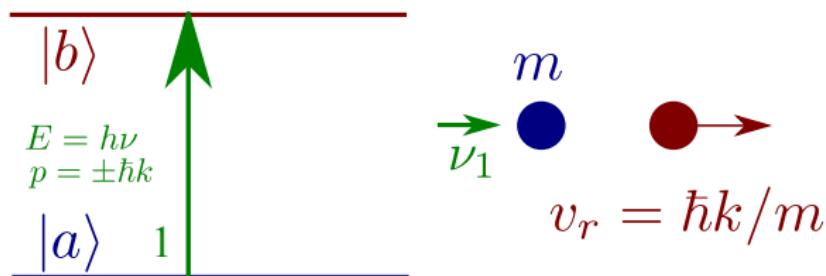




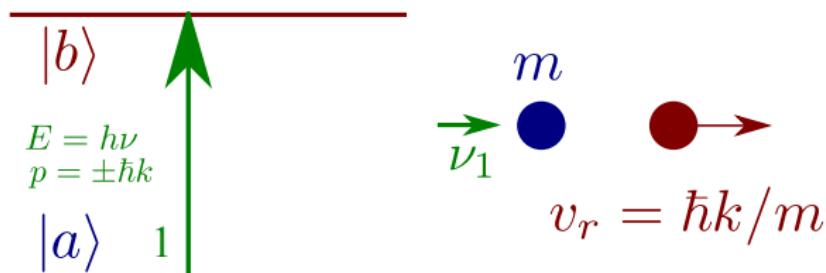
# Towards a determination of the fine-structure constant below 0.1 parts-per-billion : test of the Standard Model using the electron magnetic moment.

Léo Morel, Zhibin Yao, Pierre Cladé, Saïda Guellati-Khelifa





- $v_r = \frac{h}{m_X} \frac{1}{\lambda}$
- $v_r = 6 \text{ mm/s}$  for  $Rb$  atoms (and  $\lambda \simeq 780 \text{ nm}$ )



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The goal of our experiment is to measure precisely  $v_r$

- The ratio  $h/m_X$  and the fine structure constant
- Determination of the recoil using an atom interferometer : sensitivity and accuracy
- Test of the Standard Model using the electron magnetic moment

- The fine-structure constant :

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \simeq \frac{1}{137}$$

- The Rydberg constant  $R_\infty$

$$hcR_\infty = \frac{1}{2}m_e\alpha^2c^2$$

$$\alpha^2 = \frac{2}{c}R_\infty \frac{A_r(Rb)}{A_r(e)} \frac{h}{m_{Rb}}$$

Hydrogen spectroscopy allows to measure  $R_\infty$  with a relative uncertainty of  $2 \times 10^{-12}$

- Determination of relative atomic masses :  $A_r(X) = m_X/m_u$ 
  - Cyclotron frequencies  $A_r(Rb)$  at  $7.5 \times 10^{-11}$
  - Magnetic moment of a single electron bound to a carbon nucleus  $A_r(e)$  at  $2.9 \times 10^{-11}$

The limiting factor is the ratio  $\frac{h}{m_{Rb}}$

# The International System of Units

## ■ The new SI

Based of fundamental constants:

$\Delta\nu_{\text{Cs}}$ ,  $c$ ,  $h$ ,  $e$ ,  $N_A$ ,  $K_{cd}$

- Effective since 20 May 2019

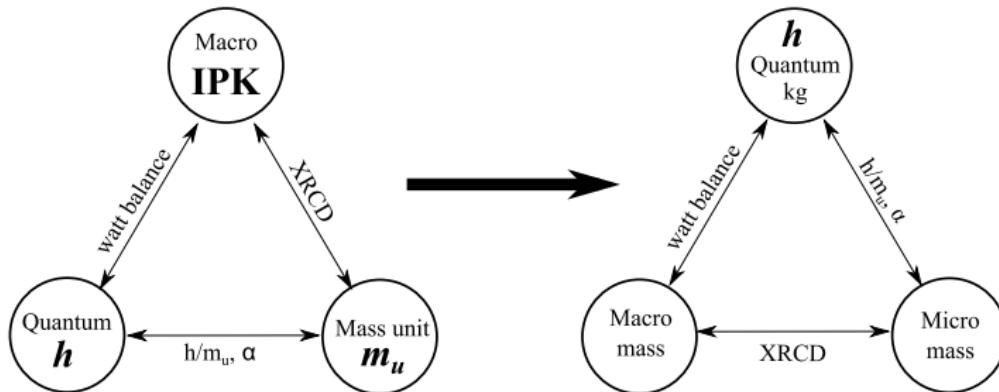


## ■ Two motivations :

- Drifts of the mass of the International Prototype of the Kilogram
- Electrical units were already based on quantum hall effect  
 $R_K = h/e^2$  and Josephson effect ( $K_J = 2e/h$ )

## ■ Both $h/m$ and $\alpha$ are important fundamental constants, especially in the new/current SI units.

# The ratio $h/m_u$ and the new SI



- *Mise en pratique* of the kilogramme at the atomic scale
- *Mise en pratique* at macroscopic scale using a silicon sphere
- The mole is now independant.  $N_A$  has a fixed value.

$$M(^{12}\text{C}) = N_A \times m(^{12}\text{C}) = \frac{12N_A h}{h/m_u} = 12,0000000xxx \text{ g/mol} \quad (1)$$

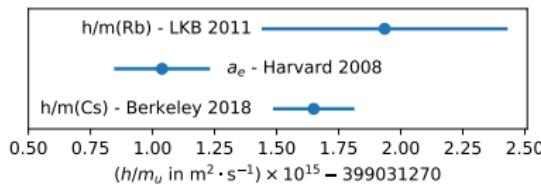
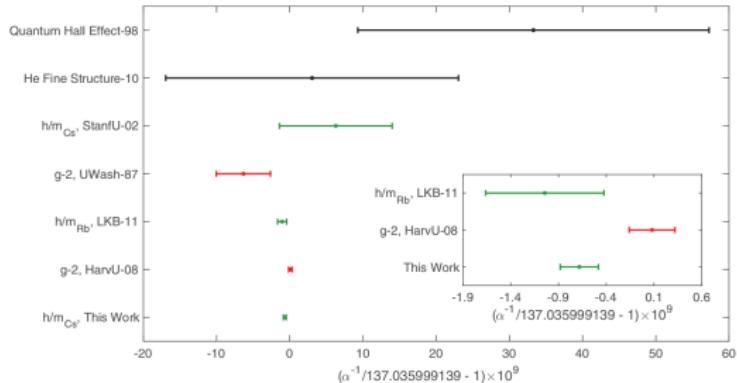
$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \quad (2)$$

