



# Search for Continuous Gravitational Wave signal with Pulsar Timing Array

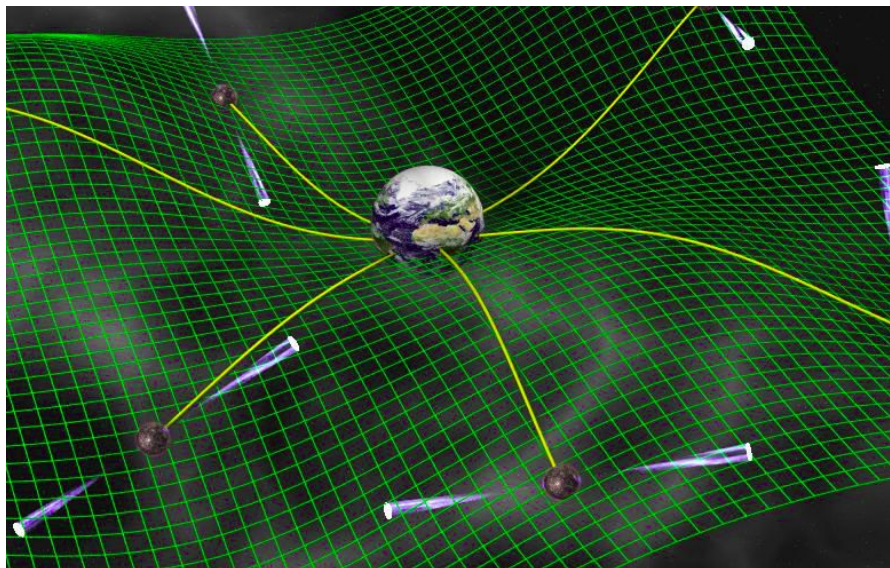
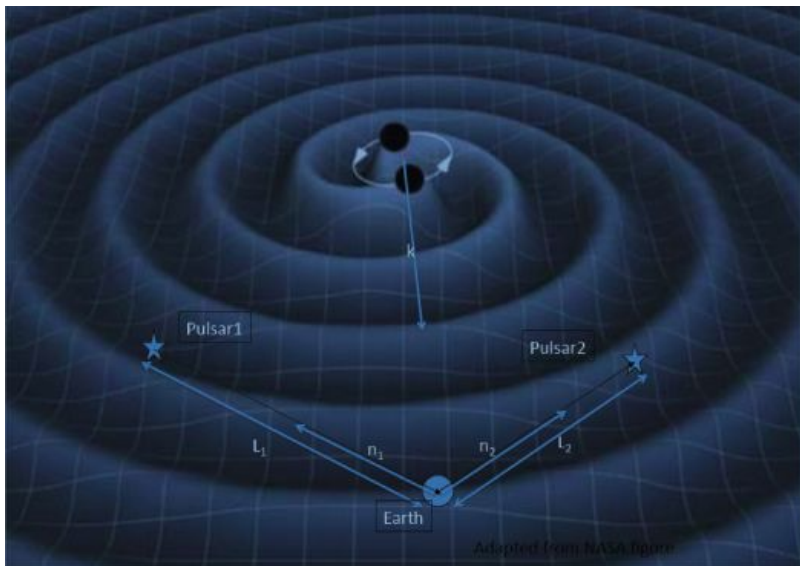
**Mikel Falxa**

