



Groupement de recherche
Ondes gravitationnelles

Parameter estimation and sky localization of massive binary black holes with LISA

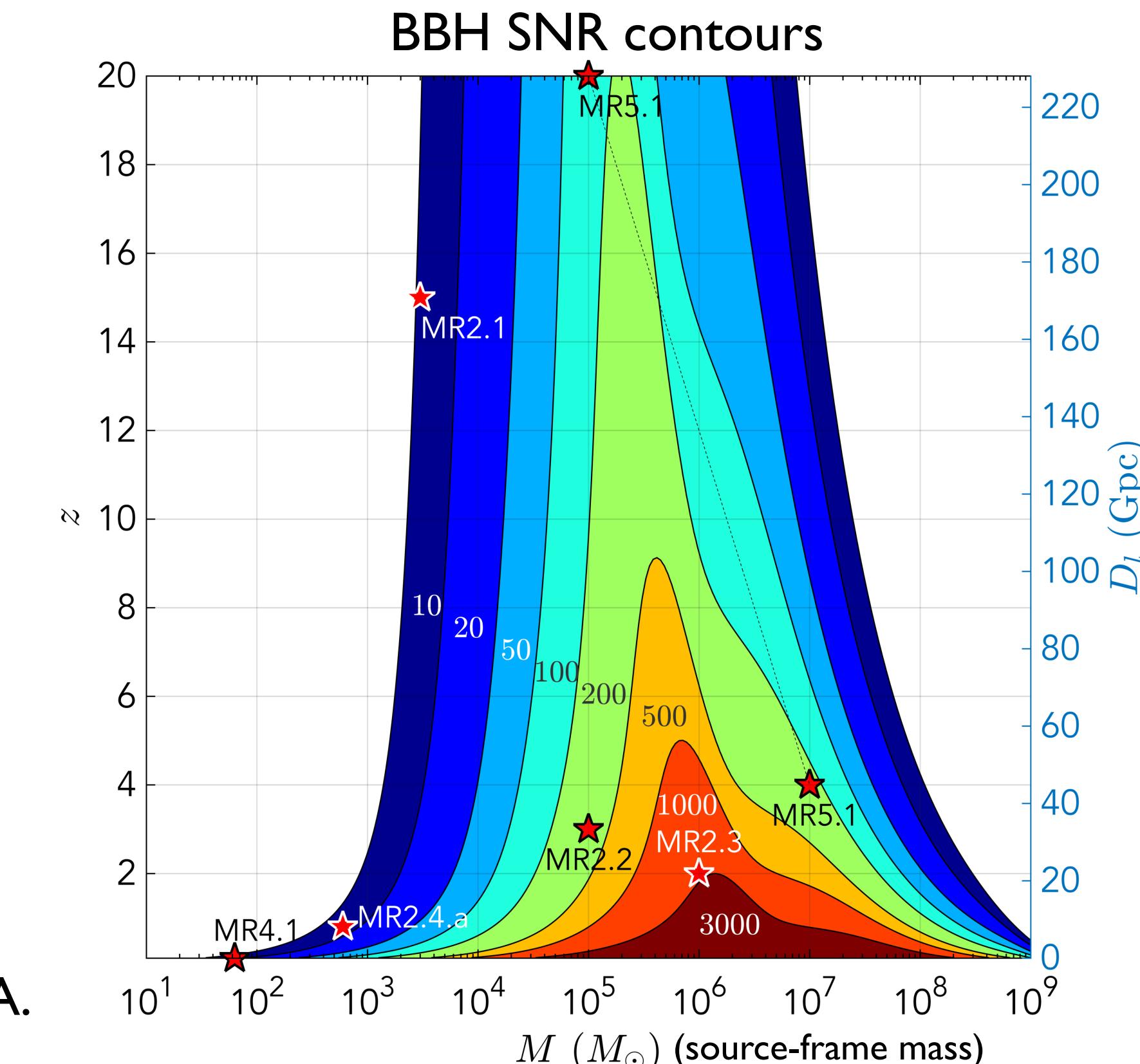
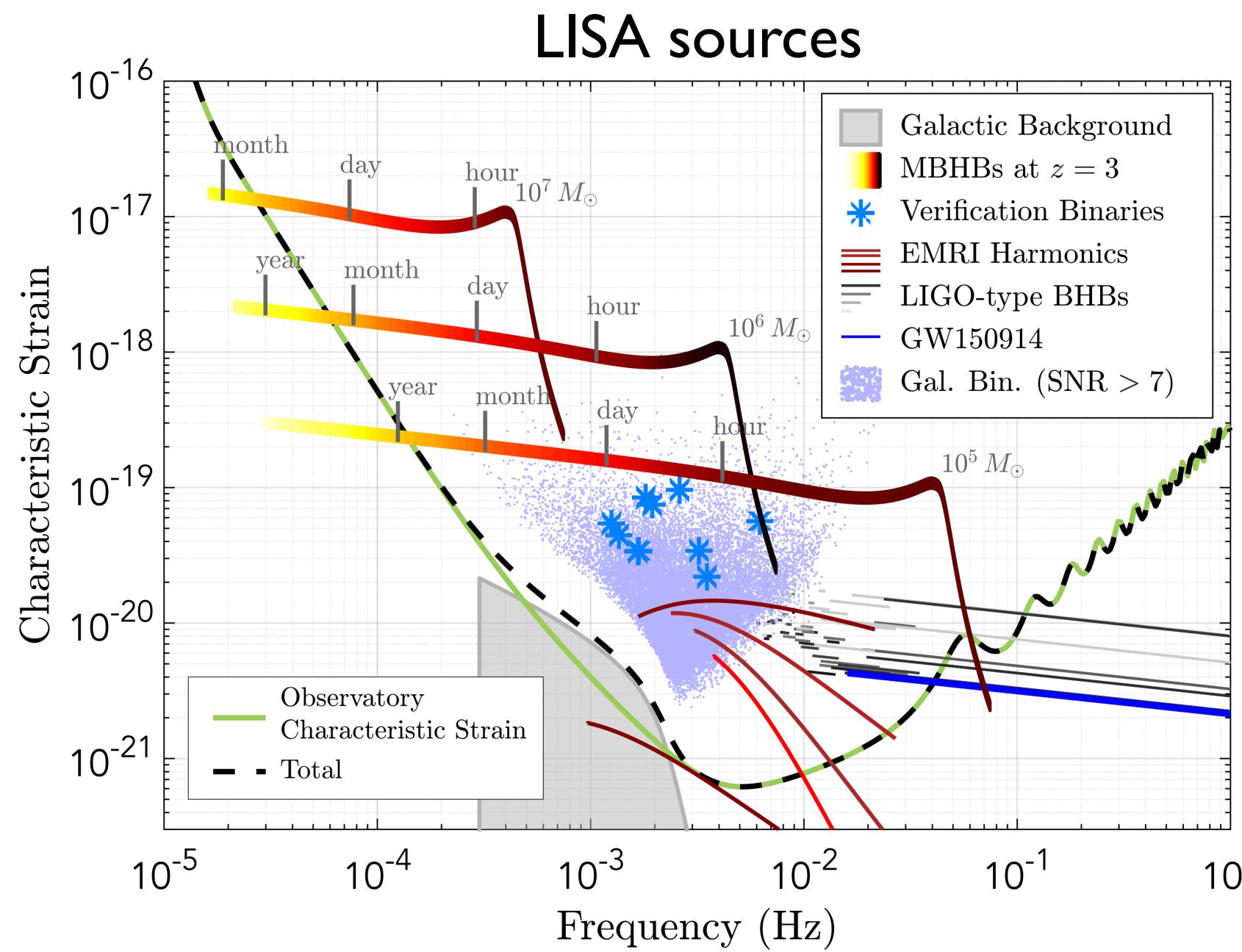
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in collaboration with J. Baker (NASA GSFC), T. Dal Canton (LAL), S. Babak (APC), A. Toubiana (APC), M. Katz (AEI)

[Marsat&Baker arXiv/1806.10734]
[Marsat, Baker, Dal Canton arXiv/2003.00357]

LISA MBHB science and motivation



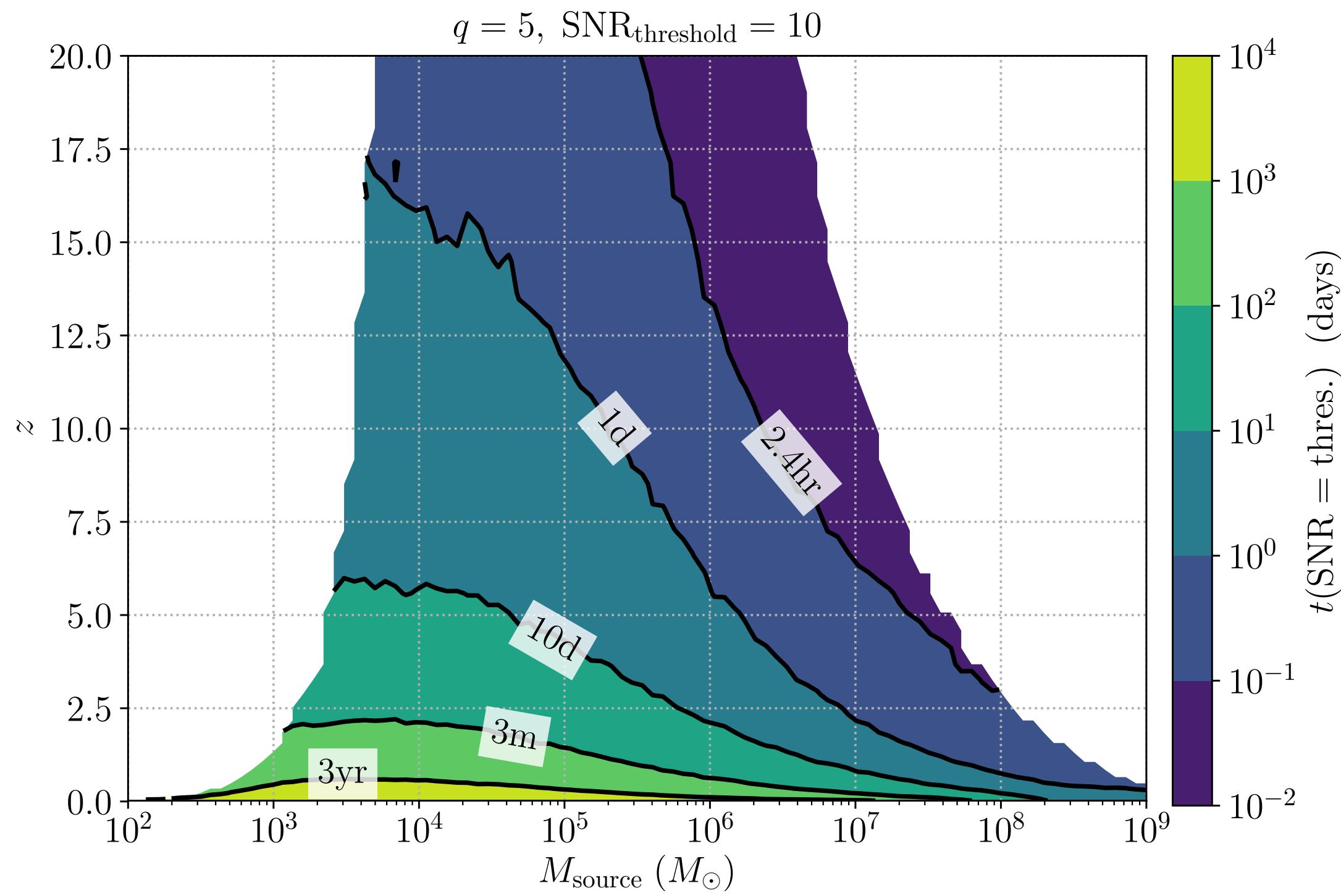
Motivation: what will LISA tell us ?

Massive black hole binaries (MBHB) are crucial sources for LISA.
How well can LISA extract their parameters ?

- Science questions: advance detection ? Localization for EM counterpart ?
Cosmology ? Constrain population and formation channels ?
- Link to instrumental choices: tradeoffs
- Aim: Bayesian parameter estimation (PE) tools
- More realistic signals: include merger and higher harmonics
- Understand degeneracies

