



Illinois Center for Advanced Studies of the Universe

Prospects on vorticity theory

Enrico Speranza

Polarization measurements in ee , ep , pp and heavy-ion collisions
IJCLab, December 17, 2020

Vorticity and spin physics in heavy-ion collisions

Vorticity

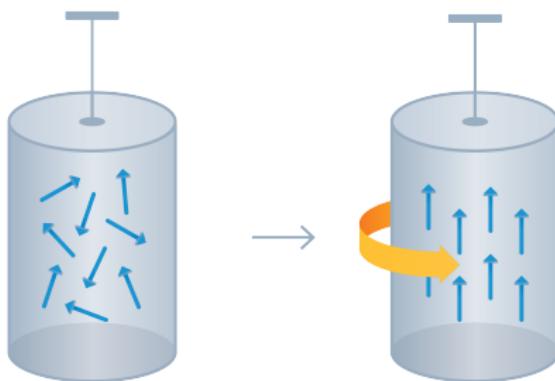


picture from Wikipedia

$$\vec{\omega} = \frac{1}{2} \vec{\nabla} \times \vec{v}$$

Rotation and polarization

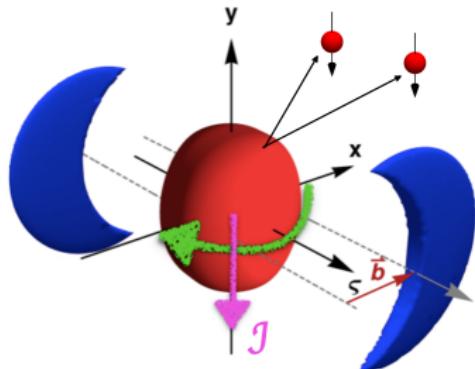
- ▶ Condensed matter: **Barnett effect**



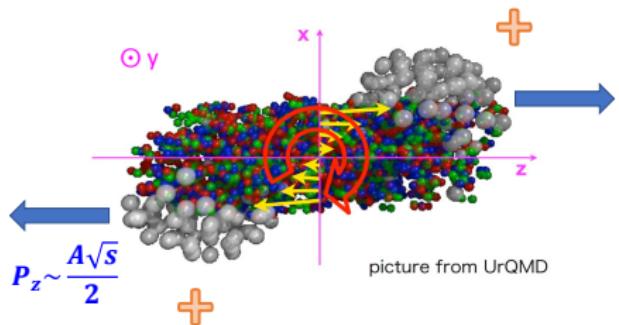
Ferromagnet gets magnetized when it rotates

Polarization effects through rotation in heavy-ion collisions? Yes!

Noncentral heavy-ion collisions



picture from Florkowski, Ryblewski, Kumar,
Prog. Part. Nucl. Phys. 108, 103709 (2019)



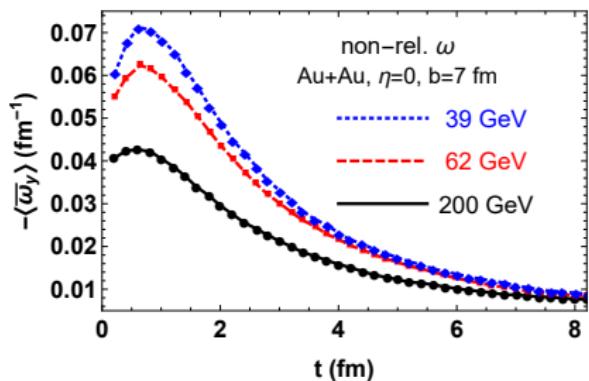
Large global angular momentum

$$J \sim \frac{A\sqrt{s}}{2} b \sim 10^5 \hbar$$

⇒ Vorticity of hot and dense matter ⇒ particle polarization along vorticity

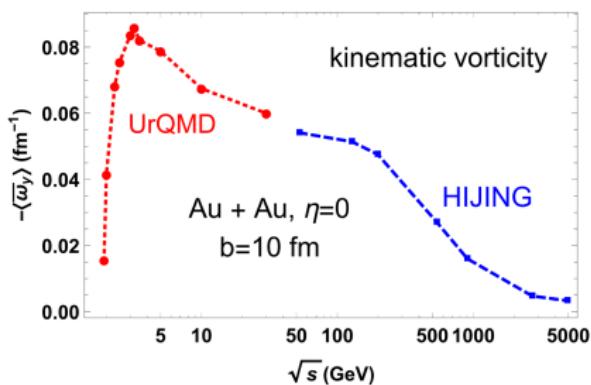
Vorticity in heavy-ion collisions

Time dependence



Jiang, Lin, Liao, PRC 94, 044910

Energy dependence



Deng, Huang, Ma, Zhang, PRC 101 064908
Deng, Huang, PRC 93 064907

see also, e.g., Huang, Liao, Wang, Xia, 2010.08937; Huang, 2002.07549; Becattini et al EPJC 75, 406;
Csernai, Magas, Wang, PRC 87, 034906; Csernai, Wang, Bleicher, Stoecker, PRC 90, 021904;
Ivanonv, Soldatov, PRC 95 054915

