{N_{H\uparrow} + N_{H\downarrow}}$$

$$P_\Lambda = \frac{\omega}{2T} + \frac{\mu_\Lambda B}{T}$$

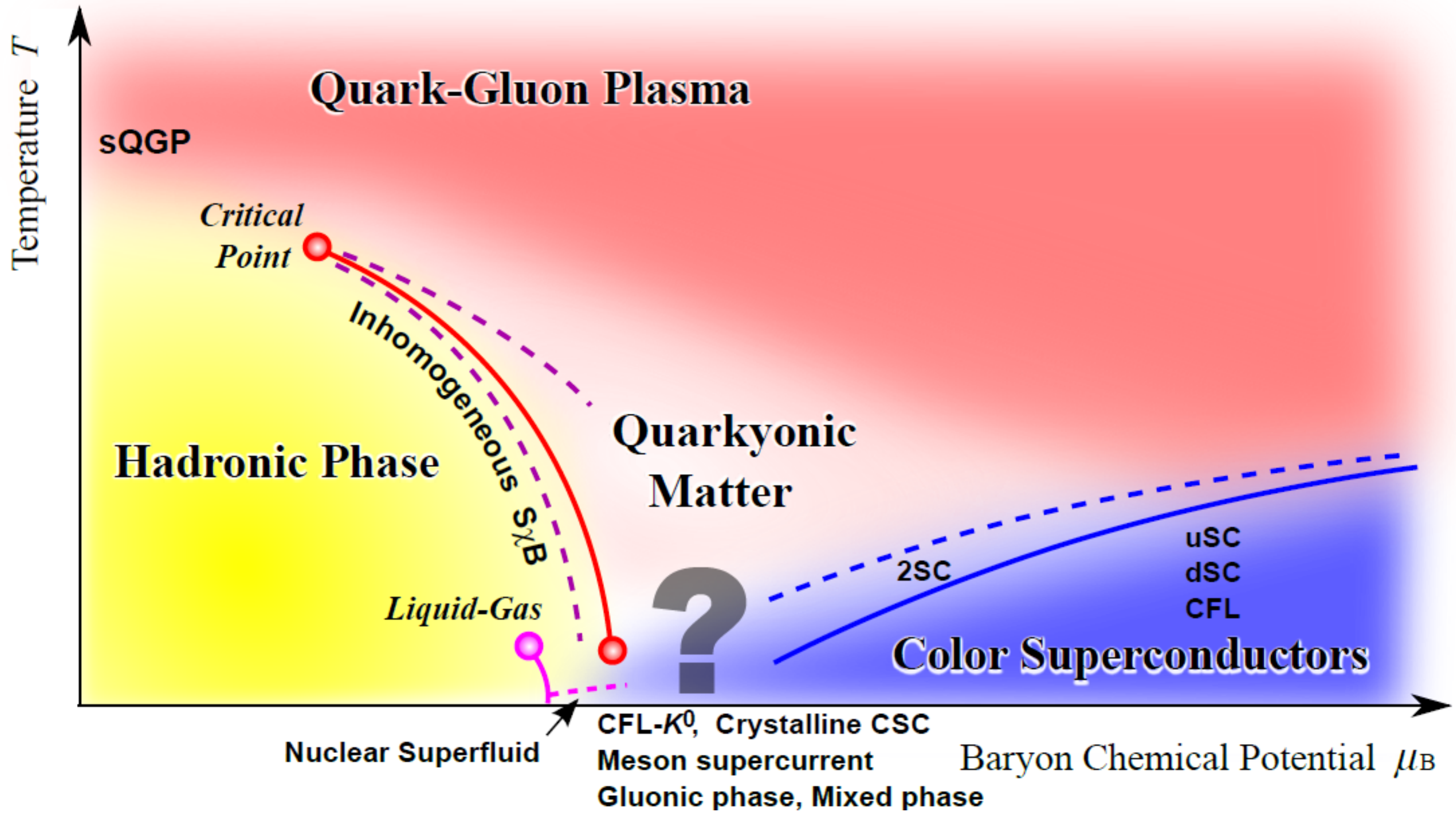
$$P_{\bar{\Lambda}} = \frac{\omega}{2T} - \frac{\mu_\Lambda B}{T}$$

Becattini-Karpenko-Lisa-Upsal-Voloshin (2016)

This Afternoon!

Effects on the QCD Phase Structures

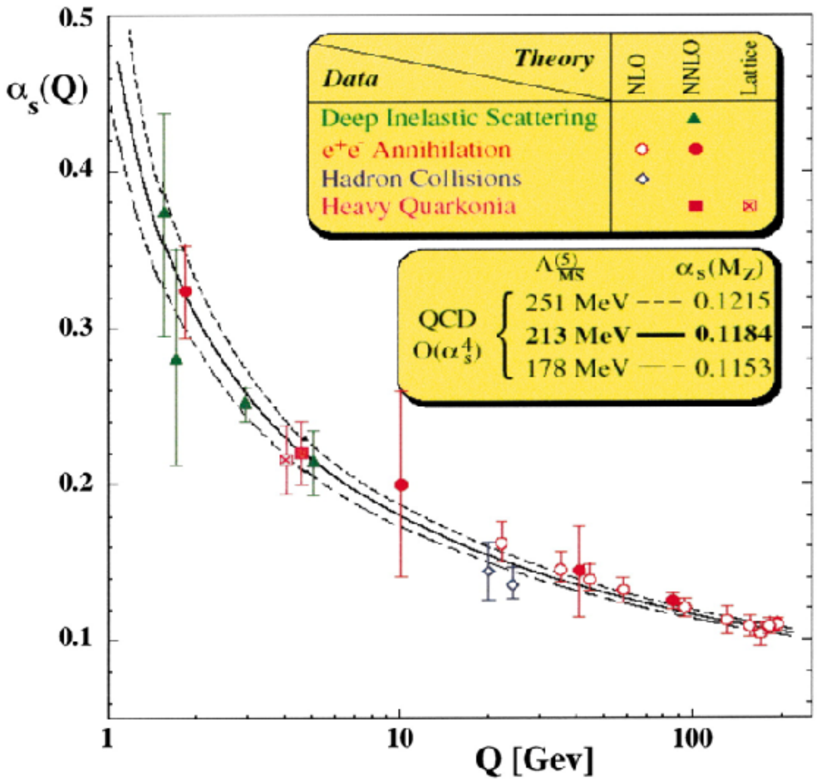
QCD Phase Diagram



Fukushima-Hatsuda (2010) / Fukushima-Sasaki (2013)

Chiral Phase Transition

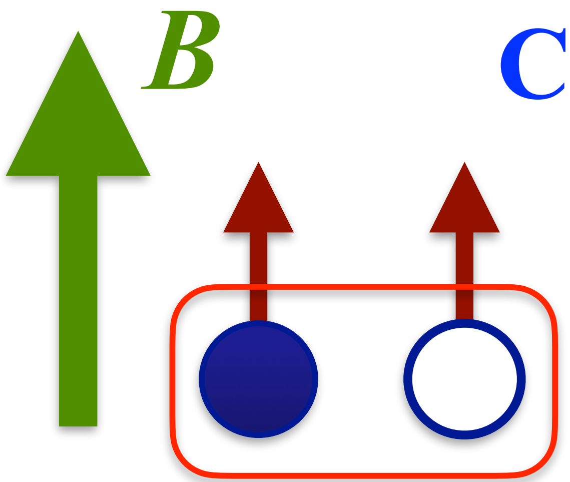
~ 350MeV **High *T*** **~ a few MeV**



$$\alpha_s(Q^2) = \frac{1}{\beta_0 \ln(Q^2/\Lambda^2_{\rm QCD})}$$

All the scales are set by $\Lambda_{\rm QCD}$
(quark mass, critical *T*, etc)

