A STATISTICAL INFERENCE APPROACH TO SPACE-BASED INTERFEROMETRY

Quentin Baghi, Ira Thorpe, John Baker, Jacob Slutsky
Gravitational Astrophysics Lab - NASA Goddard Space Flight Center

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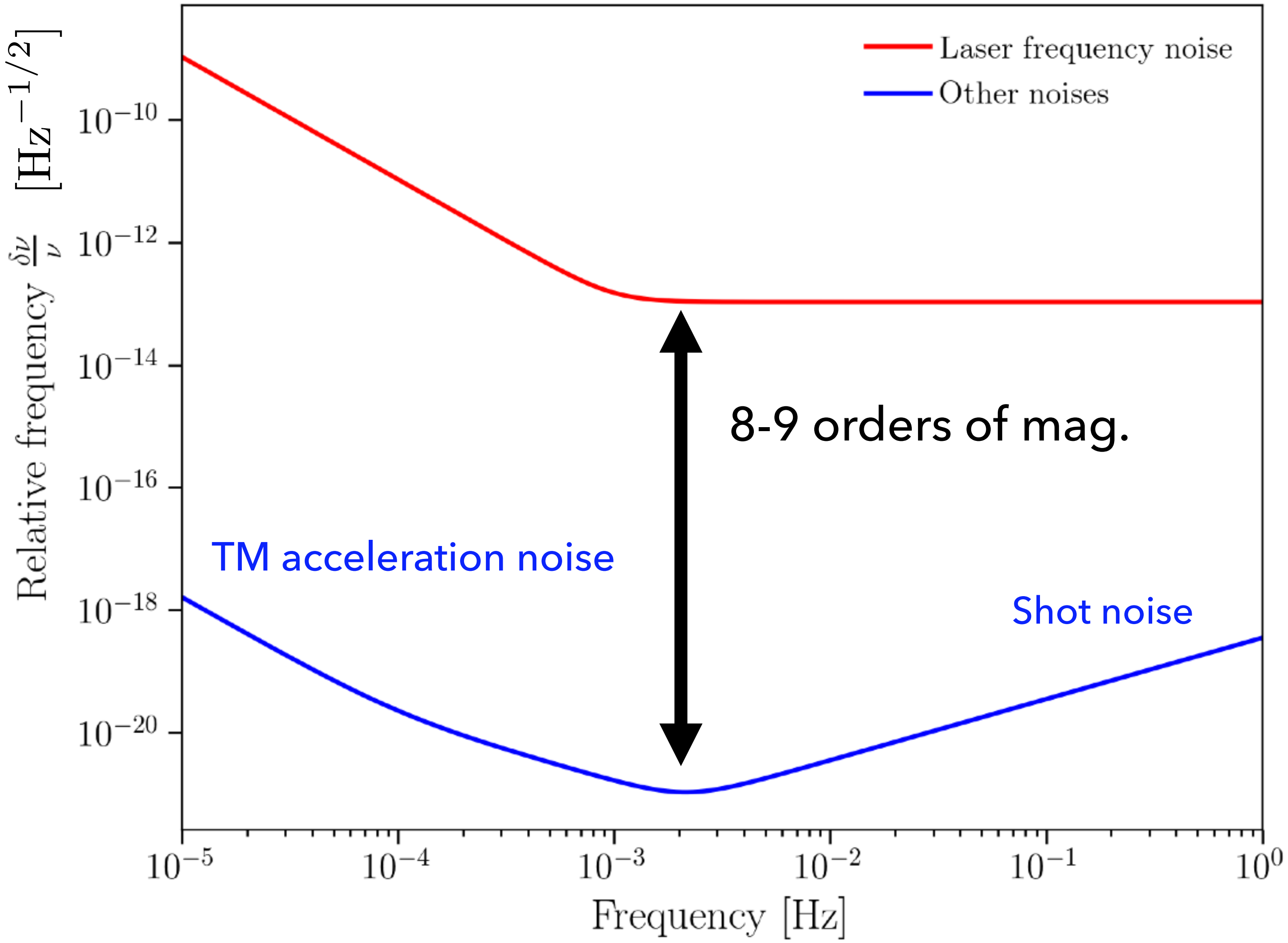
OUTLINE

1. The challenge of laser noise in space-based detection
2. A generalization of time delay interferometry
3. Numerical simulations