- ▶ Vorticity decreases at high energies
- ▶ Extremely high vorticity: $\omega_y \sim 10^{-2} \text{ fm}^{-1} \sim 10^{21} \text{ s}^{-1}$

Spin-vorticity coupling

Effective interaction $\sim -\vec{S} \cdot \vec{\omega}$
 $\sim \text{Quantum} \cdot \text{Classical}$

\vec{S} - Particle spin, $\vec{\omega}$ - Medium rotation

- ▶ Massless particles \implies Chiral Vortical Effect $\vec{J} \sim \vec{\omega}$

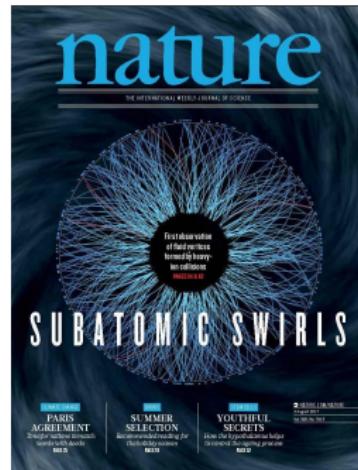
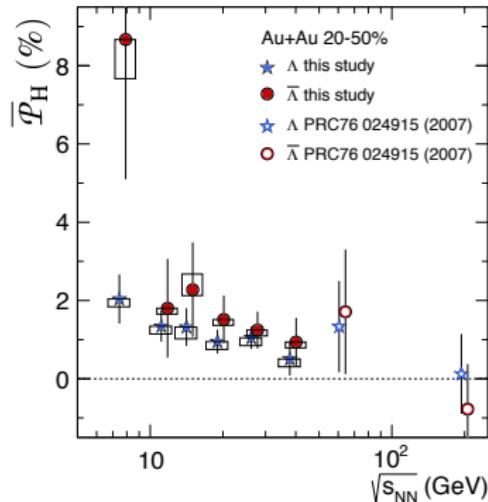
Kharzeev, Voloshin, Wang, Prog. Part. Nucl. Phys. 88, 1-28

- ▶ Massive particles \implies Λ -baryon polarization

Voloshin, 0410089; Liang, Wang, PRL 94, 102301; Betz, Gyulassy, Torrieri PRC 76, 044901;
Becattini, Piccinini, Rizzo, PRC 77, 024906

Experimental observation - Global Λ polarization

- Polarization along global angular momentum



L. Adamczyk et al. (STAR), Nature 548 62-65 (2017)

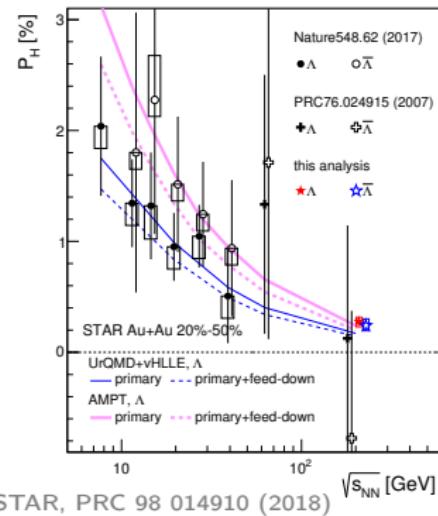
- Weak decay: $\Lambda \rightarrow p + \pi^-$ angular distr.: $dN/d\cos\theta = \frac{1}{2}(1 + \alpha|\vec{P}_H| \cos\theta)$
- Quark-gluon plasma is the "most vortical fluid ever observed"

$$\omega = (P_\Lambda + P_{\bar{\Lambda}})k_B T/\hbar \sim 10^{21} \text{ s}^{-1}$$

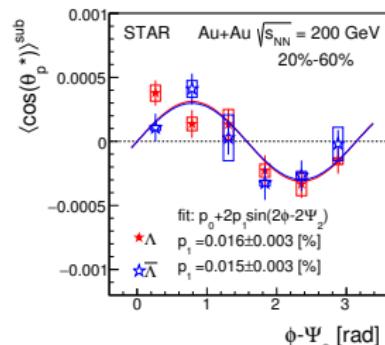
Great Red Spot of Jupiter 10^{-4} s^{-1}

Experiments vs theory: Λ polarization

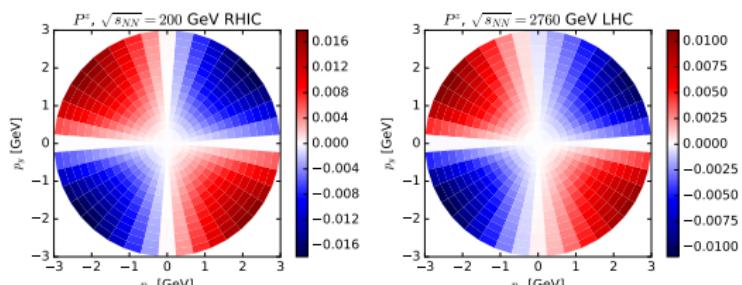
Global - along J



Longitudinal - along beam axis



J. Adam et al. [STAR Collaboration], PRL 123, 132301 (2019)



F. Becattini, I Karpenko, PRL 120, 012302

$$\Pi^\mu(x, p) \propto (1-n_F)\epsilon^{\mu\nu\rho\tau} p_\nu \varpi_{\rho\tau}$$

$$\varpi_{\rho\tau} = -\frac{1}{2}(\partial_\rho \beta_\tau - \partial_\tau \beta_\rho)$$

Becattini et al An. Phys. (2013)

- Theory assumes local equilibrium of spin degrees of freedom
- "Sign problem" between theory and experiments for longitudinal polarization!

