

Introduction to supersymmetry, the MSSM, and (a little bit) beyond

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Plan

1. Short review of motivations for supersymmetry (and unexpected bonus)
2. Ingredients and construction of supersymmetric Lagrangians
3. Essential of Minimal Supersymmetric Standard Model (MSSM)
4. Breaking supersymmetry: spontaneous, explicit, etc
5. Different popular SUSY-breaking models (minimal SUGRA, GMSB, AMSB, ...)
6. Some existing constraints on MSSM

1. Supersymmetry: Motivations

Supersymmetry: Poincaré + Fermions \leftrightarrow Bosons symmetry:

$$Q|F\rangle = |B\rangle, \quad Q|B\rangle = |F\rangle$$

numerous *independent* motivations + unexpected bonus

• Super-Poincaré: the largest possible symmetry (in 4-dim):

basic algebra (schematically):

$$\{Q, Q^\dagger\} \propto P_\mu; \quad [Q, P_\mu] = 0$$

“square-root” of translation: escape of 60’s no-go theorems

(Coleman-Mandula) for enlarged space-time+internal

symmetries [*space – time sym*] \otimes [*internal sym*]

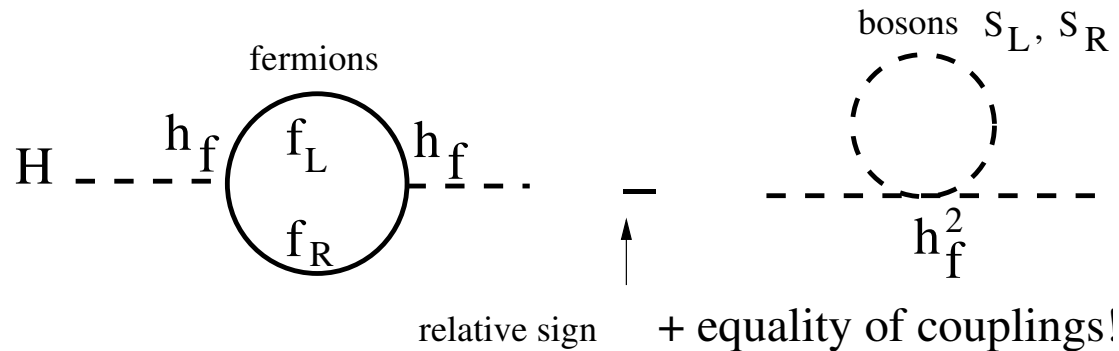
-if made a local symmetry, necessary ingredient of a quantum

gravity \rightarrow Supergravity etc

The “hierarchy” or naturalness problem

radiative corrections to Higgs mass: $\delta m_{Higgs}^2 \propto M_{GUT, Planck}^2 ??$

Stabilized!



$$\delta m_H^2 = \frac{N_c h_f^2}{16\pi^2} \left[-2M_{Pl}^2 + 3m_f^2 \ln \frac{M_{Pl}^2}{m_f^2} + 2M_{Pl}^2 - 2m_s^2 \ln \frac{M_{Pl}^2}{m_s^2} \right]$$

Moreover even the \ln terms cancel if m_f, m_s arise from sym. breaking ($m_f \sim h_f v = m_s$) (another graph then) exact SUSY \rightarrow equality of masses AND couplings.

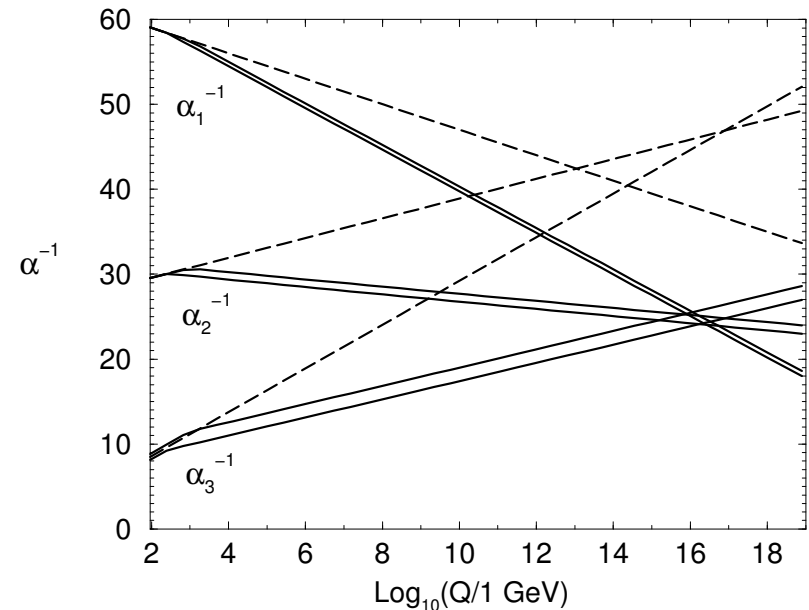
Broken SUSY: $m_f \neq m_s \rightarrow \ln$ terms survive: “fine-tuning” pb \rightarrow acceptable IF $m_{sparticles} \lesssim \mathcal{O}(1 \text{ TeV})$

NB origin of the large rad. corr. $\propto m t^4 \ln[.]$ to MSSM H mass

+Unexpected bonus (not original motivations but welcome)

- Grand Unification *consistent* with Proton lifetime limits

-Due to SUSY particle threshold
+ SUSY Renor Group Evol.
(totally excluded in SM)



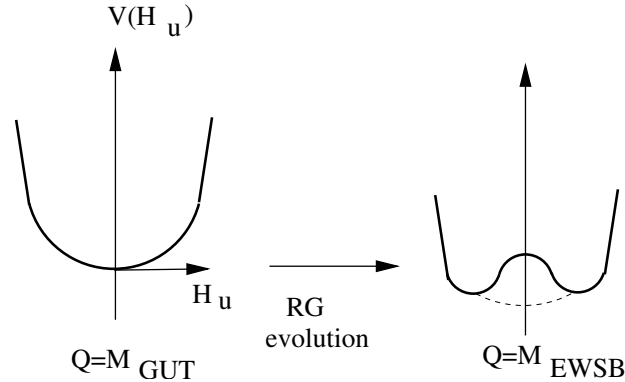
-Unification scale $M_{GUT} > 10^{16}$: large enough to escape
Proton decay limits (Superkamiokande) $\sim 1.9 \cdot 10^{33}$ years

-However, 1 – 2% mismatch $1 - \alpha_S(M_{GUT})/\alpha_1(M_{GUT})$:
hoped to be explained by GUT scale threshold corrections...