1. THE CHALLENGE OF LASER NOISE IN SPACE-BASED DETECTION

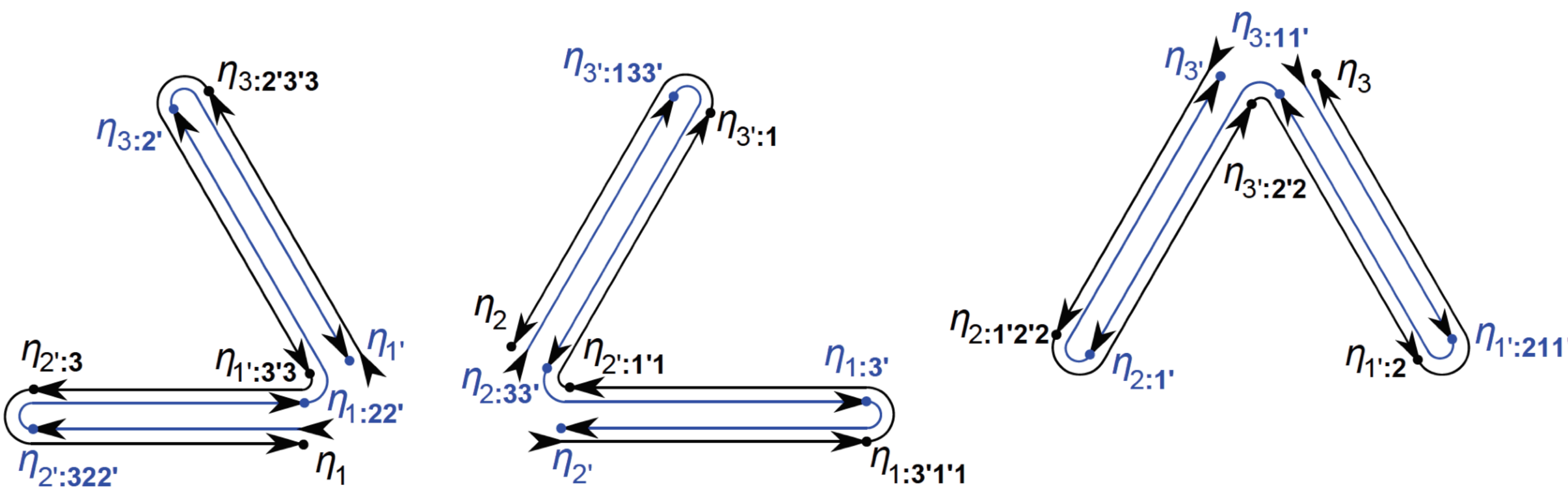
1. THE CHALLENGE OF LASER NOISE

- In LISA, each science interferometer length mismatch is of millions of kms.
- Induces a huge noise due to laser frequency random fluctuations.



1. THE CHALLENGE OF LASER NOISE

- Solution: the interferometry is done as a **post-processing step**.
- The classic algorithm is called **time-delay interferometry (TDI)** [Tinto & Armstrong1999]
- Forms linear combinations of delayed phasemeter measurements tailored to cancel laser noise
- Some of them are equivalent to a **synthetically reproducing a Michelson interferometer photon path**



TDI-X TDI-Y TDI-Z

Credit: Markus Otto, PhD thesis, 2016

- The standard formulation of TDI has some drawbacks:
  - Based on physical considerations regardless of variance optimization
  - Lengthy and hard-to-track equations
  - Accounting for additional effects leads to introducing intermediary steps

2. A GENERALIZATION OF TIME DELAY INTERFEROMETRY

2. A GENERALIZATION OF TDI

- We look for a more general formalism. A bit like [Nayak & Vinet 2004], but statistical stand point.
- We turn the science measurement equations…

$$s_i = h_{i+2} + D_{i+2} p_{i'+1} - p_i + n_i$$
$$s_{i'} = h_{i+1} + D_{i'+1} p_{i+2} - p_{i'} + n_{i'} .$$

Delay operators

$$D_i x(t) = x\left(t - c^{-1} L_i\right)$$

- Into a single matrix formulation:

$$\boxed{\boldsymbol{y} = \boldsymbol{h} + \boldsymbol{M}\boldsymbol{p} + \boldsymbol{n}}$$

Stacked interferometer measurements 6 x 1

GW signal

Combined delay matrix

Laser noises

Other noises

- We can then form a likelihood function

Full noise covariance

$$p\left(\boldsymbol{y}|\boldsymbol{\theta}\right) = \frac{\exp\left\{-\frac{1}{2}\left(\boldsymbol{y}-\boldsymbol{h}\right)^{\dagger}\boldsymbol{\Sigma}^{-1}\left(\boldsymbol{y}-\boldsymbol{h}\right)\right\}}{\sqrt{\left(2\pi\right)^{6N}\left|\boldsymbol{\Sigma}\right|}}$$

$$\boldsymbol{\Sigma} = \boldsymbol{M}\boldsymbol{\Sigma}_p\boldsymbol{M}^{\dagger} + \boldsymbol{\Sigma}_n$$

Laser noise spectrum matrix

Other noise spectrum matrix

- Problem: directly computing the likelihood may be inefficient or numerically unstable.

