

# PBHs AND SBCWS FROM MULTI-FIELDS DYNAMICS DURING INFLATION

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*October 15, 2020 troisième assemblée générale du gdr ondes gravitationnelle*

Based on

*J.F., S. Renaux-Petel, J. W. Ronayne & L. Witkowski 2004.03221,*

*J.F., S. Renaux-Petel, & L. Witkowski in Prep.*

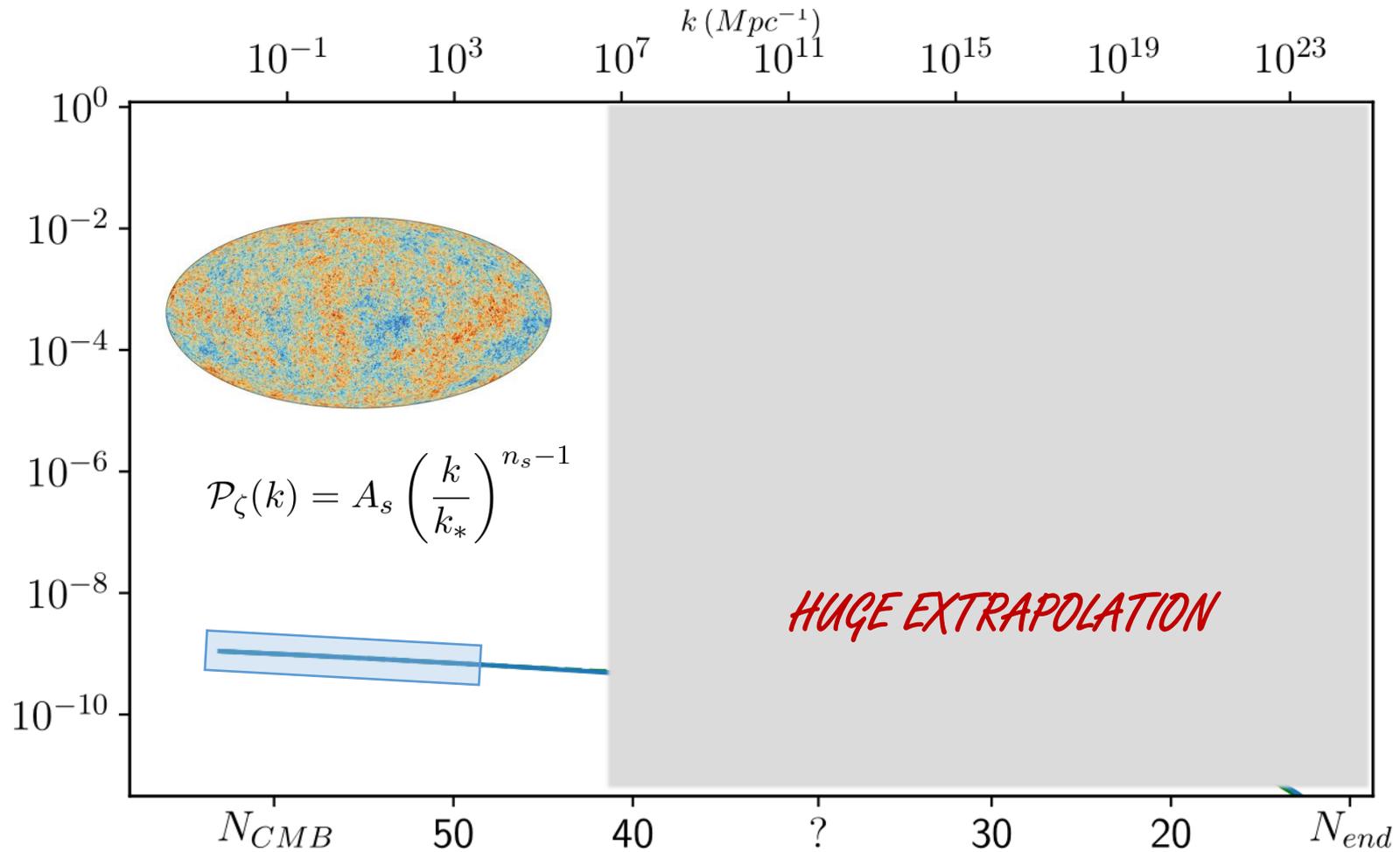
GEODESI



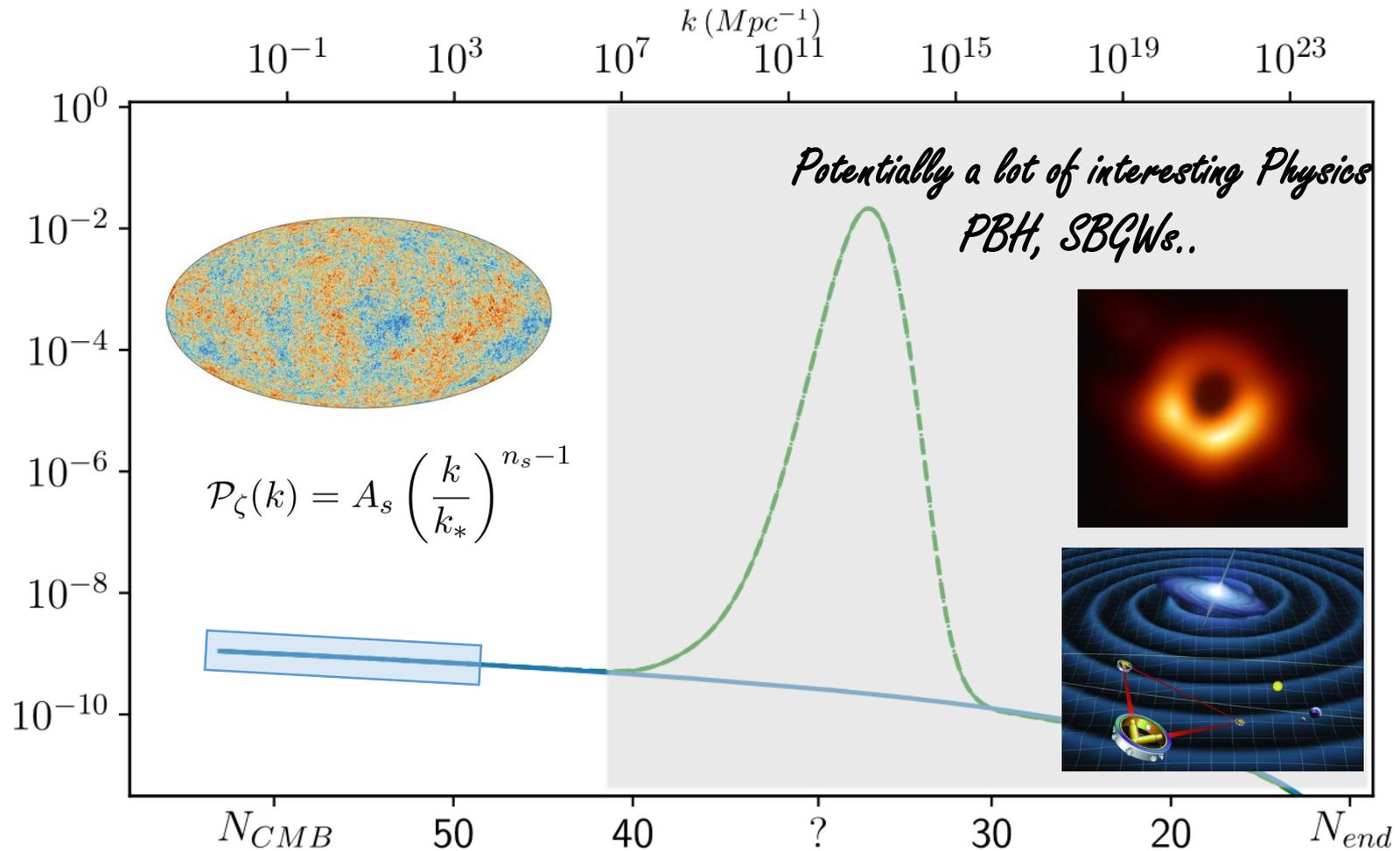
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# PRIMORDIAL SCALAR FLUCTUATIONS



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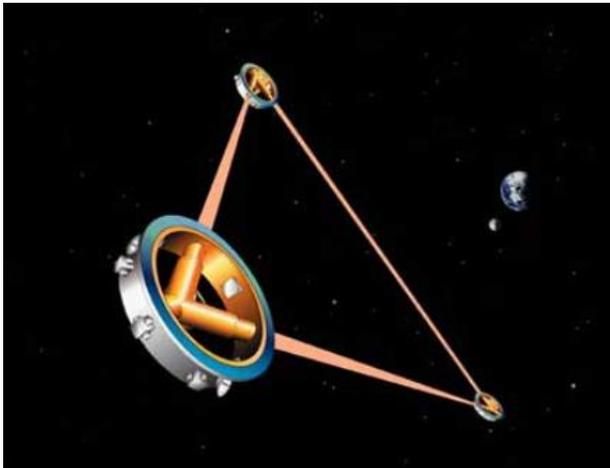


# STOCHASTIC BACKGROUND OF GWs

SCALAR PERTURBATIONS ACT AS A TENSOR SOURCE

$$h''_{ij} + 2\mathcal{H}h'_{ij} + k^2 h_{ij} = S_{ij} \sim (\partial\zeta)^2$$

$$h_{ij} \sim \int \underset{\text{GREEN}}{\text{G}} \times \overset{\text{TRANSFER}}{\text{T}}\{\zeta\zeta\} \sim \mathcal{P}_\zeta$$



Acquaviva et al. '02; Mollerach, Harari, Matarrese '03;  
Ananda, Clarkson, Wands '06;

