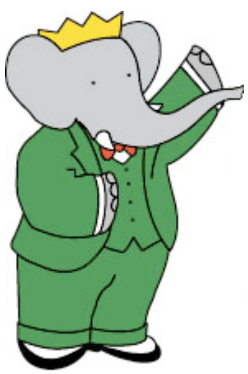
Status of CKM angle measurements,

a report from BaBar and BELLE

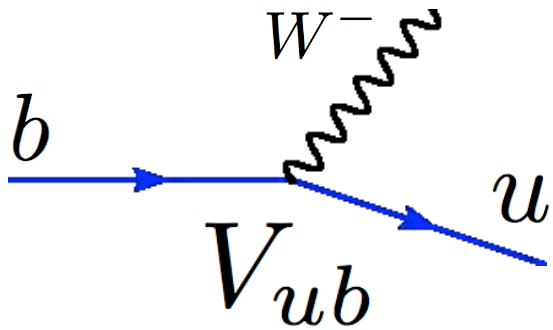
Owen Long, U. C. Riverside

Moriond EW, 2010



CP violation and the CKM matrix

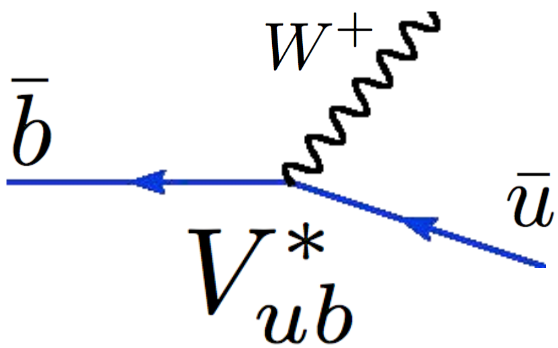
quark decay



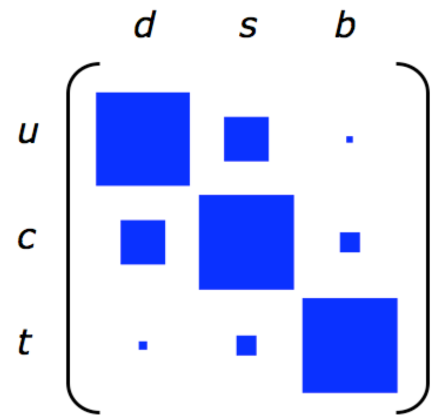
Cabibbo Kobayashi Maskawa matrix

$$V_{ij} = \begin{array}{c} \\ u \\ c \\ t \end{array} \begin{array}{c} \begin{array}{ccc} d & s & b \end{array} \\ \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} \end{array}$$

anti-quark decay



Relative magnitudes



The CP-violating phases

$$\begin{array}{c} \\ u \\ c \\ t \end{array} \begin{array}{c} \begin{array}{ccc} d & s & b \end{array} \\ \begin{pmatrix} 1 & 1 & e^{-i\gamma} \\ 1 & 1 & 1 \\ e^{-i\beta} & 1 & 1 \end{pmatrix} \end{array}$$

Non-trivial CP violating phases. Amplitude interference different for quark vs anti-quark decay. Observable rate differences.

The Unitarity Triangle

$$V_{ij} = \begin{array}{c} \\ u \\ c \\ t \end{array} \begin{array}{c} \begin{array}{ccc} d & s & b \end{array} \\ \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} \end{array}$$

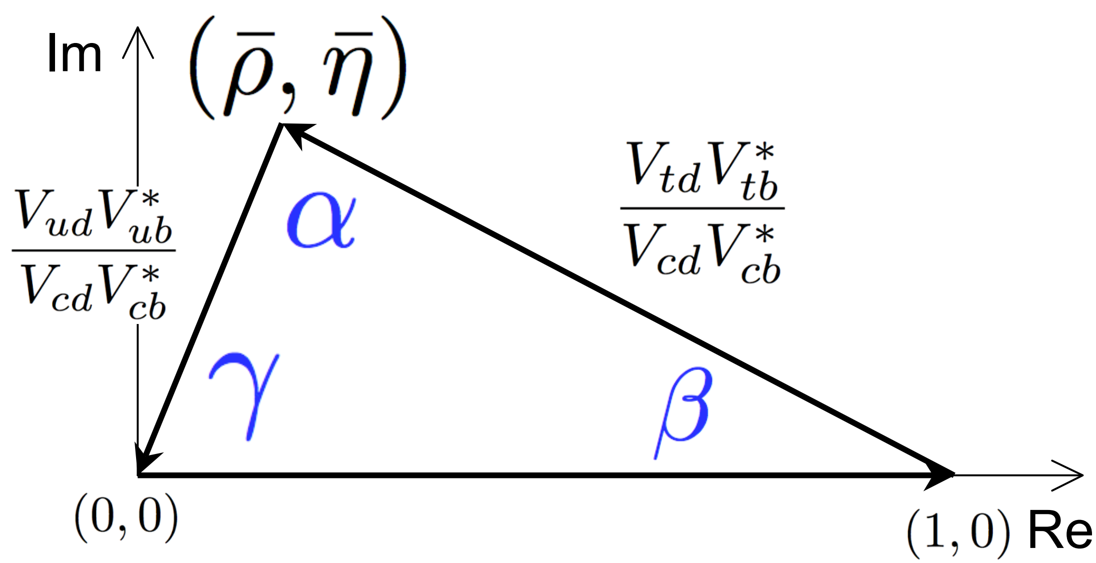
Unitarity condition from 1st and 3rd columns.

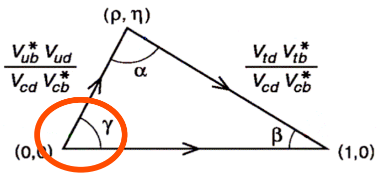
$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* \ = \ 0$$

Overconstrain apex coordinates.

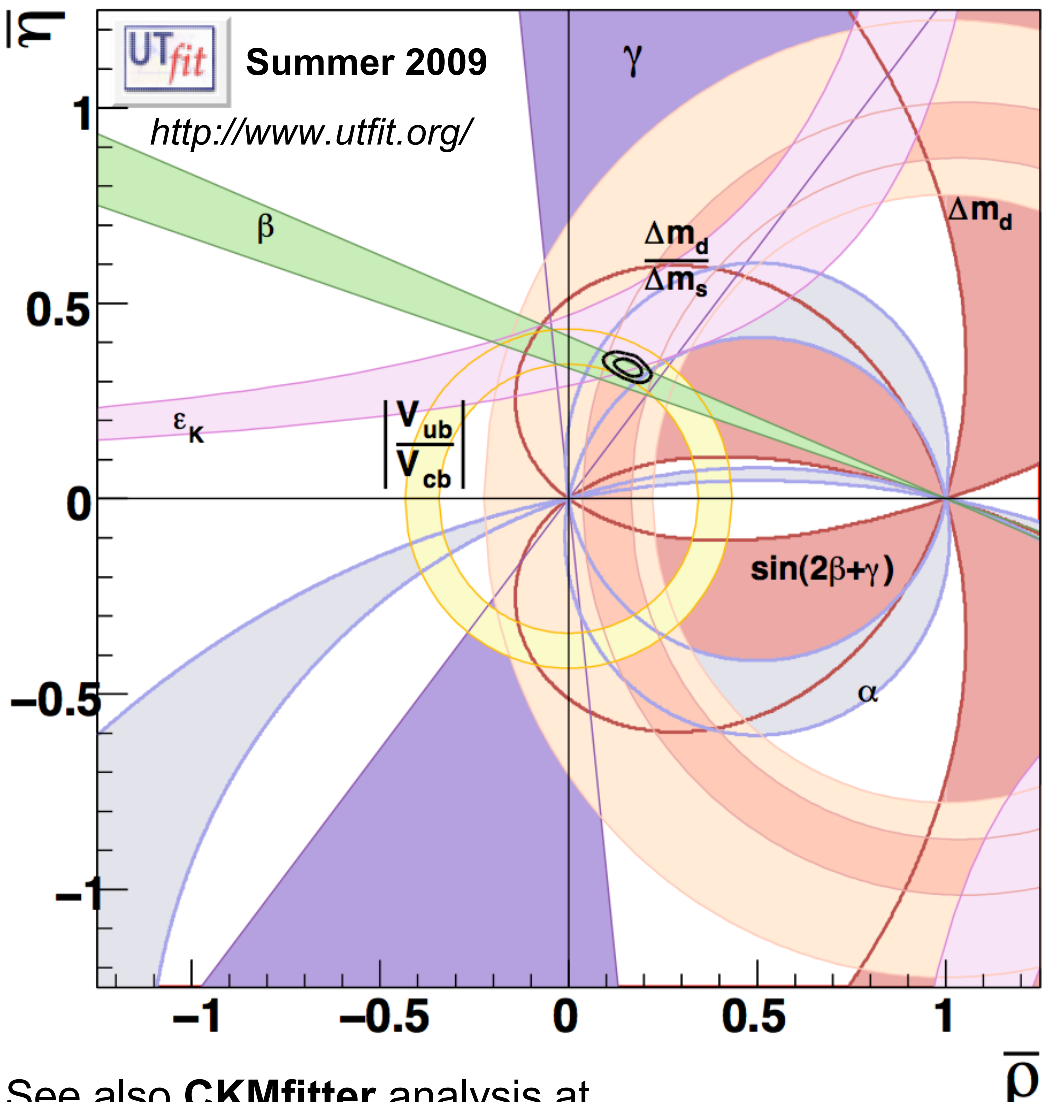
CP violation measurements give angles.