(but dim 5 operators can disturb this “conventional wisdom”!)

Another unexpected bonus..

- *Radiative* electro-weak sym. breaking: “mexican hat” scalar potential **induced** by **Renormalization Group (RG) evolution**: **GUT \rightarrow low energy**



$m_{H_u}^2(E) < 0$ by RG evolution $E_{GUT} \rightarrow E_{EWSB} (\propto m_t^2)$

made possible thanks to the large value of m_{top} !

(does not explain why m_{top} is large, though)

Yet another unexpected bonus...

- Very plausible candidate to Dark Matter (neutralino LSP)
present strong indication that $\sim 10\%$ of mass in universe is neutral, weakly interacting cold DM

But, problem: **SUSY has to be broken: what's the right model? :<...**

To date: **NO consistent model of spontaneous (or dynamical) SUSY-breaking!** (breaking has to be in a "hidden" sector)

→ proliferation of SUSY-breaking (arbitrary) parameters:
All possible gauge-invariant interactions between quite many (s)particles.. IF no more theoretical prejudices applied

2. Basics of supersymmetric gauge theories

- Supersymmetric extensions of SM follow the rules of (super)gauge theories:

based on two set of fields with specific gauge+susy transformations:

- Chiral fields: left-handed fermions + scalar partners

- Vector fields: vector gauge bosons + fermion (majorana) partners

- Right handed fermions: from charge conjugate representation of chiral fields: $(\psi_R)^c = (\psi^c)_L$

- Higgs field: described by chiral fields: \Leftrightarrow fermion partners

A bit of supersymmetric formalism

Basic ingredients: 2-components spinors $\chi_\alpha, \bar{\psi}^{\dot{\alpha}}$ $\alpha, \dot{\alpha} = 1, 2$

makes supersymmetric properties more manifest

may be contracted to form Lorentz-invariants:

$(\psi\chi) \equiv \psi^\alpha \chi_\alpha \equiv \psi^\alpha \epsilon_{\alpha\beta} \chi^\beta$, $\epsilon_{\alpha\beta}$ antisymmetric: $\epsilon_{12} = -\epsilon_{21} = 1$

Standard Dirac spinor (4-component object):

$$\Psi_D = \begin{pmatrix} \chi_\alpha \\ \bar{\psi}^{\dot{\alpha}} \end{pmatrix}, \quad \bar{\Psi}_D \equiv \Psi_D^\dagger \gamma_0 = (\psi^\alpha, \bar{\chi}^{\dot{\alpha}}), \quad \gamma_\mu = \begin{pmatrix} 0 & \sigma_\mu \\ \bar{\sigma}_\mu & 0 \end{pmatrix}$$

→ standard (Dirac) contraction e.g. $\bar{\Psi}_D \Psi_D = \psi\chi + h.c.$ etc

Majorana: $\Psi_M = \begin{pmatrix} \chi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix}$ i.e. such that $\Psi_M^c = \Psi_M$

Note $(\Psi_D)_L = \frac{1}{2}(1 - \gamma_5)\Psi_D = \chi_\alpha$, $(\Psi_D)_R = \frac{1}{2}(1 + \gamma_5)\Psi_D = \bar{\psi}^{\dot{\alpha}}$

Superspace formalism

Convenient: describe boson+ fermion by same “superfield”:
in addition to usual space coordinate x_μ , introduce new
anticommuting spinor variables $\theta_\alpha, \theta_{\dot{\alpha}}$

$$\theta_\alpha \theta_\beta = -\theta_\beta \theta_\alpha \rightarrow (\theta_\alpha)^2 = 0 \quad \text{but } \theta\theta \equiv \theta^\alpha \epsilon_{\alpha\beta} \theta^\beta \neq 0!$$

e.g chiral superfield (irreducible SUSY representation):

$$\Phi(x, \theta, \bar{\theta} = 0) = \phi(x) + \sqrt{2}\theta\psi(x) + \theta^2 F(x)$$

where ϕ scalar, ψ fermion, F scalar (auxiliary) fields

-Expansion stops at θ^2 due to anticommuting properties of θ

- F “scalar” has dim $[m]^2$ and NO kinetic term

(\Leftrightarrow function of other fields from its eq. of motion):

F assures (off-shell) matching of boson vs fermion d^0 freedom

Supersymmetric transformation of fields

Supersymmetry transformation = translation in superspace parameterized in terms of infinitesimal (Grassman) ζ
-SUSY generators expressed as derivative operators

$$Q_\alpha = -i\partial_\theta + \sigma^\mu \bar{\theta} \partial_\mu \quad (\text{analog of } P_\mu \rightarrow i\partial_\mu)$$

where extra terms originates from $\{Q, Q^\dagger\} \propto P_\mu$

Components of chiral field transform as

$$\delta\phi = \sqrt{2}\zeta\psi, \quad \delta F = -i\sqrt{2}\zeta\sigma^\mu\partial_\mu\psi$$

$$\delta\psi = -i\sqrt{2}\sigma^\mu\bar{\zeta}\partial_\mu\phi + \sqrt{2}\zeta F$$

Note F transforms as total derivative:

a basic ingredient for SUSY-invariant Lagrangians

Vector Superfield (are hermitian)

Similarly the vector superfield reads in the simplest gauge choice (so called Wess-Zumino):

$$V(x, \theta, \bar{\theta}) = -(\theta\sigma^\mu\bar{\theta})V_\mu + i\theta^2\bar{\theta}\bar{\lambda} - i\bar{\theta}^2\theta\lambda + \frac{1}{2}\theta^2\bar{\theta}^2 D$$

where V_μ usual vector field, λ its Majorana fermion partner, D auxiliary (scalar), with appropriate SUSY-transformations.

