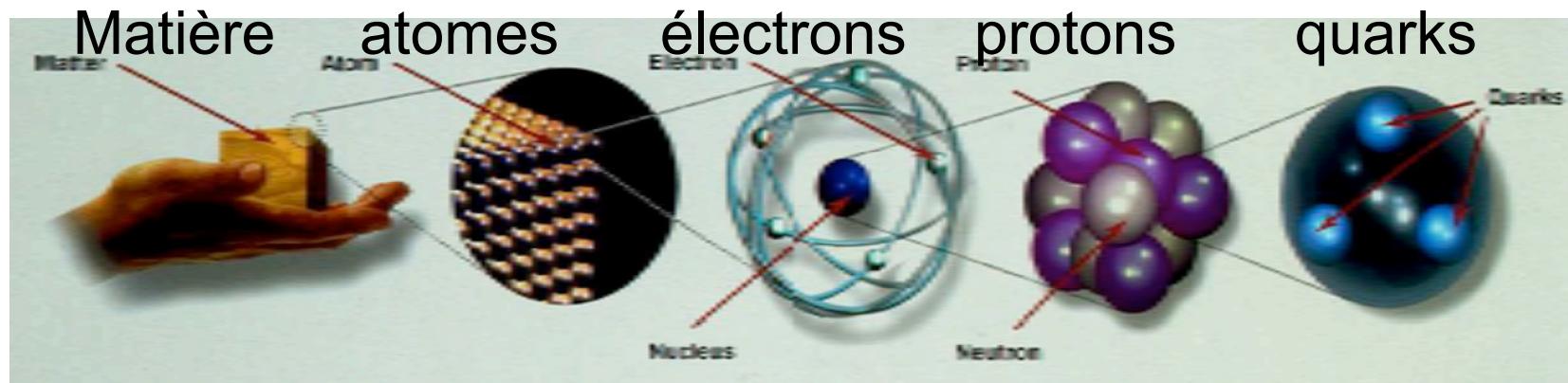


Non perturbative QCD

ECOLE PREDCTORALE REGIONALE DE PHYSIQUE SUBATOMIQUE

Annecy, 14-18 septembre 2009



- Basic notions
- Path integral
- Non-perturbative computing methods
- Some applications: beauty physics, form factors, structure functions, finite T, ...

A scientific revolution: The discovery of the standard model

1965 -1975 Quark model

Unified Electroweak Theory

Strong interaction theory (Quantum Chromodynamics -QCD)

Both are quantum field theories, with a gauge invariance.

Cabibbo-Kobayashi-Maskawa CP violation mechanism.

Successful prediction of a third generation of quarks.

Very Well verified by experiment



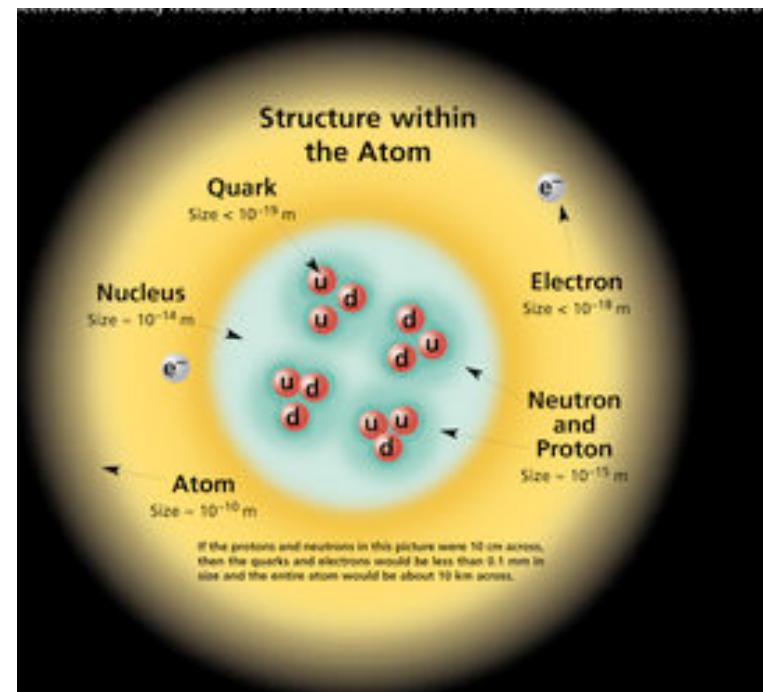
Encarta Encyclopedia, Bettmann/Corbis

However, this is not the last word. There must exist physics beyond the standard model, today unknown: neutrino masses, Baryon number of the universe, electric neutrality of the atom, quantum gravity, ...