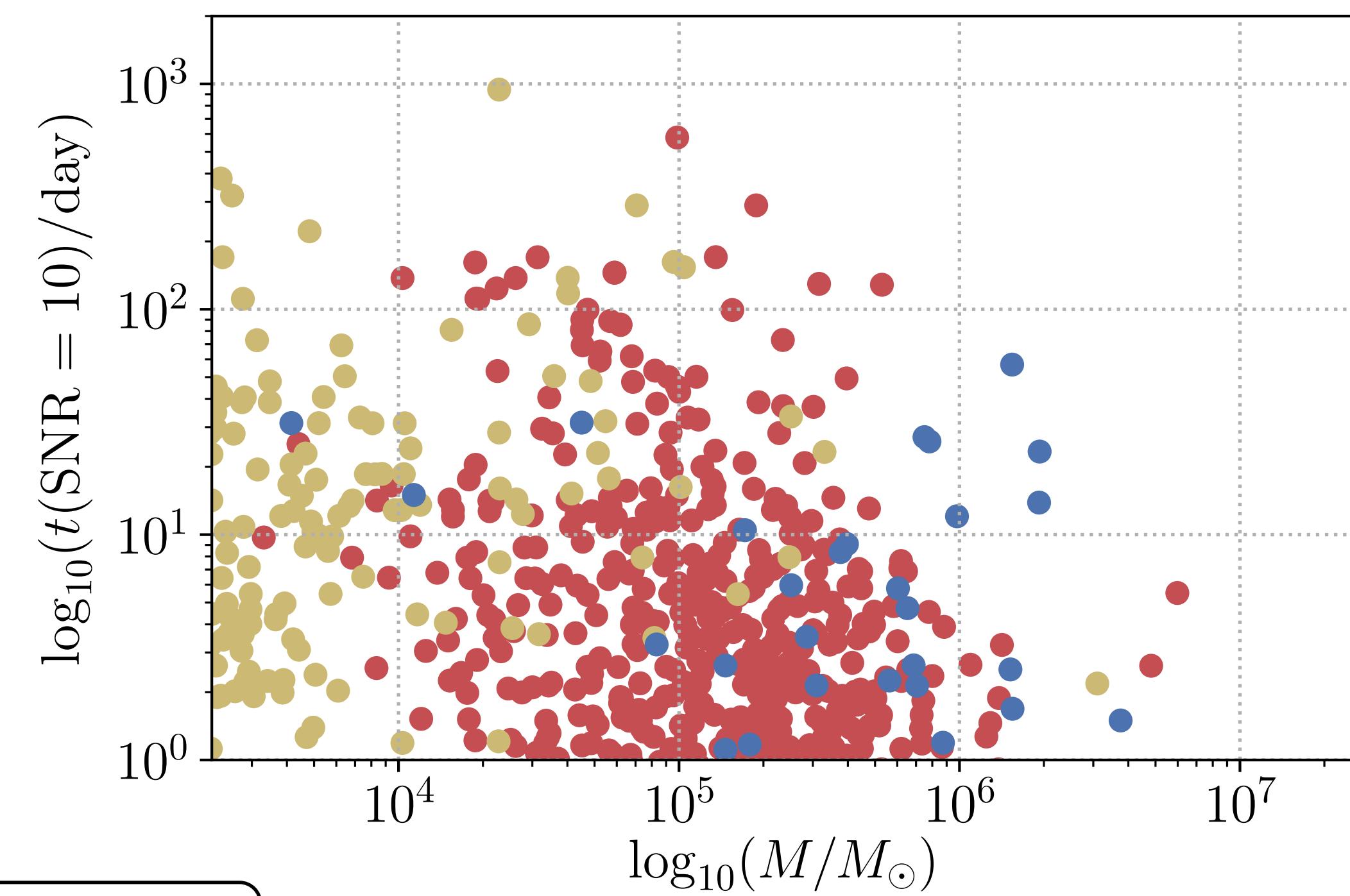
The length(s) of MBHB signals

- How long before merger can we detect the signal ?
- SNR=10 to claim detection



MBHB detected signals:
Bulk shorter than ~ 10 days
Tail extending to ~ 3 months

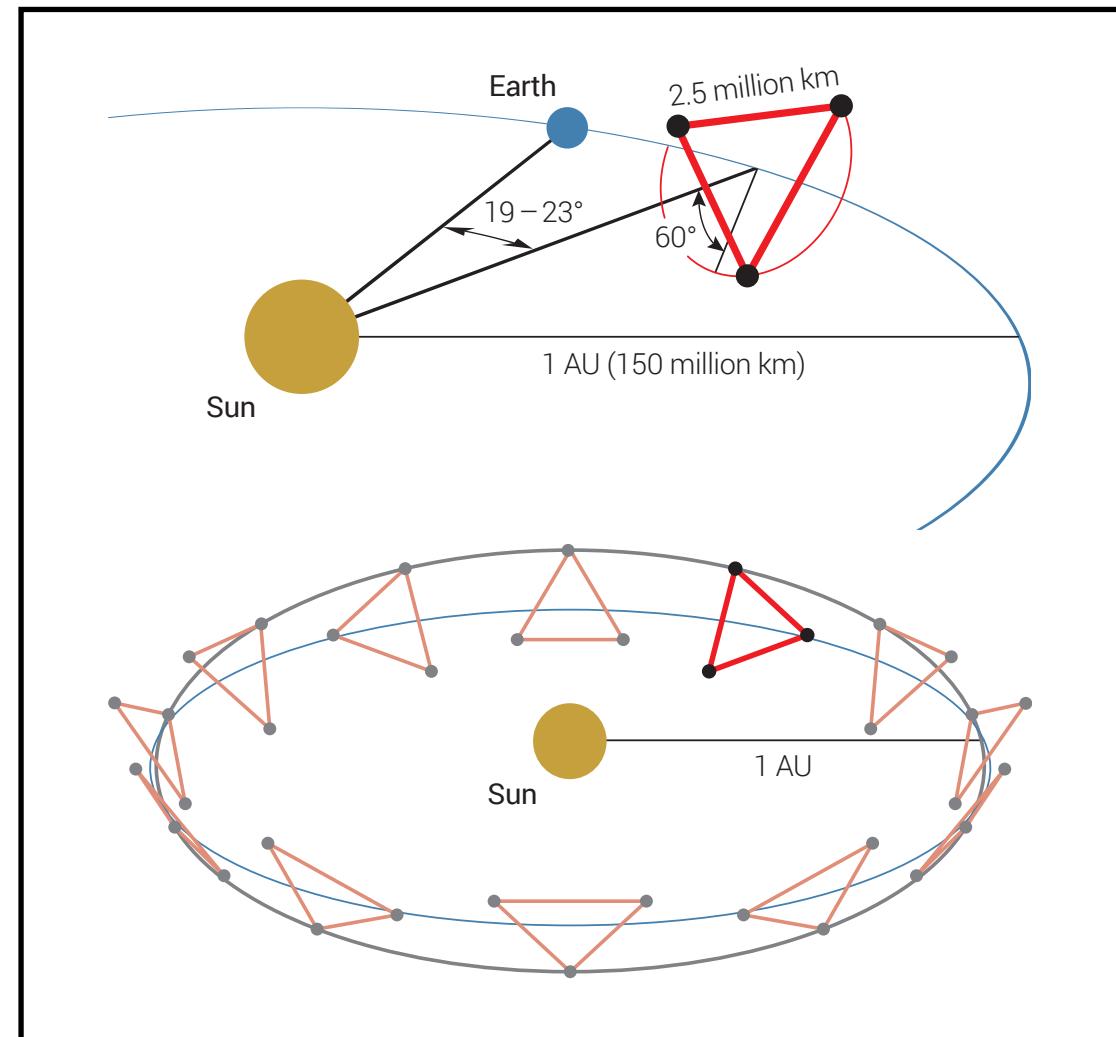
- Astrophysical models [Barausse 2012]:
- Heavy seeds - delay
 - Heavy seeds - no delay
 - PopIII seeds - delay



LISA instrumental response

LISA orbits

SSB-frame: global view of the orbits



Response

From spacecraft s to spacecraft r through link s: $y = \Delta\nu/\nu$

$$y_{slr} = \frac{1}{2} \frac{1}{1 - \hat{k} \cdot n_l} n_l \cdot (h(t_s) - h(t_r)) \cdot n_l$$

+ Time-delay interferometry (TDI)

Fourier-domain (separation of timescales [Marsat-Baker 2018])

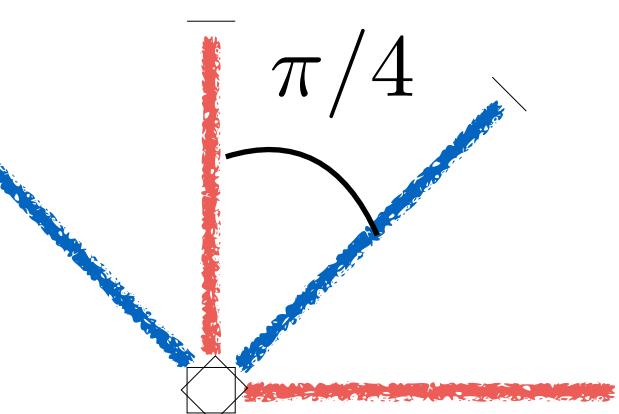
$$\mathcal{T}_{slr} = \frac{i\pi f L}{2} \text{sinc} [\pi f L (1 - k \cdot n_l)] \exp [i\pi f (L + k \cdot (p_r + p_s))] n_l \cdot P \cdot n_l(\mathbf{t}_f)$$

Time and frequency-dependency

Time: motion of LISA on its orbit

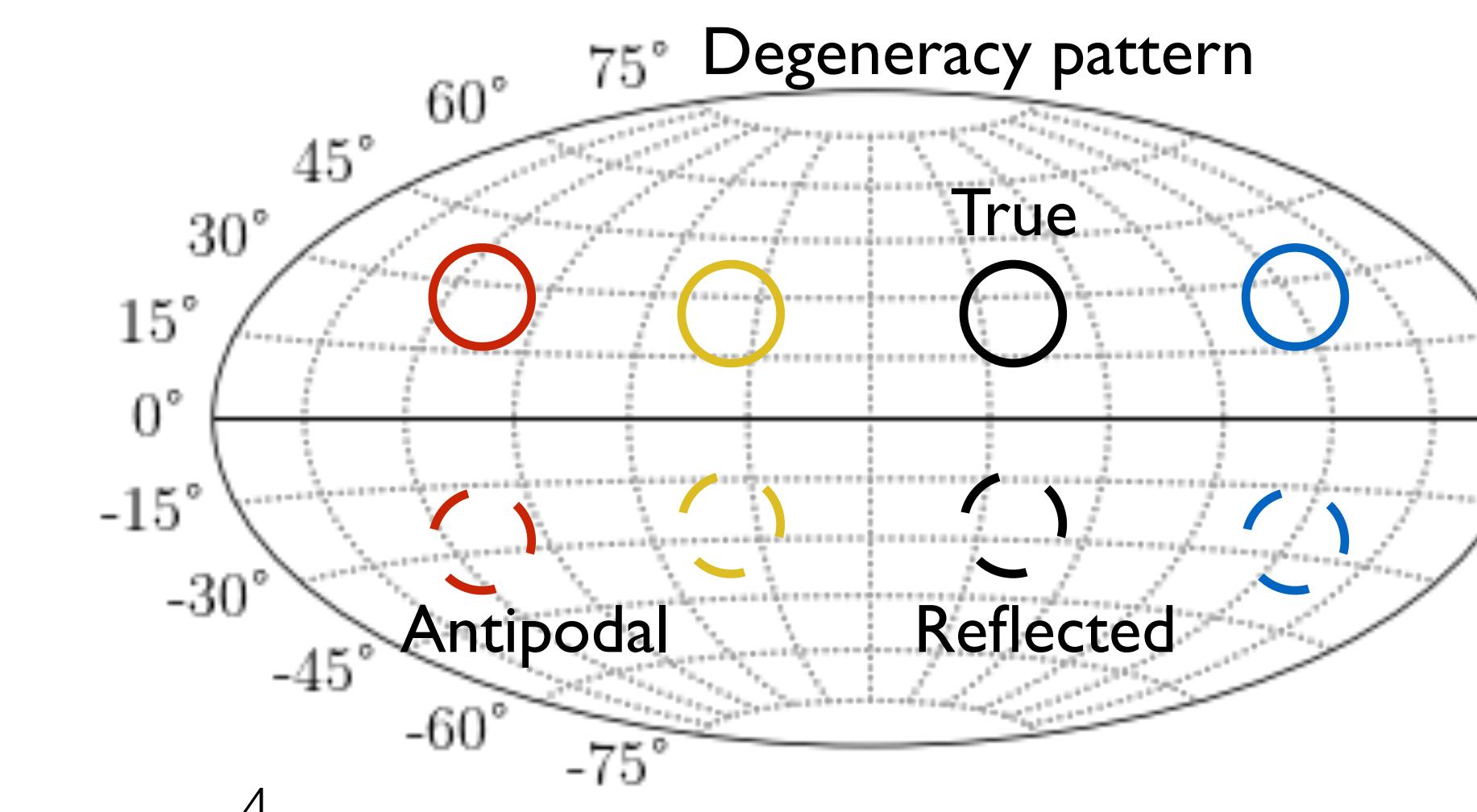
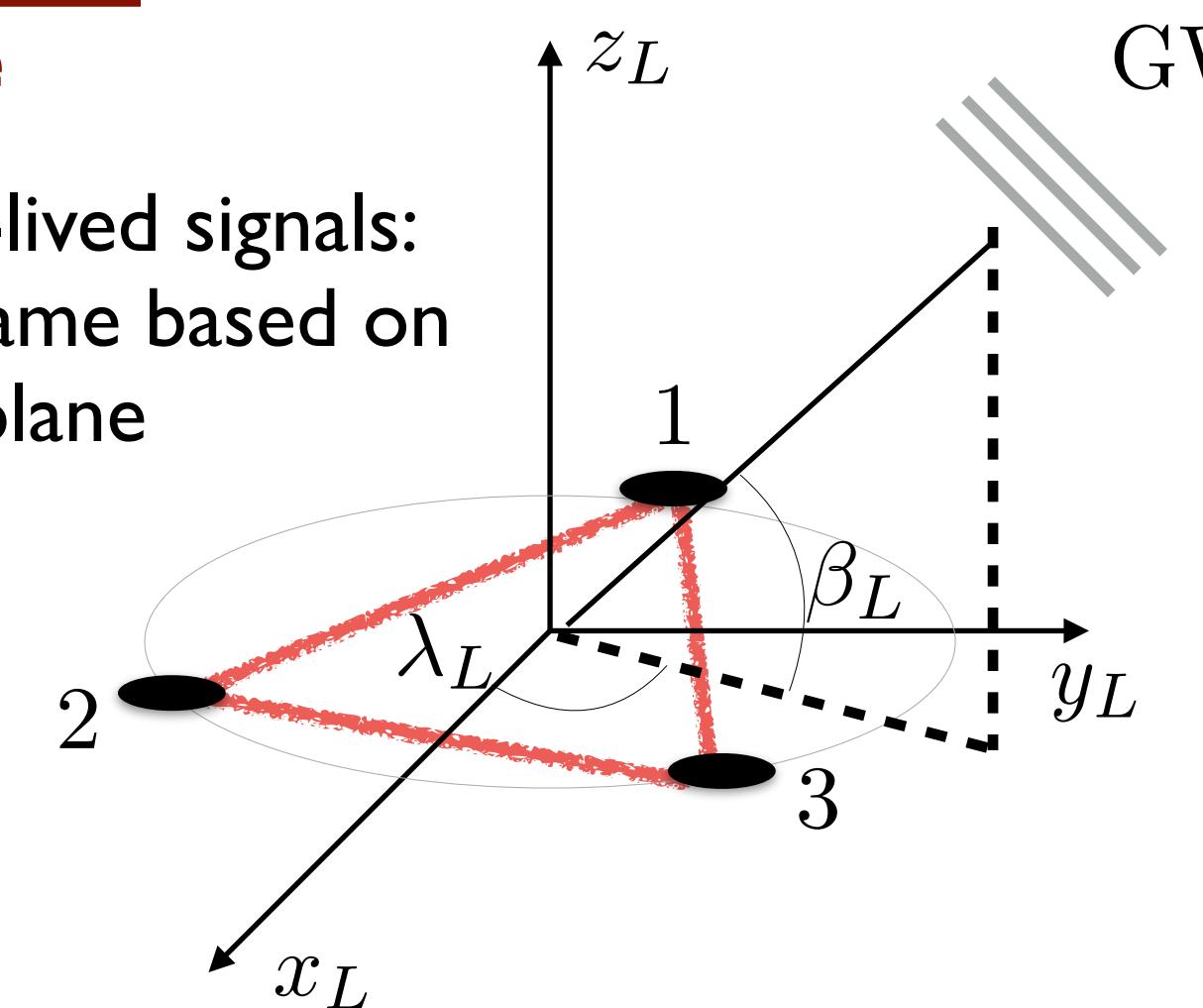
Frequency: departure from long-wavelength

Low-f approximation: two LIGO-type detectors in motion [Cutler 1997]
High-f: more complicated