2. A GENERALIZATION OF TDI

- Solution: use principal component analysis (PCA) , as suggested in [Romano & Woan, 2006]
  - Finds an orthogonal basis that concentrates only on low-variance data.
  - Efficiently approximates the likelihood by diagonalisation of the covariance:

$$\boldsymbol{\Sigma} = \boldsymbol{V}\boldsymbol{\Lambda}\boldsymbol{V}^*$$

- We find that there are:
  - 3 eigenvectors $\boldsymbol{V}_p$ with eigenvalues $\boldsymbol{\Lambda}_p$ — Laser noise-dominated
  - 3 eigenvectors $\boldsymbol{V}_n$ with small eigenvalues $\boldsymbol{\Lambda}_n$ — Laser noise-free

2. A GENERALIZATION OF TDI

- We approximate the likelihood by **restricting** the data **to the null space of the laser covariance** (see different derivation from Vallisneri et al. 2020):

$$\log p\left(\boldsymbol{y}|\boldsymbol{\theta}\right) \approx -\frac{1}{2}\left(\boldsymbol{y}-\boldsymbol{M}\boldsymbol{h}\right)^{*}\boldsymbol{V}_{n}\boldsymbol{C}_{n}^{-1}\boldsymbol{V}_{n}^{*}\left(\boldsymbol{y}-\boldsymbol{M}\boldsymbol{h}\right)-\frac{1}{2}\log\left|\boldsymbol{V}_{n}^{*}\boldsymbol{\Sigma}_{n}\boldsymbol{V}_{n}\right|.$$

Phasemeter data

Eigenvector matrix of laser noise covariance

$$\boldsymbol{C}_{n}=\boldsymbol{V}_{n}^{*}\boldsymbol{\Sigma}_{n}\boldsymbol{V}_{n}$$

Equivalent to X, Y, Z transformation

- Re-write it, making the **orthogonalization** explicit: $\boldsymbol{C}_{n}=\boldsymbol{\Phi}\boldsymbol{\Lambda}\boldsymbol{\Phi}^{*}$

$$\log p\left(\boldsymbol{y}|\boldsymbol{\theta}\right) \approx -\frac{1}{2}\left(\boldsymbol{y}-\boldsymbol{M}\boldsymbol{h}\right)^{*}\boldsymbol{V}_{n}\boldsymbol{\Phi}\boldsymbol{\Lambda}^{-1}\boldsymbol{\Phi}^{*}\boldsymbol{V}_{n}^{*}\left(\boldsymbol{y}-\boldsymbol{M}\boldsymbol{h}\right)-\frac{1}{2}\log\left|\boldsymbol{\Lambda}\right|.$$

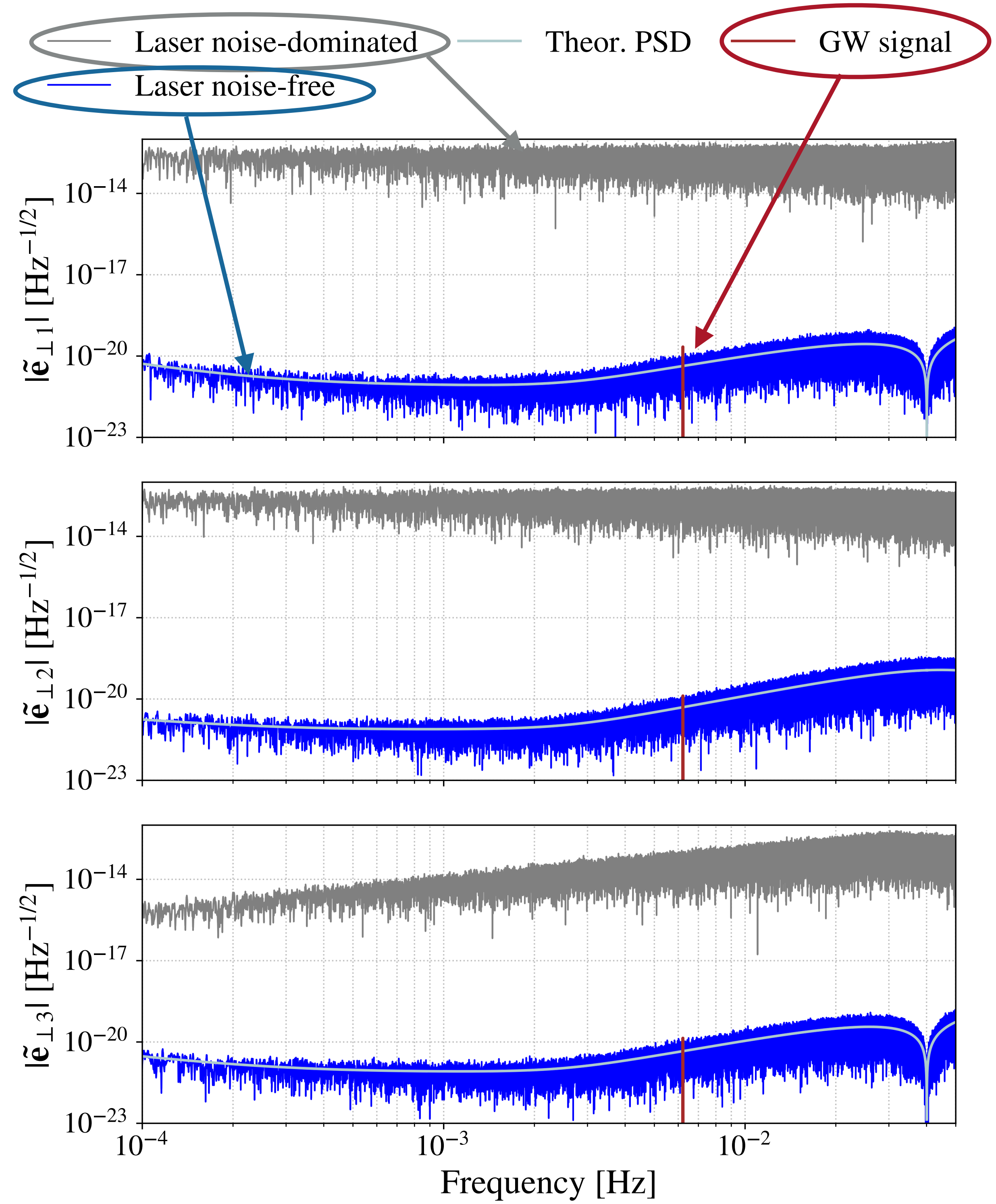
Eigenvector matrix of the projected covariance matrix