APC - 2nd year PhD

*GdR Ondes Gravitationnelles - 15 Oct 2020*

# The Pulsar Timing Array (PTA)

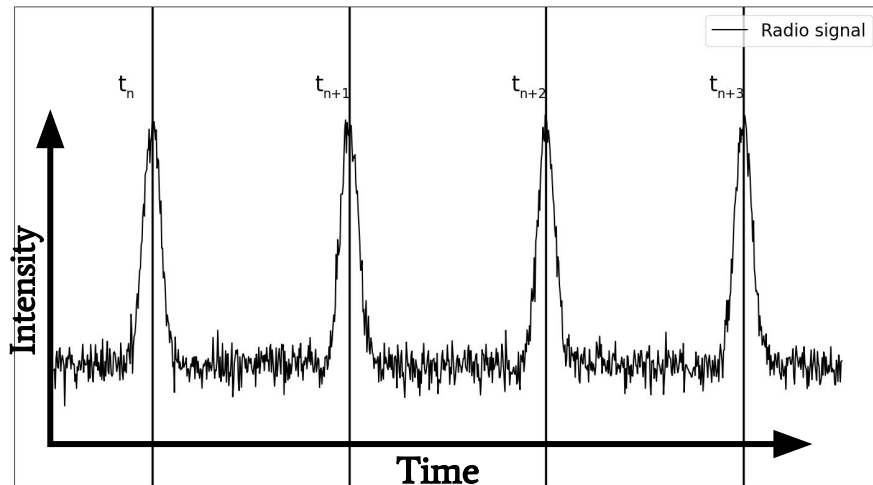
- **Millisecond pulsars** (MSP) are very **stable**
- Used as **clocks** to measure **gravitational wave** signals
- Observed in **radio frequency band**



Credits : Andrea N. Lommen, 2011, [arXiv:1112.2158v2](https://arxiv.org/abs/1112.2158v2)

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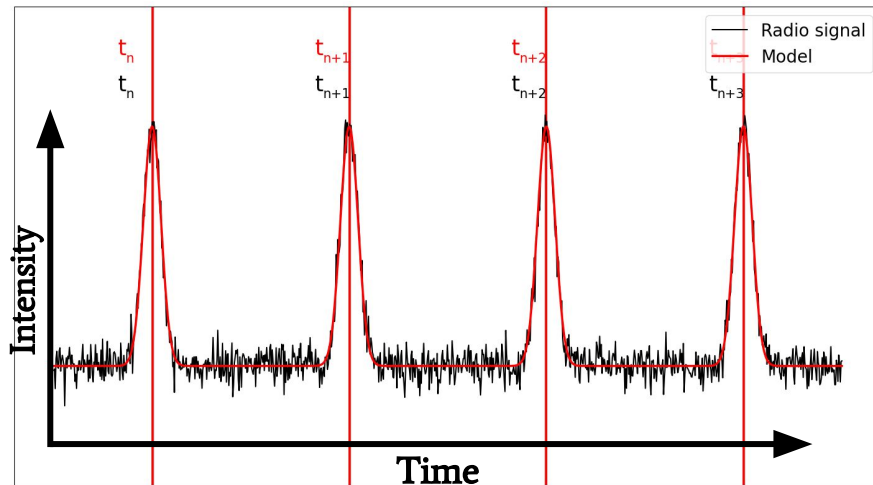
The gravitational wave signal modulates the **time of arrivals** of pulses...



