Microscopic description of ^{76}Ge $\beta\beta$ decay

Spote Alexandra
Tutor : Frédéric Nowacki

University of Strasbourg, IPHC

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- Matrix element

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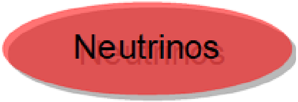
Context



2 issues :

$\rightarrow$ Which mass hierarchy is satisfied ?

Mass hierarchies

Schrödinger's equation

$\mathcal{H}\Psi = E\Psi \implies \Psi =$ eigenstates

eigenstates : flavor $\neq$ mass ?

- flavor eigenstates $\neq$ mass eigenstates
- type of neutrinos, observable $\neq$ purely mathematical object

$\rightarrow$ linked by the matrix PMNS :

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}$$

mass eigenstates $\leftrightarrow$ mass

Neutrino mass ?

= mixing of m_1, m_2, m_3, associated to mass eigenstates

Mass hierarchies

Oscillation probability $\rightarrow \Delta m^2$

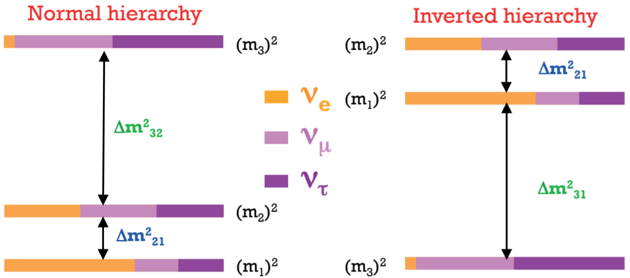
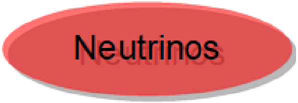


Figure: https://sfp2015.sciencesconf.org/74897/document

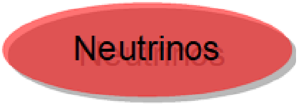
Contexte



2 issues :

- Which mass hierarchy is satisfied ?
- What is the nature of neutrinos ?

Contexte



2 issues :

- Which mass hierarchy is satisfied ?
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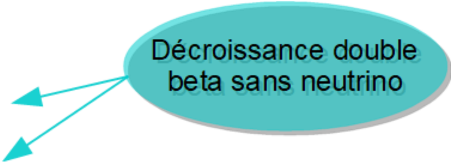
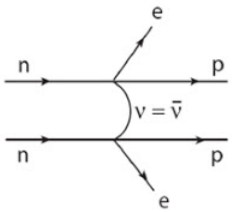


Neutrinoless double beta decay



$$^{A}_{Z}X_{N} \longrightarrow \, ^{A}_{Z+2}Y_{N-2} + 2e^{-}$$

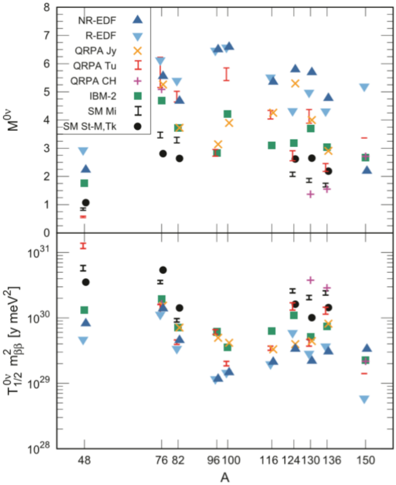
Experimental observation ?

theoretical decay rate :

$$\begin{aligned}[T^{0\nu}_{1/2}]^{-1} &= G_{0\nu}|M_{0\nu}|^{2}m^{2}_{\beta\beta} \\ &= G_{0\nu}|M^{GT}_{0\nu} + M^{F}_{0\nu} + M^{T}_{0\nu}|^{2}m^{2}_{\beta\beta}\end{aligned}$$

Theoretical predictions depend on Models

Theoretical predictions



Extrait de : J.Engel and J.Menéndez 2017 Rep.Prog. Phys. 80 046301

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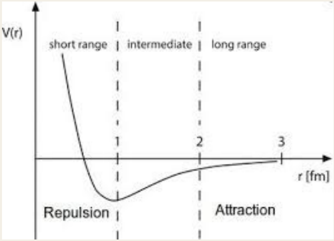
Basic notions : nucleus

Nucleus



- Atomic nucleus
  = many-body system, high complexity
- Nuclear force ?
  $\rightarrow$ not explicitly known

Basic notions : deformation

Collective motion

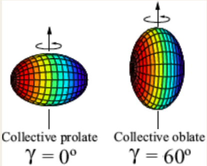
Rotations, vibrations (monopole, quadrupole, etc.), Giant resonances ...
$\rightarrow$ Deformation :

$$R(\theta,\phi) = R_0 \left[1 + \sum_{\lambda=0}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \alpha^*_{\lambda\mu} Y_{\lambda\mu}(\theta,\phi) \right]$$