LISA frame

Short-lived signals:
use frame based on
LISA plane

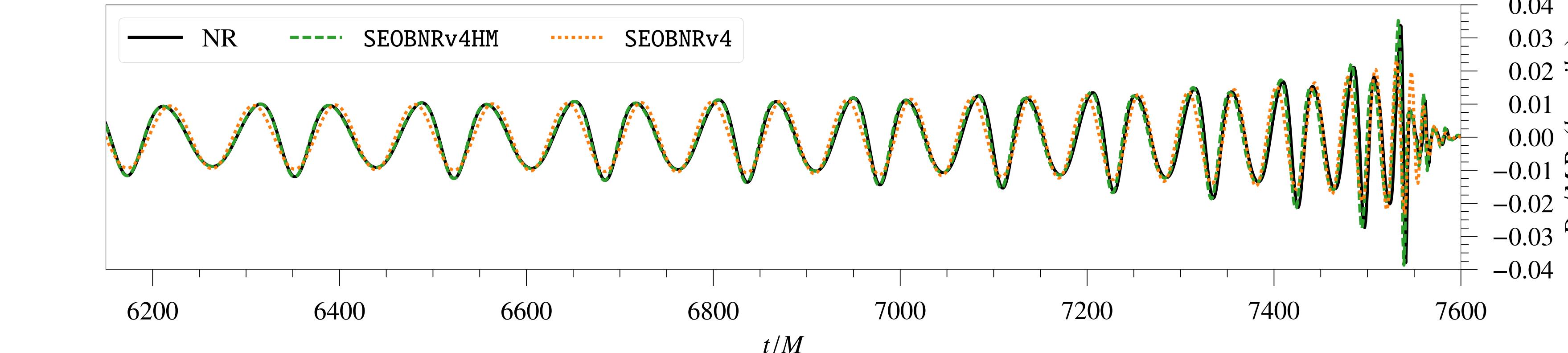


Higher harmonics in GW signals

Higher harmonics hlm

Example in time domain:

$$(q = 8, \chi_1 = 0.5, \chi_2 = 0, \iota = \pi/2, \varphi_0 = 1.2)$$

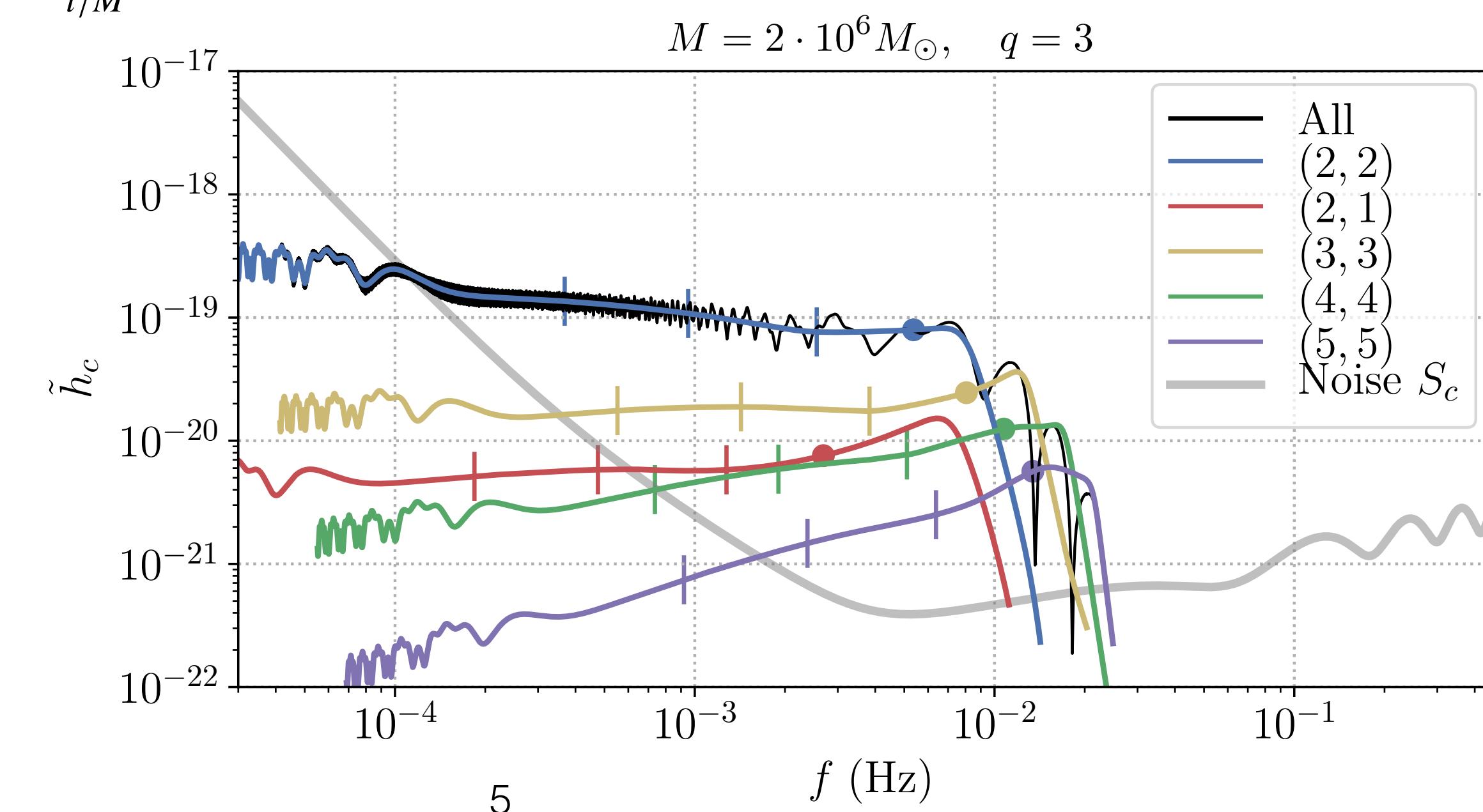


- Dominant harmonic h22
- Higher harmonics stronger at merger/ringdown
- Higher harmonics more important for high q and edge-on

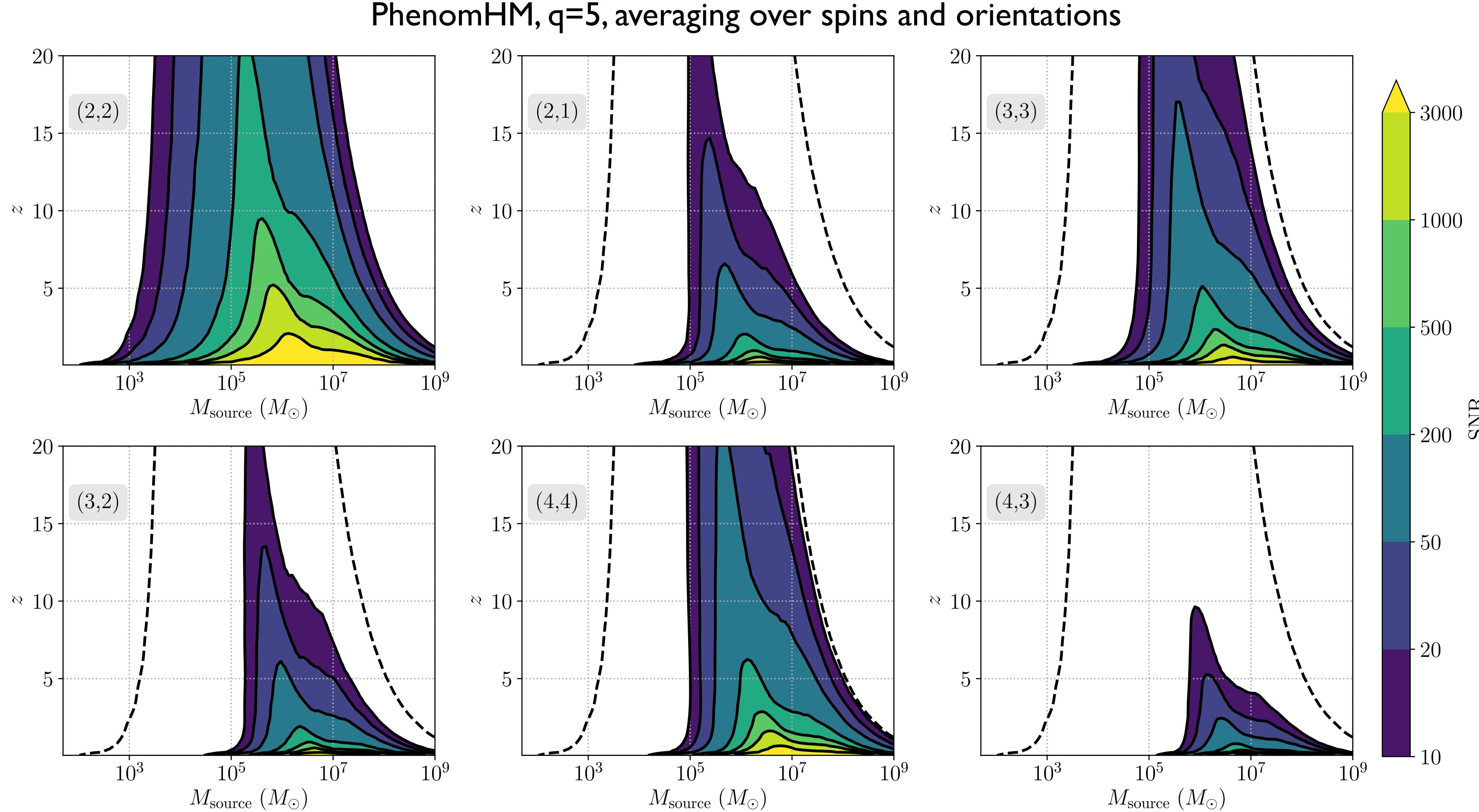
MBHB with higher harmonics

Ticks:

- SNR/64 (40h)
- SNR/16 (2.5h)
- SNR/4 (7min)
- merger



The SNR of higher harmonics



[Preliminary]

'Mode SNR':

$$\sqrt{(s_{\ell m} | s_{\ell m})}$$

Cross-terms not negligible

$$\sum_{(\ell m) \neq (\ell' m')} (s_{\ell m} | s_{\ell' m'})$$

Bayesian methodology

Bayesian analysis

$$\text{Posterior: } p(\theta|d) = \frac{\mathcal{L}(d|\theta)p_0(\theta)}{p(d)}$$

$$\text{Likelihood: } \ln \mathcal{L}(d|\theta) = - \sum_{\text{channels}} \frac{1}{2} (h(\theta) - d | h(\theta) - d)$$

$$\text{Data: signal+noise } d = s + n$$

- PE example: 0-noise simulation, n=0
- LDC: noise included

Samplers:

- MultiNest: Nested Sampling
- Parallel Tempering MCMC, differential evolution

Millions of likelihoods
needed

Computational performance of
waveforms/likelihoods crucial

Waveforms

$$h_+ - i h_\times = \sum_{\ell,m} {}_{-2}Y_{\ell m}(\iota, \varphi) h_{\ell m}$$

- PE example: EOBNRv2HM (Non-spinning model, includes modes (22, 21, 33, 44, 55) + Reduced Order Model (ROM) for sub-ms evaluation)
- LDC: PhenomD (22, spins aligned)
- Fast Fourier-domain LISA response [Marsat-Baker 2018]

Accelerated likelihoods

Likelihood for n=0:

$$(h_1|h_2) = 4\text{Re} \int df \frac{\tilde{h}_1(f)\tilde{h}_2^*(f)}{S_n(f)}$$

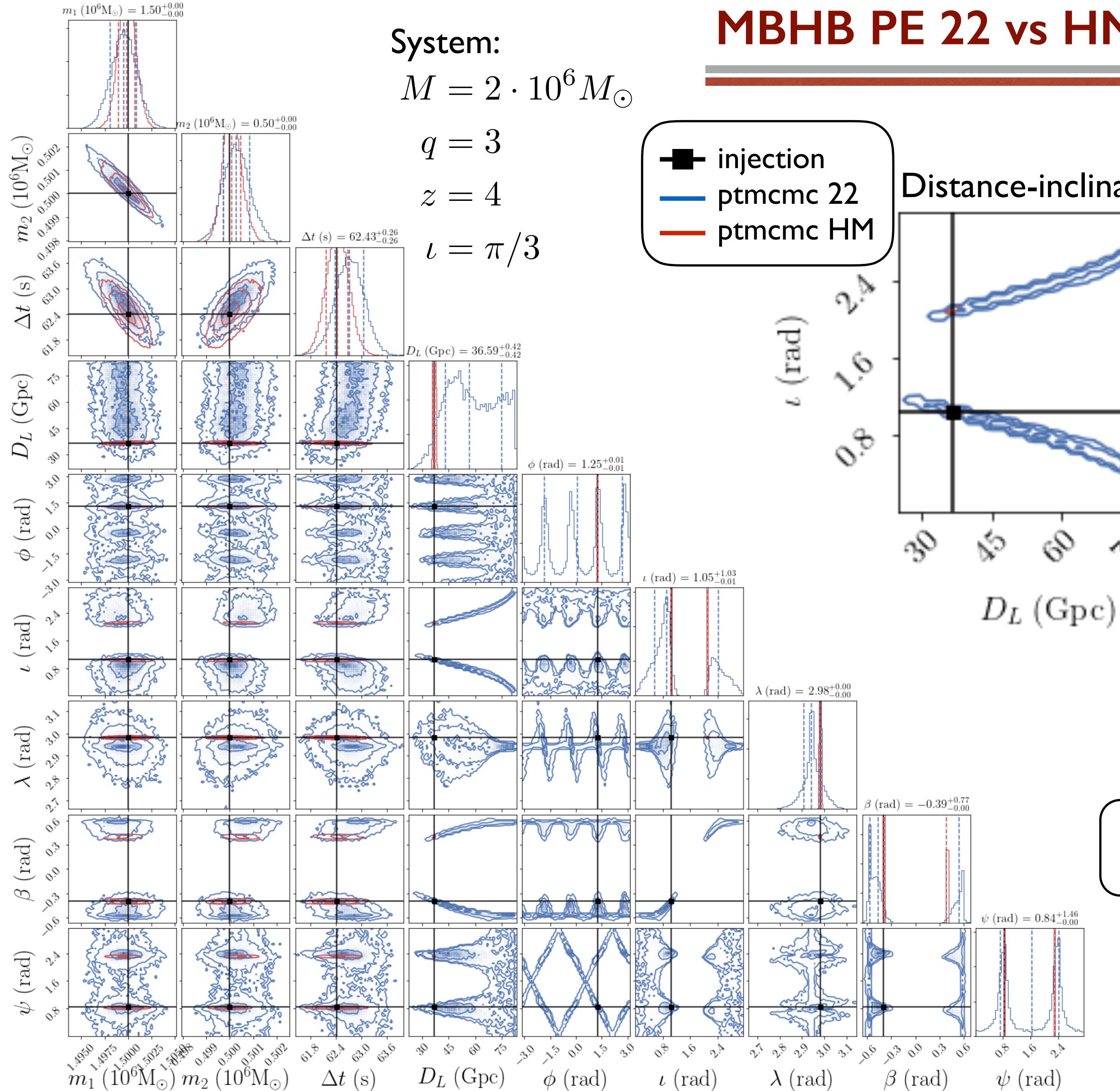
- Sparse grids: amplitude/phase and response
- Cubic spline representation 300-800 pts
- **Cost: h22: 1-2ms, 5 modes hlm: 15ms**

Likelihood with noise:

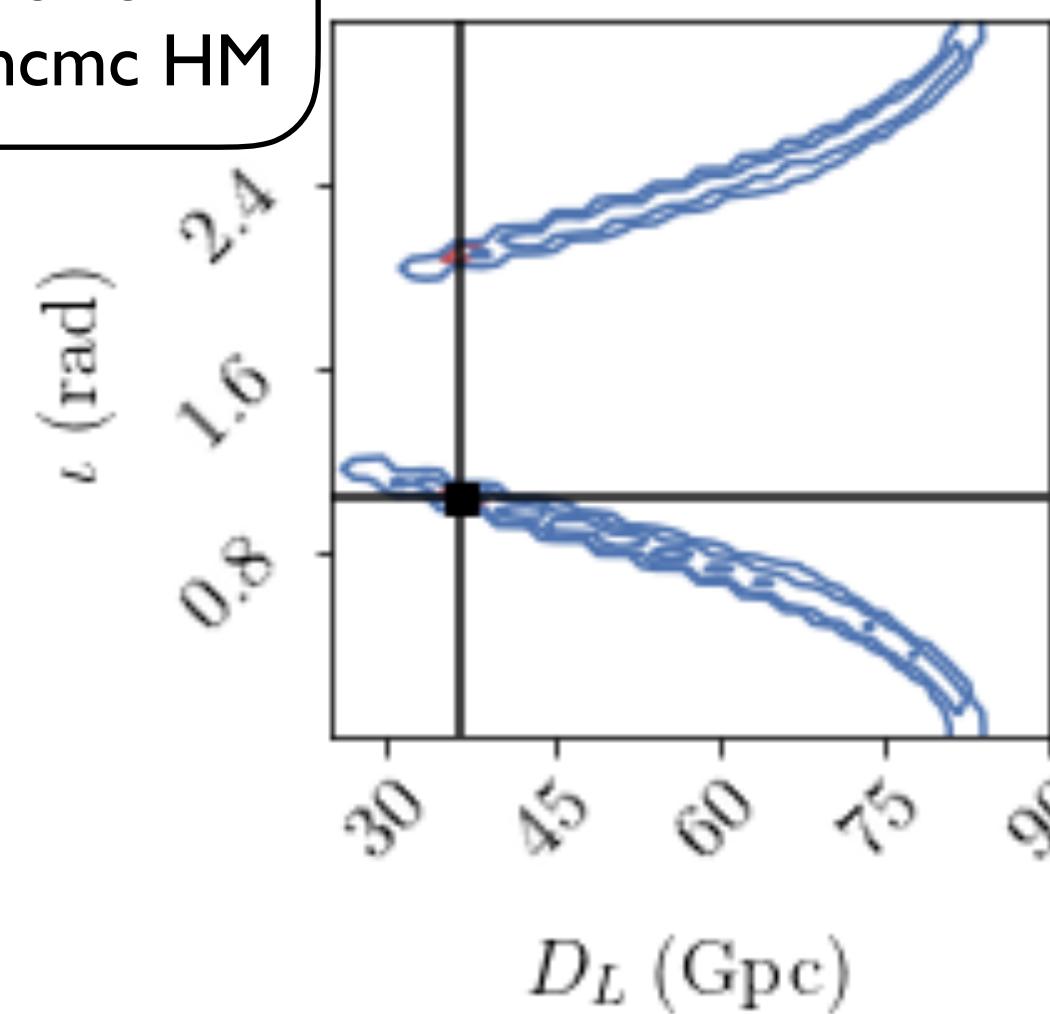
- Downsample for a short MBHB signal
- **Cost: h22: 3-5ms**

+ GPU acceleration:
[Katz&al 2020]

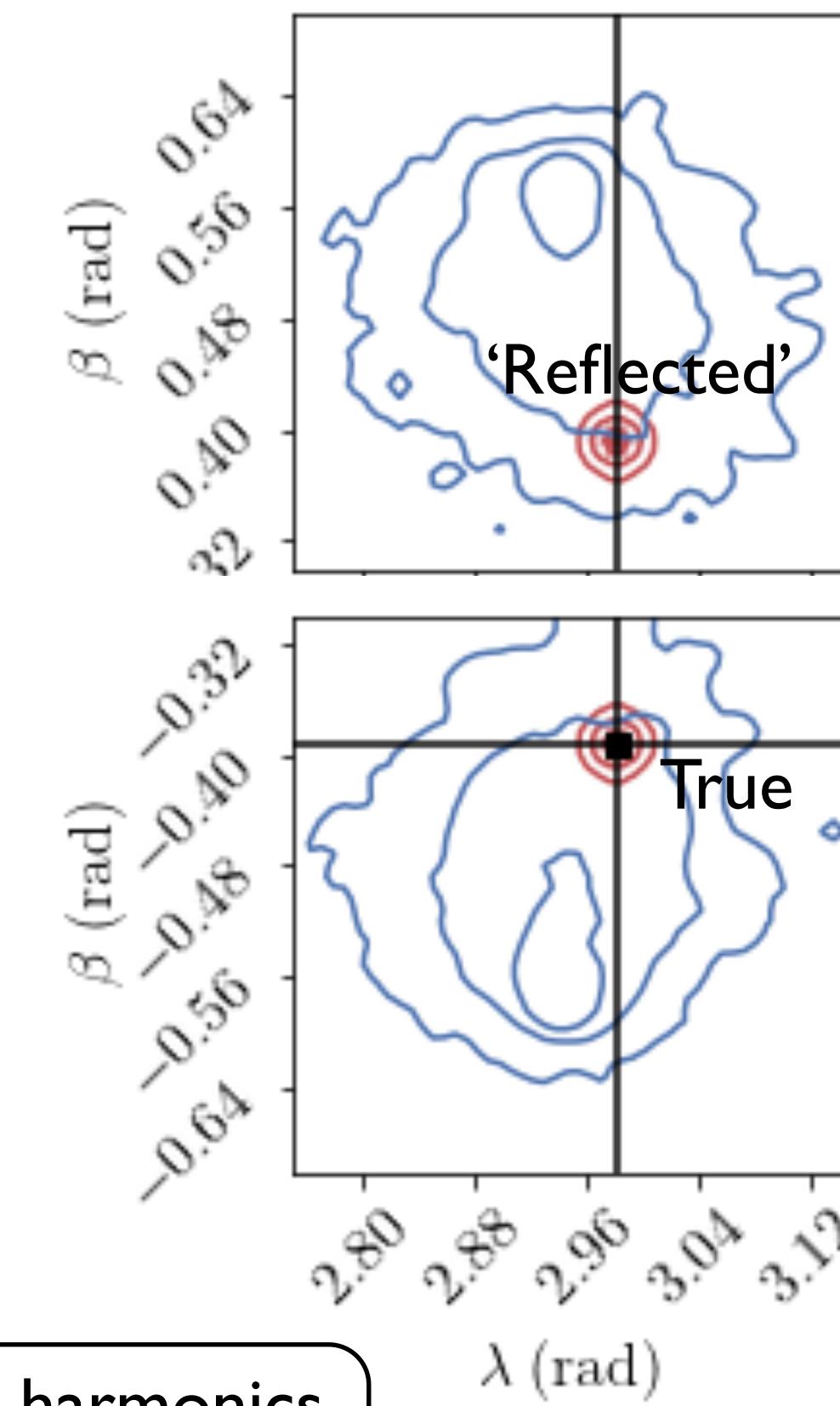
MBHB PE 22 vs HM: degenerate case



Distance-inclination

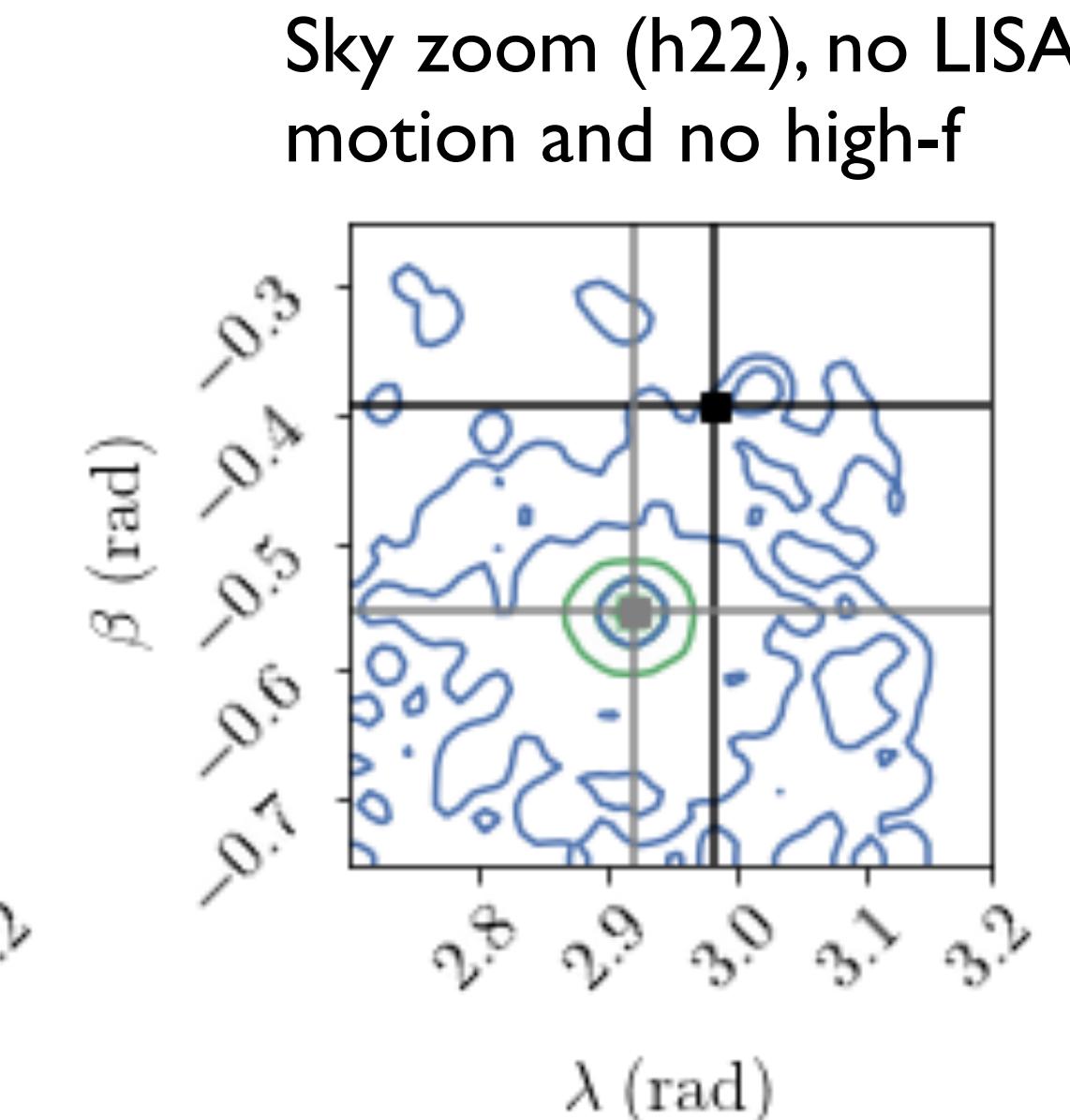
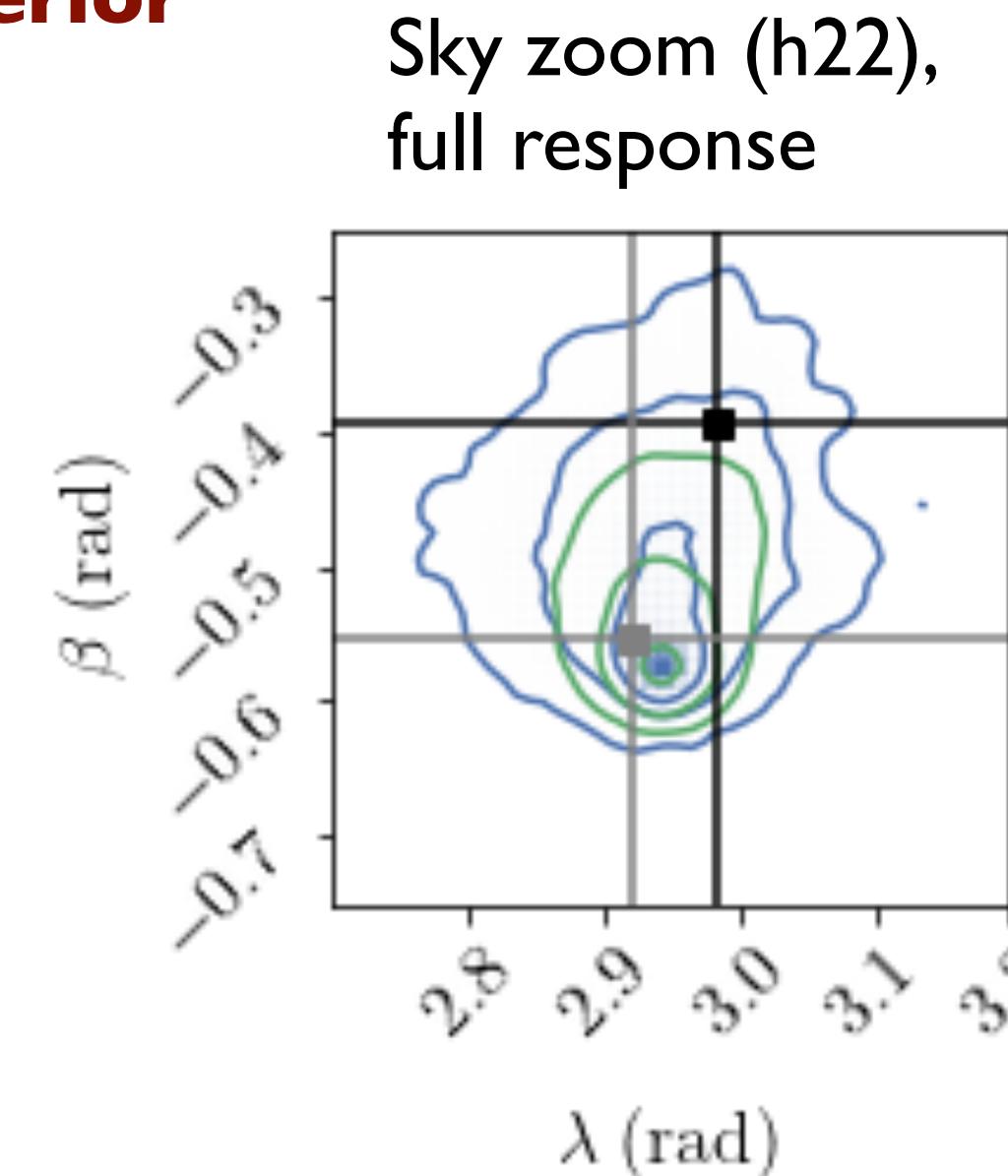
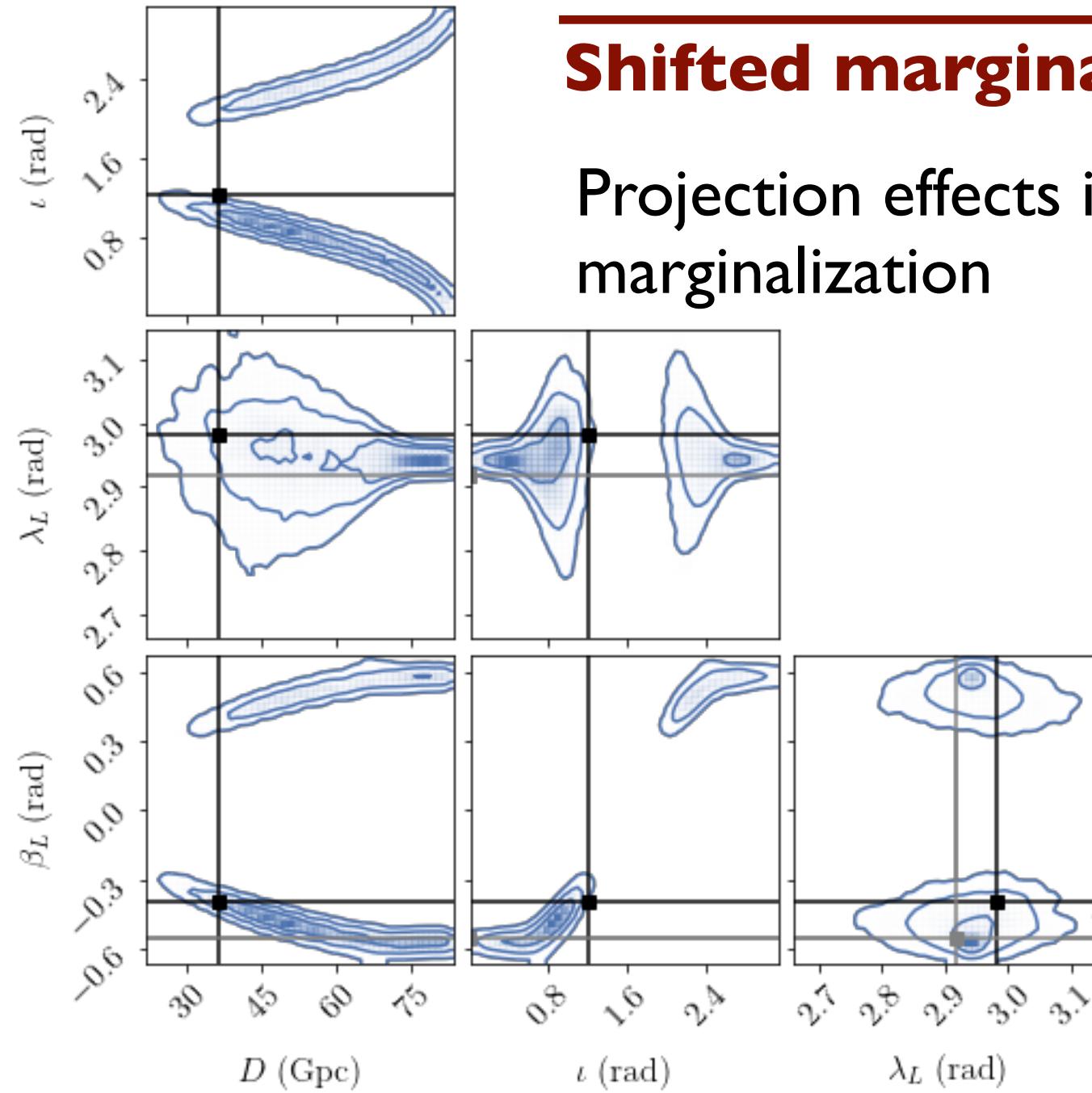