Does spin play a dynamical role in hydro?

- ▶ Relativistic hydrodynamics is a good effective theory: $\partial_\mu T^{\mu\nu} = 0$

Goal: Relativistic hydrodynamics (classical) with spin (quantum) as dynamical variable

Florkowski, Friman, Jaiswal, ES, PRC 97, no. 4, 041901 (2018)

Florkowski, Friman, Jaiswal, Ryblewski, ES, PRD 97, no. 11, 116017 (2018)

Florkowski, Becattini, ES, Acta Phys. Polon. B 49, 1409 (2018)

Becattini, Florkowski, ES, PLB 789, 419 (2019)

Bhadury, Forkowski, Jaiswal, Kumar, Ryblewski, 2002.03937, 2008.10976 (2020)

Shi, Gale, Jeong, 2008.08618 (2020)

Starting point: Kinetic theory from quantum field theory

Weickgenannt, Sheng, ES, Wang, Rischke, PRD 100, no. 5, 056018 (2019)

Weickgenannt, ES, Sheng, Wang, Rischke, 2005.01506 (2020)

- ▶ Alternative approaches: Lagrangian formulation, entropy current

Montenegro, Tinti, Torrieri, PRD 96, 056012 (2017)

Montenegro, Torrieri, 2004.10195 (2020)

Hattori, Hongo, Huang, Matsuo, Taya, PLB795, 100 (2019)

Fukushima, Pu, 2010.01608 (2020)

Li, Stephanov, Yee, 2011.12318 (2020)

Anticipation of our results

Weickgenannt, ES, Sheng, Wang, Rischke, 2005.01506 (2020); ES, Weickgenannt, 2007.00138 (2020)

- ▶ How do we describe the orbital-to-spin angular momentum conversion in kinetic theory?

Nonlocal particle scatterings (finite impact parameter)

- ▶ And in hydrodynamics?

Antisymmetric part of energy-momentum tensor

Quantum kinetic theory

Wigner function - Quantum mechanics

"Quantum extension" of classical distribution function

$$W(x, p) = \int \frac{dy}{2\pi\hbar} e^{-\frac{i}{\hbar} p \cdot y} \psi^* \left(x + \frac{y}{2} \right) \psi \left(x - \frac{y}{2} \right)$$

Properties: $\int dp W(x, p) = |\psi(x)|^2$, $\int dx W(x, p) = |\psi(p)|^2$

Connected to probability!

- Expectation value of any operator \hat{A}

$$\langle \hat{A} \rangle = \int dx dp W(x, p) a(x, p)$$

Wigner function - Quantum field theory

$$W_{\chi\sigma}(x, p) = \int \frac{d^4 y}{(2\pi\hbar)^4} e^{-\frac{i}{\hbar} p \cdot y} \left\langle : \bar{\Psi}_\sigma \left(x + \frac{y}{2} \right) \Psi_\chi \left(x - \frac{y}{2} \right) : \right\rangle$$

- Dirac equation \implies Equation of motion for Wigner function

Elze, Gyulassy, Vasak, Ann. Phys. 173 (1987) 462

de Groot, van Leeuwen, van Weert, Relativistic Kinetic Theory. Principles and Applications

$$\left[\gamma \cdot \left(p + i \frac{\hbar}{2} \partial \right) - m \right] W(x, p) = \hbar \int \frac{d^4 y}{(2\pi\hbar)^4} e^{-\frac{i}{\hbar} p \cdot y} \left\langle : \rho \left(x - \frac{y}{2} \right) \bar{\psi} \left(x + \frac{y}{2} \right) : \right\rangle$$

$$\rho = -(1/\hbar) \partial \mathcal{L}_I / \partial \bar{\psi}, \mathcal{L}_I = \text{interaction Lagrangian}$$

\implies Boltzmann equation \implies Kinetic theory

$$p \cdot \partial W_{\chi\sigma}(x, p) = C_{\chi\sigma}$$

Polarization observable in heavy-ion collisions

- ▶ Polarization vector for particle with momentum p^μ (e.g. Λ -hyperon)

Becattini, 2004.04050; ES, Weickgenannt, 2007.00138; Tinti, Florkowski, 2007.04029

$$\Pi_\mu(p) = \frac{\hbar}{2m} \frac{\int d\Sigma_\lambda p^\lambda \mathcal{A}_\mu}{\int d\Sigma_\lambda p^\lambda \text{tr}[W]}$$

$\mathcal{A}_\mu = \text{tr}[\gamma_\mu \gamma_5 W]$, Σ_λ - Hypersurface

- ▶ Equilibrium

$$\Pi_\mu(p) = -\frac{\hbar^2}{8m} \epsilon_{\mu\nu\alpha\beta} p^\nu \frac{\int d\Sigma_\lambda p^\lambda f(1-f) \varpi^{\alpha\beta}}{\int d\Sigma_\lambda p^\lambda f}$$

$\varpi^{\alpha\beta} = -\frac{1}{2}(\partial^\alpha \beta^\beta - \partial^\beta \beta^\alpha)$ - Thermal vorticity

f - Distribution function

Becattini, Chandra, Del Zanna, Grossi, Annals. Phys. 338, 32 (2013)

What are nonequilibrium effects on $\Pi_\mu(p)$?