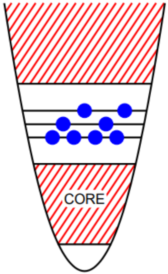
Hill-Wheeler parameters β, γ:

$$a_0 = \beta \cos\gamma \qquad a_2 = \frac{1}{\sqrt{2}}\beta \sin\gamma$$

$$\sum_\mu |\alpha_{2\mu}|^2 = \sum_\mu |\alpha'_{2\mu}|^2 = a_0^2 + 2a_2^2 = \beta^2$$



Microscopic Shell Model



neutrons + protons move freely in a central potential
satisfy Pauli principal

- external space
- valence space
  = supposed to give properties of nuclei
- core = supposed to be inert

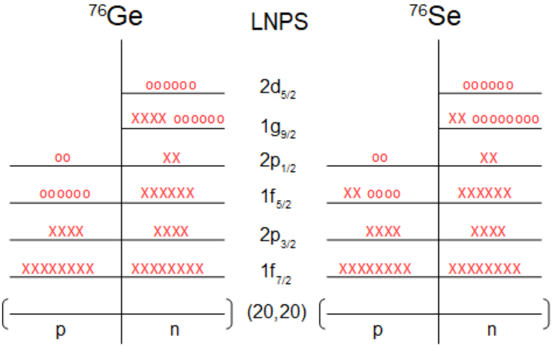
Theoretical approach

To study ^{76}Ge and ^{76}Se nuclei :

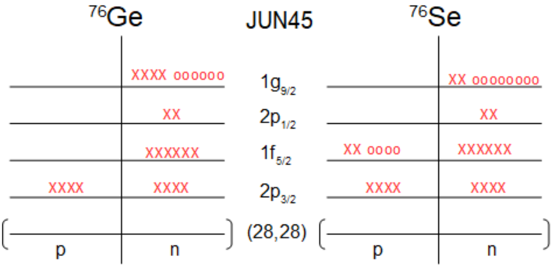
- Define a valence space
- Define an effective interaction such that :
  $\mathcal{H}\Psi = E\Psi \rightarrow \mathcal{H}^{eff}\Psi^{eff} = E\Psi^{eff}$
  For operators : multiplication of proton and neutron parts by effective charges

Valence space-interaction

Interactions : LNPS and JUN45.



Valence space-interaction



Theoretical approach

To study ^{76}Ge and ^{76}Se nuclei :

- Define a valence space
- Define an effective interaction such that :
  $\mathcal{H}\Psi = E\Psi \rightarrow \mathcal{H}^{eff}\Psi^{eff} = E\Psi^{eff}$
  For operators : multiplication of proton and neutron parts by effective charges
- Diagonalize the hamiltonian matrix to get E and Ψ^{eff} $\rightarrow$ ANTOINE code

ANTOINE code

It uses Lanczos method to diagonalize the hamiltonian :

$$\mathcal{H}\left|n\right\rangle = E_{n,n-1}\left|n-1\right\rangle + E_{n,n}\left|n\right\rangle + E_{n,n+1}\left|n+1\right\rangle$$

with :

- $E_{n,n-1} = E_{n-1,n}$ by hermiticity of $\mathcal{H}$
- $E_{n,n} = \left\langle n\right|\mathcal{H}\left|n\right\rangle$
- $E_{n,n+1} = \mathcal{H}\left|n\right\rangle - E_{n,n}\left|n\right\rangle - E_{n,n-1}\left|n-1\right\rangle$
- All terms E_{ij} such that $|i-j| > 1$ are zero.

The obtained matrix = tridiagonal, symmetric in basis $|n>$

$\Longrightarrow D_{(H)} = U^{-1}HU$ in basis Ψ^{eff}

Computing time

ANTOINE code = iterative methode to compute E et Ψ (Lanczos method)

Nuclei of reference

Ni56 (Z=28,N=28) : NZME = 955511317808 $\Longrightarrow$ $3437s \approx 1h$

	t	0	2	4	6	8
^{76}Ge	time	0.0018	0.062	44.02	1019.78 (16 min)	7542.76 (2 hours)
	NZME	517 026	17 360 268	12 237 904 794	283 507 139 576	2 096 943 456 180
^{76}Se	time	0.00074	0.24	24.23	35.96	3432.10 (57 min)
	NZME	206 498	65 558 946	6 736 575 108	9 996 648 224	954 149 961 758

Analysis of 76Ge and 76Se spectroscopic properties

Spectroscopic properties

- E*(2+) : Excitation energy from 0+ to 2+ states
- B(E2) : 2+ to 0+ transition probability
- Q_{spec} : describes non spherical charge distribution in the nucleus
- deformation (β, γ)

		^{76}Ge			^{76}Se		
	(e_p, e_n)	E*(2+)	B(E2)	Q_{spec}	E*(2+)	B(E2)	Q_{spec}
JUN45	0.8, 1.8	0.745	540.9	3.97	0.574	676.65	49.46
	1.1, 1.5	-	568.3	5.56	-	717.42	51.05
LNPS	0.46, 1.31	0.529	534.6	-14.21	0.544	488.28	32.95
Expt.	-	0.563	554.54	-19.6	0.559	838.14	-35.4

Table 3.4 – Excitation energies (MeV), E2 transition probabilities ($e^2 fm^4$) and quadrupole moments ($e fm^2$) for ^{76}Ge and ^{76}Se elements for LNPS and for different effective charges (e_n, e_p) for JUN45 interaction. Experimental values are extracted from [2][3][4][5].