- In the new SI, the constants  $c$ ,  $\hbar$  and  $e$  are fixed.
- The constants  $\epsilon_0$  et  $\mu_0$  are measured.
- $\frac{\mu_0}{4\pi \times 10^{-7}} = 1,00000000xxx \text{ m} \cdot \text{kg} \cdot \text{s}^{-2} \cdot \text{A}^{-2}$
- Exact relationship between  $\epsilon_0$ ,  $\mu_0$  and  $\alpha$  (quantum electrodynamics)

# Most precise determinations

$$\alpha^2 = \frac{2}{c} R_\infty \frac{A_r(Rb)}{A_r(e)} \frac{h}{m_{Cs}}$$

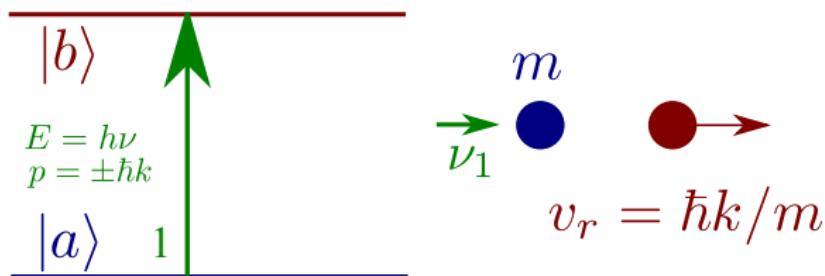
R. Parker et al., Science  
360, 191–195 (2018)



$$\frac{h}{m_u} = \frac{h}{m_X} A_r(X) = \frac{\alpha^2 c A_r(e)}{2R_\infty}$$

Anomalous magnetic moment of electron  $a_e$  (function of  $\alpha$ )

# Measurement of the recoil velocity

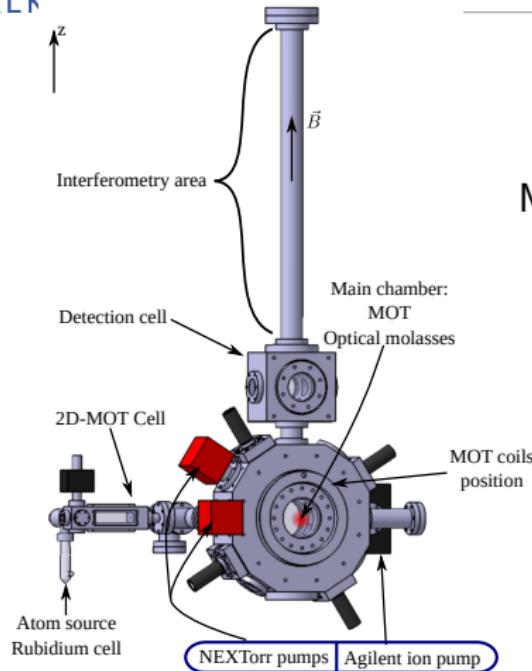


- Differential velocity sensor  
⇒ Atom interferometer based on Raman transitions

Sensitivity:  $\sigma_v$

- Transfer a large number  $N$  of photon momenta  
⇒ Bloch Oscillations technique

$$\sigma_{v_r} = \frac{\sigma_v}{N}$$

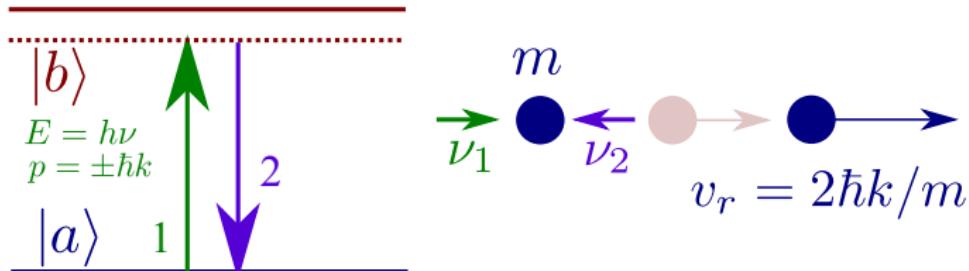


# Experimental setup

## Main characteristics:

- $T \simeq 4 \mu\text{K}$
- $r \simeq 600 \mu\text{m}$
- $\sim 10^8$  atoms
- Vacuum  $\sim 10^{-10} \text{ mbar}$

# Measurement of the recoil velocity

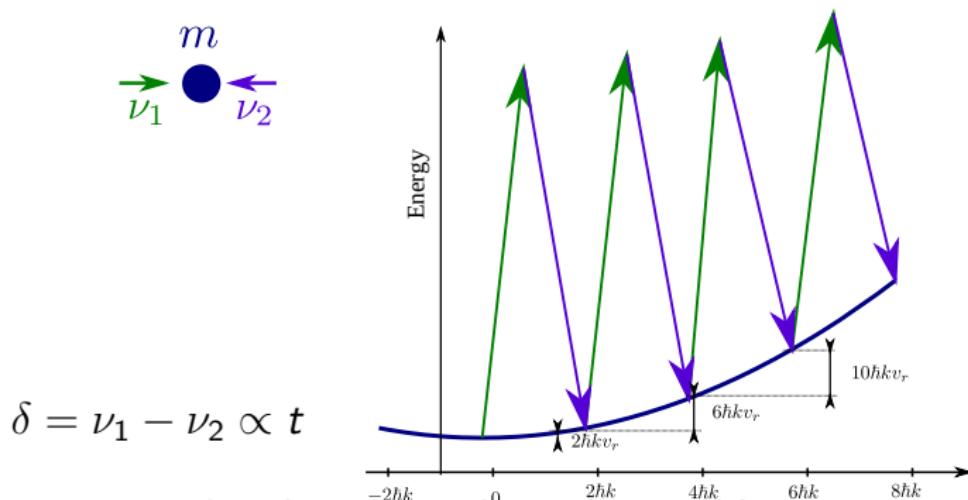


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  - Sensitivity:  $\sigma_v$
- Transfer a large number  $N$  of photon momenta
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# Coherent acceleration of atoms

Succession of stimulated Raman transitions  $\pi$  pulses in the same hyperfine level.