\hbar -expansion \iff Gradient expansion

Weickgenannt, ES, Sheng, Wang, Rischke, 2005.01506 (2020)

- ▶ Semiclassical expansion of Wigner function

$$W = W^{(0)} + \hbar W^{(1)} + \mathcal{O}(\hbar^2)$$

- ▶ Semiclassical and **nonlocal** expansion of collision kernel

$$C = C_{\text{local}}^{(0)} + \hbar C_{\text{local}}^{(1)} + \hbar C_{\text{nonlocal}}^{(1)} + \mathcal{O}(\hbar^2)$$

- ▶ Expansion **around equilibrium**

$$W - W_{\text{equilibrium}} = \delta W \sim \mathcal{O}(\hbar)$$

$$\mathcal{O}(\hbar) \sim \mathcal{O}(\partial)$$

Spin in phase space

Weickgenannt, ES, Sheng, Wang, Rischke, 2005.01506 (2020)

- ▶ In order to account for spin dynamics enlarge phase space

J. Zamanian, M. Marklund, and G. Brodin, NJP 12, 043019 (2010)

W. Florkowski, R. Ryblewski, and A. Kumar, Prog. Part. Nucl. Phys. 108, 103709 (2019)

- ▶ Introduce new phase-space variable \mathfrak{s}^μ

$$\mathfrak{f}(x, p, \mathfrak{s}) \equiv \frac{1}{2} [\bar{\mathcal{F}}(x, p) - \mathfrak{s} \cdot \mathcal{A}(x, p)]$$

- ▶ Components of Wigner fct. $\bar{\mathcal{F}} = m/p^2 \text{tr}[p \cdot \gamma W]$, $\mathcal{A}^\mu = \text{tr}[\gamma^\mu \gamma_5 W]$

$$\bar{\mathcal{F}} = \int dS(p) \mathfrak{f}(x, p, \mathfrak{s}) \quad \mathcal{A}^\mu = \int dS(p) \mathfrak{s}^\mu \mathfrak{f}(x, p, \mathfrak{s})$$

$$\text{with } dS(p) \equiv \frac{\sqrt{p^2}}{\sqrt{3}\pi} d^4 \mathfrak{s} \delta(\mathfrak{s}^2 + 3) \delta(p \cdot \mathfrak{s})$$

- ▶ Boltzmann equation

$$p \cdot \partial \mathfrak{f}(x, p, \mathfrak{s}) = m \mathfrak{C}[\mathfrak{f}]$$

All dynamics in one scalar equation!

Nonlocal collisions

Weickgenannt, ES, Sheng, Wang, Rischke, 2005.01506 (2020)

$$\mathfrak{C}[\mathfrak{f}] = \mathfrak{C}_{\text{local}}[\mathfrak{f}] + \hbar \mathfrak{C}_{\text{nonlocal}}[\mathfrak{f}]$$

- ▶ Long calculation \implies Intuitive result in low-density approximation:

$$\begin{aligned}\mathfrak{C}[\mathfrak{f}] &= \int d\Gamma_1 d\Gamma_2 d\Gamma' \mathcal{W} [\mathfrak{f}(x + \Delta_1, p_1, \mathfrak{s}_1) \mathfrak{f}(x + \Delta_2, p_2, \mathfrak{s}_2) - \mathfrak{f}(x + \Delta, p, \mathfrak{s}) \mathfrak{f}(x + \Delta', p', \mathfrak{s}')] \\ &\quad + \int d\Gamma_2 dS_1(p) \mathfrak{W} \mathfrak{f}(x + \Delta_1, p, \mathfrak{s}_1) \mathfrak{f}(x + \Delta_2, p_2, \mathfrak{s}_2)\end{aligned}$$

$$d\Gamma \equiv d^4 p dS(p)$$

- ▶ Structure: Momentum and spin exchange + Spin exchange only
- ▶ Nonlocal Collisions \implies Displacement $\Delta \sim \mathcal{O}(\hbar) \sim \mathcal{O}(\partial)$
- ▶ $\mathcal{W}, \mathfrak{W}$ vacuum transition probabilities, depend on phase-space spins

Equilibrium distribution function

Weickgenannt, ES, Sheng, Wang, Rischke, 2005.01506 (2020)