What will we learn from LHC ?

Fundamental Particles

Elementary Particles		
Quarks	Leptons	Force Carriers
u up	c	t top
d down	s strange	b bottom
ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino
e electron	μ muon	τ tau
Three Families of Matter		

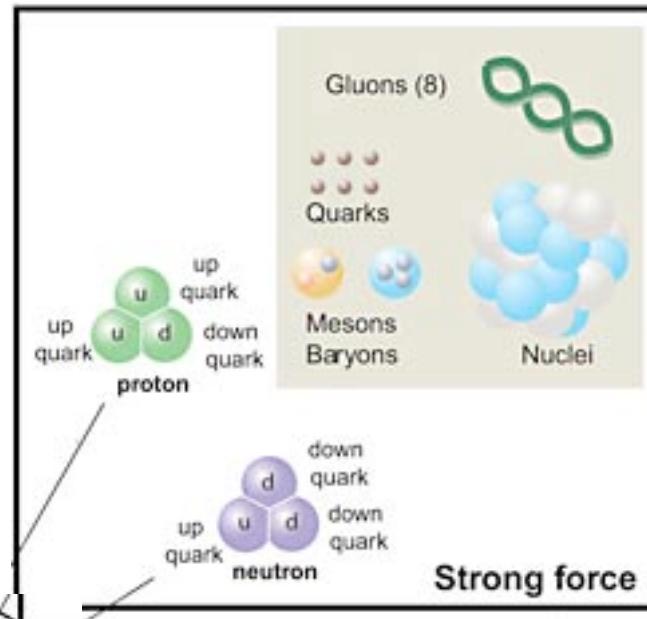
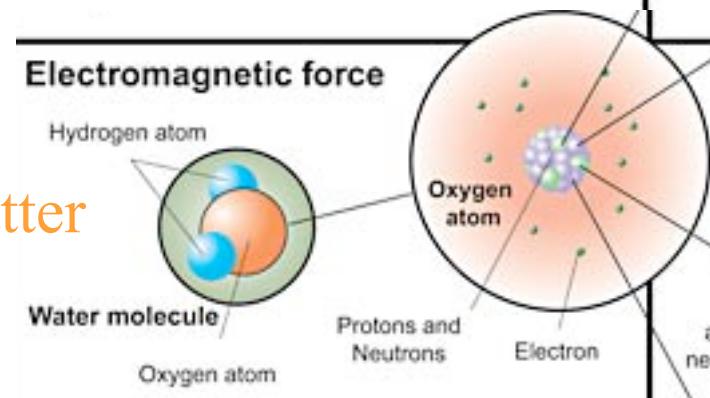


+ Higgs boson, to be discovered; at LHC ?

QCD: Theory of the strong subnuclear interaction

How do quarks and gluons combine to build-up protons, neutrons, pions and other hadrons.

Hadronic matter represents 99% of the visible matter of universe



How do protons and neutrons combine to Build-up atomic nuclei ?

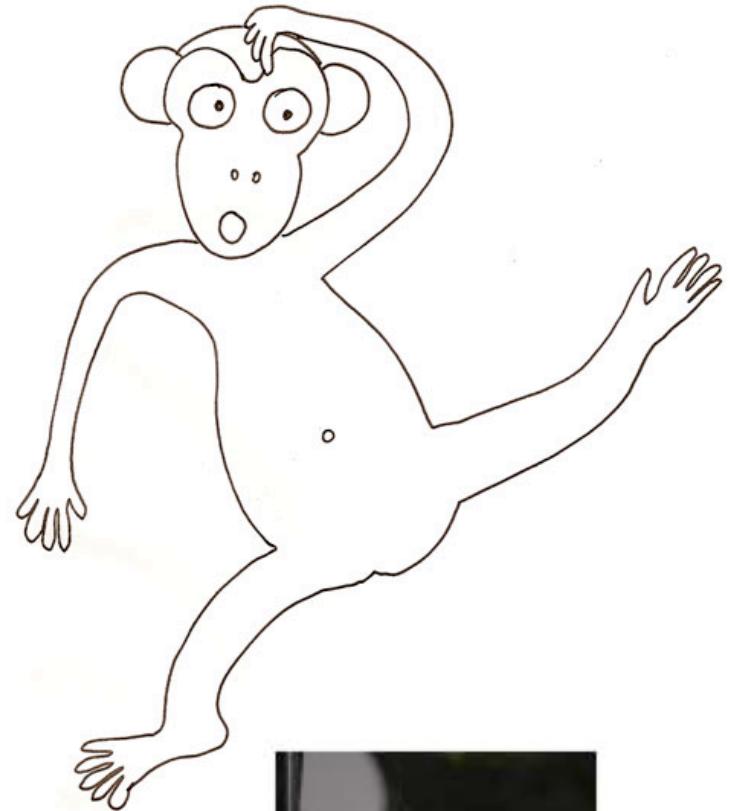
**During the 60's,
understanding strong
interactions seemed to be
an insurmountable
challenge !**

and yet. . .