Magnetic Catalysis



Condensate enhanced

Chiral condensate
(scalar - isoscalar)

$L = 1$ and $S = 1$ making $J = 0$
more favored by strong B

Chiral Perturbation Theory **(Shushpanov-Smilga 1997)**

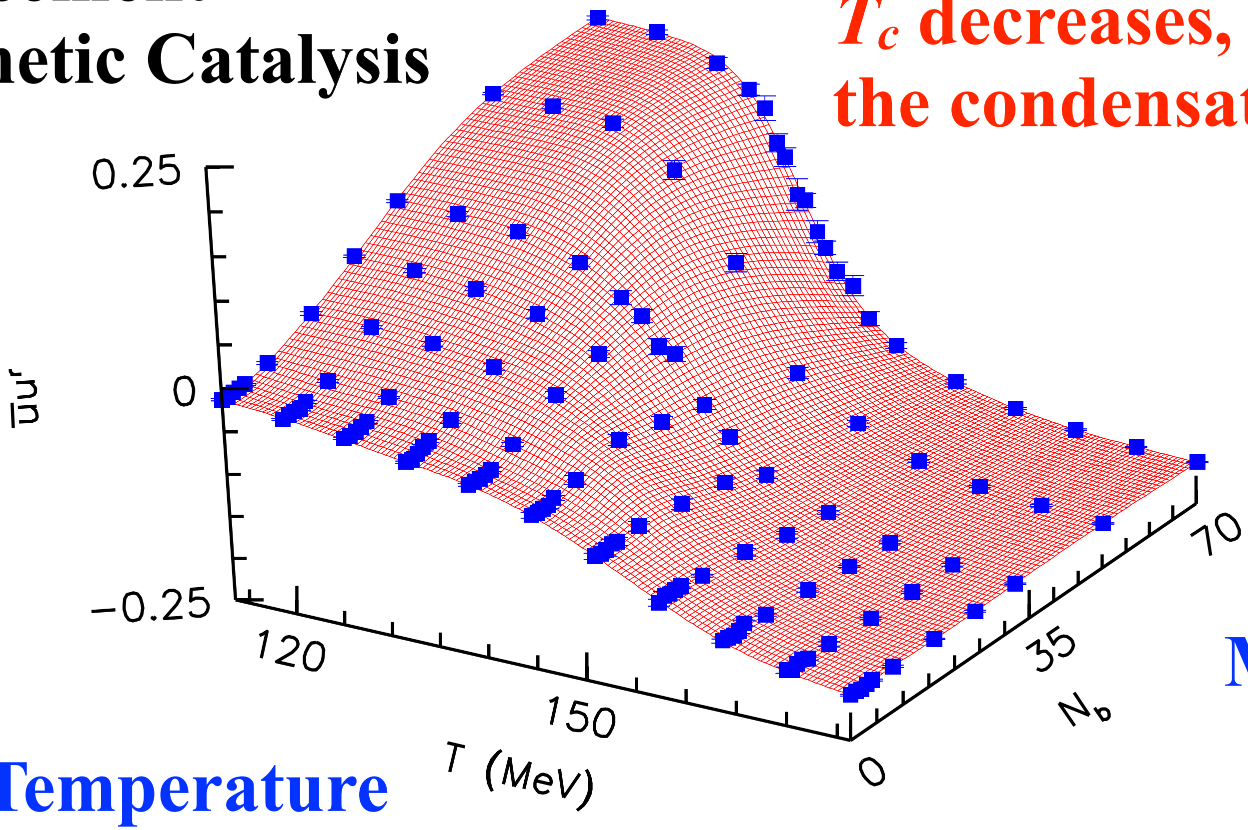
$$\Sigma(B) = \Sigma(0)\left(1 + \boxed{\frac{\ln 2}{16\pi^2 f_\pi^2}} eB + \cdots\right)$$

Postive coefficient

Inverse Magnetic Catalysis

Enhancement
~ Magnetic Catalysis

T_c decreases, while the condensate increases



Temperature

Magnetic Field

Bali *et al.* (2011)

Inverse Magnetic Catalysis

Difficulty in understanding the IMC

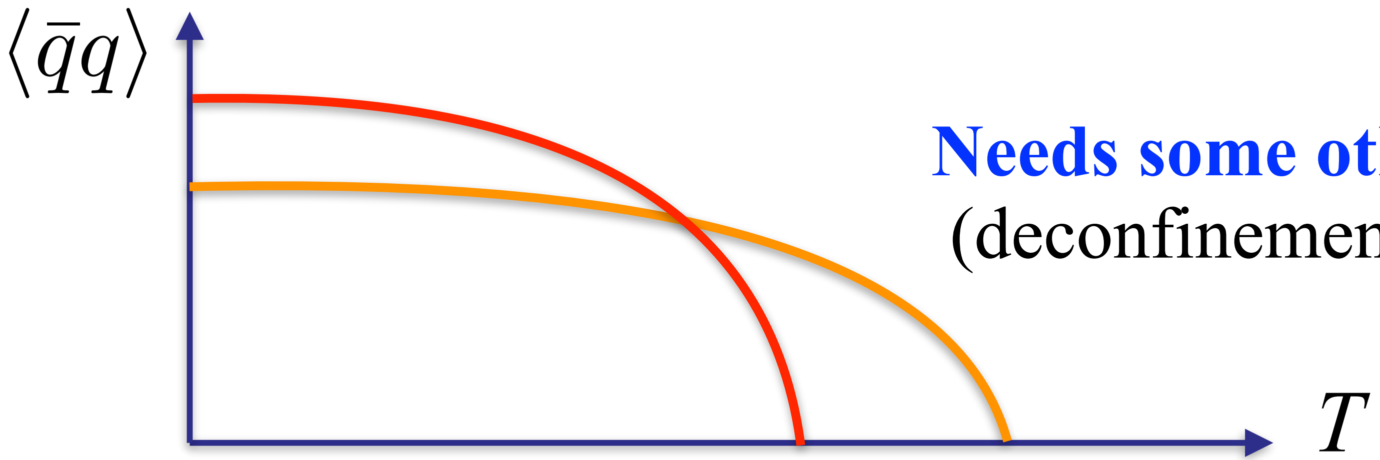
Critical T in BCS: $\quad T_c \propto \underline{\Delta(T=0)}$

$\langle \bar{q}q \rangle$ in QCD

Reconcile?