Again, auxiliary field D transforms as total derivative

There is also a chiral superfield, derived from V , generalizing "gauge field strength":

$$W_\alpha(x, \theta, \bar{\theta}) = -i\lambda_\alpha + (\theta\sigma_{\mu\nu})^\alpha F^{\mu\nu} + \theta^\alpha D - \theta^2(\bar{\sigma}^\mu\mathcal{D}_\mu\bar{\lambda})^\alpha$$

transforms like usual $V_{\mu\nu}$ under gauge symmetry

→ building blocks to construct SUSY-invariant Lagrangian.

Supersymmetric Lagrangian

Armed with this formalism, “straightforward” to construct SUSY- and gauge-invariant Lagrangians

$$\mathcal{L}_{SUSY} = \frac{1}{4g^2}(\text{Tr}[W^\alpha W_\alpha]_F + h.c) + \sum_i [\bar{\Phi} e^{(gV)} \Phi]_D + [W(\Phi)]_F$$

where $[\dots]_{F,D}$ means appropriate “projection”

$(\theta^2, \theta^2 \bar{\theta}^2$ coefficients resp.) that transform as total derivative.

$W(\Phi)$ superpotential = dim-3 gauge-invariant polynomial function of chiral field Φ :

$$W(\Phi) = c_i \Phi_i + \frac{m_{ij}}{2} \Phi_i \Phi_j + \frac{\lambda_{ijk}}{3!} \Phi_i \Phi_j \Phi_k$$

Scalar potential:

$$V(F_i, F_i^*, D^a) = \sum_i F_i^* F_i + \frac{1}{2} \sum_a (D^a)^2$$

$$F_i^* = \frac{\partial W(\Phi)}{\partial \Phi_i}, \quad D^a = -g \sum_i (\phi_i^* T^a \phi_i)$$

3. Minimal Supersymmetric Standard Model (MSSM) in short

Table 1: Chiral Supermultiplet of MSSM

(s)particles		spin 0	spin 1/2	$SU(3)_c, SU(2)_L, U(1)_Y$
squarks, quarks (x 3 families)	Q	$(\tilde{u}_L, \tilde{d}_L)$	(u_L, d_L)	$(3, 2, 1/6)$
	\bar{u}	\tilde{u}_R^*	u_R^\dagger	$(\bar{3}, 1, -2/3)$
	\bar{d}	\tilde{d}_R^*	d_R^\dagger	$(\bar{3}, 1, 1/3)$
sleptons, leptons (x 3 families)	L	$(\tilde{\nu}, \tilde{e}_L)$	(ν, e_L)	$(1, 2, -1/2)$
	\bar{e}	\tilde{e}_R^*	e_R^\dagger	$(1, 1, 1)$
Higgs, Higgsinos	H_u	(H_u^+, H_u^0)	$(\tilde{H}_u^+, \tilde{H}_u^0)$	$(1, 2, 1/2)$
	H_d	(H_d^0, H_d^-)	$(\tilde{H}_d^0, \tilde{H}_d^-)$	$(1, 2, -1/2)$

Table 2: Vector Supermultiplet of MSSM

(s)particles	spin 1/2	spin 1	$SU(3)_c, SU(2)_L, U(1)_Y$
gluino, gluon	\tilde{g}	g	$(8, 1, 0)$
Winos, W boson	$\tilde{W}^\pm, \tilde{W}^0$	W^\pm, W^0	$(1, 3, 0)$
Binos, B boson	\tilde{B}	B	$(1, 1, 0)$

MSSM Superpotential (R-parity conserving!)

$$W = \sum_{i,j=gen} -Y_{ij}^u \hat{u}_{Ri} \hat{H}_u \cdot \hat{Q}_j + Y_{ij}^d \hat{d}_{Ri} \hat{H}_d \cdot \hat{Q}_j + Y_{ij}^l \hat{l}_{Ri} \hat{H}_d \cdot \hat{L}_j + \mu \hat{H}_u \cdot \hat{H}_d ,$$

$\mathcal{L}_{SUSY} = \text{kin. terms (SUSY +gauge)} + F^2, D^2 \text{ terms} \propto \partial_{\phi_i} W, \text{ etc}$

Note at this (exact supersymmetric SM) stage:

$-m_{fermions} = m_{bosons}$? Yes, before EWSB, but all masses zero!

some amount of F/B mass diff. due to EWSB! (see later)

-quartic couplings determined by gauge couplings

-equality of fermion and boson couplings:

essential for cancellation of all quadratic UV div.

\Rightarrow only logarithmic div (wave fctn and gauge cpling

renormalization, superpotential $W(\Phi)$ NOT renormalized)

-Only new parameter: μ

Clearly unrealistic! must introduce supersymmetry breaking...