- ▶ Equilibrium condition: $\mathcal{C}[f] = 0$

- ▶ Ansatz for distribution function

F. Becattini, V. Chandra, L. Del Zanna, and E. Grossi, AP. 338, 32 (2013)

W. Florkowski, R. Ryblewski, and A. Kumar, Prog. Part. Nucl. Phys. 108, 103709 (2019)

$$f_{eq}(x, p, \mathfrak{s}) \propto \exp \left[\underbrace{-\beta(x) \cdot p}_{\text{Energy-momentum}} + \underbrace{\frac{\hbar}{4} \Omega_{\mu\nu}(x) \Sigma_{\mathfrak{s}}^{\mu\nu}}_{\text{Total angular momentum}} \right] \delta(p^2 - M^2)$$

- ▶ M - mass (possibly modified by interactions)
- ▶ $\beta^\mu = u^\mu / T$, Spin potential $\Omega^{\mu\nu}$
- ▶ Spin-dipole-moment tensor $\Sigma_{\mathfrak{s}}^{\mu\nu} \equiv -\frac{1}{m} \epsilon^{\mu\nu\alpha\beta} p_\alpha \mathfrak{s}_\beta$
- ▶ Insert into $\mathcal{C}[f]$ and expand up to $\mathcal{O}(\hbar)$

Condition for $\mathcal{C}[f] = 0 \implies$ Global equilibrium

$$\partial_\mu \beta_\nu + \partial_\nu \beta_\mu = 0 \quad \Omega_{\mu\nu} = -\frac{1}{2} (\partial_\mu \beta_\nu - \partial_\nu \beta_\mu) = \text{const.}$$

System gets polarized through rotations!

Equilibrium distribution function

Weickgenannt, ES, Sheng, Wang, Rischke, 2005.01506 (2020)

- ▶ Equilibrium condition: $\mathcal{C}[f] = 0$

- ▶ Ansatz for distribution function

F. Becattini, V. Chandra, L. Del Zanna, and E. Grossi, AP. 338, 32 (2013)

W. Florkowski, R. Ryblewski, and A. Kumar, Prog. Part. Nucl. Phys. 108, 103709 (2019)

$$f_{eq}(x, p, \mathfrak{s}) \propto \exp \left[\underbrace{-\beta(x) \cdot p}_{\text{Energy-momentum}} + \underbrace{\frac{\hbar}{4} \Omega_{\mu\nu}(x) \Sigma_{\mathfrak{s}}^{\mu\nu}}_{\text{Total angular momentum}} \right] \delta(p^2 - M^2)$$

- ▶ M - mass (possibly modified by interactions)
- ▶ $\beta^\mu = u^\mu / T$, Spin potential $\Omega^{\mu\nu}$
- ▶ Spin-dipole-moment tensor $\Sigma_{\mathfrak{s}}^{\mu\nu} \equiv -\frac{1}{m} \epsilon^{\mu\nu\alpha\beta} p_\alpha \mathfrak{s}_\beta$
- ▶ Insert into $\mathcal{C}[f]$ and expand up to $\mathcal{O}(\hbar)$

Condition for $\mathcal{C}[f] = 0 \implies \text{Global equilibrium}$

$$\partial_\mu \beta_\nu + \partial_\nu \beta_\mu = 0 \quad \Omega_{\mu\nu} = -\frac{1}{2} (\partial_\mu \beta_\nu - \partial_\nu \beta_\mu) = \text{const.}$$

System gets polarized through rotations!

Spin hydrodynamics

Spin hydrodynamics

ES, Weickgenannt, 2007.00138 (2020)

Energy-momentum tensor: $\hat{T}^{\lambda\nu}$

Total angular momentum tensor:

$$\hat{j}^{\lambda,\mu\nu} \equiv x^\mu \hat{T}^{\lambda\nu} - x^\nu \hat{T}^{\lambda\mu} + \hbar \hat{S}^{\lambda,\mu\nu}$$

Additional dynamical tensor: Spin tensor $\hat{S}^{\lambda,\mu\nu}$

- ▶ Hydrodynamic densities from quantum field theory

$$T^{\mu\nu} = \langle \hat{T}^{\mu\nu} \rangle \quad S^{\lambda,\mu\nu} = \langle \hat{S}^{\lambda,\mu\nu} \rangle$$

- ▶ **10 hydro eqs.:** 4 Energy-momentum + 6 Total angular momentum cons.

$$\partial_\mu T^{\mu\nu} = 0 \quad \hbar \partial_\lambda S^{\lambda,\mu\nu} = T^{[\nu\mu]}$$

$$T^{[\nu\mu]} = T^{\nu\mu} - T^{\mu\nu}$$

- ▶ **10 unknowns:** $\beta^\mu = u^\mu / T$ and $\Omega^{\mu\nu}$

- ▶ $T^{\mu\nu}$ and $S^{\lambda,\mu\nu}$ are NOT uniquely defined \implies Pseudo-gauge transformations
 - ▶ Canonical - Problem: Total spin is not a tensor for free fields
 - ▶ Solution: Hilgevoord, Wouthuysen, NP 40 (1963) 1

Spin hydrodynamics

ES, Weickgenannt, 2007.00138 (2020)