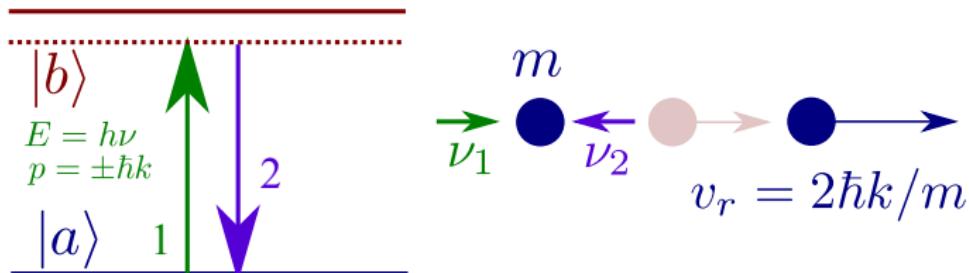


The atoms are placed in an accelerated optical lattice: in its frame, they are submitted to an inertial force

→ Bloch oscillations in a periodic potential

1000 oscillations → 6 m/s

# Measurement of the recoil velocity



- Differential velocity sensor  
⇒ Atom interferometer based on Raman transitions

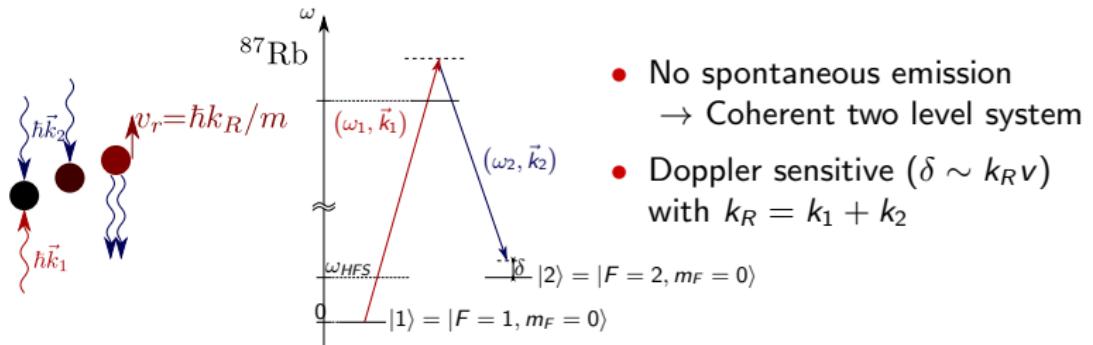
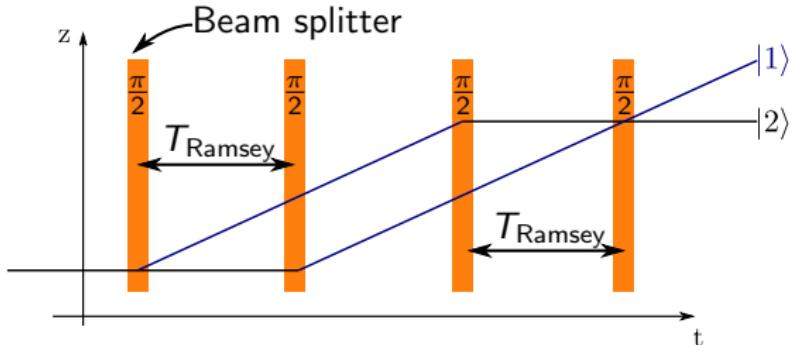
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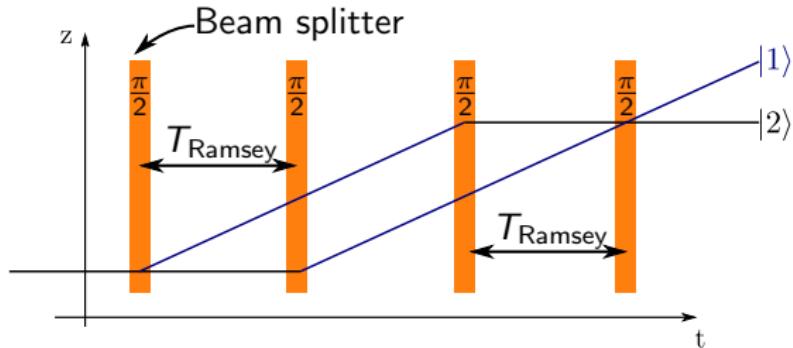
# Differential velocity sensor

Interferometer based on counterpropagating Raman transitions



# Differential velocity sensor

Interferometer based on counterpropagating Raman transitions

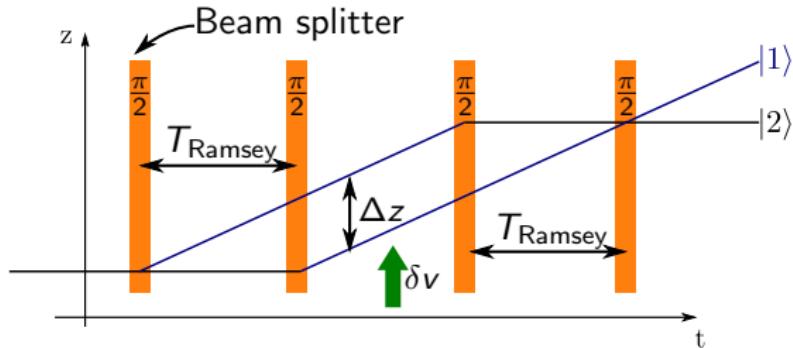


Phase difference between the two arms  $\Phi$   
Probability to find an atom in  $|2\rangle$ :

$$P_2 = \frac{1 + \cos(\Phi)}{2}$$

# Differential velocity sensor

Interferometer based on counterpropagating Raman transitions



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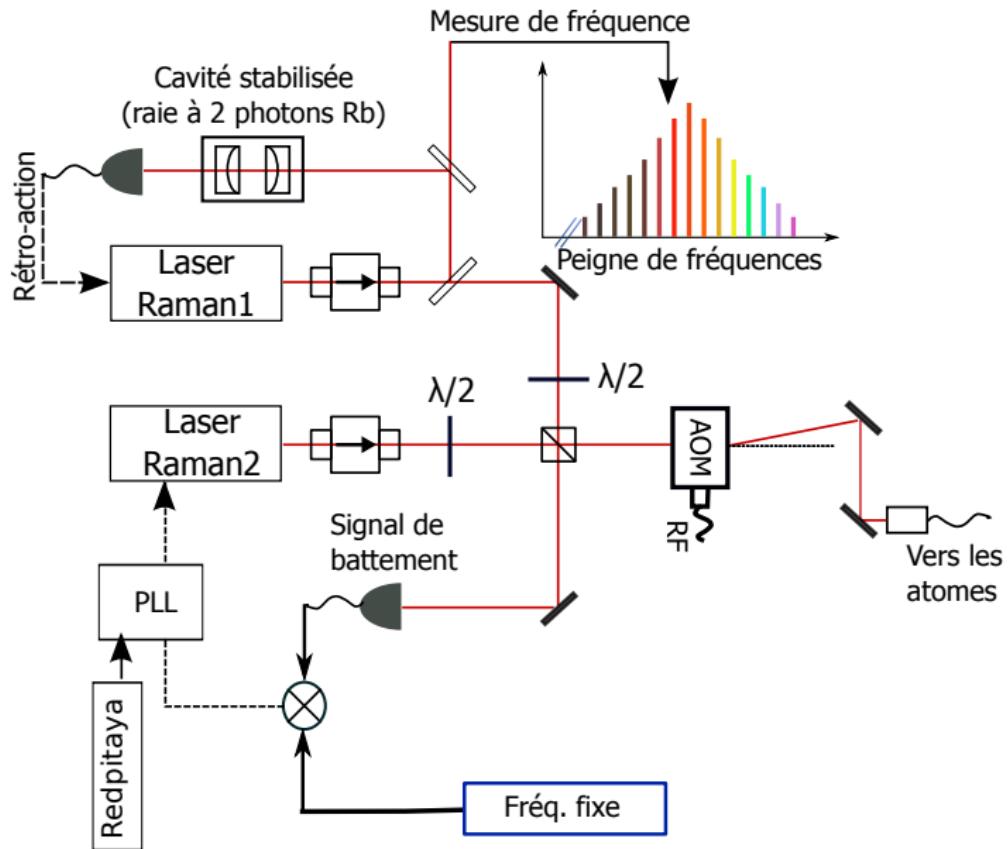
Velocity transfer  $\delta v$ :