**BEGINNING OF THE 70'S QCD WAS
DISCOVERED AND VERY FAST
CONFIRMED BY EXPERIMENT**

A splendid scientific epic.

cf Patrick Aurenche



Quantum Field theory (QFT)

Lagrange



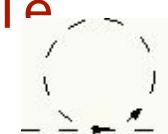
QCD a QFT (synthesis of special relativity and quantum mechanics):

- 1) We must first define fields and the corresponding particles.**
- 2) We must define the dynamics (the Lagrangian has the advantage of a manifest Lorentz invariance (the Hamiltonien does not) and the symmetries.**
- 3) Last but not least: we must learn how to compute physical quantities. This is the hard part for QCD.**

Example, the $\lambda\phi^4$ theory: the field is a real function of space-time. The Lagrangian defines its dynamics (we shall see how):

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \phi(x))^2 - \frac{1}{2} m^2 \phi^2(x) - \frac{\lambda}{4!} \phi^4(x)$$

The action is defined for all field theory by $S = \int d^4x \mathcal{L}(x)$



QCD's Dynamics : Lagrangien

Three « colors » a kind of generalised charge related to the « gauge group» SU(3).

Action: $S_{QCD} = \int d^4x \mathcal{L}_{QCD}(x)$

On every space-time point: 3(colors)x6(u,d,s,c,b,t)
quarks/antiquark fields [Dirac spinors] $q(x)$ and 8 real gluon fields
[Lorentz vectors] $A_\mu^a(x)$

$$\mathcal{L} = -1/4 G_{\mu\nu}^a G_a^{\mu\nu} + i \sum_f \bar{q}_f^i \gamma_\mu (D^\mu)_{ij} q_f^j - m_f \bar{q}_f^i q_f^i$$



Where $a=1,8$ gluon colors, $i,j=1,3$ quark colors,

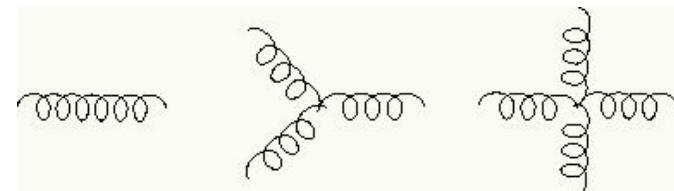
$F=1,6$ quark flavors, $\mu \nu$ Lorentz indices

$$G_{\mu\nu}^a = \partial_\mu A_{\nu}^a - \partial_\nu A_{\mu}^a + g f_{abc} A_\mu^b A_\nu^c$$

$$(D_\mu)_{ij} = \delta_{ij} \partial_\mu - i g \lambda_{ij}^a / 2 A_\mu^a$$

f_{abc} is SU(3)'s structure constant, λ_{ij}^a are Gell-Mann matrices

$$\bar{q} = q^t \gamma_0$$



The Lagrangian of QED is obtained from the same formulae after withdrawing color indices a,b,c,i,j ; $f_{abc} \rightarrow 0$ et $\lambda_{ij}^a / 2 \rightarrow 1$

The major difference is the gluon-gluon interaction



An astounding consequence of this Lagrangian

Confinement

- One never observes isolated quarks neither gluons. They only exist in bound states, **hadrons** (color singlets) made up of:
 - three quarks or three anti-quarks, the **(anti-)baryons**, example: the proton, neutron, lambda,
 - one quark and one anti-quark, **mésons**, example: the pion, kaon, B, the J/psi,..

CONFINEMENT HAS NOT YET BEEN DERIVED FROM QCD

Image: we pull afar two heavy quarks, a strong « string » binds them (linear potential). At som point the string breaks, a quark-antiquark pair jumps out of the vacuum to produce two mesons. You never have separated quarks and antiquarks.

Imagine you do the same with the electron and proton of H atom. The force is less and at some point e and pare separated (ionisation).

Strong interaction is omnipresent

It explains:

Hadrons structure and masses

The properties of atomic nuclei

The « form factors » of hadrons (ex: $p+e \rightarrow p+e$)

The final states of $p+e \rightarrow e+ \text{hadrons}$ (pions, nucleons...)