Semileptonic decay rates (and other methods) give sides.





The big picture

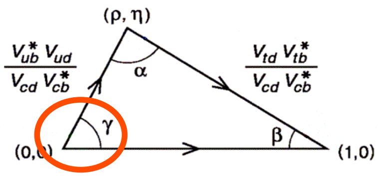


See also **CKMfitter** analysis at
http://ckmfitter.in2p3.fr/

- Apex of the triangle over constrained.
- Very impressive consistency.

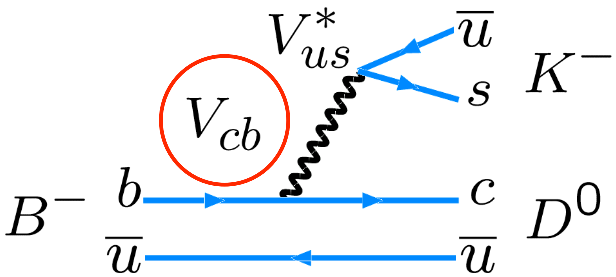
Kobayashi and Maskawa awarded half of 2008 N.P.

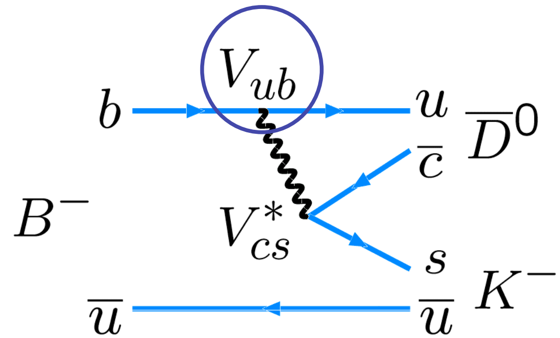
- Still have some work to do on γ (a.k.a ϕ_3).
- The γ constraint comes from *tree-level* $B^{\pm} \to D^{(*)} K^{(*)\pm}$ decays.
  - Insensitive to most types of NP.



Gamma: how it's done

Measure interference of these decay amplitudes





$V_{ub} \propto e^{-i\gamma}$

Hadronic parameters

$$r_b = \left| \frac{A(b \to u)}{A(b \to c)} \right|$$

Strong (CP conserving) phase difference δ between $A(b \to u)$ and $A(b \to c)$

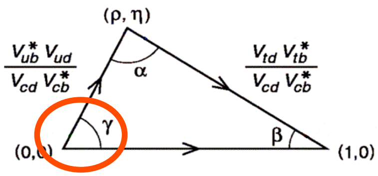
Determined experimentally.

Use final state accessible from *both* D^0 *and* $\bar{D}^0$.

Many ways to do this (GLW, ADS, GGSZ, …).

Complementary methods share the same hadronic parameters (r_b, δ) and gamma.

Difficult because BFs small (not many events) and r_b is small (small interference).

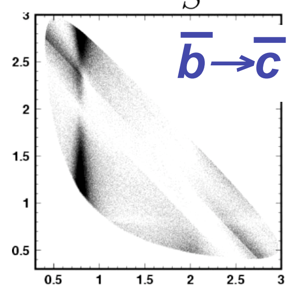


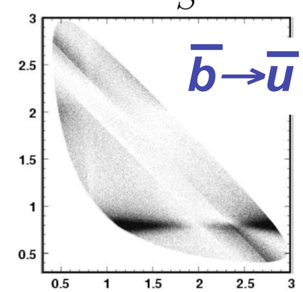
Gamma from $B^{\pm} \to DK^{\pm}$

*The D Dalitz approach**

decay amplitude

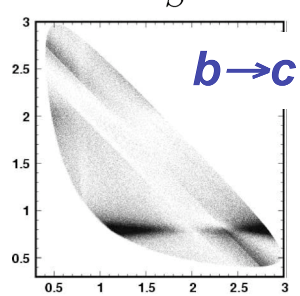
$$A(B^+) = \left(\overline{D}^0 \to K^0_S \pi^+ \pi^- \quad + r_b e^{+i\gamma + i\delta_b} \quad D^0 \to K^0_S \pi^+ \pi^- \right)$$

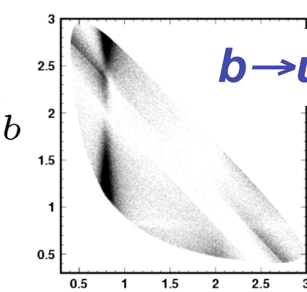




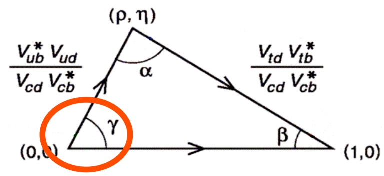
Relative weak phase (gamma) changes sign

$$A(B^-) = \left(D^0 \to K^0_S \pi^+ \pi^- \quad + r_b e^{-i\gamma + i\delta_b} \quad \overline{D}^0 \to K^0_S \pi^+ \pi^- \right)$$





* *A. Giri, Y. Grossman, A. Soffer, J. Zupan PRD 68, 054018 (2003); A. Bondar, unpublished.*

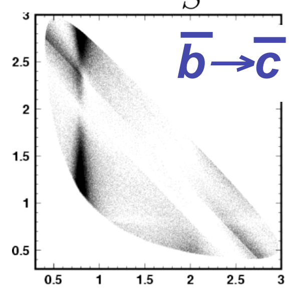


Gamma from $B^{\pm} \rightarrow DK^{\pm}$

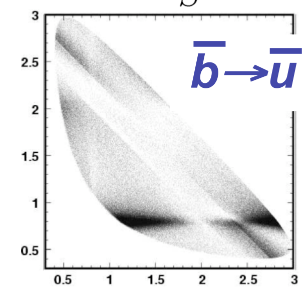
*The D Dalitz approach**

decay amplitude

$A(B^+) =$ ($\overline{D}^0 \to K_S^0 \pi^+ \pi^-$



$+ r_b e^{+i\gamma + i\delta_b}$ $D^0 \to K_S^0 \pi^+ \pi^-$



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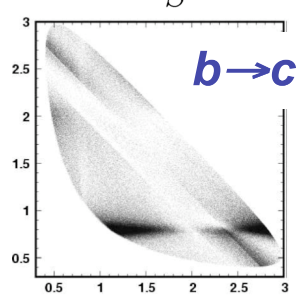
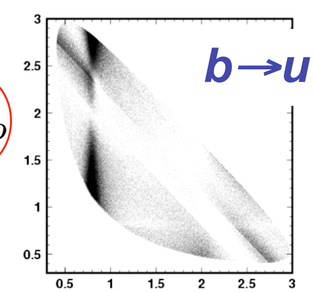
r_b = CKM and color-suppression factor expected to be in range 0.1 to 0.2.