$\langle \bar{q}q \rangle (T=0)$ is increased at finite B

T_c is decreased at finite B



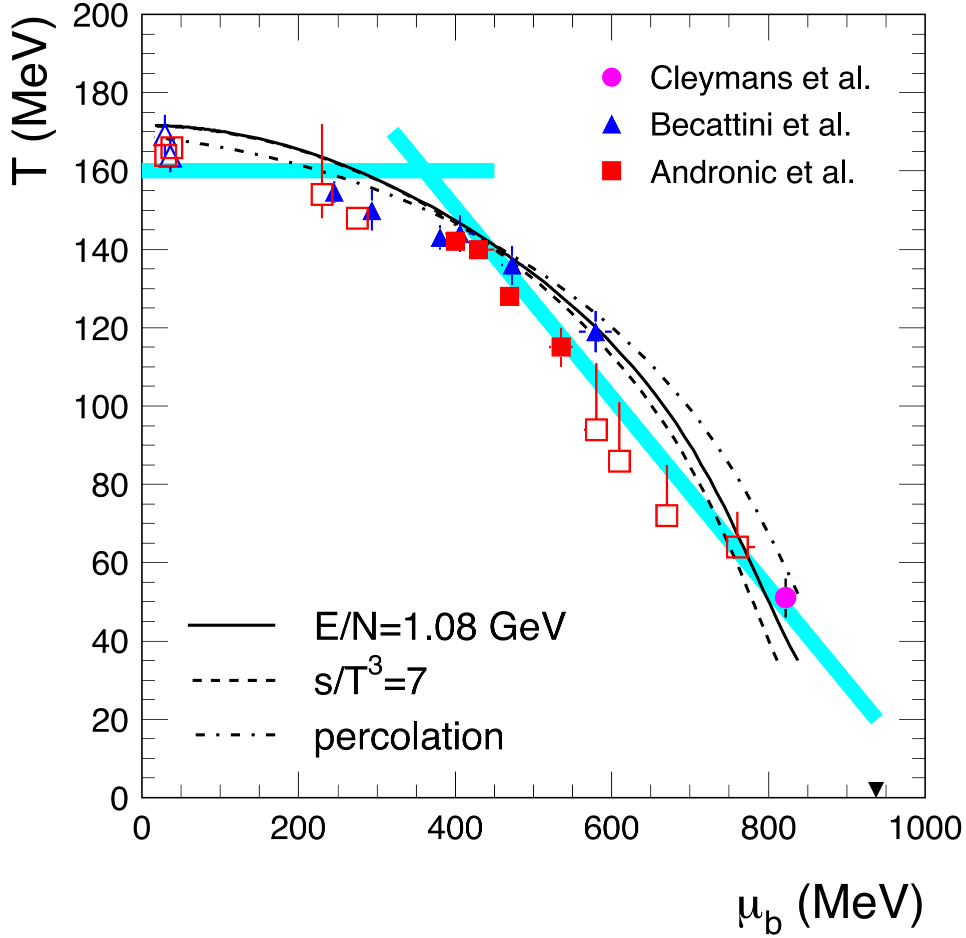
Needs some other dynamics?
(deconfinement / IR meson)

Phenomenological Approach

Chemical Freezeout

Experimentally seen "Phase Diagram"

Point (T, μ_B, etc) where "inelastic" scattering is turned off (due to changes in inter-particle distance)



Hadron Resonance Gas

$$p_{b/f} = \pm T \sum_{s_z=-s}^{s} \sum_{n=0}^{\infty} \frac{qB}{2\pi} \int \frac{dp_z}{2\pi} \ln(1 \pm e^{-(E-\mu_i Q_i)/T})$$

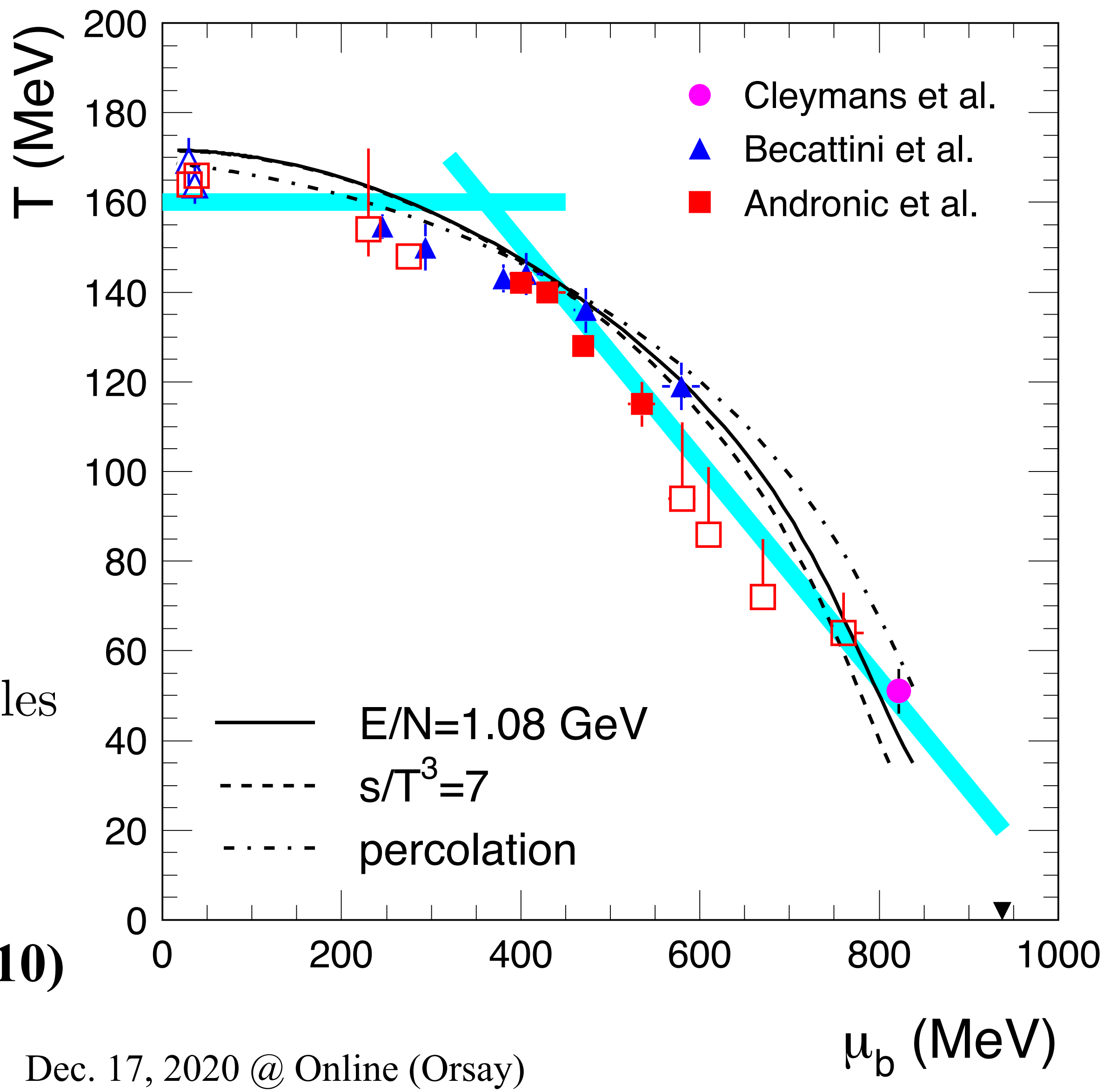
Phenomenological Approach

Cleymans-Redlich (1998)

