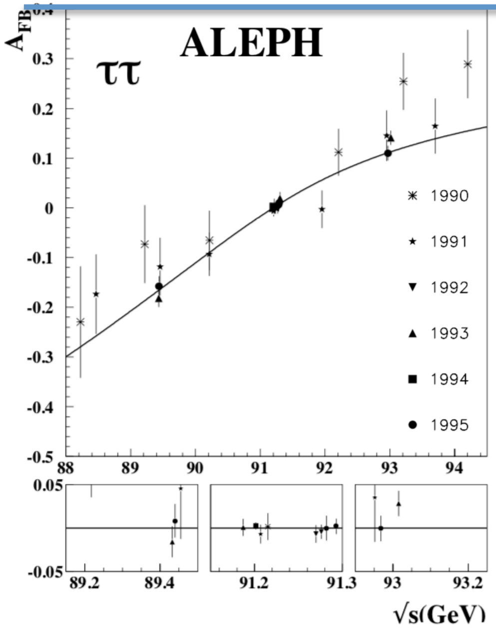
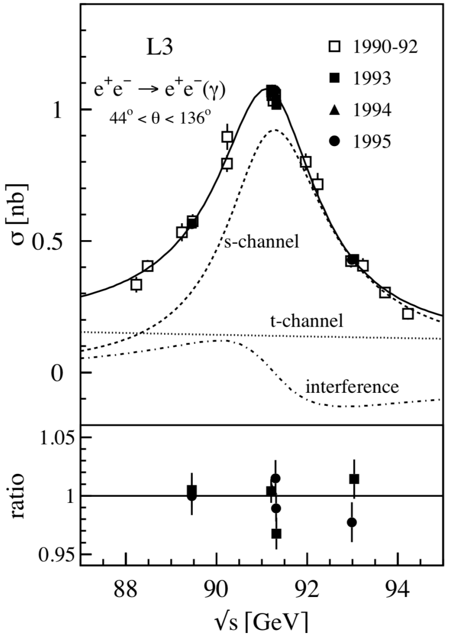
Systematics on EWPO measurements: Z line-shape + $A_{FB}^{0\ell}$ ($\sin^2\vartheta_W$)

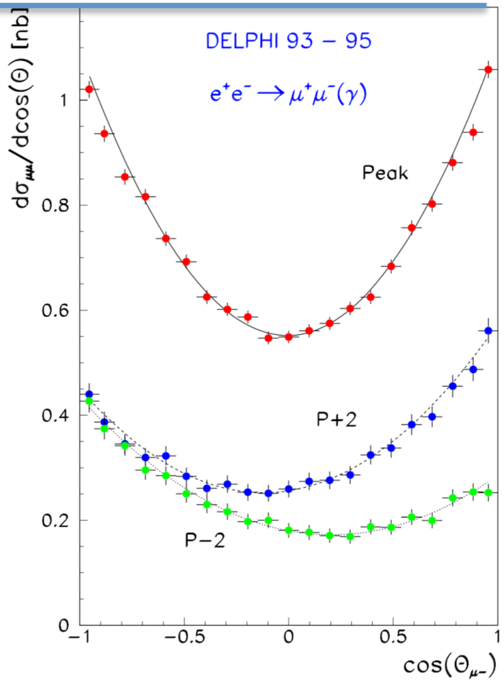


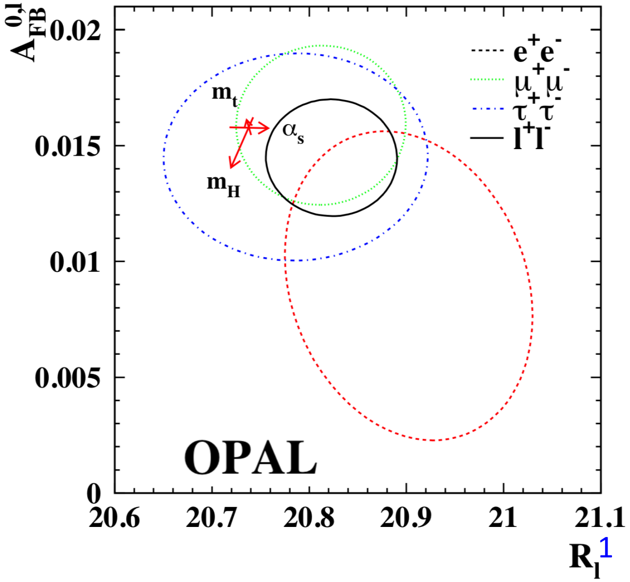


Lucia Di Ciaccio - USMB & LAPP
FCC France, Paris 14 May 2020

Outline

- Motivations
- EWPO & basic observables
- σ: statistical & experimental errors
- σ: selection & acceptance
- A_{FB}
- Extraction of $\sin^2\vartheta_{eff}$
- Common errors:
  luminosity, √s, theory
- Work in progress
- Conclusions

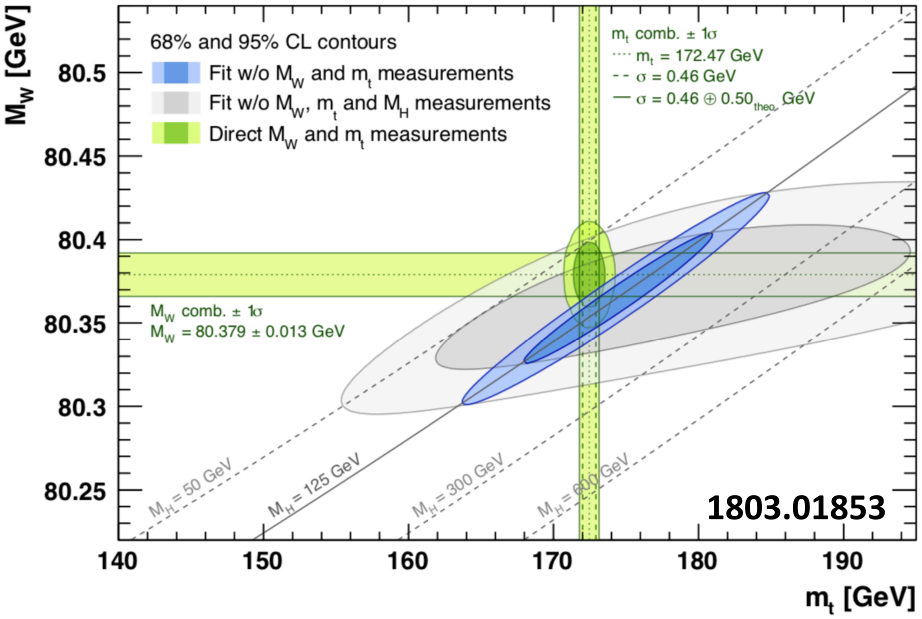




Motivations for looking at the systematics on EWPO

- FCCee: 5 order of magnitude luminosity more than LEP
  → need to match the statistical uncertainty

- Search for new physics effects by measuring very precisely key SM observables :
  any significant inconsistency →existence of New Physics



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- Probe energy scales beyond the direct kinematic reach

- Slightly different situation wrt pre-LEP era, certainly not less interesting

ElectroWeak Precision$^{(*)}$ Observables: which & how precise?

- Choice reduce correlation
- Single experiment

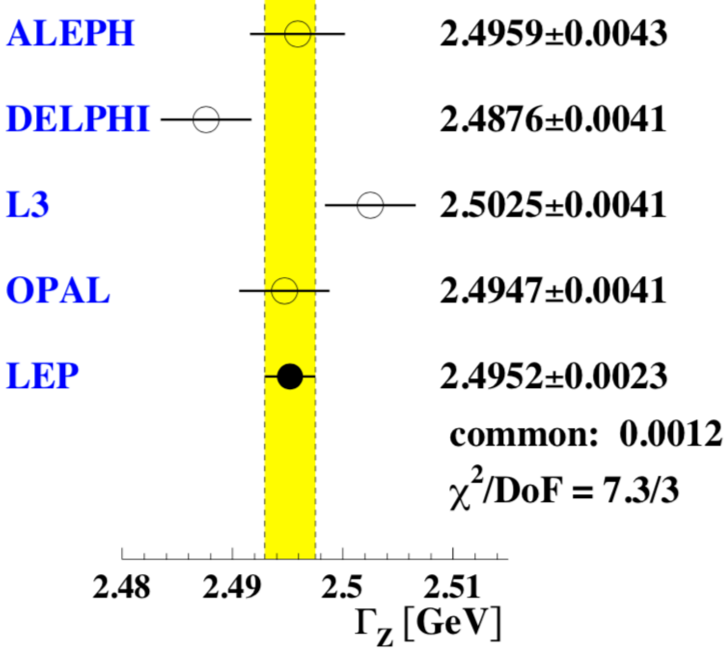
$\chi^2/\text{dof} = 158/198$ (OPAL)		Rel. unc.
m_Z [GeV]	91.1853 ± 0.0029	3 $*10^{-5}$
Γ_Z [GeV]	2.4947 ± 0.0041	2 $*10^{-3}$
σ^0_{had} [nb]	41.502 ± 0.055	1 $*10^{-3}$
$R^0_\ell \equiv \Gamma_{\text{had}}/\Gamma_{\ell\ell}$	20.822 ± 0.044	2 $*10^{-3}$
$A^{0,\ell}_{\text{FB}}$	0.0145 ± 0.0017	$12*10^{-2}$