The products of high energy collisions:

$e^- e^+ \rightarrow \text{hadrons}$ (beaucoup de hadrons)

The products of $pp \rightarrow X$ (hadrons)

Heavy ions collisions ($\text{Au} + \text{Au} \rightarrow X$), new states of matter (quark gluon plasmas)

And all which includes heavier quarks (s,c,b,t)

.....

Apology of QCD

Prototype of a « beautiful theory »: Newton's

A « beautiful theory » contains an *input* precise and condensed, principles, postulates, free parameters (QCD: simple Lagrangian of quarks and gluons, 7 parameters).



A very rich *output*, many physical observables (QCD: millions of experiments implying hundreds of « hadrons »: baryons, mesons, nuclei).

QCD is noticeable by the unequaled number and variety of its « outputs »

Confinement : « input » speaks about a few quarks and gluons, et la « output », hundred's of hadrons, of nuclei. This **metamorphosis** is presumably the reason of that rich variety of « outputs ».



BUT the accuracy of the predictions is rather low

Gauge invariance

redondance des degrés de liberté

drastically reduces the size of the input, reduces the “ultraviolet” singularities, makes the theory renormalisable

- Finite / Infinitesimal : $g(x) \approx \exp[i\epsilon^a(x)\lambda_a/2]$
- Huit fonctions réelles ϵ^a , $a=1,8$

$$\delta A_\mu^a(x) = \frac{1}{g_s} \partial_\mu \epsilon^a(x) + f^{abc} A_\mu^b(x) \epsilon^c(x),$$

$$\delta q(x) = i\epsilon^c(x) \frac{\lambda^c}{2} q(x)$$

•Finite gauge transformation

$$q(x) \rightarrow g(x) q(x), \quad W(x, y) \rightarrow g(x) W(x, y) g^{-1}(y)$$

$$A_\mu = \sum_a A_\mu^a \lambda^a / 2$$

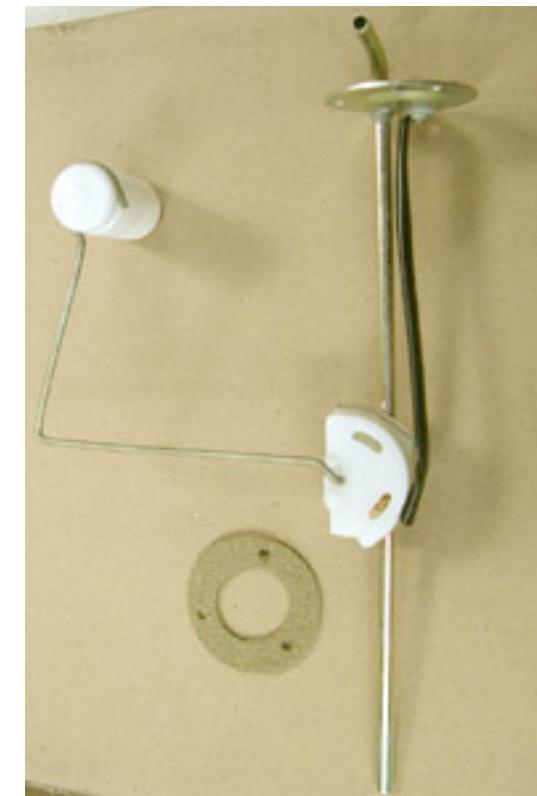
where

$$W(x, y) = P \left[\exp \left\{ i g_s \int_{C_{x,y}} dz^\mu A_\mu(z) \right\} \right]$$

gauge Invariants

$$\bar{q}(x) W(x, y) q(y), \quad \text{et} \quad Tr[W(x, x)]$$

Jauge 2CV



Symmetries

$$\mathcal{L}_{QCD} = -\frac{1}{4}G_{\mu\nu}^a G_a^{\mu\nu} + i \sum_f \bar{q}_f^i \gamma^\mu (D_\mu)_{ij} q_f^j - \sum_f m_f \bar{q}_f^i q_{fi},$$

Symmetric for:

✓TCP

✓Charge Conjugation

✓Chiral (approximate symmetry)

✓flavour (approximate symmetry)

✓Heavy quark symmetry (approximate symmetry)

✓Parity

✓CP

$$+ \frac{g^2}{64\pi^2} \theta \epsilon_{\mu\nu\rho\sigma} G_{\mu\nu}^a G_{\rho\sigma}^a$$

Mystery of strong CP violation,
never observed

What to compute and how ?

≥ What objects are we interested in ?

Green functions

≥ What formula allows to compute them ?

Path integral

≥ How to tame path integral ?