$$E/N \sim 1\text{GeV}$$

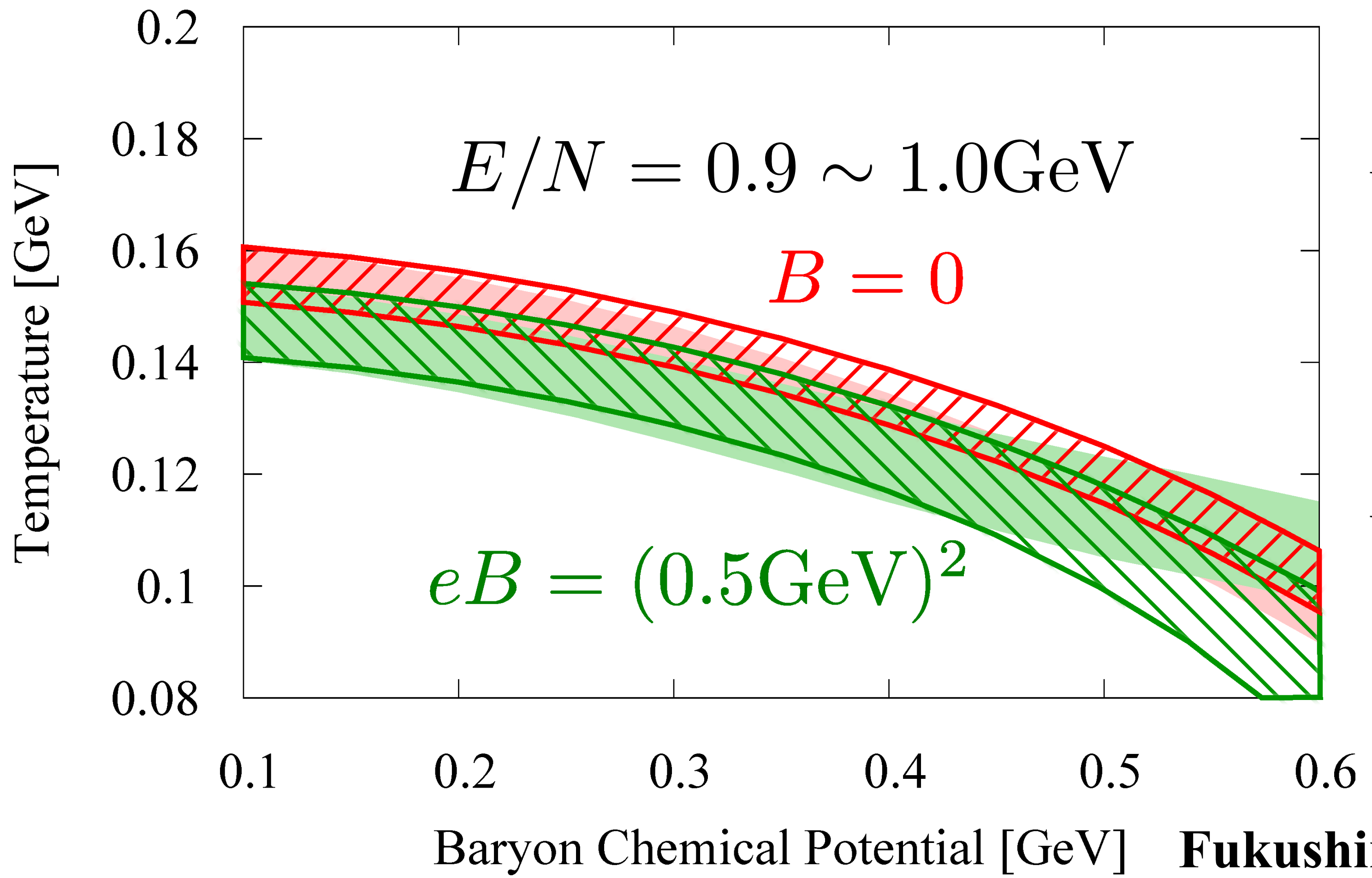
E : internal energy

N : particles + antiparticles



Andronic et al. (2010)

Phenomenological Approach



Slant lines

With conservation of S and Q

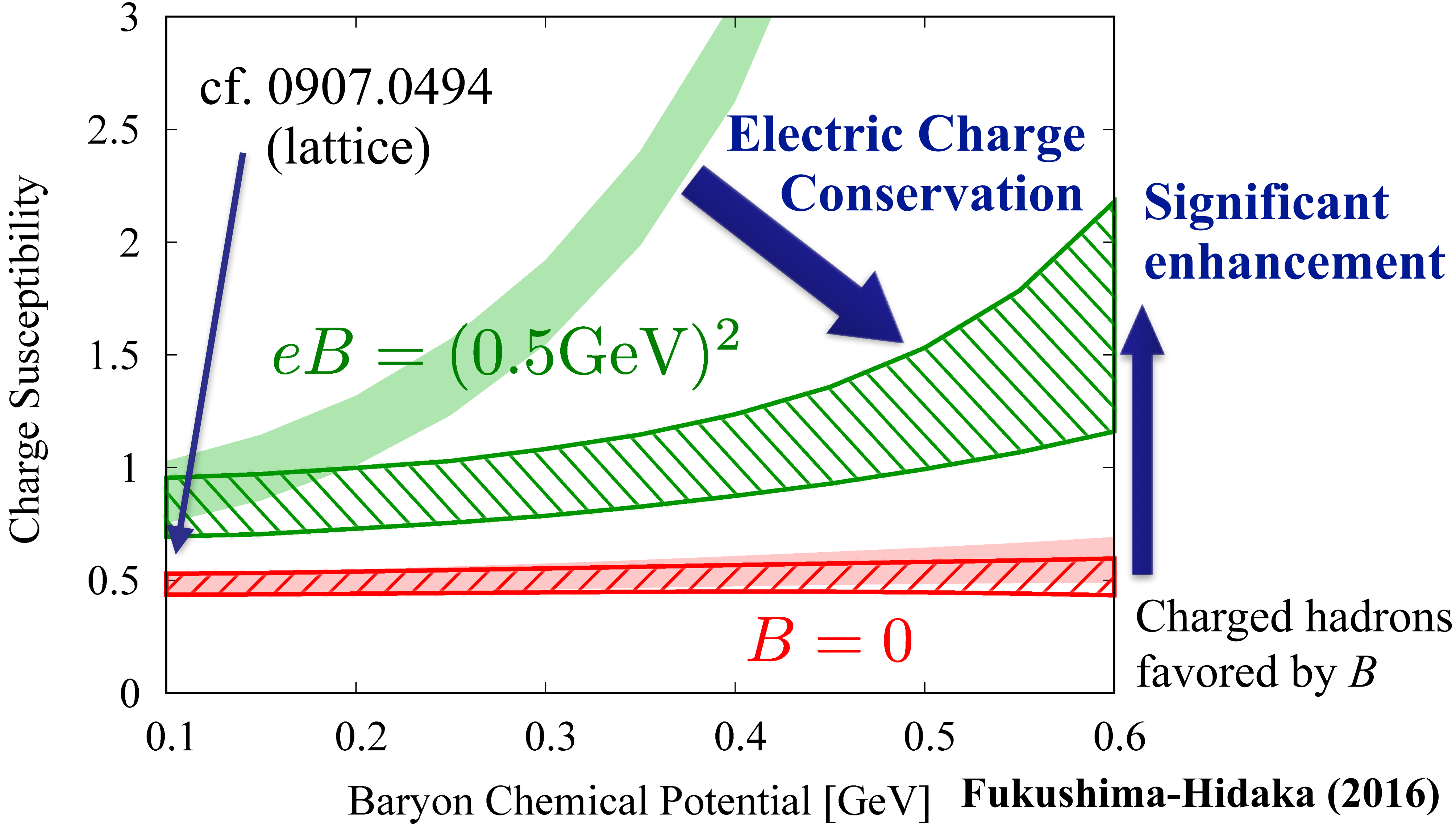
Shaded regions

Without conserv. of S and Q

Fukushima-Hidaka (2016)