Hadronic parameters (r_b, δ_b) determined from data.

$A(B^-) =$ ($D^0 \to K_S^0 \pi^+ \pi^-$



$+ r_b e^{-i\gamma + i\delta_b}$ $\overline{D}^0 \to K_S^0 \pi^+ \pi^-$



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* *A. Giri, Y. Grossman, A. Soffer, J. Zupan PRD 68, 054018 (2003); A. Bondar, unpublished.*

$B^{\pm} \to D^{(*)}K^{(*)\pm}$ Dalitz: analysis samples

BaBar 425 fb^{-1}
(468 M$B\bar{B}$)

BELLE 605 fb^{-1}
(657 M$B\bar{B}$)*

$$B^{\mp} \to DK^{\mp}, \quad D^{*}(D^{0}\pi^{0}, D^{0}\gamma)K^{\mp}, \quad DK^{*\mp}(K^{0}_{S}\pi^{\mp})$$

$$D \to K^{0}_{S}\pi^{+}\pi^{-}, \boxed{K^{0}_{S}K^{+}K^{-}}$$

BaBar analysis only

Signal separated from combinatoric background using

$$m_{\rm es} = \sqrt{E^{2}_{\rm beam} - p^{2}_{B}} \quad \text{and} \quad \Delta E = E_{B} - E_{\rm beam}$$

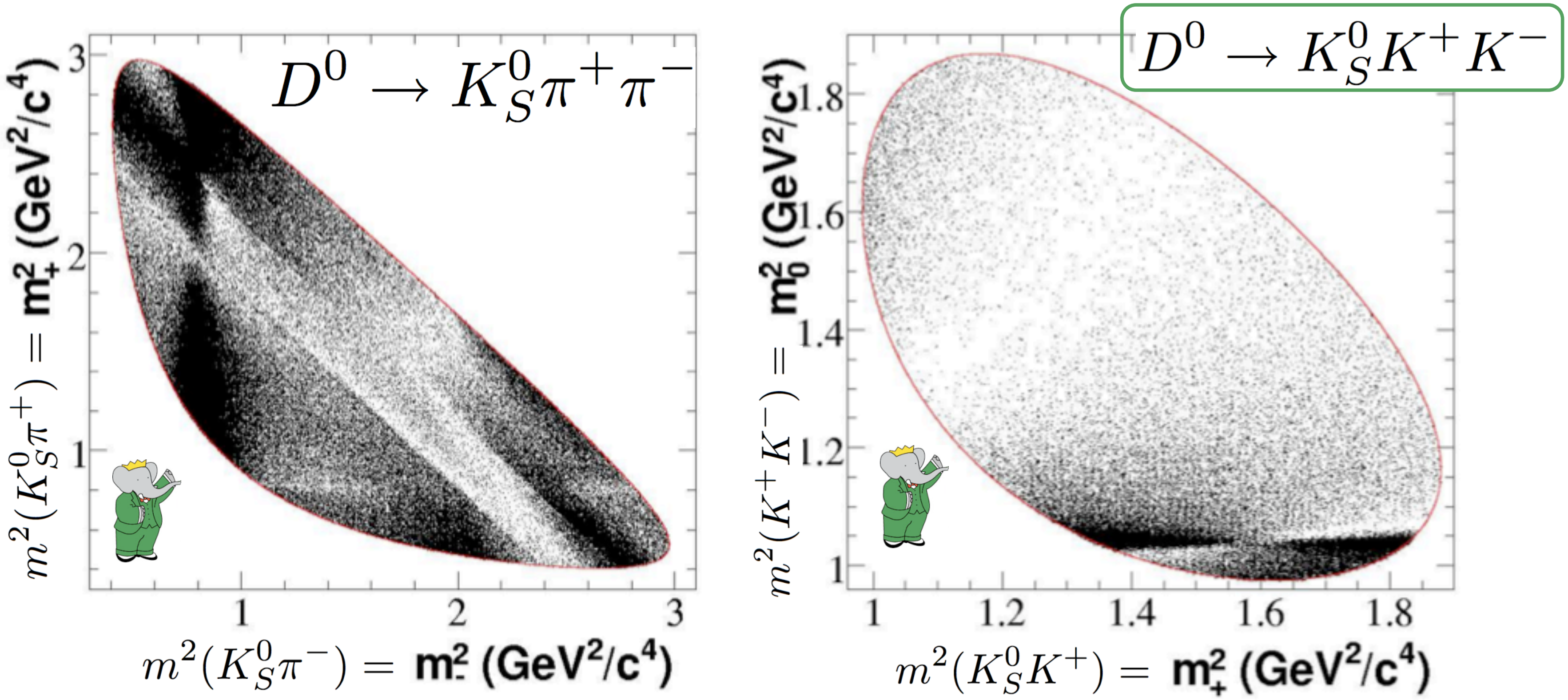
Continuum ($e^{+}e^{-} \to q\bar{q}$) BG rejected using *event shape variables* combined in optimal linear combination (Fisher discriminant).

Large $B^{+} \to D^{(*)}\pi^{+}$ data control sample (r_b~0.01, x10 smaller than *DK*).

Very large, clean sample of flavor-tagged D^{0} from $D^{*+} \to D^{0}\pi^{+}$ produced in continuum ($e^{+}e^{-} \to c\bar{c}$) used for D.P. amplitude determination.

**BELLE $B \to DK^{*}$ analysis uses 387 M$B\bar{B}$.*

$D^0 \to K_S h^+ h^-$ Amplitude analysis



- Fit for amplitudes relative to CP eigenstates [$K_s\rho^0(770)$ or $K_s a_0(980)$].
- Main differences in $D^0 \to K_s\pi^+\pi^-$:
  - *BaBar:* K-matrix formalism for $\pi\pi$ S-wave, LASS model for $K\pi$ S-wave.
  - ***BELLE:*** Includes σ_1 and σ_2 $\pi\pi$ scalar resonances. $K^*_0(1430)$ for $K\pi$ S-wave.

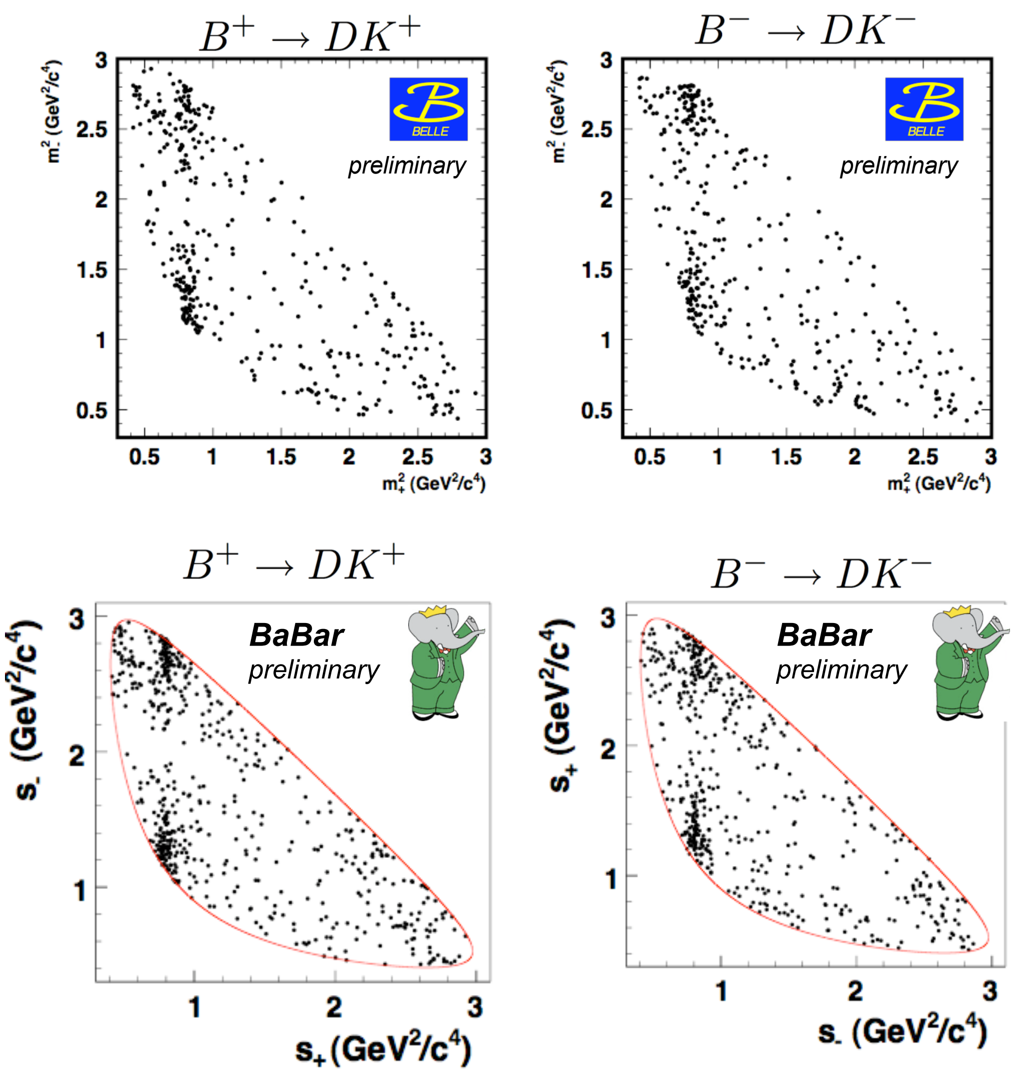
BaBar: PRD 78, 034023 (2008), J Garra Tico at this meeting. *BELLE:* PRD 73, 112009 (2006), arXiv:0803.3375, A. Poluektov at EPS 2009.