Energy-momentum tensor: $\hat{T}^{\lambda\nu}$

Total angular momentum tensor:

$$\hat{j}^{\lambda,\mu\nu} \equiv x^\mu \hat{T}^{\lambda\nu} - x^\nu \hat{T}^{\lambda\mu} + \hbar \hat{S}^{\lambda,\mu\nu}$$

Additional dynamical tensor: Spin tensor $\hat{S}^{\lambda,\mu\nu}$

- ▶ Hydrodynamic densities from quantum field theory

$$T^{\mu\nu} = \langle \hat{T}^{\mu\nu} \rangle \quad S^{\lambda,\mu\nu} = \langle \hat{S}^{\lambda,\mu\nu} \rangle$$

- ▶ 10 hydro eqs.: 4 Energy-momentum + 6 Total angular momentum cons.

$$\partial_\mu T^{\mu\nu} = 0 \quad \hbar \partial_\lambda S^{\lambda,\mu\nu} = T^{[\nu\mu]}$$

$$T^{[\nu\mu]} = T^{\nu\mu} - T^{\mu\nu}$$

- ▶ 10 unknowns: $\beta^\mu = u^\mu / T$ and $\Omega^{\mu\nu}$
- ▶ $T^{\mu\nu}$ and $S^{\lambda,\mu\nu}$ are NOT uniquely defined \implies Pseudo-gauge transformations
 - ▶ Canonical - Problem: Total spin is not a tensor for free fields
 - ▶ Solution: Hilgevoord, Wouthuysen, NP 40 (1963) 1

Spin hydro with Hilgevoord-Wouthuysen currents

Weickgenannt, ES, Sheng, Wang, Rischke, 2005.01506 (2020); ES, Weickgenannt, 2007.00138 (2020)

- ▶ Equations of motion from kinetic theory

$$\partial_\mu T_{\text{HW}}^{\mu\nu} = \int d\Gamma p^\nu \mathfrak{C}[\mathfrak{f}] = 0$$

$$\hbar \partial_\lambda S_{\text{HW}}^{\lambda,\mu\nu} = \int d\Gamma \frac{\hbar}{2} \Sigma_{\mathfrak{s}}^{\mu\nu} \mathfrak{C}[\mathfrak{f}] = T_{\text{HW}}^{[\nu\mu]}$$

$$\Sigma_{\mathfrak{s}}^{\mu\nu} \equiv -\frac{1}{m} \epsilon^{\mu\nu\alpha\beta} p_\alpha \mathfrak{s}_\beta$$

Spin hydro with Hilgevoord-Wouthuysen currents

Weickgenannt, ES, Sheng, Wang, Rischke, 2005.01506 (2020); ES, Weickgenannt, 2007.00138 (2020)

- ▶ Equations of motion from kinetic theory

$$\partial_\mu T_{\text{HW}}^{\mu\nu} = \int d\Gamma p^\nu \mathfrak{C}[\mathfrak{f}] = 0$$

$$\hbar \partial_\lambda S_{\text{HW}}^{\lambda,\mu\nu} = \int d\Gamma \frac{\hbar}{2} \Sigma_{\mathfrak{s}}^{\mu\nu} \mathfrak{C}[\mathfrak{f}] = T_{\text{HW}}^{[\nu\mu]}$$

$$\Sigma_{\mathfrak{s}}^{\mu\nu} \equiv -\frac{1}{m} \epsilon^{\mu\nu\alpha\beta} p_\alpha \mathfrak{s}_\beta$$

- ▶ Energy-momentum conserved in a collision

Spin-dipole $\Sigma_{\mathfrak{s}}^{\mu\nu}$ not conserved in nonlocal collisions $\Rightarrow T_{\text{HW}}^{[\nu\mu]} \neq 0$
 \Rightarrow Conversion between spin and orbital angular momentum

Spin hydro with Hilgevoord-Wouthuysen currents

Weickgenannt, ES, Sheng, Wang, Rischke, 2005.01506 (2020); ES, Weickgenannt, 2007.00138 (2020)

- ▶ Equations of motion from kinetic theory

$$\partial_\mu T_{\text{HW}}^{\mu\nu} = \int d\Gamma p^\nu \mathfrak{C}[\mathfrak{f}] = 0$$

$$\hbar \partial_\lambda S_{\text{HW}}^{\lambda,\mu\nu} = \int d\Gamma \frac{\hbar}{2} \Sigma_s^{\mu\nu} \mathfrak{C}[\mathfrak{f}] = T_{\text{HW}}^{[\nu\mu]}$$

$$\Sigma_s^{\mu\nu} \equiv -\frac{1}{m} \epsilon^{\mu\nu\alpha\beta} p_\alpha \mathfrak{s}_\beta$$

- ▶ Energy-momentum conserved in a collision

Spin-dipole $\Sigma_s^{\mu\nu}$ not conserved in **nonlocal collisions** $\Rightarrow T_{\text{HW}}^{[\nu\mu]} \neq 0$
 \Rightarrow Conversion between spin and orbital angular momentum

- ▶ $T_{\text{HW}}^{[\nu\mu]} = 0$:
 - (i) for **local collisions** (**spin** is collisional invariant)
 - (ii) in global equilibrium ($\mathfrak{C}[\mathfrak{f}] = 0$)
- ▶ **Nonlocal collisions** away from global equilibrium \Rightarrow Dissipative dynamics
- ▶ Nonrelativistic limit agrees with known results

Hess, Waldmann (1966); G. Lukaszewicz, Micropolar Fluids, Theory and Applications (1999)

Fluid gets polarized through rotation!