Inverse Magnetic Catalysis naturally reproduced

Phenomenological Approach



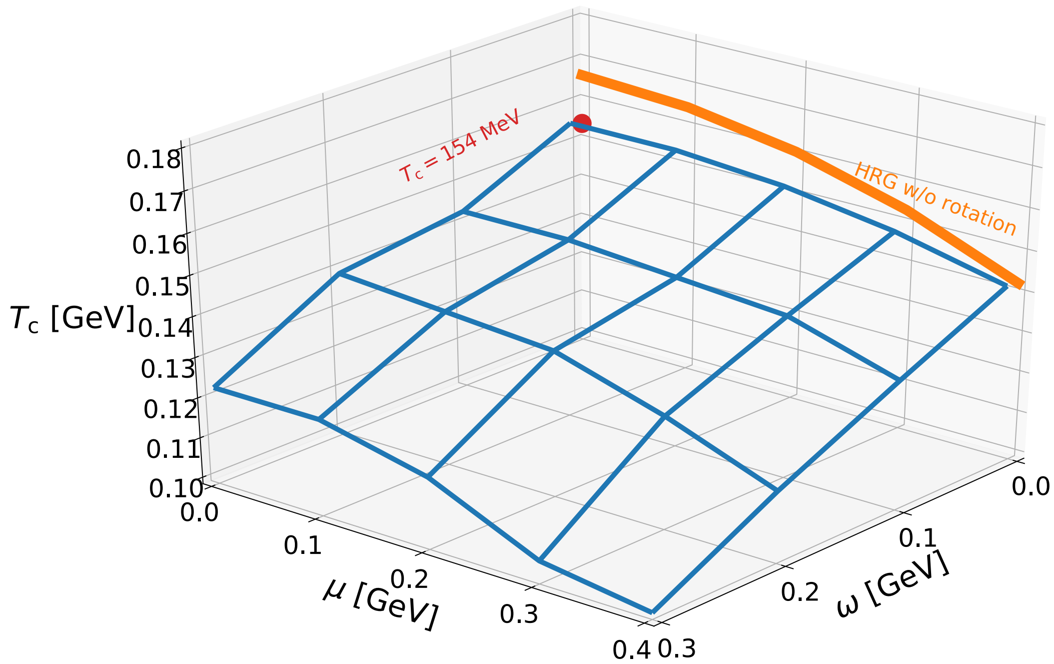
Fukushima-Hidaka (2016)

Phenomenological Approach

Phase Diagram with Finite Rotation?

T_c decreases in model studies
T_c increases (!?) in lattice QCD (by **Braguta *et al.***)

Fujimoto-Fukushima-Hidaka (2020 maybe)



Surface of
$P/P_{\mathrm{SB}} = $ (const.)

T_c **decreases…**

Effects on the transport / polarisation

Conserved Current

Axial rotation by $\theta(x)$

$$\delta S = \int dx\, \theta(x) \left[\partial_\mu j^\mu_A + \frac{q_e^2}{16\pi^2} \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \right]$$

$$= \int dx\, \partial_i \theta(x) \left[-j^i_A - \frac{q_e^2}{2\pi^2} \underline{\varepsilon^{0ijk} A_0 \partial_j A_k} \right]$$

$$= -\mu_q B^i$$

Anomaly induced transport (chiral separation effect)

Conserved Current

$$j^i = -\frac{e^2}{4\pi^2}\varepsilon^{i0jk}A_0\partial_j A_k = -\frac{e^2}{4\pi^2}A_0 B^i$$

In electromagnetism a constant vector potential is irrelevant

$$A_\mu \to A_\mu + \partial_\mu \varphi$$

***A_0* can be non-trivial in Euclidean spacetime**

$$i\partial_0 + eA_0(x) - \mu \quad \to \quad -\partial_4 + ieA_4(x) - \mu$$

See: A. Yamamoto, 1210.8250

Conserved Current

$$A_0 \;\leftrightarrow\; iA_4 \sim -\mu$$

gauged away→anomaly

$$\boldsymbol{j} = \frac{e^2}{4\pi^2}\mu\boldsymbol{B}$$

Especially if μ_A for right-handed and -μ_A for left-handed:

Chiral Magnetic Effect

$$\boldsymbol{j}_{\rm V} = \frac{e^2}{2\pi^2}\mu_{\rm A}\boldsymbol{B}$$

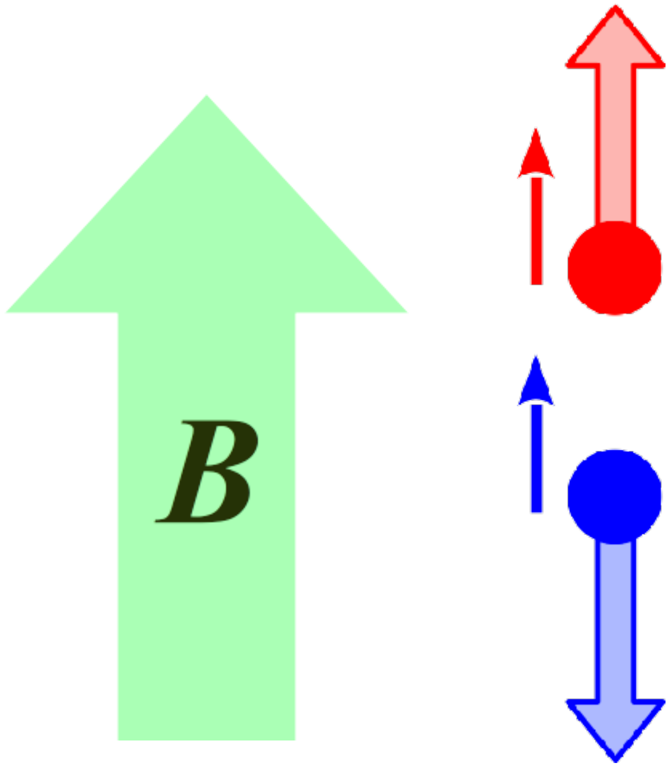
Especially if μ_V for right-handed and μ_V for left-handed:

Chiral Separation Effect

$$\boldsymbol{j}_{\rm A} = \frac{e^2}{2\pi^2}\mu_{\rm V}\boldsymbol{B}$$

Chiral Magnetic Effect

Chiral Magnetic Effect



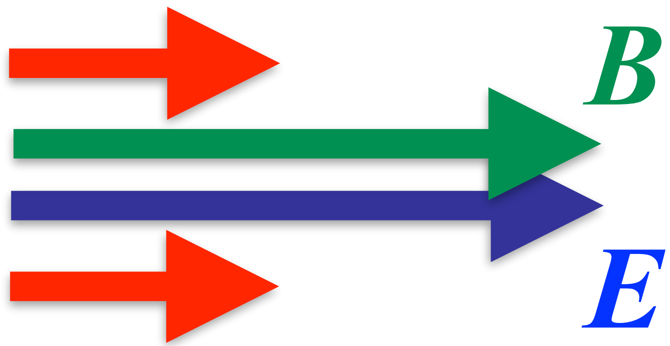
Right-handed particles
Momentum parallel to Spin

Left-handed particles
Momentum anti-parallel to Spin

Only LLL contributes to the topological current

Chiral Magnetic Effect

$$\boldsymbol{j}_{\text{CME}} = (\boldsymbol{E}\cdot\boldsymbol{B})\boldsymbol{B} \quad \propto \quad B^2$$



