DE LA RECHERCHE À L'INDUSTRIE

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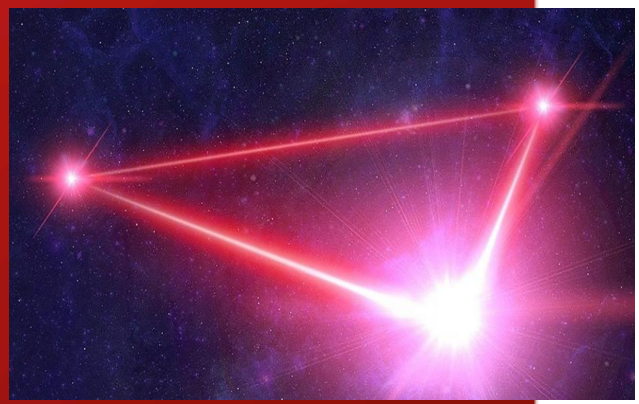
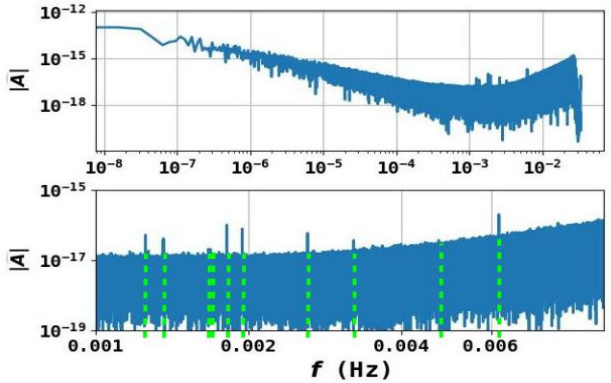
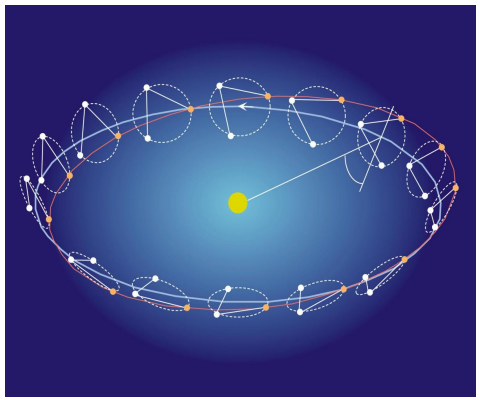
Sparse data inpainting for the recovery of gapped gravitational wave signals







GDR OG Assembly | Hervé MOUTARDE

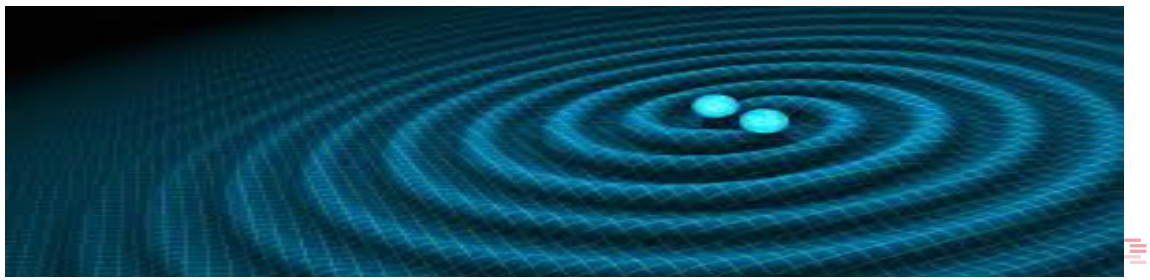
15 Oct. 2020

université PARIS-SACLAY

Galactic binaries.
Physics and data analysis aspects.

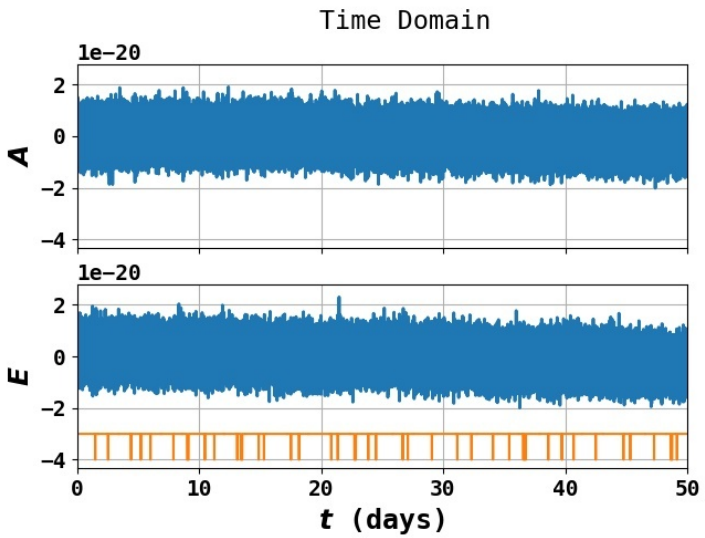
- Physics perspective:
  - Expect $\sim 10^6$ sources.
  - Aim: **characterize** a subset of $\sim 10^4$ sources.
  - **Foreground** of many other signals of physical interest.
- Data analysis perspective:
  - Characterization of individual sources.
  - Determination of a **confusion noise**.
  - Assessment of impact of **gaps**, **glitches** or **non-stationary** power spectral density.

    Baghi *et al.*, Phys. Rev. **D100**, 022003 (2019)
  - Low signal to noise ratio requires long observations.

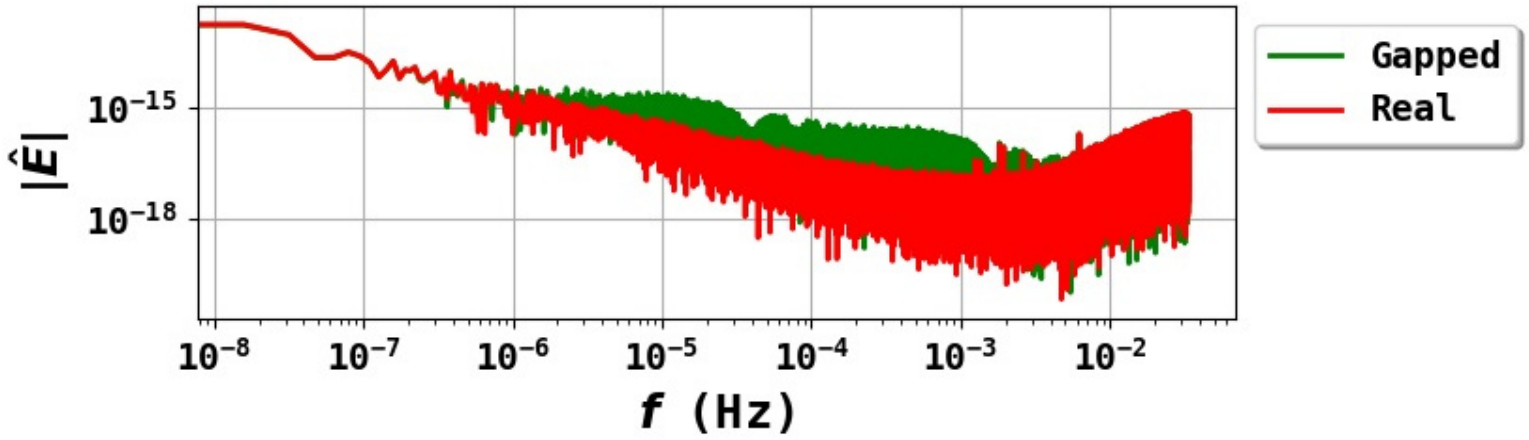


Galactic binaries from realistic LISA data.

Focus on gaps.



- Origins : antennas, laser frequency, orbit corrections, ...
- Both **planned** and **unplanned** gaps.
- Features : 1h / 5 days, 10 min. / day and up to 5 days (unplanned, LISA Pathfinder).



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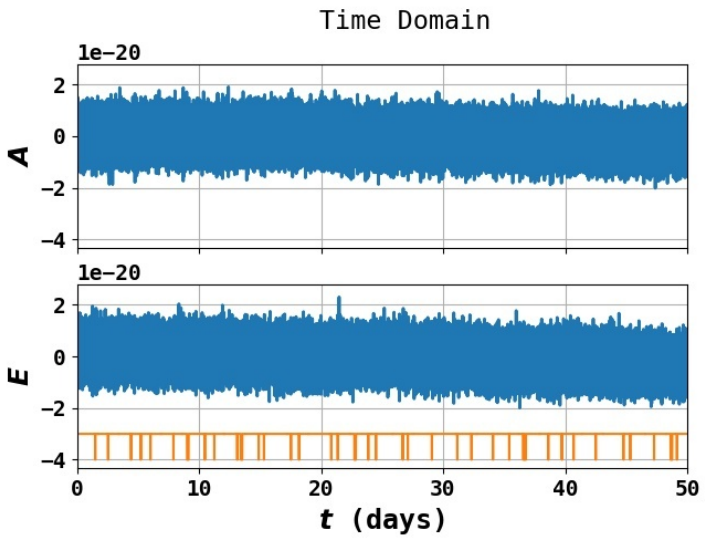