Continuation to imaginary time

Green functions, a couple of examples

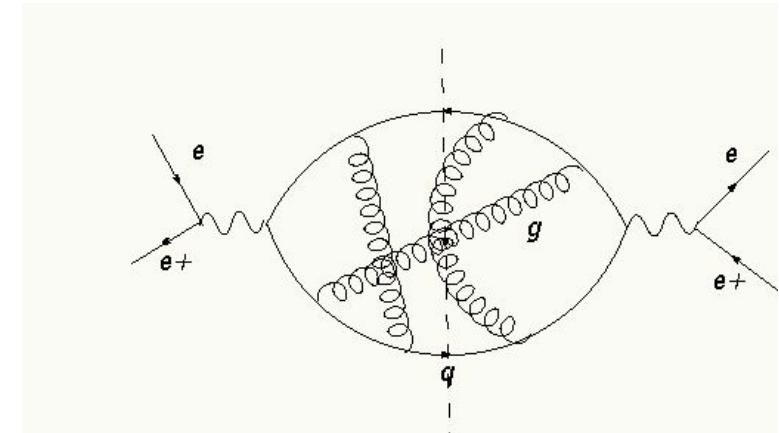
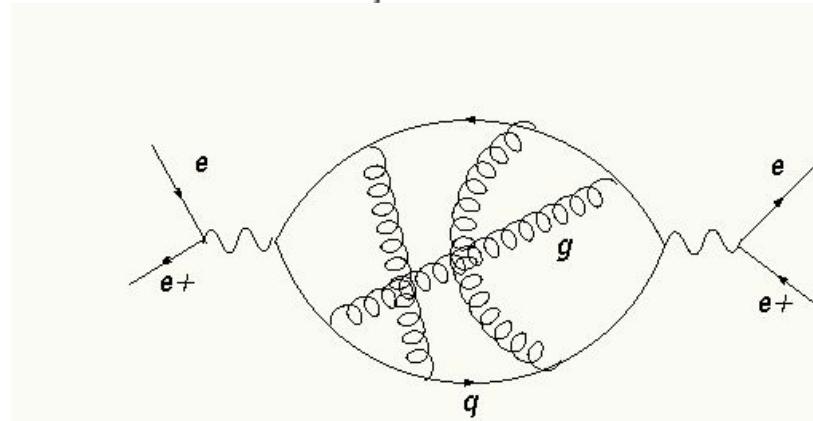
Quark propagator (non gauge-invariant)

$$S(x, y) \equiv \langle 0 | T [q(x) \bar{q}(y)] | 0 \rangle$$

$S(x,y)$ is a 12×12 matrix (spin x color)