Only this is external

$$\boldsymbol{j}_{\text{Ohm}} = \sigma\boldsymbol{E}$$

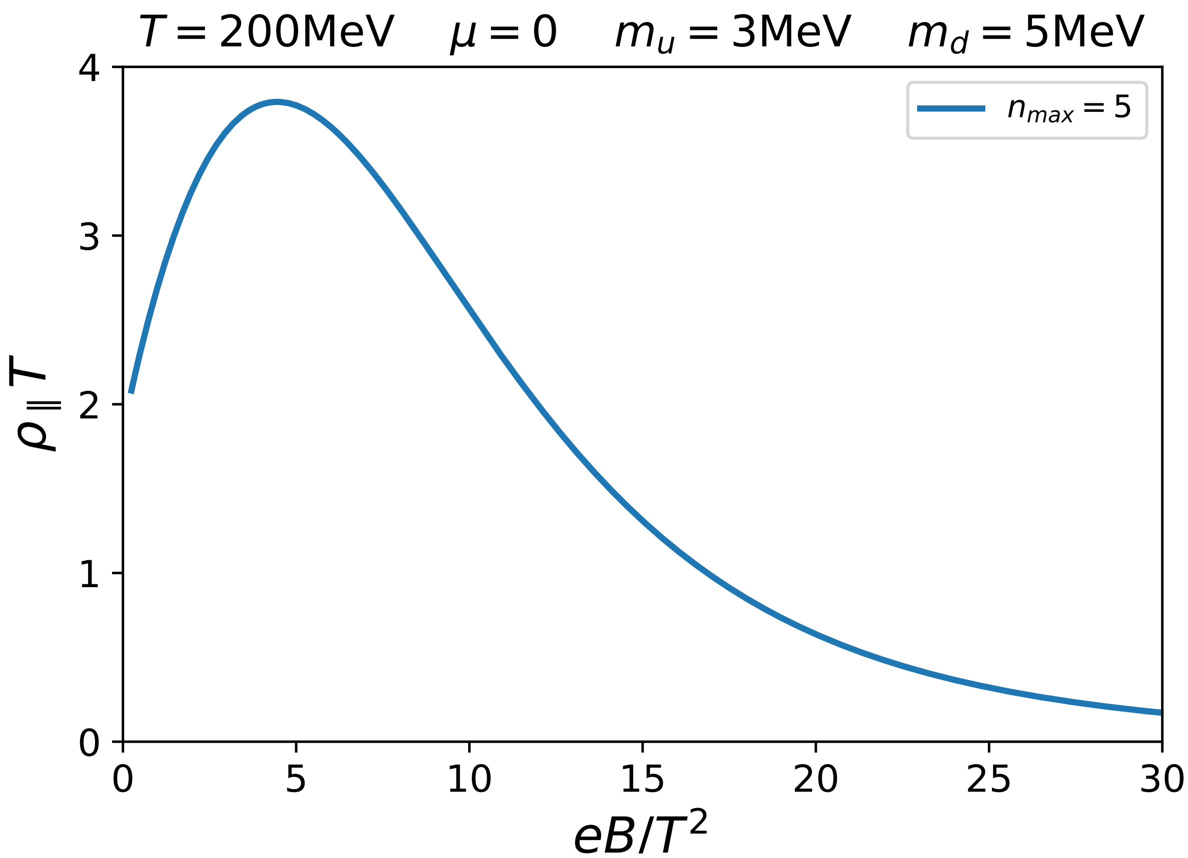
$$j = (\sigma_{\text{Ohm}} + \sigma_{\text{CME}})E \qquad \sigma_{\text{CME}} \propto B^2$$

Electric conductivity once again!

Son-Spivak (2012)

Chiral Magnetic Effect

Fukushima-Hidaka (2017/2019)



Hydro modes lead to divergences in the Kubo formula.

Hydro modes should be projected out.

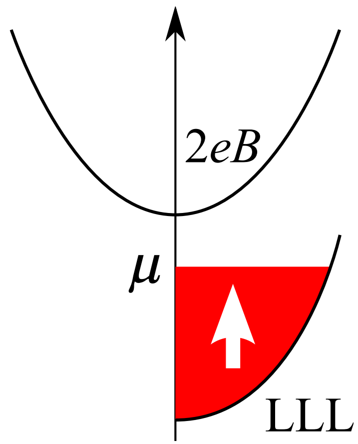
For small quark mass the axial charge should be a hydro mode, but it is "dropped" in this calculation.

Spin Polarisation

$$j_A^i = \langle \bar{\psi}\gamma^i\gamma_5\psi \rangle = \phi_R^\dagger \sigma^i \phi_R + \phi_L^\dagger \sigma^i \phi_L$$

Gauge-invariant quark spin operator

$$\frac{q_e B}{2\pi} \times q_e \frac{\mu_q}{\pi}$$



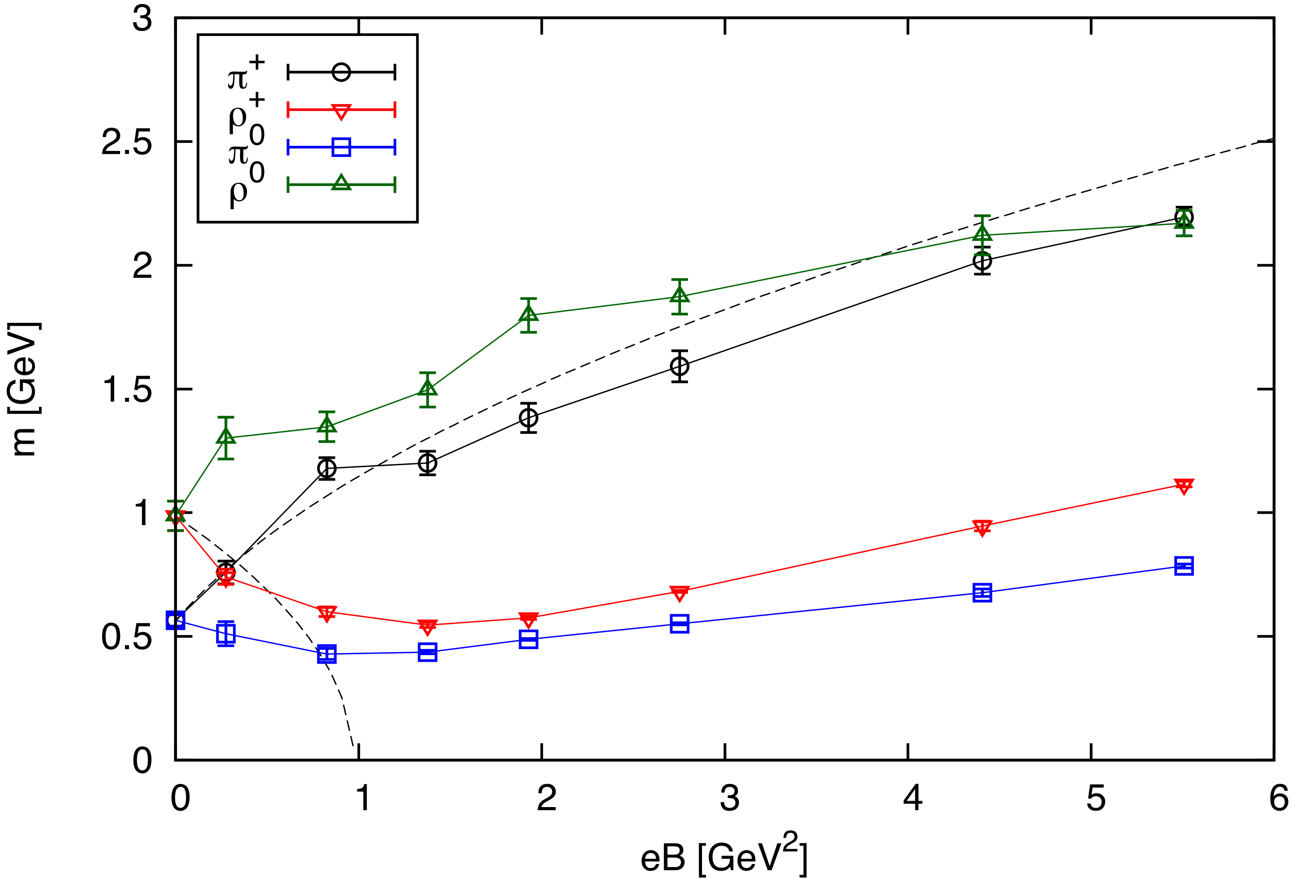
Density of states **1D charge density**

Implication to the polarisation measurement?