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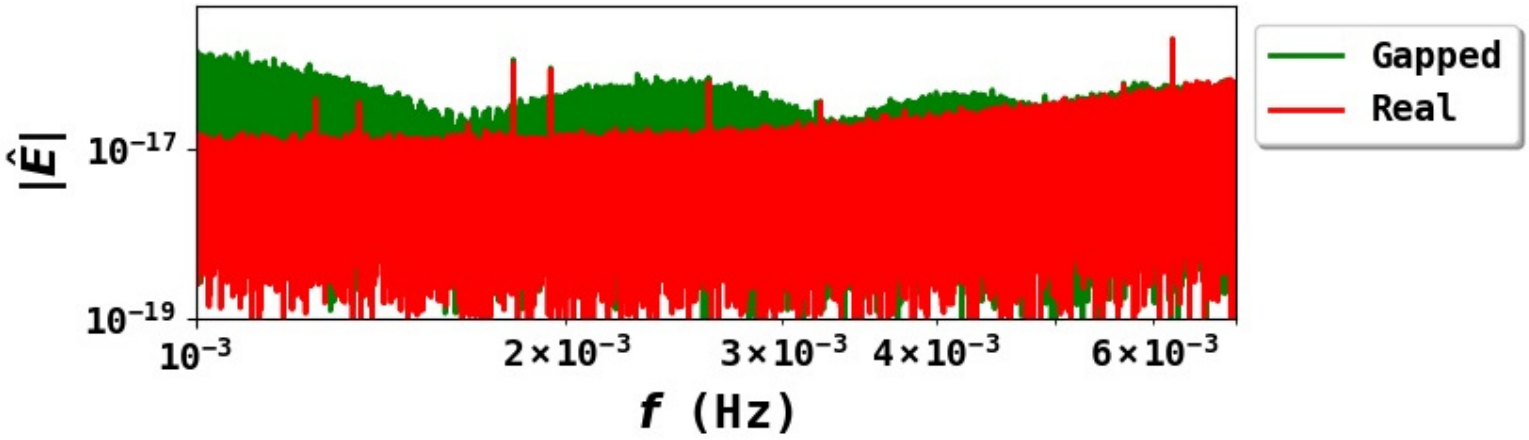
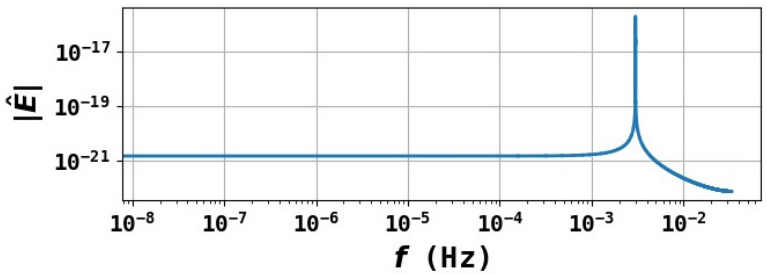
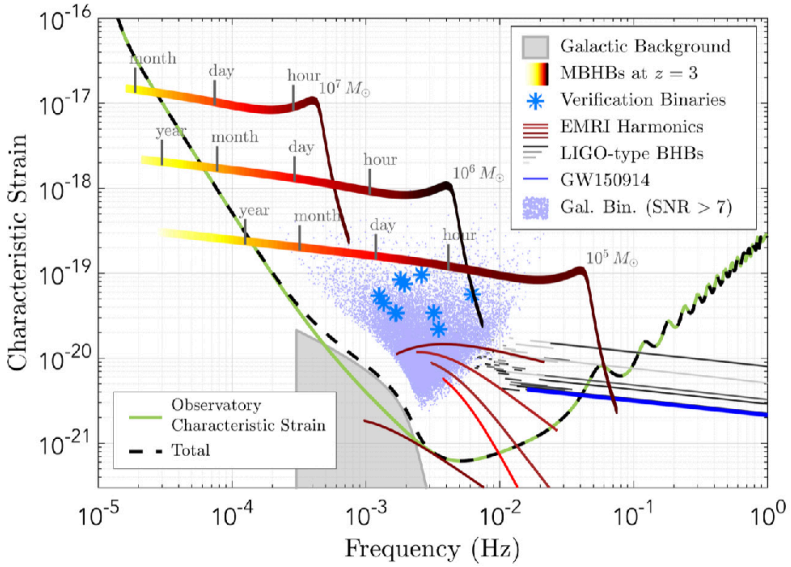


Sparse data representation.

Exploit distinctive structural features of signal and noise.



- Expected signal is **sparse in Fourier basis**.



- Instrumental noise is **Gaussian** with known power spectral density Σ.

- Detection through **non-parametric** approach.
- Possible extension to **other types of signals** by **adequate choice of basis** with sparse representation.

Ungapped data

Statistical estimator of the signal.
Construction by resolution of an optimization problem.

- Introduce:
  - Data: time series V of length N.
  - **Whitened** variables $\widehat{V} = \Sigma^{-1/2} V$ in frequency domain.
  - **Threshold** γ to separate signal from noise.
  - Compact notation: $\gamma \odot \widehat{V} = \sum_k \gamma[k] \widehat{V}[k]$.
- Signal estimator:

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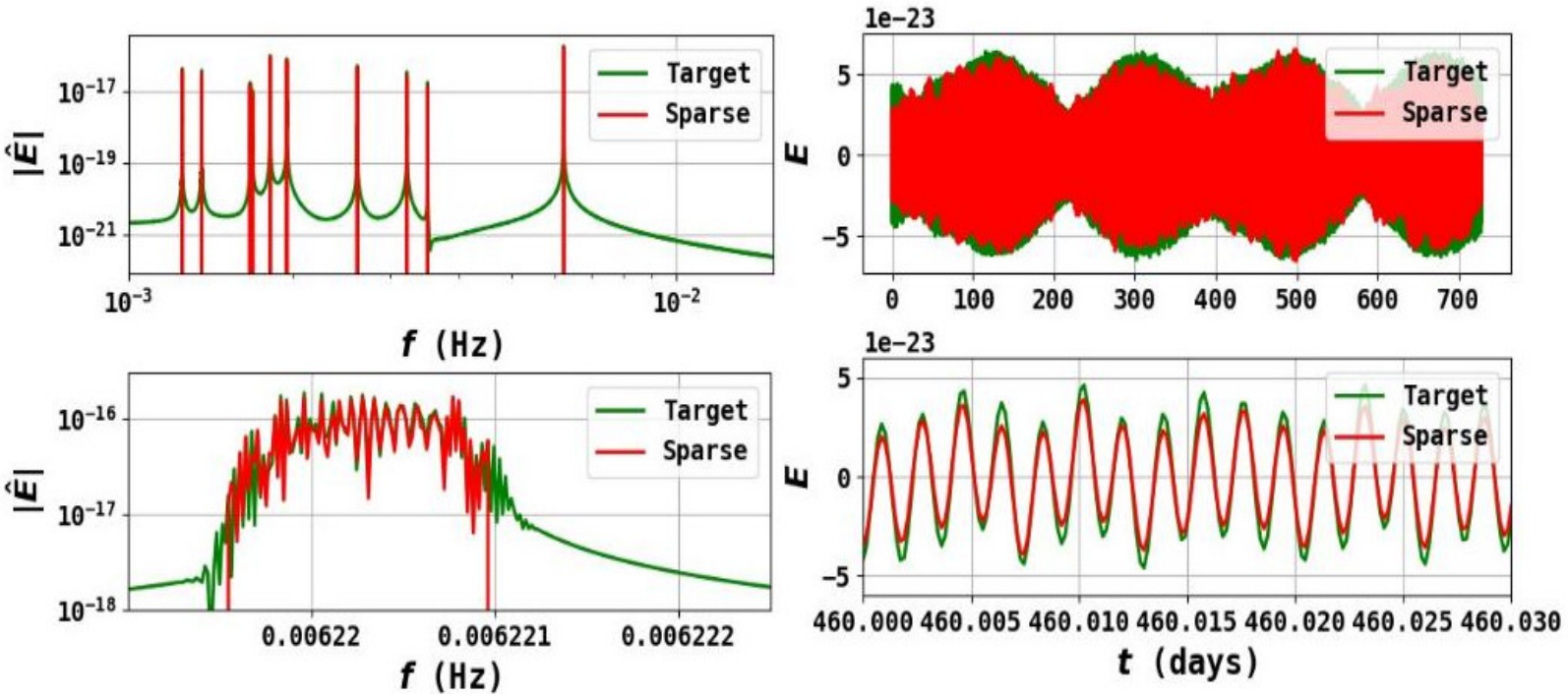
- Extension to coupled TDI channels, frequency blocks, etc.

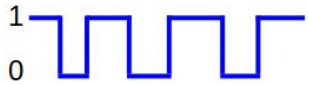
Blelly *et al.* [arXiv:2005.03696 [gr-qc]], Phys. Rev. D

Results.
Test on LDC1-3: verification galactic binaries.



- **All peaks detected**, even for weak amplitudes.
- **No false positive signals**.
- **Accurate** phase description.
- Amplitude can be improved by adapted basis choice.

Gapped data

- Introduce:
  - Data mask M: 

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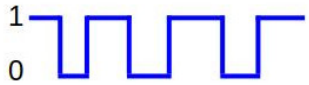
  - Data mask M: 

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Statistical estimators of signal and noise.

Construction by resolution of an optimization problem.

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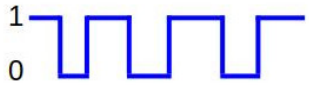
  - Data mask M: 

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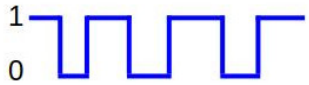
  - Data mask M: 

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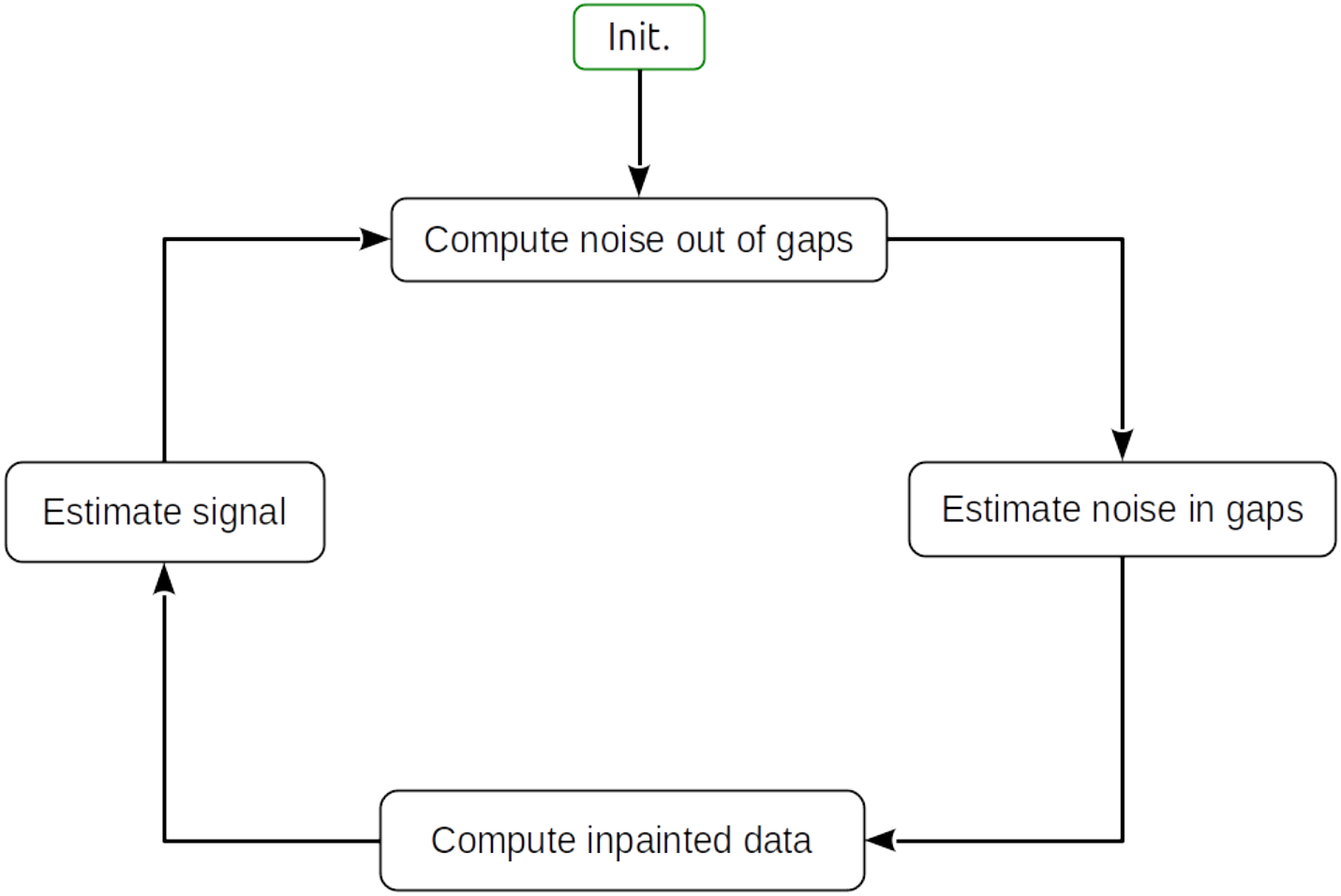
- Resolution alternates between signal and noise estimations.