Digression: R-parity and its violation business

In MSSM, Higgs superfields H_u, H_d have same quantum numbers as leptons: \rightarrow SUSY+gauge-inv allow mixing:

$\mu^i L_i H_u, \lambda^{ijk} L_i L_j \bar{e}_k$ etc \rightarrow L -violation + ν -mass contributions !!

similarly trilinear quark terms allowed: $\bar{u} \bar{d} \bar{d} \rightarrow$ B -violation

Some couplings very constrained by rare decays, P decay, etc, but not all

\rightarrow introduce discrete symmetry: R-parity (Fayet 1976)

$$R = (-1)^{2s+3B+L}$$

$\rightarrow R_P(\text{matter fermions}) = +1, R_P(\text{all spartners}) = -1$

ensure that superpartners produced by pairs

lightest R_P -odd partner (LSP) stable (DM candidate)

Rk: R_P is discrete version of $U(1)$ R-sym in extended models

General (arbitrary) parameters of “soft” SUSY-breaking:

soft SUSY-breaking = that do not reintroduce quadratic UV divergences

• Mass Terms for Gluinos, Winos and Binos:

$$- \mathcal{L}_{\text{gaugino}} = \frac{1}{2} \left[M_1 \tilde{B} \tilde{B} + M_2 \sum_{a=1}^3 \tilde{W}^a \tilde{W}_a + M_3 \sum_{a=1}^8 \tilde{G}^a \tilde{G}_a + \text{h.c.} \right]$$

minimal SUGRA universality: $M_1(E_{GUT}) = M_2(E_{GUT}) = M_3(E_{GUT}) \equiv m_{1/2}$

• Mass terms for sfermions:

$$- \mathcal{L}_{\text{sfermions}} = \sum_{i=\text{gen}} m_{\tilde{Q}_i}^2 \tilde{Q}_i^\dagger \tilde{Q}_i + m_{\tilde{L}_i}^2 \tilde{L}_i^\dagger \tilde{L}_i + m_{\tilde{u}_i}^2 |\tilde{u}_{R_i}|^2 + m_{\tilde{d}_i}^2 |\tilde{d}_{R_i}|^2 + m_{\tilde{l}_i}^2 |\tilde{l}_{R_i}|^2$$

mSUGRA universality: $m_{\tilde{Q}_i}(E_{GUT}) = \dots = m_{\tilde{l}_i}(E_{GUT}) \equiv m_0$

Mass and bilinear terms for Higgs scalars:

$$-\mathcal{L}_{\text{Higgs}} = m_{H_u}^2 H_u^\dagger H_u + m_{H_d}^2 H_d^\dagger H_d + B\mu(H_u \cdot H_d + \text{h.c.})$$

mSUGRA universality: $m_{H_u}^2(E_{GUT}) = m_{H_d}^2(E_{GUT}) \equiv m_0^2$

- Finally, some trilinear interactions between scalars (sfermions and Higgs bosons):

$$-\mathcal{L}_{\text{tril.}} = \sum_{i,j=\text{gen}} \left[-A_{ij}^u Y_{ij}^u \tilde{u}_{R_i} H_u \cdot \tilde{Q}_j + A_{ij}^d Y_{ij}^d \tilde{d}_{R_i} H_d \cdot \tilde{Q}_j + A_{ij}^l Y_{ij}^l \tilde{l}_{R_i} H_d \cdot \tilde{L}_j + \text{h.c.} \right]$$

mSUGRA universality: $A_{ij}^u(E_{GUT}) = A_{ij}^d(E_{GUT}) = A_{ij}^l(E_{GUT}) \equiv A_0 \delta_{ij}$

Sparticle spectrum: • 5 Higgs scalars: h, H, H^\pm, A

• 2 Charginos: $\tilde{\chi}_{1,2}^\pm$; 4 neutralinos $\tilde{\chi}_{1-4}^0$, 1 gluino \tilde{g}

• Numerous sfermions: sleptons $(\tilde{e}, \tilde{\mu}, \tilde{\nu}_e, \dots, \tilde{\tau}_{1,2})$,

squarks: $(\tilde{u}, \tilde{d}, \dots, \tilde{b}_{1,2}, \tilde{t}_{1,2})$

An Extension of MSSM: N(ext)MSSM

- μ -parameter problem in MSSM:

$$\mu \stackrel{?}{\sim} M_{\text{susy}} \sim M_{\text{weak}}$$

- $\mu = 0$? (could be if "R-symmetry")

But experimentally excluded

- $\mu = M_{\text{Pl}}$ \longrightarrow "hierarchy" problem

- Solution: add a singlet S coupled to H_u, H_d

$$W_{\text{NMSSM}} = \cancel{\mu H_u H_d} + \lambda S H_u H_d + \frac{1}{3} \kappa S^3 \quad (+ \text{Yukawas})$$

After potential minimization: $\mu_{\text{eff}} \equiv \lambda \langle S \rangle \sim M_{\text{susy}}$

- $\lambda, \kappa \rightarrow 0, \mu_{\text{eff}} \neq 0$: MSSM + decoupled singlet sector

The NMSSM in short

- particle content:

- \tilde{S} : one more neutralino $\longrightarrow \tilde{\chi}_{i=1..5}^0$
- S_R : one more neutral (CP even) scalar $\longrightarrow h_{i=1,2,3}$
- S_I : one more (CP odd) scalar $\longrightarrow A_{i=1,2}$

\Rightarrow New physics beyond MSSM

- Parameters: $V_{\text{Higgs}} = V_F + V_D + V_{\text{soft}}$

$$V_{\text{soft}} = \left(\lambda A_\lambda H_u H_d S + \frac{1}{3} \kappa A_\kappa S^3 + \text{hc} \right) + m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 + m_S^2 |S|^2$$

+ 3 minimization conditions:

$$\mu_{\text{eff}} = \lambda \langle S \rangle, \quad \tan\beta = \frac{\langle H_u \rangle}{\langle H_d \rangle}, \quad M_Z^2 = \bar{g}^2 (\langle H_u \rangle^2 + \langle H_d \rangle^2)$$

\Rightarrow 6 free parameters: $\lambda, \kappa, A_\lambda, A_\kappa, \mu_{\text{eff}}, \tan\beta$

compared to MSSM: 2 free parameters ($m_A, \tan\beta$)

4. How to break supersymmetry?

Why is it so difficult to break SUSY *spontaneously*?