current-current Green function

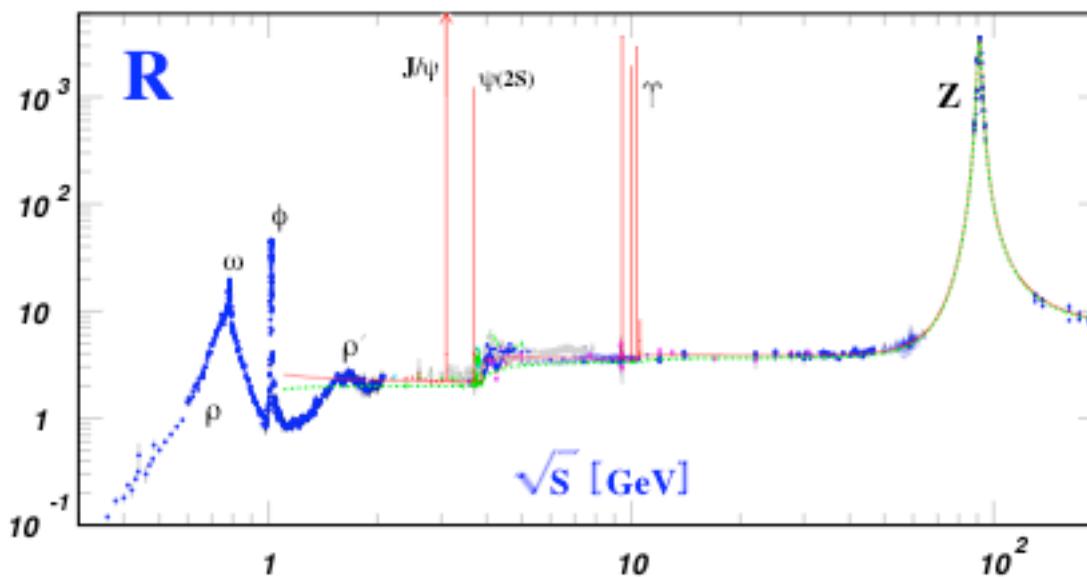
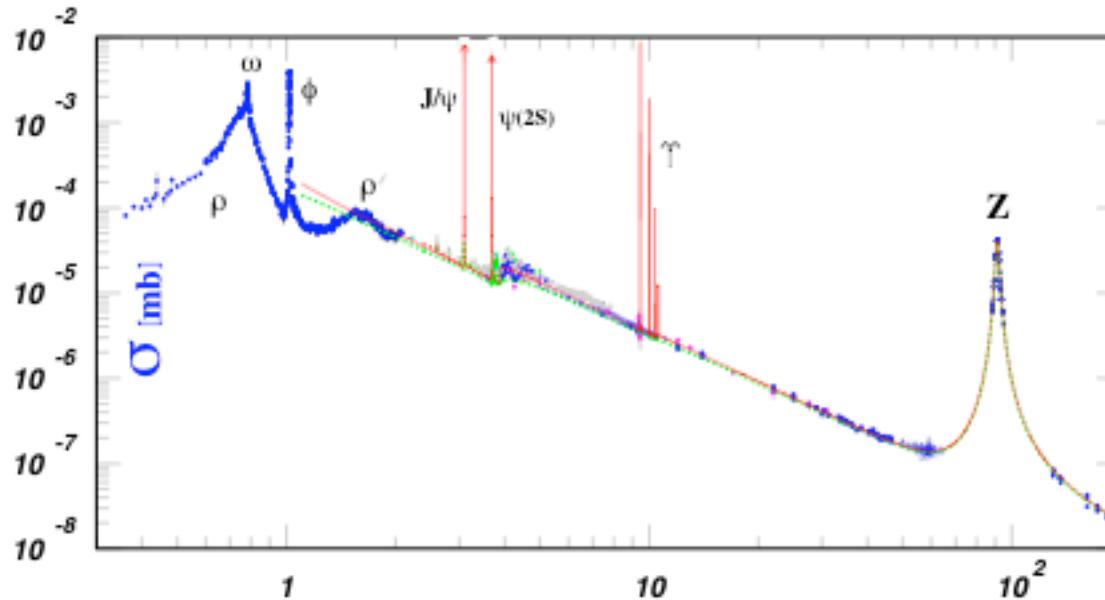
$$T_{\mu\nu}(x, y) \equiv \sum_f e_f^2 \langle 0 | T [J_\mu^f(x) J_\nu^f(y)] | 0 \rangle \quad \text{ou} \quad J_\mu(x) \equiv \bar{q}(x) \gamma_\mu q(x)$$



$T_{\mu\nu}$ is gauge-invariant.

$\text{Im}\{T_{\mu\nu}\}$ related to the ($e^+e^- \rightarrow \text{hadrons}$) total cross-section

Cross section $e^+e^- \rightarrow$ hadrons (PDG) $\propto \text{Im}(T_\mu^\mu)$



Path intégral

In a generic quantum field theory, the vacuum expectation value of an operator \mathcal{O} is given by

ϕ Is a generic bosonic field

$$\mathcal{L} \equiv \frac{1}{2} (\partial_\mu \phi(x))^2 - \frac{1}{2} m^2 \phi^2(x) - \frac{\lambda}{4!} \phi^4(x)$$
$$\langle \mathcal{O} \rangle = \frac{\int \prod_x \mathcal{D}\phi(x) \mathcal{O} \exp \{iS[\phi]\}}{\int \prod_x \mathcal{D}\phi(x) \exp \{iS[\phi]\}}$$

The action $S[\phi]$ is:

$$S[\phi] \equiv \int d^4x \mathcal{L}[\phi(x)]$$

The « i » in the exponential accounts for quantum interferences between paths. Extremely painful numerically

For example the propagator of the particle « ϕ » is given by:

$$\langle T[\phi(x) \phi(y)] \rangle = \frac{\int \prod_x \mathcal{D}\phi(x) T[\phi(x) \phi(y)] \exp \{iS[\phi]\}}{\int \prod_x \mathcal{D}\phi(x) \exp \{iS[\phi]\}}$$

The path integral of a fermion with an action $\int d^4x d^4y \bar{\psi}(x) M(x,y) \psi(y)$ is given by $\text{Det}[M]$