$$\Phi = T_{\text{Ramsey}} k_R \delta v = \frac{\Delta z \cdot m \delta v}{\hbar}$$

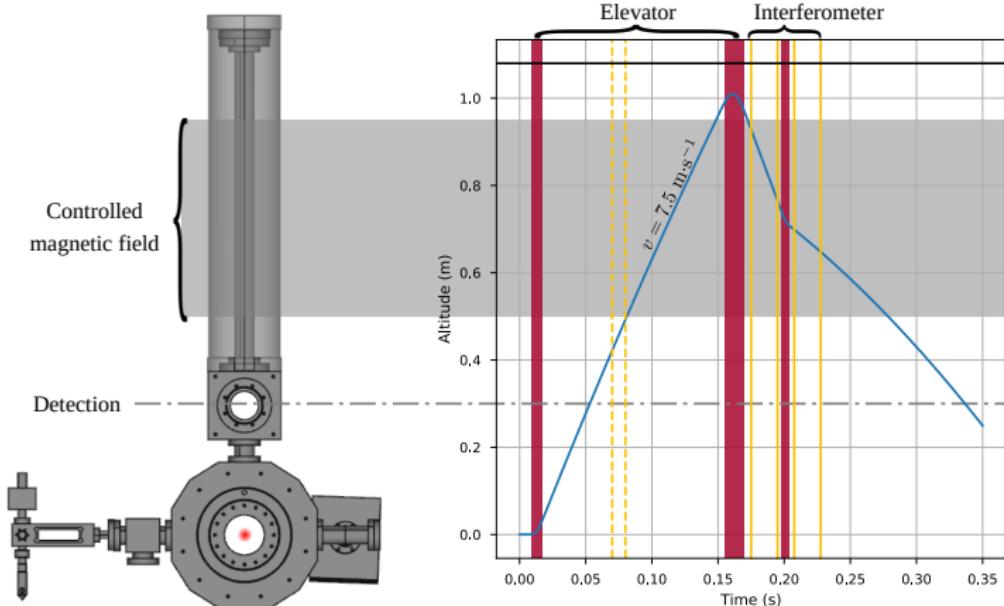
Sensitivity :  $\Delta z = 250 \mu\text{m} \rightarrow 3 \mu\text{m/s/rad}$

Inertial sensor : Gravimeter, gyrometer, ...

