GENERATION OF Z BOSONS IN EMISSION PROCESSES BY NEUTRINOS IN EARLY UNIVERSE

Mihaela Băloi and Cosmin Crucean

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Universitatea de Vest din Timişoara

Abstract

Production of Z bosons in emission processes by neutrinos in the expanding de Sitter universe is studied. We use perturbative methods to investigate emission processes that are forbidden in flat spacetime electroweak theory by the energy and momentum conservation. The amplitude and probability for the spontaneous emission of a Z boson by a neutrino or an antineutrino are computed analytically, then we perform a graphical analysis in terms of the expansion parameter. Our results prove that this process is possible only for large expansion conditions of the early Universe. The limit of large space expansion when the expansion parameter is much more larger than the mass of the Z boson is also obtained and the results prove that in this limit the emission probability increase.

The general formalism

- For studying the emission of Z bosons by neutrinos in early universe we start with the de Sitter line element:

$$ds^2 = dt^2 - e^{2\omega t}d\vec{x}^{\,2} = \frac{1}{(\omega t_c)^2}(dt_c^2 - d\vec{x}^{\,2}), \qquad (1)$$

- where the conformal time is $t_c = \frac{-e^{-\omega t}}{\omega}$ and $\omega > 0$ is the expansion factor.
- The first order transition amplitude in the theory of perturbations for the interaction between Z bosons and neutrino-antineutrino field reads:

$$\mathcal{A}_{\nu\to Z\nu} = -\int d^4x\sqrt{-g}\left(\frac{e_0}{\sin(2\theta_W)}\right)\overline{\psi}_{\nu_e}\,\gamma^{\hat{\mu}}e^{\alpha}_{\hat{\mu}}\left(\frac{1-\gamma^5}{2}\right)\psi_{\nu_e}A_\alpha(Z), \qquad (2)$$

- The tetrad fields $e^{\alpha}_{\hat{\mu}}$ expressed in the Cartesian tetrad gauge have the following non-vanishing components:

$$e^{0}_{\hat{0}} = -\omega t_c; \quad e^{i}_{\hat{j}} = -\delta^{i}_{\hat{j}}\,\omega t_c. \qquad (3)$$

- The neutrino field in de Sitter geometry is described by the following solution of the Dirac equation:

$$(U_{\vec{p},\sigma}(x))_\nu = \left(-\frac{\omega t_c}{2\pi}\right)^{3/2}\begin{pmatrix}(\frac{1}{2}-\sigma)\xi_\sigma(\vec{p})\\ 0\end{pmatrix}e^{i\vec{p}\cdot\vec{x}-ipt_c}. \qquad (4)$$

- The massive Z boson filed in the case of the transversal polarization $\lambda = \pm 1$ is the solution of the Proca equation:

$$\vec{f}_{\vec{\mathcal{P}},\lambda}(x) = \frac{\sqrt{\pi}e^{-\pi k/2}}{2(2\pi)^{3/2}}\sqrt{-t_c}H^{(1)}_{ik}(-\mathcal{P}t_c)e^{i\vec{\mathcal{P}}\vec{x}}\vec{\epsilon}(\vec{n}_{\mathcal{P}},\lambda). \qquad (5)$$

- For $\lambda = \pm 1$ the polarization vectors satisfy $\vec{\mathcal{P}}\cdot\vec{\epsilon}(\vec{n}_{\mathcal{P}},\lambda = \pm 1) = 0$. The notation for the mass of the Z boson is M_Z, and the parameter $k = \sqrt{\left(\frac{M_Z}{\omega}\right)^2 - \frac{1}{4}}$ is dependent on the ratio $\frac{M_Z}{\omega}$, and the condition $\frac{M_Z}{\omega} > \frac{1}{2}$ assures that the index of the Hankel functions are imaginary.

The calculation

- In the case of the transversal modes $\lambda = \pm 1$ the transition amplitude takes the following form:

$$\mathcal{A}_{\nu\to Z\nu} = -\int d^4x\sqrt{-g}\left(\frac{e_0}{\sin(2\theta_W)}\right)(\overline{U}_{p',\sigma'})_\nu(x)\gamma^{\hat{i}}e^{j}_{\hat{i}}\left(\frac{1-\gamma^5}{2}\right)(U_{p\sigma})_\nu(x)f^{*}_{j\,\mathcal{P}}(x). \qquad (6)$$

- After we replace the solutions of the Dirac equation and Proca equation we obtain:

$$\mathcal{A}_{\nu\to Z\nu}(\lambda = \pm 1) = \frac{e_0}{\sin(2\theta_W)}\delta^3(\vec{\mathcal{P}}+\vec{p}\,'-\vec{p})\frac{1}{\sqrt{\pi}(2\pi)^{3/2}}\left(\frac{1}{2}-\sigma\right)\left(\frac{1}{2}-\sigma'\right)$$
$$\times\left\{B(t_c)\xi^{+}_{\sigma'}(\vec{p}\,')\vec{\sigma}\cdot\vec{\epsilon}^{\,*}(\vec{n}_{\mathcal{P}},\lambda=\pm 1)\xi_\sigma(\vec{p})\right\}. \qquad (7)$$

- The temporal integral depends on the Bessel K function and is expressed as follows:

$$B(t_c) = i\int_0^\infty dz\sqrt{z}\,e^{-i(p'-p)z}K_{-ik}(i\mathcal{P}z). \qquad (8)$$

- The above integral was obtained by considering the new variable $z = -t_c$ and the following relation:

$$H^{(1,2)}_\nu(z) = \mp\left(\frac{2i}{\pi}\right)e^{\mp i\pi\nu/2}K_\nu(\mp iz). \qquad (9)$$