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R.P.Feynman

Fermionic Determinants

The « quark » part of QCD Lagrangien is

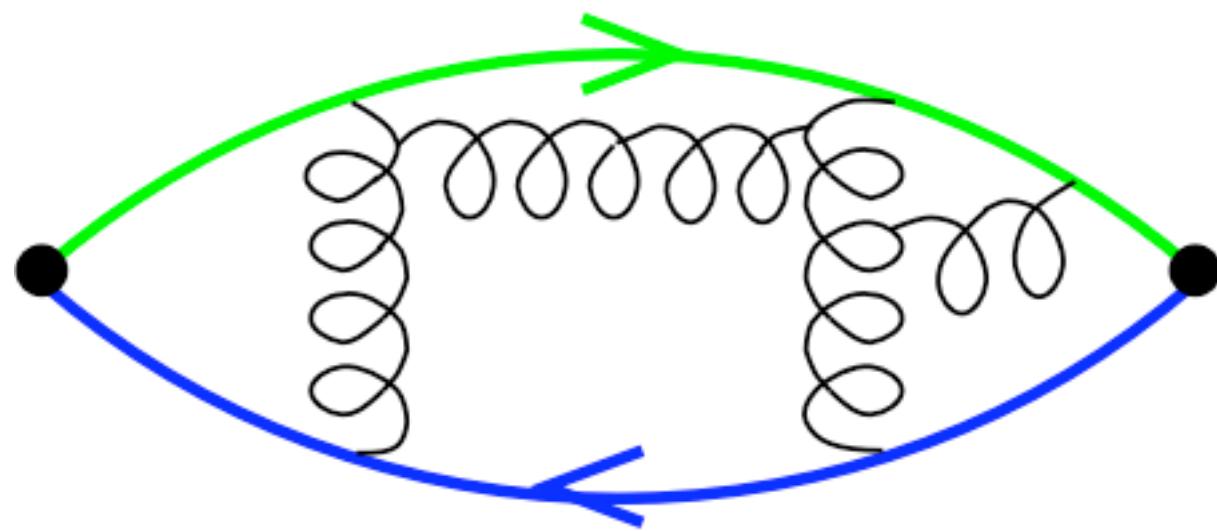
$$\mathcal{L}_{q_f} = \bar{q}_f(x) \left[\gamma_\mu \left(i \partial_\mu + g_s \frac{\lambda_a}{2} A_\mu^a \right) - m_f \right] q_f(x) \equiv \sum_{x,y} \bar{q}_f(x) M_f(x,y) q_f(y)$$

Where $M_f(x,y)$ is a matrix in the space direct product of
space-time x spin x color

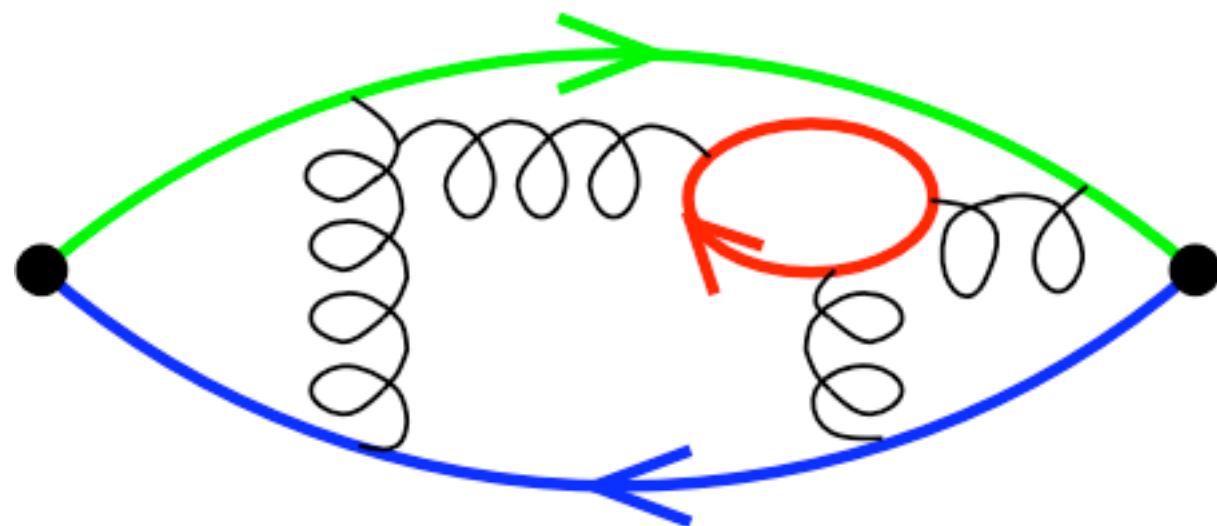
$$M_f(x,y) = \sum_\mu \gamma^\mu \left[\frac{i}{2a} (\delta_{x+\hat{\mu},y} - \delta_{x-\hat{\mu},y}) + g_s \frac{\lambda_a}{2} A_\mu^a \delta_{x,y} \right] - m_f \delta_{x,y}$$