Baumann et al. '07

Caprini and Figueroa '18

## QWS SPECTRUM

$$- \Omega_{\text{GW}}(k) = \frac{1}{\rho_c} \frac{d\rho_{\text{GW}}}{d \ln k} \quad \frac{m_{\text{pl}}^2}{16a^2} \langle (\nabla h_{ij})^2 \rangle$$

$$\propto \int_0^\infty dv \int_{|1-v|}^{1+v} du I(u, v) \mathcal{P}_\zeta(ku) \mathcal{P}_\zeta(kv)$$

See for instance:

K. Kohri, T Terada. 18

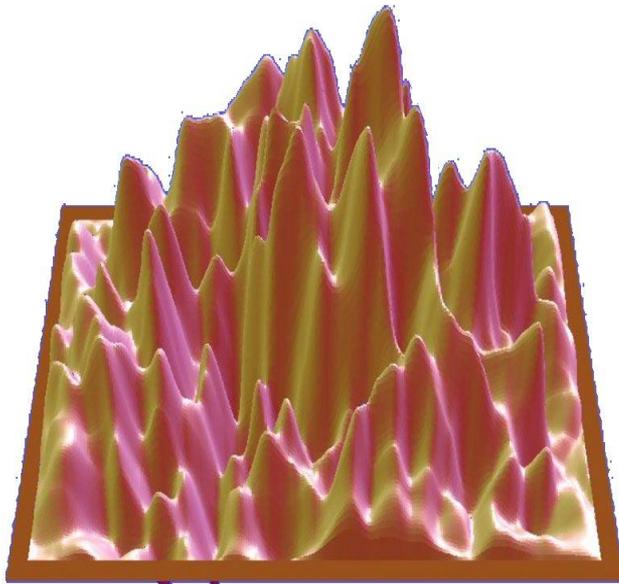
J. Espinosa, D. Racco, A. Riotto '18

M. Maggiore 2000

# TURNING IN THE LANDSCAPE

VARIOUS IDEAS TO ACHIEVE THE DESIRED ENHANCEMENT ...

*Inflection point, ultra slow-roll, varying speed of sounds, gauge fields, hybrid inflation etc. etc.*



*GENERIC/MODEL-INDEPENDENT PROPOSAL,*

- ✗ UNIQUE TO MULTI-FIELD*
- ↳ THEORETICAL ADVANTAGES*
- ✗ CHARACTERISTIC SIGNATURES*

*See also:*

*G. Palma, S. Sypsas, C. Zenteno '20*

# PLAN

- INFLATION IN NON-GEODESIC MOTION (MECHANISM)
- POWER SPECTRUM (PECULIARITY)
- CONSEQUENCES FOR
  - PRIMORDIAL BLACK HOLES
  - STOCHASTIC BACKGROUND OF GRAVITATIONAL WAVES

# INFLATION IN NON-GEODESIC MOTION

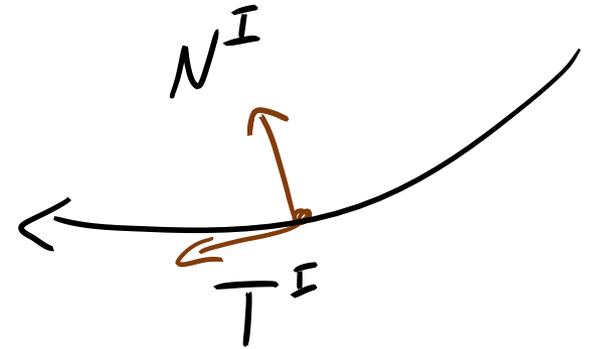
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$$S = \int d^4x \sqrt{-g} \left[ \frac{M_{\text{Pl}}^2}{2} R - \frac{1}{2} G_{IJ} \partial^\mu \phi^I \partial_\mu \phi^J - V(\phi) \right]$$

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$$N^I(\text{EOM}) \rightarrow D_t T^I = \gamma_\perp H N^I$$



$$\gamma_\perp = 0 \Rightarrow \text{GEODESIC MOTION, } D_t T^I = 0$$

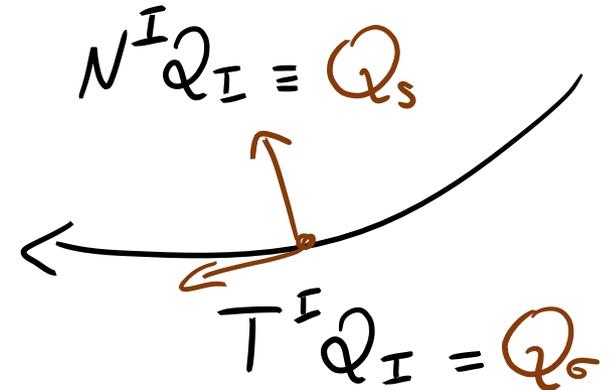
$$\gamma_\perp \gg 1 \Rightarrow \text{STRONGLY NON-GEODESIC}$$



# MULTI-FIELD PERTURBATIONS

- *EDM ENTROPIC FLUCTUATION*

$$\ddot{Q}_s + 3H\dot{Q}_s + \left(\frac{k^2}{a^2} + m_s^2\right) Q_s = -\alpha\eta_\perp\dot{\zeta}$$



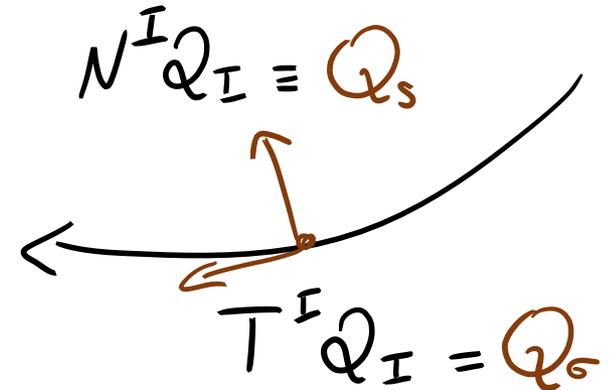
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*POTENTIAL + GEOMETRY* —  $\eta_{\perp}^2 H^2$

- N.B.  $\eta_{\perp}^2 \gg 1 \implies m_s^2 < 0$

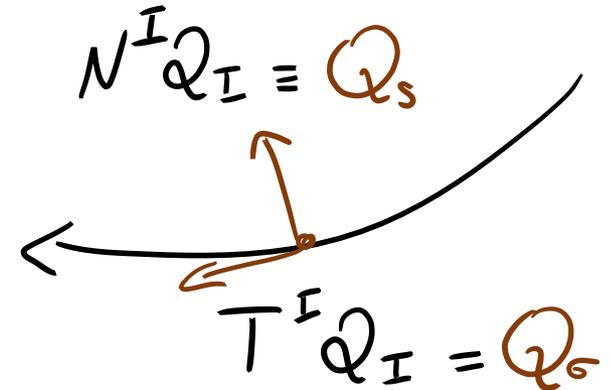


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### MOTIVATED SET-UP

SIDETRACKED, HYPERINFLATION,  
ANGULAR INFLATION .....

S. Garcia Saenz, S. Renaux-Petel & J.  
Rouayre '18, A. Brown '17, D. Marsh &  
T. Bjorkmo '19, P. Christodoulidis, D. Rest, E. Sfakianidis

.....

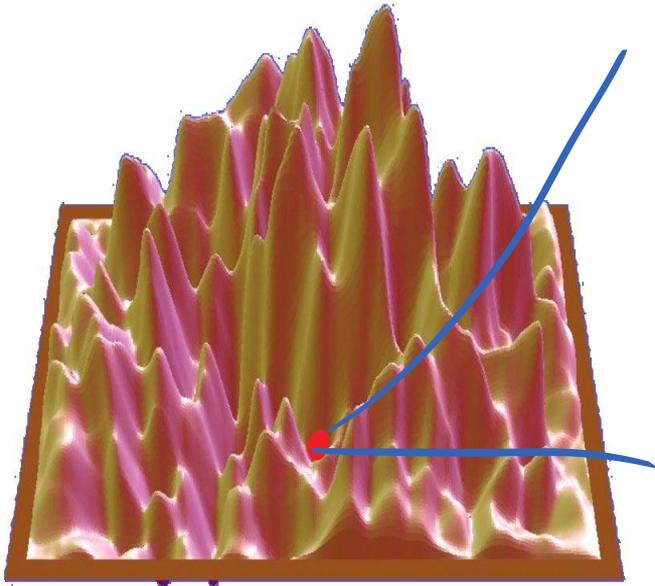
### GEOMETRICAL DESTABILIZATION OF INFLATION

