

# $\alpha_s$ at FCC-ee

**1<sup>st</sup> FCC-France meeting**  
Paris, 14<sup>th</sup> May 2020

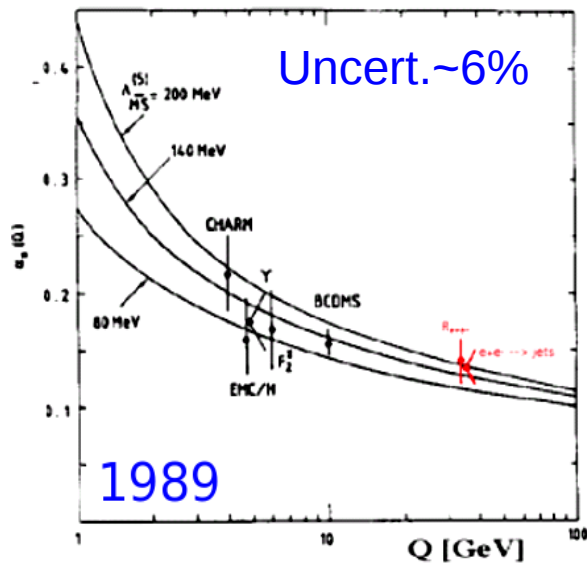
**David d'Enterria**

**CERN**

Latest materials from: *D. d'Enterria, V. Jacobsen "Improved strong coupling determinations from hadronic decays of electroweak bosons at N<sup>3</sup>LO accuracy", <https://arxiv.org/abs/2005.04545> [hep-ph]*

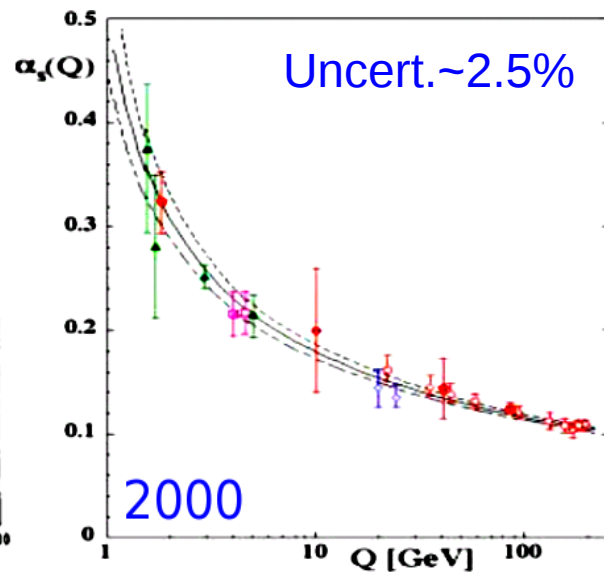
# QCD coupling $\alpha_s$

- Determines **strength of the strong interaction** between quarks & gluons.
- **Single free parameter of QCD** in the  $m_q \rightarrow 0$  limit.
- Determined at a ref. scale ( $Q=m_Z$ ), decreases as  $\alpha_s \sim \ln(Q^2/\Lambda^2)^{-1}$ ,  $\Lambda \sim 0.2$  GeV



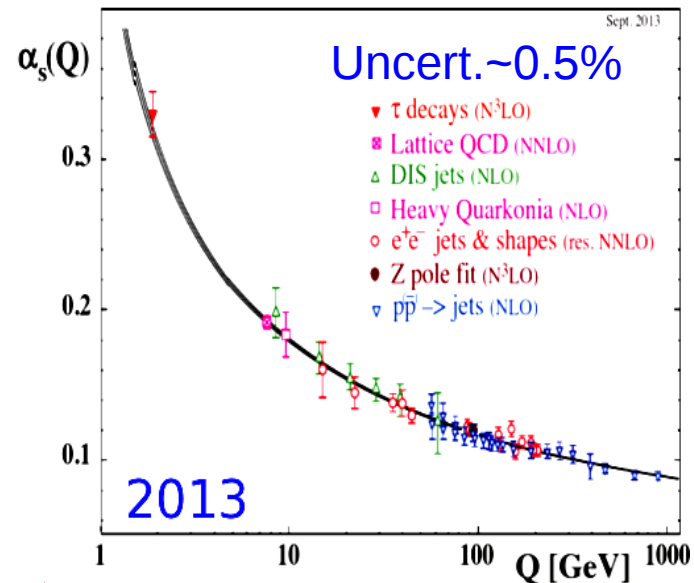
$$\alpha_s(M_Z) = 0.110^{+0.006}_{-0.008} \text{ (NLO)}$$

G. Altarelli, Ann. Rev. Nucl. Part. Sci. 39, 1989



$$\alpha_s(M_Z) = 0.1184 \pm 0.0031 \text{ (NNLO)}$$

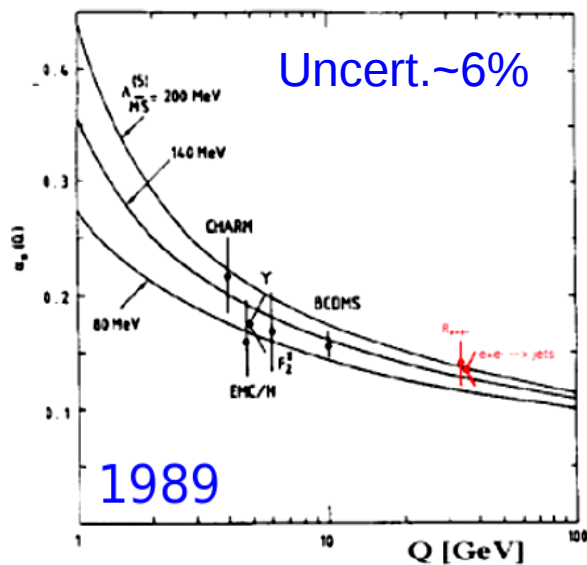
S. B. , J. Phys. G 26, 2000



$$\alpha_s(M_Z) = 0.1185 \pm 0.0006 \text{ (NNLO)}$$

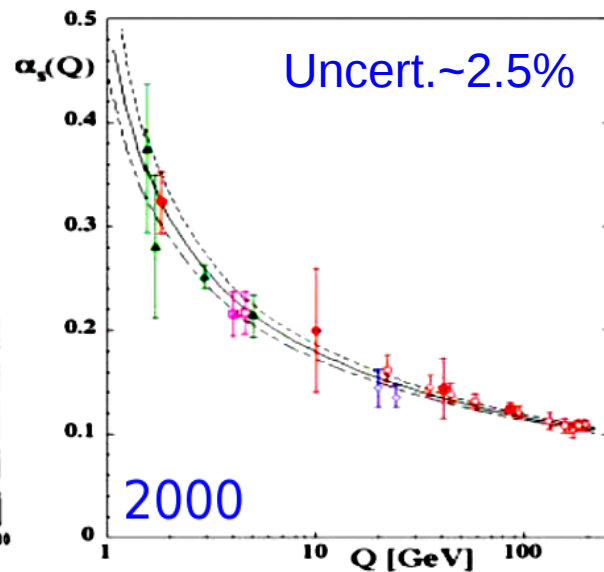
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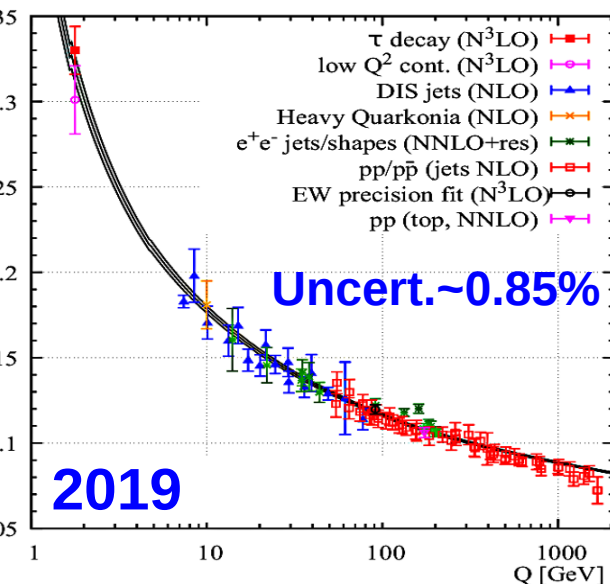
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$$\equiv \alpha_s(M_Z^2) = 0.1179 \pm 0.0010$$

- **Least precisely known** of all interaction **couplings** !

$$\delta\alpha \sim 10^{-10} \ll \delta G_F \ll 10^{-7} \ll \delta G \sim 10^{-5} \ll \delta\alpha_s \sim 10^{-3}$$

# Importance of the QCD coupling $\alpha_s$

Impacts all QCD x-sections & decays (H), precision top & parametric EWPO:

Process	$\sigma$ (pb)	$\delta\alpha_s$ (%)	PDF + $\alpha_s$ (%)	Scale (%)
ggH	49.87	$\pm 3.7$	-6.2 +7.4	-2.61 + 0.32
ttH	0.611	$\pm 3.0$	$\pm 8.9$	-9.3 + 5.9