The intégral is performed with integration variables defined in Grassman algebra

$$\begin{aligned} & \int \Pi_{x,y,f} \mathcal{D}\bar{\eta}_f(x) \mathcal{D}\eta_f(y) \exp \left[\sum_f \int d^4z d^4t \bar{\eta}_f(z) i M_f(z,t) \eta_f(t) \right] \\ &= \Pi_f \text{Det} [i M_f] \end{aligned}$$



(A) Quenched QCD: quark loops neglected



(B) Full QCD

Path integral of gauge fields

$\langle \mathcal{O} \rangle =$

$$\frac{\int \prod_{\mu,a,x} \mathcal{D}A_\mu^a(x) B[\partial_\mu A_\mu^a] \text{Det}[\mathcal{F}] \prod_f \text{Det} [M_f] \exp[-S_G] \mathcal{O}}{\int \prod_{\mu,a,x} \mathcal{D}A_\mu^a(x) B[\partial_\mu A_\mu^a] \text{Det}[\mathcal{F}] \prod_f \text{Det} [M_f] \exp[-S_G]}$$

Where $B[\partial_\mu A_\mu^a]$ fixes the gauge: $B[\partial_\mu A_\mu^a] \equiv \exp \left[-\frac{i}{2\xi} \int d^4x (\partial_\mu A_\mu^a)^2 \right]$

$\xi=0$, Landau gauge : $\partial_\mu A_\mu^a = 0$

$\text{Det}[\mathcal{F}] \mathcal{F} = \partial_\mu D_\mu$ Is the Faddeev Popov determinant, necessary to protect gauge invariance of the final result

$$S_G = -1/4 \int d^4x G_{\mu\nu}^a G_a^{\mu\nu}$$

Flipping to imaginary time

Continuation to imaginary time

$$t = -i\tau, \quad \exp[i\beta S_G] \rightarrow \exp[-\beta S_G]$$

S_G is positive, $\exp[-\beta S_G]$ is a probability distribution

$$\langle O \rangle = \int D\Gamma O \exp[-\beta S_G] \Pi_f \text{Det}[M_f] / \int D\Gamma \exp[-\beta S_G] \Pi_f \text{Det}[M_f]$$

Is a Boltzman distribution in 4 dimensions:
 $\exp[-\beta S_G] \Leftrightarrow \exp[-\beta H]$

The passage to imaginary time has turned the quantum field theory into a classical thermodynamic theory at equilibrium. The metric becomes Euclidian.

Once the Green functions computed with imaginary time, one must return to the quantum field theory, one must perform an analytic continuation in the complex variable faire t or p^0 . Using the analytic properties of quantum field theory.

Simple case, the propagator in time of a particle of energy E :

$$\begin{aligned} t: \text{real time} \\ \exp[-iEt] \end{aligned}$$

$$\begin{aligned} \tau: \text{imaginary time} \\ \Leftrightarrow \\ \exp[-E\tau] \end{aligned}$$



Maupertuis (1744)

Maintenant, voici ce principe, si sage, si digne de l'Être suprême lorsqu'il arrive quelque changement dans la Nature, la quantité d'Action employée pour ce changement est toujours la plus petite qu'il soit possible. »

Suite au
prochain épisode

Caractères spéciaux

➤ $\partial_\mu \mathcal{L} \propto \phi \nabla^2 \phi - \frac{1}{2} m^2 \phi^2 - \frac{\lambda}{4!} \phi^4$

➤ $\mathcal{L} = \frac{1}{2} (\partial_\mu \phi(x))^2 - \frac{1}{2} m^2 \phi^2(x) - \frac{\lambda}{4!} \phi^4(x)$

➤ $\mathcal{L} = -\frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu} + i \sum_f q_f^i \gamma_\mu (D^\mu)_{ij} q_f^j - m_f q_f^i q_i$

➤ $G_{\mu}^a = \partial_\mu A_a^\nu - \partial_\nu A_a^\mu + g f_{abc} A_b^\mu A_c^\nu$

➤ $(D_\mu)_{ij} = \delta_{ij} \partial_\mu - i g \lambda^a_{ij} / 2 A_a^\mu$

$$\mathcal{L}_{QCD} = -\frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu} + i \sum_f \bar{q}_f^i \gamma^\mu (D_\mu)_{ij} q_f^j - \sum_f m_f \bar{q}_f^i q_{fi}, \quad (1)$$

a indice de couleur

i, j indices de spin,

f est la saveur du quark ($f = u, d, s, c, b, t$),

μ, ν indices de Lorentz.