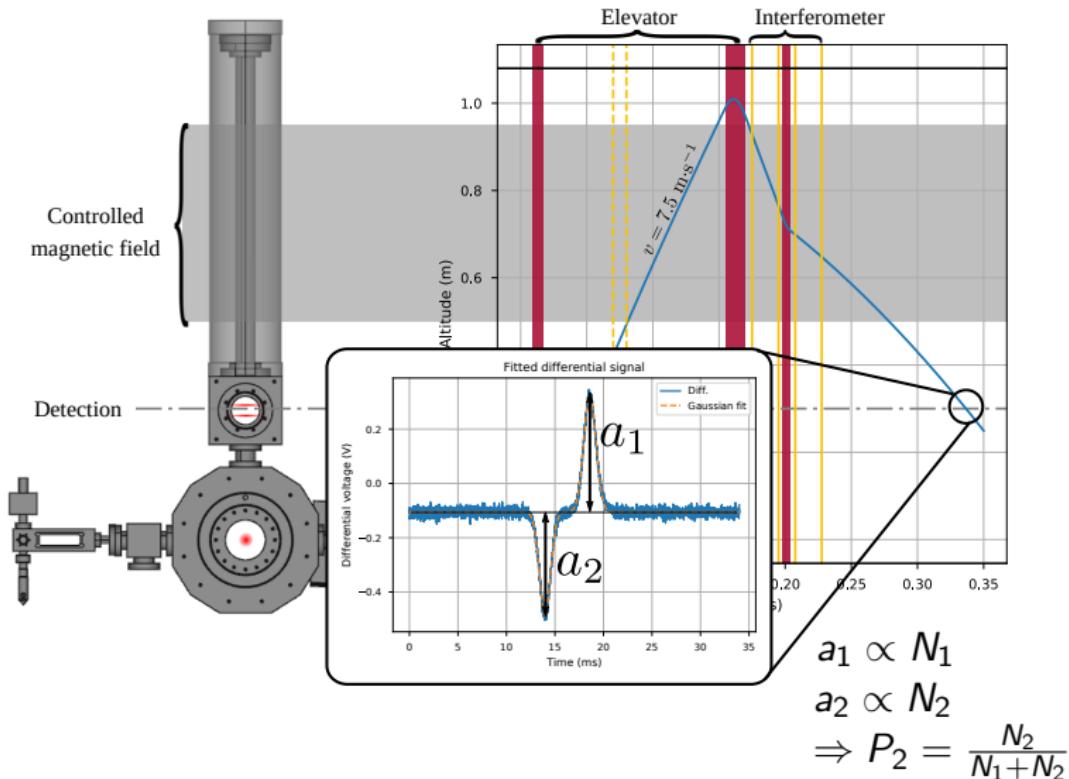
## Raman laser



# Atom trajectories

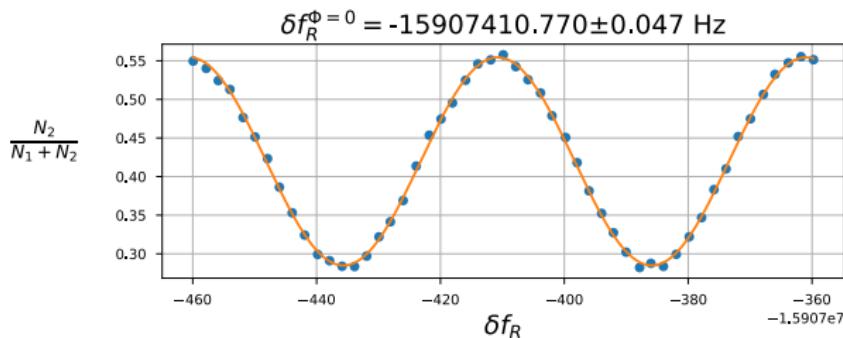


# Atom trajectories



# Data acquisition

- $\sim 1$  point per second ( $\sim 1$  minute)
- $T_{\text{Ramsey}} = 20 \text{ ms}$ , Number of Bloch Oscillations  $N_B = 500$

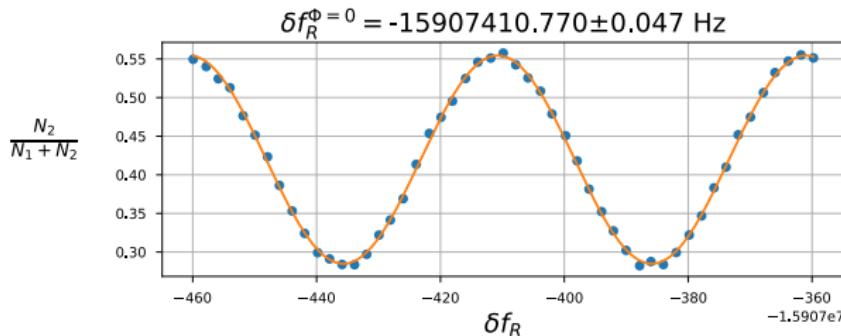


$$\Phi = T_{\text{Ramsey}} \left( k_R 2 N_B \frac{\hbar}{m} k_B - 2\pi \delta f_R \right)$$

- One photon momentum:  $\sim 15 \text{ kHz} \rightarrow 1000$  photon momenta:  $\sim 15 \text{ MHz}$
- $\sigma_v = 0.047 \text{ Hz} \sim 3 \cdot 10^{-6} v_r \sim 20 \text{ nm} \cdot \text{s}^{-1} \rightarrow 3 \cdot 10^{-9} \text{ on } h/m$

# Data acquisition

- $\sim 1$  point per second ( $\sim 1$  minute)
- $T_{\text{Ramsey}} = 20$  ms. Number of Bloch Oscillations  $N_D = 500$

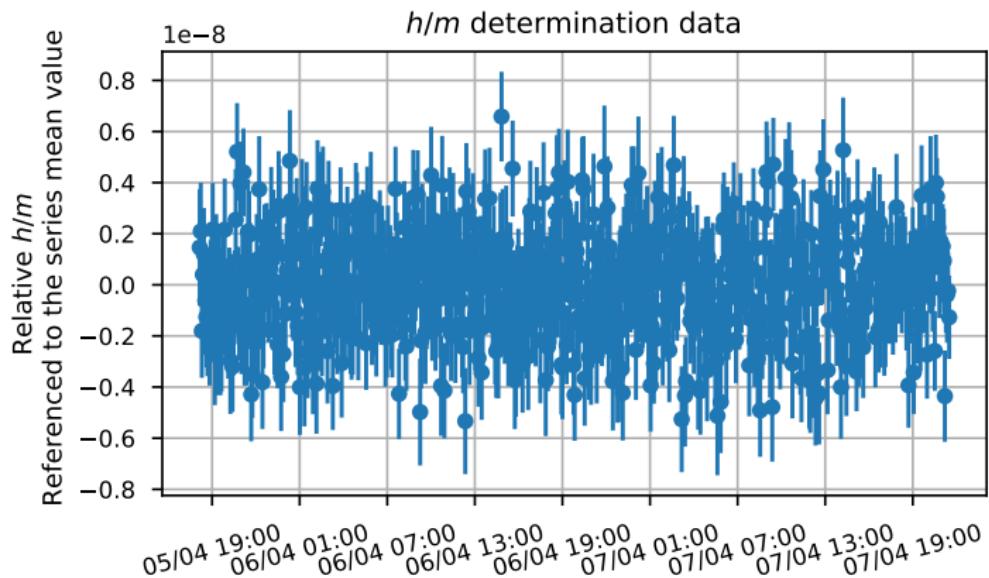


$$\Phi = T_{\text{Ramsey}} \left( k_R \left( 2N_B \frac{\hbar}{m} k_B - gT \right) - 2\pi\delta f_R \right) + \phi_{LS}$$

- One photon momentum:  $\sim 15$  kHz  $\rightarrow$  1000 photon momenta:  $\sim 15$  MHz
- $\sigma_v = 0.047 \text{ Hz} \sim 3 \cdot 10^{-6} v_r \sim 20 \text{ nm} \cdot \text{s}^{-1} \rightarrow 3 \cdot 10^{-9}$  on  $h/m$
- Contribution of  $g$
- Contribution of light shifts  $\phi_{LS}$