Channel	$M_H$ [GeV]	$\delta\alpha_s$ (%)	$\Delta m_b$	$\Delta m_c$
H $\rightarrow c\bar{c}$	126	$\pm 7.1$	$\pm 0.1\%$	$\pm 2.3\%$
H $\rightarrow gg$	126	$\pm 4.1$	$\pm 0.1\%$	$\pm 0\%$

Msbar mass error budget (from threshold scan)

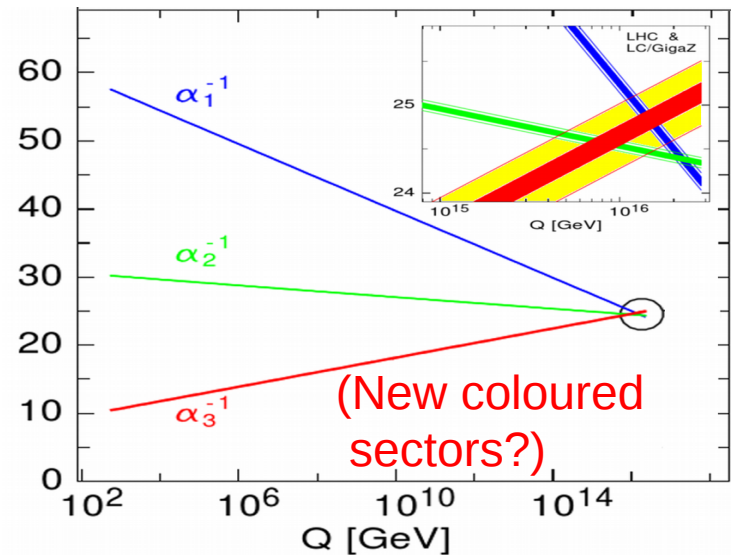
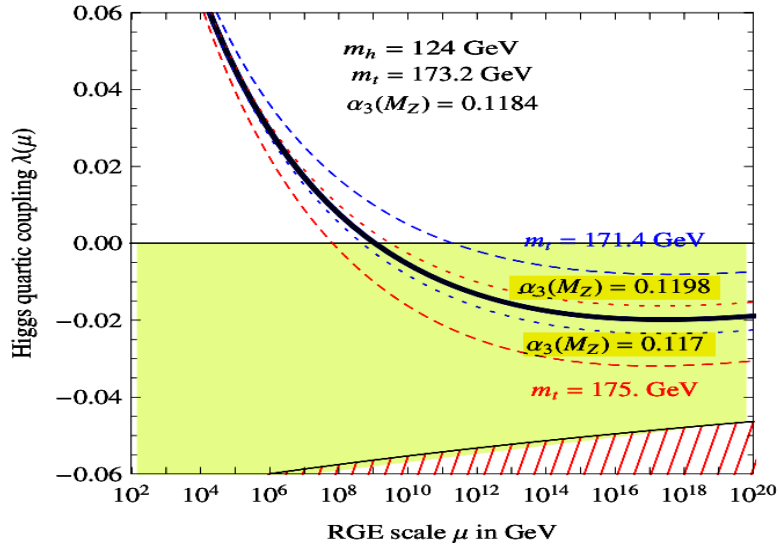
$(\delta M_t^{SD-low})^{exp}$	$(\delta M_t^{SD-low})^{theo}$	$(\delta \overline{m}_t(\overline{m}_t))^{conversion}$	$(\delta \overline{m}_t(\overline{m}_t))^{\alpha_s}$
40 MeV	50 MeV	7 – 23 MeV	70 MeV

$\Rightarrow$  improvement in  $\alpha_s$  crucial  $\delta\alpha_s(M_Z) = 0.001$

Quantity	FCC-ee	future param.unc.	Main source
$\Gamma_Z$ [MeV]	0.1	0.1	$\delta\alpha_s$
$R_b$ [ $10^{-5}$ ]	6	< 1	$\delta\alpha_s$
$R_\ell$ [ $10^{-3}$ ]	1	1.3	$\delta\alpha_s$

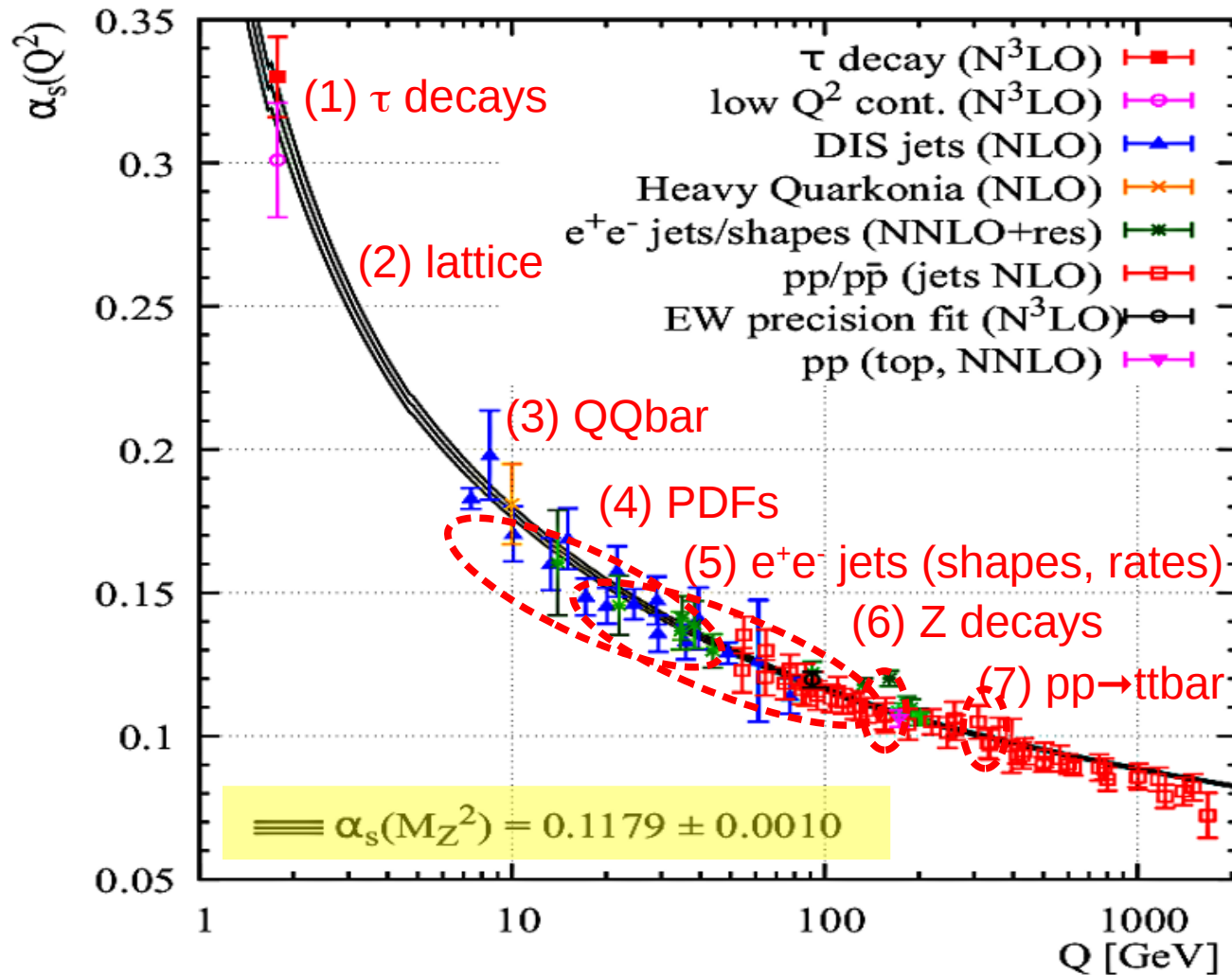
Sven Heinemeyer – 1st FCC physics workshop, CERN, 17.01.2017