S. Renaux-Petel & K. Turzinsky '15 ...

### LARGE BENDING - SWAMPLAND

A. Achucarro & G. Palma '18

# TURNING IN THE LANDSCAPE



*MODEL-INDEPENDENT TREATMENT*

$$\eta_{\perp}(N), \quad m_s(N), \quad H(N)$$

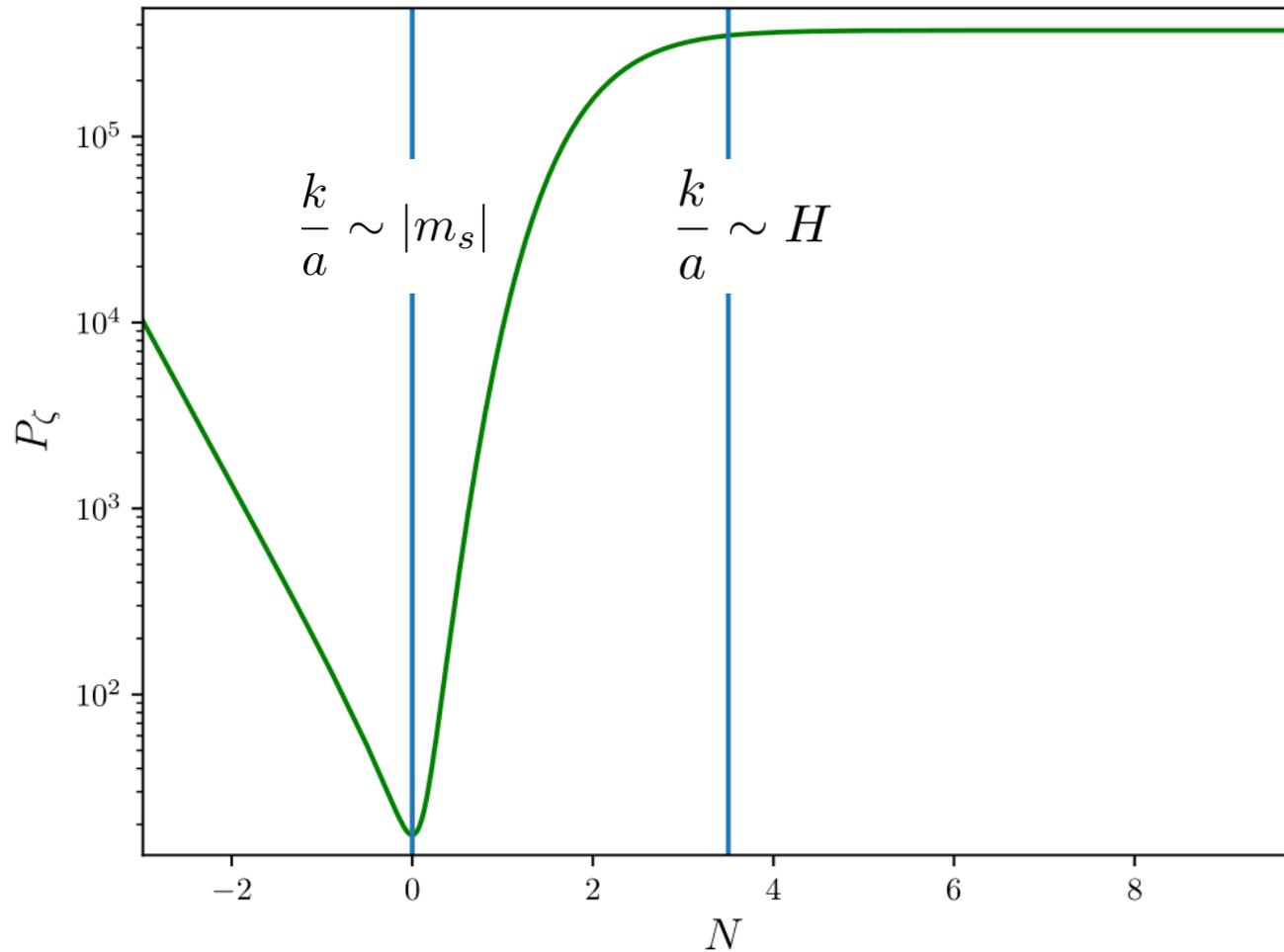
*NO NEED FOR A WATERFALL PHASE*

*J. Garcia-Bellido, A. D. Linde, and D. Wands '96*

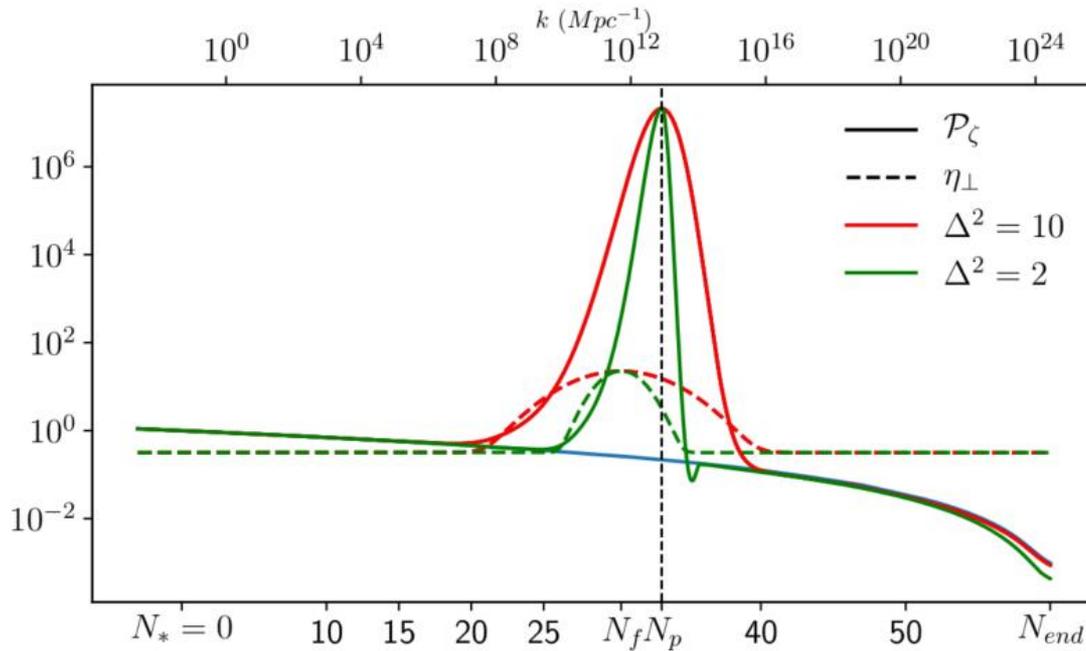
*S. Clesse, J. Garcia-Bellido '15*

*NO NEED FOR USR and/or STOCHASTIC  
TREATMENT*

# TIME EVOLUTION OF $\mathcal{P}_\zeta(k)$



# TURNING IN THE LANDSCAPE



$$\eta_{\perp} = \eta_{\perp}^{\max} e^{-\frac{(N - N_f)^2}{2\Delta^2}}$$



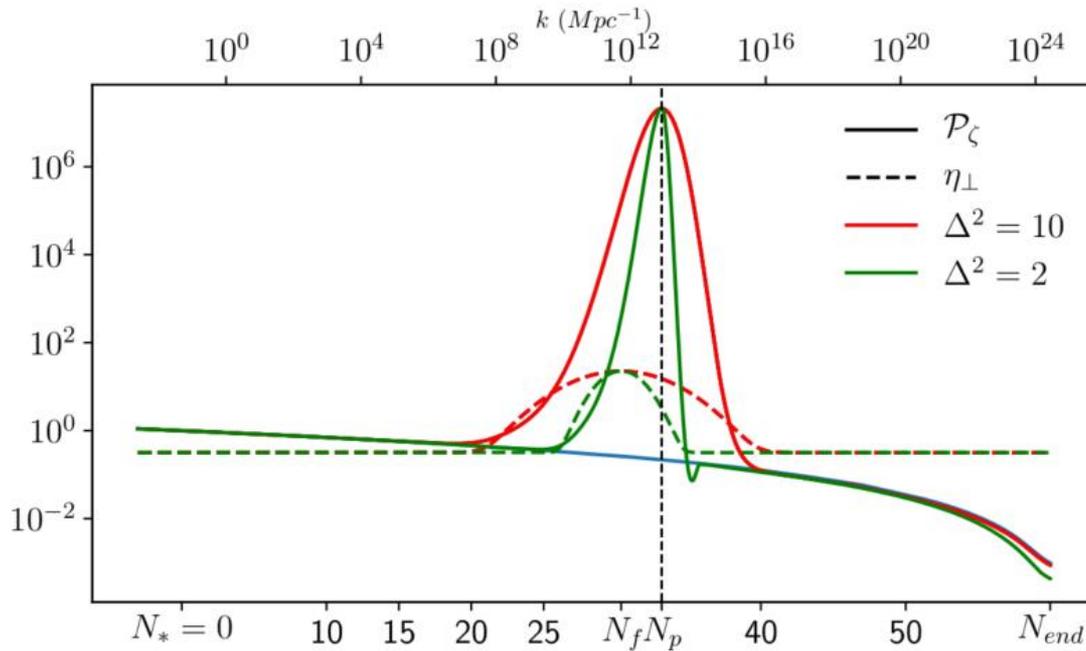
$$\mathcal{P}(k) = \mathcal{P}_0(k) e^{2x} \quad x \propto \eta_{\perp}$$

*J.F., L. Witkowski, S. Renaux-Petel, J. Ronayne '20*

*ANALYTICALLY*

- *POSITION/HEIGHT*
- *SHAPE*
- *GROWTH OF  $\mathcal{P}_{\zeta}$*

# TURNING IN THE LANDSCAPE



$$\eta_{\perp} = \eta_{\perp}^{\max} e^{-\frac{(N - N_f)^2}{2\Delta^2}}$$



**N.B. BEYOND SINGLE-FIELD THEOREM ON STEEPNESS OF THE POWER SPECTRUM**

*C. T. Byrnes, P. S. Cole, and S. P. Patil '18  
P. Carrilho, K. A. Malik, and D. J. Malmgren, '19*