Signal event yields

B decay mode	***BELLE*** $(K_s\pi^+\pi^-)$ ***657 MB$\bar{B}$***	***BaBar*** $(K_s\pi^+\pi^-)$ ***468 MB$\bar{B}$***	***BaBar*** $(K_sK^+K^-)$ ***468 MB$\bar{B}$***
$B^\pm \to DK^\pm$	757 ± 30	920 ± 35	142 ± 14
$B^\pm \to D^*(D\pi^0)K^\pm$	168 ± 15	246 ± 22	53 ± 11
$B^\pm \to D^*(D\gamma)K^\pm$	83 ± 10	191 ± 19	31 ± 7
$B^\pm \to DK^{*\pm}$	*(not updated to 657 MB$\bar{B}$)*	163 ± 17	28 ± 6

- Signal yields are for sample used in final fit for CP parameters.
- ***BaBar*** efficiencies improved substantially (20% to 40% relative) with respect to previous *BaBar* measurement (383 MB$\bar{B}$).
  - Reprocessed dataset with improved track reconstruction.
  - Improved particle ID
  - Revised K_s selection criteria.

$B^{\pm} \rightarrow DK^{\pm}$: $D \rightarrow K_s \pi^+ \pi^-$ Dalitz plots



Unbinned maximum likelihood fits.

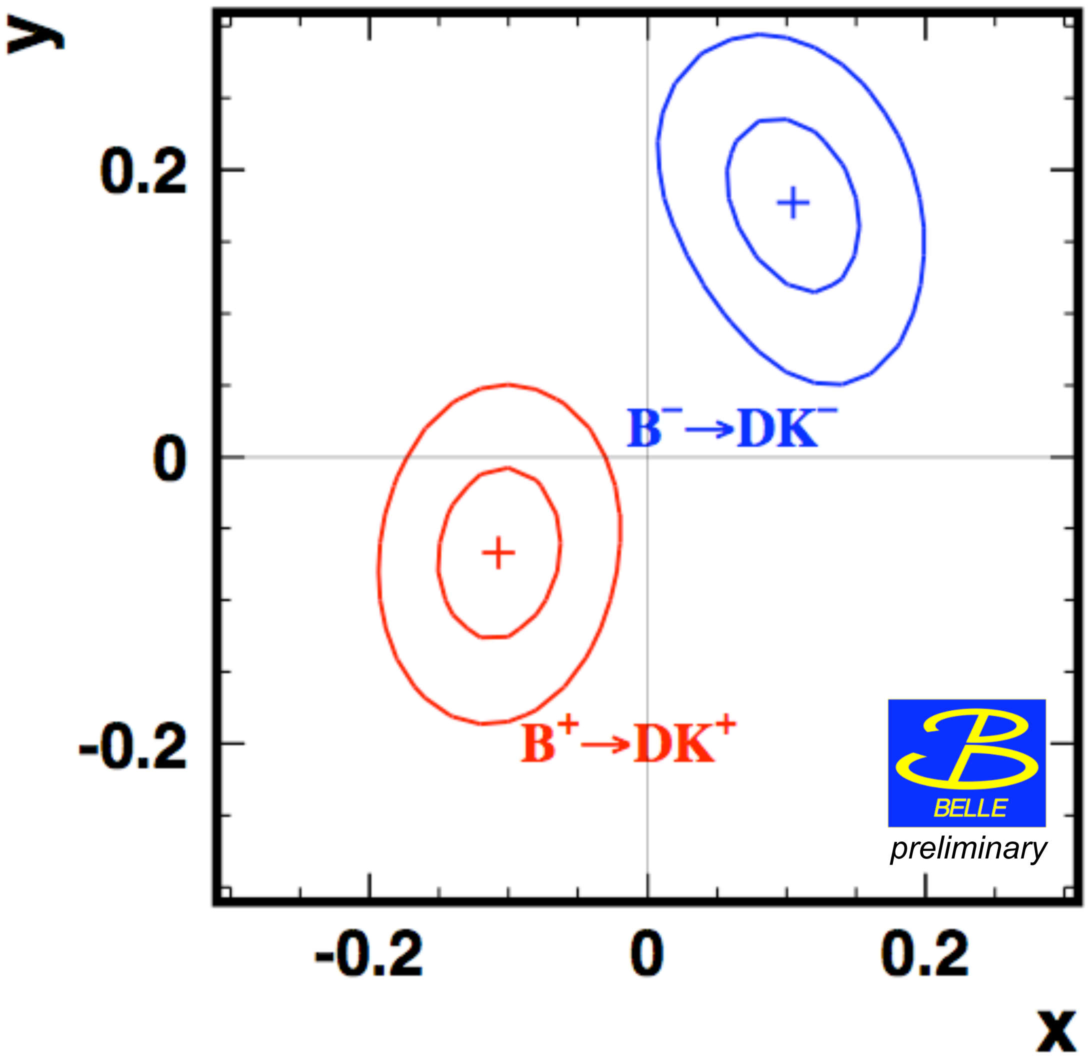
Probability density functions in the likelihood depend on
- ΔE, m_{es},
- continuum rejection var(s),
- Dalitz plot position.

Interference terms in intensity proportional to

$$x_{\pm} = r_B \cos(\delta_B \pm \gamma)$$
$$y_{\pm} = r_B \sin(\delta_B \pm \gamma)$$

Fit for $x_{\pm}$ and $y_{\pm}$ for each B decay mode.