Equivalent to A, E, T transformation [Prince et al. 2002]

But valid of all types of assumptions on noise levels and arm lengths

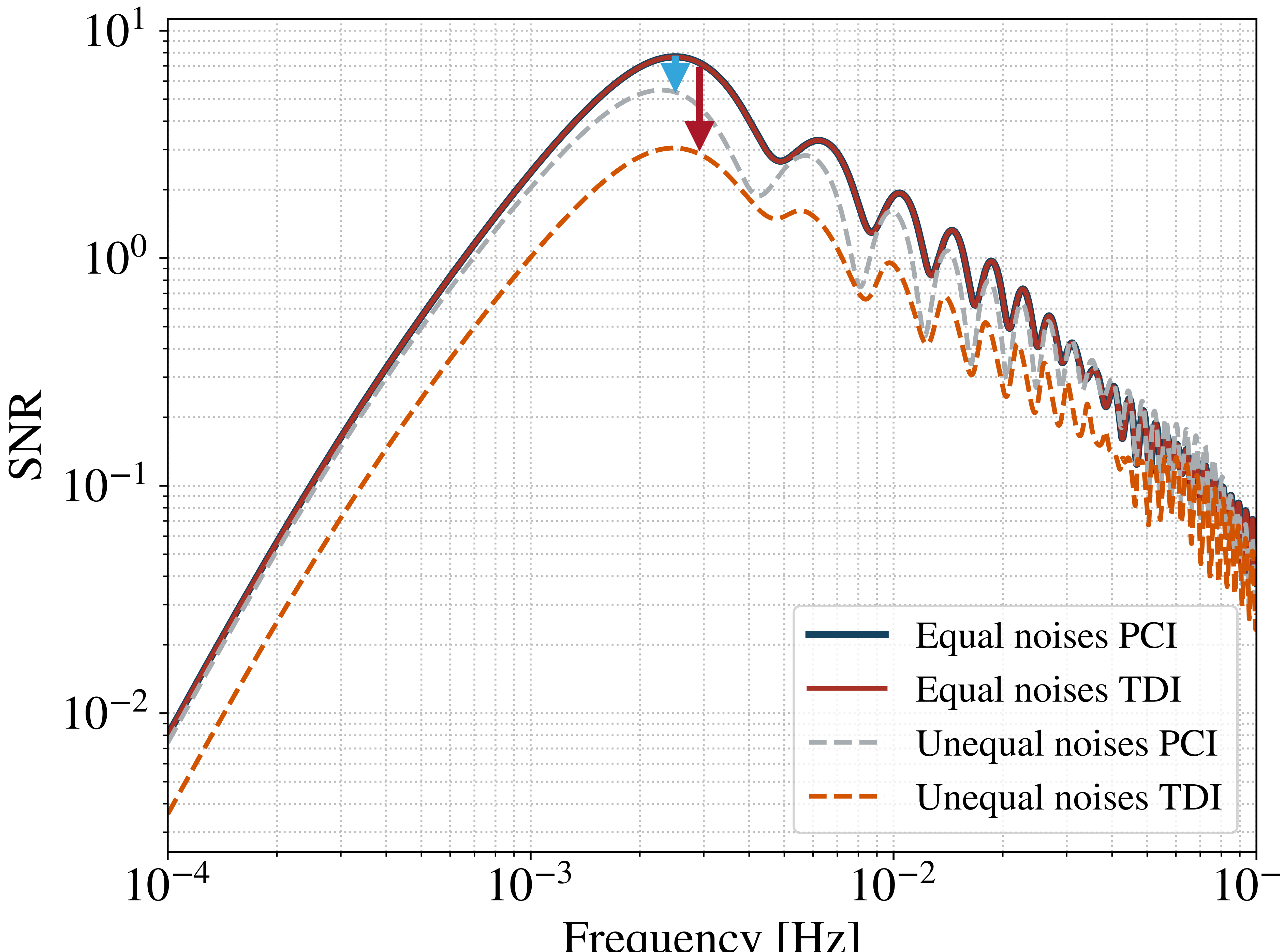
3. NUMERICAL SIMULATIONS

- We simulate 1-month long phasemeter measurements with:
  - Laser noise + TM acceleration noise + OMS noise
  - Rigid, rotating LISA
  - One GW source (galactic binary HM Cnc)