Sky position



Higher harmonics
crucial

Understanding degeneracy breaking by higher harmonics



■ injection
— ptmcmc 22
— multinest 22
■ analytic degeneracy

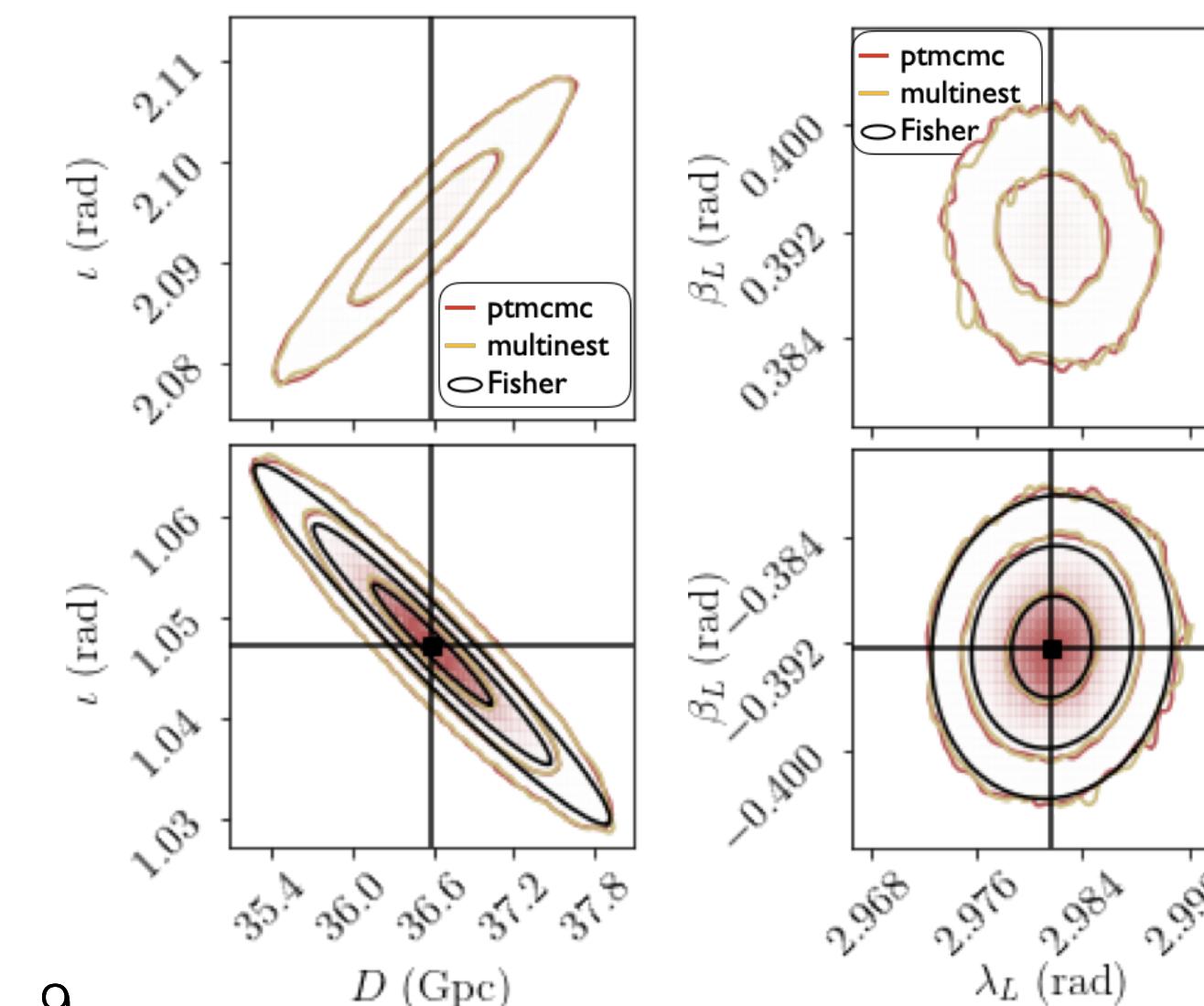
The role of higher harmonics

$$h_+ - ih_\times = \sum {}_{-2} Y_{\ell m}(\iota, \varphi) h_{\ell m}$$

$${}_{-2} Y_{\ell m}(\iota, \varphi) = {}_{-2} Y_{\ell m}(\iota, 0) e^{im\varphi}$$

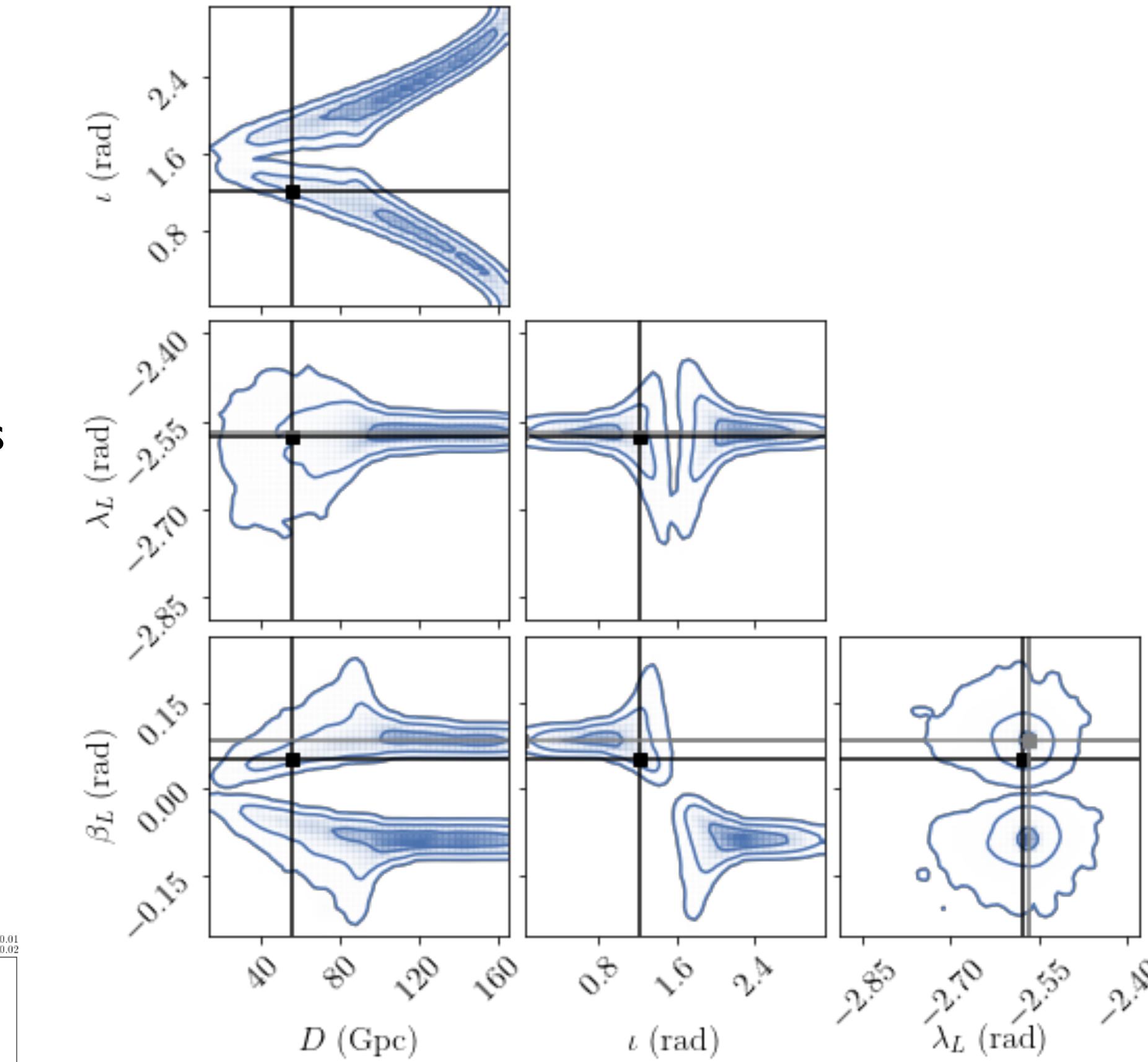
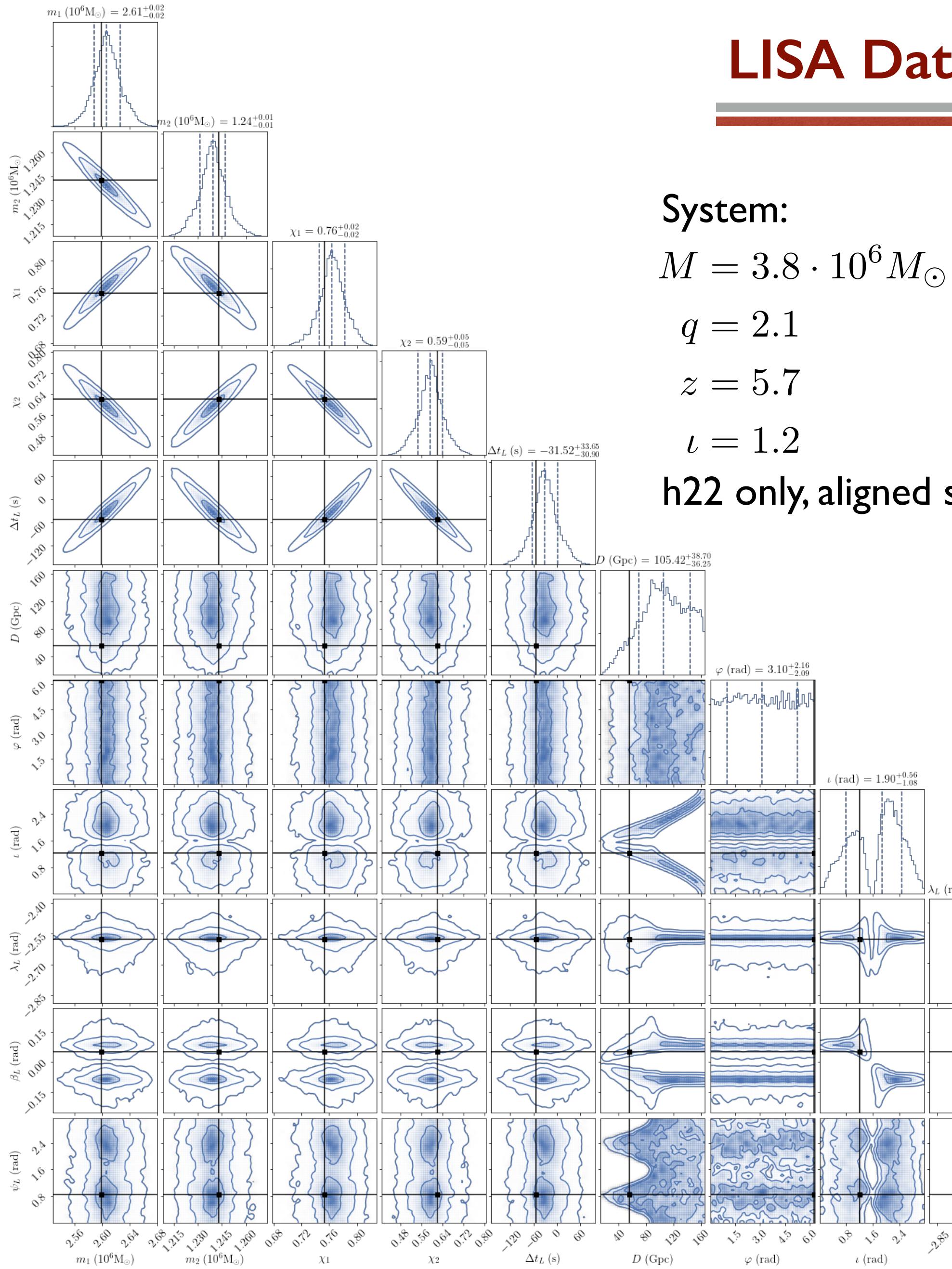
When measuring several modes $h_{\ell m}$:

- Distance/inclination degeneracy broken
- Phase independently measured
- Better sky localization (caveat: edge-on)