...the measured quantities are the **timing residuals**

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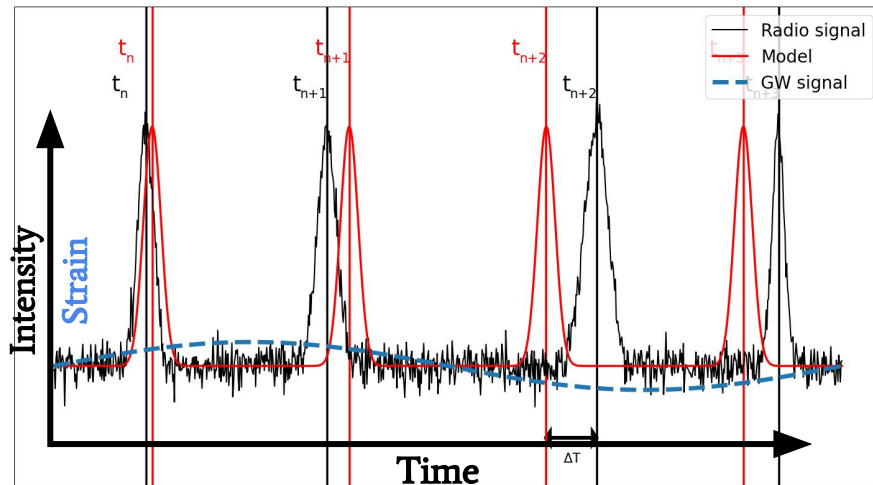
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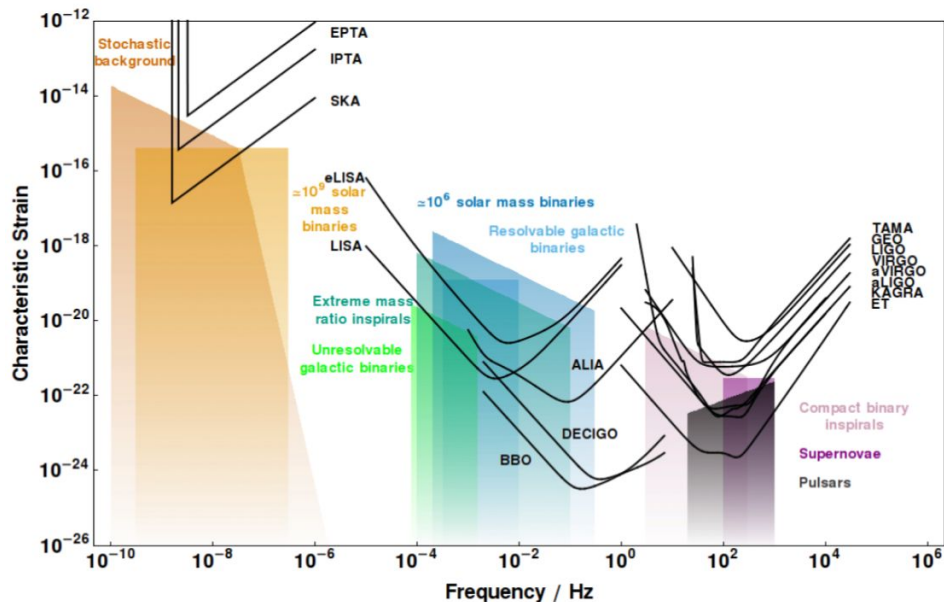
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# The Pulsar Timing Array (PTA)

Two main categories of signals:

- **Gravitational wave background (GWB)**
- **Continuous waves (CW)**

 **Data analysis** for CW detection



Credits : Gravitational-wave sensitivity curves, C J Moore et al., 2014

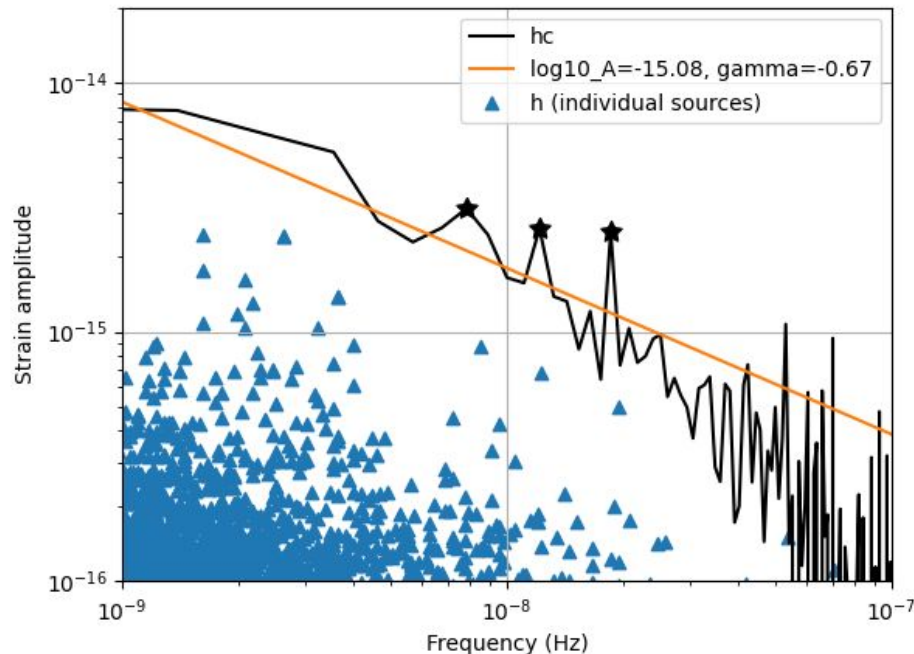


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**↳ Data analysis for CW detection**