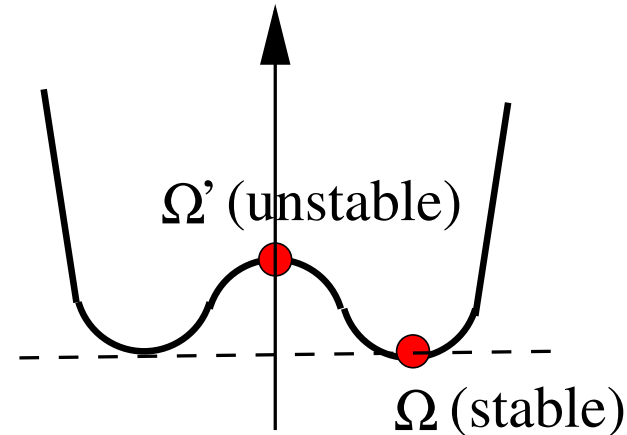
SUSY algebra involves the Hamiltonian: $H = P_0 = \sum Q_\alpha^2 \geq 0$

→ expect (in global SUSY)

$$\langle H \rangle_{\Omega \text{ supersymmetric}} = 0;$$

$$\langle H \rangle_{\Omega' \text{ non-supersymmetric}} > 0$$

$$V \sim \frac{1}{2} \sum (F^2 + D^2) > 0$$



from SUSY-transformation (schematically):

$$\delta\psi \sim (\sigma^\mu \partial_\mu \phi + F)\zeta, \quad \delta\lambda \sim (\sigma^\mu \sigma^\nu V_{\mu\nu} + D)\zeta$$

$\langle F \rangle$ and/or $\langle D \rangle \neq 0 \leftrightarrow \langle \delta\psi \rangle$ and/or $\langle \delta\lambda \rangle \neq 0$ spont. breaking
with sfermion ψ or gaugino λ Goldstone fermion resp.

(Analogy with usual SSB: $\delta\phi_2 = \theta\phi_1$, so $\langle \phi_1 \rangle \neq 0 \rightarrow \langle \delta\phi_2 \rangle \neq 0$)

Only way to get spontaneous SUSY-breaking:

look for models where $F_i = 0$ and/or $D^a = 0$ cannot be *simultaneously* satisfied for *any* field values.

Toy models do exist, but turn to be both

-contrived and exceptional situations

-phenomenologically unrealistic

(can't match SM gauge etc structure *and/or strongly*

already excluded e.g due to sparticle mass limits)

Toy models of spontaneous SUSY-breaking

-O’Raifeartaigh: (F-term breaking) superpotential W such that

$$V = |m\phi_1|^2 + |\lambda(\phi_1^2 - a^2)|^2 + |m\phi_2 + 2\lambda\phi_3\phi_1|^2$$

(ϕ_i are chiral superfields)

immediate that the first two terms can’t be *both* zero \rightarrow SSSB.

More precisely if $|m|^2 > 2|\lambda^2 a^2|$ global min at $\phi_1 = \phi_2 = 0$;
 $\rightarrow \langle F_3 \rangle \neq 0$: *flat* direction along ϕ_3 (so-called “moduli” field)

SUSY-breaking manifests as **fermion ψ_1 mass m**

while ϕ_1^+, ϕ_1^- mass $m^2 \pm 2\lambda^2 a^2$.

However note the sum rule (a generic feature):

$$m_{\phi_1^-}^2 + m_{\phi_1^+}^2 = 2m_{\psi_1}^2 \quad \text{just like exact SUSY...}$$

Clearly excluded in MSSM!

D-term spontaneous SUSY-breaking

Fayet-Iliopoulos model: for $U(1)$ gauge symmetry

$$V = |mQ|^2 + |m\bar{Q}|^2 + \frac{1}{8}|Q^\dagger Q - \bar{Q}^\dagger \bar{Q} + 2\kappa_{FI}|^2, \quad Q, \bar{Q} \text{ chiral Sfields.}$$

Linear term in κ_{FI} allows SSSB (for $m^2 > \kappa_{FI}/2$):

only OK for $U(1)$ (non-abelian sym: no invariant linear term!).

-maybe possible for extra $U(1)$ beyond SM: Z' models

(Still, not sufficient for realistic MSSM spectrum)

-Note D-term and F-term present in MSSM:

some $m_F \neq m_B$ amount triggered by EWSB...

(e.g. in sfermions mass terms) but not consistent alone

(tachyonic and/or obviously excluded) sfermion masses

typically \rightarrow MSSM really needs soft terms!

\rightarrow SUSY-breaking in hidden sector, communicated to SM

5. Generic features of hidden sector SUSY-breaking

Analogy with EWSB in SM: parameterized by $\langle v \rangle$

EWSB sector	Mediating interactions (= Yukawa couplings)	Observable sector
$h \rightarrow \langle v \rangle$	h, q, l	q, l

"Hidden" SUSY-breaking sector	Mediating interactions	Observable sector
$Z \rightarrow \langle F \rangle$	Z, Q, L	Q, L

SUSY-breaking parameterized by $\langle F \rangle$ of dim $[m]^2$

3 popular patterns: gravity-, gauge-, and anomaly-mediated

Actually all appear in a complete Supergravity picture!

Distinction arise from assumption on dominant mechanisms

Gravity-mediated susy breaking (minimal SuperGRAvity)

Start from Supergravity with “Kähler potential” $K(\phi, \phi^*)$
(Non-renormalizable terms) \rightarrow suppressed by $1/M_{Planck}$
 \rightarrow soft terms of order $\sim \langle F \rangle / M_{Planck}$ when $Z \rightarrow \langle F \rangle$:

$$c_{ij} \frac{Z^\dagger Z}{M_{Planck}^2} \phi_i^* \phi_j \rightarrow m_0^2 \text{ scalar masses}$$
$$c_a \frac{Z}{M_{Planck}} \lambda_a \lambda_a \rightarrow m_{1/2} \text{ gaugino masses}$$
$$c_{ijk} \frac{Z}{M_{Planck}} \phi_i \phi_j \phi_k \rightarrow A_0 \text{ trilinear terms}$$

$F \sim M_{weak} M_{Planck} \sim [10^{10} GeV]^2$: high scale SUSY-breaking
(but $\langle F \rangle$ may also be triggered by gaugino condensation)

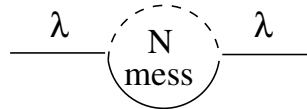
-Caution: famous universality in mSUGRA comes from minimal assumptions on Kähler and Super potential
(i.e. *separable* hidden/visible $K(\phi, \phi^*)$, $W(\phi)$ contributions)

Non-universal terms are there in more general scenario...