Optimization problem: algorithm.
Description of an iteration.



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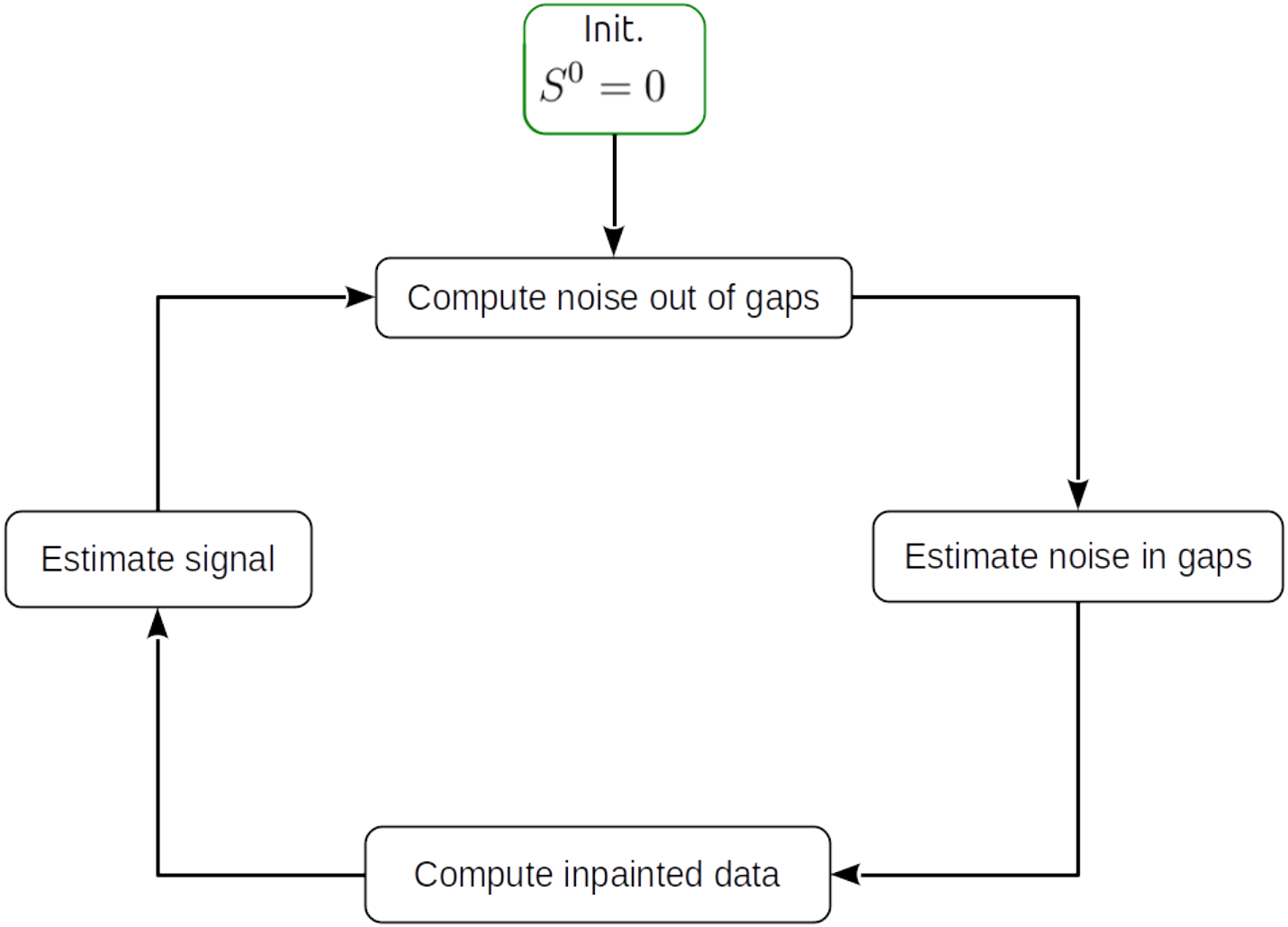
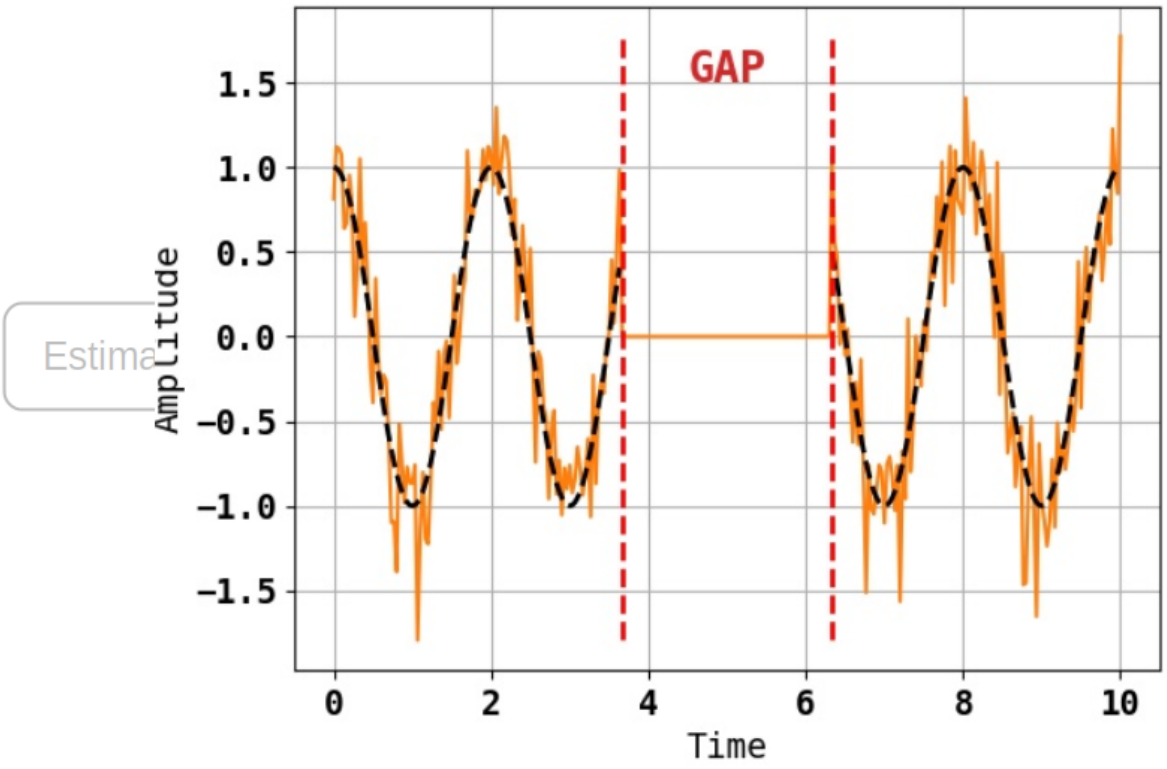
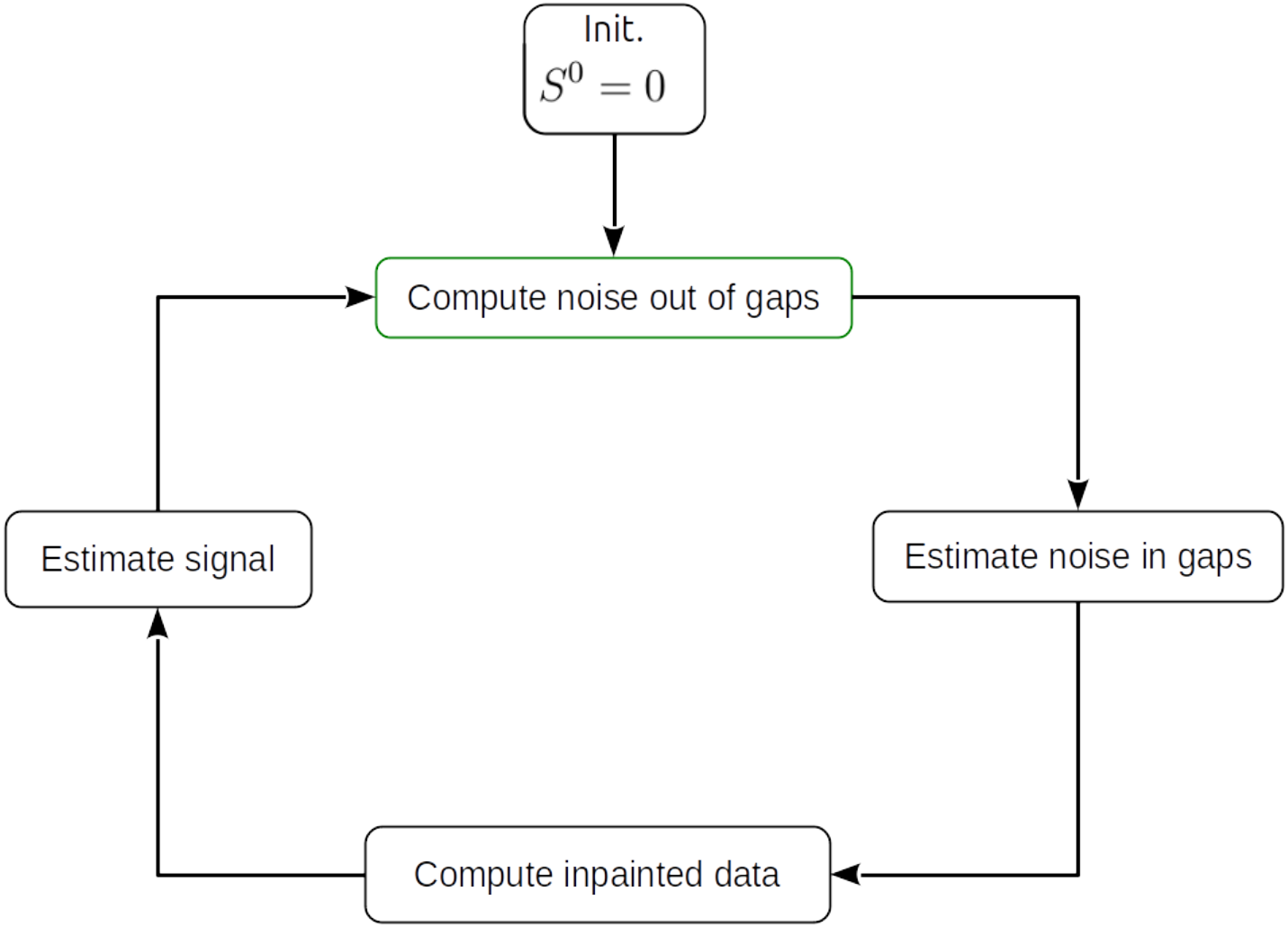