$B^{\pm}\rightarrow DK^{\pm}$: fit results



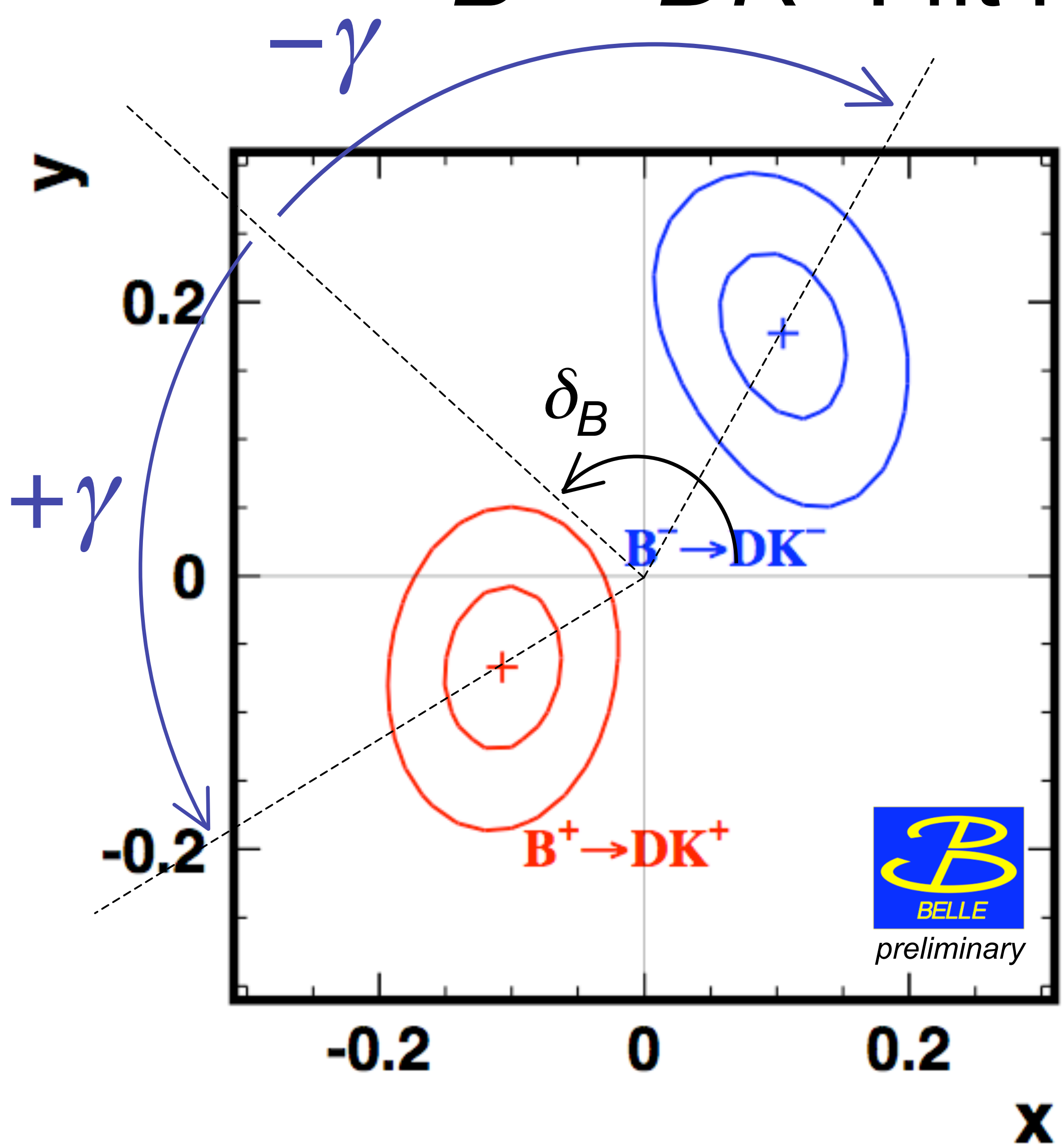
Interference terms in intensity proportional to

$$x_{\pm} = r_B \ \cos(\delta_B \pm \gamma)$$

$$y_{\pm} = r_B \ \sin(\delta_B \pm \gamma)$$

Contours are 1 and 2 sigma.

$B^{\pm}\to DK^{\pm}$: fit results



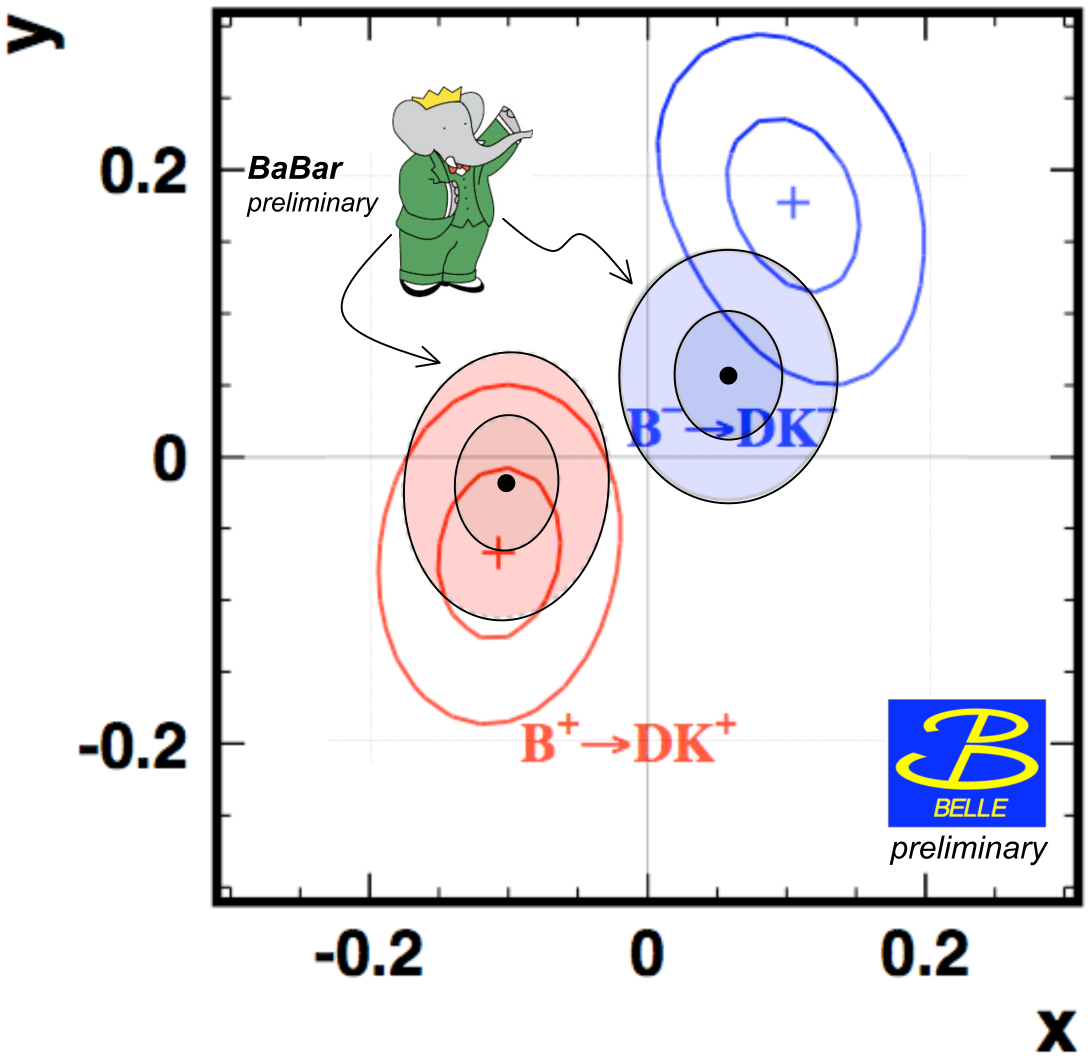
Interference terms in intensity proportional to

$$x_{\pm} = r_B \cos(\delta_B \pm \gamma)$$
$$y_{\pm} = r_B \sin(\delta_B \pm \gamma)$$

Discrete ambiguity: can add 180° to γ and δ_B.

Contours are 1 and 2 sigma.