- Combination experiments (ADLO)

With lepton universality		Rel. unc.	Target FCC
$\chi^2/\text{dof} = 36.5/31$			
m_Z [GeV]	91.1875 ± 0.0021	2 $*10^{-5}$	→ 10^{-6}
Γ_Z [GeV]	2.4952 ± 0.0023	1 $*10^{-3}$	→ 4 $*10^{-5}$
σ^0_{had} [nb]	41.540 ± 0.037	0.9 $*10^{-3}$	→ 10^{-4}
$R^0_\ell \equiv \Gamma_{\text{had}}/\Gamma_{\ell\ell}$	20.767 ± 0.025	1 $*10^{-3}$	→ 3 $* 10^{-5}$
$A^{0,\ell}_{\text{FB}}$	0.0171 ± 0.0010	6 $*10^{-2}$	

$^{(*)}$ pseudo-observable 'X^0'
with theoretical corrections

Γ_Z statistically dominated (even after combination)



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$N_\nu = 2.9840 \pm 0.0082$ → 0.001

$\sin^2\theta^{\text{lept}}_{\text{eff}} = 0.23153 \pm 0.00016$ →0.000005

Basic: cross section $e^+e^- \rightarrow f\bar{f}$ measurements vs √s

- m_Z , Γ_Z, σ^0_{had} , R^0_ℓ , $A^{0\,\ell}_{FB}$

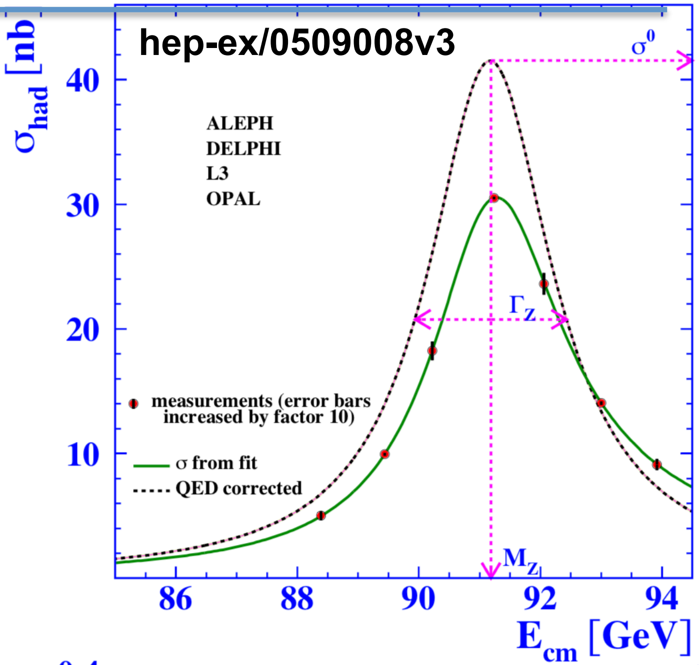
$$R_\ell = \frac{\sigma^0_{\mathrm{had}}}{\sigma^0_{\ell^+\ell^-}}$$

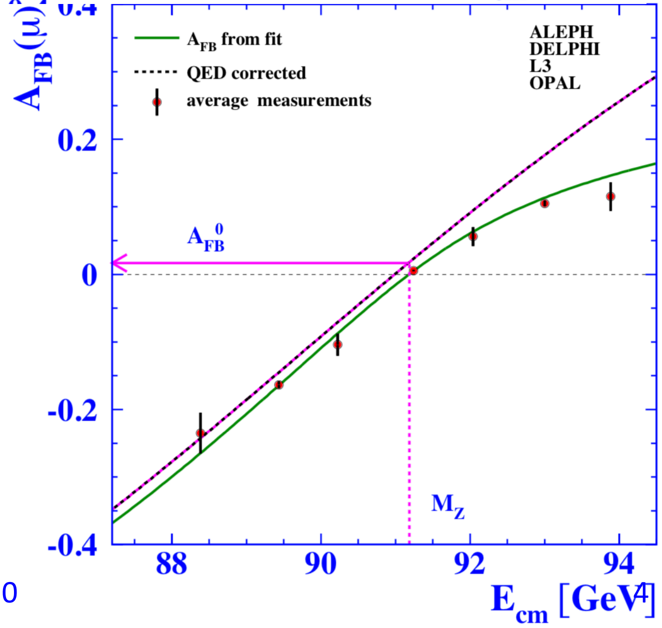
$$\frac{d\sigma}{d\cos\theta^*} \propto \left(1 + \cos^2\theta^* + \frac{8}{3}A_{\mathrm{FB}}\cos\theta^*\right)$$

- Basic observables: $\sigma = \dfrac{N_{\mathrm{sel}} - N_{\mathrm{bg}}}{\epsilon_{\mathrm{sel}}\mathcal{L}}$ & √s

- Ingredients:
  - Bkg & selection corrections
  - Luminosity determination
  - Acceptance & theory corrections
  - Center of mass energy determination

- **@LEP clever/hard work to get small systematics**
  → statistics could be fully exploited

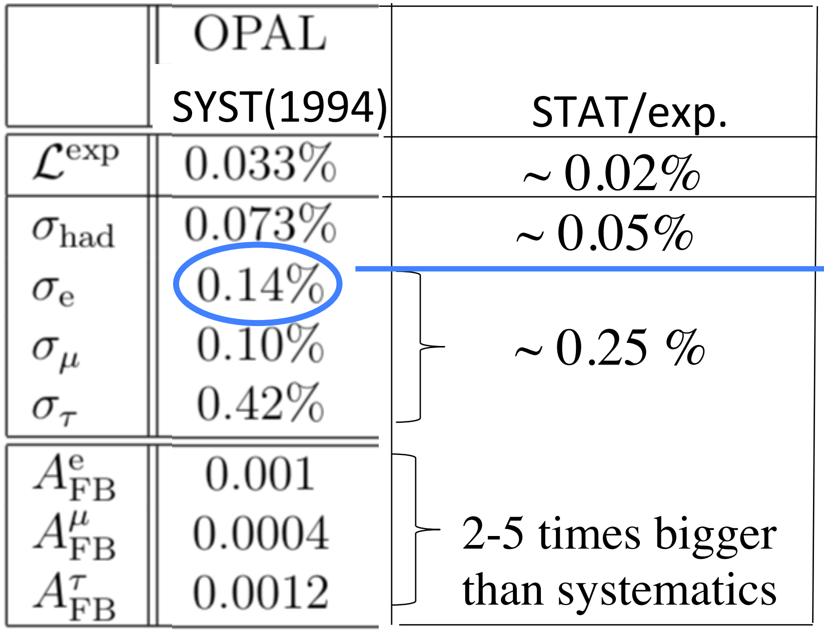




Statistics & experimental errors

- @ LEP experimental error on “basic” observables (mainly acceptance & bkg) mostly uncorrelated among experiments

@Z peak:



	OPAL SYST(1994)	STAT/exp.
$\mathcal{L}^{\mathrm{exp}}$	0.033%	~ 0.02%
σ_{had}	0.073%	~ 0.05%
σ_{e}	0.14%	~ 0.25 %
σ_{μ}	0.10%	
σ_{τ}	0.42%	
$A_{\mathrm{FB}}^{\mathrm{e}}$	0.001	2-5 times bigger than systematics
A_{FB}^{μ}	0.0004	
A_{FB}^{τ}	0.0012	

detailed in the next slide

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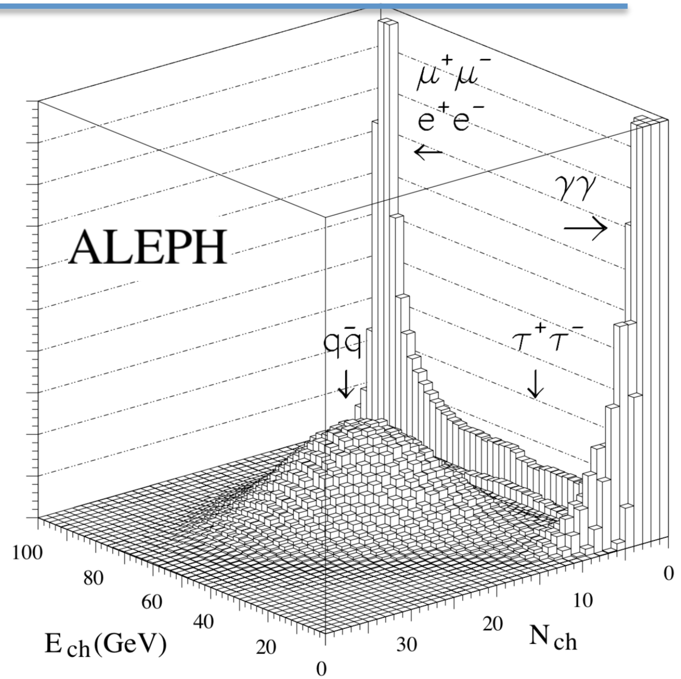
- From ~ 2 *10 ^{7} Z (LEP) → 5 * 10^{12} Z (FCC-ee) 5 orders of magnitude