Spin hydro with Hilgevoord-Wouthuysen currents

Weickgenannt, ES, Sheng, Wang, Rischke, 2005.01506 (2020); ES, Weickgenannt, 2007.00138 (2020)

- ▶ Equations of motion from kinetic theory

$$\partial_\mu T_{\text{HW}}^{\mu\nu} = \int d\Gamma p^\nu \mathfrak{C}[\mathfrak{f}] = 0$$

$$\hbar \partial_\lambda S_{\text{HW}}^{\lambda,\mu\nu} = \int d\Gamma \frac{\hbar}{2} \Sigma_s^{\mu\nu} \mathfrak{C}[\mathfrak{f}] = T_{\text{HW}}^{[\nu\mu]}$$

$$\Sigma_s^{\mu\nu} \equiv -\frac{1}{m} \epsilon^{\mu\nu\alpha\beta} p_\alpha \mathfrak{s}_\beta$$

- ▶ Energy-momentum conserved in a collision

Spin-dipole $\Sigma_s^{\mu\nu}$ not conserved in **nonlocal collisions** $\Rightarrow T_{\text{HW}}^{[\nu\mu]} \neq 0$
 \Rightarrow Conversion between spin and orbital angular momentum

- ▶ $T_{\text{HW}}^{[\nu\mu]} = 0$:
 - (i) for **local collisions** (**spin** is collisional invariant)
 - (ii) in global equilibrium ($\mathfrak{C}[\mathfrak{f}] = 0$)
- ▶ **Nonlocal collisions** away from global equilibrium \Rightarrow Dissipative dynamics
- ▶ Nonrelativistic limit agrees with known results

Hess, Waldmann (1966); G. Lukaszewicz, Micropolar Fluids, Theory and Applications (1999)

Fluid gets polarized through rotation!

Polarization observable in heavy-ion collisions

- ▶ Polarization vector for particle with momentum p^μ (e.g. Λ -hyperon)

Becattini, 2004.04050; ES, Weickgenannt, 2007.00138; Tinti, Florkowski, 2007.04029

$$\Pi_\mu(p) = \frac{\hbar}{2m} \frac{\int d\Sigma_\lambda p^\lambda \mathcal{A}_\mu}{\int d\Sigma_\lambda p^\lambda \text{tr}[W]}$$

$\mathcal{A}_\mu = \text{tr}[\gamma_\mu \gamma_5 W]$, Σ_λ - Hypersurface

- ▶ Equilibrium

$$\Pi_\mu(p) = -\frac{\hbar^2}{8m} \epsilon_{\mu\nu\alpha\beta} p^\nu \frac{\int d\Sigma_\lambda p^\lambda f(1-f) \varpi^{\alpha\beta}}{\int d\Sigma_\lambda p^\lambda f}$$

$\varpi^{\alpha\beta} = -\frac{1}{2}(\partial^\alpha \beta^\beta - \partial^\beta \beta^\alpha)$ - Thermal vorticity

f - Distribution function

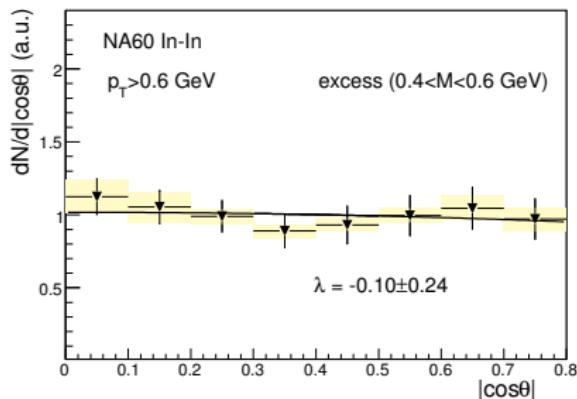
Becattini, Chandra, Del Zanna, Grossi, Annals. Phys. 338, 32 (2013)

What are nonequilibrium effects on $\Pi_\mu(p)$?

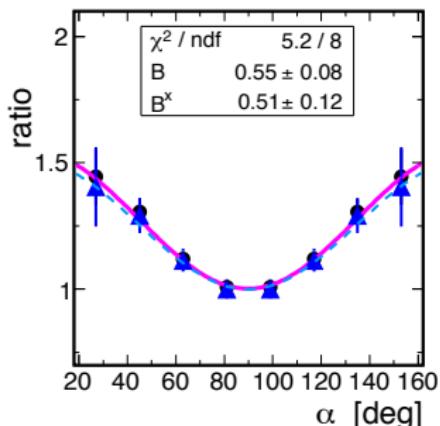
Vector meson polarization

Experimental results (NA60 and HADES)

Collins-Soper frame



Helicity frame



In-In at $158A$ GeV

NA60, PRL 96, 222301 (2009)