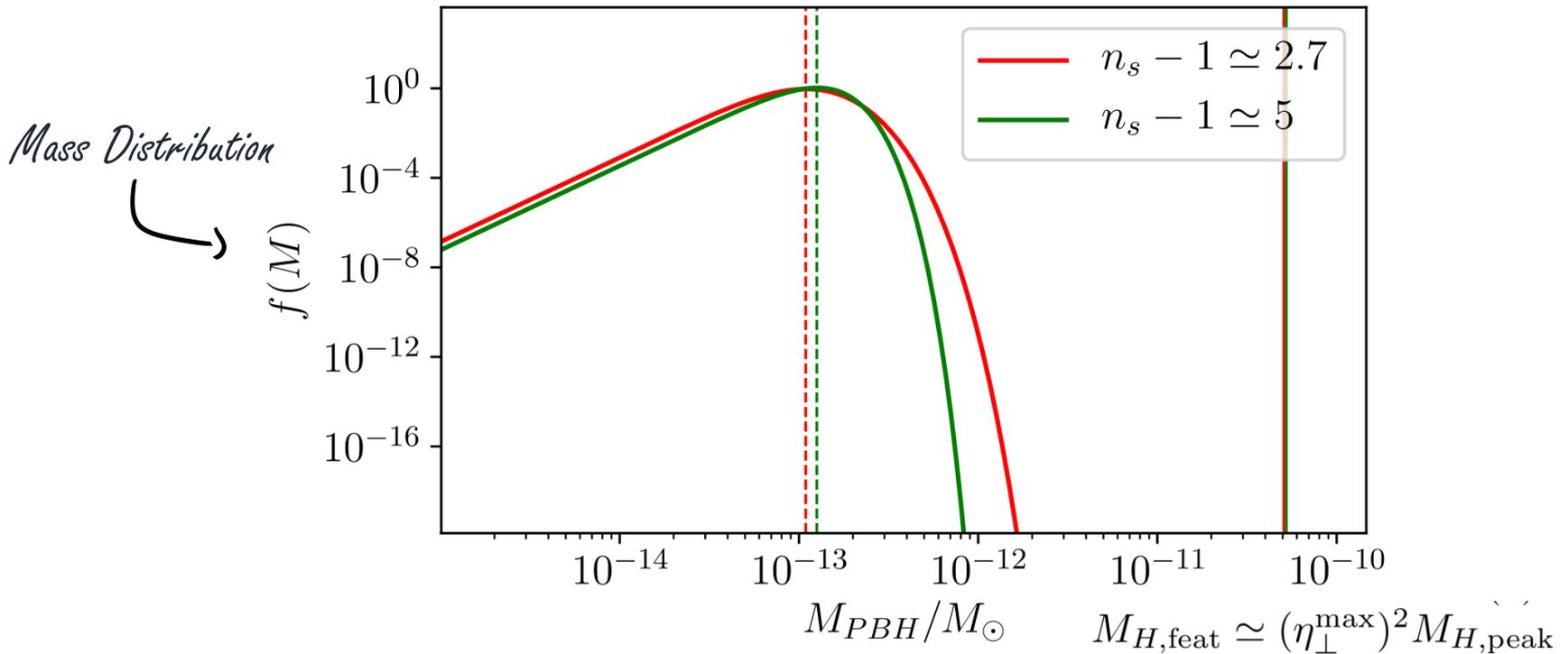
ANALYTICALLY

- POSITION/HEIGHT
- SHAPE
- GROWTH OF  $\mathcal{P}_{\zeta}$

# PBHs - Mass Distribution

(ASSUMING GAUSSIAN STATISTICS)

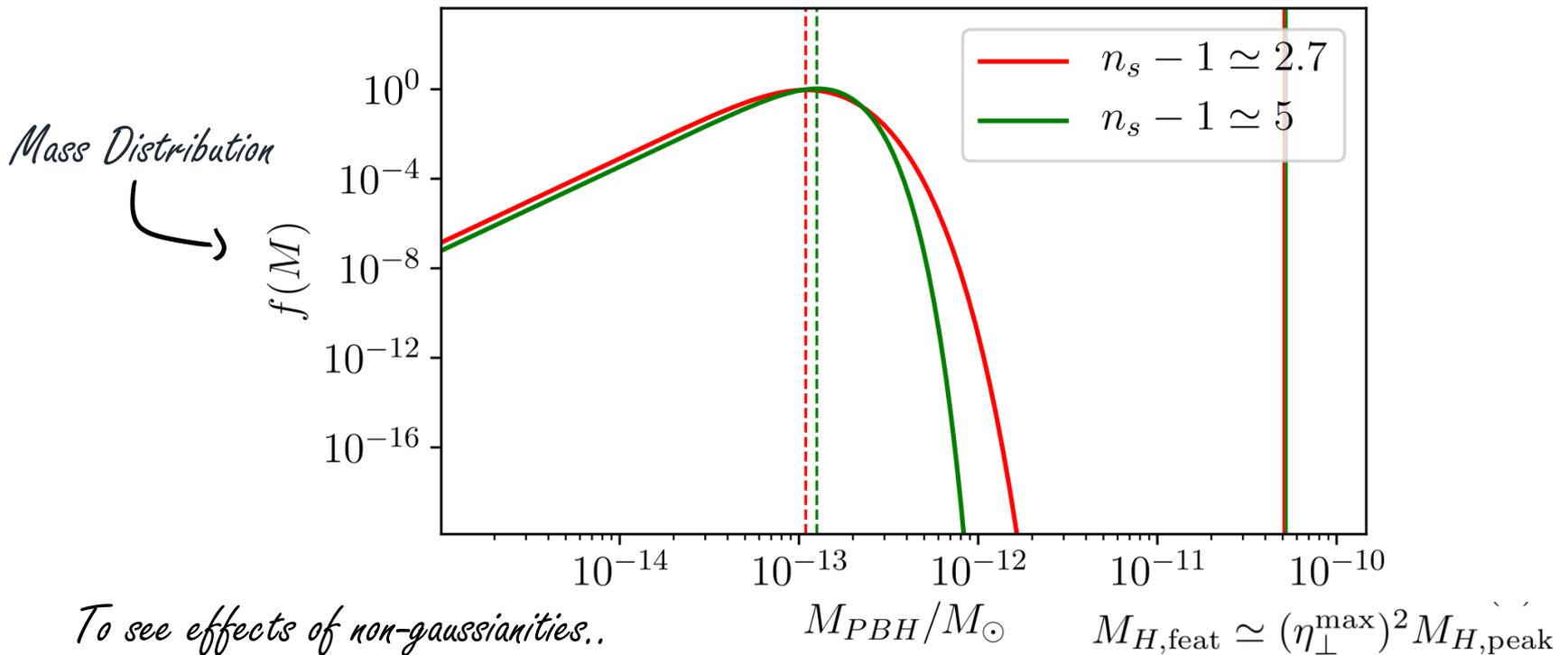
BEYOND SINGLE-FIELD GROWTH OF  $\mathcal{P}_\zeta(k) \implies$  NARROWER PEAK FOR  $f(M)$



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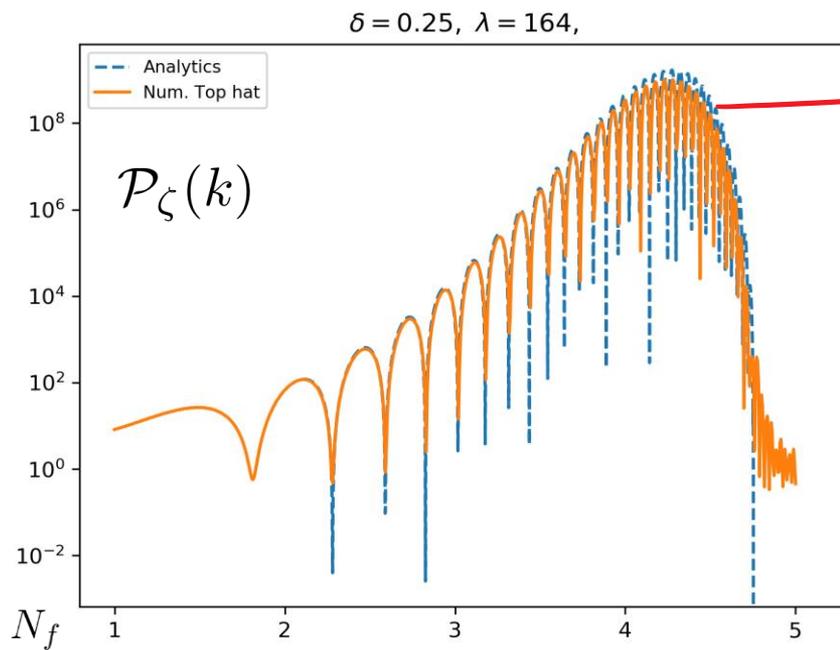


J.F., S. Garcia Saenz, L. Pinol, S. Renaux-Petel, J. Ronayne PRL '19

STRONGLY NON-GEODESIC MOTION  $\implies$  HYPER-NON-GAUSSIANITIES

# SHARP TURNING

- SHARPER BENDING  $\Delta t \ll H^{-1}$



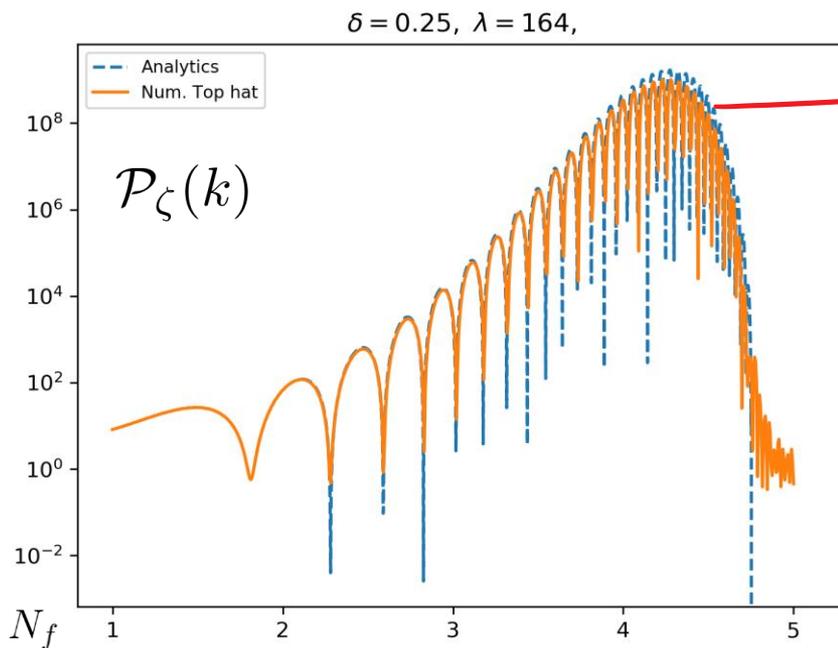
OSCILLATORY  
PATTERN IN  $\mathcal{P}_\zeta(k)$