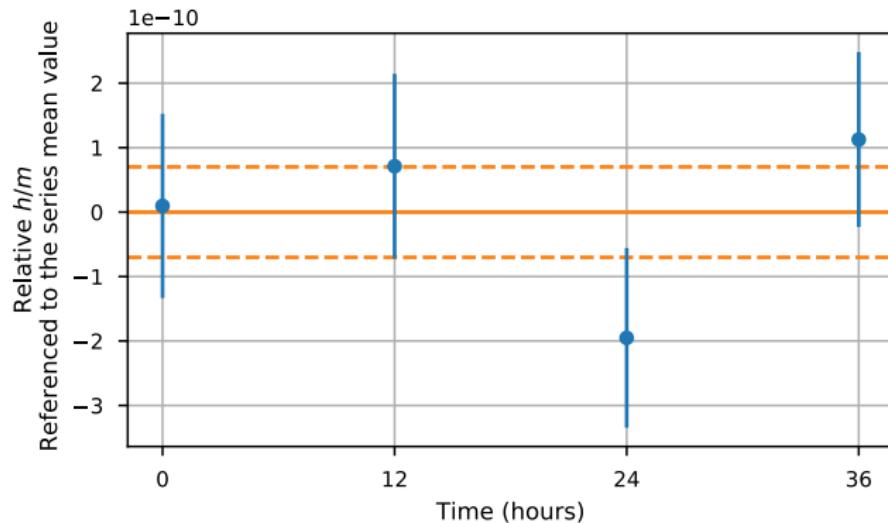
## 48 hours integration

Stable and reliable device  $\Rightarrow$  Long measurement periods



From Friday to ..... Sunday

# Estimation of statistical uncertainty

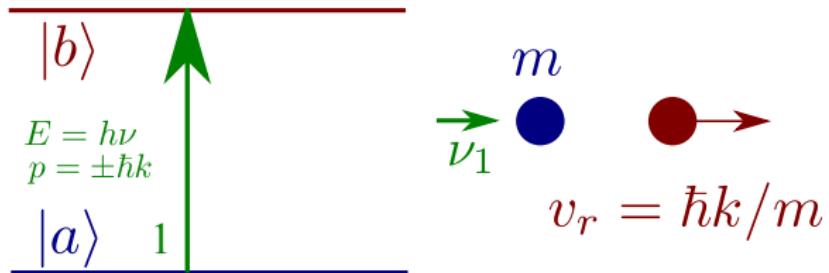


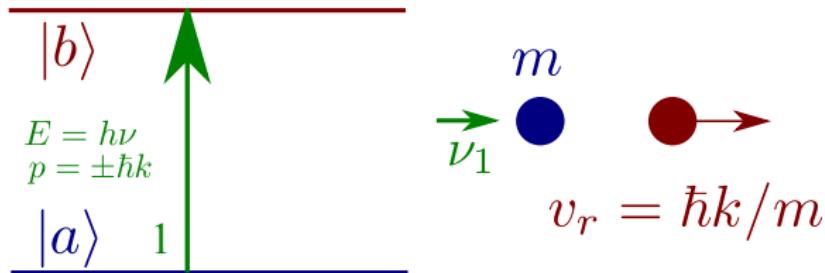
48h integration:  $8.5 \cdot 10^{-11}$  on  $\frac{h}{m}$

→  $4.3 \cdot 10^{-11}$  on  $\alpha$

# Error budget

Source	Corr. [ $10^{-11}$ ]	Uncert. [ $10^{-11}$ ]
Gravity gradient	-0.6	0.1
Beams alignment	0.5	0.5
Coriolis acceleration		1.2
Lasers frequencies		0.3
Wave front curvature	0.6	0.3
Wave front distortion	3.9	1.9
Gouy phase	108.2	5.4
Residual Raman phase shift	2.3	2.3
Index of refraction	0	< 0.1
Internal interaction	0	< 0.1
Light shift (two-photon transition)	-11.0	2.3
Second order Zeeman effect		0.1
Phase shifts in Raman phase lock loop	-39.8	0.4
Global systematic effects	64.2	6.8
Statistical uncertainty		2.4
Relative mass of $^{87}\text{Rb}$		3.5
Relative mass of the electron		1.5
Rydberg constant		0.1
Total :		8.1





What is the recoil velocity ?

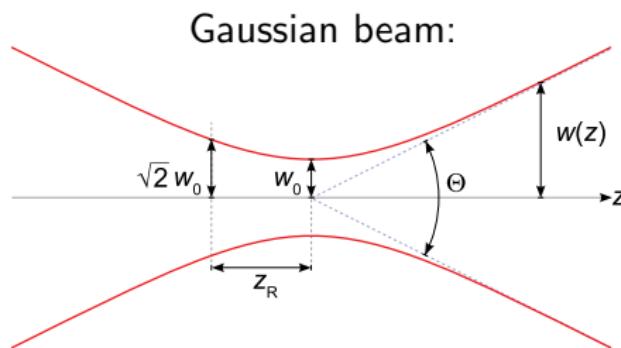
- Poynting vector (density of momentum)
- Photon (plane wave) :  $p = \hbar k = h\nu/c$
- General case:  $p_z = \hbar \frac{d\phi}{dz}$

# Atom recoil in a gaussian beam

Electric field  $E(\vec{r}, t) = A(\vec{r}, t)e^{i\phi(\vec{r})} \rightarrow \vec{k}_{\text{eff}} = \vec{\nabla}\phi$

Plane wave model:  $k = \nu/c$

Correction:  $k_{\text{eff},z} = k + \delta k$



Correction to the effective wavevector:

$$\frac{\delta k}{k} = -\frac{2}{k^2 w^2(z)} \left( 1 - \frac{\langle r^2 \rangle}{w^2(z)} \right) - \frac{\langle r^2 \rangle}{2R^2(z)}$$

Related to the dispersion of wavevectors  $\sim -\frac{\Theta^2}{2}$

# Effect of distortions

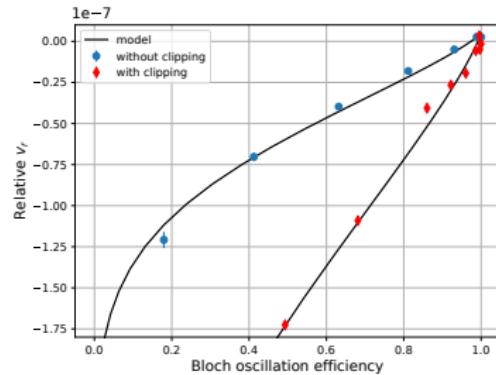
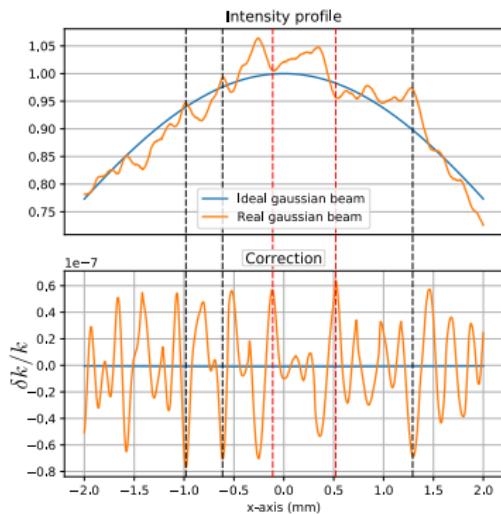
Momentum correction due to transverse phase and amplitude fluctuations:

$$\delta k_{\text{rel}} = \frac{\delta k}{k} = -\frac{1}{2} \left\| \frac{\vec{\nabla}_{\perp} \phi}{k} \right\|^2 + \frac{1}{2k^2} \frac{\Delta_{\perp} A}{A} \quad (I = A^2)$$

During the Bloch Oscillation pulse, the survival probability  $P(I)$  is governed by Landau-Zener Losses:

$$\langle \delta k_{\text{rel}} \rangle = \langle \delta k_{\text{rel}} P(I) \rangle / \langle P(I) \rangle$$