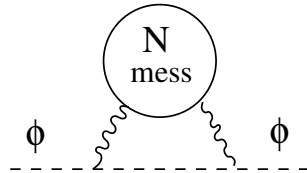
Gauge-mediated SUSY-breaking (GMSB)

Add N “messenger” Q, L heavy fields with mass M_{mess} and SUSY-breaking vev $\langle F \rangle$ that couple to SM gauge fields

$$M_\lambda \sim N \frac{g^2}{16\pi^2} \frac{\langle F \rangle}{M_{mess}}$$



$$m_\phi^2 \sim N \left[\frac{g^2}{16\pi^2} \right]^2 \left[\frac{\langle F \rangle}{M_{mess}} \right]^2$$



Trilinear terms $A_i(M_{mess}) \sim 0$ (2-loop; but much suppressed)

choose $M_{mess} \ll M_{Planck}$:

$\frac{F}{M_{mess}} \gg \frac{F}{M_{Planck}} \rightarrow$ gravity-mediated contributions negligible

Scalar masses determined by gauge quantum nbs:

solve SUSY flavor pb

Low scale SUSY breaking $F \sim M_{mess}^2$, $\sqrt{F} \sim 10^4$ GeV

but $10^4 \text{ GeV} \lesssim M_{mess} \lesssim 10^{14} \text{ GeV}$ possible

NB LSP can be (very light) gravitino: $M_{3/2} \sim \langle F \rangle / M_{Planck}$

Anomaly-mediated SUSY-breaking (AMSB)

The anomaly (symmetry breaking at quantum level) of the (super)conformal symmetry induces soft SUSY breaking!
NB was always present; but assumed sub-dominant (loop-suppressed) in standard "mSUGRA"

gauginos: $M_i \sim b_i \frac{g_i^2}{16\pi^2} M_{3/2}$ $b_i(\text{RGE}) = (33/5, 1, -3)$

squarks, sleptons: $(m^2)_j^i \sim (\dot{\gamma})_j^i \left[\frac{M_{3/2}}{16\pi^2} \right]^2$; also $A_i \sim \frac{M_{3/2}}{16\pi^2}$

γ_j^i standard RGE anomalous mass dimensions

e.g. $\gamma_Q = -Y_u^\dagger Y_u - Y_d^\dagger Y_d + \sum_i c_i g_i^2$

Almost flavor blind!

But generally tachyonic $\tilde{l}_L, \tilde{l}_R \rightarrow$ add a m_0 term by hand...

however some recent criticisms (e.g. Dine+Seiberg '07)

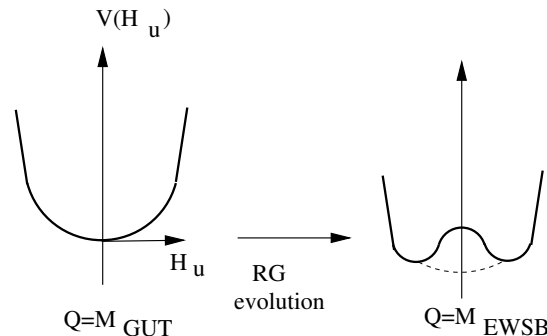
perhaps more consistent " m_0 " terms will soon emerge??..

6. Some constraints on MSSM

Unescapable constraint: *consistent* electro-weak symmetry breaking (EWSB) $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$

In MSSM: *produced* by RG evolution of $m_{H_u}^2(E)$, $m_{H_d}^2(E)$:

\sim **OK** if $m_{H_u}^2(E) < 0$ by **RG evolution** $E_{GUT} \rightarrow E_{EWSB} (\propto m_t^2)$



AND $|\mu|$ **determined** by minimization of the scalar potential:

$$2\mu^2 = \tan(2\beta)(\hat{m}_{H_u}^2 \tan\beta - \hat{m}_{H_d}^2 \cot\beta - M_Z^2)$$

$$2B\mu = \sin 2\beta (\hat{m}_{H_u}^2 + \hat{m}_{H_d}^2 + 2\mu^2)$$

$$\tan\beta \equiv \frac{v_u}{v_d}, \quad \hat{m}_{H_i}^2 = m_{H_i}^2 + \partial_{v_i} V_{loop}^{eff}(m_{particles}, \mu)$$

\rightarrow **not always consistent solution for μ** \rightarrow **excluded domains**

μ very sensitive to rad. corr., m_t, \dots via Renorm. Group Evolution (RGE):

$$\frac{d(m_{H_u}^2)}{d \ln E} \propto m_t^2 (m_{H_u}^2 + \dots)$$

and $\mu^2 \sim -m_{H_u}^2 - m_Z^2/2$ (for $\tan \beta \gg 1$),

• μ enters everywhere in MSSM spectrum:

Higgses, $\tilde{\chi}^\pm, \tilde{\chi}^0$ (via Higgsinos \tilde{H}_u, \tilde{H}_d), \tilde{q}, \tilde{l} (via mixing)

Also: "CCB" minima (Charge and/or Color breaking)

deeper than electroweak min. can appear

(CCB domains to exclude e.g if trilin. cpling A_i too large)

Ingredients of spectrum calculation in MSSM

–for example **SuSpect 2.35** (A. Djouadi, JLK, G. Moultaka)

•Low energy input $\alpha(M_Z), \alpha_S(M_Z), M_t^{\text{pôle}}, M_\tau^{\text{pôle}}, m_b^{\overline{\text{MS}}}(m_b); \tan \beta(M_Z)$

via radiative corrections $\Rightarrow g_{1,2,3}^{\overline{\text{DR}}}(M_Z), Y_\tau^{\overline{\text{DR}}}(M_Z), Y_b^{\overline{\text{DR}}}(M_Z), Y_t^{\overline{\text{DR}}}(M_Z)$

•Choice of SUSY-breaking model (mSUGRA, GMSB, AMSB,...)