# Data analysis

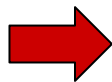
- Build a **model** that takes into account all possible **noise components**
- Main **noise** components: **white** noise, **red** noise, **dispersion variation** noise
- Modelled as **gaussian process**, encoded in the **covariance matrix**  $\Sigma$
- **Bayesian** analysis

**Likelihood:**

$$p(\delta t | \vec{\phi}) = \frac{1}{\sqrt{\det(2\pi\Sigma)}} \exp\left(-\frac{1}{2}\delta t^T \Sigma^{-1} \delta t\right)$$

**Residuals:**

$$\delta t \rightarrow \delta t - \sum_{i=1}^{N_{\text{signals}}} s_i(\vec{\lambda}_i)$$



Continuous waves



# Data analysis

$$\Sigma^{\alpha} = \boxed{\sigma_{\alpha,WN}^2 \delta_{ij}} + \Sigma_{RN}^{\alpha} + \Sigma_{DM}^{\alpha} (+\Sigma_{GW}^{\alpha\beta})$$

**White noise** : measurement errors  
(radiometer noise) + systematics

$$S_{WN} = \sigma^2 \delta(f - f')$$

**Red noise** : low frequency noise on  
pulsar rotation

$$S_{RN} = A_{RN} f^{-\gamma_{RN}}$$

**Dispersion noise** : dispersion due to  
propagation through interstellar medium

$$S_{DM} = \left(\frac{K_{DM}}{\nu^2}\right) A_{DM} f^{-\gamma_{DM}}$$

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# Data analysis

**Stochastic Gravitational Wave Background** : noise term, correlated across pulsars in array

$$S_{GW} = \Gamma_{\alpha\beta} A_{GW} f^{-\gamma_{GW}}$$

$$\Sigma^\alpha = \sigma_{\alpha,WN}^2 \delta_{ij} + \Sigma_{RN}^\alpha + \Sigma_{DM}^\alpha (+ \boxed{\Sigma_{GW}^{\alpha\beta}})$$

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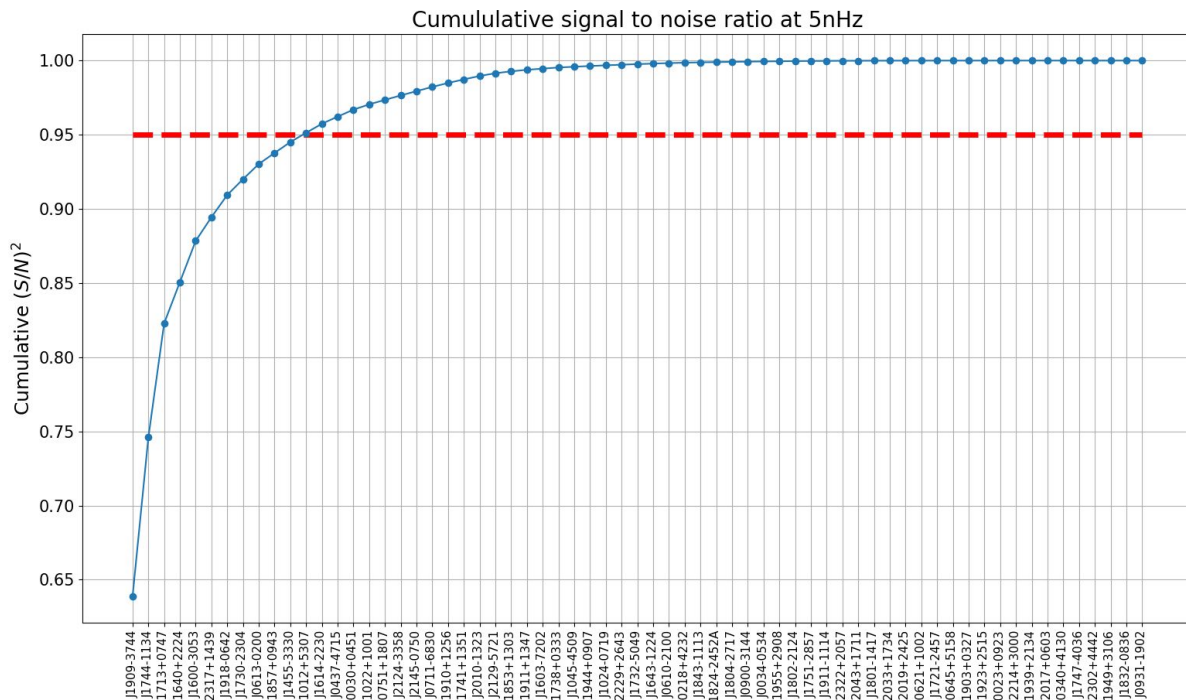
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# Ranking