Init.
$S^0 = 0$



Estima

n gaps



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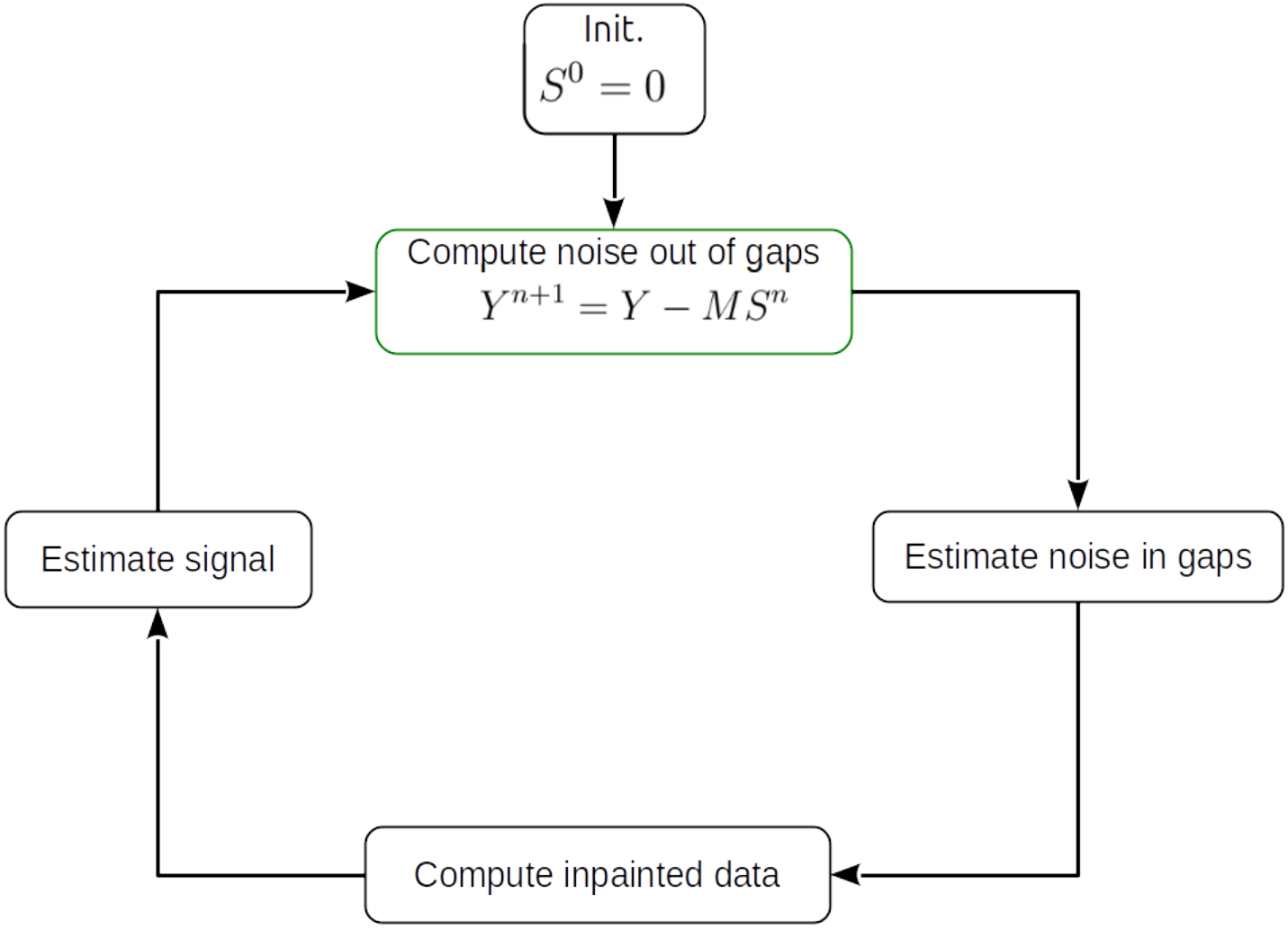

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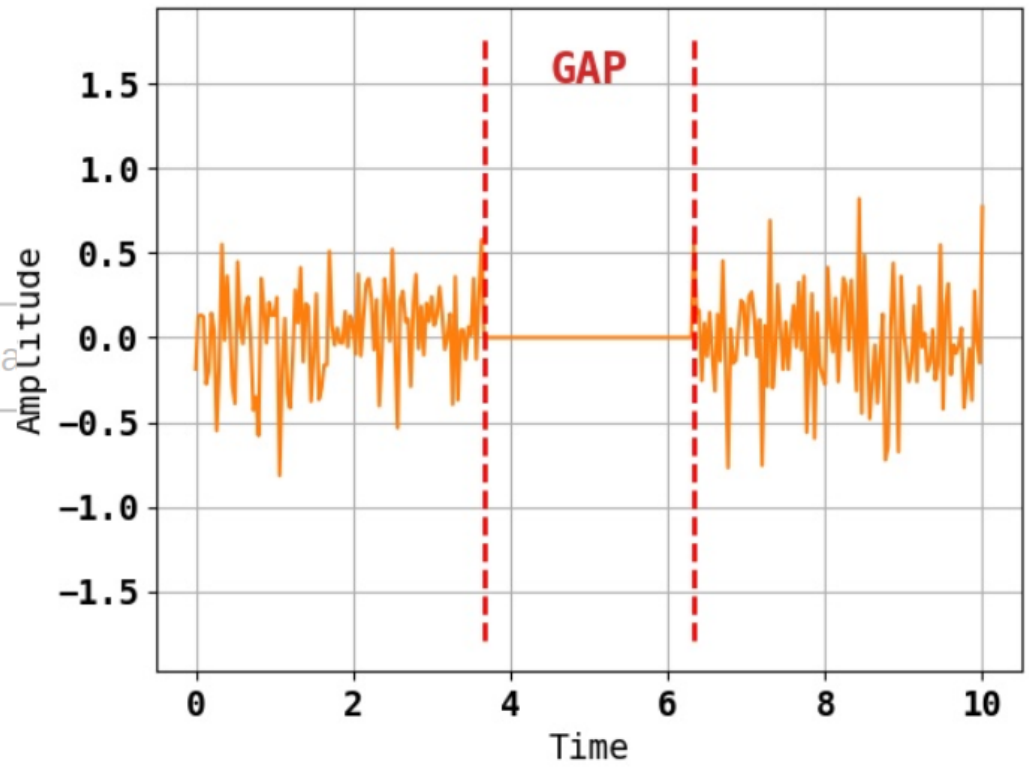

Estima

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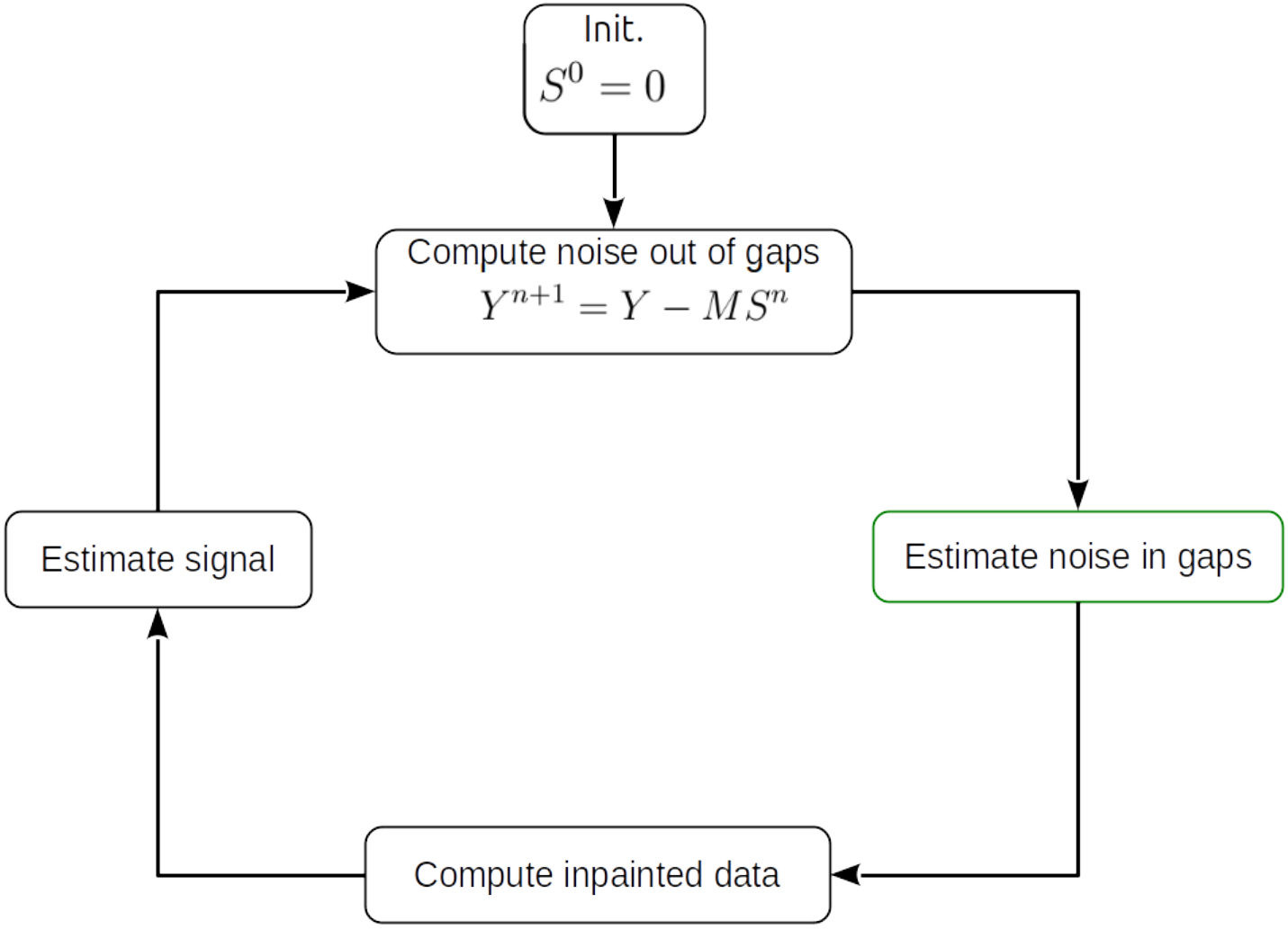