Fixes initial condition at high energy (mSUGRA: $m_0, m_{1/2}, A_0, \text{sign}(\mu)$, etc...).

•Evolution of parameters by RGE down to $M_{\text{EWSB}} \sim \mathcal{O}(100\text{GeV} - 1\text{TeV})$

•Control of EWSB consistency (convergence of μ , no CCB minima, etc...)

•Diagonalisation of mass mixing matrices and pole mass calculation (Including Rad. Corrections for Higgses, sfermions, gauginos)

Experimental Constraints on MSSM

- previous LEP limits on sparticle masses:

$$m_{\chi_1^+} \gtrsim 104 \text{ GeV}$$

$$m_{\tilde{\tau}^\pm} \gtrsim 100 \text{ GeV}$$

$$m_{\tilde{t}_1, \tilde{b}_1} \gtrsim 100 \text{ GeV}$$

(Latest TeVatron limits: see Laurent Duflot presentation!!)

Direct (LEP) limits on Higgs mass:

$$M_h \gtrsim 114 \text{ GeV}$$

(but th. uncertainty on M_h : $\sim 3 \text{ GeV}$)

-not valid if A light: $\rightarrow M_{h,A} \gtrsim 90 \text{ GeV}$ (limits from $e^+e^- \rightarrow hA$)

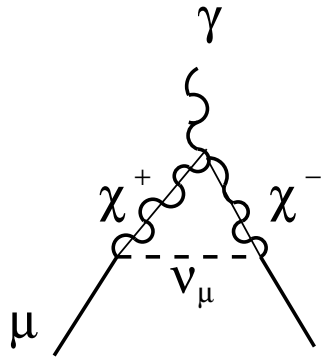
Indirect constraints: from virtual SUSY contributions:



- $g_\mu - 2$ constraints: SUSY loop contributions

Charginos+ sneutrino (leading);

(Also Neutralinos + smuon)



Standard Model (SM) contributions: hadron vacuum polarization (from dispersion relation: $\sigma(e^+e^-)$, τ decays)

Recent re-emergence of a $2-3\sigma$ -discrepancy

if taking only $\sigma(e^+e^-)$ data (see Fabio Zwirner's talk)

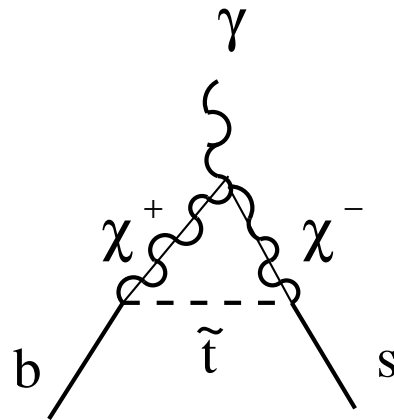
$$\text{approx.: } 10.6 \cdot 10^{-10} < \Delta a_\mu^{SUSY} < 43.6 \cdot 10^{-10}$$

→ Rather constraining: ($\mu < 0$ not favored)

- $b \rightarrow s\gamma$ constraints:

SM contributions: W^\pm and t essentially

SUSY contributions: Charginos + stops; H^+ + top



+ potentially large NLO contributions

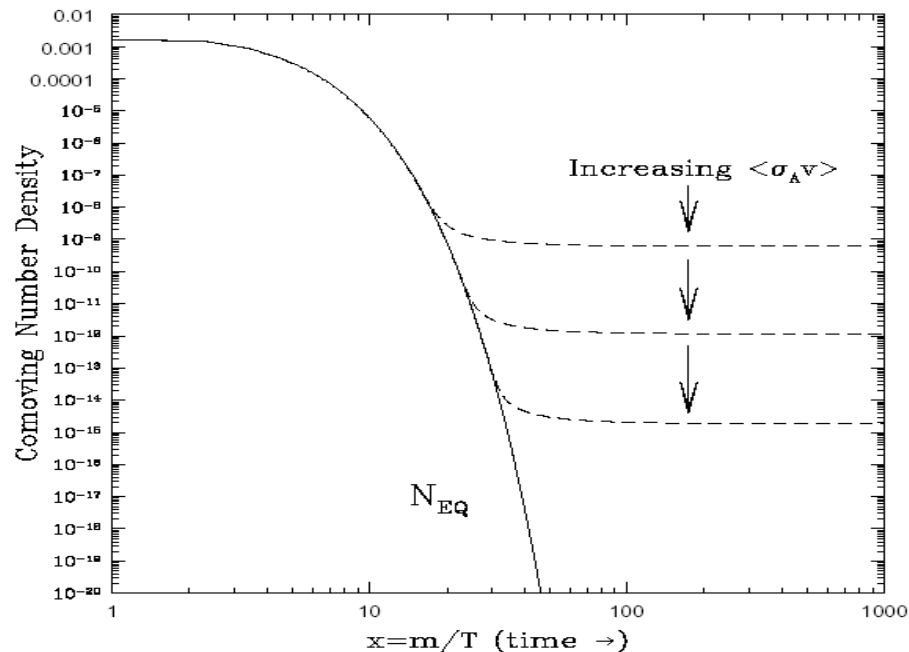
IF enhanced by large $\tan \beta$ and/or $\ln(m_{sparticles}/M_W)$

e.g. approx.: $2.65 \leq 10^4 \cdot B.R.(b \rightarrow s\gamma) \leq 4.45$

+ constraint on amplitude sign! (\simeq constraints on

$BR(b \rightarrow sl^+l^-)$ (i.e. requires SM sign)

Dark Matter relic density constraints:



- In early universe, “WIMP” (χ^0) are in thermal equilibrium
- As universe expanded and cool down, their density reduced through pair annihilation
- Eventually, density too low for annihilation to keep up with expansion rate: $\rightarrow T_{Freeze-out}$ (i.e. χ^0 decouple from SM)

$$\frac{dn}{dt} = -3Hn - \langle\sigma v\rangle[n^2 - n_{eq}^2]$$

Experimental (WMAP) evidence for Ωh^2

WMAP: $0.087 < \Omega h^2 < 0.138$: conservative (99% C.L.)