- Study on **IPTA DR2** data, **65 pulsars**, combining data from **worldwide collaborations**
- We **inject fake GW** signal to estimate **relative contribution** of each **pulsar** to the **total SNR**



We can express the SNR as :

$$\rho^2 \propto \frac{h^2}{\pi^2 f_{gw}^2} \langle s | s \rangle_{\phi_0, i, \psi, \theta, \phi}$$

with :  $\langle s | s \rangle = s^T \Sigma^{-1} s$

the **covariance matrix** is a **function** of **model parameters**, we use **maximum likelihood** parameter values for ranking

# Sensitivity curves

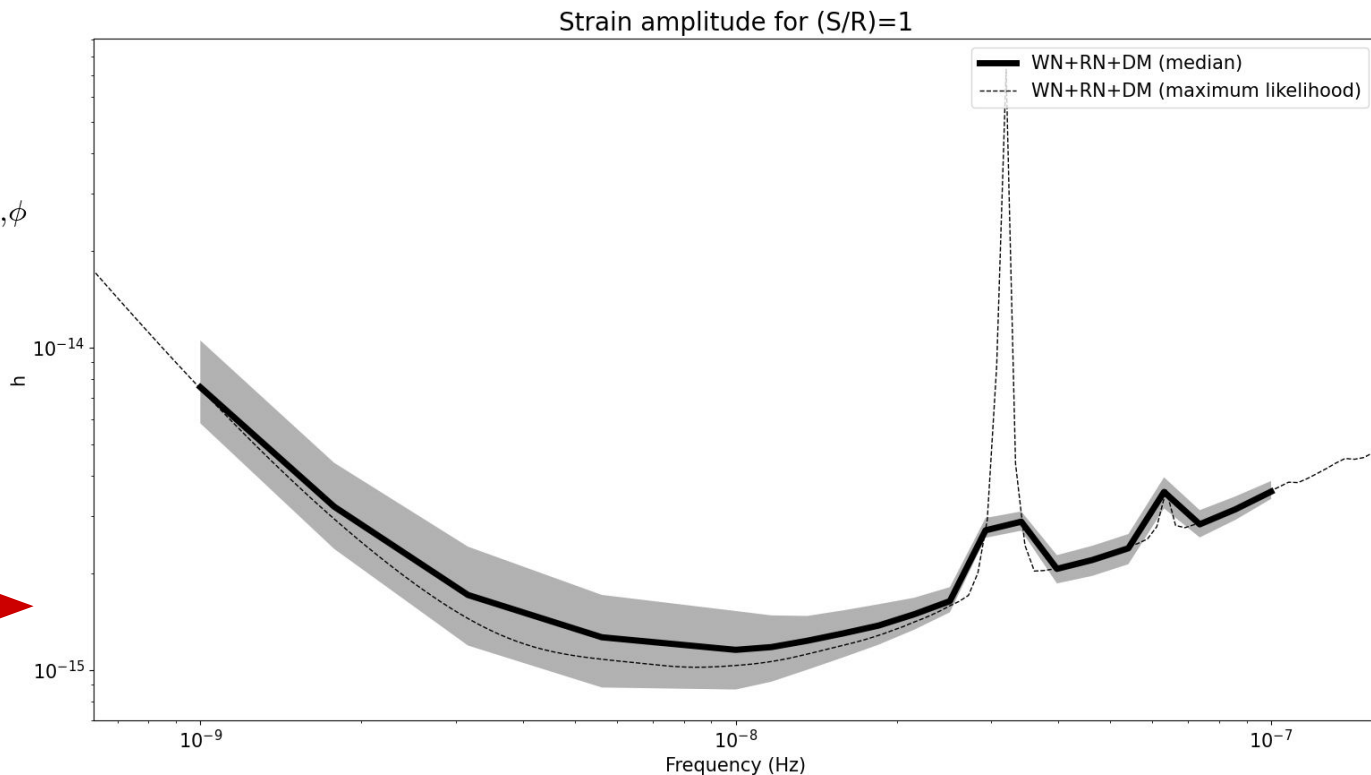
Given the previous expression, we have :