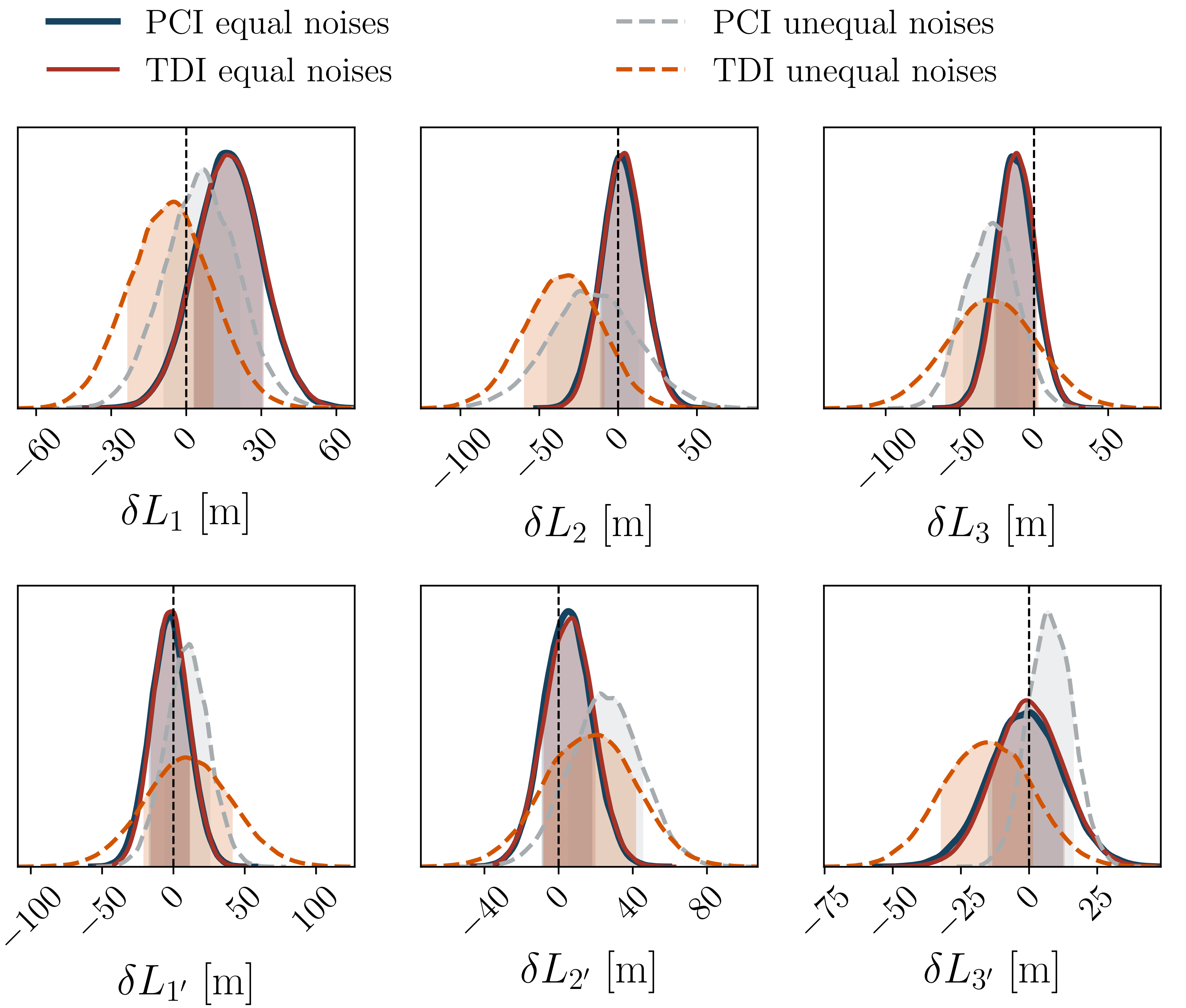
3. NUMERICAL SIMULATIONS

- Now assume that **acceleration noise levels are <u>not</u> equal** on all science interferometer: example they have ratios between 0.1 to 20 with a mean of 4 x the baseline