→ Statistical uncertainty ~ **300** times smaller
(challenging demand on systematics)

Selection & acceptance systematics @ LEP

- Clean environment: “easy” separation
  background from 2γ (ee → $\bar{e}e\bar{f}f$), 4 fermions,
  for Z_{lep} also cosmics, beam-gas

- Definition of the Phase Space of the measurement
  (≈ detector fiducial phase space)



$e^+e^- \rightarrow e^+e^-$ @ peak OPAL	$\Delta f/f$ (%)
Monte Carlo	
$e^+e^- \rightarrow e^+e^-$ Monte Carlo	0.02
Acceptance Correction	
Electromagnetic energy	0.07
Electron identification	0.05
Acceptance definition	0.09
Low multiplicity	0.01
Other Corrections	
Four-fermion events	0.02
Signal Correction	0.13
Backgrounds	
$e^+e^- \rightarrow \tau^+\tau^-$	0.06
$e^+e^- \rightarrow \gamma\gamma$	0.01
$e^+e^- \rightarrow q\bar{q}$	0.02
$e^+e^- \rightarrow e^+e^-\ell^+\ell^-$	0.01
Background Correction	0.06
Total Correction Factor	0.14

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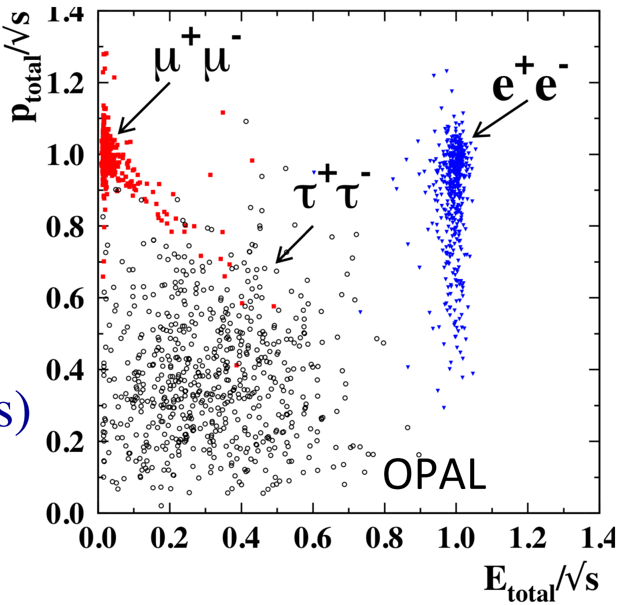
f = corr. factor

$f * N_{meas} = N_{PS}$

- MC modelling
  geometrical boundary
- $\Delta f/f$ ~ 1/3 smaller in Z→had
  ~ 3 higher in Z→ ττ

→ @FCC challenge:
control of acceptance (vs √s)
important for $R_\ell = \dfrac{\sigma^0_{had}}{\sigma^0_{\ell^+\ell^-}}$



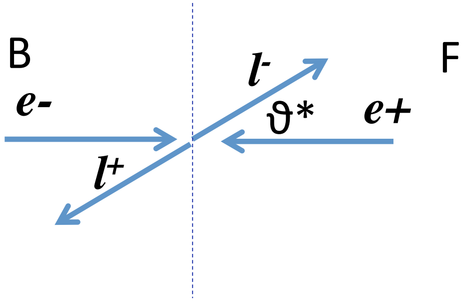
A^{ℓ}_{FB} measurement @ LEP

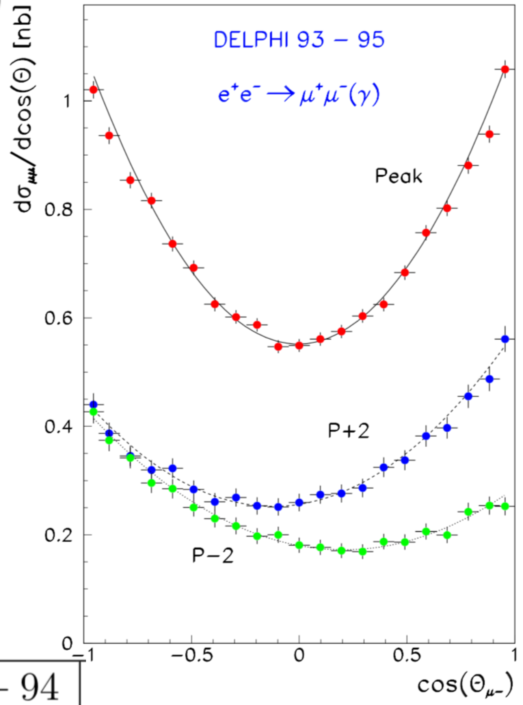
- Important to extract g^{ℓ}_{V} , g^{ℓ}_{A} and $\sin^2 \vartheta^{\ell}_{eff}$

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

- For s channel:

$$\frac{d\sigma}{d\cos\theta^*} \propto \left(1 + \cos^2\theta^* + \frac{8}{3} A_{FB} \cos\theta^*\right)$$





- No dependence on luminosity or overall efficiency but dependence on efficiency vs cos θ & asymmetry in charge mis-identification

- Basic measurement systematics @ LEP:

- Statistical uncertainty: 2-5 times bigger

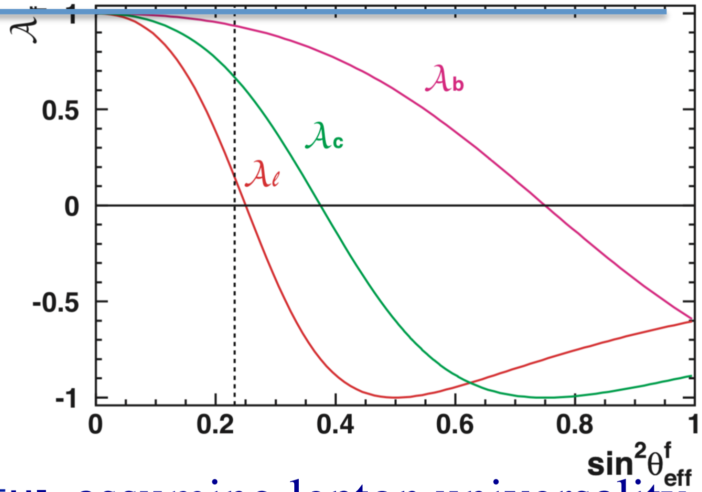
Source L3 $e^+e^- \to \mu^+\mu^-$	1993 – 94
Fit procedure	< 0.0003
Detector asymmetry	0.0006
Charge confusion	0.0004
Momentum reconstruction	0.0004
Background	0.0001 – 0.0005
Total uncertainty	0.0008 – 0.0009

data driven

- @FCCee: fiducial acceptance asymmetric: beam crossing angle 30 mrad can be measured from $e^+e^- \to \mu^+\mu^-$ 10^6 dimuon events (5' $\langle\alpha\rangle = 29.9998 \pm 0.0003\,\text{mrad}$) (**1909.12245**)

Extraction of $\sin^2 \vartheta^{\ell}_{\text{eff}}$

$$A^{0,\,\mathrm{f}}_{\mathrm{FB}} \;=\; \frac{3}{4}\mathcal{A}_{\mathrm{e}}\mathcal{A}_{\mathrm{f}} \qquad \mathcal{A}_{\mathrm{f}} \;=2\frac{g_{\mathrm{Vf}}/g_{\mathrm{Af}}}{1+(g_{\mathrm{Vf}}/g_{\mathrm{Af}})^2}$$



@LEP all asymmetries used to extract $\sin^2 \vartheta_{\text{eff}}$

$$\sin^2\theta^{\text{lept}}_{\text{eff}} \;=\; 0.23153 \pm 0.00016$$

- @FCC → extraction of $\sin^2 \vartheta^{\ell}_{\text{eff}}$ from $e^+e^- \to \mu^+\mu^-$ assuming lepton universality
  Uncertainty ~ 0.000005 (= 5 *10^{-6}, factor ~ 30 wrt LEP)
  (main uncertainty point-to-point energy error)
- Using from τ polarisation avoids assumption on lepton universality

$$\mathcal{P}_\tau \;\equiv\; (\sigma_+ - \sigma_-)/(\sigma_+ + \sigma_-) \qquad \mathcal{P}_{\mathrm{f}}(\cos\theta) \;=\; -\,\frac{\mathcal{A}_{\mathrm{f}}(1+\cos^2\theta) + 2\mathcal{A}_{\mathrm{e}}\cos\theta}{(1+\cos^2\theta) + 2\mathcal{A}_{\mathrm{f}}\mathcal{A}_{\mathrm{e}}\cos\theta}$$