Effects on the mesons and baryons

Lattice QCD

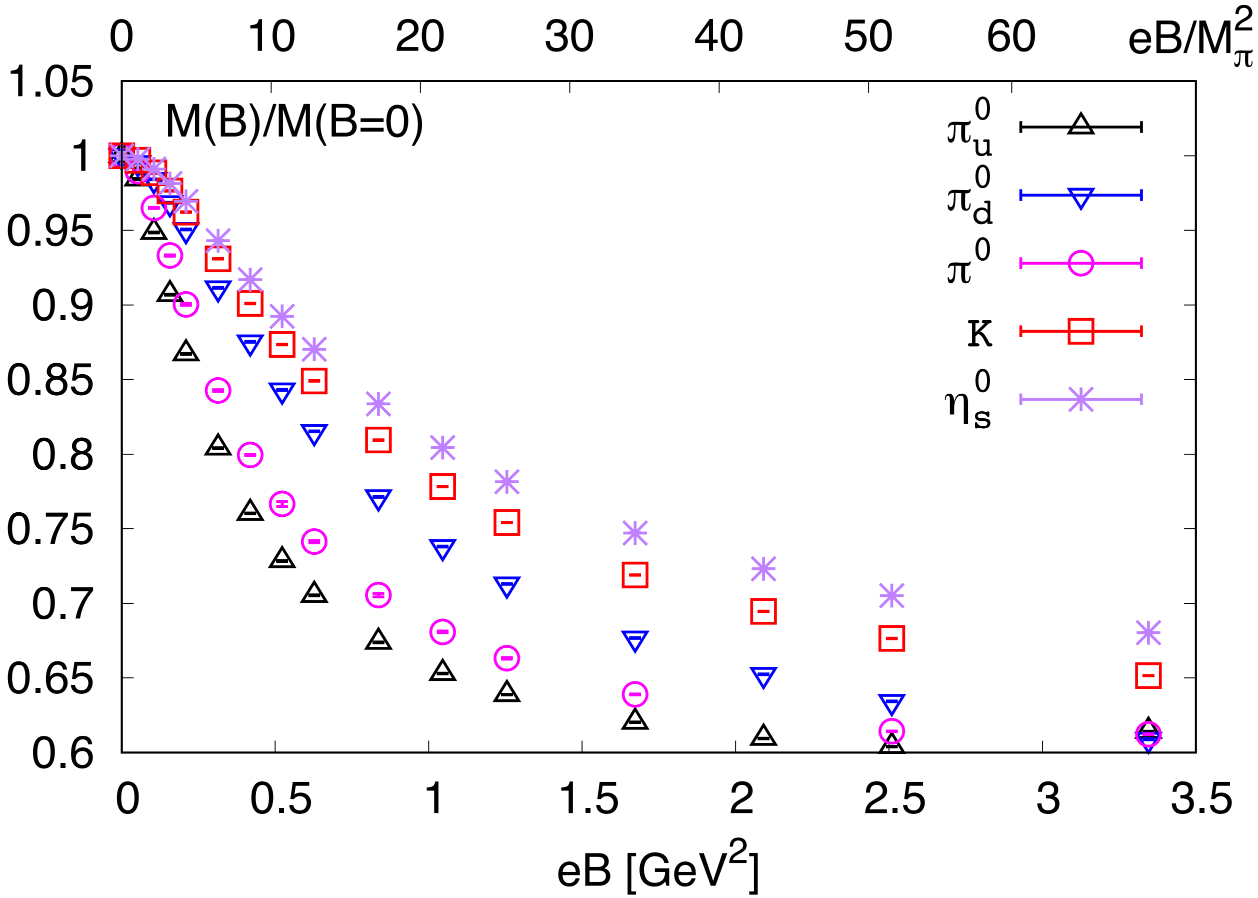
Hidaka-Yamamoto (2012)



Disfavors Chernodub's scenario of "Vacuum Superconductor"

Lattice QCD

Ding-Li-Tomiya-Wang-Zhang (2020)



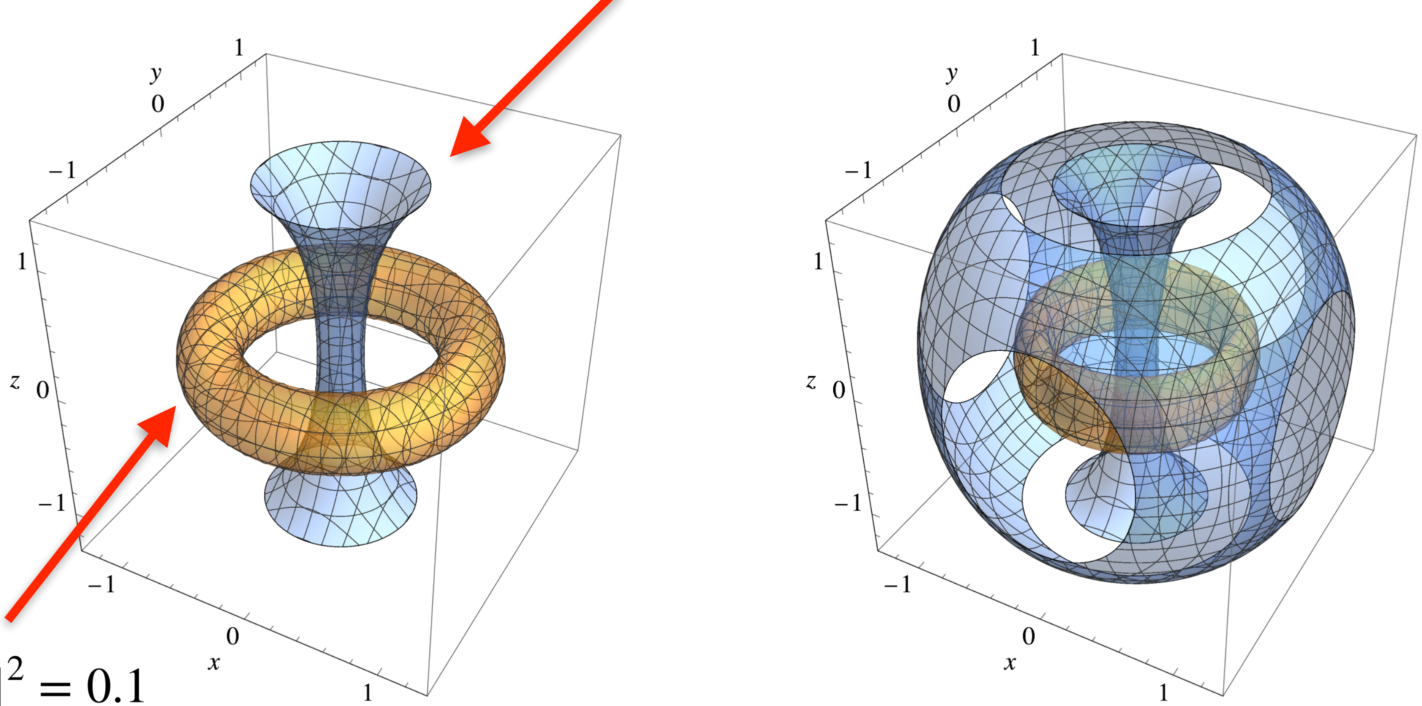
More confinement? (Remember quarks are more massive)

Baryon (in the Skyrme model)