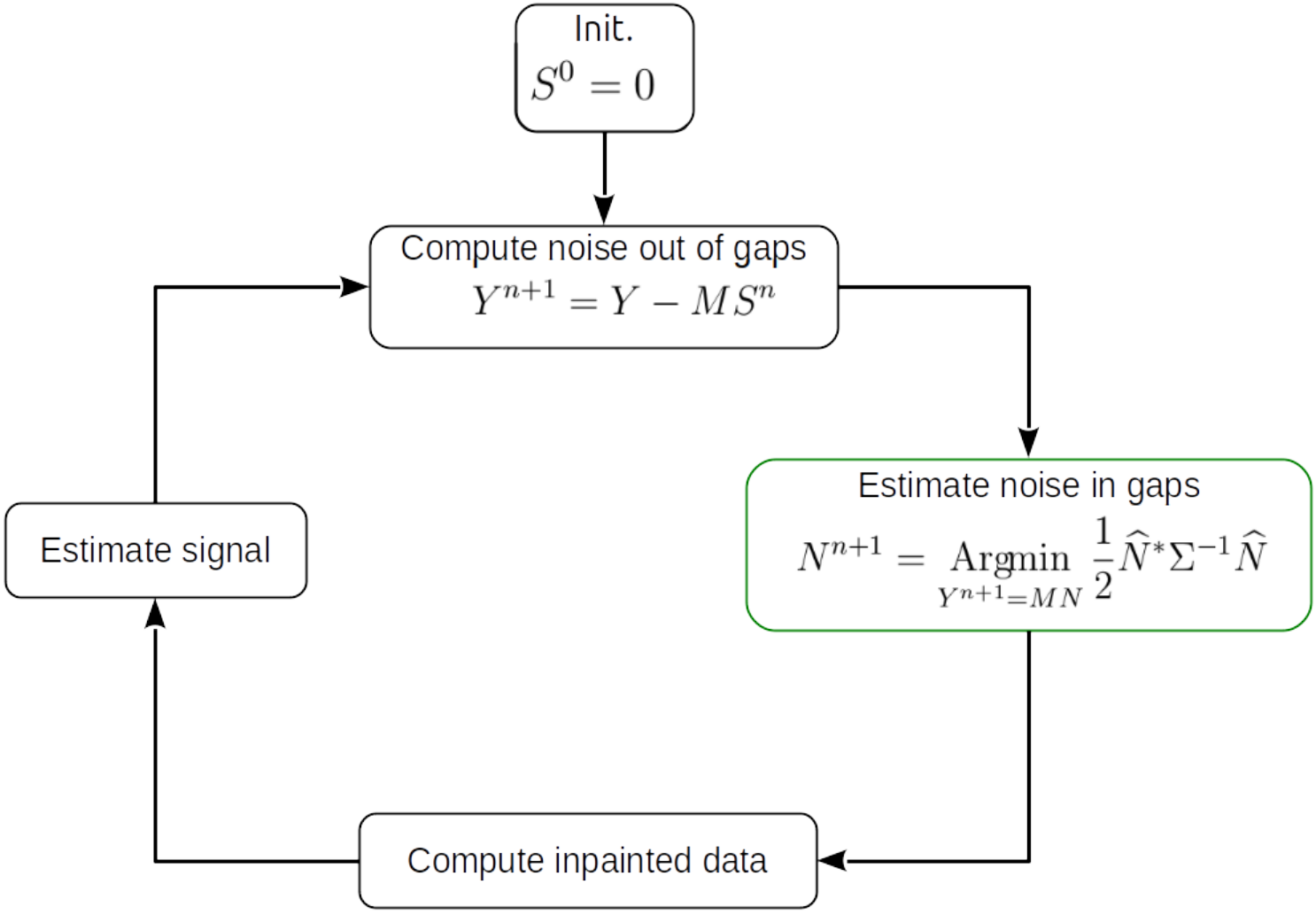

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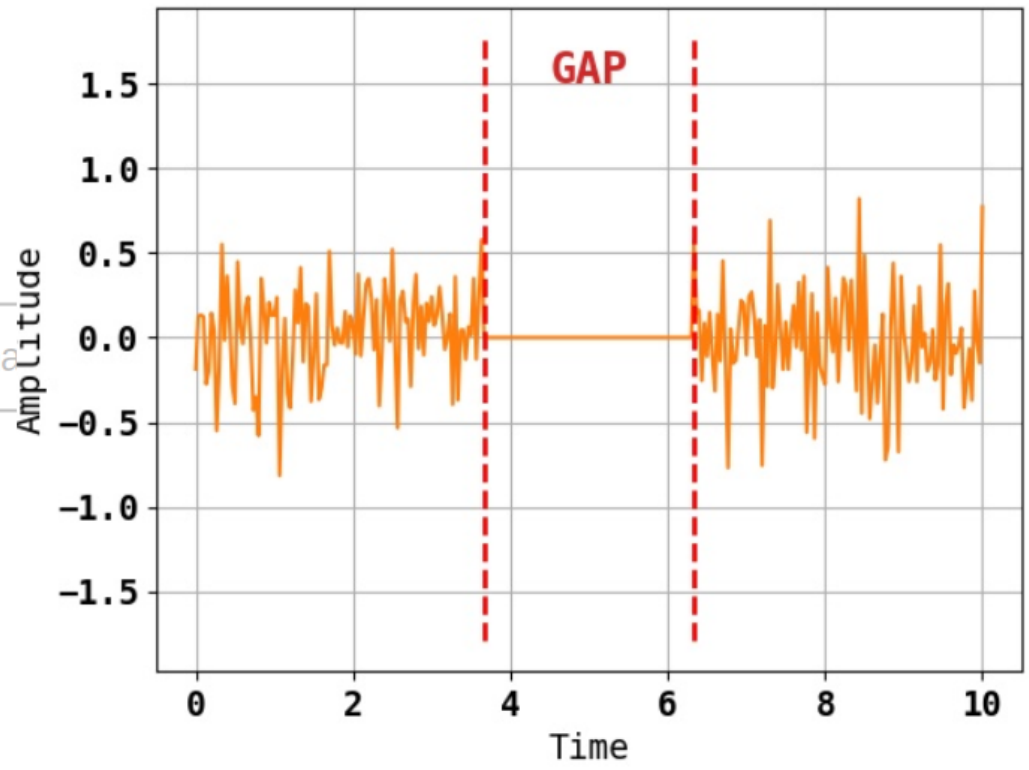

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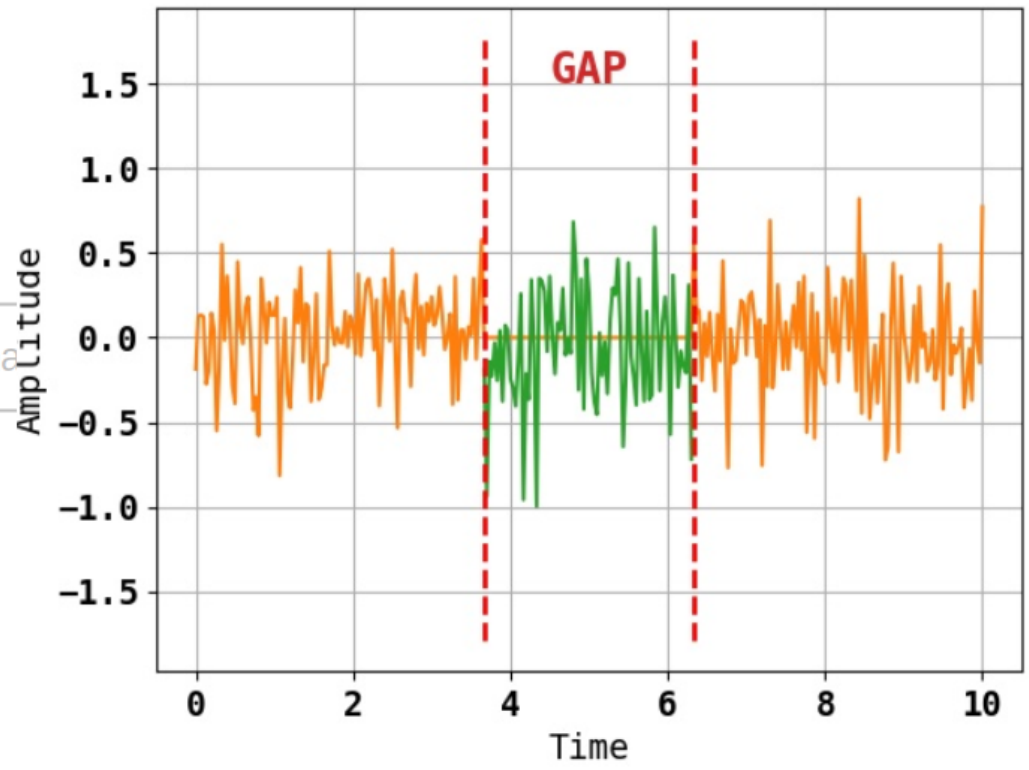
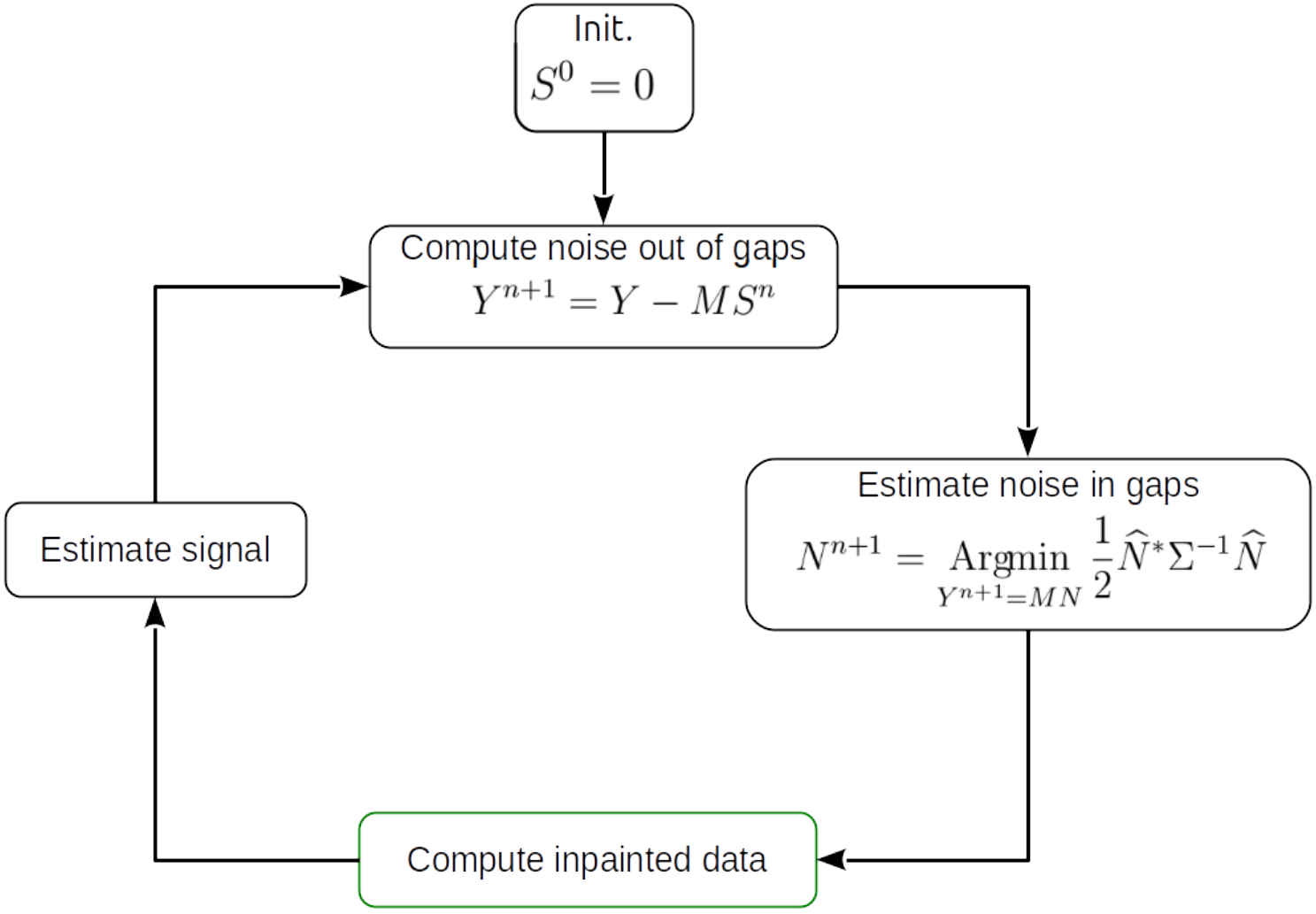