⇒ Effect on the measured recoil velocity

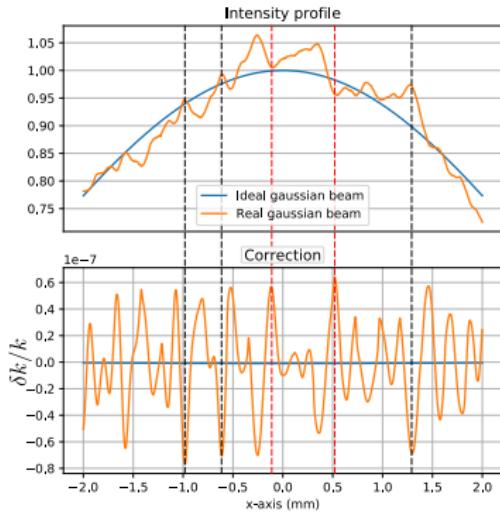


# Effect of distortions

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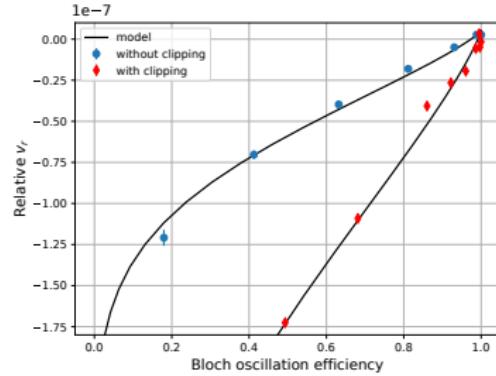
Correlation between  $I$  and  $\delta k$



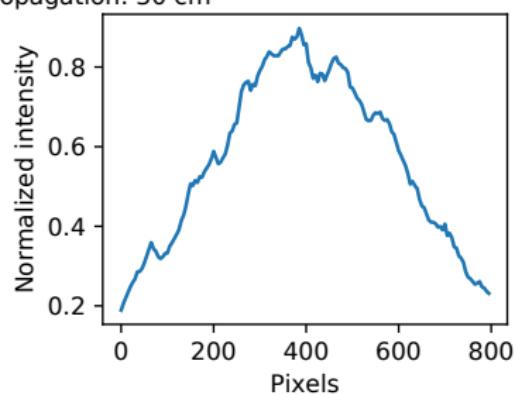
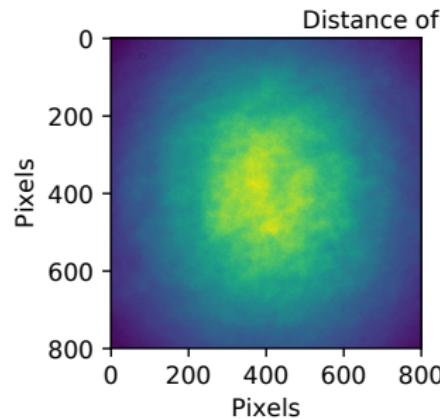
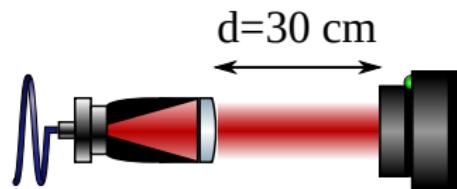
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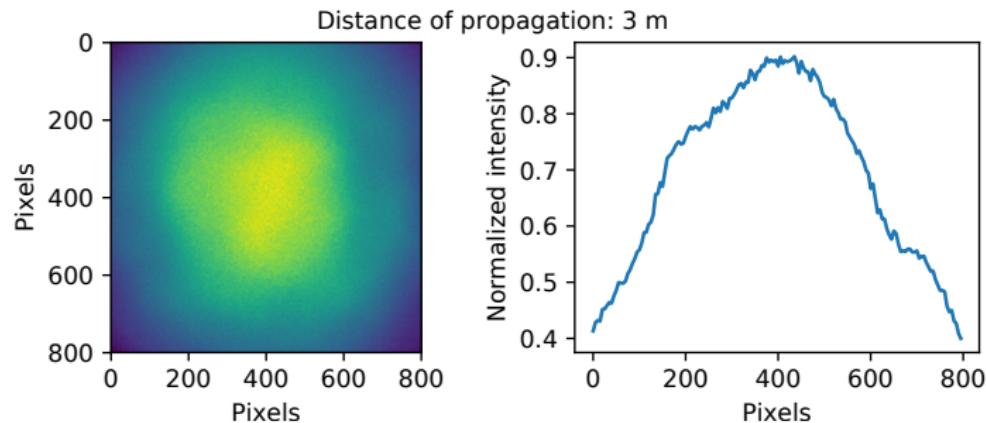
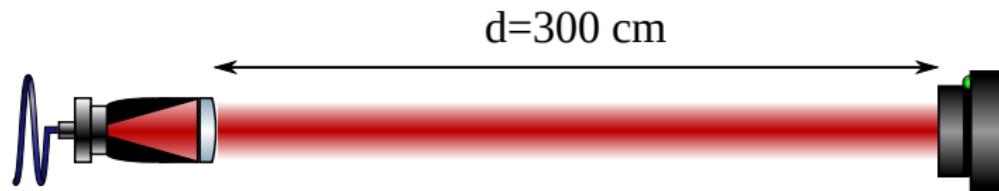
⇒ Effect on the measured recoil velocity



# Reduction of short scale distortions



# Reduction of short scale distortions



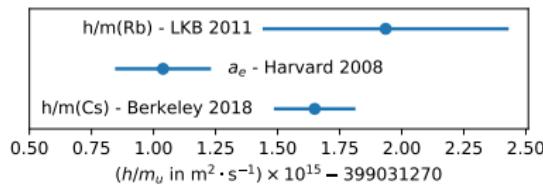
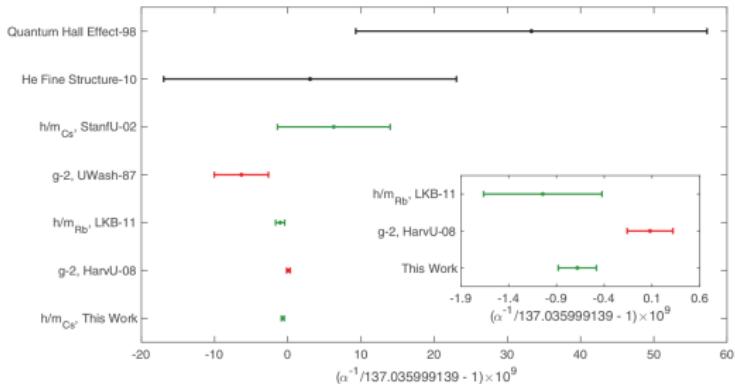
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# Most precise determinations