IF LSP = χ_0 : $\Omega_\chi h^2 \equiv$ relic density $\sim 3 \cdot 10^{-27} \text{ cm}^3 \text{ s}^{-1} \times$
 $[\langle \sigma(\chi_0 \chi_0 \rightarrow \text{all}) + \text{co-annihilation processes} \rangle]^{-1}$

$\rightarrow \sigma$ large $\rightarrow \Omega h^2 \ll .1$ too small;
 σ small $\rightarrow \Omega h^2$ too large

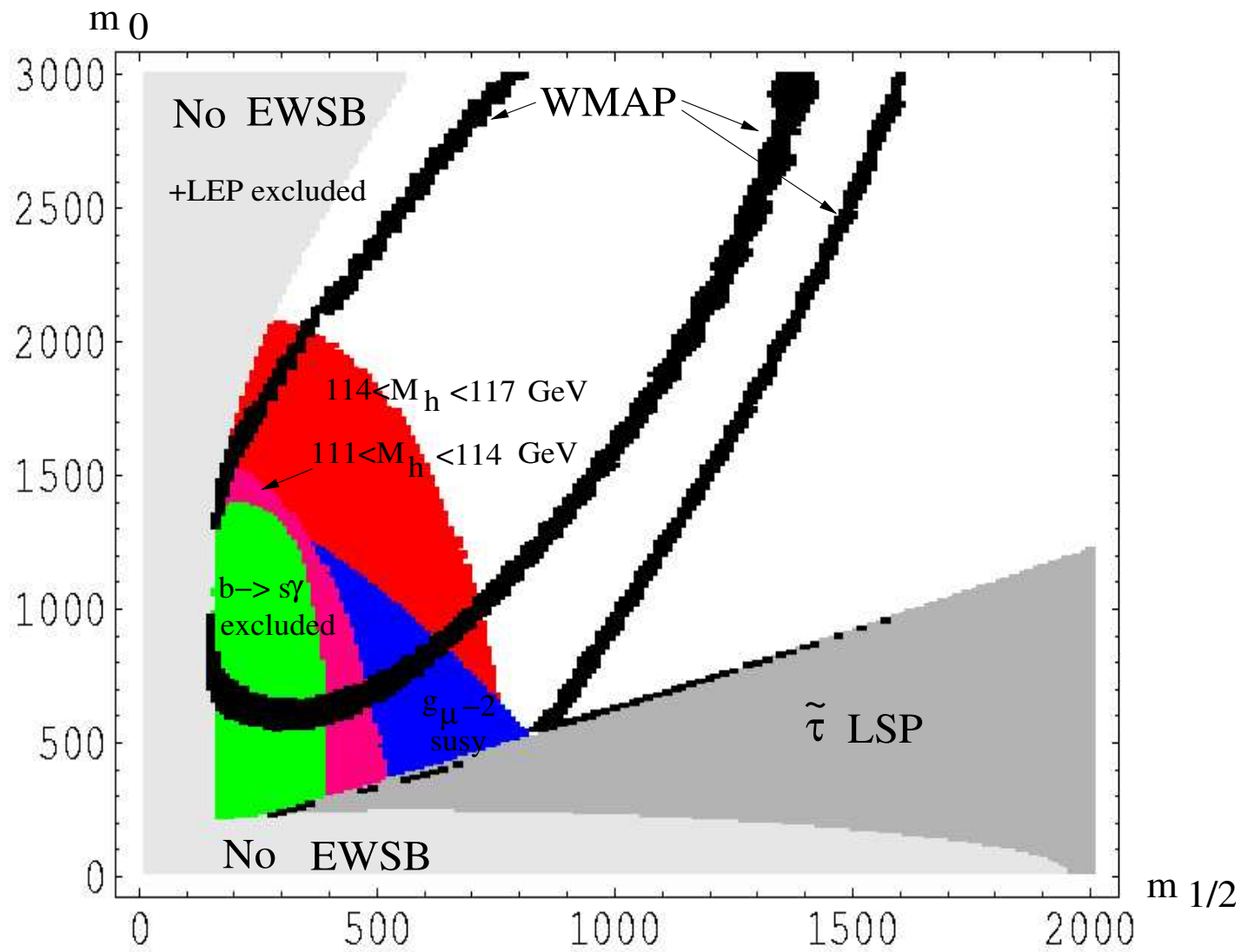
NB over 3000 processes $\sigma(\chi_0 \chi_0 \rightarrow \dots)$ can contribute!

But most relevant contributions depend on nature of LSP χ_1^0 :

$$\chi_1^0 = N_{11} \tilde{B} + N_{12} \tilde{W}_3 + N_{13} \tilde{H}_d + N_{14} \tilde{H}_u$$

e.g for $M_1 \ll \mu$, χ_1^0 is mainly "Bino", etc

Example of constraints in mSUGRA



$A_0 = 0, \tan \beta = 50, m_t = 173$ GeV. [Djouadi, Drees, JLK '06]

Summary

-Problem of SUSY-breaking: no final convincing model
-would be better guide if a truly consistent picture of *dynamical* SUSY-breaking

-about 35 years of “waiting for SUSY”: shall we start skepticism?

(and she missed already some rendez-vous:
LEP1,2,TeVatron,..)

-embarrassing fine tuning pbs, what if the spartners are very heavy, etc...

-embarrassing flavor mixing, R-parity, etc

-On aimerait bien surfer avec SUSY sur la vague LHC!

LHC should guide our prejudices on SUSY-breaking models