$$\eta_\perp = \begin{array}{c} \lambda/2 \\ \delta \end{array}$$

See also: G. Palma, S. Sypas, C. Zenero '20

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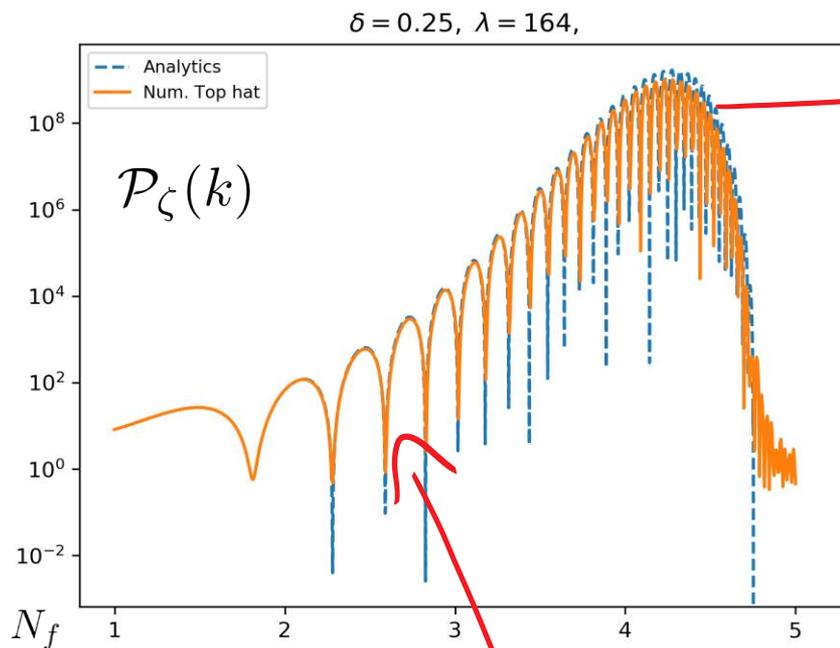
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$$P_\zeta(x \equiv k/k_*) \propto P_0 e^{\lambda \delta Y(x)} (1 + \alpha(x) \cos(e^{-\delta/2} \lambda x) + \beta(x) \sin(e^{-\delta/2} \lambda x))$$

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OSCILLATORY PATTERN IN  $\mathcal{P}_\zeta(k)$

$$\eta_\perp = \begin{array}{c} \lambda/2 \\ \delta \end{array}$$

CONSTANT FREQUENCY  $\frac{\Delta k}{k_*} \simeq \frac{2\pi}{e^{-\delta/2}\lambda} \simeq \frac{2\pi}{\lambda}$

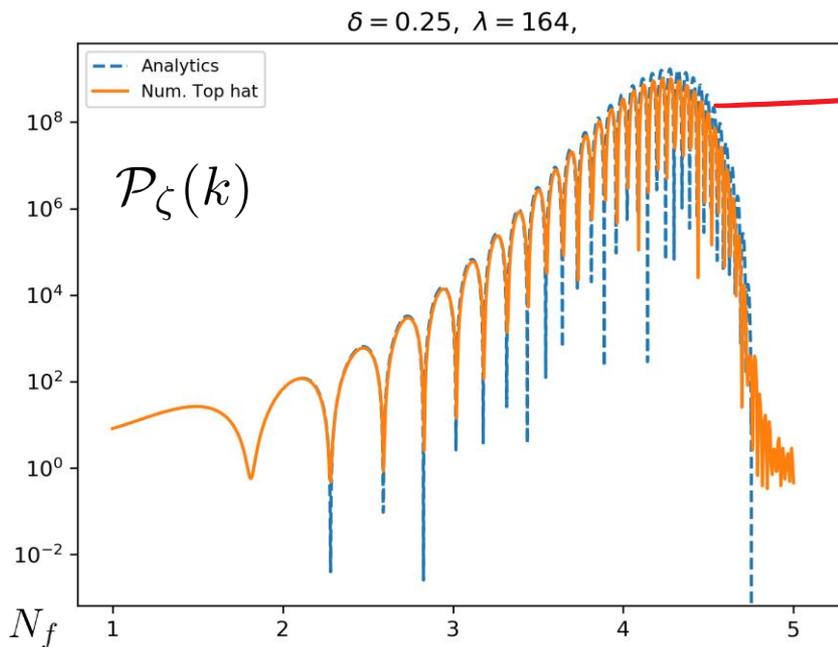
ENVELOPE DEPENDS ON  $m_s^2$

$$P_\zeta(x \equiv k/k_*) \propto P_0 e^{\lambda \delta Y(x)} (1 + \alpha(x) \cos(\underline{e^{-\delta/2} \lambda x}) + \beta(x) \sin(e^{-\delta/2} \lambda x))$$

ANGLE SWEEP IN FIELD-SPACE

# SHARP TURNING

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See also: G. Palma, S. Sypsas, C. Zenteno '20



DO OSCILLATIONS LEAVE IMPRINTS IN THE GRAVITATIONAL WAVES SPECTRUM?

# STOCHASTIC BACKGROUND OF GWs

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$$\Omega_{\text{GW}}(k) \propto \int \int I(u, v) \mathcal{P}_\zeta(ku) \mathcal{P}_\zeta(kv)$$

*Ex.*  $\mathcal{P}_\zeta = A_p \delta(\log k/k_*)$

*MANY RECENT ANALYTICAL STUDIES*

*K. Kolari, T Terada '18,*

*J. Espinosa, D. Racco, A. Riotto '18,*

*R. Cai, S. Pi, M. Sasaki '19...*

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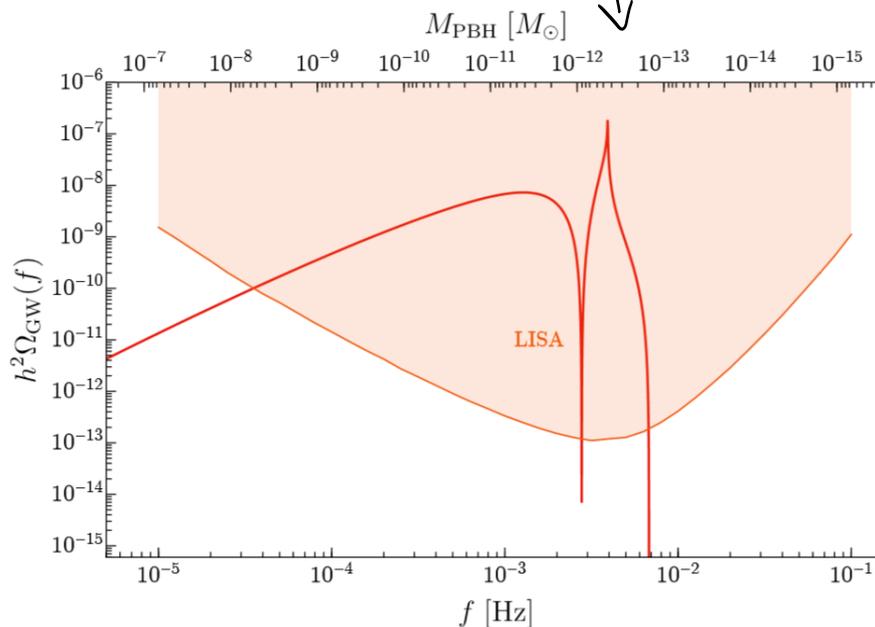
$$\text{Ex. } \mathcal{P}_\zeta = A_p \delta(\log k/k_*) \implies k_p = \frac{2}{\sqrt{3}} k_*$$

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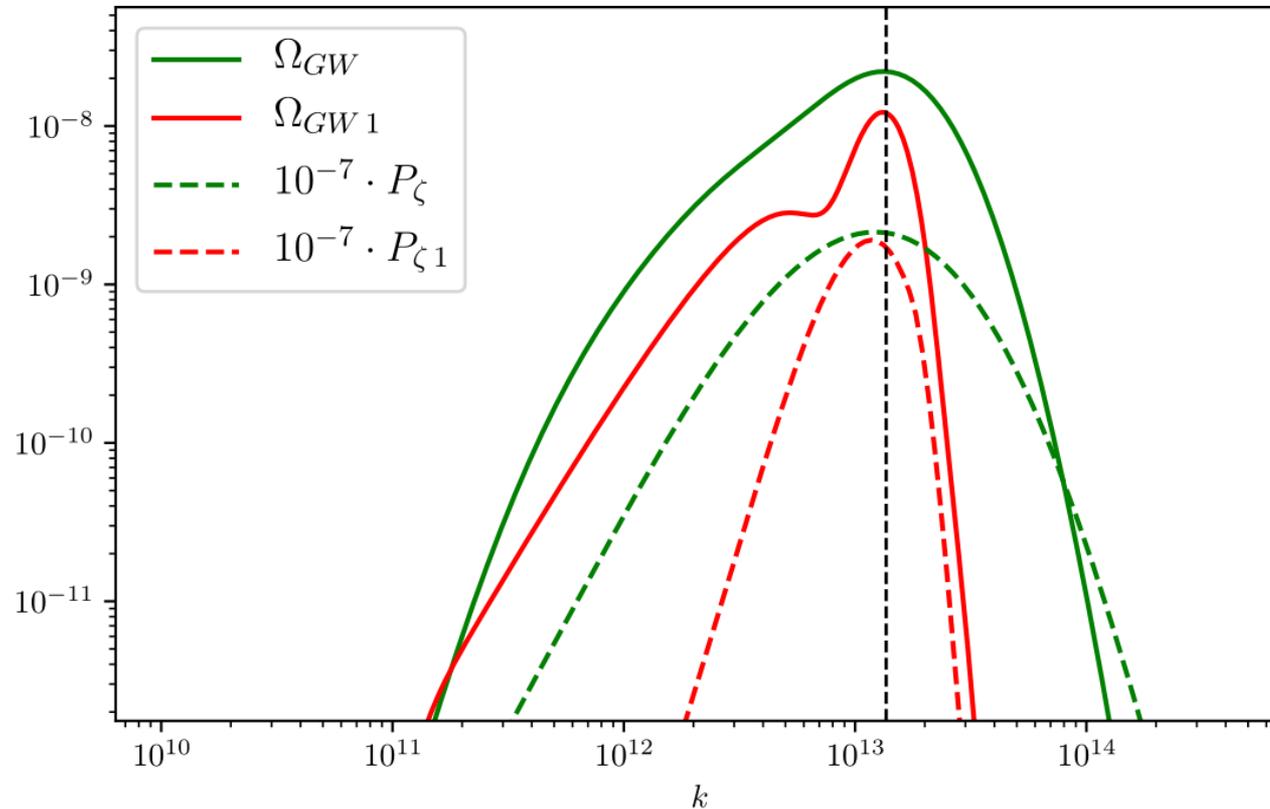