$$\alpha^2 = \frac{2}{c} R_\infty \frac{A_r(Rb)}{A_r(e)} \frac{h}{m_{Cs}}$$

R. Parker et al., Science  
360, 191–195 (2018)



$$\frac{h}{m_u} = \frac{h}{m_X} A_r(X) = \frac{\alpha^2 c A_r(e)}{2R_\infty}$$

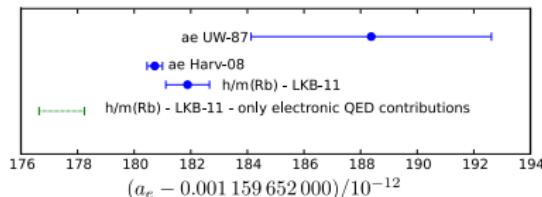
Anomalous magnetic moment of electron  $a_e$  (function of  $\alpha$ )

$$a_e = \frac{g_e - 2}{2}$$

- QED calculation

$$a_e (\text{QED}) = \sum_{n=1}^{\infty} A^{(2n)} \left( \frac{\alpha}{2\pi} \right)^n + \sum_{n=1}^{\infty} A_{\mu,\tau}^{(2n)} \left( \frac{m_e}{m_\mu}, \frac{m_e}{m_\tau} \right) \left( \frac{\alpha}{2\pi} \right)^n$$

- Measurement:  $a_e(\text{exp}) = 0.00115965218073(28)$  (Harv-08)
- Calculation of the  $A^{10}$  term : 12672 Feynman diagrams (Kinoshita *et al.*)
- LKB 2011 : observation of the muonic contribution



- Other contributions

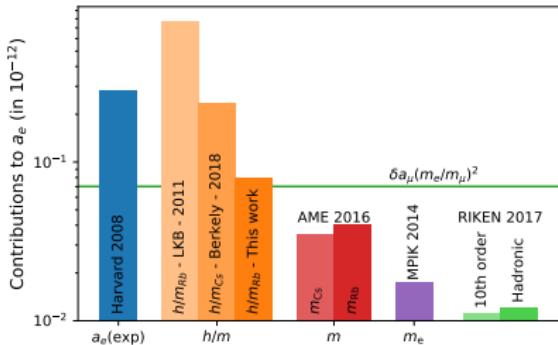
$$a_e (\text{theo}) = a_e (\text{QED}) + a_e (\text{Hadron}) + a_e (\text{Weak})$$

# LKB Testing the muon $a_\mu$ discrepancy in the electron sector

$$\delta a_\mu = a_\mu(\text{exp}) - a_\mu(\text{theo}) = 2.50(0.86) \cdot 10^{-9} \ (3.4\sigma)$$

Naive scaling:  $\left| \frac{\delta a_e}{\delta a_\mu} \right| = \left( \frac{m_e}{m_\mu} \right)^2 \sim 2.3 \cdot 10^{-5}$

$$\sigma_{a_e} = 2.5 \cdot 10^{-9} \cdot \left( \frac{m_e}{m_\mu} \right)^2 \sim 5.8 \cdot 10^{-14} \ (\text{0.05 ppb})$$



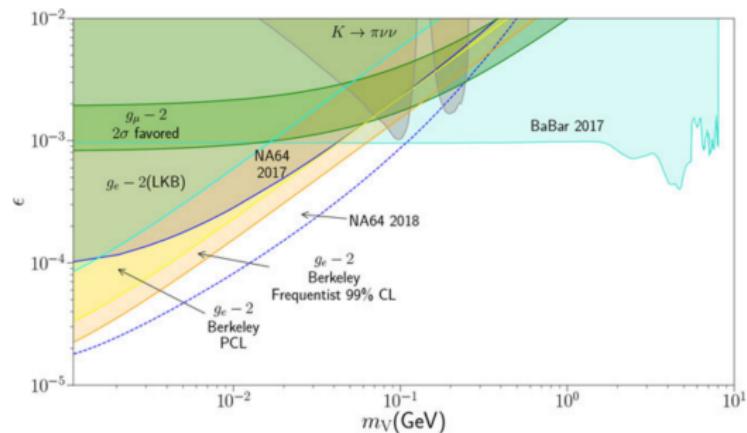
G. W. Bennett *et al.*, PRL **92**, 161802 (2004)  
F. Terranova and G. M. Tino, PRA **89**, 052118 (2014)

# Dark photon or dark bosons

"Dark particle" of mass  $m_V$  and coupling  $\epsilon$  with electrons.

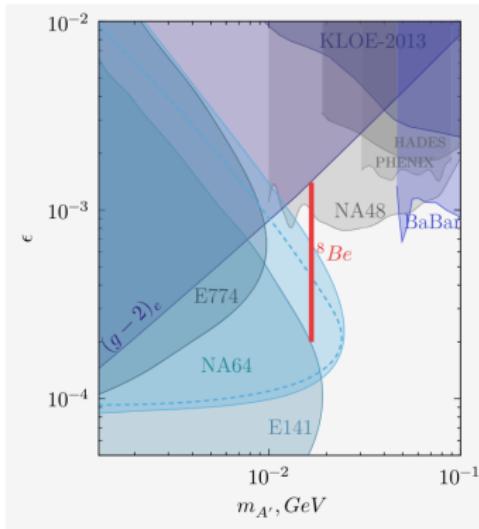
$$\delta a_e = \frac{\alpha}{2\pi} \times \epsilon^2 \int_0^1 dz \frac{2m_e^2 z(1-z)^2}{m_e^2(1-z)^2 + m_V^2 z} \simeq \frac{\alpha \epsilon^2}{3\pi} \frac{m_e^2}{m_V^2} \text{ for } m_V \gg m_e$$

Dark photon :



# The hypothetical X(16.7) boson

- Anomaly in the angular distribution of  $e^+e^-$  produced in  ${}^8Be$  nuclear transition
- Hypothetical protophobic gauge boson X with a mass of 16.7 MeV followed by a decay through  $X \rightarrow e^+e^-$



- NA64 : electron beam dump experiment
- $g_e$  more precise than  $g_\mu$ .
- Comparison limited by  $a_e(\text{exp})$  and not  $a_e(\text{theo})$

PRD 101, 071101(R) (2020)

## Permanent staff:

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- Saïda Guellati-Khelifa
- François Biraben (emeritus)

## PhD students (since 2000):

- L. Morel
- Z. Yao
- M. Andia
- R. Jannin
- C. Courvoisier
- R. Bouchendira
- M. Cadoret
- P. Cladé
- R. Battesti