Higher harmonics break degeneracies, still bimodal in sky

LISA Data Challenge - I MBHB results



Pre-merger analysis: accumulation of information with time

Cutting signals before merger

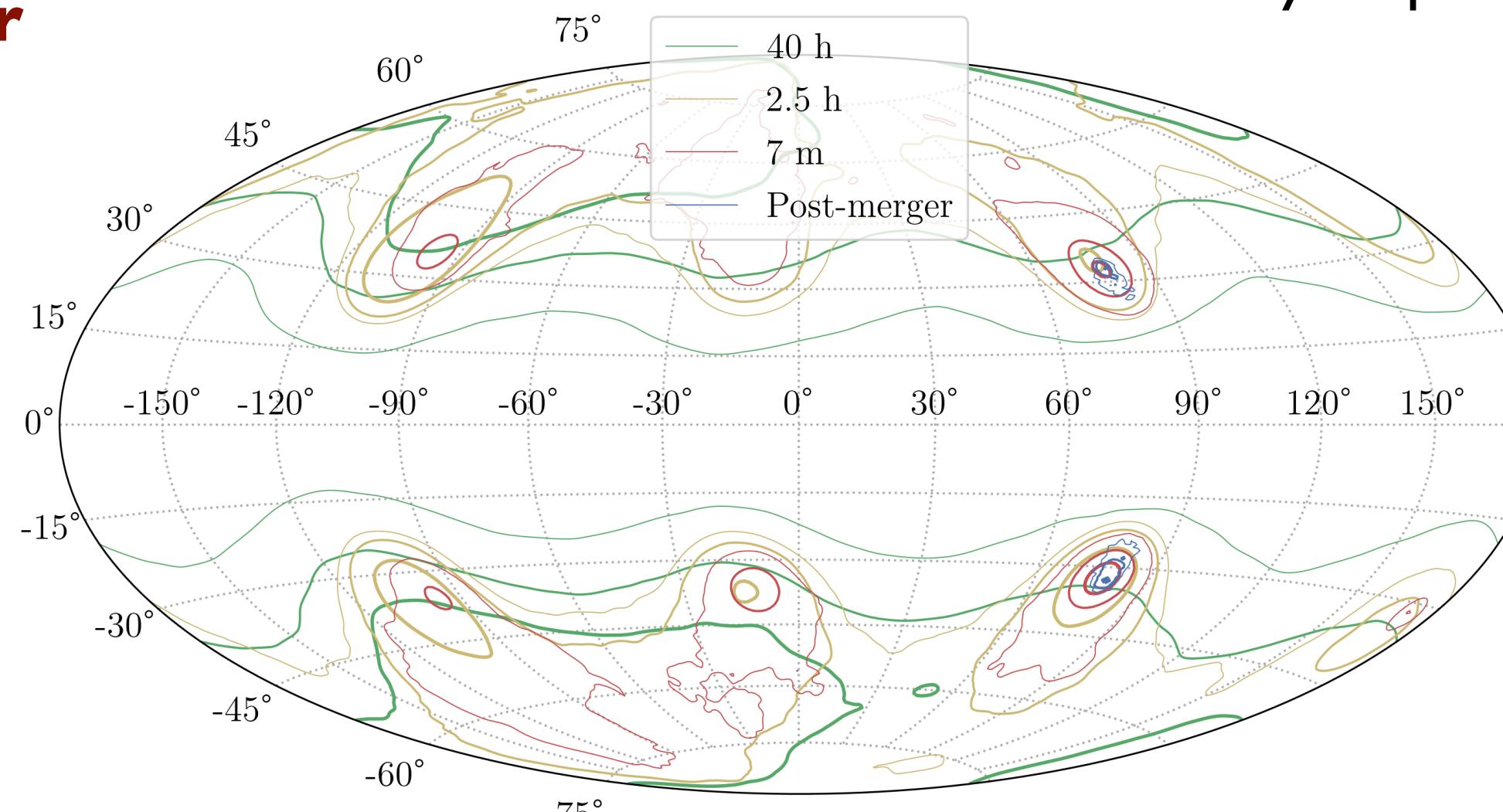
SNR-based cuts:

SNR	DeltaT
10	40h
42	2.5h
167	7min
666	-

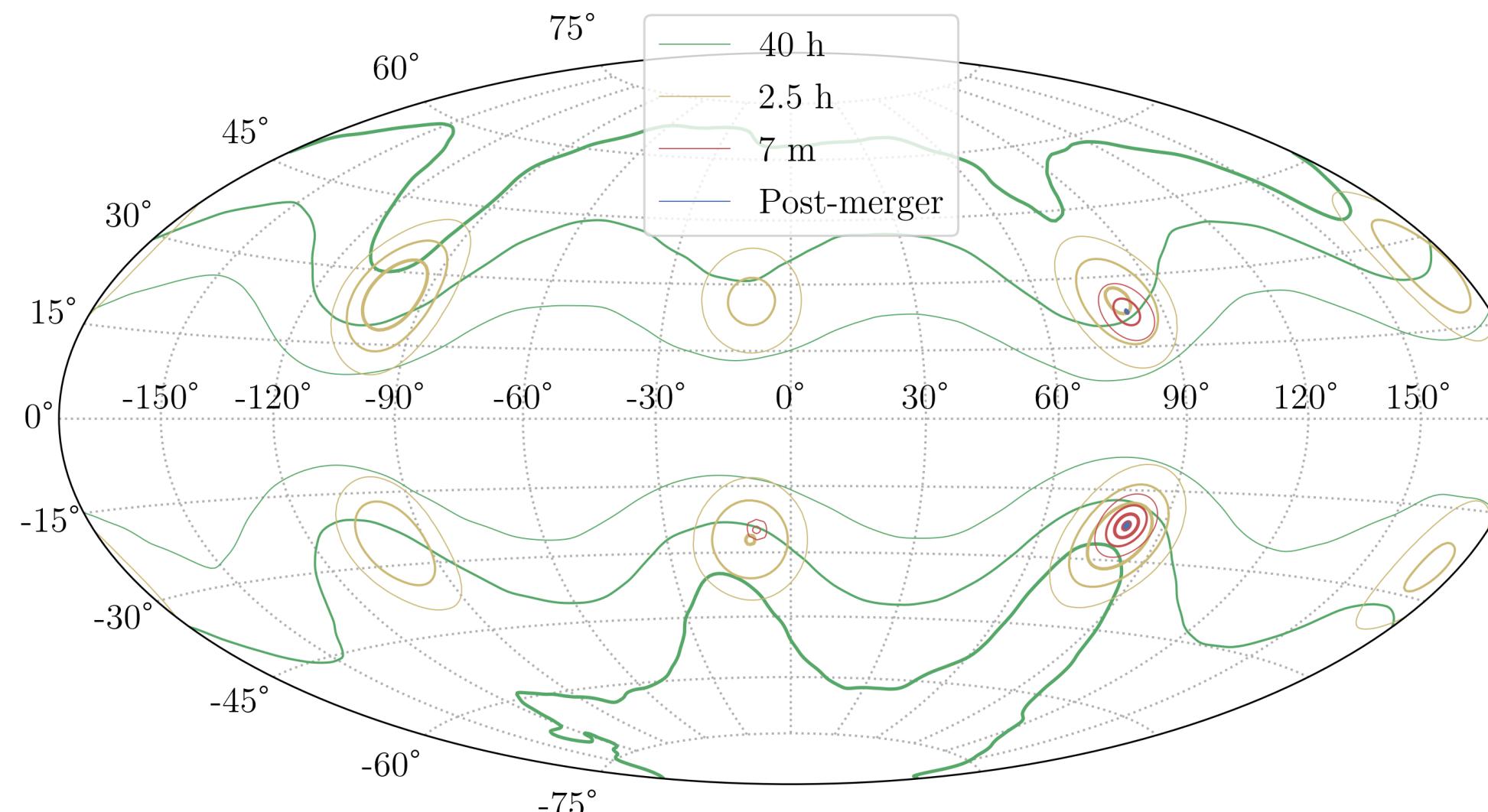
Note: z=4, short signal

Premerger localization (Fisher):
[Mangiagli&al 2020]

LISA-frame sky map 22



LISA-frame sky map hm



8-maxima sky degeneracy
only broken shortly before merger
2-maxima sky degeneracy
survives after merger ('Reflected')

Multimodal degeneracies
pre-merger
Not a golden source !

Pre-merger analysis: decomposing the instrument response

Decomposing the response

$$\mathcal{T}_{slr} = \frac{i\pi f L}{2} \text{sinc} [\pi f L (1 - k \cdot n_l)] \exp [i\pi f (L + k \cdot (p_r + p_s))] n_l \cdot P \cdot n_l(t_f)$$

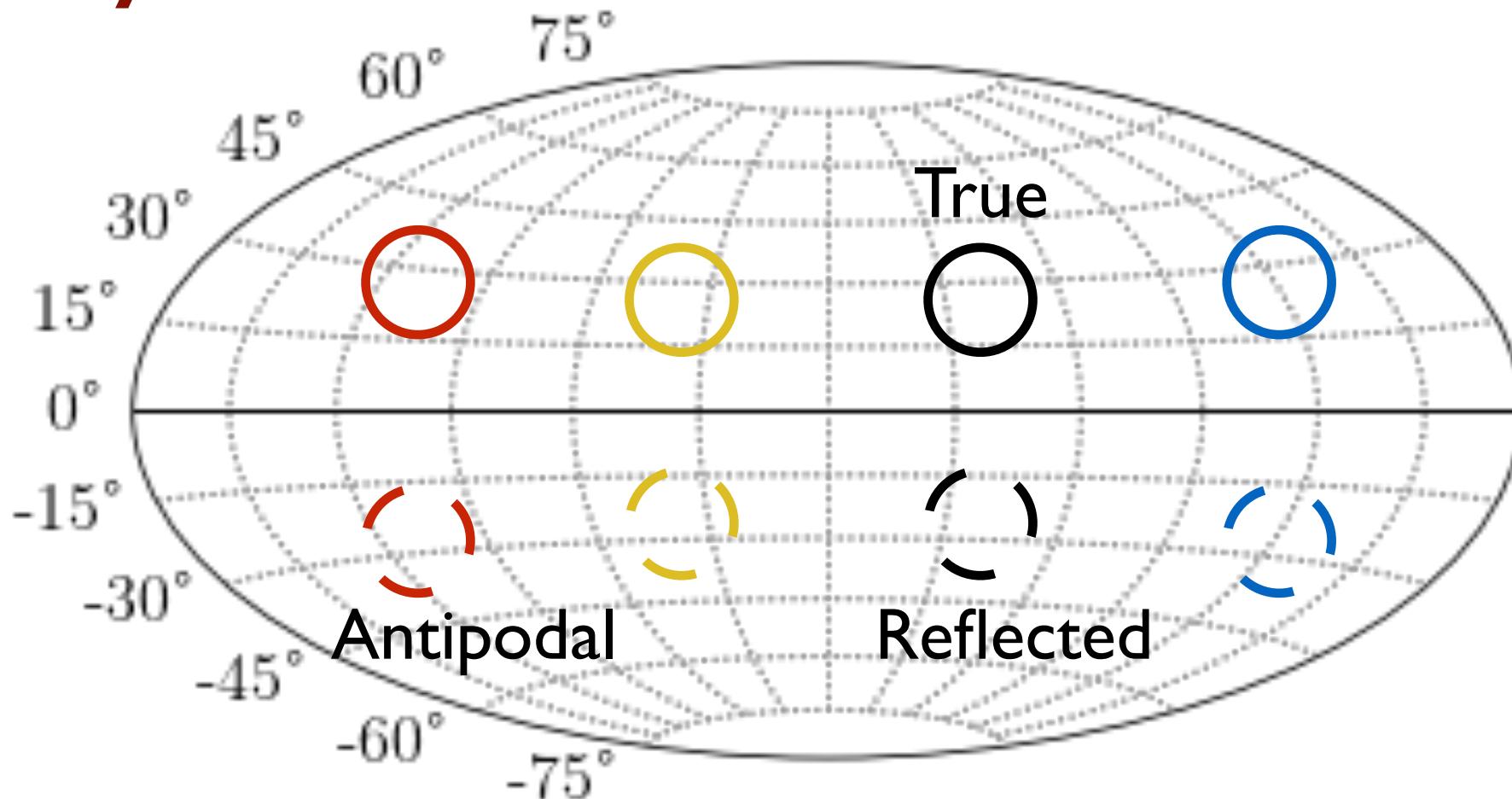
Time and frequency-dependency in transfer functions

Time: motion of LISA on its orbit

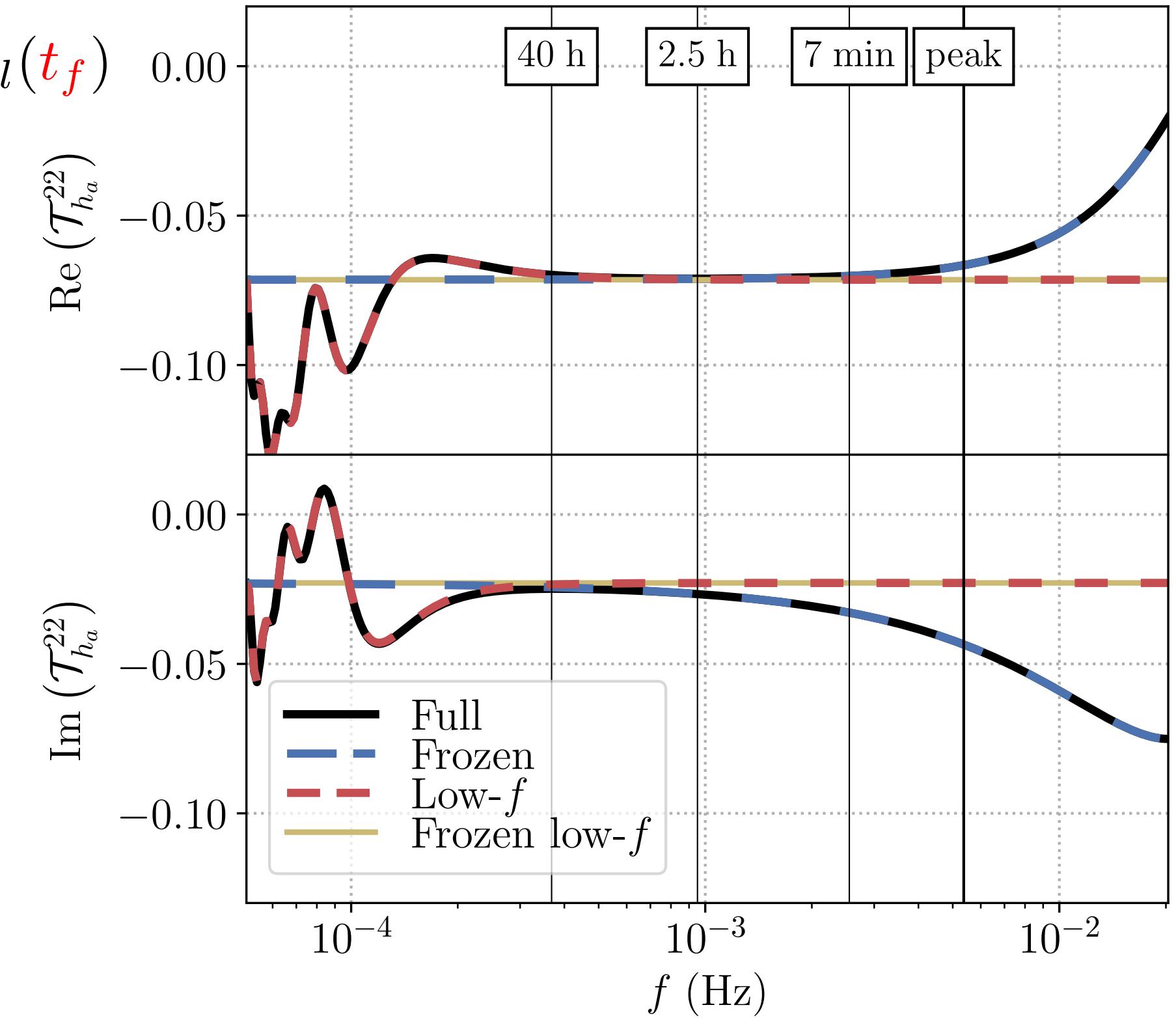
Frequency: departure from long-wavelength approx.