Impacts physics approaching Planck scale: EW vacuum stability, GUT



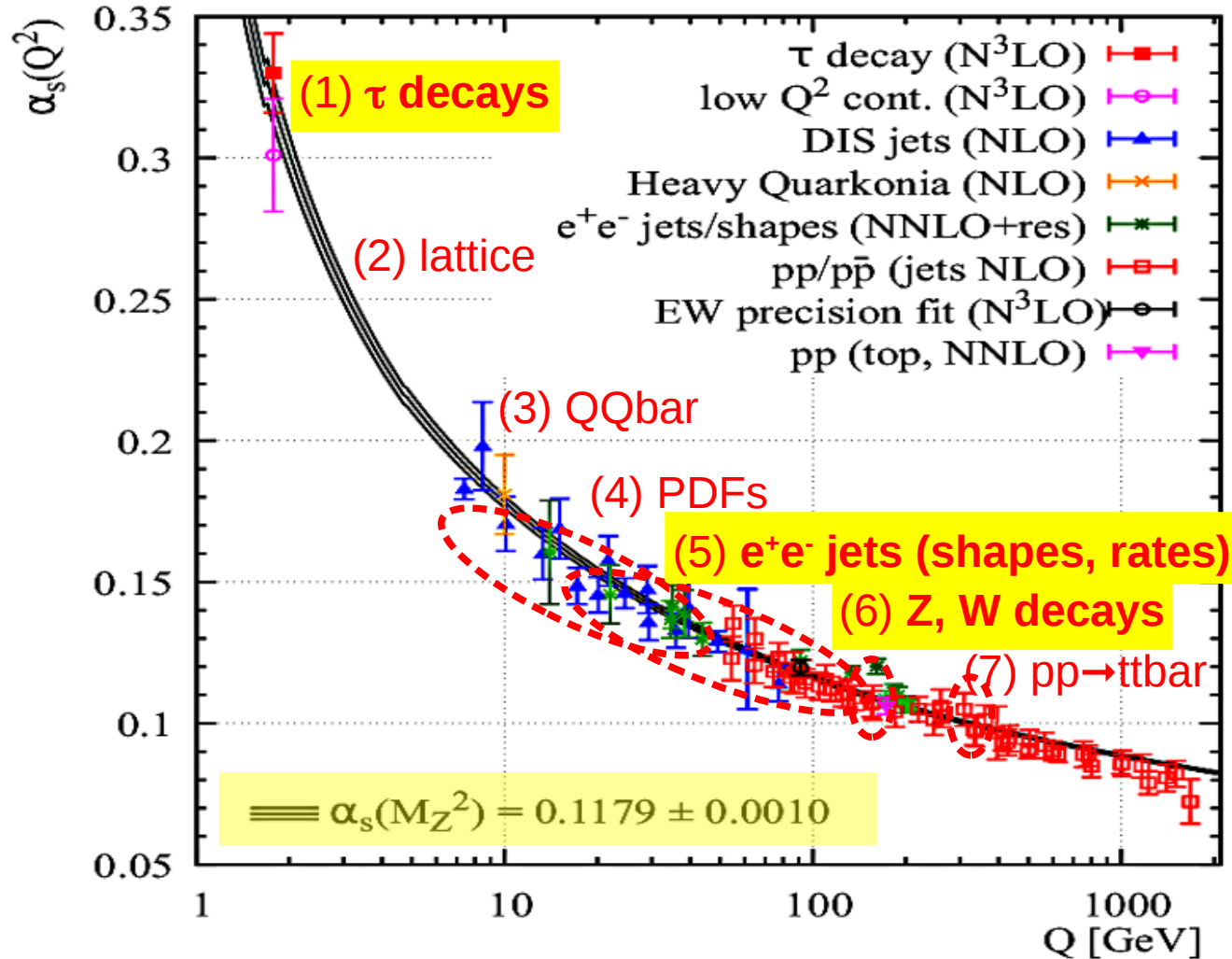
# World $\alpha_s$ determination (PDG 2019)

- Determined today by comparing 7 experimental observables to pQCD NNLO, N<sup>3</sup>LO predictions, plus global average at the Z pole scale:



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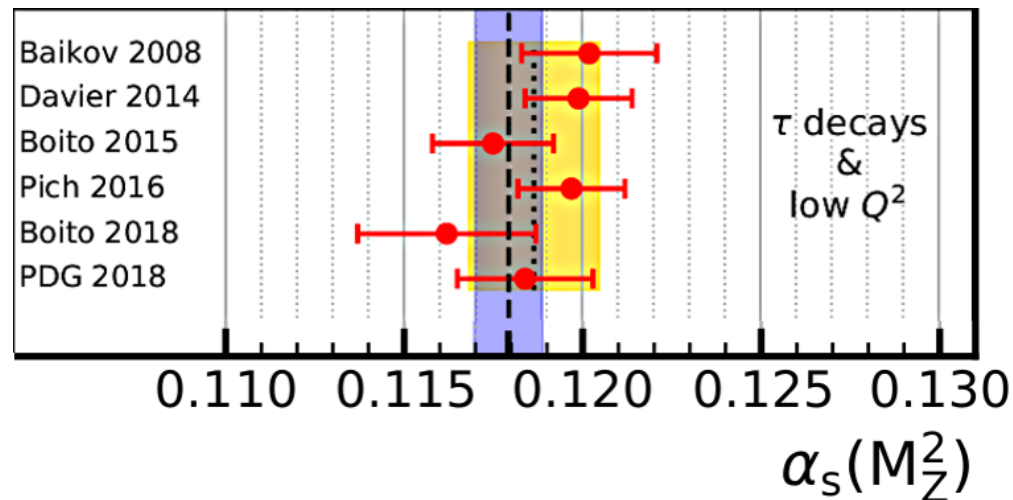
FCC-ee

# $\alpha_s$ from hadronic $\tau$ -lepton decays

➔ Computed at **N<sup>3</sup>LO**:  $R_\tau \equiv \frac{\Gamma(\tau^- \rightarrow \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)} = S_{\text{EW}} N_C (1 + \sum_{n=1}^4 c_n \left(\frac{\alpha_s}{\pi}\right)^n + \mathcal{O}(\alpha_s^5) + \delta_{\text{np}})$

➔ Experimentally:  $R_{\tau, \text{exp}} = 3.4697 \pm 0.0080$  ( $\pm 0.23\%$ )

➔ Various pQCD approaches (FOPT vs CIPT) & treatment of non-pQCD corrections  $(\Lambda/m_\tau)^2 \sim 2\%$ , yield different results.



Uncertainty slightly increased:  
2013 ( $\pm 1.3\%$ )  $\rightarrow$  2019 ( $\pm 1.5\%$ )

$$\alpha_s(M_Z^2) = 0.1187 \pm 0.0018 \quad (\pm 1.5\%)$$

➔ Future :

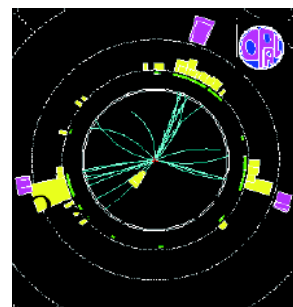
- TH: Better understanding of FOPT vs CIPT differences.
- Better spectral functions needed (high stats & better precision): B-factories (BELLE-II)?
- High-stats:  $\mathcal{O}(10^{11})$  from  $Z \rightarrow \tau\tau$  at FCC-ee(90) :  $\delta\alpha_s/\alpha_s \ll 1\%$

# $\alpha_s$ from $e^+e^-$ event shapes & jet rates (today)

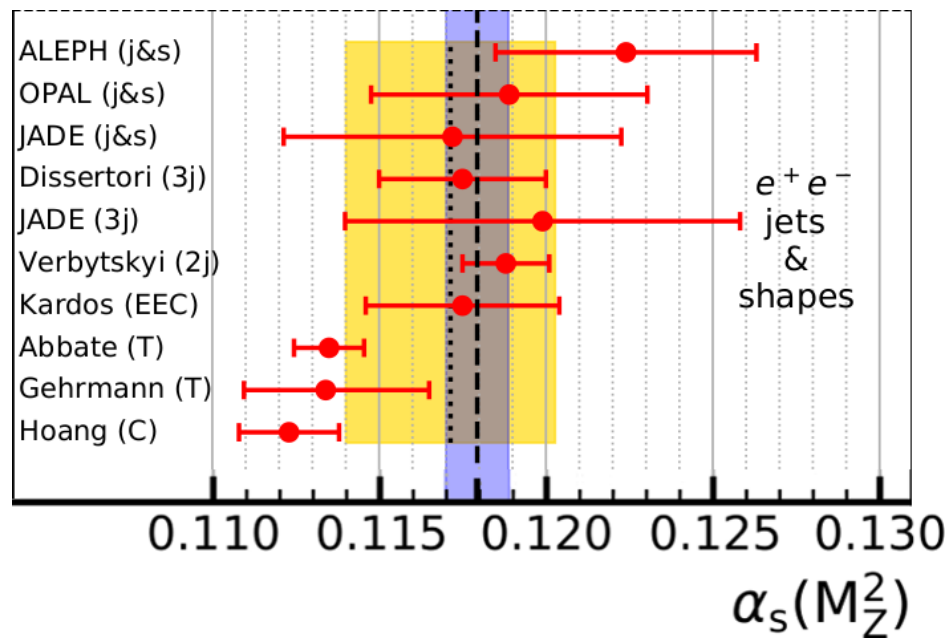
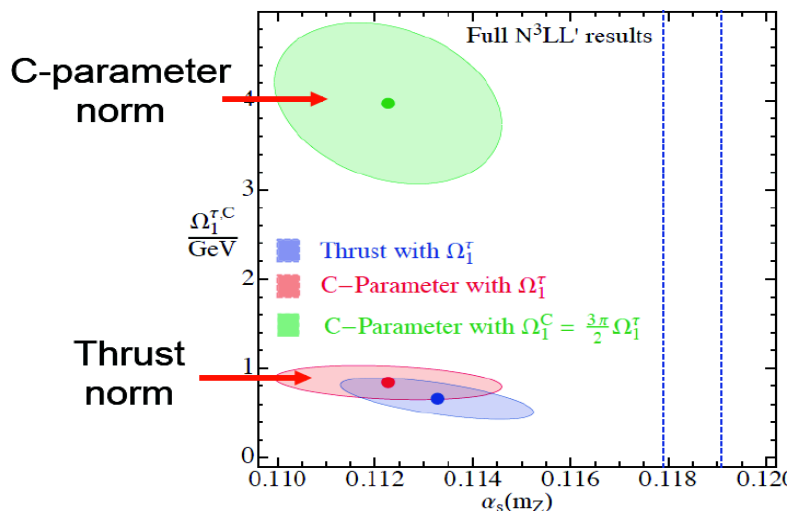
- Computed at  $N^{2,3}LO+N^{(2)}LL$  accuracy.
- Experimentally (LEP):
  - Thrust, C-parameter, jet shapes
  - n-jet x-sections
- Results sensitive to non-pQCD (hadronization) accounted for via MCs or analytically:

$$\tau = 1 - \max_{\hat{n}} \frac{\sum |\vec{p}_i \cdot \hat{n}|}{\sum |\vec{p}_i|}$$

$$C = \frac{3}{2} \frac{\sum_{i,j} |\vec{p}_i| |\vec{p}_j| \sin^2 \theta_{ij}}{(\sum_i |\vec{p}_i|)^2}$$



OPAL 3 jet event



Wide span of TH extractions...

$\alpha_s(M_Z^2) = 0.1171 \pm 0.0031$  ( $\pm 2.6\%$ )

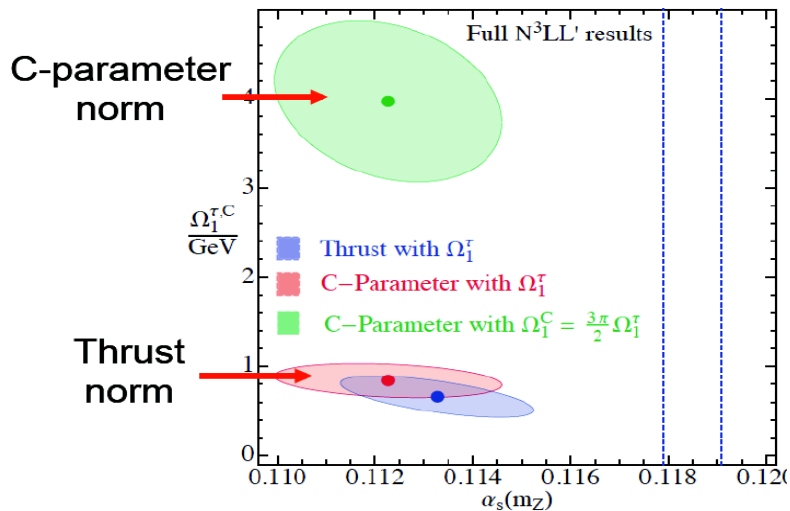
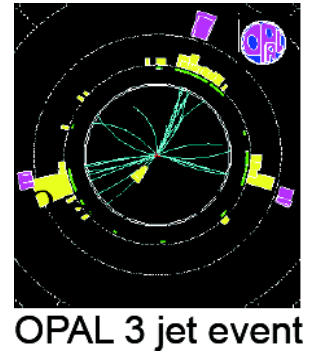


# $\alpha_s$ from $e^+e^-$ event shapes & jet rates (FCC-ee)

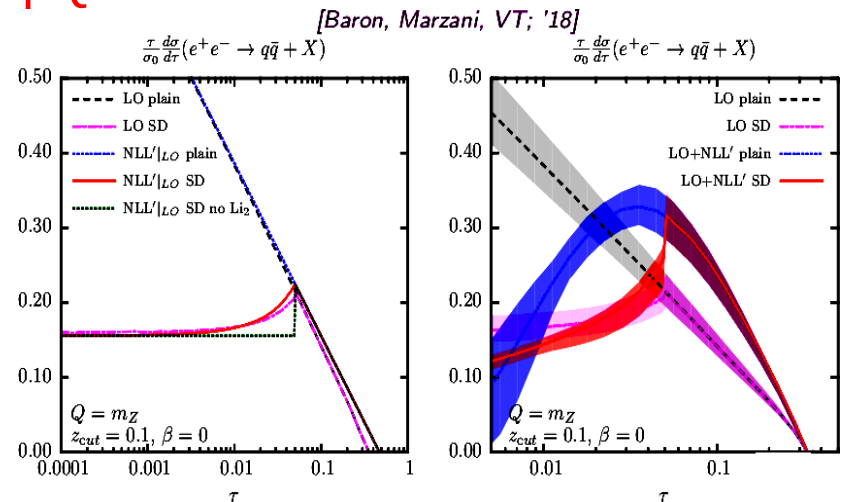
- Computed at  $N^{2,3}LO+N^{(2)}LL$  accuracy.
- Experimentally (LEP):
  - Thrust, C-parameter, jet shapes
  - 3-jet x-sections
- Results sensitive to non-pQCD (hadronization) accounted for via MCs or analytically:

$$\tau = 1 - \max_{\hat{n}} \frac{\sum |\vec{p}_i \cdot \hat{n}|}{\sum |\vec{p}_i|}$$

$$C = \frac{3}{2} \frac{\sum_{i,j} |\vec{p}_i| |\vec{p}_j| \sin^2 \theta_{ij}}{(\sum_i |\vec{p}_i|)^2}$$



- Modern jet substructure techniques:
  - “Soft drop” can help reduce non-pQCD corrections for thrust:



- Future:  $\delta\alpha_s/\alpha_s < 1\%$
- FCC- $e^+e^-$ : Lower- $\sqrt{s}$  (ISR) for shapes, higher- $\sqrt{s}$  for jet rates
- TH: Improved ( $N^{2,3}LL$ ) resummation for rates, hadronization for shapes

# $\alpha_s$ from hadronic Z, W decays

DdE, Jacobsen:  
arXiv:2005.04545

→ Z & W pseudo-observ. theoretically known at N<sup>3</sup>LO accuracy:

- The W and Z hadronic widths :

$$\Gamma_{W,Z}^{\text{had}}(Q) = \Gamma_{W,Z}^{\text{Born}} \left( 1 + \sum_{i=1}^4 a_i(Q) \left( \frac{\alpha_S(Q)}{\pi} \right)^i + \mathcal{O}(\alpha_S^5) + \delta_{\text{EW}} + \delta_{\text{mix}} + \delta_{\text{np}} \right)$$

TH uncertainties:  
( $\alpha^2, \alpha^3$  included for Z):  
±0.015–0.03% (Z)  
±0.015–0.04% (W)

- The ratio of W, Z hadronic-to-leptonic widths :

$$R_{W,Z}(Q) = \frac{\Gamma_{W,Z}^{\text{had}}(Q)}{\Gamma_{W,Z}^{\text{lep}}(Q)} = R_{W,Z}^{\text{EW}} \left( 1 + \sum_{i=1}^4 a_i(Q) \left( \frac{\alpha_S(Q)}{\pi} \right)^i + \mathcal{O}(\alpha_S^5) + \delta_{\text{mix}} + \delta_{\text{np}} \right)$$

Param. uncerts.:  
( $m_{Z,W}, \alpha, V_{\text{cs,ud}}$ ):

- In the Z boson case, the hadronic cross section at the resonance peak in  $e^+e^-$ :

$$\sigma_Z^{\text{had}} = \frac{12\pi}{m_Z} \cdot \frac{\Gamma_Z^e \Gamma_Z^{\text{had}}}{(\Gamma_Z^{\text{tot}})^2}$$

±0.01–0.03% (Z)  
±1.1–1.7% (W)  
±0.03% (W, CKM unit)

→ Measured at LEP with ±0.1–0.3% (Z), ±0.9–2% (W) exp. uncertainties:

	theory			experiment		
	previous	new (this work)	change	previous [6]	new [20, 21]	change
$\Gamma_Z^{\text{tot}}$ (MeV)	2494.2 ± 0.8 <sub>th</sub>	2495.2 ± 0.6 <sub>par</sub> ± 0.4 <sub>th</sub>	+0.04%	2495.2 ± 2.3	2495.5 ± 2.3	+0.012%
$R_Z$	20.733 ± 0.007 <sub>th</sub>	20.750 ± 0.006 <sub>par</sub> ± 0.006 <sub>th</sub>	+0.08%	20.767 ± 0.025	20.7666 ± 0.0247	-0.040%
$\sigma_Z^{\text{had}}$ (pb)	41 490 ± 6 <sub>th</sub>	41 494 ± 5 <sub>par</sub> ± 6 <sub>th</sub>	+0.01%	41 540 ± 37	41 480.2 ± 32.5	-0.144%