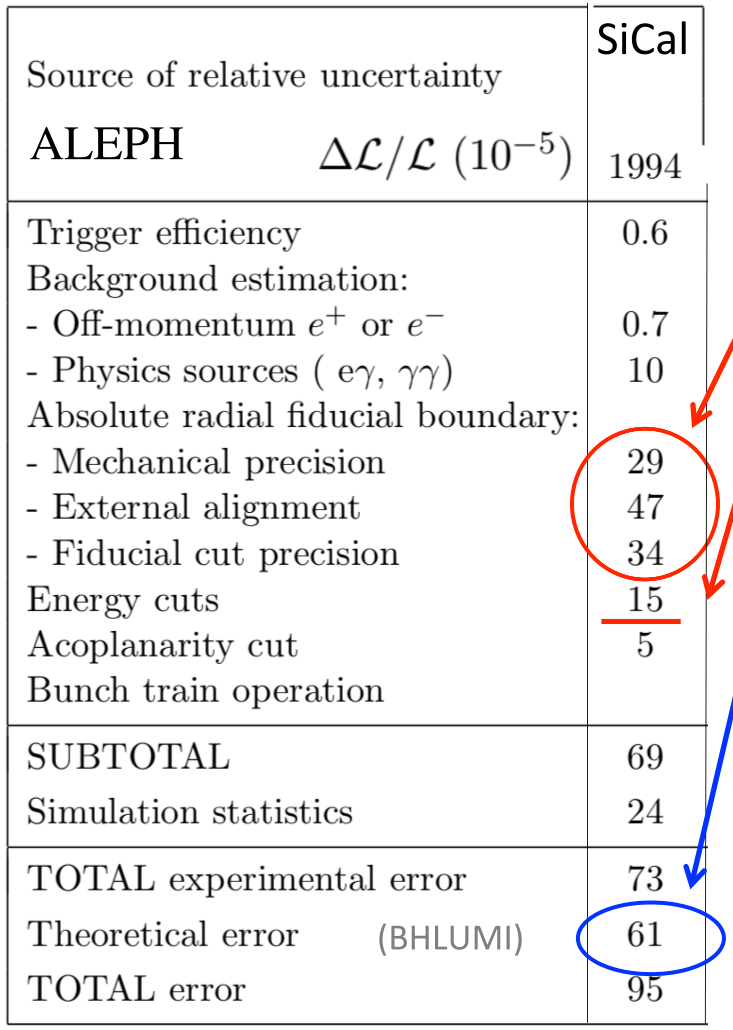
@ LEP several decay modes were used. Main uncertainties: from τ BR and hadronic τ decay modelling

→ @FCC use τ → ϱν Uncertainty on $\sin^2 \vartheta_{\text{eff}} \sim 6.6* 10^{-6}$

- e^+e^- colliders unique power for $\sin^2 \vartheta_{\text{eff}}$ determination @ m_Z

Luminosity systematics

- Luminosity from small-angle Bhabha-scattered electrons



Source of relative uncertainty ALEPH $\Delta\mathcal{L}/\mathcal{L}$ (10^{-5})	SiCal 1994
Trigger efficiency	0.6
Background estimation:	
- Off-momentum e^+ or e^-	0.7
- Physics sources ($e\gamma$, $\gamma\gamma$)	10
Absolute radial fiducial boundary:	
- Mechanical precision	29
- External alignment	47
- Fiducial cut precision	34
Energy cuts	15
Acoplanarity cut	5
Bunch train operation	
SUBTOTAL	69
Simulation statistics	24
TOTAL experimental error	73
Theoretical error (BHLUMI)	61
TOTAL error	95

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- Uncertainty important for σ^0_{had} @LEP after combination contributes to ~ half its total error
- Definition of geometrical acceptance (use of special methods, or W mask)
- Steep variation of cross section with E_{sum} (cut to fight beam background)
- Theory error biggest single contribution
- Total error: $\Delta L/L \sim 10^{-3}$ (ADL)

! (reanalysis **1908.01704** beam-beam effect: 10^{-3} bias)

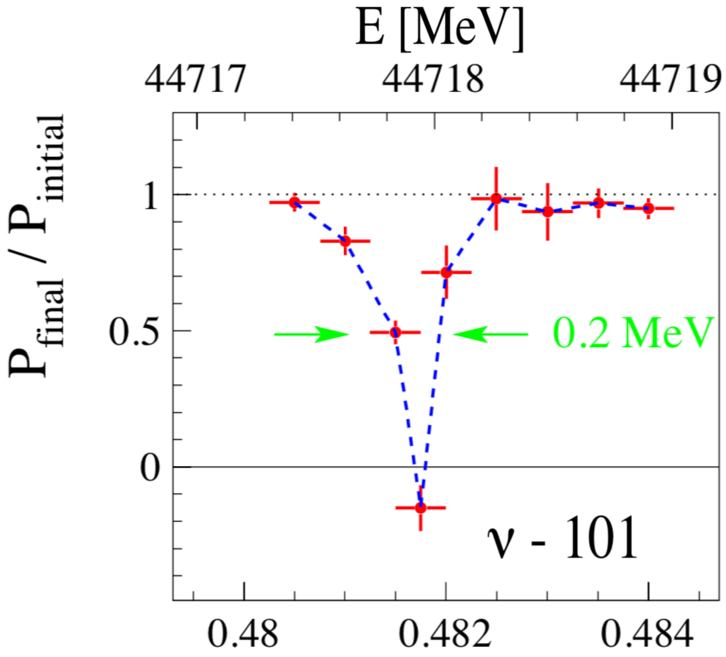
- @ FCC-ee (**1812.01004**) (use also ee->γγ):
  $\Delta L/L$: absolute ~ 10^{-4}
  (→reduction of factor 8 on ΔN_ν)
  $\Delta L/L$: point-to-point $5*10^{-5}$ (relevant for Γ_Z)

Impact of the energy measurement

- Uncertainty common across experiments
- $\sqrt{s}$ calibration from spin tune (ν) measurements via resonant depolarisation

$$\frac{E}{m_e} = \frac{2\ \nu}{g_e - 2}$$

- Control of $\sqrt{s}$ uncertainties essential for precision on m_Z , Γ_Z , A_{FB}



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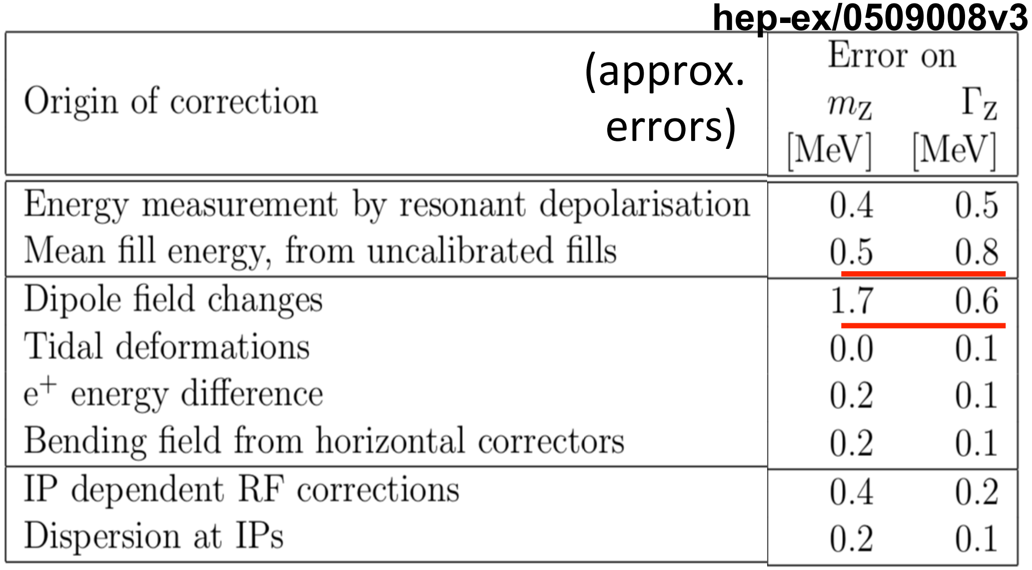


Origin of correction (approx. errors)	Error on m_Z [MeV]	Error on Γ_Z [MeV]
Energy measurement by resonant depolarisation	0.4	0.5
Mean fill energy, from uncalibrated fills	0.5	0.8
Dipole field changes	1.7	0.6
Tidal deformations	0.0	0.1
e^+ energy difference	0.2	0.1
Bending field from horizontal correctors	0.2	0.1
IP dependent RF corrections	0.4	0.2
Dispersion at IPs	0.2	0.1

- Impact on EWPO uncertainty also from beam energy spread: δE
- @ FCC challenging: δE increases wrt LEP and may fluctuate (beamstrahlung)
  Measured with $e^+e^- \rightarrow \mu^+\mu^-$ (5') → $\Delta\Gamma_Z$ ~ 25 keV (**arXiv:1812.01004**)