Deformation

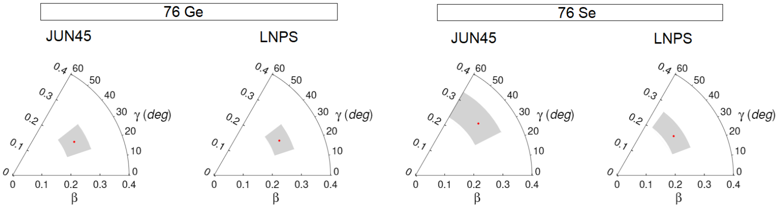


Figure 3.2 – Hill-Wheeler parameters (red dots) for Ge with JUN45 (left) and LNPS (middle left), and for Se with LNPS (middle right) and JUN45 (right). The grey rectangles are the variance of these parameters

Results

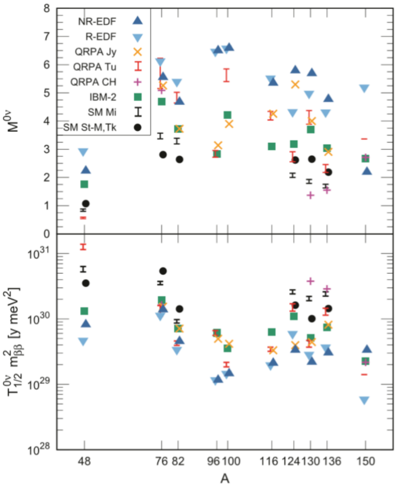
t	0	2	4	6	8
$M_{0\nu}^{GT}$	0.7731	1.352	1.846	2.422	2.618
$M_{0\nu}^{F}$	0.095	0.185	0.256	0.335	0.361
$M_{0\nu}^{T}$	$4.672 \cdot 10^{-2}$	$1.912 \cdot 10^{-2}$	$8.375 \cdot 10^{-4}$	$-9.005 \cdot 10^{-3}$	$-10.55 \cdot 10^{-3}$
$M_{0\nu}$	0.915	1.556	2.103	2.748	2.969

Table 3.6 – Evolution of the matrix elements $M_{0\nu}^{GT}$, $M_{0\nu}^{F}$, $M_{0\nu}^{T}$, $M_{0\nu}$ as function of the truncation, computed with JUN45 interaction.

t	0	2	4	6	8
$M_{0\nu}^{GT}$	0.550	1.348	2.080	1.799	2.067
$M_{0\nu}^{F}$	0.083	0.249	0.324	0.324	0.424
$M_{0\nu}^{T}$	$2.455 \cdot 10^{-2}$	$3.602 \cdot 10^{-2}$	$4.794 \cdot 10^{-2}$	$3.062 \cdot 10^{-2}$	$2.383 \cdot 10^{-2}$
$M_{0\nu}$	0.657	1.632	2.452	2.176	2.514

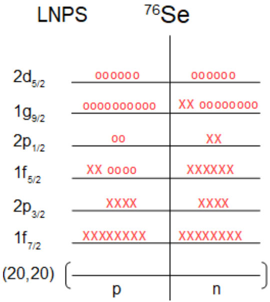
Table 3.7 – Evolution of the matrix elements $M_{0\nu}^{GT}$, $M_{0\nu}^{F}$, $M_{0\nu}^{T}$, $M_{0\nu}$ as function of the truncation, computed with LNPS interaction.

Results


NR-EDF
R-EDF
QRPA Jy
QRPA Tu
QRPA CH
IBM-2
SM Mi
SM St-M,Tk
$M^{0\nu}$
$T^{0\nu}_{1/2}\, m^2_{\beta\beta}$ [y meV^2]
48
76 82
96 100
116 124 130 136
150
A

Improvements

LNPS underestimates experimental values of B(E2) for ^{76}Se
$\rightarrow$ adding $1g_{9/2}$ and $1d_{5/2}$ orbitals to proton valence space :



$\rightarrow$ Computation of the B(E2) in case of 2 protons excited in $1g_{9/2}$ and $2d_{5/2}$.
Results :

- $B(E2) = 644\ e^2 fm^4$ with $(e_p, e_n) = (0.43, 1.31)$
- $B(E2) = \mathbf{821}\ e^2 fm^4$ with $(e_p, e_n) = (0.5, 1.5)$

Experimental value : $\mathbf{838}\ e^2 fm^4$

Conclusion

- LNPS well describes Q_{spec} of 76Ge → thanks to 2d5/2 neutron shell
- for Se, LNPS underestimates experimental values → need to add $1g_{9/2}$ and $2d_{5/2}$ to proton valence space.
- matrix elements a priori underestimated : due to proton valence space ? → extension of the space to reproduce BE2 increase from 76Kr to 76Sr.
  Preliminary results seem to support this mecanism.
- next step :double beta decay of 82Se into 82Kr.