$B^{\pm} \rightarrow DK^{\pm}$: fit results



$$x_{\pm} = r_B \ \cos(\delta_B \pm \gamma)$$
$$y_{\pm} = r_B \ \sin(\delta_B \pm \gamma)$$

	x_-
BaBar	(5.7 ± 3.8 ± 0.7 ± 0.6)%
BELLE	(10.5 ± 4.7 ± 1.1)%
	y_-
BaBar	(5.9 ± 4.5 ± 0.4 ± 0.6)%
BELLE	(17.7 ± 6.0 ± 1.8)%

	x_+
BaBar	(-10.2 ± 3.7 ± 0.6 ± 0.7)%
BELLE	(-10.7 ± 4.3 ± 1.1)%
	y_+
BaBar	(-1.8 ± 4.7 ± 0.4 ± 0.9)%
BELLE	(-6.7 ± 5.9 ± 1.8)%

(*value* ± *stat.* ± *syst.* ± *model*)%

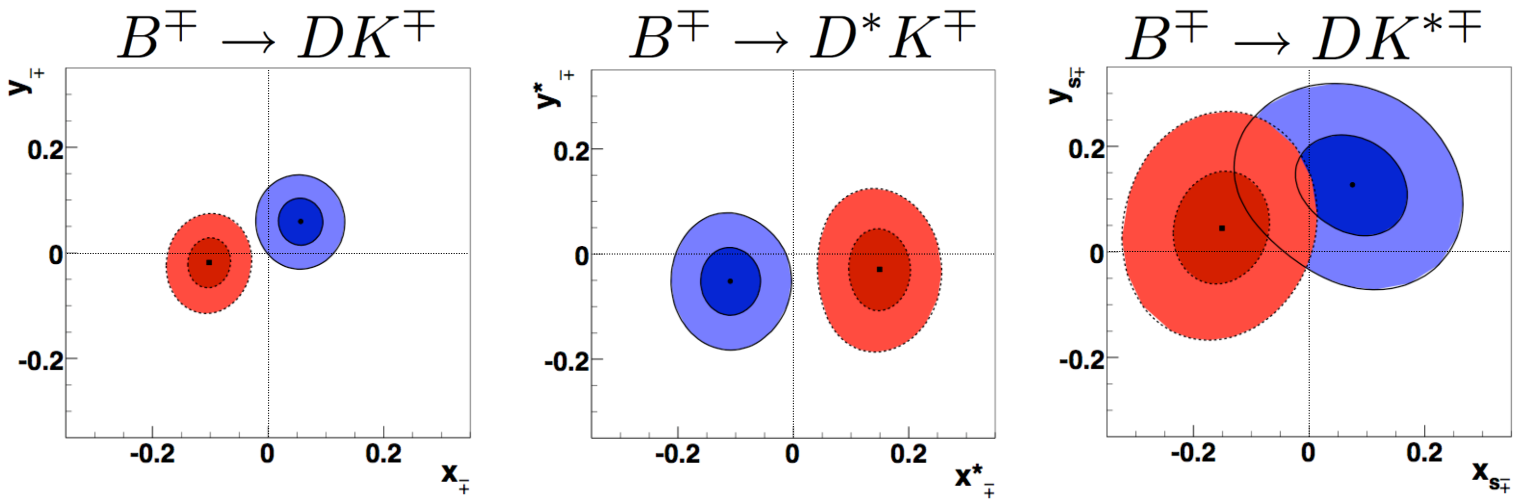
Contours are 1 and 2 sigma.

*BaBar result is **new** and still preliminary.*

All $x_{\pm}, y_{\pm}$ measurements

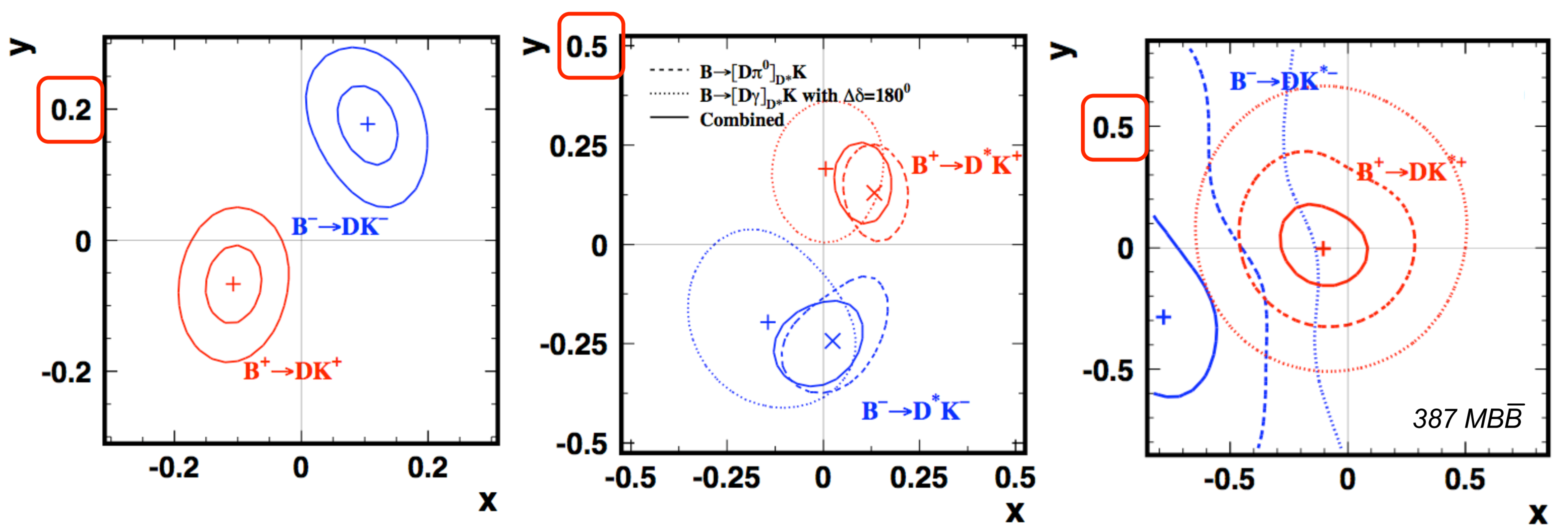
$$x_{\pm} = r_B \cos(\delta_B \pm \gamma)$$
$$y_{\pm} = r_B \sin(\delta_B \pm \gamma)$$

BaBar
preliminary



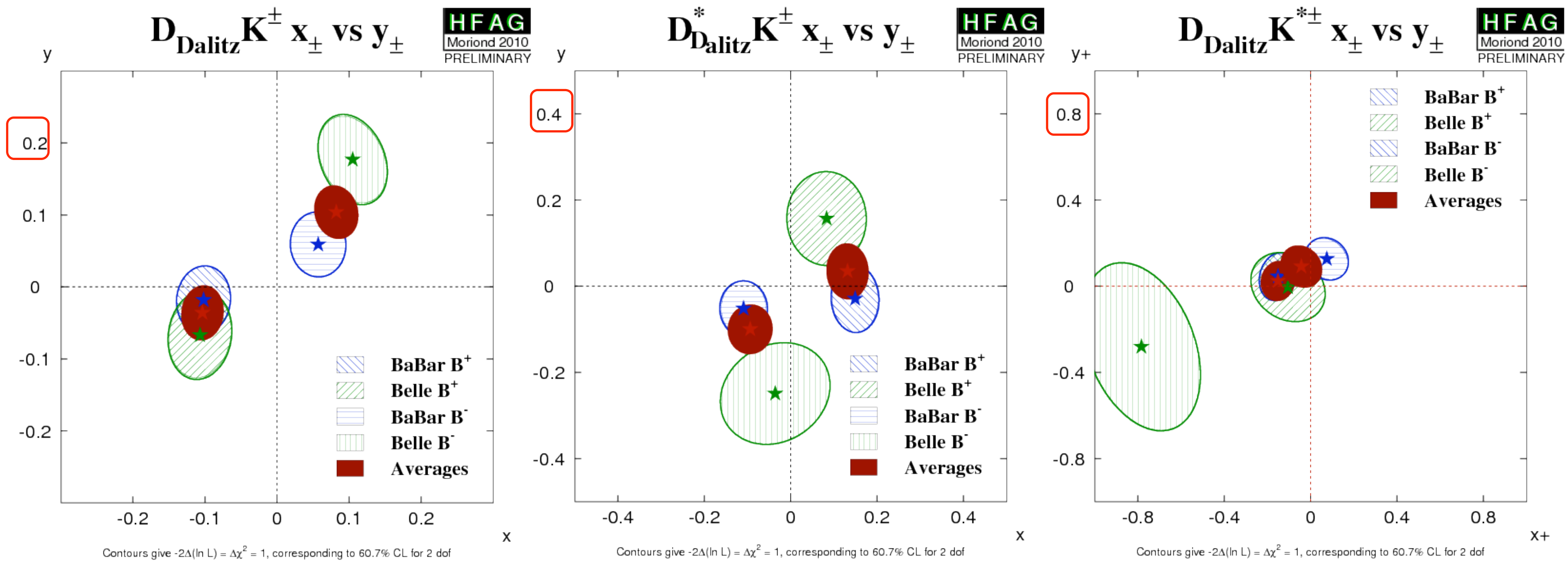
preliminary

Note: scale not same for 3 plots.