- Response:
- ‘Full’: keep all terms
 - ‘Frozen’: ignore LISA motion
 - ‘Low-f’: ignore f-dependency
 - ‘Frozen Low-f’: ignore both

Sky modes L-frame



- ‘True’: true position
- ‘Reflected’: eliminated by motion, degenerate for high-f
- ‘Antipodal’: degenerate for motion, eliminated by high-f



Pre-merger analysis: likelihood with decomposed response

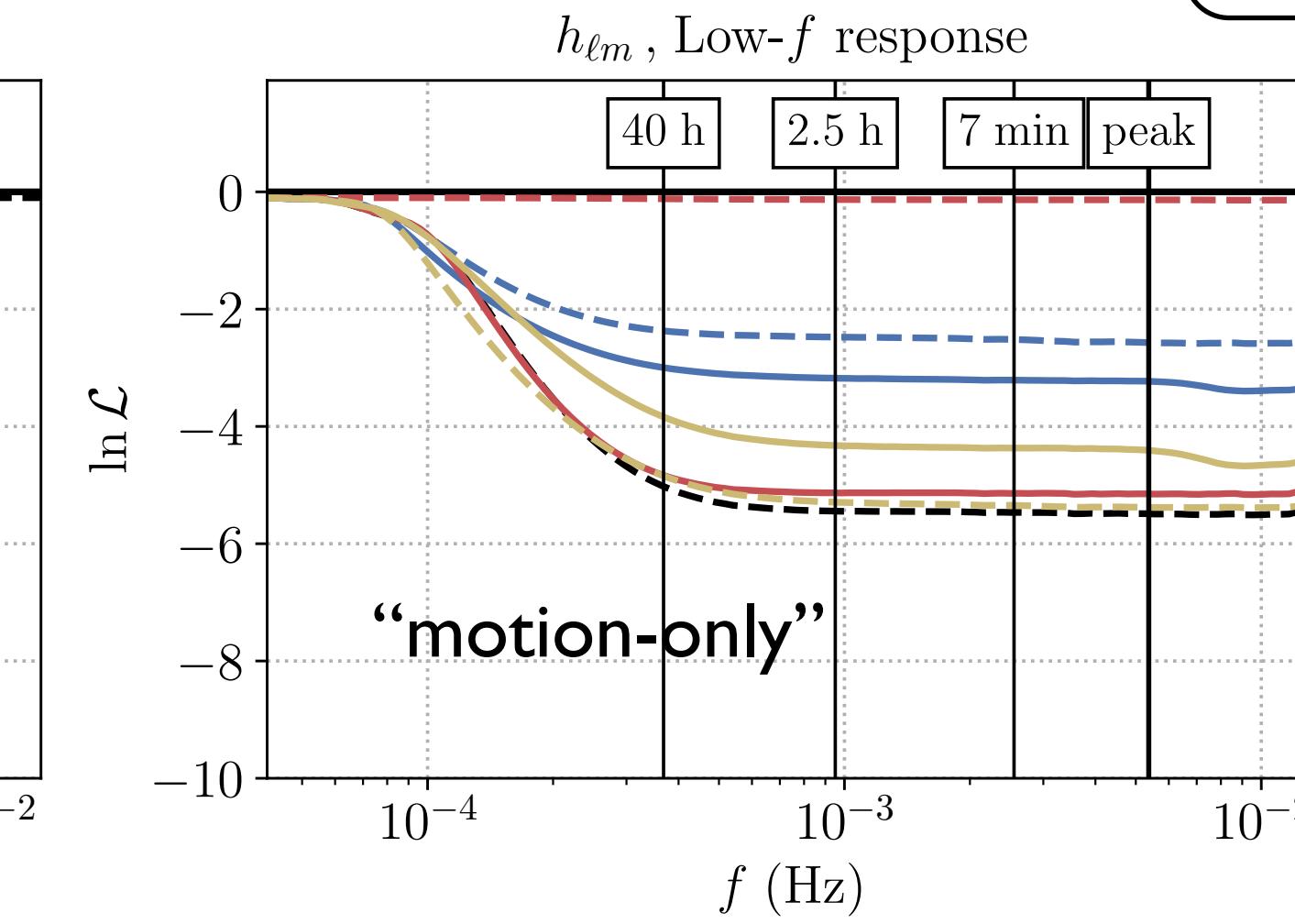
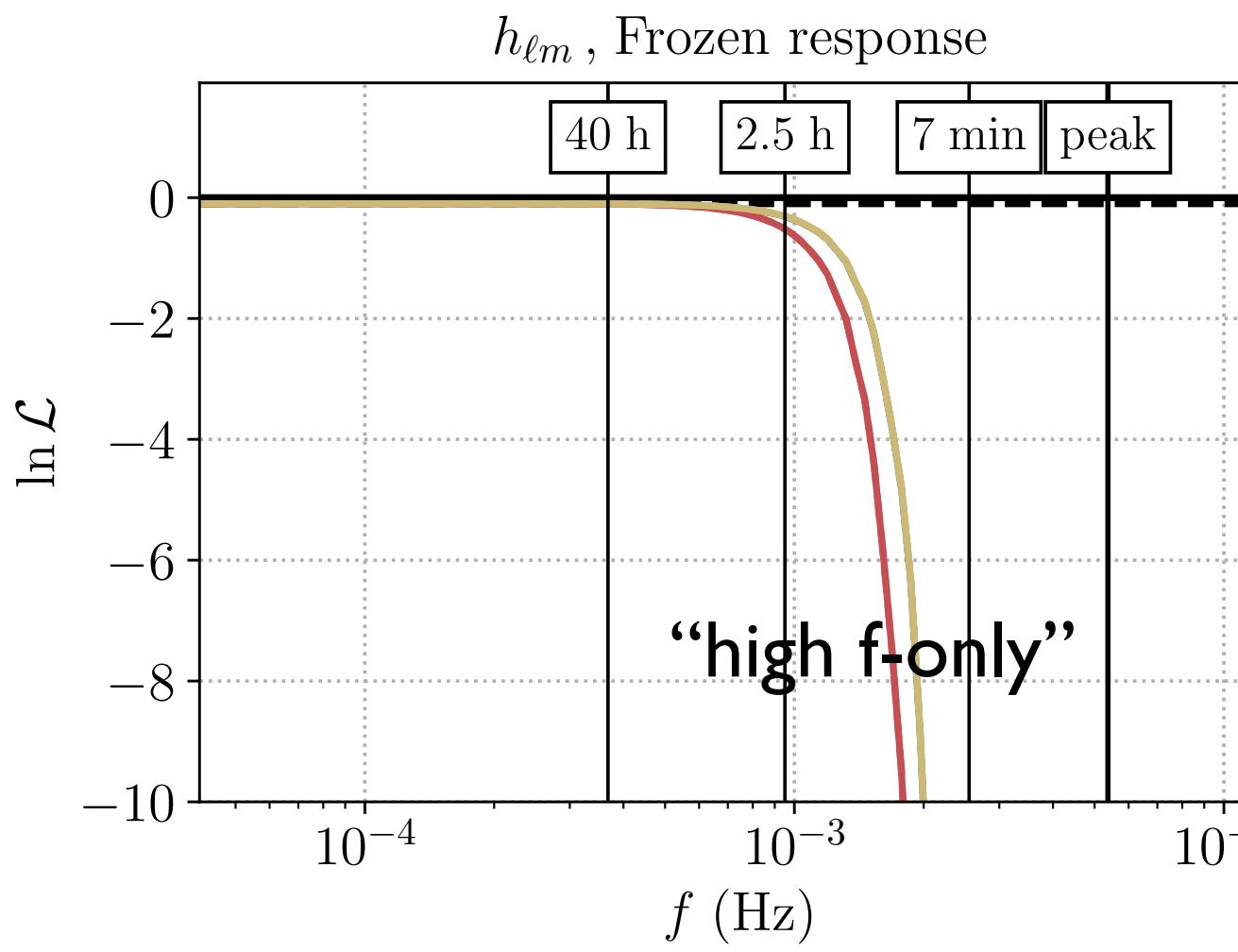
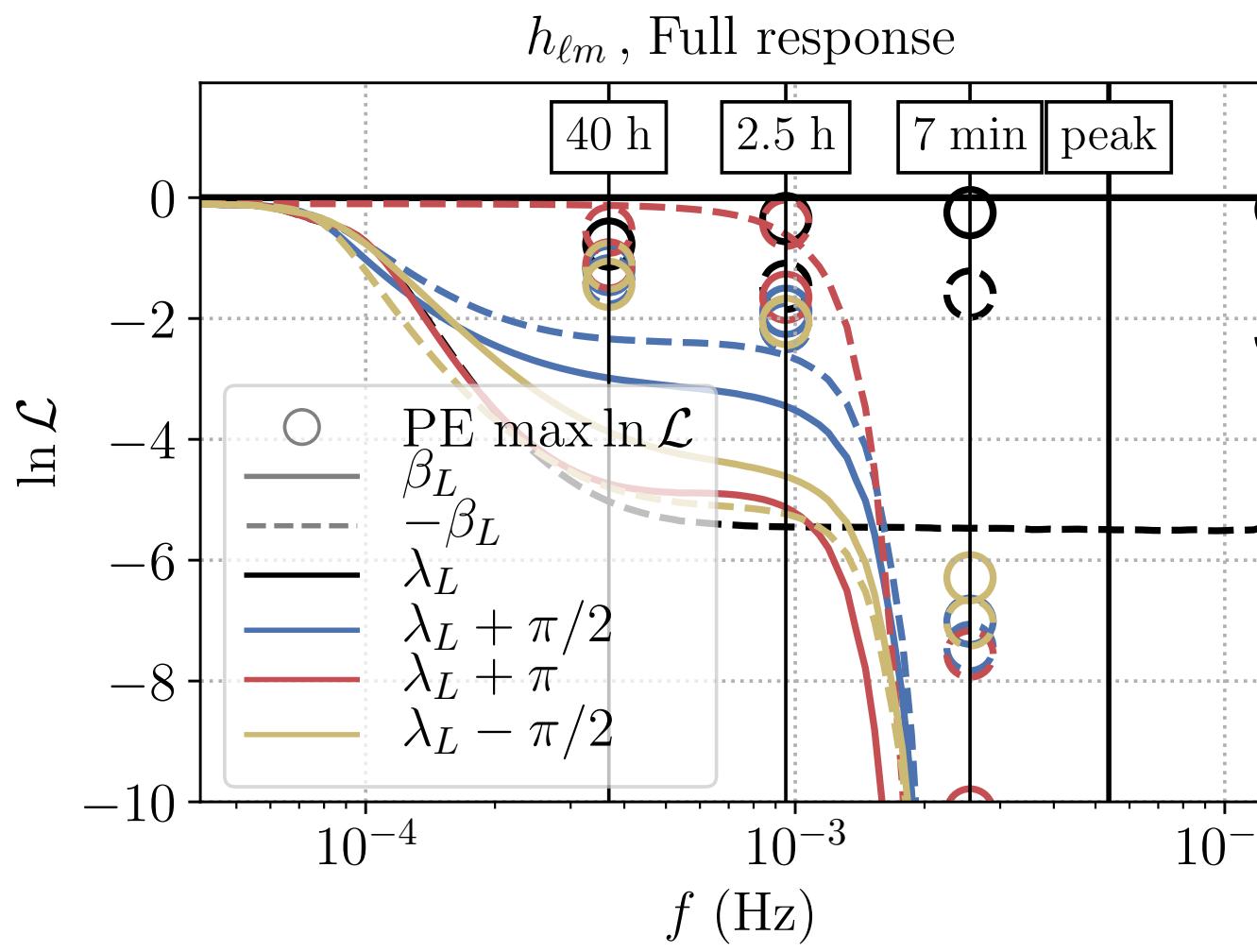
Degeneracy breaking for 8 sky maxima

Instrument response:

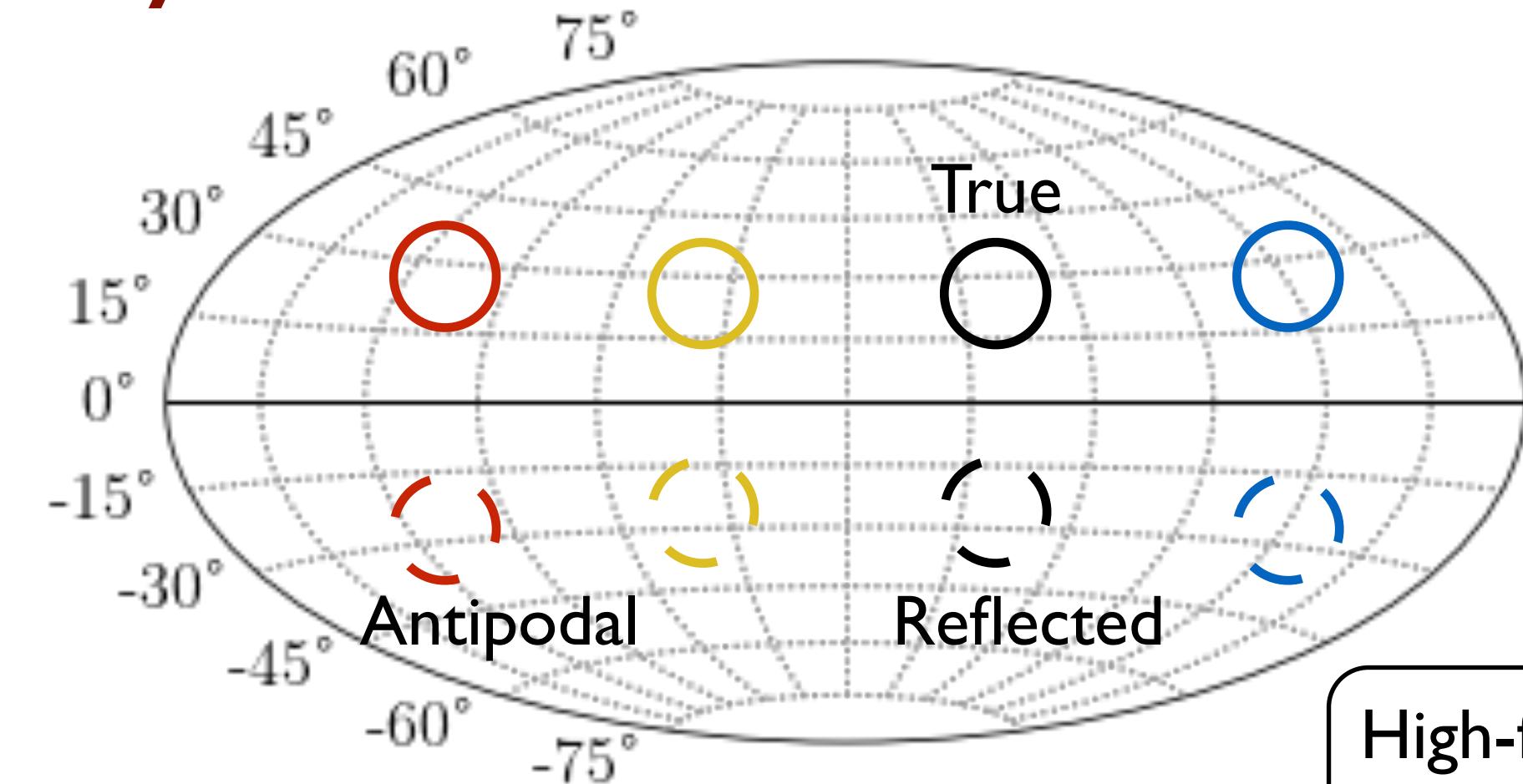
- ‘Full’: keep all terms
- ‘Frozen’: ignore LISA motion
- ‘Low-f’: ignore f-dependency
- ‘Frozen Low-f’: ignore both

$$\text{Likelihood: } \ln \mathcal{L}(d|\theta) = - \sum_{\text{channels}} \frac{1}{2} (h(\theta) - d | h(\theta) - d)$$