Recent update of LEP luminosity bias(\*) change the Z values by few permil

W boson observables	GFITTER 2.2 (NNLO)	this work (N <sup>3</sup> LO)		experiment
		(exp. CKM)	(CKM unit.)	
$\Gamma_W^{\text{had}}$ (MeV)	–	1440.3 ± 23.9 <sub>par</sub> ± 0.2 <sub>th</sub>	1410.2 ± 0.8 <sub>par</sub> ± 0.2 <sub>th</sub>	1405 ± 29
$\Gamma_W^{\text{tot}}$ (MeV)	2091.8 ± 1.0 <sub>par</sub>	2117.9 ± 23.9 <sub>par</sub> ± 0.7 <sub>th</sub>	2087.9 ± 1.0 <sub>par</sub> ± 0.7 <sub>th</sub>	2085 ± 42
$R_W$	–	2.1256 ± 0.0353 <sub>par</sub> ± 0.0008 <sub>th</sub>	2.0812 ± 0.0007 <sub>par</sub> ± 0.0008 <sub>th</sub>	2.069 ± 0.019

(\*) Voutsinas et al.  
arXiv:1908.01704,  
Janot et al.  
arXiv:1912.02067

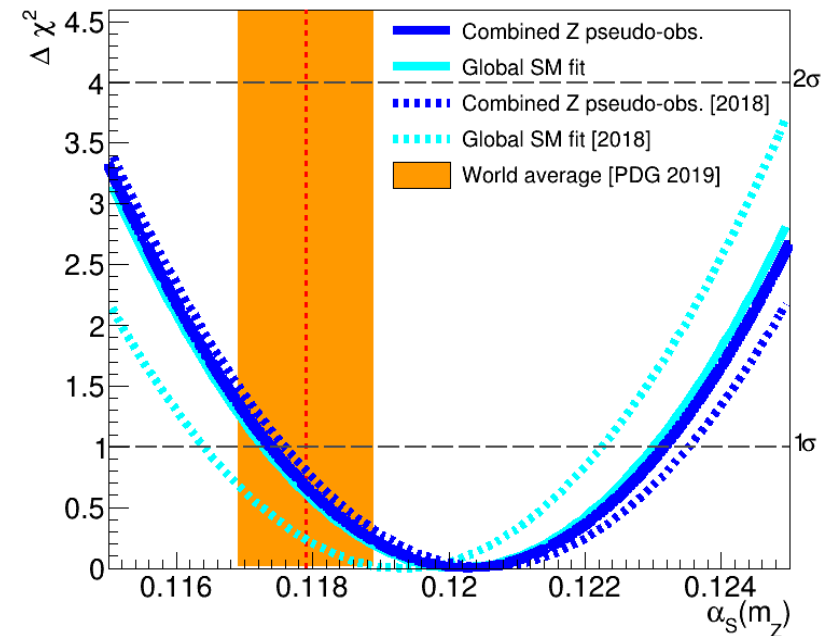
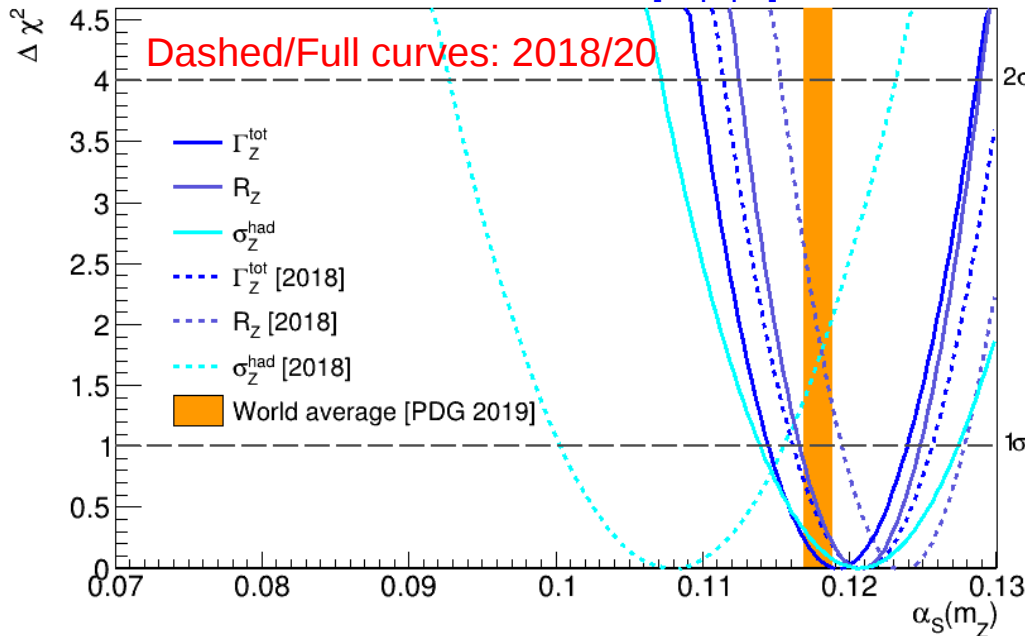
# $\alpha_s$ from hadronic Z decays (today)

➔ QCD coupling extracted from:

- (i) combined fit of 3 Z pseudo-observ:
- (ii) full SM fit (with  $\alpha_s$  free parameter)

Z boson observable	$\alpha_s(m_Z)$		uncertainties	
	extraction	exp.	param.	theor.
$\Gamma_Z^{\text{tot}}$	$0.1192 \pm 0.0047$	$\pm 0.0046$	$\pm 0.0005$	$\pm 0.0008$
$R_Z$	$0.1207 \pm 0.0041$	$\pm 0.0041$	$\pm 0.0001$	$\pm 0.0009$
$\sigma_Z^{\text{had}}$	$0.1206 \pm 0.0068$	$\pm 0.0067$	$\pm 0.0004$	$\pm 0.0012$
All combined	$0.1203 \pm 0.0029$	$\pm 0.0029$	$\pm 0.0002$	$\pm 0.0008$
Global SM fit	$0.1202 \pm 0.0028$	$\pm 0.0028$	$\pm 0.0002$	$\pm 0.0008$

DdE, Jacobsen: arXiv:2005.04545 [hep-ph]



➔ LEP lumi-bias updates lead to much **better agreement** among  $\Gamma_Z$ ,  $R_Z$ ,  $\sigma_0$  extractions:

➔ Improved  $\alpha_s(m_Z) = 0.1203 \pm 0.0028$  ( $\pm 2.3\%$ )

PDG'19:  $\alpha_s(m_Z) = 0.1205 \pm 0.0030$  ( $\pm 2.5\%$ )

➔ EXP/TH updates lead to **better agreement** with full SM fit:

➔  $\alpha_s(m_Z) = 0.1202 \pm 0.0028$

PDG'19:  $\alpha_s(m_Z) = 0.1194 \pm 0.0029$

# $\alpha_s$ from hadronic Z decays (FCC-ee)

→ QCD coupling extracted from:

- (i) combined fit of 3 Z pseudo-observ:
- (ii) full SM fit (with  $\alpha_s$  free parameter)

Z boson observable	$\alpha_s(m_Z)$ extraction	uncertainties		
		exp.	param.	theor.
All combined	$0.1203 \pm 0.0029$	$\pm 0.0029$	$\pm 0.0002$	$\pm 0.0008$
Global SM fit	$0.1202 \pm 0.0028$	$\pm 0.0028$	$\pm 0.0002$	$\pm 0.0008$
All combined (FCC-ee)	$0.12030 \pm 0.00026$	$\pm 0.00013$	$\pm 0.00005$	$\pm 0.00022$
Global SM fit (FCC-ee)	$0.12020 \pm 0.00026$	$\pm 0.00013$	$\pm 0.00005$	$\pm 0.00022$

→ FCC-ee:

- Huge Z pole stats. ( $\times 10^5$  LEP)
- Exquisite systematic/parametric precision (stat. uncert. much smaller):