Tables of fitted $x_{\pm}, y_{\pm}$ for 3 modes given in backup slides.

HFAG averages of x,y parameters



Note: scale not same for 3 plots.

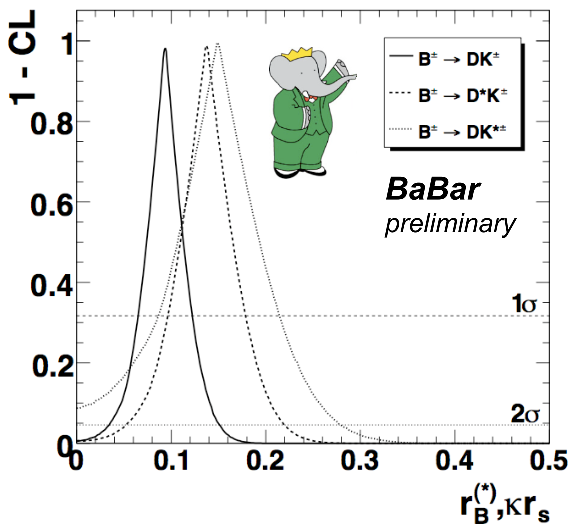
Disclaimers for HFAG averages

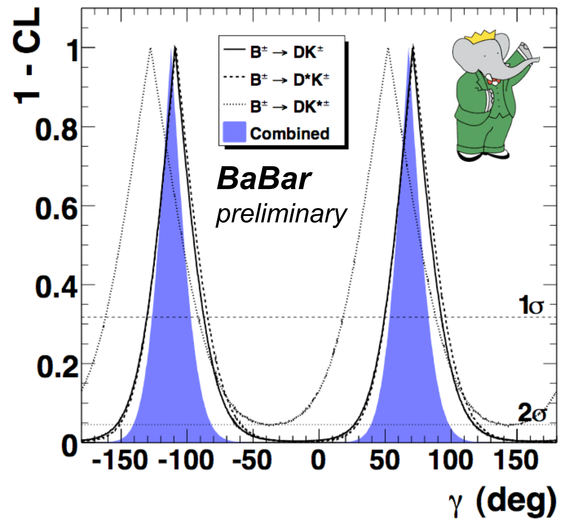
Averages are performed using the following procedure.

- It is assumed that both experiments use the same D decay model. Therefore, we do not rescale the results to a common model.
- It is further assumed that the model uncertainty is 100% correlated between experiments, and therefore this source of error is not used in the averaging procedure.
- Note that while the above two assumptions may be reasonable in the case that both experiments are using the same decay mode (as was the case until recently with both using only $D \to K_S\pi^+\pi^-$), now that BaBar include also $D \to K_SK^+K^-$; this is certainly invalid. However, separate results for the two D decay modes are not available, making a better treatment difficult at present.
- We include in the average the effect of correlations within each experiments set of measurements.
- At present it is unclear how to assign an average model uncertainty. We have not attempted to do so. Our average includes only statistical and systematic error. An unknown amount of model uncertainty should be added to the final error.
- We follow the suggestion of Gronau in making the DK* averages. Explicitly, we assume that the selection of $K^{*-} \to K_S\pi^-$ is the same in both experiments (so that κ, r_s and δ_s are the same), and drop the additional source of model uncertainty assigned by Belle due to possible nonresonant decays.
- We do not consider common systematic errors, other than the D decay model.

For more info, please see http://www.slac.stanford.edu/xorg/hfag/triangle/moriond2010

$B^{\pm}\to D^{(*)}K^{(*)\pm}$ Dalitz: Interpretation





BaBar *preliminary*

$\gamma = (68 \pm 14 \pm 4 \pm 3)^{o}$

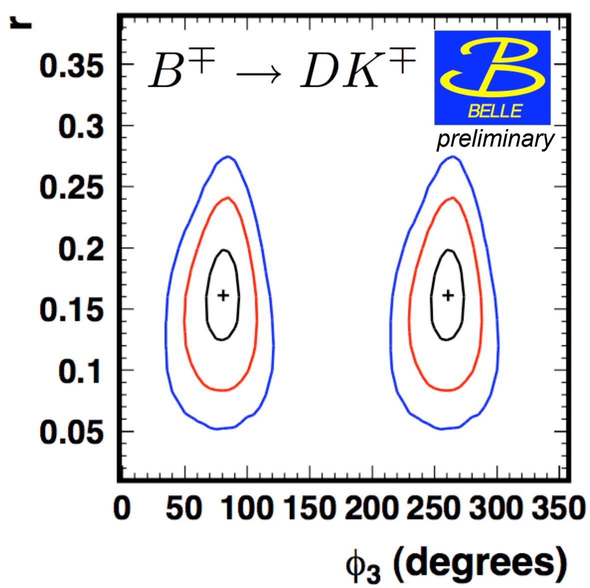
(*value* ± *stat.* ± *sys.* ± *model*)^{o}

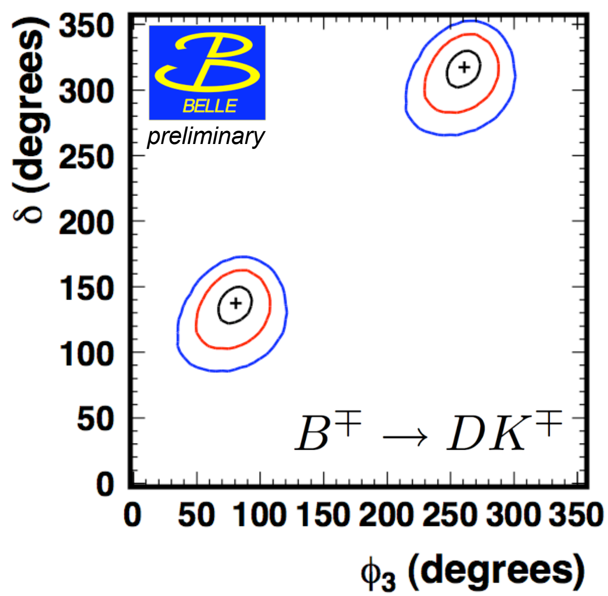
Excludes γ =0 at 3.5 std.dev.

$r_B(DK) = (9.4\ ^{+2.8}_{-2.9})\%$

(*value* ± *total error*)%

Error breakdown (±0.5 *expt.*, ±0.4 *model*)%





BELLE *preliminary*

$\gamma = (78.4\ ^{+10.8}_{-11.6} \pm 3.6 \pm 8.9)^{o}$

(*value* ± *stat.* ± *sys.* ± *model*)^{o}

*B→DK and B→D*K only, 657 MB$\bar{B}$*

Excludes γ =0 at 3.5 std.dev.

$r_B(DK) = (16.0\ ^{+4.0}_{-3.8} \pm 0.011\ ^{+5.0}_{-1.0})\%$

(*value* ± *stat.* ± *sys.* ± *model*)$^{\%}$

More results for individual modes in backup slides.

BaBar ADS* analysis

- The "ADS*" technique equalizes the magnitude of the interfering amplitudes

$B^- \to D^0 K^-$ *followed by* $D^0 \to K^+ \pi^-$

favored $(b \to c)$ suppressed$(c \to d)$

interferes with

$B^- \to \bar{D}^0 K^-$ *followed by* $\bar{D}^0 \to K^+ \pi^-$

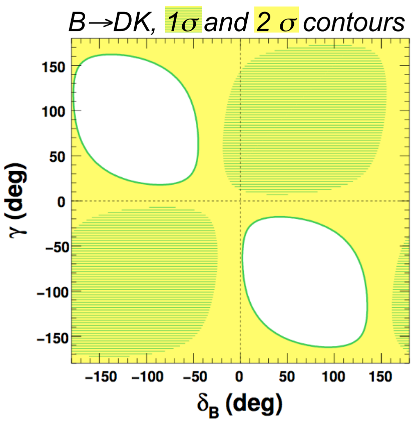
suppressed $(b \to u)$ favored $(\bar{c} \to \bar{s})$

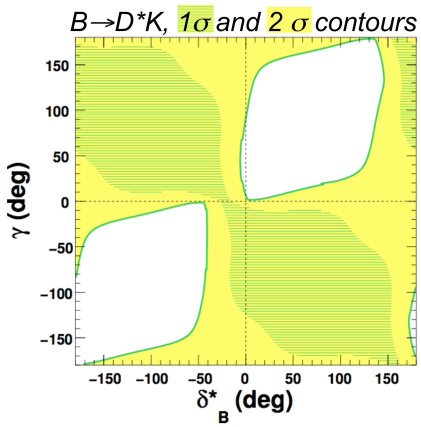
- CP asymmetry can be *very* large.
- Very sensitive to r_B.