$$\rho^2 \propto \frac{h^2}{\pi^2 f_{gw}^2} \langle s|s \rangle_{\phi_0, i, \psi, \theta, \phi}$$

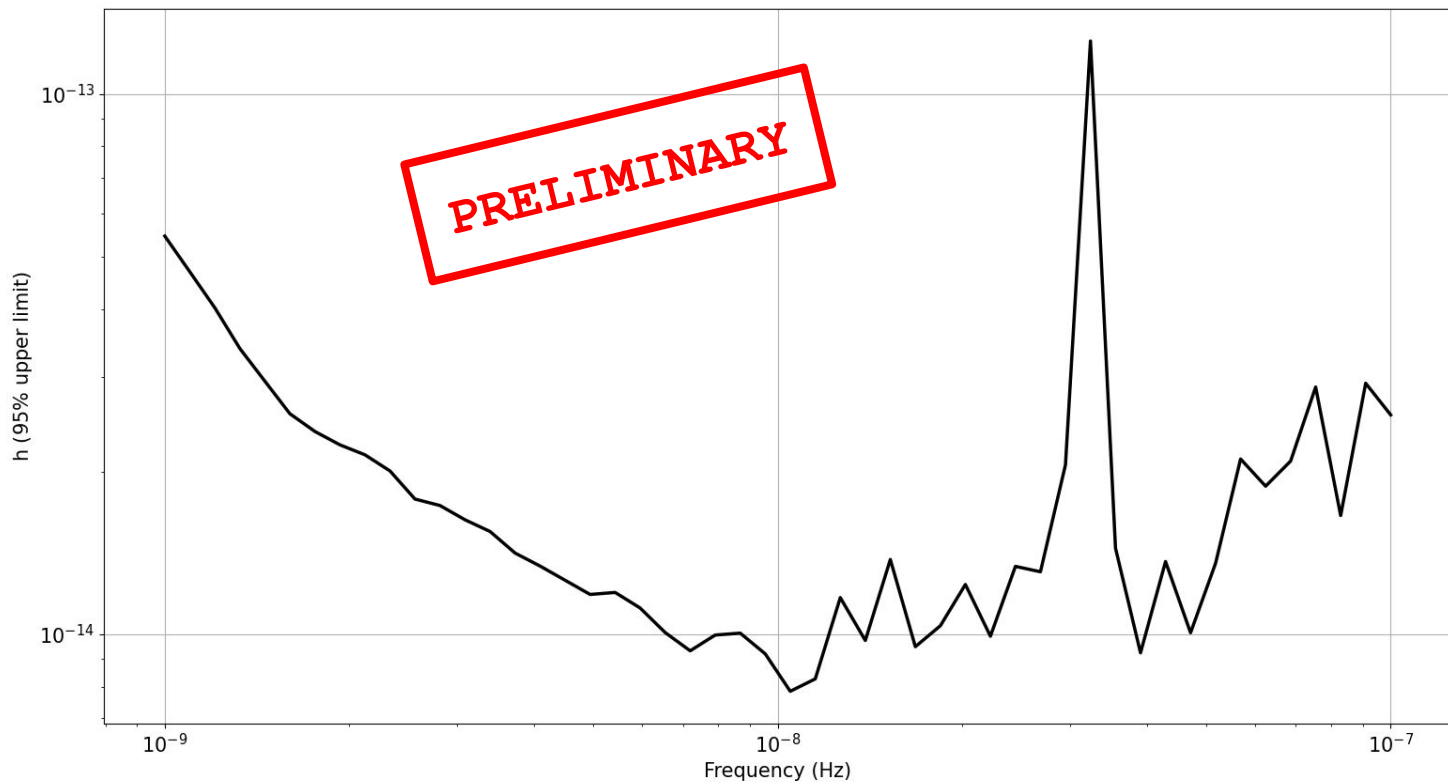


$$h^2 \propto \frac{\rho^2 \pi^2 f_{gw}^2}{\langle s|s \rangle_{\phi_0, i, \psi, \theta, \phi}}$$

$$\rho^2 = 1$$



# 95% upper limit on strain amplitude for CW

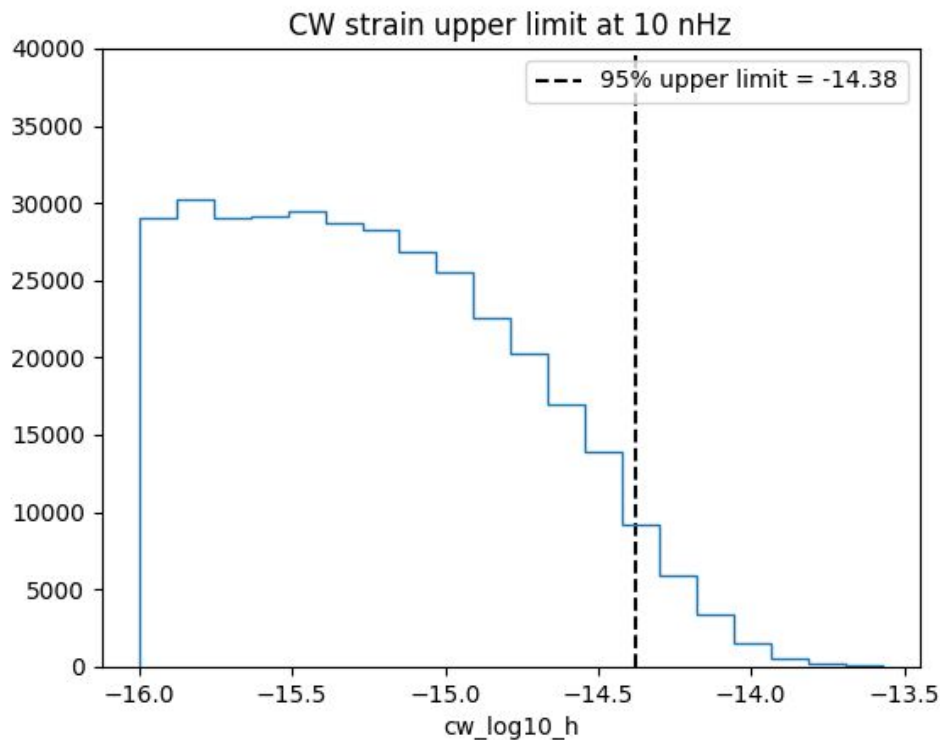




# Summary

- The **PTA collaboration** aims to **detect low frequency GW** signals in the **nHz band**
- We perform **search for continuous GWs** standing **above stochastic signal** (possible nearby **SMBHB**)
- **Ranking** allows to **reduce computational cost** by selecting pulsar which contribute (on average) **95% of total SNR: reduction in number of fitting parameters without significant loss in detectability**
- Preliminary results **do not indicate presence of continuous GW** signal: setting **upper limit** on GW amplitude
- **IPTA data** is most **sensitive** around **10 nHz** where  $h_{95\%} \sim 7.8 \cdot 10^{-15}$

# 95% upper limit on strain amplitude for CW



- The **95% upper limit** is computed by **integration** of the **posterior probability distribution** of our **strain amplitude** parameter
- These posterior probability distributions are obtained with **Monte Carlo Markov Chain** (MCMC) algorithms