Ar-KCl at $1.76A$ GeV

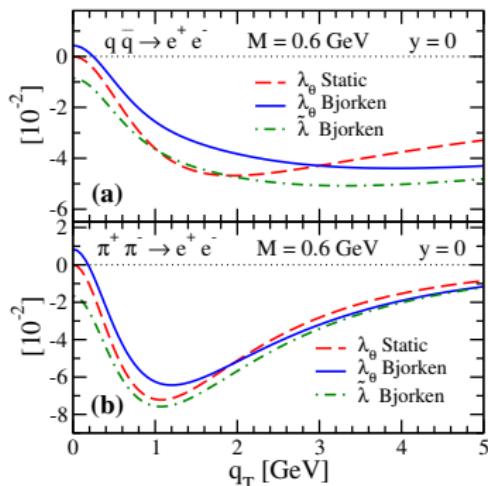
HADES, PRC 84, 014902 (2011)

- ▶ NA60: $\lambda_\theta \simeq 0$, but large error bars
- ▶ HADES: large polarization $\lambda_\theta \simeq 0.5$

Virtual photon polarization without vorticity

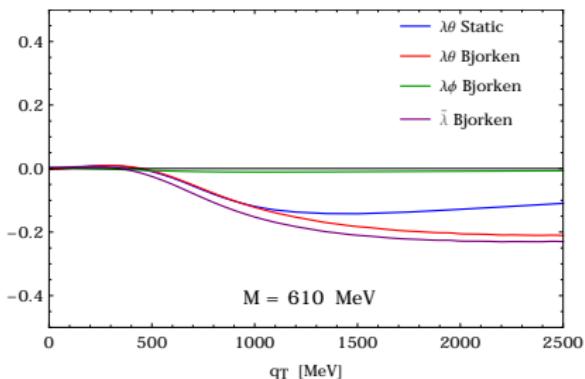
- Thermalized medium without rotation emits **tensor polarized** virtual photons
 $\gamma^* \rightarrow e^+ e^-$

$$\frac{d\Gamma}{d^4 q d\Omega_e} = \mathcal{N} \left(1 + \lambda_\theta \cos^2 \theta_e + \lambda_\phi \sin^2 \theta_e \cos 2\phi_e + \lambda_{\theta\phi} \sin 2\theta_e \cos \phi_e \right)$$



ES, Jaiswal, Friman, PLB 782, 395 (2018)

More realistic models



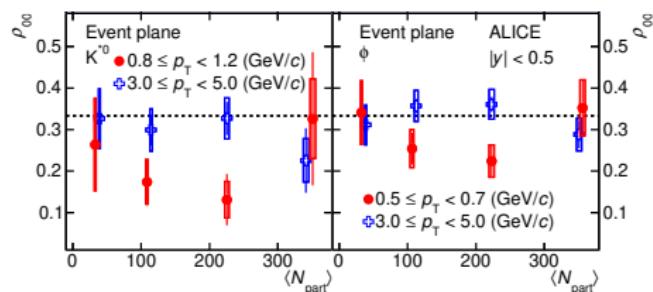
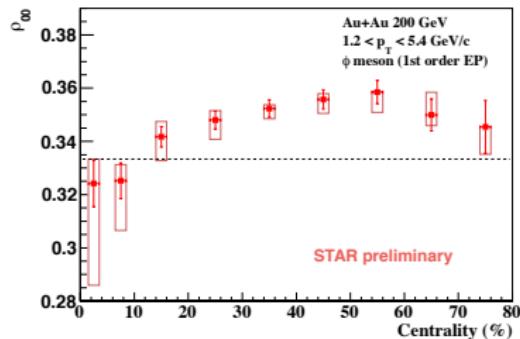
Friman, Galatyuk, Rapp, ES, van Hees, Wambach
(in preparation)

- Calculations in **helicity frame**
- What happens with **vorticity**?

ϕ and K^{*0} spin alignment

$$\lambda_\theta = \frac{3\rho_{00} - 1}{1 - \rho_{00}}$$

Unpolarized vector meson: $\rho_{00} = \frac{1}{3}$



ALICE, PRL 125, 012301 (2020)

C. Zhou (STAR), Proceedings of Quark Matter (2018)

- Theoretical prediction: $\rho_{00} \sim \frac{1}{3} - \frac{1}{9} \left(\frac{\hbar \omega}{k_B T} \right)^2$ Yang et al, PRC 97, 034917
From Λ -polarization $\frac{\hbar \omega}{k_B T} \sim 10^{-2} \Rightarrow \rho_{00} \sim \left(\frac{1}{3} - 10^{-4} \right) \neq \text{data}$
- Better theoretical models are needed!

Conclusions

- ▶ Vorticity and spin polarization are inherently connected
- ▶ New era where spin degrees of freedom must be taken into account in hydrodynamics/kinetic theory - New experimental observables
- ▶ Quantum field theory calculations suggest that spin hydrodynamics from kinetic theory with nonlocal collisions is always dissipative
- ▶ Vector meson spin alignment can help us better understand vorticity and spin dynamics in heavy-ion collisions
- ▶ Study nonequilibrium effects on spin observables