Estima

gaps
$\widehat{V}^* \Sigma^{-1} \widehat{N}$



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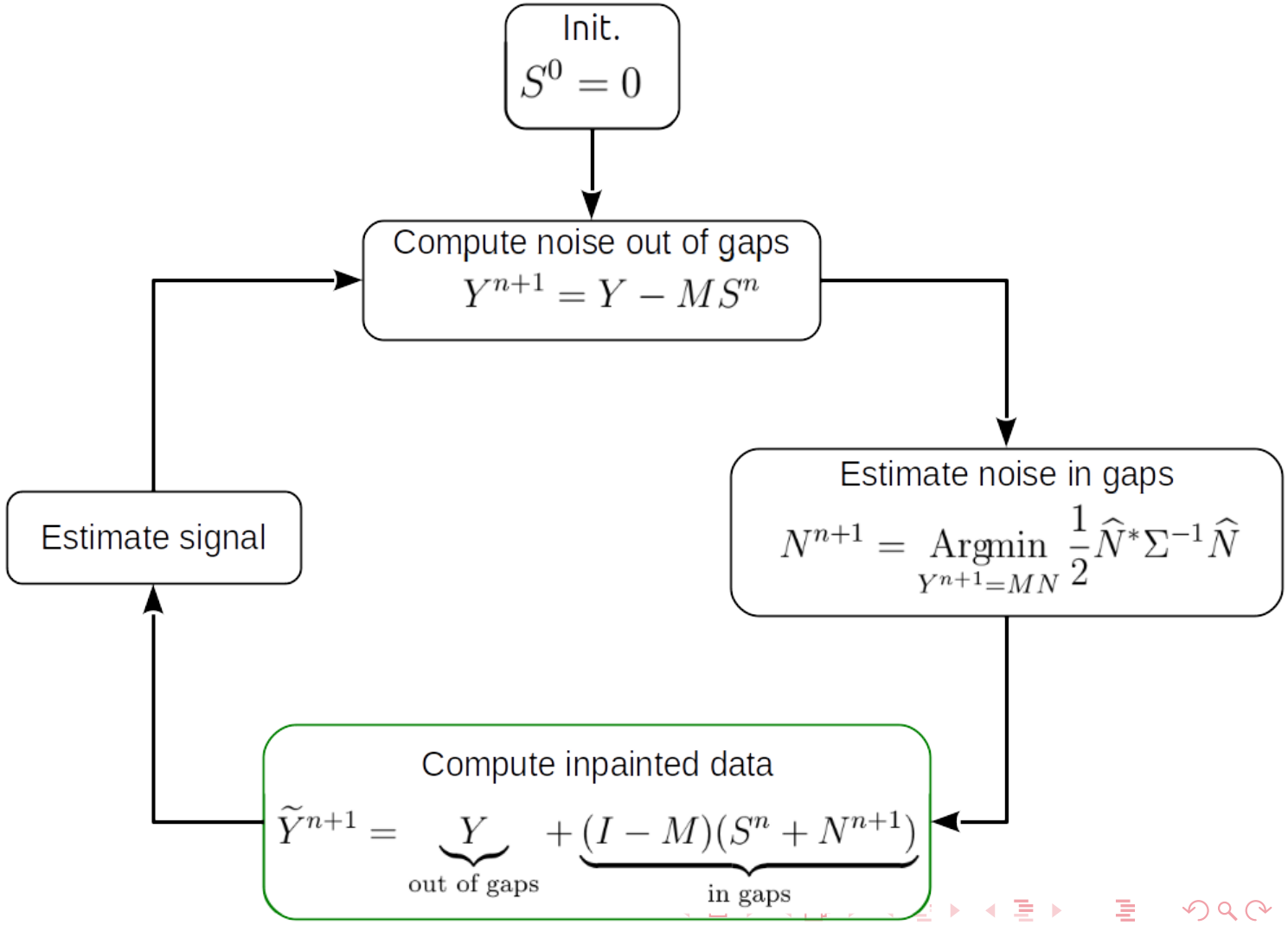

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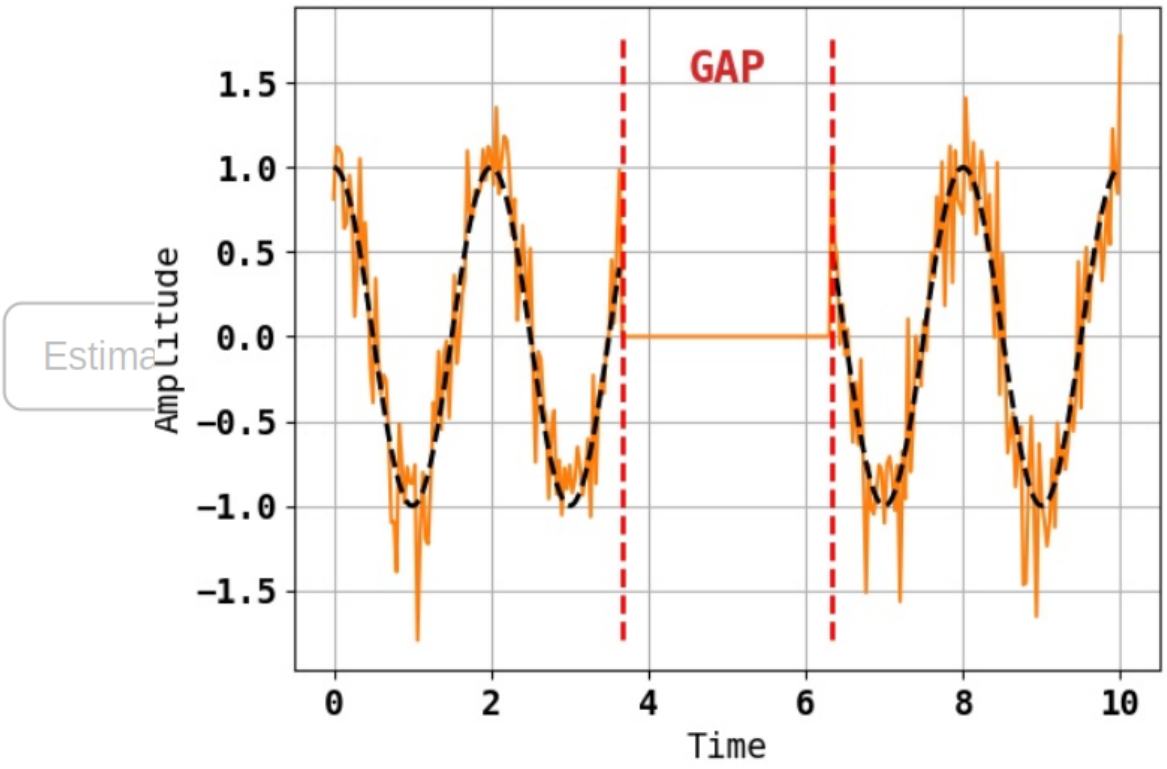

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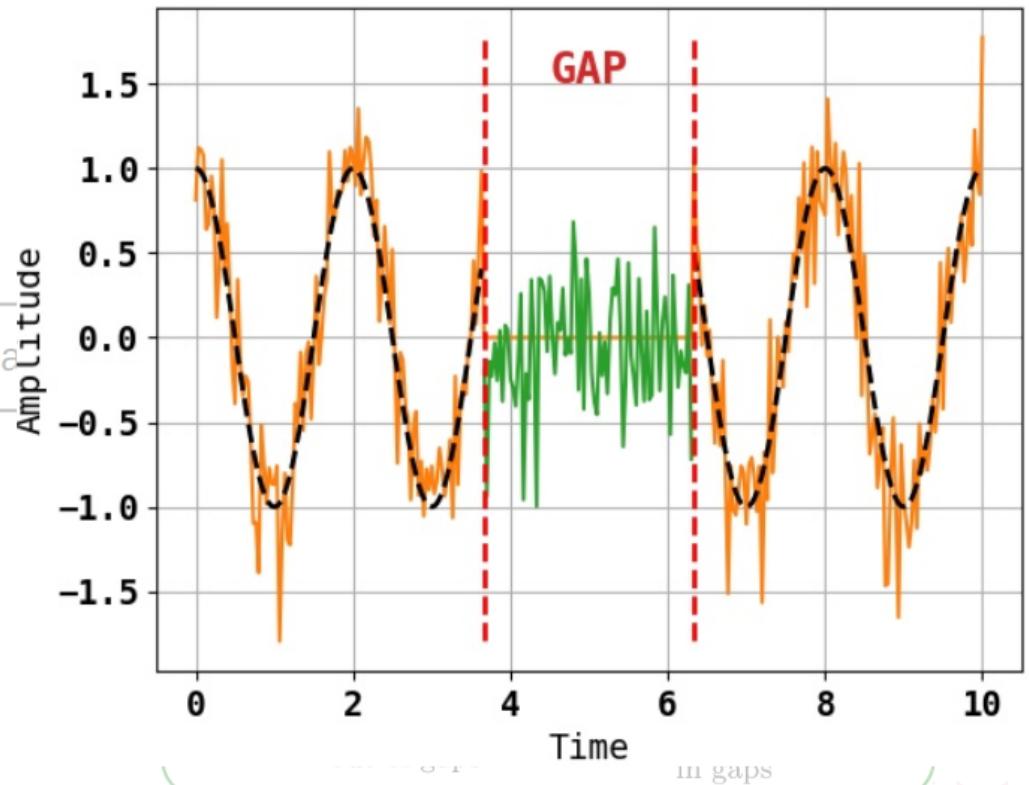

Estima

gaps
$\widehat{N}^* \Sigma^{-1} \widehat{N}$

in gaps

Optimization problem: algorithm.