*Ex Passant: LISA SERENDIPITY*

*Franciolini et al. '18*

# STOCHASTIC BACKGROUND OF GWs

FEATURES FROM STEEPER GROWTHS OF  $\mathcal{P}_\zeta(k)$

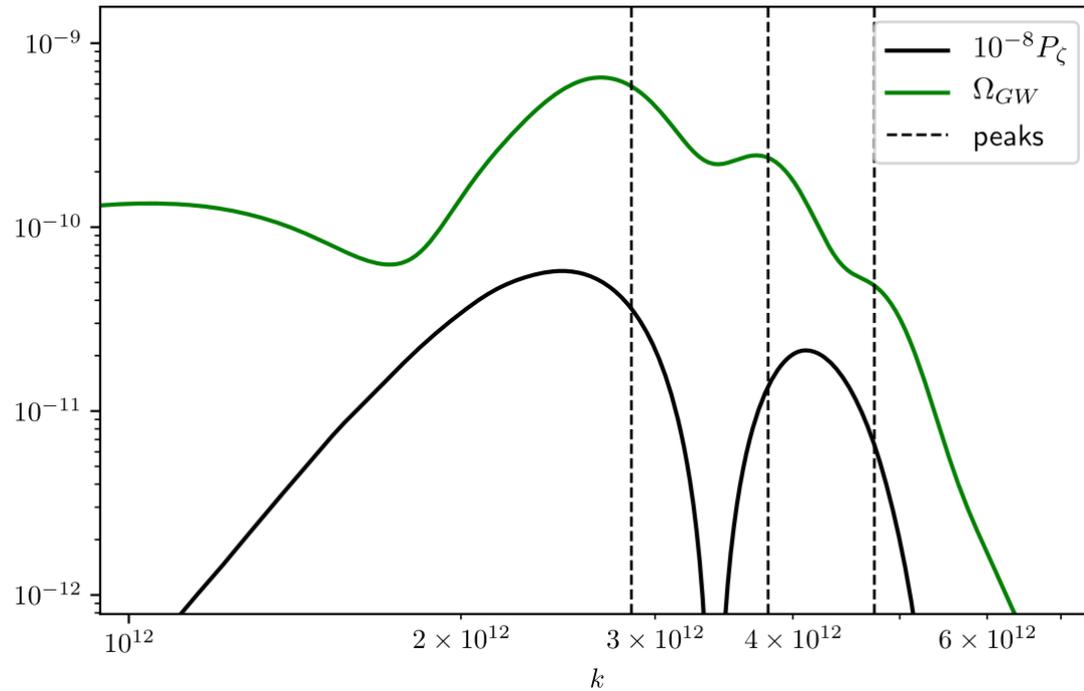


# STOCHASTIC BACKGROUND OF GWs

$N$  - PEAKS IN  $\mathcal{P}_\zeta(k)$   $\implies$   $N(N+1)/2$  PEAKS IN  $\Omega_{\text{GW}}$

$$k_{pij} = \frac{k_i + k_j}{\sqrt{3}}, \quad i, j = 1, \dots, n$$

*R. Cai, S. Pi, S. Wang, & X. Yang*



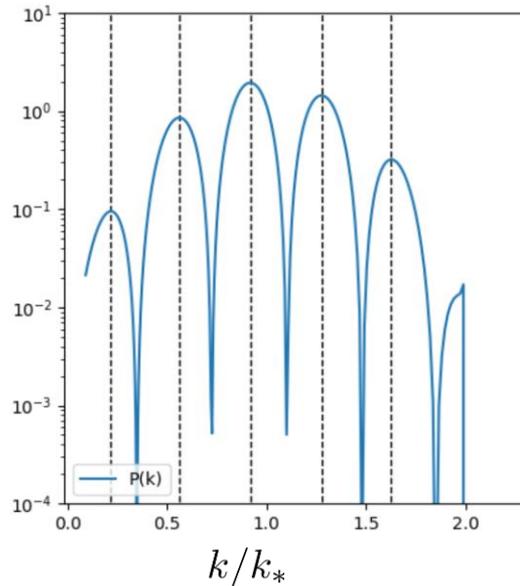
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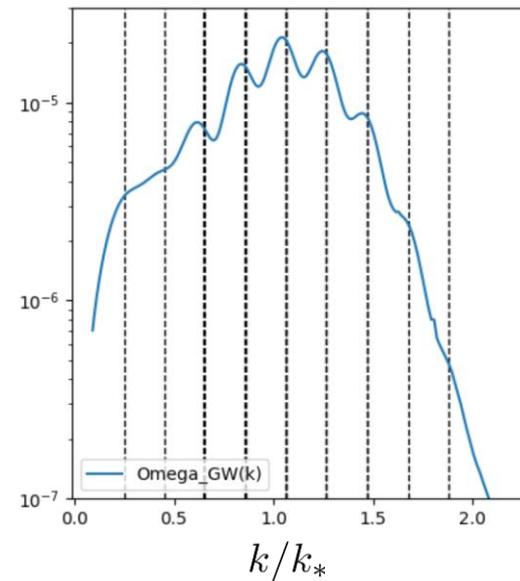
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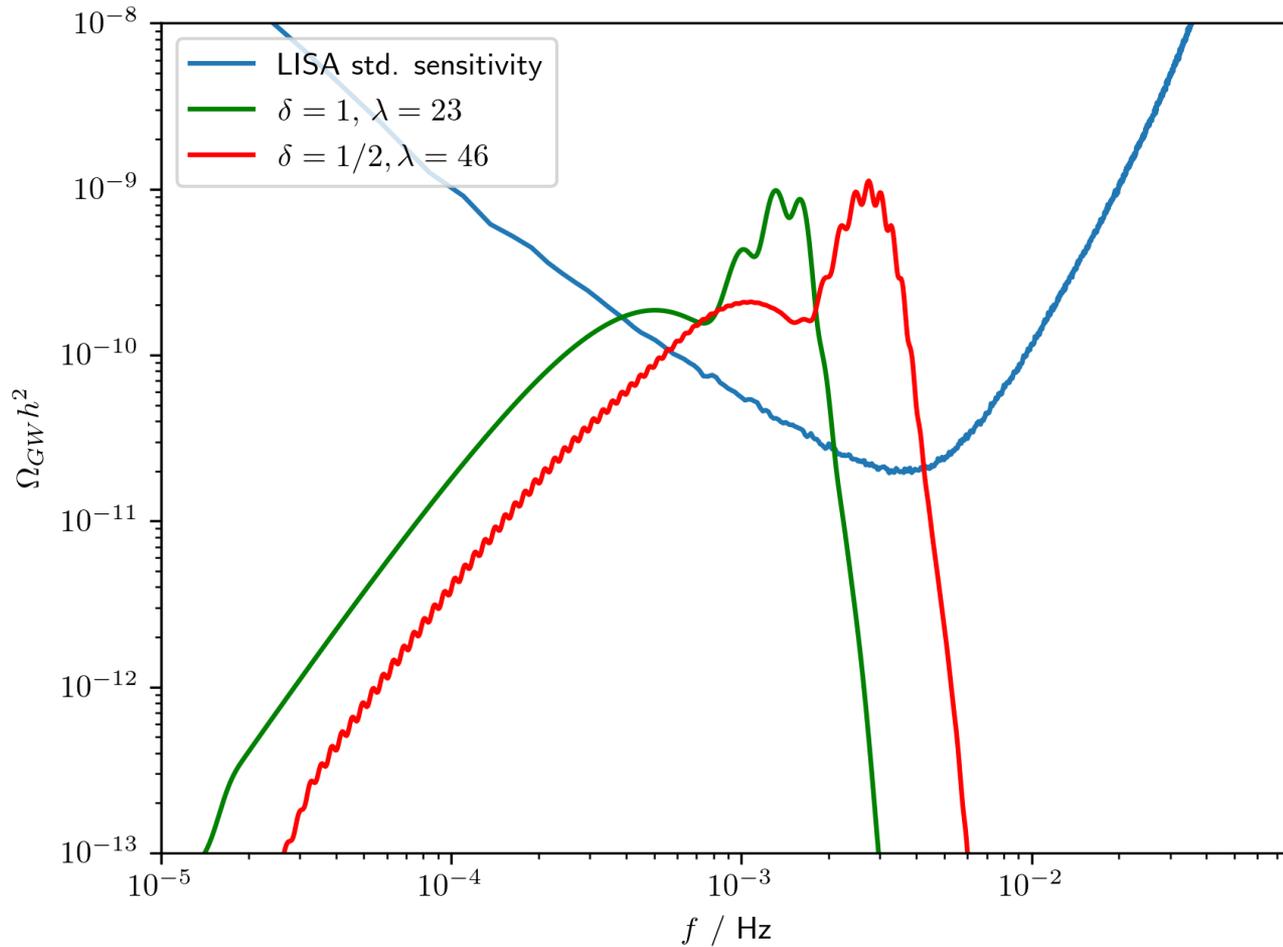
$\mathcal{P}_\zeta(k)$