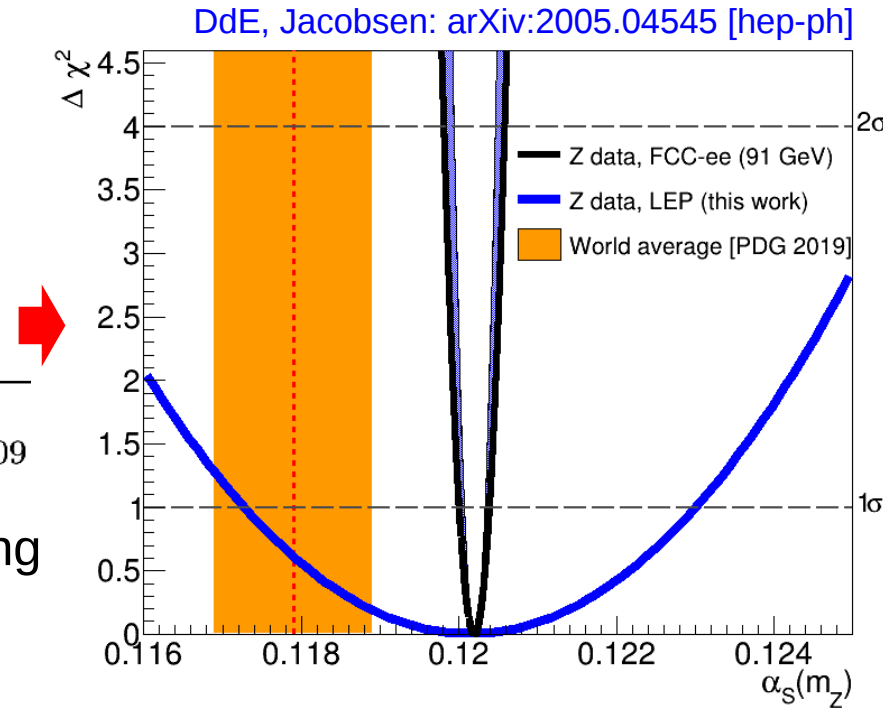
$$\begin{array}{l} \Delta R_Z = 10^{-3}, \quad R_Z = 20.7500 \pm 0.0010 \\ \Delta \Gamma_Z^{\text{tot}} = 0.1 \text{ MeV}, \quad \Gamma_Z^{\text{tot}} = 2495.2 \pm 0.1 \text{ MeV} \\ \Delta \sigma_Z^{\text{had}} = 4.0 \text{ pb}, \quad \sigma_Z^{\text{had}} = 41\,494 \pm 4 \text{ pb} \\ \hline \Delta m_Z = 0.1 \text{ MeV}, \quad m_Z = 91.18760 \pm 0.00001 \text{ GeV} \\ \Delta \alpha = 3 \cdot 10^{-5}, \quad \Delta \alpha_{\text{had}}^{(5)}(m_Z) = 0.0275300 \pm 0.0000009 \end{array}$$

- TH uncertainty reduced by  $\times 4$  computing missing  $\alpha_s^5, \alpha^3, \alpha\alpha_s^2, \alpha\alpha_s^2, \alpha^2\alpha_s$  terms

→ 10 times better precision than today:

$$\delta\alpha_s/\alpha_s \sim \pm 0.2\% \text{ (tot)}, \pm 0.1\% \text{ (exp)}$$

Strong (B)SM consistency test.



$$\alpha_s(m_Z) = 0.12030 \pm 0.00028 \text{ } (\pm 0.2\%)$$

# $\alpha_s$ from hadronic W decays (today)

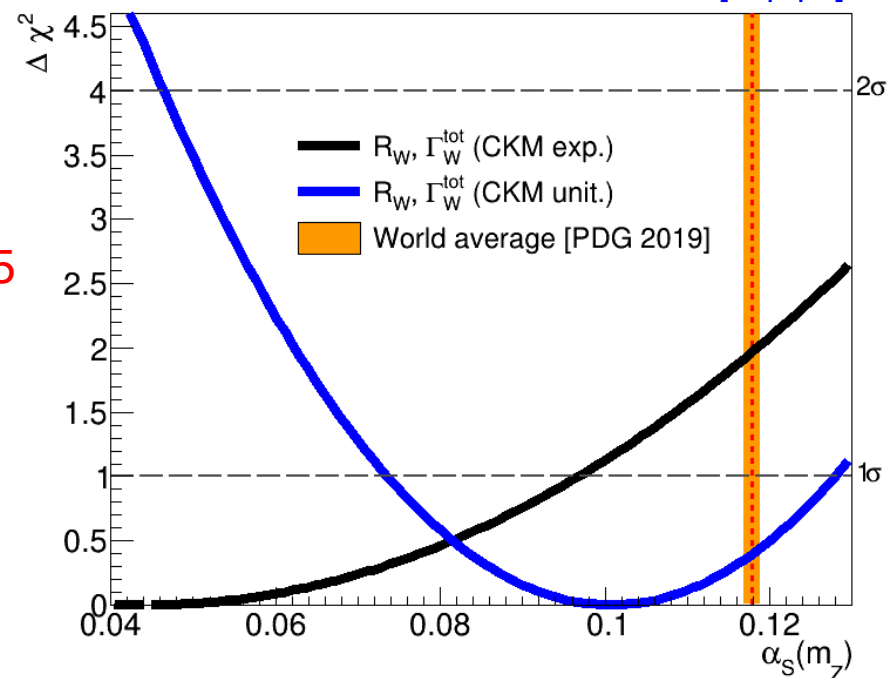
♦ QCD coupling extracted from **new N<sup>3</sup>LO fit of combined  $\Gamma_W$ ,  $R_W$  pseudo-observ.**:

W boson observables	$\alpha_s(m_Z)$	uncertainties		
	extraction	exp.	param.	theor.
$\Gamma_W^{\text{tot}}, R_W$ (exp. CKM)	<b><math>0.044 \pm 0.052</math></b>	$\pm 0.024$	$\pm 0.047$	$(\pm 0.0014)$
$\Gamma_W^{\text{tot}}, R_W$ (CKM unit.)	<b><math>0.101 \pm 0.027</math></b>	$\pm 0.027$	$(\pm 0.0002)$	$(\pm 0.0016)$
$\Gamma_W^{\text{tot}}, R_W$ (FCC-ee, CKM unit.)	$0.11790 \pm 0.00023$	$\pm 0.00012$	$\pm 0.00004$	$\pm 0.00019$

♦ **Very imprecise extraction:**

- Large propagated parametric uncert. from **poor  $V_{cs}$  exp. precision ( $\pm 2\%$ )**:  
QCD coupling unconstrained:  **$0.04 \pm 0.05$**
- Imposing CKM unitarity: **large exp. uncertainties** from  $\Gamma_W, R_W$  (0.9–2%):  
QCD extracted with  **$\sim 27\%$  precision**
- **Propagated TH uncertainty** much smaller today:  **$\sim 1.5\%$**

DdE, Jacobsen: arXiv:2005.04545 [hep-ph]



$$\alpha_s(m_Z) = 0.101 \pm 0.027 \quad (\pm 27\%)$$

# $\alpha_s$ from hadronic W decays (FCC-ee)

→ QCD coupling extracted from new N<sup>3</sup>LO fit of combined  $\Gamma_W$ ,  $R_W$  pseudo-observ.:

W boson observables	$\alpha_s(m_Z)$	uncertainties		
	extraction	exp.	param.	theor.
$\Gamma_W^{\text{tot}}, R_W$ (exp. CKM)	$0.044 \pm 0.052$	$\pm 0.024$	$\pm 0.047$	$(\pm 0.0014)$
$\Gamma_W^{\text{tot}}, R_W$ (CKM unit.)	$0.101 \pm 0.027$	$\pm 0.027$	$(\pm 0.0002)$	$(\pm 0.0016)$
$\Gamma_W^{\text{tot}}, R_W$ (FCC-ee, CKM unit.)	$0.11790 \pm 0.00023$	$\pm 0.00012$	$\pm 0.00004$	$\pm 0.00019$

→ FCC-ee extraction:

– Huge W pole stats. ( $\times 10^4$  LEP-2).

– Exquisite syst./parametric precision:

$$\Gamma_W^{\text{tot}} = 2088.0 \pm 1.2 \text{ MeV}$$

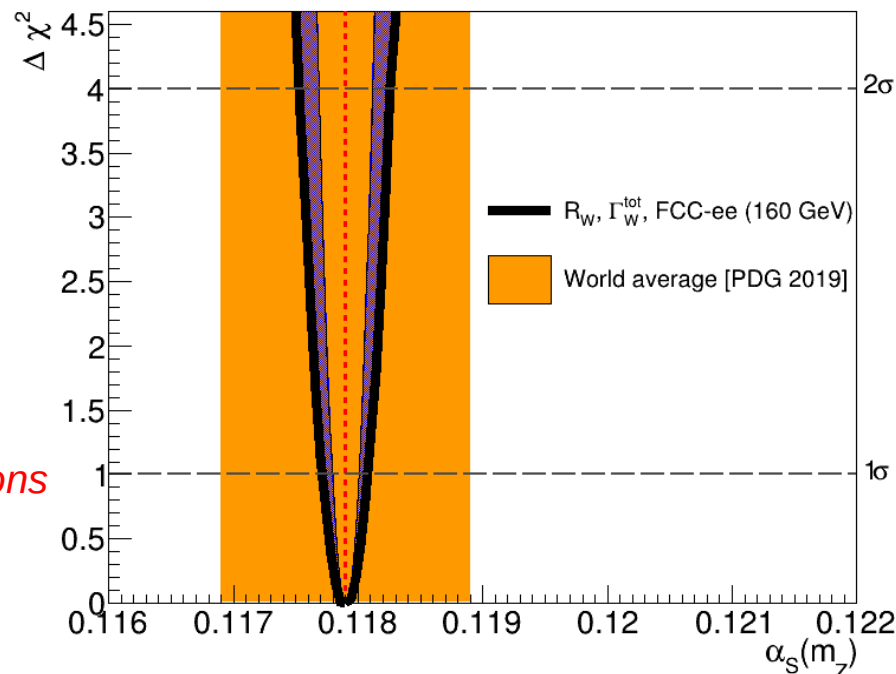
$$R_W = 2.08000 \pm 0.00008$$

$$m_W = 80.3800 \pm 0.0005 \text{ GeV}$$

$$|V_{cs}| = 0.97359 \pm 0.00010 \leftarrow O(10^{12}) \text{ D mesons}$$

– TH uncertainty reduced by  $\times 10$   
after computing missing  $\alpha_s^5, \alpha_s^2, \alpha_s^3,$   
 $\alpha\alpha_s^2, \alpha\alpha_s^2, \alpha^2\alpha_s$  terms