Impact of the energy measurement

- From: “Polarization and Centre-of-mass Energy Calibration at FCC-ee”
  A. Blondel et al (**1909.12245**)
- Impact of error on √s for 3 point scan:

$$\frac{\Delta m_Z}{m_Z} = \left\{\frac{\Delta\sqrt{s}}{\sqrt{s}}\right\}_{\rm abs} \oplus \left\{\frac{\Delta(\sqrt{s_+}+\sqrt{s_-})}{\sqrt{s_+}+\sqrt{s_-}}\right\}_{\rm ptp-syst} \oplus_i \left\{\frac{\Delta\sqrt{s^i_\pm}}{\sqrt{s^i_\pm N^i_\pm}}\right\}_{\rm sampling},$$

$$\frac{\Delta \Gamma_Z}{\Gamma_Z} = \left\{\frac{\Delta\sqrt{s}}{\sqrt{s}}\right\}_{\rm abs} \oplus \left\{\frac{\Delta(\sqrt{s_+}-\sqrt{s_-})}{\sqrt{s_+}-\sqrt{s_-}}\right\}_{\rm ptp-syst} \oplus_i \left\{\frac{\Delta\sqrt{s^i_\pm}}{\sqrt{s^i_\pm N^i_\pm}}\right\}_{\rm sampling},$$

$$\Delta A^{\mu\mu}_{\rm FB}({\rm pole}) = \frac{\partial A^{\mu\mu}_{\rm FB}}{\partial\sqrt{s}}\{\Delta(\sqrt{s_0}-0.5(\sqrt{s_+}+\sqrt{s_-}))\}_{\rm ptp-syst} \oplus_i \frac{\partial A^{\mu\mu}_{\rm FB}}{\partial\sqrt{s}}\left\{\frac{\Delta\sqrt{s^i_{0,\pm}}}{\sqrt{N^i_{0,\pm}}}\right\}_{\rm sampling},$$

$$\frac{\Delta\alpha_{\rm QED}(m_Z^2)}{\alpha_{\rm QED}(m_Z^2)} = \left\{\frac{\Delta\sqrt{s}}{\sqrt{s}}\right\}_{\rm abs} \oplus \left\{\frac{\Delta(\sqrt{s_+}-\sqrt{s_-})}{\sqrt{s_+}-\sqrt{s_-}}\right\}_{\rm ptp-syst} \oplus_i \left\{\frac{\Delta\sqrt{s^i_\pm}}{\sqrt{s^i_\pm N^i_\pm}}\right\}_{\rm sampling},$$

Three categories:

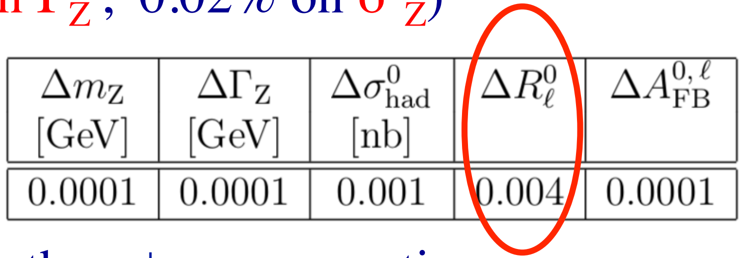
- Absolute dominate for Z mass
- Point-to-point dominate for Γ_Z & $A_{FB}^{\mu\mu}$ (peak and off-peak)
- Due to sampling negligible for 1 Resonant Depolarisation every 15’ (= 1000s) → 10^4 measts

- With an improved systematics error evaluation

Observable	statistics	$\Delta\sqrt{s}_{\rm abs}$ 100 keV	$\Delta\sqrt{s}_{\rm syst-ptp}$ **40 keV**	calib. stats. $200\,{\rm keV}/\sqrt{N^i}$	$\sigma_{\sqrt{s}}$ $85 \pm \mathbf{0.05}$ MeV
m_Z (keV)	4	100	**28**	1	–
Γ_Z (keV)	4	2.5	**22**	1	**10**
$\sin^2\theta_W^{\rm eff} \times 10^6$ from $A_{FB}^{\mu\mu}$	2	–	**2.4**	0.1	–
$\frac{\Delta\alpha_{\rm QED}(m_Z^2)}{\alpha_{\rm QED}(m_Z^2)} \times 10^5$	3	0.1	**0.9**	–	**0.1**

Theoretical uncertainties

- Common across the experiments
- Theory uncertainties related to the knowledge of :
  - QED radiative effects
    (@LEP: ± 0.3 MeV on m_Z, ± 0.3 MeV on Γ_Z , 0.02% on $\sigma^0{}_Z$)
  - parametrization of line shape and A_{FB} in term of the pseudo observables →



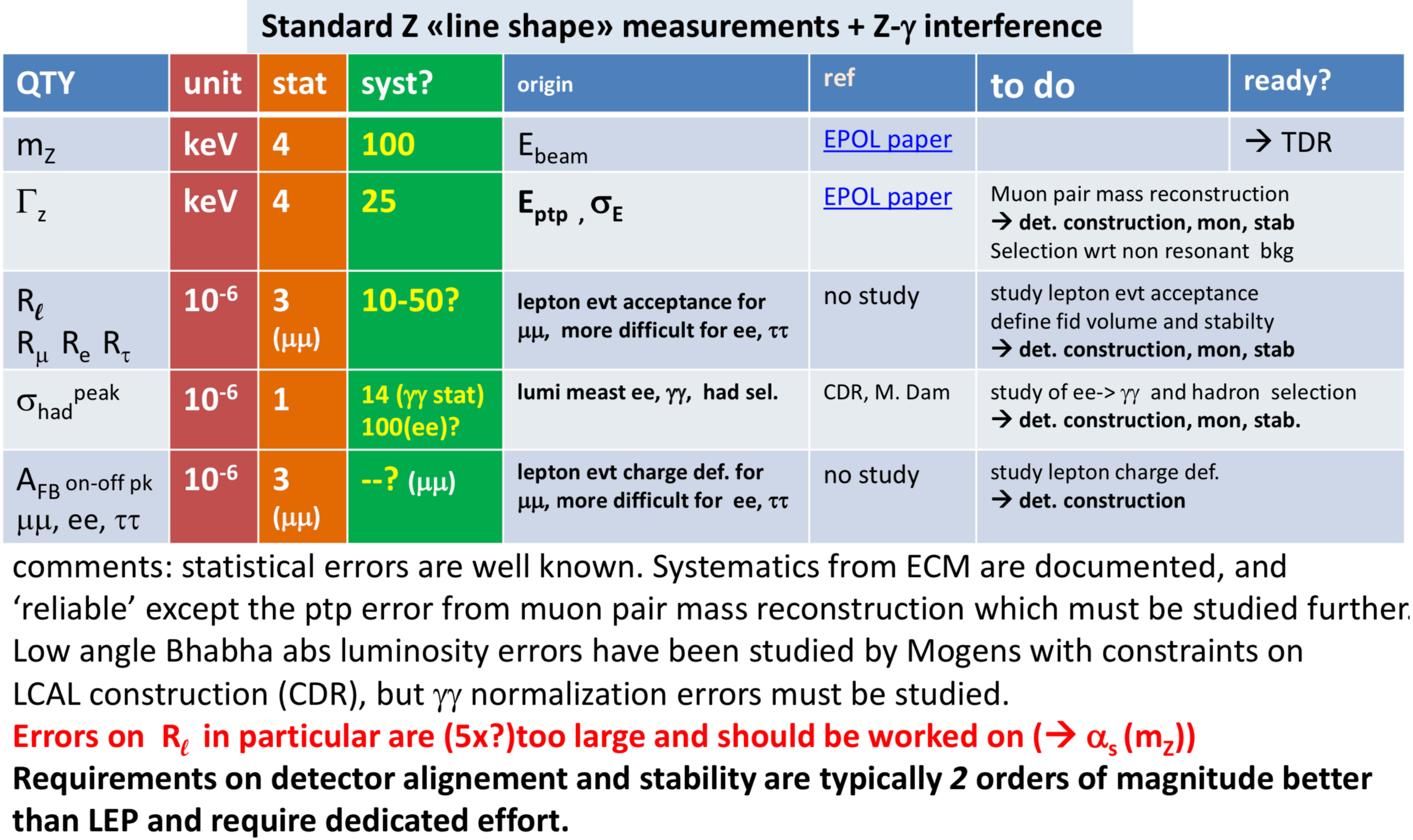
Δm_Z [GeV]	$\Delta\Gamma_Z$ [GeV]	$\Delta\sigma^0_{\mathrm{had}}$ [nb]	ΔR^0_ℓ	$\Delta A^{0,\ell}_{\mathrm{FB}}$
0.0001	0.0001	0.001	0.004	0.0001

  - t-channel and s-t interference contribution to the e^+e^- cross section
  - small angle Bhabha cross section for the luminosity (@LEP ~ 0.061%)
- Need to match experimental uncertainties: dedicated workshop, 2018 → @FCC need:
  * New approach for the extraction of the observables
  * Three loop calculations (EW, QCD, mixed)
  * Dedicated generators
- Need close collaboration theorists-experimentalists

Work in progress

- From talk: “Detector requirement”, Alain Blondel, Mogens Dam (https://indico.cern.ch/event/887925/) FCC physics, 30 March 2020

Thanks to Alain



Standard Z «line shape» measurements + Z-γ interference

QTY	unit	stat	syst?	origin	ref	to do	ready?
m_Z	**keV**	**4**	**100**	E_{beam}	EPOL paper		→ TDR
Γ_z	**keV**	**4**	**25**	$\mathbf{E_{ptp}}$, σ_E	EPOL paper	Muon pair mass reconstruction → **det. construction, mon, stab** Selection wrt non resonant bkg	
R_ℓ R_μ R_e R_τ	**10^{-6}**	**3** **(μμ)**	**10-50?**	**lepton evt acceptance for μμ, more difficult for ee, ττ**	no study	study lepton evt acceptance define fid volume and stabilty → **det. construction, mon, stab**	
σ_{had}^{peak}	**10^{-6}**	**1**	**14 (γγ stat)** **100(ee)?**	**lumi meast ee, γγ, had sel.**	CDR, M. Dam	study of ee-> γγ and hadron selection → **det. construction, mon, stab.**	
A_{FB} on-off pk μμ, ee, ττ	**10^{-6}**	**3** **(μμ)**	**--? (μμ)**	**lepton evt charge def. for μμ, more difficult for ee, ττ**	no study	study lepton charge def. → **det. construction**	

comments: statistical errors are well known. Systematics from ECM are documented, and ‘reliable’ except the ptp error from muon pair mass reconstruction which must be studied further. Low angle Bhabha abs luminosity errors have been studied by Mogens with constraints on LCAL construction (CDR), but γγ normalization errors must be studied.