$\Omega_{\text{GW}}$



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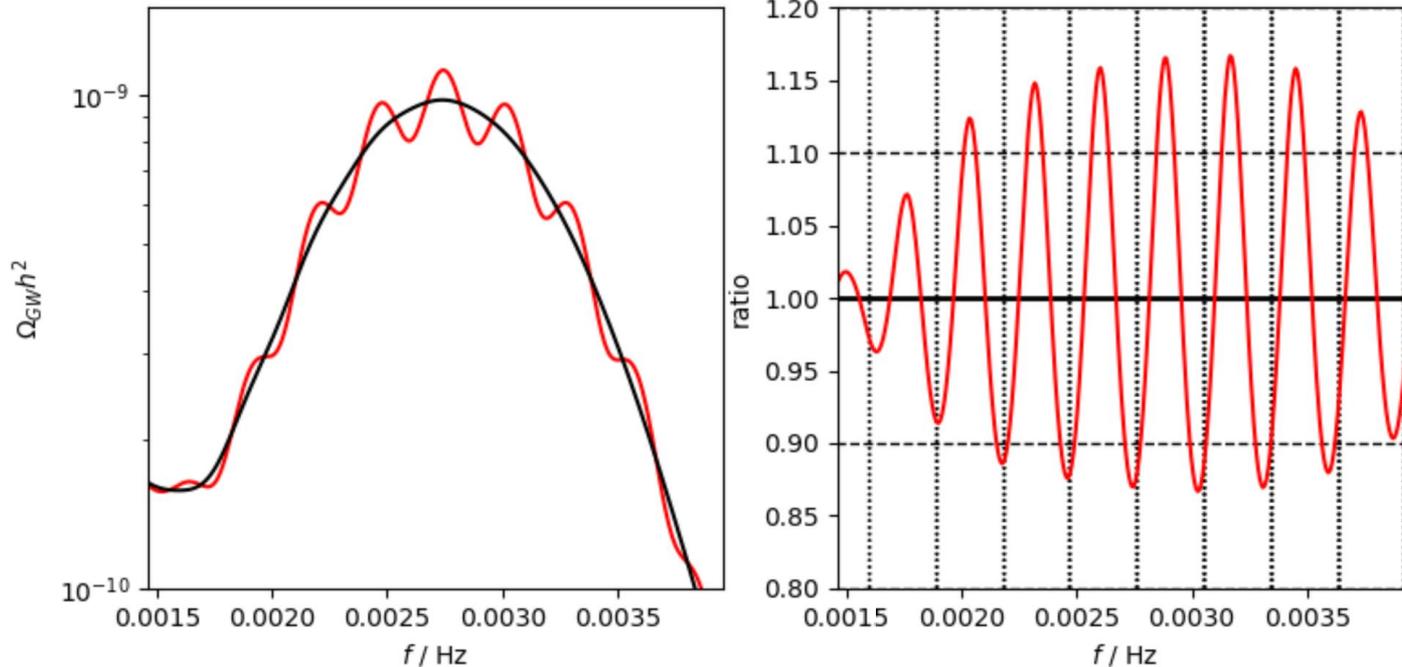


*J. F., S. Renaux-Petel, L. Witkowski, IN PREP.*

# STOCHASTIC BACKGROUND OF GWs

MANY PEAKS & CHARACTERISTIC OSCILLATIONS

$$\delta = 1/2, \lambda = 46$$



*J. F., S. Renaux-Petel, L. Witkowski, IN PREP.*

# CONCLUSIONS

- ENHANCEMENT of CURVATURE PERTURBATIONS DURING INFLATION LEADS TO PBHs & GWs SIGNALS

PROPOSED MECHANISM (Turning in the Landscape)

EVASIVE SINGLE-FIELD BOUNDS

UNIQUE TO MULTI-FIELD  
INFLATION

See also: G. Palma, S. Sypsas, C. Zenteno '20

CHARACTERISTIC SIGNATURES IN  
SBCWs

EFFECTS FROM  
DISTINCTIVE PATTERN OF  
NON-GAUSSIANITIES  
(to do)

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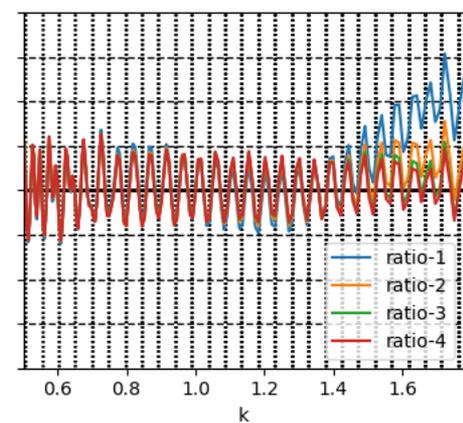
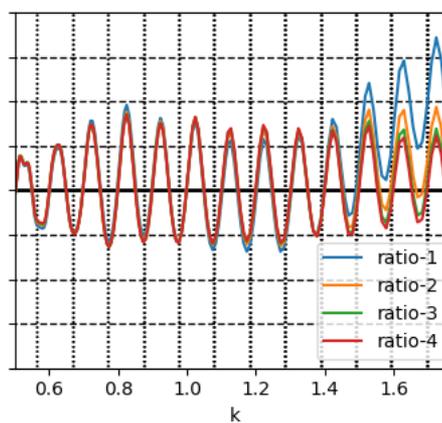
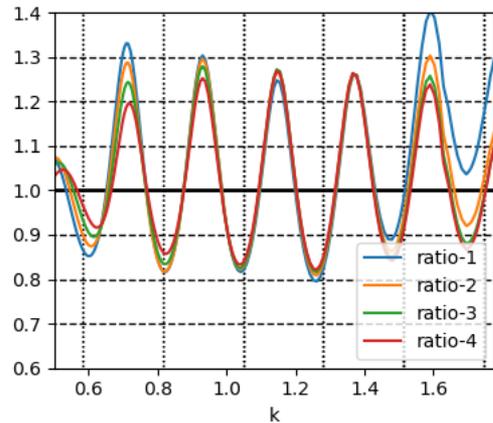
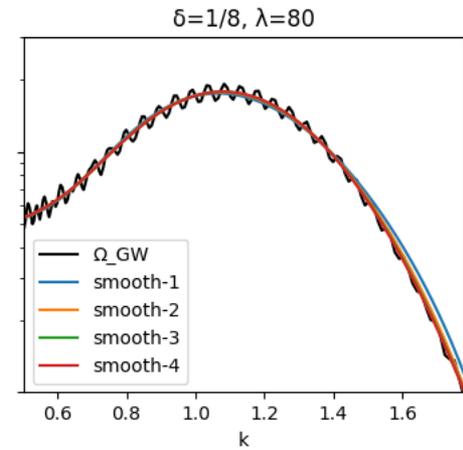
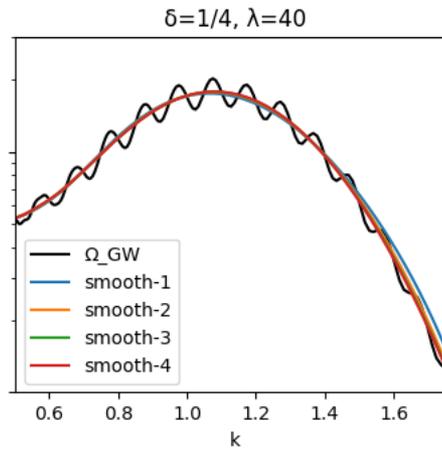
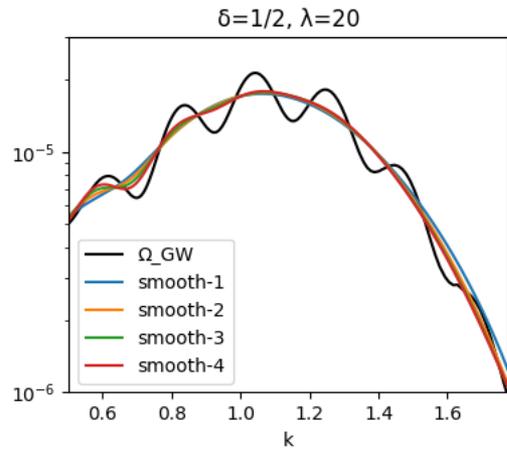
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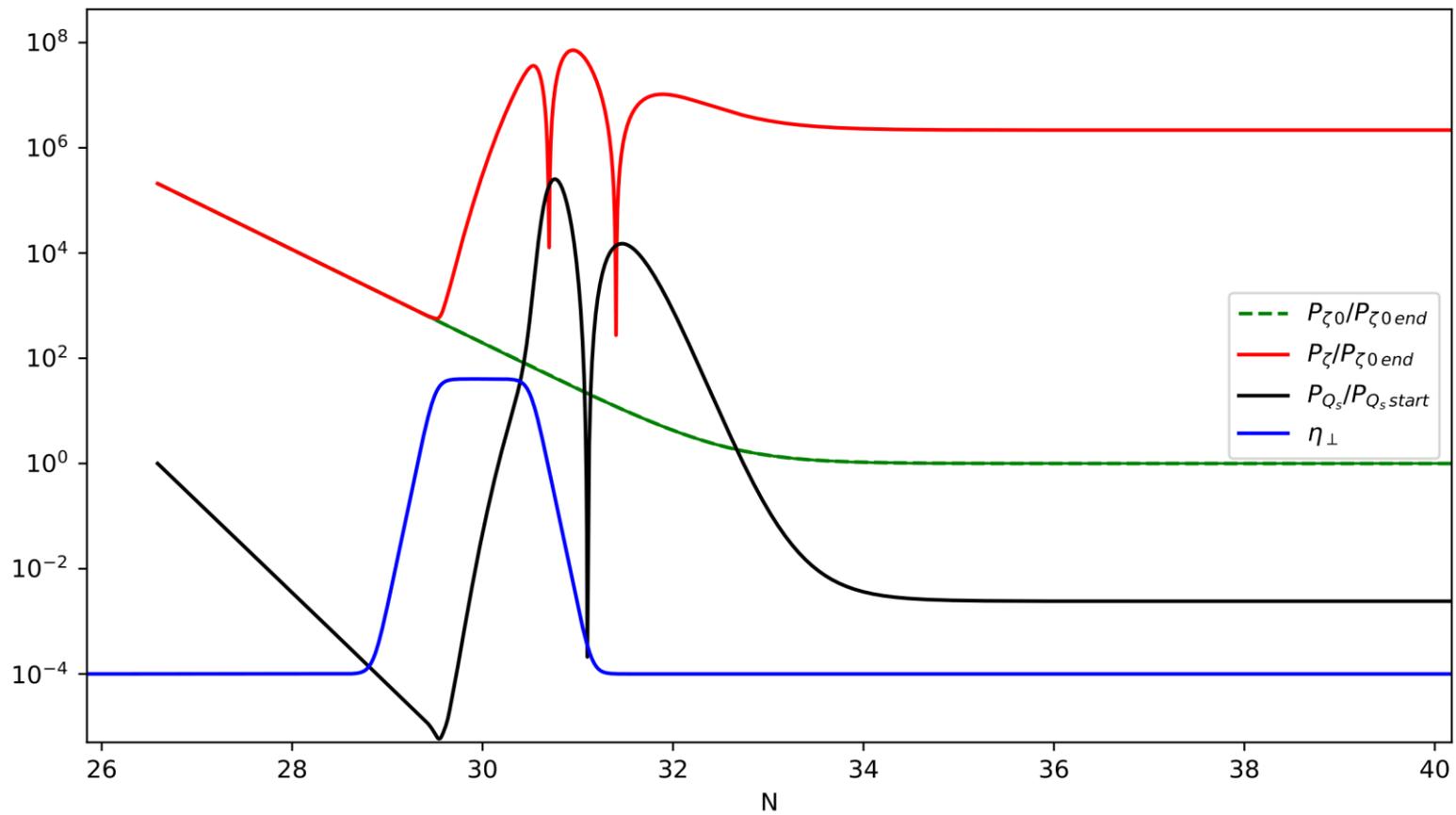
GWs → NEW WINDOW TO MULTI-FIELD EFFECTS DURING INFLATION