DdE, Jacobsen: arXiv:2005.04545 [hep-ph]



$$\alpha_s(m_Z) = 0.11790 \pm 0.00023 \quad (\pm 0.2\%)$$

# Summary: $\alpha_s$ at FCC-ee

- World-average QCD coupling at N<sup>2,3</sup>LO today:
  - Determined from **7 observables** with combined **0.85% uncertainty** (least well-known gauge coupling).
  - Impacts all **LHC QCD x-sections & decays**.
  - Role **beyond SM**: GUT, EWK vacuum stability, New colored sectors?

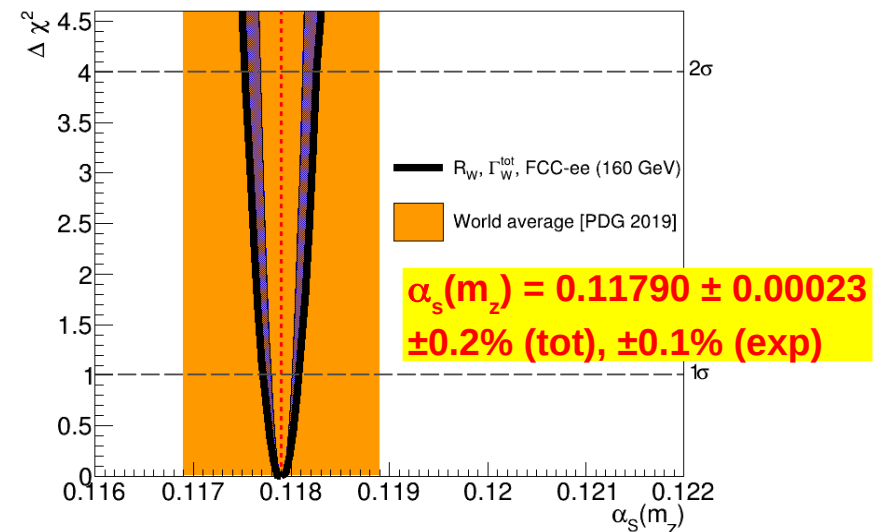
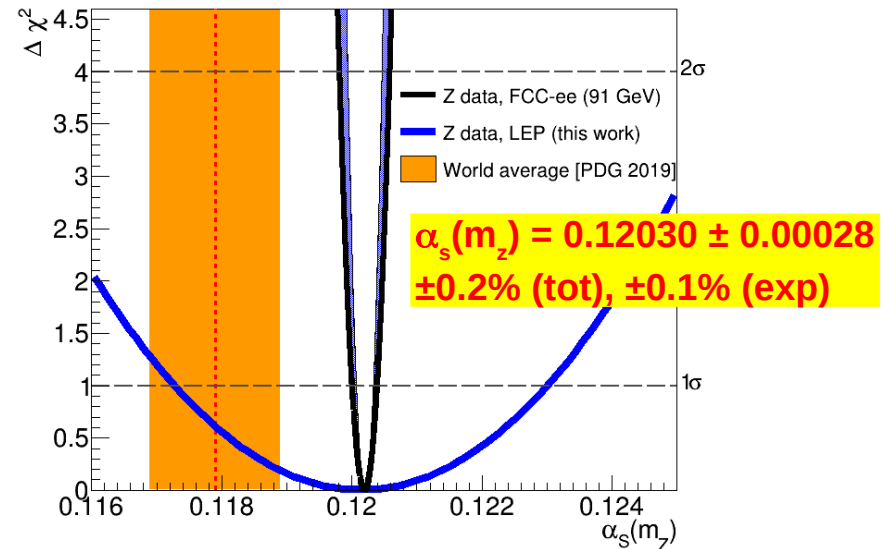
## ■ e<sup>+</sup>e<sup>-</sup> extractions:

- Hadronic tau decays:  $\pm 1\%$  TH
- Event shapes, jet rates:  $\pm 1\%$  TH
- Z&W pseudo-observ.:  $\pm 0.1\%$  TH

## ■ State-of-the-art extractions:

- Z boson: New fit with high-order EW corrections + updated LEP data:  **$\sim 2.3\%$  (exp.) uncertainty today.**
- W boson: New **N<sup>3</sup>LO** fit to  $\Gamma_W$ ,  $R_W$   **$\sim 27\%$  (exp.) uncertainty today.**

- **Permil uncertainty** only possible with a machine like **FCC-e<sup>+</sup>e<sup>-</sup>**