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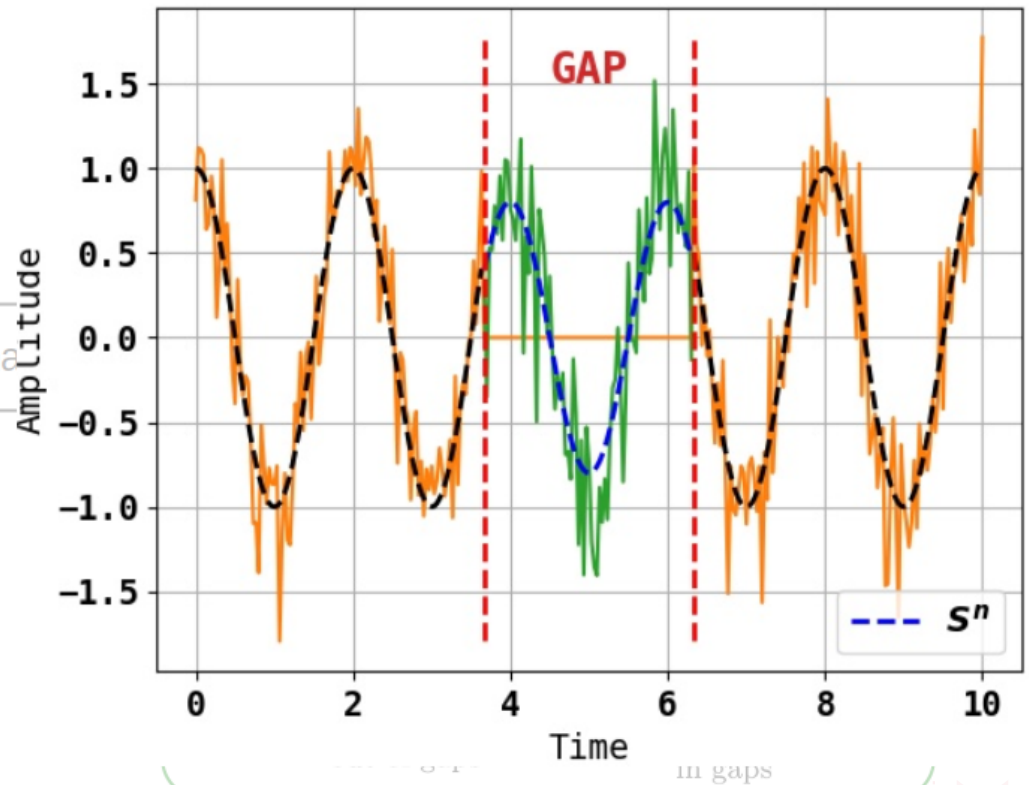

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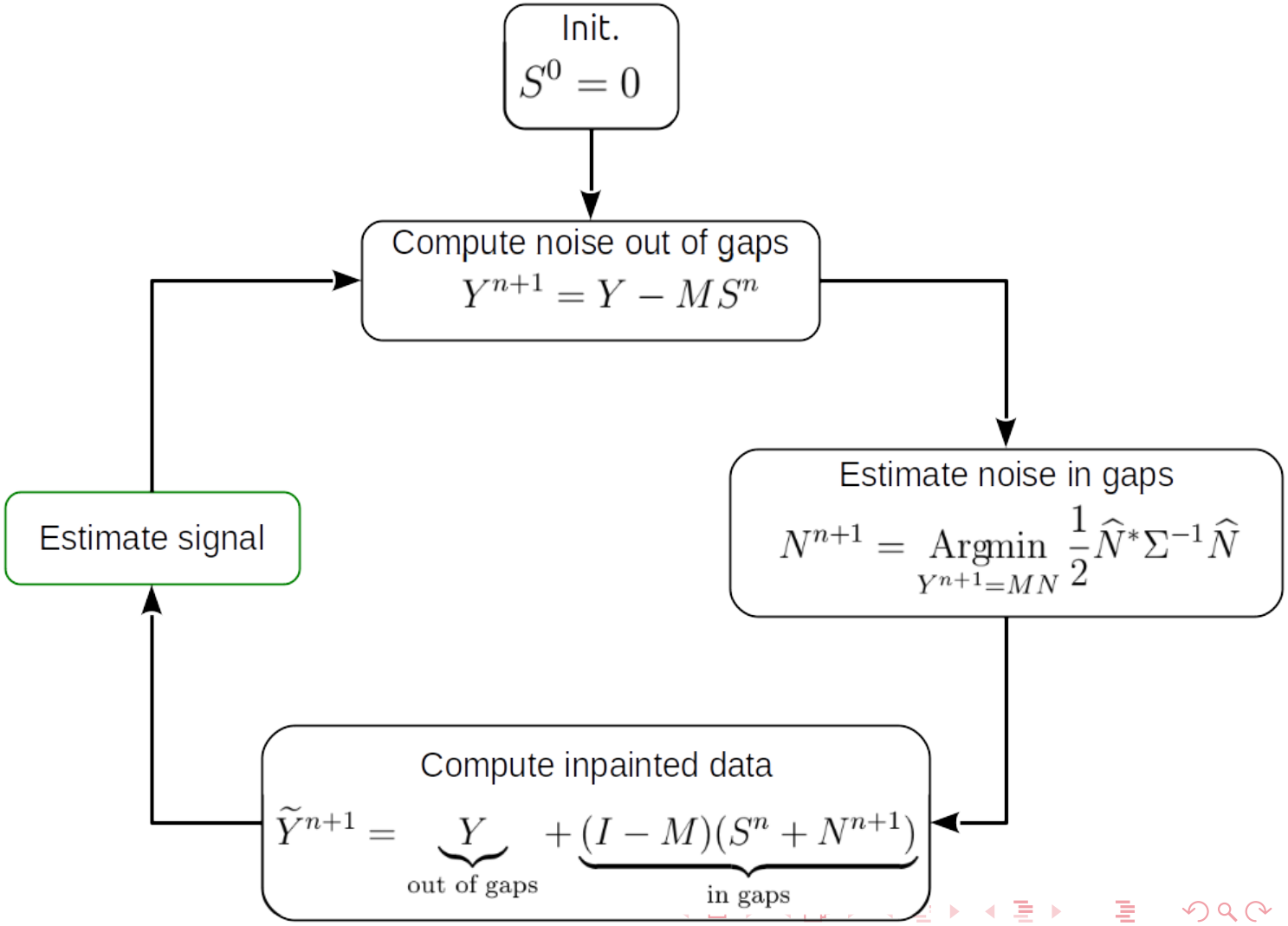

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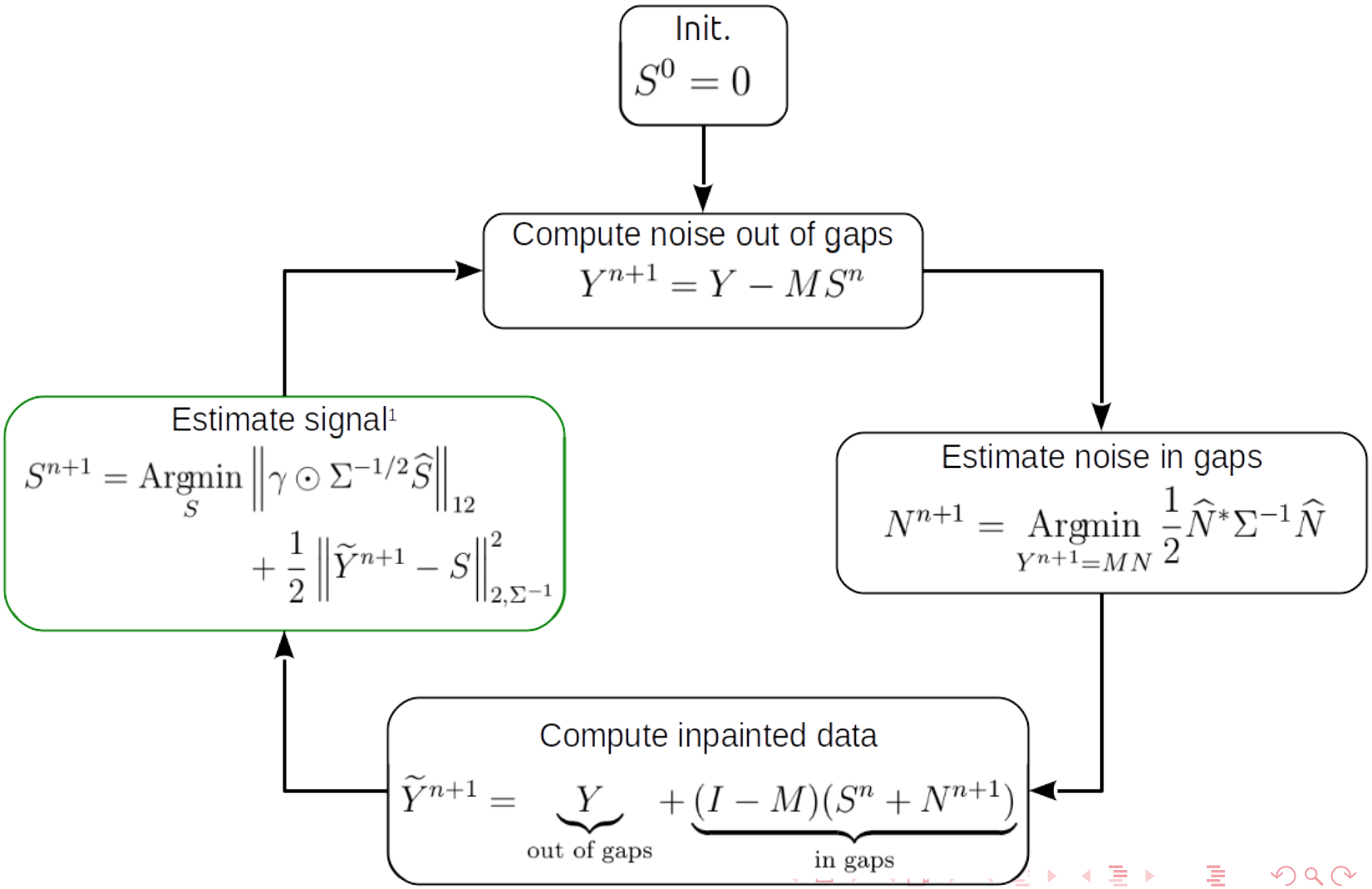

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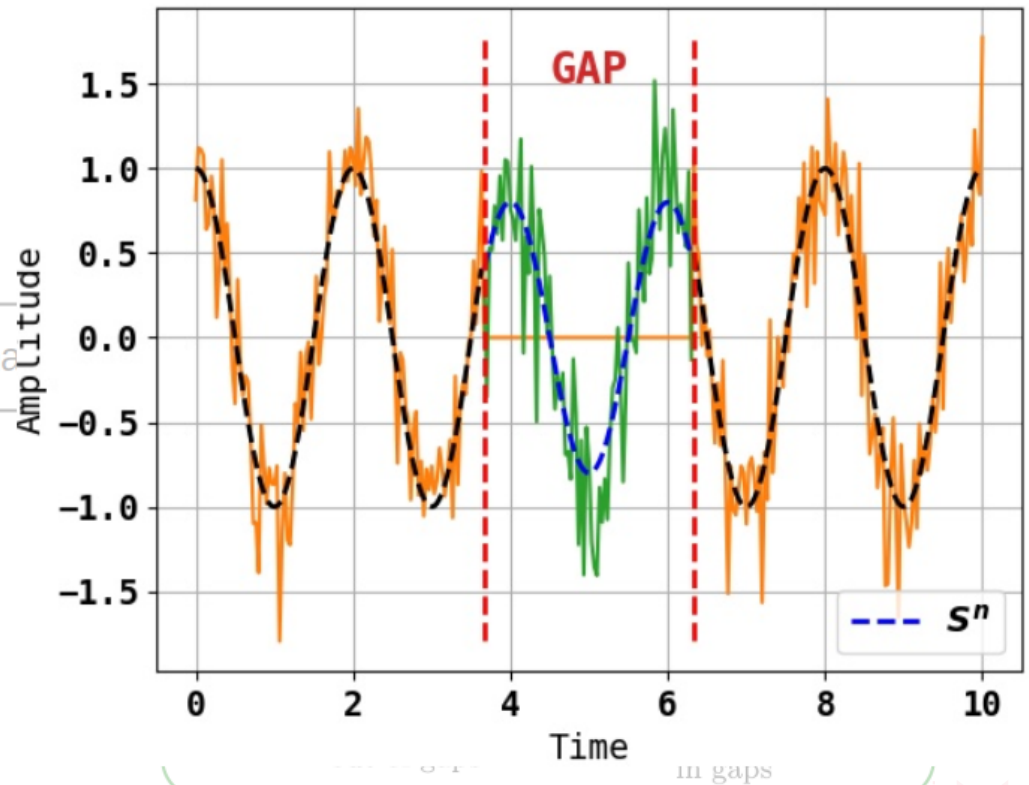