Approximate degeneracy measure: likelihood at the other sky positions

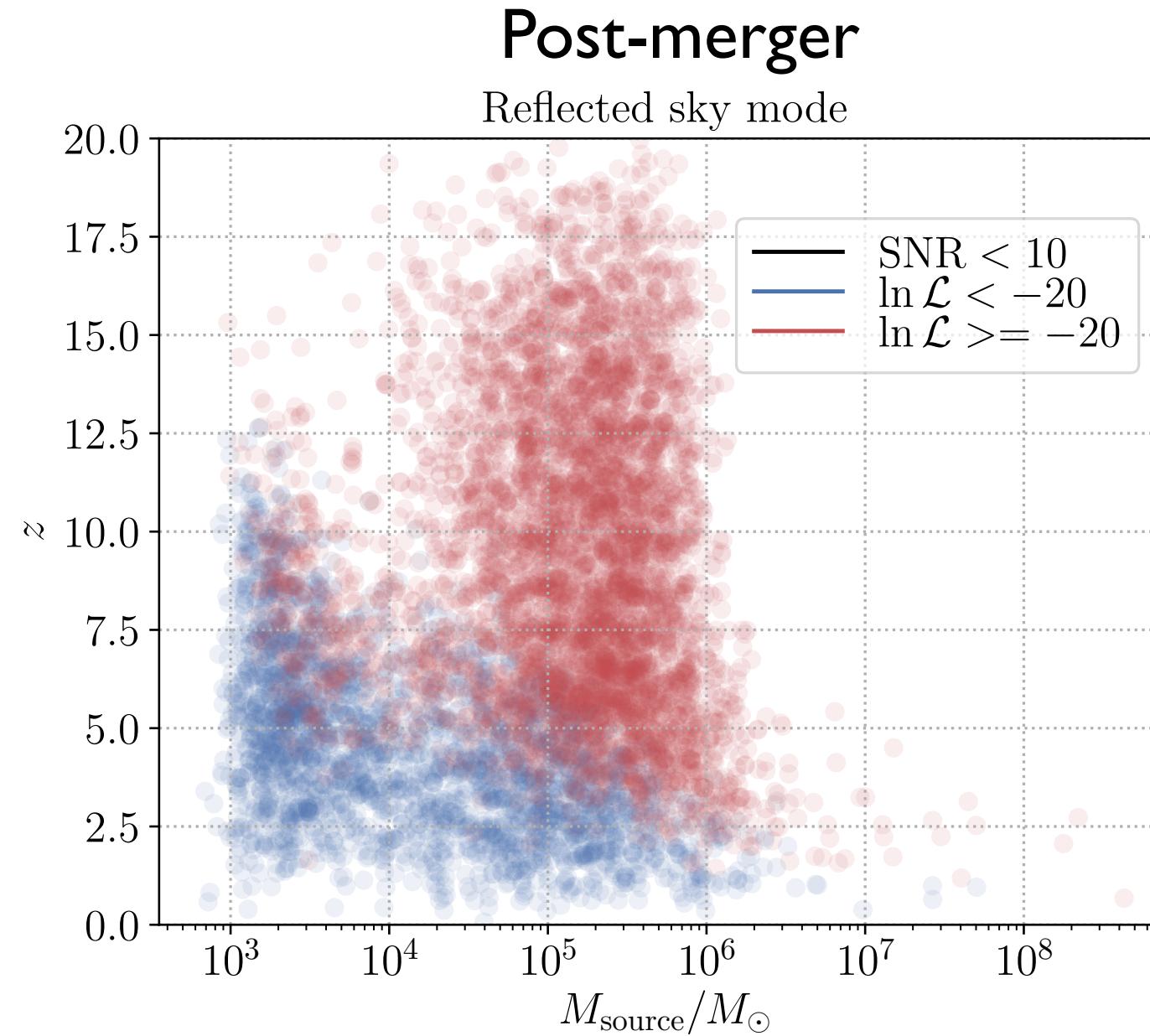


Sky modes L-frame



High-f features crucial
in reducing
multimodality (8->2)

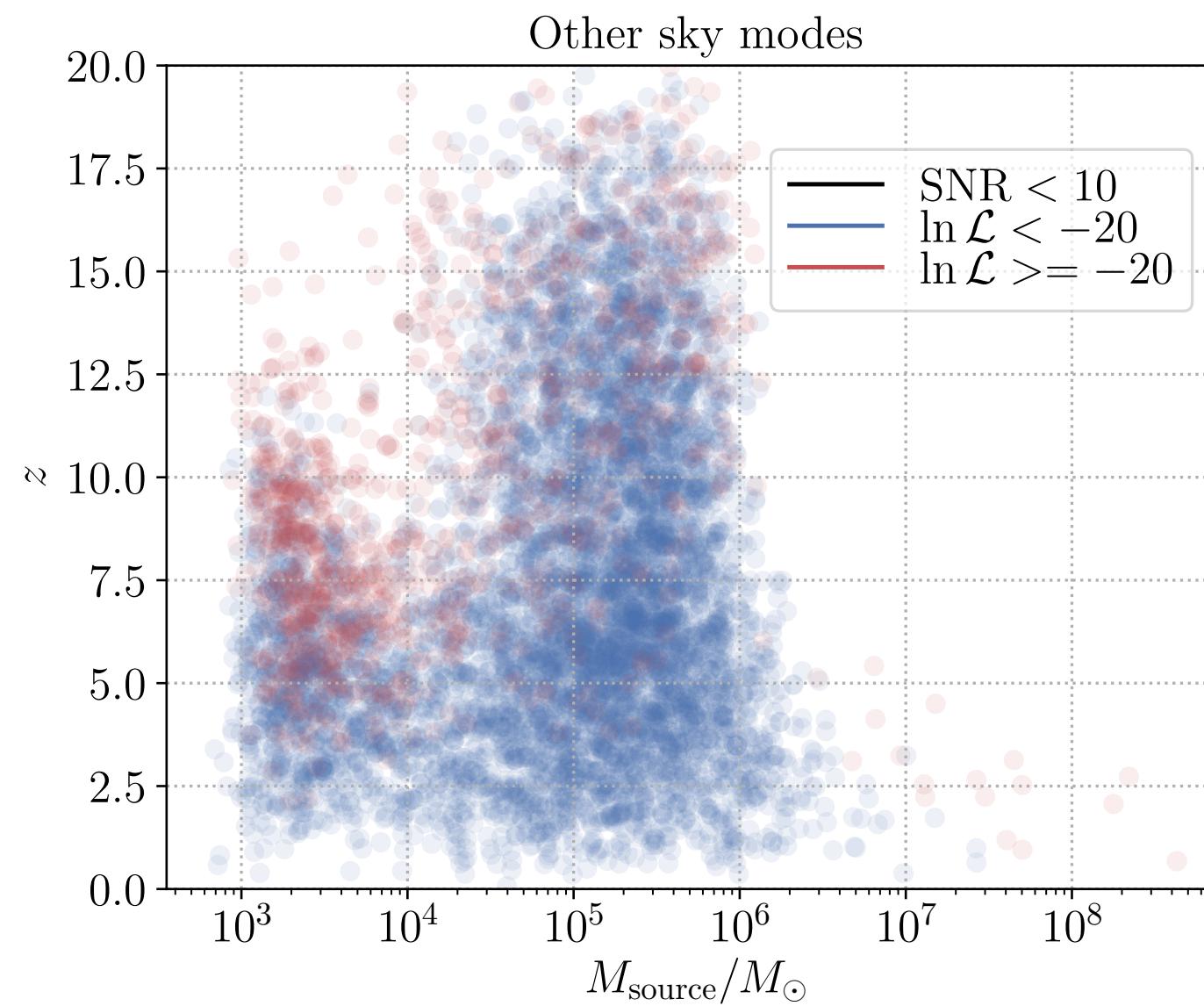
Multimodality of the sky localization



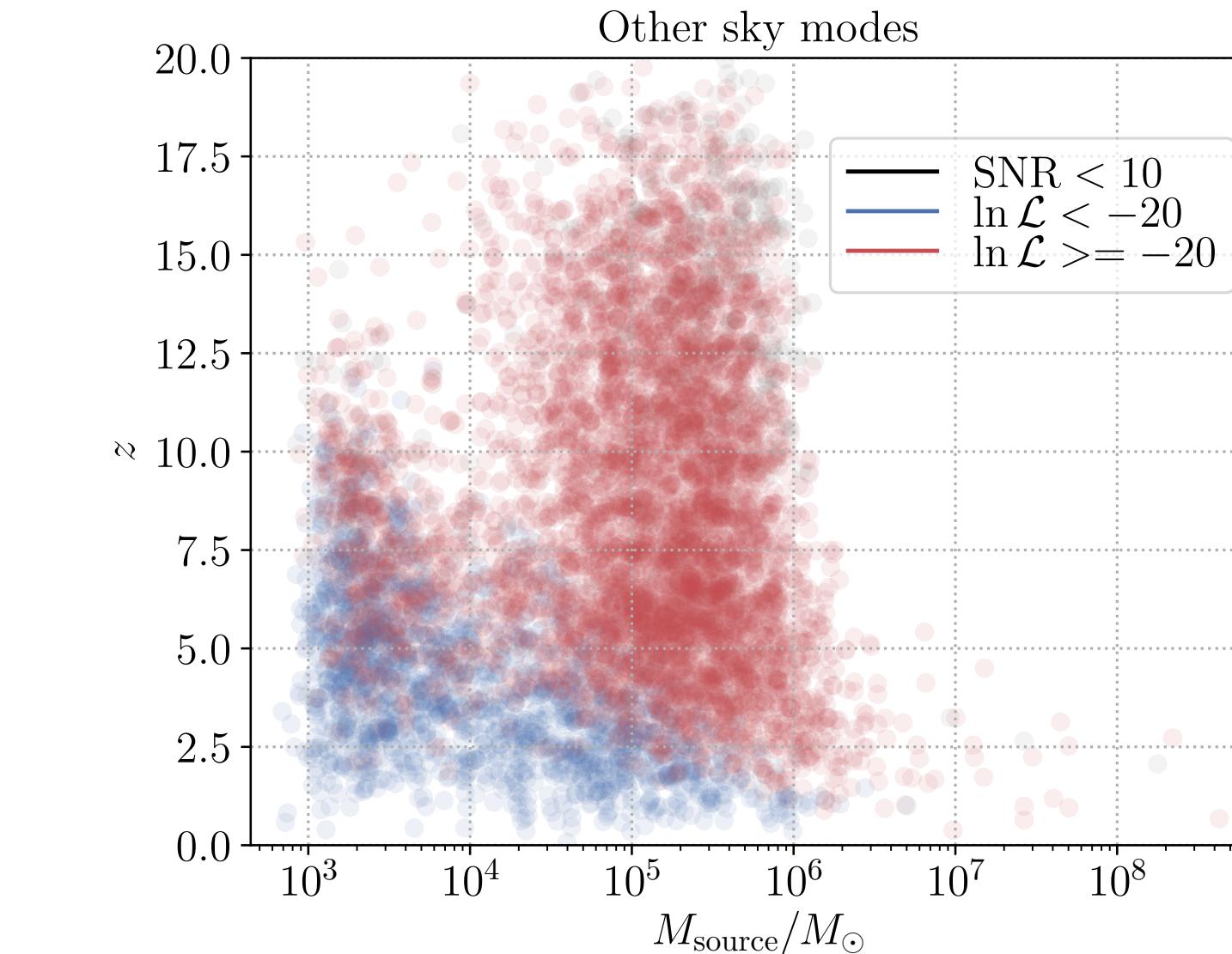
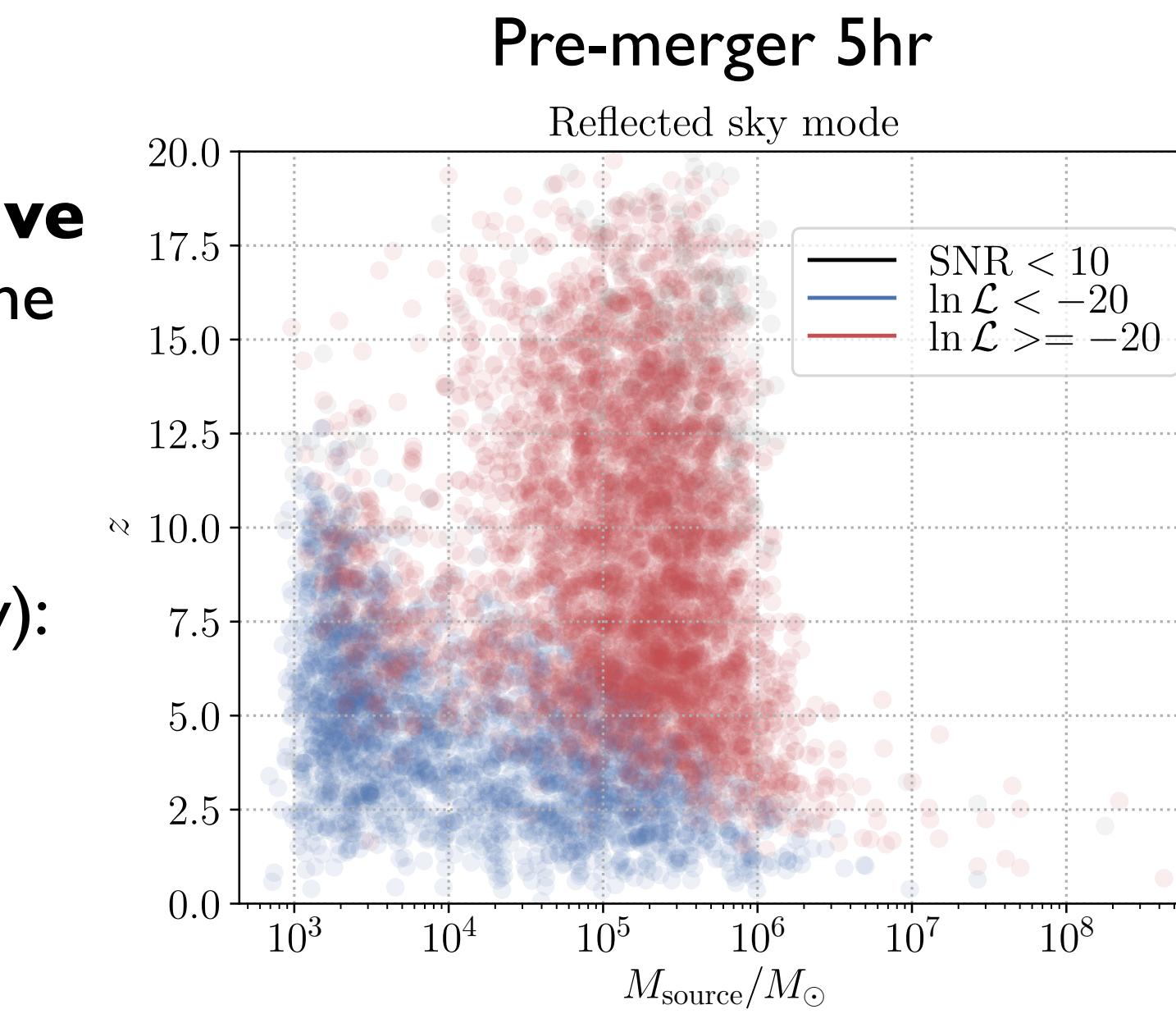
Approximate and conservative
degeneracy measure: likelihood at the
other sky positions

Threshold used here (a bit arbitrary):

$$\ln \mathcal{L} > -20$$



[Preliminary]



Conclusion and outlook

Highlights

- Bayesian tools for fast PE of MBHB signals
- Explored the LISA parameter recovery of MBHBs
- Analyzed role of higher harmonics in breaking parameter space degeneracies
- Analyzed features of the instrumental response and breaking of multimodal sky degeneracies
- Successful analysis of MBHB in LISA Data Challenge I (Radler)

Outlook

- Explore the parameter space
- Optimize samplers for known degeneracies (MCMC jump proposals)
- More realistic analysis: multiple signals, realistic noise
- More realistic waveforms: precession and eccentricity
- Assess waveform requirements: how accurate do the waveforms need to be ?