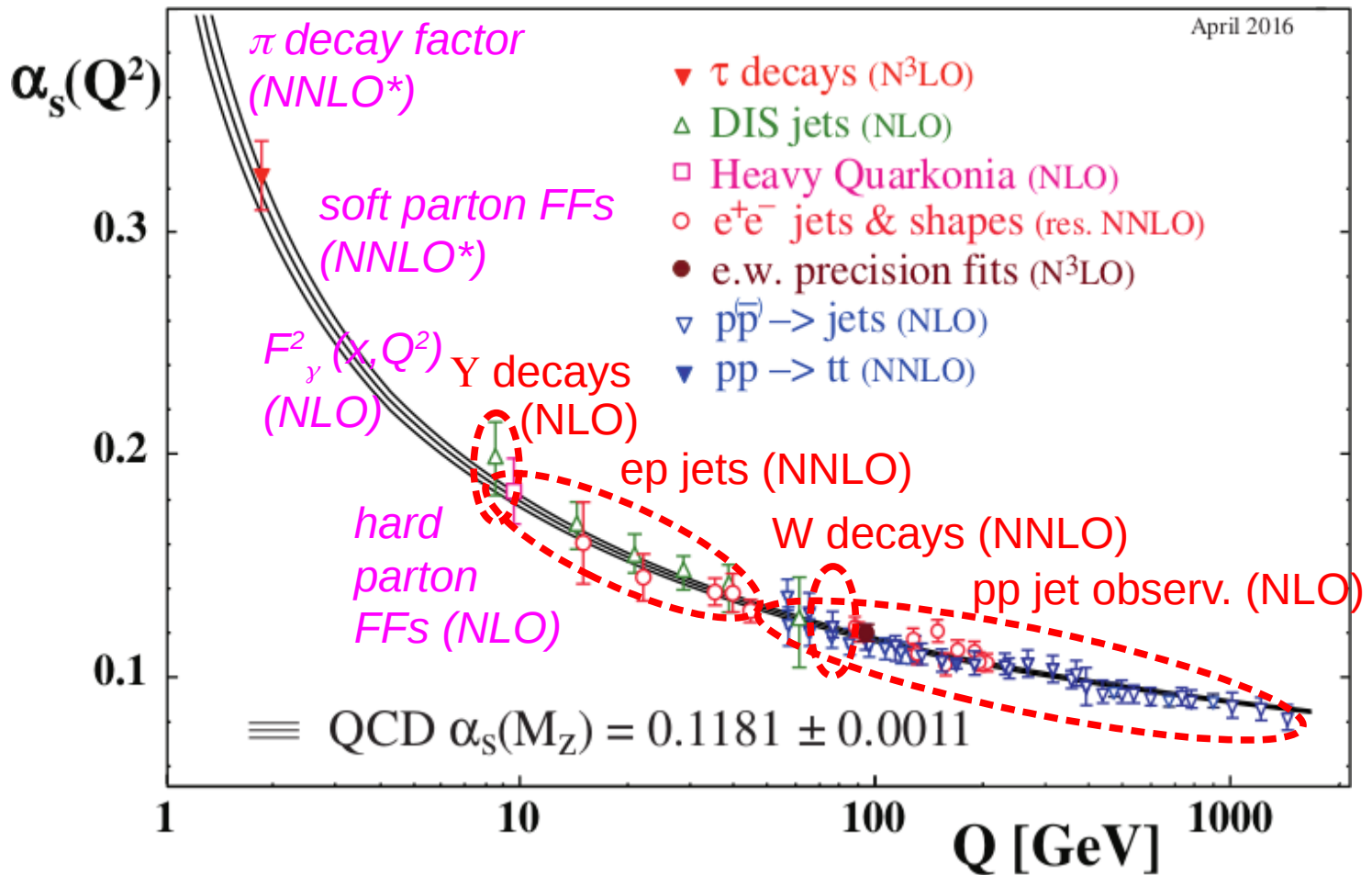


# Backup slides



# Other $\alpha_s$ extractions (not yet in world average)

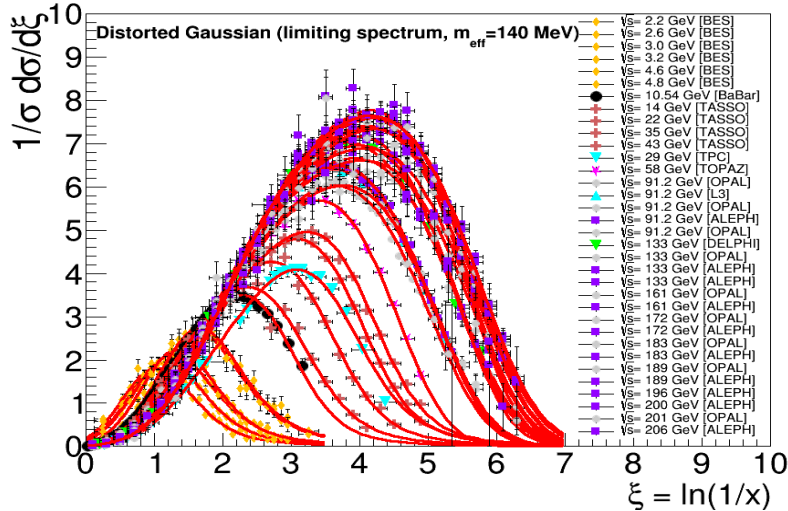
- There exist **at least 8 other classes of observables**, computed at lower accuracy (**NLO, NNLO\***), used to extract the QCD coupling:



# Other $\alpha_s$ extractions from jets (NLO, NNLO\*)

➔ Soft parton-to-hadron FFs (NNLO\*+NNLL):

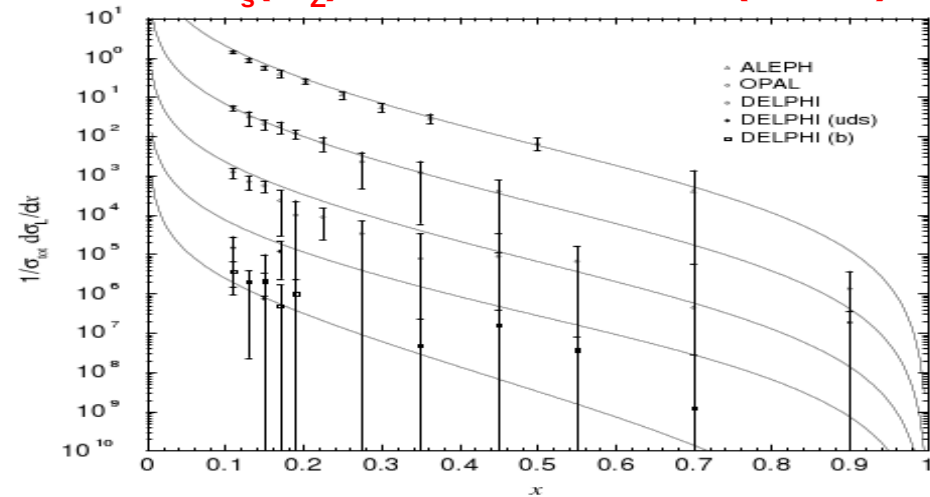
$$\alpha_s(m_Z) = 0.1205 \pm 0.0022 (\pm 2\%)$$



[D.d'E.,R.Perez-Ramos, arXiv:0505.02624 ]

➔ Hard parton-to-hadron FFs (NLO):

$$\alpha_s(m_Z) = 0.1176 \pm 0.0055 (\pm 4.7\%)$$



[AKK, B. Kniehl et al., NPB 803(2008)42]

# $\alpha_s$ from $\gamma$ QCD structure function

➔ Computed at NNLO:  $\int_0^1 dx F_2^\gamma(x, Q^2, P^2) = \frac{\alpha}{4\pi} \frac{1}{2\beta_0} \left\{ \frac{4\pi}{\alpha_s(Q^2)} c_{LO} + c_{NLO} + \frac{\alpha_s(Q^2)}{4\pi} c_{NNLO} + \mathcal{O}(\alpha_s^2) \right\}$

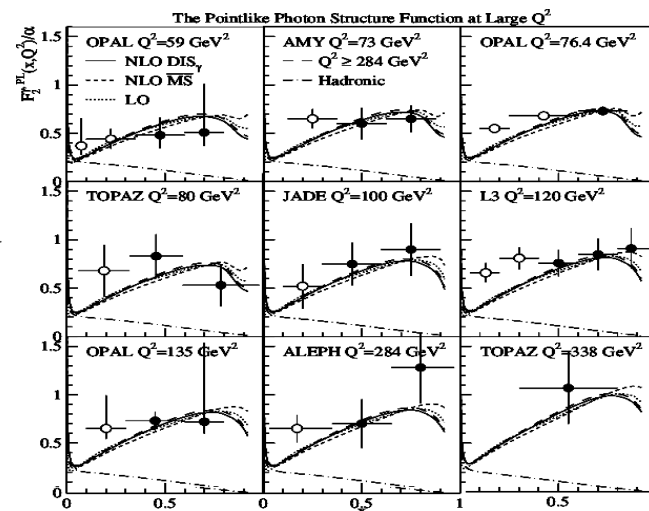
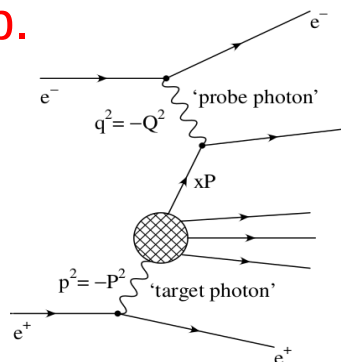
➔ Poor  $F_2^\gamma(x, Q^2)$  experimental measurements:

➔ Extraction (NLO) with large exp. uncertainties today:

$$\alpha_s(M_Z) = 0.1198 \pm 0.0054$$

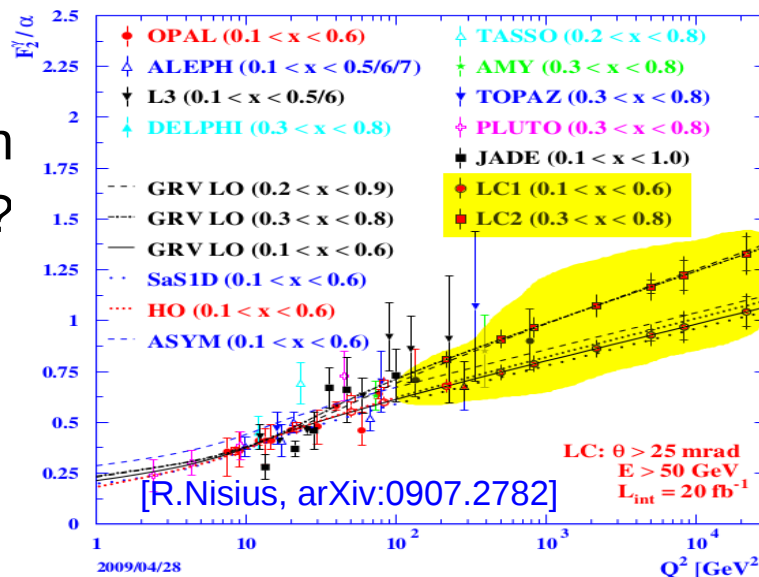
( $\pm 4.5\%$ )

[M.Klasen et al. PRL89 (2002)122004]



➔ Future prospects:

- Fit with NNLO  $F_2^\gamma$  evolution (ongoing)
- Better data badly needed: Belle-II ?
- Dedicated studies at ILC exist:
- Huge  $\gamma\gamma$  (EPA) stats at FCC-ee will lead to:  $\delta\alpha_s/\alpha_s < 1\%$



# $\alpha_s$ from lattice QCD

- Comparison of short-distance quantities (Wilson loops, q static potential, vacuum polariz.,...) computed at NNLO in pQCD, to lattice QCD “data”:

$$K^{\text{NP}} = K^{\text{PT}} = \sum_{i=0}^n c_i \alpha_s^i$$

[FLAG Collab. <http://itpwiki.unibe.ch/flag>]

- Currently, it's extraction with **smallest uncertainties:  $\pm 1\%$**  (lattice spacing & statistics).

Extracted value depends on observables:

Uncertainty **increased:**  
2013 ( $\pm 0.4\%$ )  $\rightarrow$  2017 ( $\pm 1.0\%$ )

- Future prospects:

- **Uncertainty in  $\alpha_s$  could be halved** with (much) better numerical data.
- Reaching  **$\pm 0.1\%$  requires 4<sup>th</sup>-loop** perturbation theory (~10 years?)