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gaps
$\widehat{N}^*\Sigma^{-1}\widehat{N}$

in gaps

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Init.
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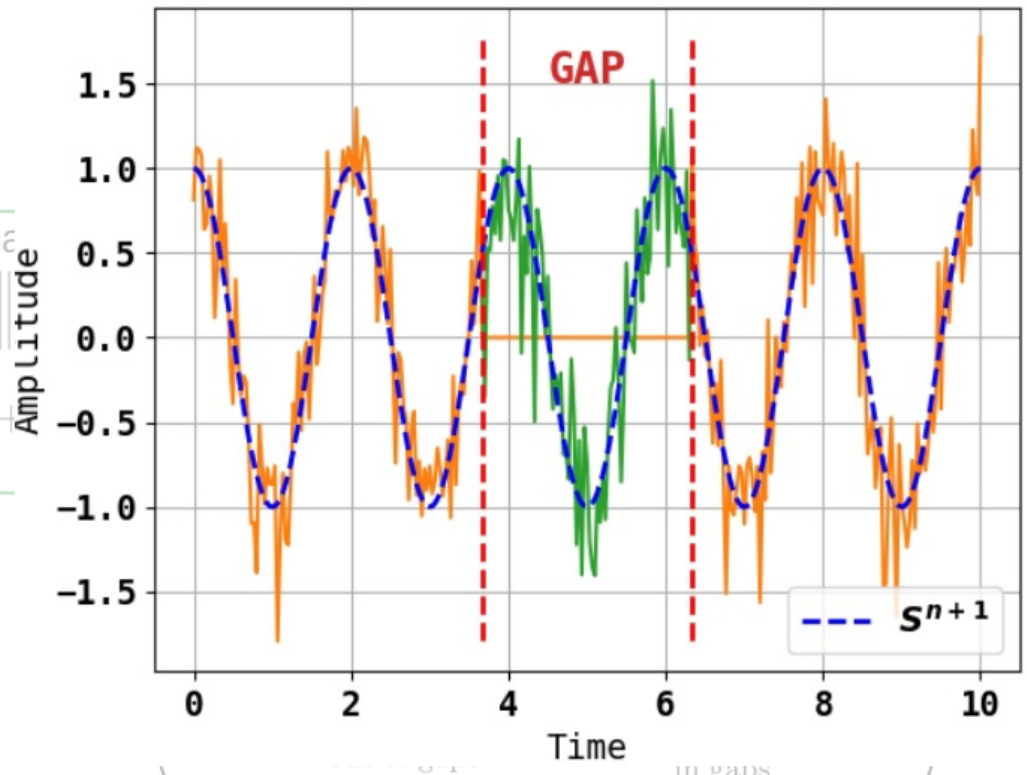
Estima
$S^{n+1} = \underset{S}{\text{Argmin}}$

gaps
$\widehat{V}^* \Sigma^{-1} \widehat{N}$



in gaps

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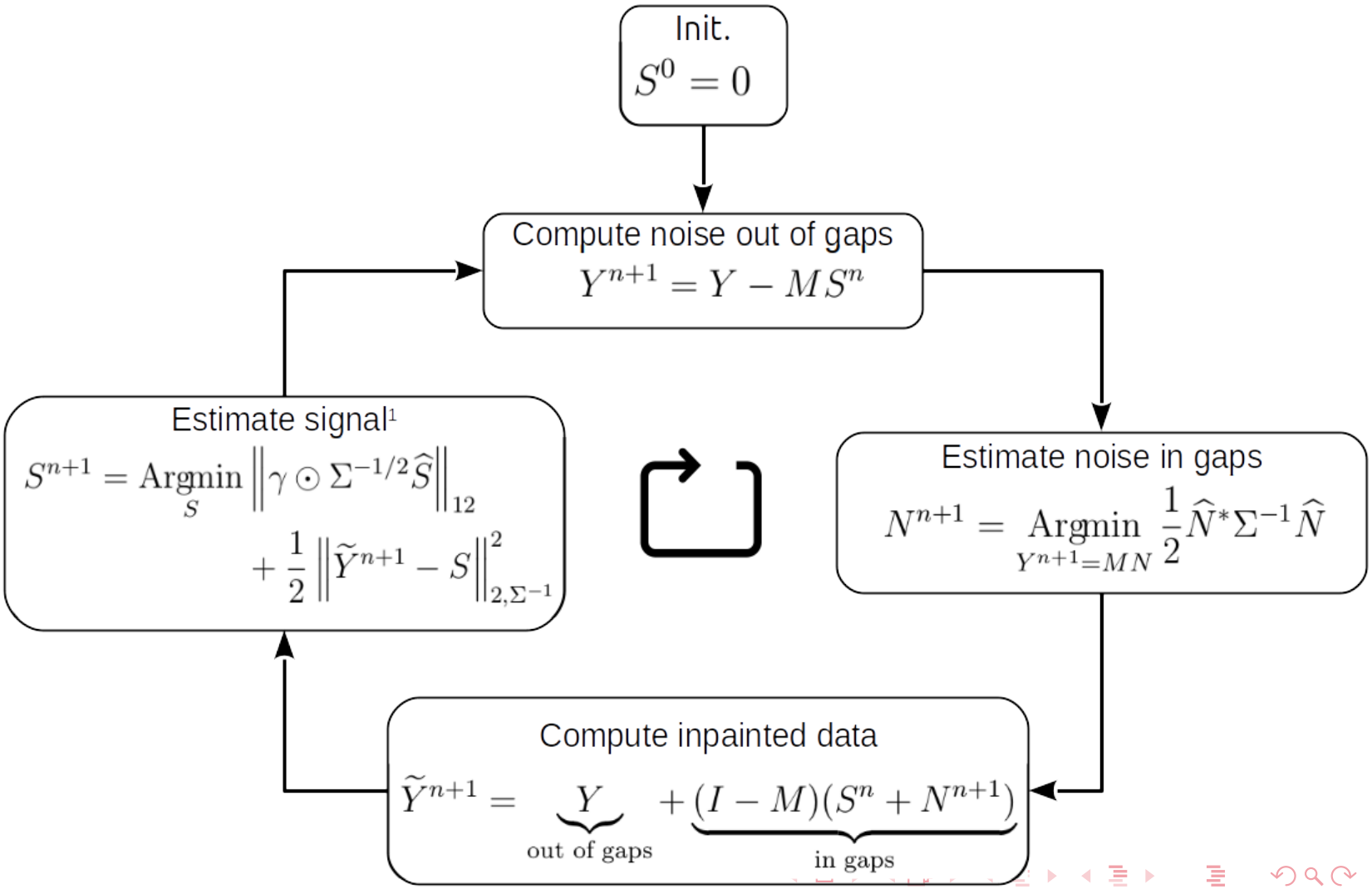
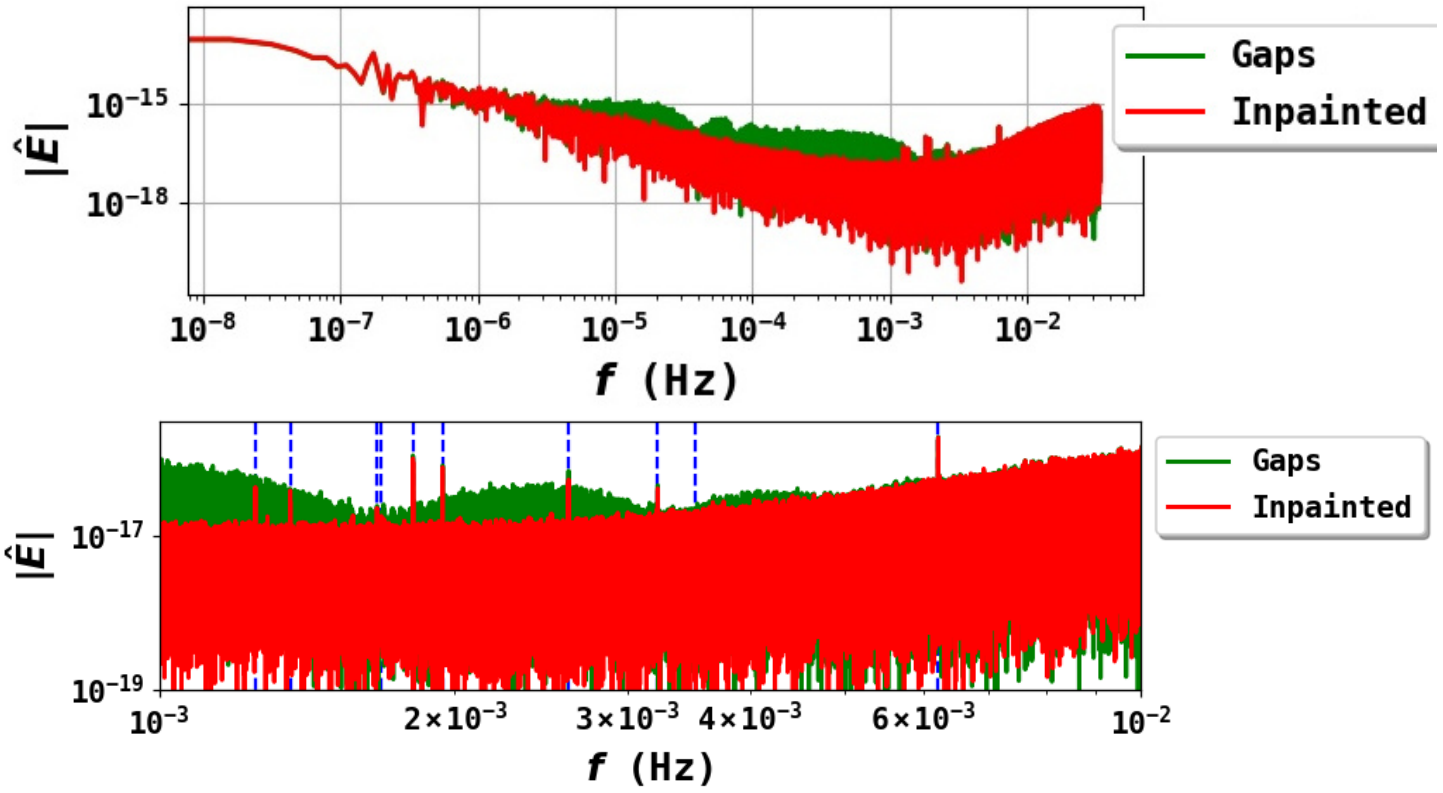


Results.

Test on LDC1-3: verification galactic binaries.

- Gaussian noise.
- Gaps: 7h/2 weeks (planned), 10 min / 1 day (unplanned).
- **Flattened** effective noise power spectral density and **limited** leakage between signal and noise.



Conclusion

Conclusion.

Development and evaluation of a new non-parametric method.

- Tests with both **gapped** and **ungapped** data.
- **Efficient reconstruction** of the signal in the gaps.
- **Fast**: $\sim$ 1 hour for 2 years of data on a laptop.
- Works as well with a **large number of separated sources**.
- **Flexible framework** which could (in principle) be adapted to:
  - Estimation of power spectral density.
  - Other types of sources.
  - Sources of different types.

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