Errors on R_ℓ in particular are (5x?)too large and should be worked on (→ α_s (m_Z))

Requirements on detector alignement and stability are typically *2* orders of magnitude better than LEP and require dedicated effort.

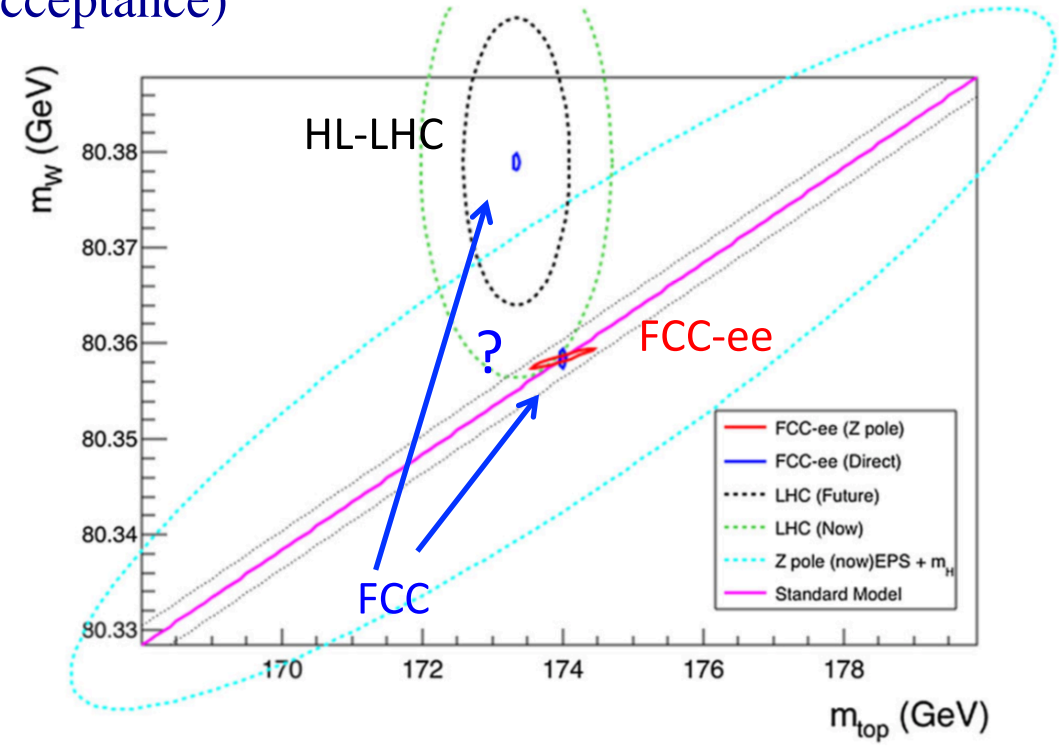
Conclusions

- EWPO measurements are very valuable probes of BSM physics
- LEP has set very high standards for the evaluation of systematic errors
- @FCC it must be improved a lot to benefit from the huge jump in statistics

- The target precision is challenging: need clever ideas and work on systematic common uncertainties (beam related & theory) but also on detectors and analysis techniques (control of acceptance)

- Machine and detector challenge: high power environment

- Important to work closely:
  - * theory -experiment
  - * experiment-machine



From CDR

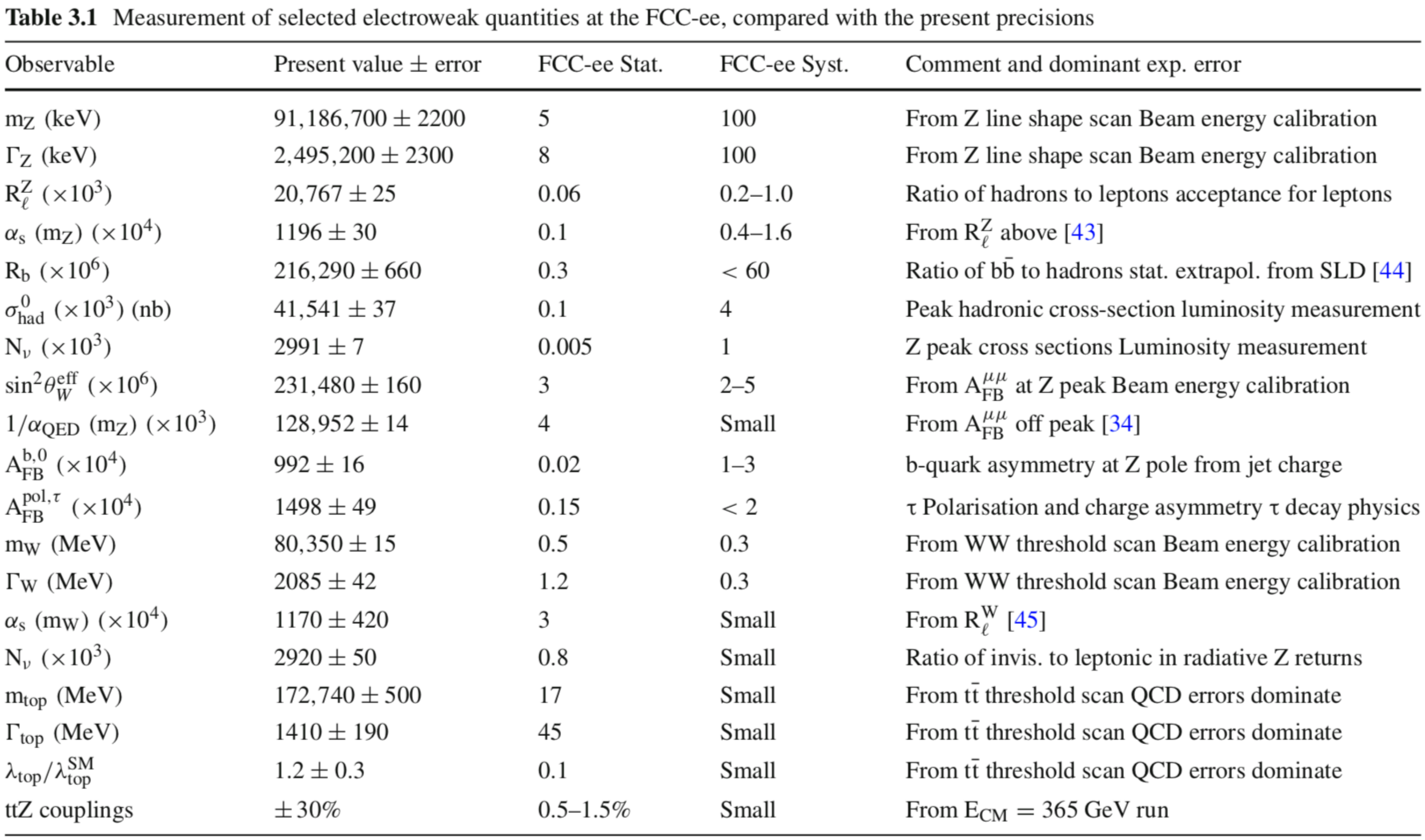


Table 3.1 Measurement of selected electroweak quantities at the FCC-ee, compared with the present precisions