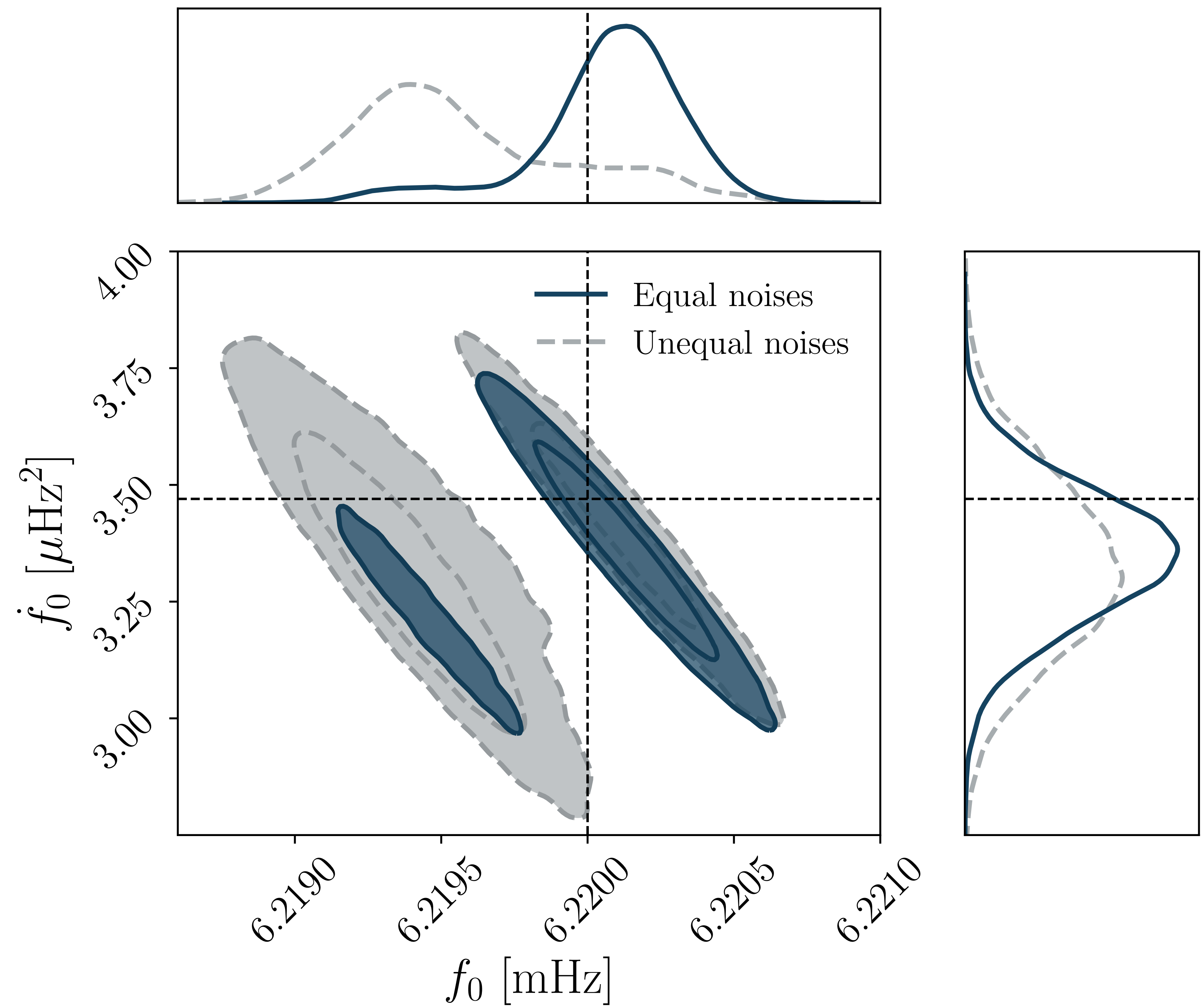
[Baghi et al, to be submitted]

Joint estimation of **delays**, source parameters and covariance elements



3. NUMERICAL SIMULATIONS

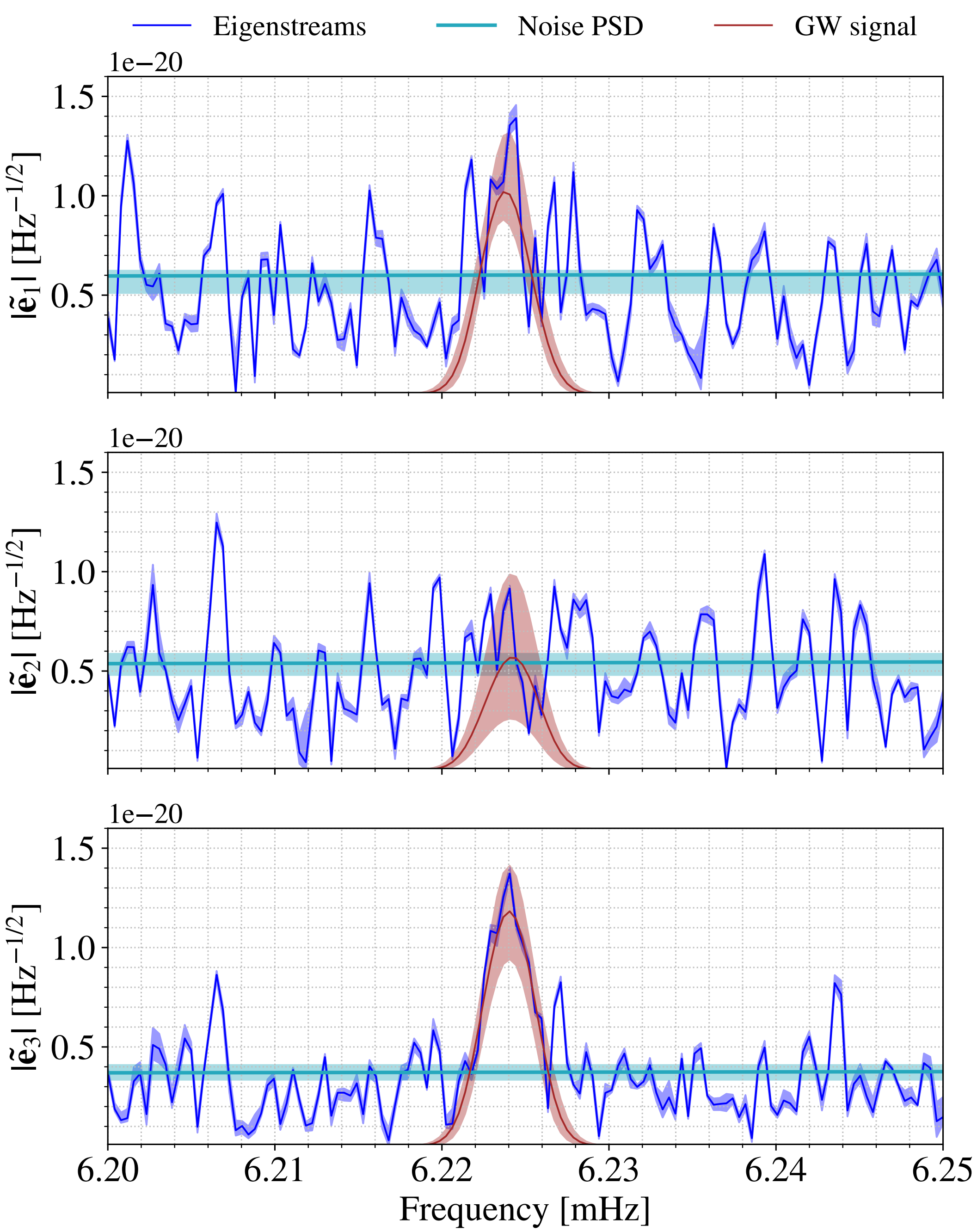
Joint estimation of delays, **source parameters** and covariance elements