- The temporal integrals defining the amplitude have the following general form:

$$\int_0^\infty dz z^{\mu-1}e^{-\alpha z}K_\nu(\beta z) = \frac{\sqrt{\pi}(2\beta)^\nu}{(\alpha+\beta)^{\mu+\nu}}\frac{\Gamma(\mu+\nu)\Gamma(\mu-\nu)}{\Gamma\left(\mu+\frac{1}{2}\right)}$$
$$\times\,{}_2F_1\left(\mu+\nu,\nu+\frac{1}{2};\mu+\frac{1}{2};\frac{\alpha-\beta}{\alpha+\beta}\right),$$
$$Re(\alpha+\beta) > 0\,, |Re(\mu)| > |Re(\nu)|. \qquad (10)$$

- After we apply (10) we obtain:

$$B_k(\mathcal{P},p,p') = \frac{i^{-1/2}(2\mathcal{P})^{-ik}}{(\mathcal{P}+p'-p)^{3/2-ik}}\Gamma\left(\frac{3}{2}-ik\right)\Gamma\left(\frac{3}{2}+ik\right)$$
$$\times\,{}_2F_1\left(\frac{3}{2}-ik,\frac{1}{2}-ik;2;\frac{p'-\mathcal{P}-p}{\mathcal{P}+p'-p}\right). \qquad (11)$$

- The probability density is calculated as the square modulus of the transition amplitude:

$$P_{\nu\to Z\nu}(\lambda = \pm 1) = \frac{e_0^2}{\sin^2(2\theta_W)}\delta^3(\vec{\mathcal{P}}+\vec{p}\,'-\vec{p})\frac{1}{(2\pi)^3}$$
$$\times\left(\frac{1}{2}-\sigma\right)^2\left(\frac{1}{2}-\sigma'\right)^2\left\{\frac{1}{2}\sum_\lambda|B_k(\mathcal{P},p,p')|^2|\xi^{+}_{\sigma'}(\vec{p}\,')\vec{\sigma}\cdot\vec{\epsilon}^{\,*}(\vec{n}_{\mathcal{P}},\lambda)\xi_\sigma(\vec{p})|^2\right\}. \qquad (12)$$

- The probability is nonzero in the case when $\sigma = \sigma' = -\frac{1}{2}$. If the helicity of the neutrino before emitting a Z boson is σ, then after emission the helicity of the neutrino will remain the same and do not change sign, i.e. $\sigma' = \sigma$.

Graphical results

- The graphs that follows shows the behaviour of the functions that define the probability in terms of ratio $\frac{M_Z}{\omega}$.

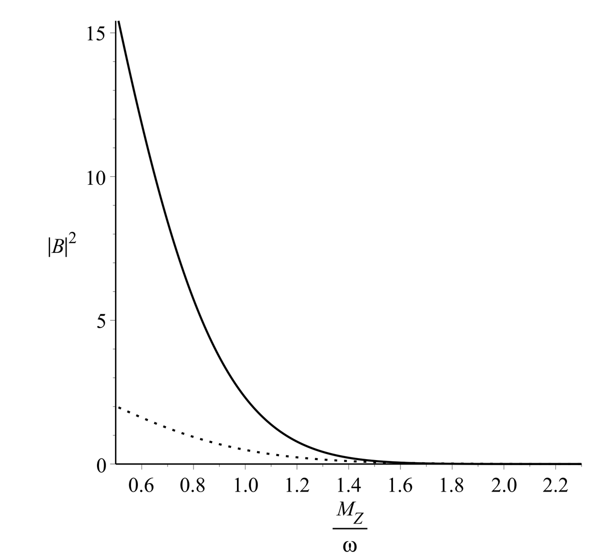

Fig. 1: $|B_k|^2$ as a function of parameter M_Z/ω; *solid line*: $p = 0.3, p' = 0.6, \mathcal{P} = 0.1$ and *point line*: $p = 0.2, p' = 0.6, \mathcal{P} = 0.3$

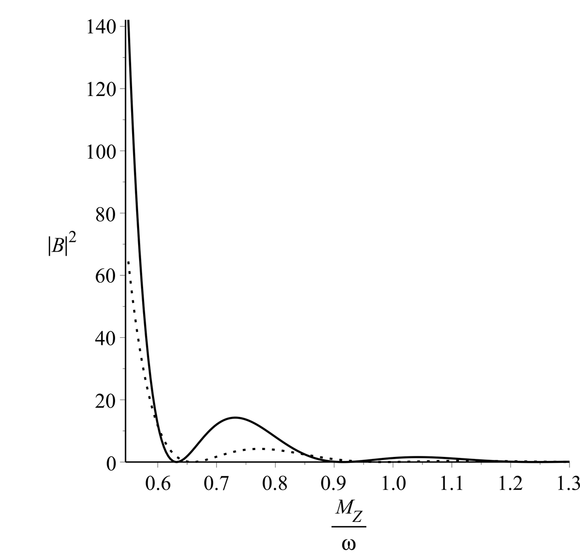

Fig. 2: $|B_k|^2$ as a function of parameter M_Z/ω;*solid line*: $p = 0.3, p' = 0.6, \mathcal{P} = 0.0001$; *point line*: $p = 0.2, p' = 0.6, \mathcal{P} = 0.0003$.

- The probability densities increase as the Z boson momenta $\mathcal{P}$ take small values resulting that, there are favoured the processes where emission of soft Z bosons occurs.

Conclusions

- We studied the process $\nu \to \nu + Z$ using the theory of electroweak interactions on de Sitter expanding universe.
- We obtained that this process of Z bosons production is possible in the large expansion conditions from early universe.
- This mechanism for the generation of massive Z bosons in emission process could explain the abundance of massive bosons in early universe.