# STOCHASTIC BACKGROUND OF GWs

$\delta \cdot \lambda = 10$



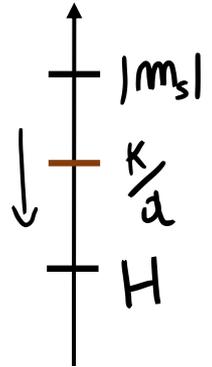
32.59



# SINGLE FIELD EFFECTIVE FIELD THEORY

- INTEGRATING OUT THE ENTROPIC MODE  $|m_s^2| \gg \frac{k^2}{a^2}$

$$Q_s^{\text{EFT}} = -\frac{(2\dot{\sigma})\eta_{\perp}}{m_s^2} \dot{\zeta} \longrightarrow \mathcal{S}_2^{\text{EFT}}[\zeta] = \int d\tau d^3x \epsilon a^2 \left[ \frac{(\dot{\zeta}')^2}{c_s^2} - (\partial_i \zeta)^2 \right]$$



- $\frac{1}{c_s^2} = 1 + \frac{4H^2 \eta_{\perp}^2}{m_s^2}$

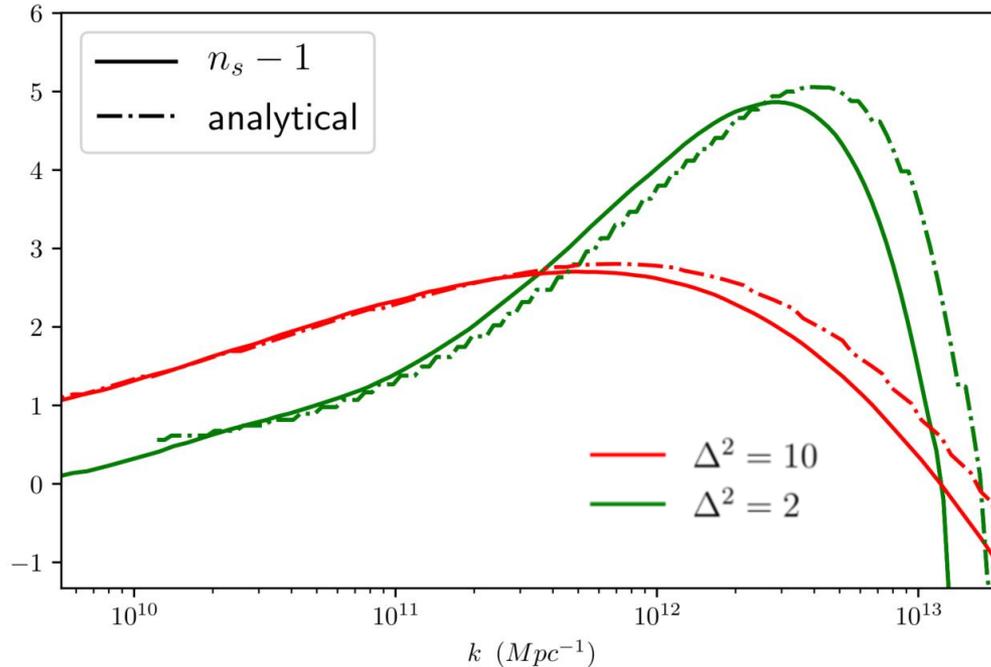
*A. Achúcarro et al. '11*

# GROWTH OF THE POWER SPECTRUM

## N.B. BEYOND SINGLE-FIELD THEOREM ON GROWTH OF THE POWER SPECTRUM

*C. T. Byrnes, P. S. Cole, and S. P. Patil '18*

*P. Carrilho, K. A. Malik, and D. J. Mulryne, '19*



$$\eta_{\perp} = \eta_{\perp}^{\max} e^{-\frac{(N-N_f)^2}{2\Delta^2}}$$

$$\bullet (n_s - 1) \simeq K \frac{(\tilde{N} - N_f)}{(\tilde{N} - N_f) - \Delta^2} \eta_{\perp}$$

# PBH - NON-GAUSSIANITIES

$$\beta_H = 2 \int_{\delta_c}^{\infty} P_H(\delta) d\delta$$

PBHs SENSITIVE TO THE TAIL OF THE DISTRIBUTION

$\Rightarrow$  NON-GAUSSIANITIES ARE RELEVANT (NG)

LOCAL N-G, C.T. Byrnes '12 S. Young and C.T. Byrnes '13 '15, Y. Tada and S. Yokoyama '15...

N-G SMALL SCALES, G. Franciolini et al. '18, V. Atal, C. Germani '18,

FULL PDF THROUGH NON-PERTURBATIVE METHODES C. Pattison et al. '17, J.M. Ezquiaga. '19....

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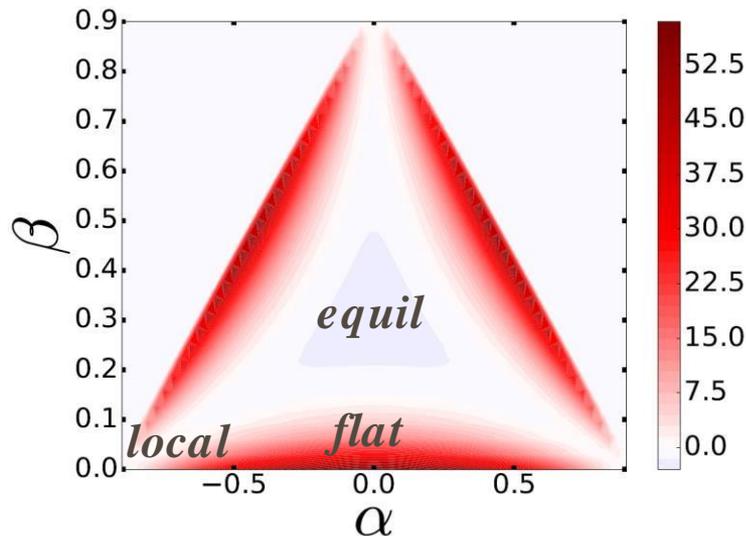
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J.F., S. Garcia Saenz, L. Pitol, , S. Renaux-Petel, J. Rouayne PRL '19

STRONGLY NON-GEODESIC MOTION  $\Rightarrow$  HYPER-NON-GAUSSIANITIES



See also: T. Bjorkmo, D. Marsh, R. Ferreira '19, R. Ferreria '20

$\times$  HIERARCHICAL ENHANCEMENT OF THE N-POINT CORRELATION FUNCTIONS

# TURNING IN THE LANDSCAPE