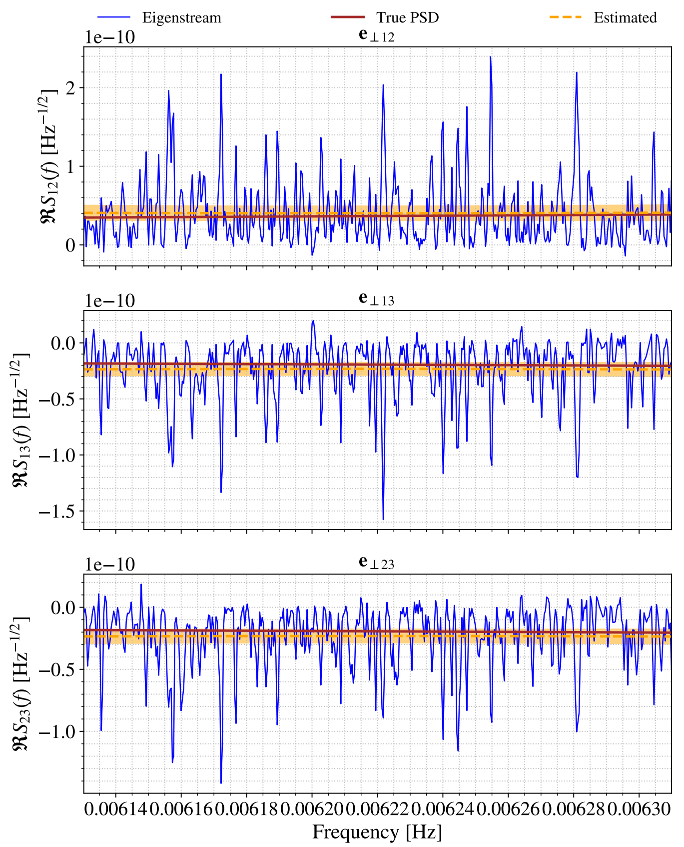
3. NUMERICAL SIMULATIONS

Joint estimation of delays, source parameters and **covariance elements** $\mathrm{Cov}\,(e)$