Observable	Present value ± error	FCC-ee Stat.	FCC-ee Syst.	Comment and dominant exp. error
m_Z (keV)	$91,186,700 \pm 2200$	5	100	From Z line shape scan Beam energy calibration
Γ_Z (keV)	$2,495,200 \pm 2300$	8	100	From Z line shape scan Beam energy calibration
R_ℓ^Z $(\times 10^3)$	$20,767 \pm 25$	0.06	0.2–1.0	Ratio of hadrons to leptons acceptance for leptons
α_s (m_Z) $(\times 10^4)$	1196 ± 30	0.1	0.4–1.6	From R_ℓ^Z above [43]
R_b $(\times 10^6)$	$216,290 \pm 660$	0.3	< 60	Ratio of $b\bar{b}$ to hadrons stat. extrapol. from SLD [44]
σ^0_{had} $(\times 10^3)$ (nb)	$41,541 \pm 37$	0.1	4	Peak hadronic cross-section luminosity measurement
N_ν $(\times 10^3)$	2991 ± 7	0.005	1	Z peak cross sections Luminosity measurement
$\sin^2\theta_W^{eff}$ $(\times 10^6)$	$231,480 \pm 160$	3	2–5	From $A_{FB}^{\mu\mu}$ at Z peak Beam energy calibration
$1/\alpha_{QED}$ (m_Z) $(\times 10^3)$	$128,952 \pm 14$	4	Small	From $A_{FB}^{\mu\mu}$ off peak [34]
$A_{FB}^{b,0}$ $(\times 10^4)$	992 ± 16	0.02	1–3	b-quark asymmetry at Z pole from jet charge
$A_{FB}^{pol,\tau}$ $(\times 10^4)$	1498 ± 49	0.15	< 2	τ Polarisation and charge asymmetry τ decay physics
m_W (MeV)	$80,350 \pm 15$	0.5	0.3	From WW threshold scan Beam energy calibration
Γ_W (MeV)	2085 ± 42	1.2	0.3	From WW threshold scan Beam energy calibration
α_s (m_W) $(\times 10^4)$	1170 ± 420	3	Small	From R_ℓ^W [45]
N_ν $(\times 10^3)$	2920 ± 50	0.8	Small	Ratio of invis. to leptonic in radiative Z returns
m_{top} (MeV)	$172,740 \pm 500$	17	Small	From $t\bar{t}$ threshold scan QCD errors dominate
Γ_{top} (MeV)	1410 ± 190	45	Small	From $t\bar{t}$ threshold scan QCD errors dominate
$\lambda_{top}/\lambda_{top}^{SM}$	1.2 ± 0.3	0.1	Small	From $t\bar{t}$ threshold scan QCD errors dominate
ttZ couplings	$\pm 30\%$	0.5–1.5%	Small	From $E_{CM} = 365$ GeV run

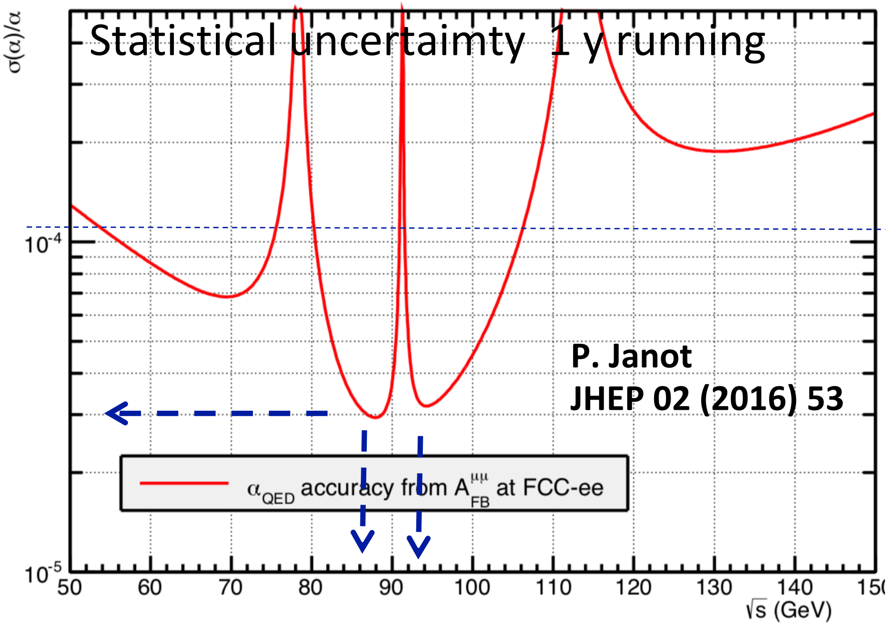
Ancillary measurements: $\alpha_{QED}(m_Z)$

- $\alpha_{QED}(m_Z)$ important ingredient of the EW fits (Gfitter, 2014):

$$\begin{aligned}\sin^2\theta_W^{\rm eff} &= 0.231488 \pm 0.000029_{m_{\rm top}} \pm 0.000015_{m_Z} \pm \boxed{0.000035_{\alpha_{\rm QED}}}\\ &\quad \pm 0.000010_{\alpha_S} \pm 0.000001_{m_H} \pm 0.000047_{\rm theory}\\ &= 0.23149 \pm 0.00007_{\rm total},\end{aligned}$$

- At present: $\Delta\alpha/\alpha \sim 10^{-4}$ Dominated by hadron vacuum polarisation corrections
- Proposal to use $A_{FB}^{\mu\mu}(s_\pm)$ just below/above the peak

$$A_{\rm FB}^{\mu\mu}(s) \simeq \frac{3}{4}\mathcal{A}_e\mathcal{A}_\mu \times \left[1 + \frac{8\pi\sqrt{2}\alpha_{\rm QED}(s)}{m_Z^2 G_F\left(1-4\sin^2\theta_W^{\rm eff}\right)^2}\,\frac{s-m_Z^2}{2s}\right]$$



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- Total uncertainty (except missing EW high orders, so far few 10^{-4}) dominated by √s calibration :

Systematics	1.2×10^{-5}
Statistics	3×10^{-5}

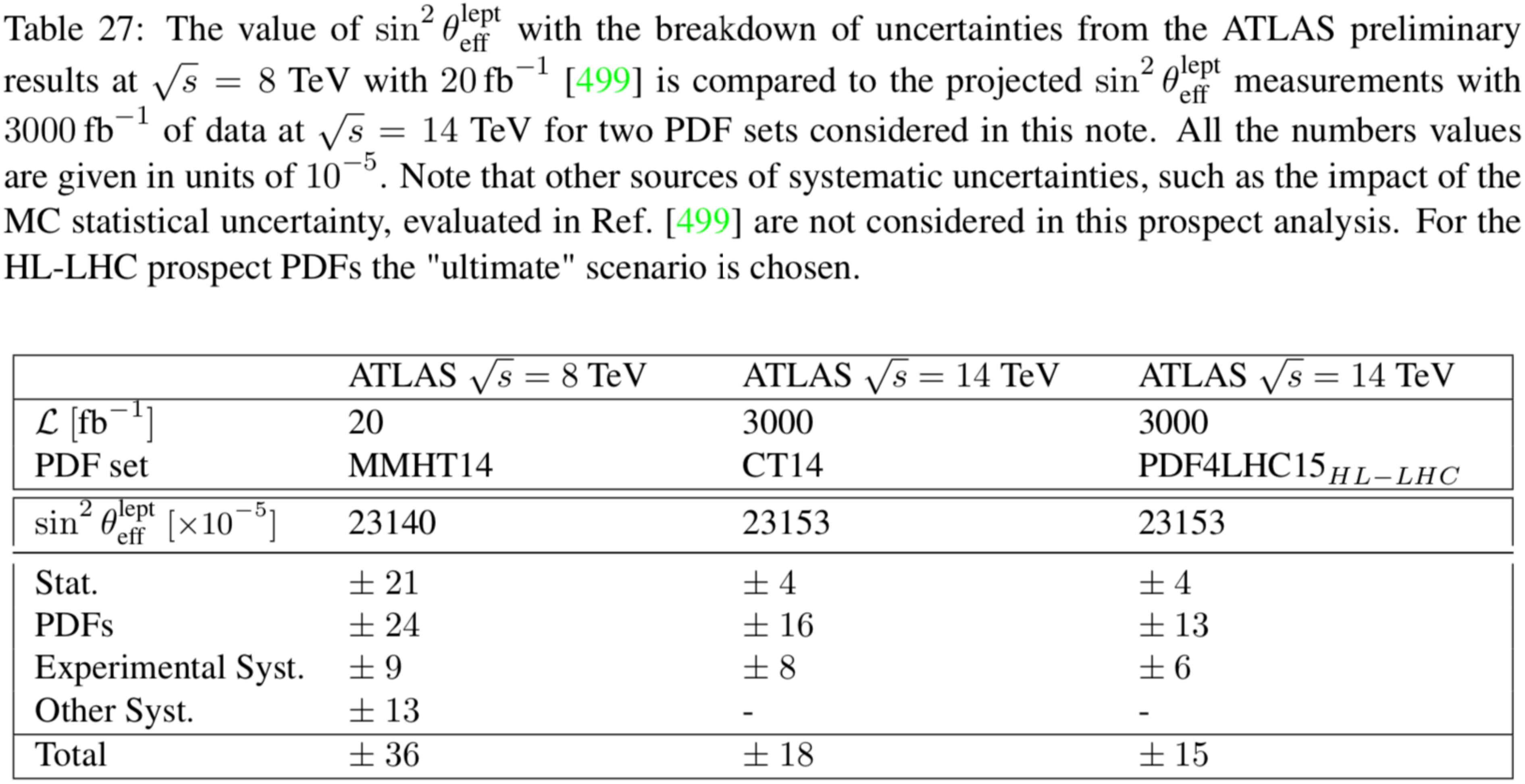
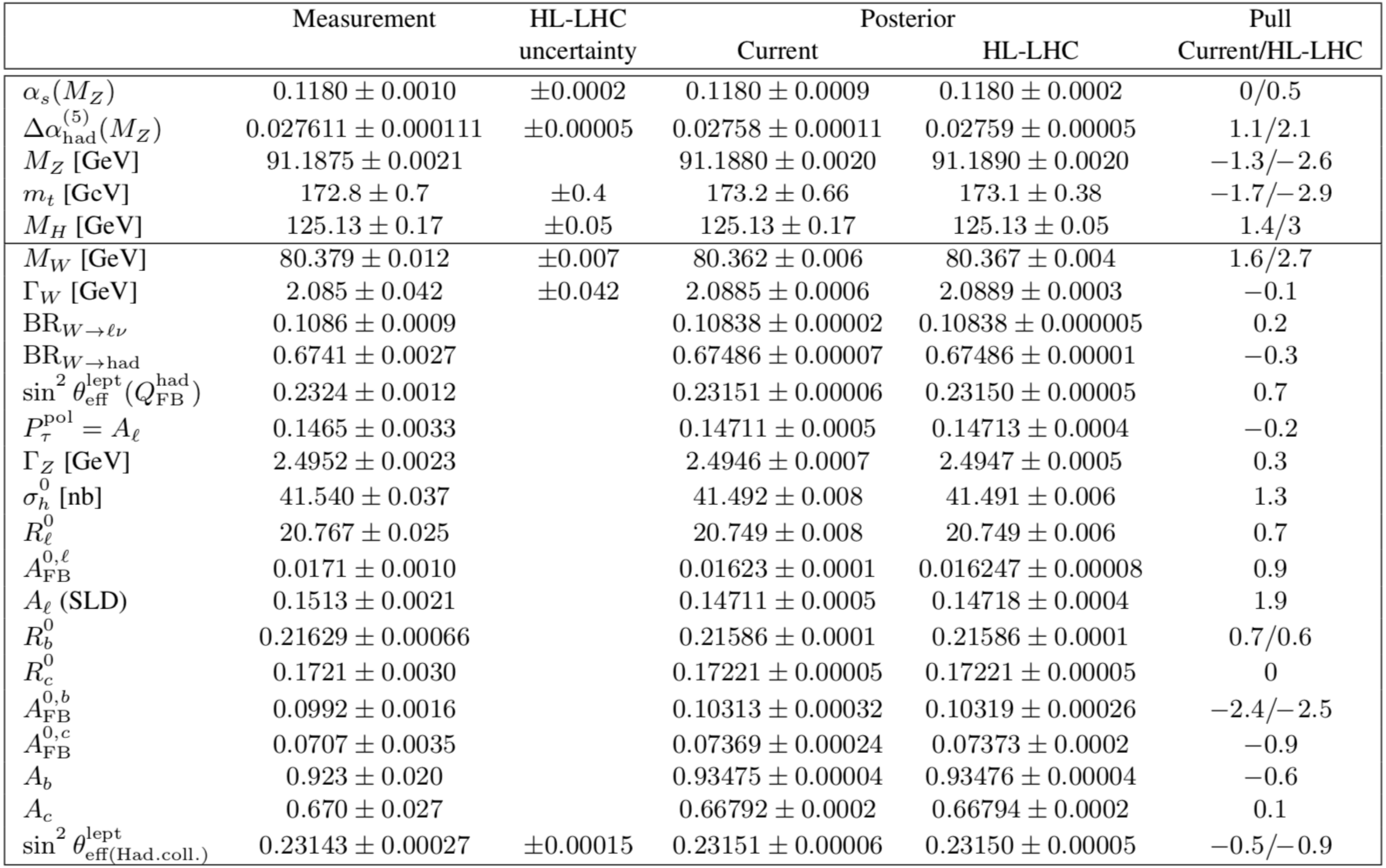