Chen-Fukushima-Qiu (2020 maybe)

$$U = \Sigma + \tau^a \Pi^a$$

$|\Pi^1|^2 + |\Pi^2|^2 = 0.1$

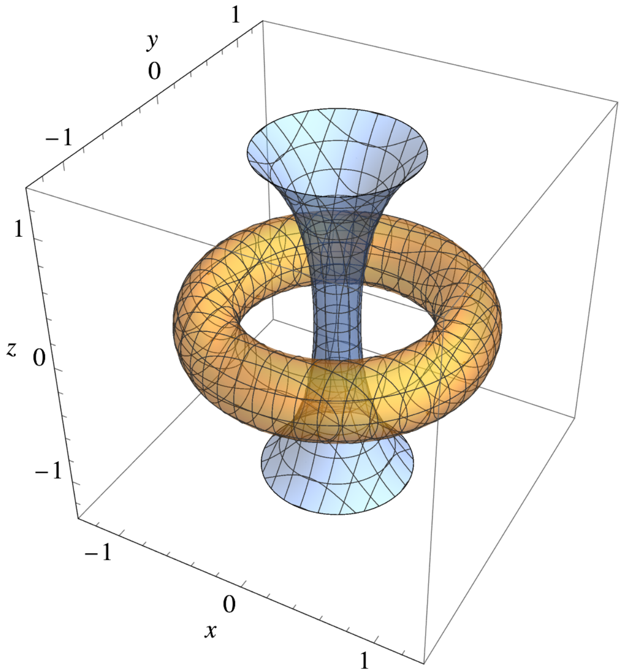


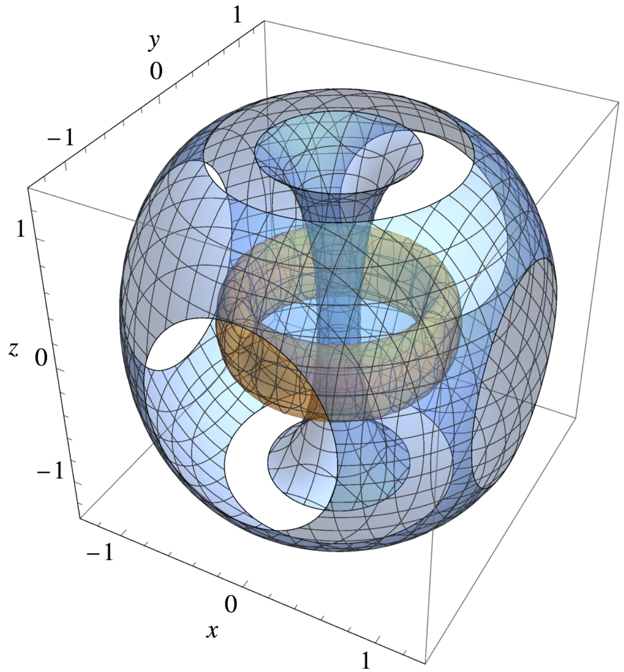
$|\Sigma|^2 + |\Pi^3|^2 = 0.1$

$B = 0$ $\quad$ $B > 0$

Baryon (in the Skyrme model)

Chen-Fukushima-Qiu (2020 maybe)





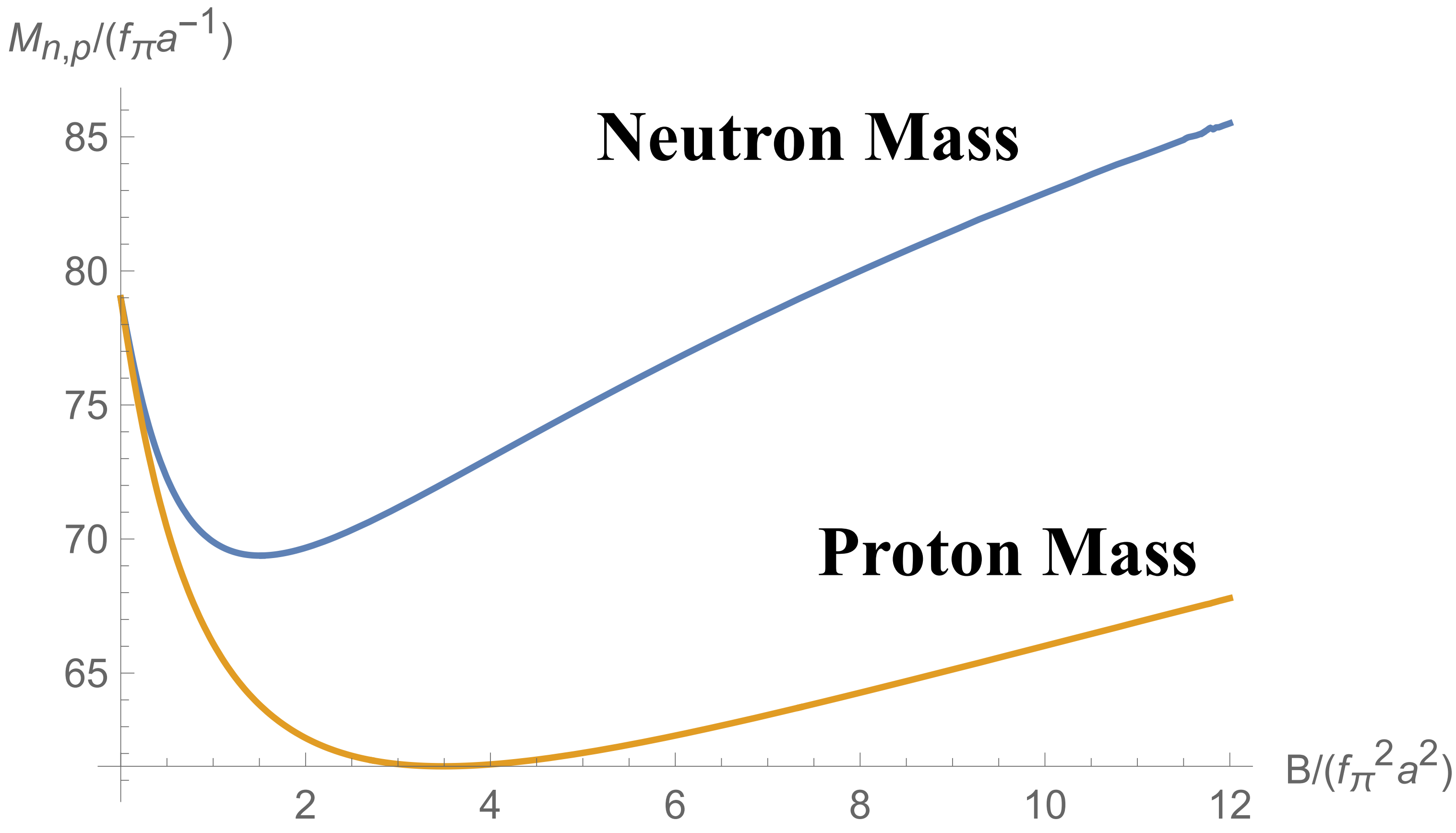
$\Pi_3(SU(2)) = \mathbb{Z}$ winding never loosened

Baryons become compact (with prolate deformation)

More confinement?

Baryon (in the Skyrme model)

Chen-Fukushima-Qiu (2020 maybe)



Summary

- **Strong magnetic field expected in HI collisions**
  - □ Conductivity is a key quantity (seems to be small)
  - □ Flow and polarisation can quantify *B*
- **QCD phase (chiral) structures affected by *B***
  - □ Chiral condensate enhanced (catalyzed)
  - □ Inverse magnetic catalysis was (is?) a surprise
- **Transport / polarisation measurable**
  - □ Chiral magnetic and related effects
  - □ Axial current is nothing but a spin expectation value
- **Still, little is known about *B* effects on bulk thermodynamics and individual mesons/baryons**