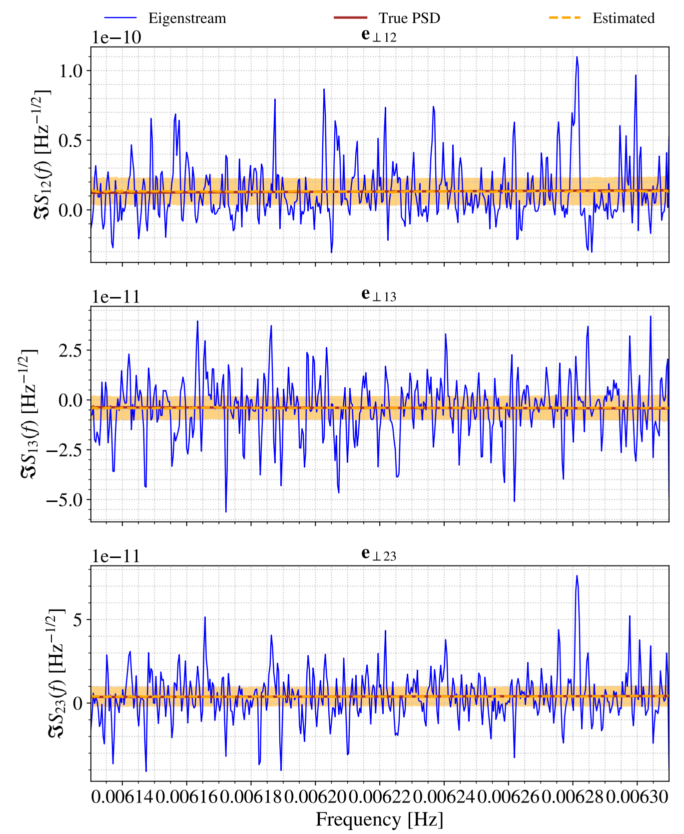
Diagonal



Off-diagonal, real part



Off-diagonal, imaginary part

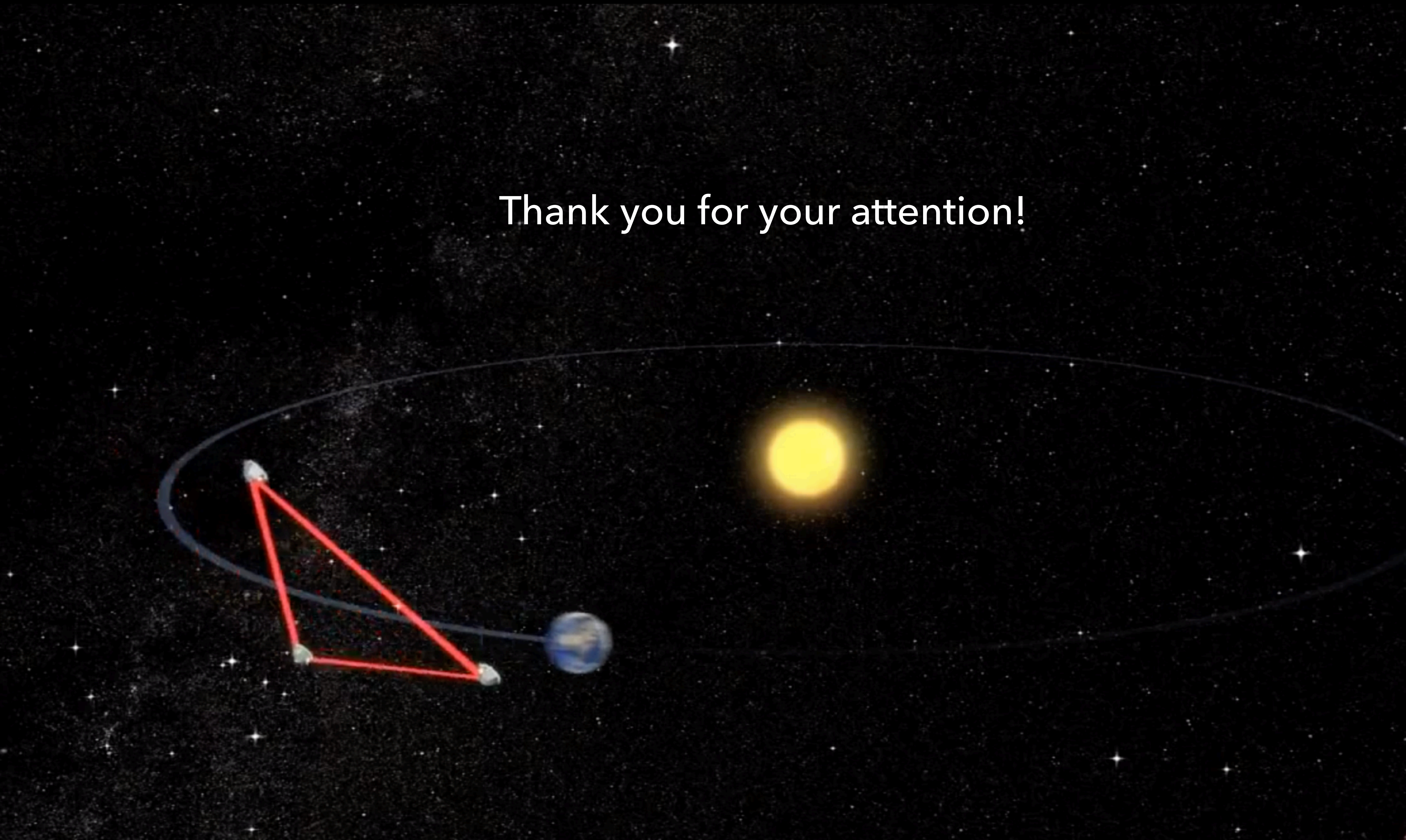


4. TOWARDS A MORE DATA-DRIVEN APPROACH

- We pave the way for a more **flexible**, **general** TDI based on PCA → "PCI"
- Provides a **systematic way to process L0 data**, regardless of the number of channels
- Performs frequency-dependent orthogonalization on the fly
- We can **increase the robustness** of LISA data analysis a more data-driven approach

https://arxiv.org/abs/2010.07224

Credit: AEI/Milde Marketing



Thank you for your attention!