Table 27: The value of $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$ with the breakdown of uncertainties from the ATLAS preliminary results at $\sqrt{s} = 8$ TeV with $20\,\mathrm{fb}^{-1}$ [499] is compared to the projected $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$ measurements with $3000\,\mathrm{fb}^{-1}$ of data at $\sqrt{s} = 14$ TeV for two PDF sets considered in this note. All the numbers values are given in units of 10^{-5}. Note that other sources of systematic uncertainties, such as the impact of the MC statistical uncertainty, evaluated in Ref. [499] are not considered in this prospect analysis. For the HL-LHC prospect PDFs the "ultimate" scenario is chosen.

	ATLAS $\sqrt{s} = 8$ TeV	ATLAS $\sqrt{s} = 14$ TeV	ATLAS $\sqrt{s} = 14$ TeV
$\mathcal{L}$ [fb^{-1}]	20	3000	3000
PDF set	MMHT14	CT14	PDF4LHC15$_{HL-LHC}$
$\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$ [$\times 10^{-5}$]	23140	23153	23153
Stat.	± 21	± 4	± 4
PDFs	± 24	± 16	± 13
Experimental Syst.	± 9	± 8	± 6
Other Syst.	± 13	-	-
Total	± 36	± 18	± 15

0.00015



	Measurement	HL-LHC uncertainty	Posterior		Pull Current/HL-LHC
			Current	HL-LHC	
$\alpha_s(M_Z)$	0.1180 ± 0.0010	± 0.0002	0.1180 ± 0.0009	0.1180 ± 0.0002	$0/0.5$
$\Delta\alpha^{(5)}_{\rm had}(M_Z)$	0.027611 ± 0.000111	± 0.00005	0.02758 ± 0.00011	0.02759 ± 0.00005	$1.1/2.1$
M_Z [GeV]	91.1875 ± 0.0021		91.1880 ± 0.0020	91.1890 ± 0.0020	$-1.3/-2.6$
m_t [GeV]	172.8 ± 0.7	± 0.4	173.2 ± 0.66	173.1 ± 0.38	$-1.7/-2.9$
M_H [GeV]	125.13 ± 0.17	± 0.05	125.13 ± 0.17	125.13 ± 0.05	$1.4/3$
M_W [GeV]	80.379 ± 0.012	± 0.007	80.362 ± 0.006	80.367 ± 0.004	$1.6/2.7$
Γ_W [GeV]	2.085 ± 0.042	± 0.042	2.0885 ± 0.0006	2.0889 ± 0.0003	-0.1
${\rm BR}_{W\to\ell\nu}$	0.1086 ± 0.0009		0.10838 ± 0.00002	0.10838 ± 0.000005	0.2
${\rm BR}_{W\to{\rm had}}$	0.6741 ± 0.0027		0.67486 ± 0.00007	0.67486 ± 0.00001	-0.3
$\sin^2\theta^{\rm lept}_{\rm eff}(Q^{\rm had}_{\rm FB})$	0.2324 ± 0.0012		0.23151 ± 0.00006	0.23150 ± 0.00005	0.7
$P^{\rm pol}_\tau = A_\ell$	0.1465 ± 0.0033		0.14711 ± 0.0005	0.14713 ± 0.0004	-0.2
Γ_Z [GeV]	2.4952 ± 0.0023		2.4946 ± 0.0007	2.4947 ± 0.0005	0.3
σ^0_h [nb]	41.540 ± 0.037		41.492 ± 0.008	41.491 ± 0.006	1.3
R^0_ℓ	20.767 ± 0.025		20.749 ± 0.008	20.749 ± 0.006	0.7
$A^{0,\ell}_{\rm FB}$	0.0171 ± 0.0010		0.01623 ± 0.0001	0.016247 ± 0.00008	0.9
A_ℓ (SLD)	0.1513 ± 0.0021		0.14711 ± 0.0005	0.14718 ± 0.0004	1.9
R^0_b	0.21629 ± 0.00066		0.21586 ± 0.0001	0.21586 ± 0.0001	$0.7/0.6$
R^0_c	0.1721 ± 0.0030		0.17221 ± 0.00005	0.17221 ± 0.00005	0
$A^{0,b}_{\rm FB}$	0.0992 ± 0.0016		0.10313 ± 0.00032	0.10319 ± 0.00026	$-2.4/-2.5$
$A^{0,c}_{\rm FB}$	0.0707 ± 0.0035		0.07369 ± 0.00024	0.07373 ± 0.0002	-0.9
A_b	0.923 ± 0.020		0.93475 ± 0.00004	0.93476 ± 0.00004	-0.6
A_c	0.670 ± 0.027		0.66792 ± 0.0002	0.66794 ± 0.0002	0.1
$\sin^2\theta^{\rm lept}_{\rm eff(Had.coll.)}$	0.23143 ± 0.00027	± 0.00015	0.23151 ± 0.00006	0.23150 ± 0.00005	$-0.5/-0.9$

Additional measurement: Neutrino Light Species

$$R^0_{\text{inv}} \equiv \frac{\Gamma_{\text{inv}}}{\Gamma_{\ell\ell}} = N_\nu \left(\frac{\Gamma_{\nu\bar{\nu}}}{\Gamma_{\ell\ell}}\right)_{\text{SM}}$$

$$\Gamma_{\text{inv}} = \Gamma_{\text{Z}} - \Gamma_{\text{had}} - \Gamma_{\ell\ell}\ (3+\delta_m)$$

$$R^0_{\text{inv}} = \left(\frac{12\pi R^0_\ell}{\boxed{\sigma^0_{\text{had}}} m^2_{\text{Z}}}\right)^{\frac{1}{2}} - R^0_\ell - (3+\delta_\tau)$$

$$N_\nu = 2.9840 \pm 0.0082$$

$$\delta N_\nu \simeq 10.5 \frac{\delta n_{\text{had}}}{n_{\text{had}}} \oplus 3.0 \frac{\delta n_{\text{lep}}}{n_{\text{lep}}} \oplus 7.5 \frac{\delta \mathcal{L}}{\mathcal{L}}$$

Largest contribution from lumi (0.061% @ LEP)

Also from radiative return Zγ