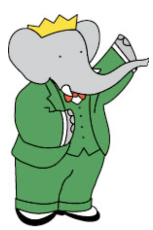
**ADS stands for D. Atwood, I. Dunietz, A. Soni, PRL 78, 3257 (1997); PRD 63, 036005 (2001).*

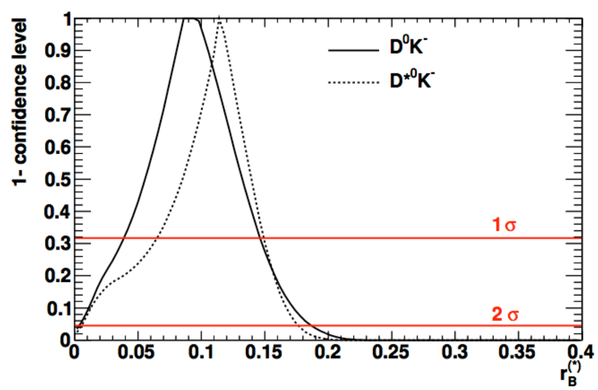
BaBar ADS analysis results

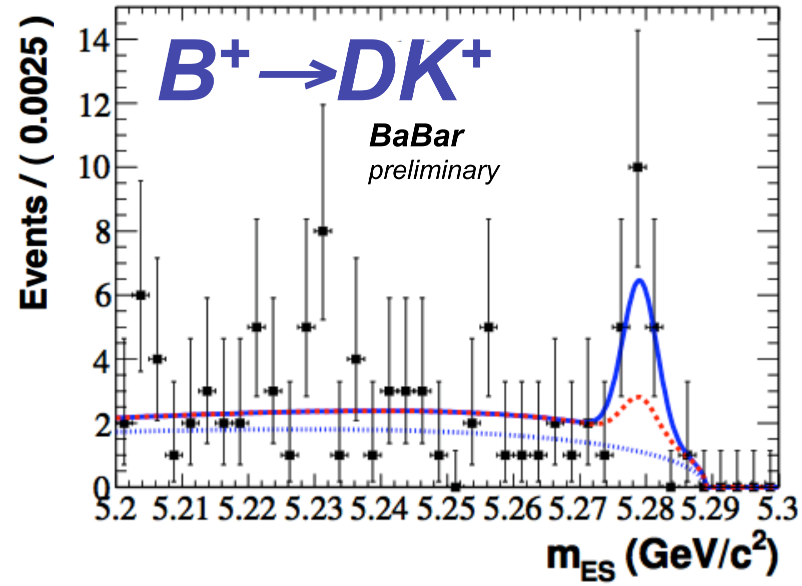
- Update with final dataset (468 $MB\bar{B}$).
- First sign of an ADS signals in $B^{\pm}\to DK^{\pm}$ and $B^{\pm}\to D^{*}K^{\pm}$!
- Large CP asymmetries.

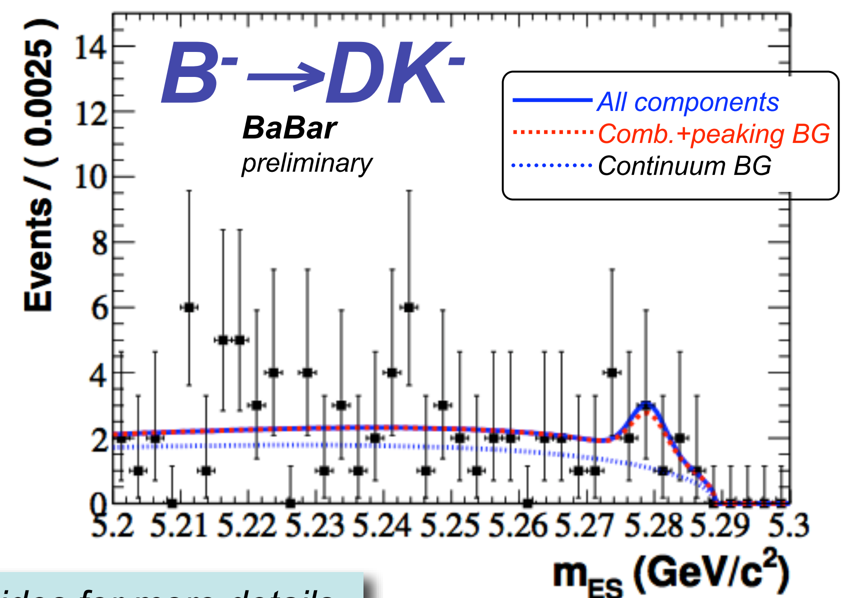












See talk by Neus Lopez March at EPS 2009 and backup slides for more details.

BaBar ADS analysis results

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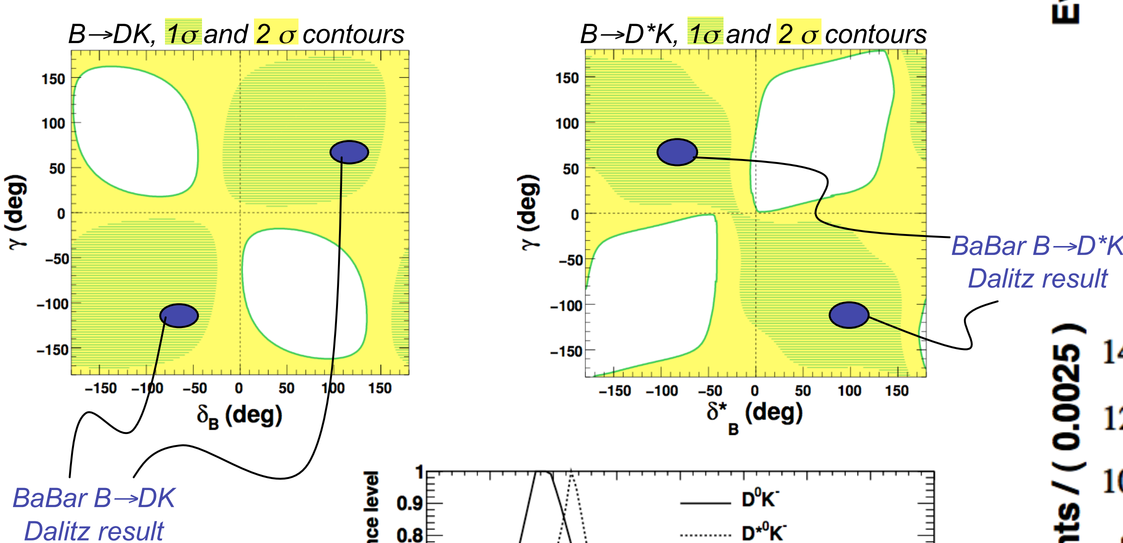
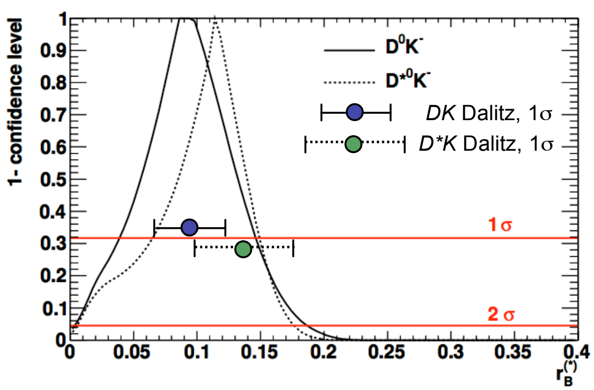
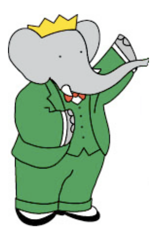
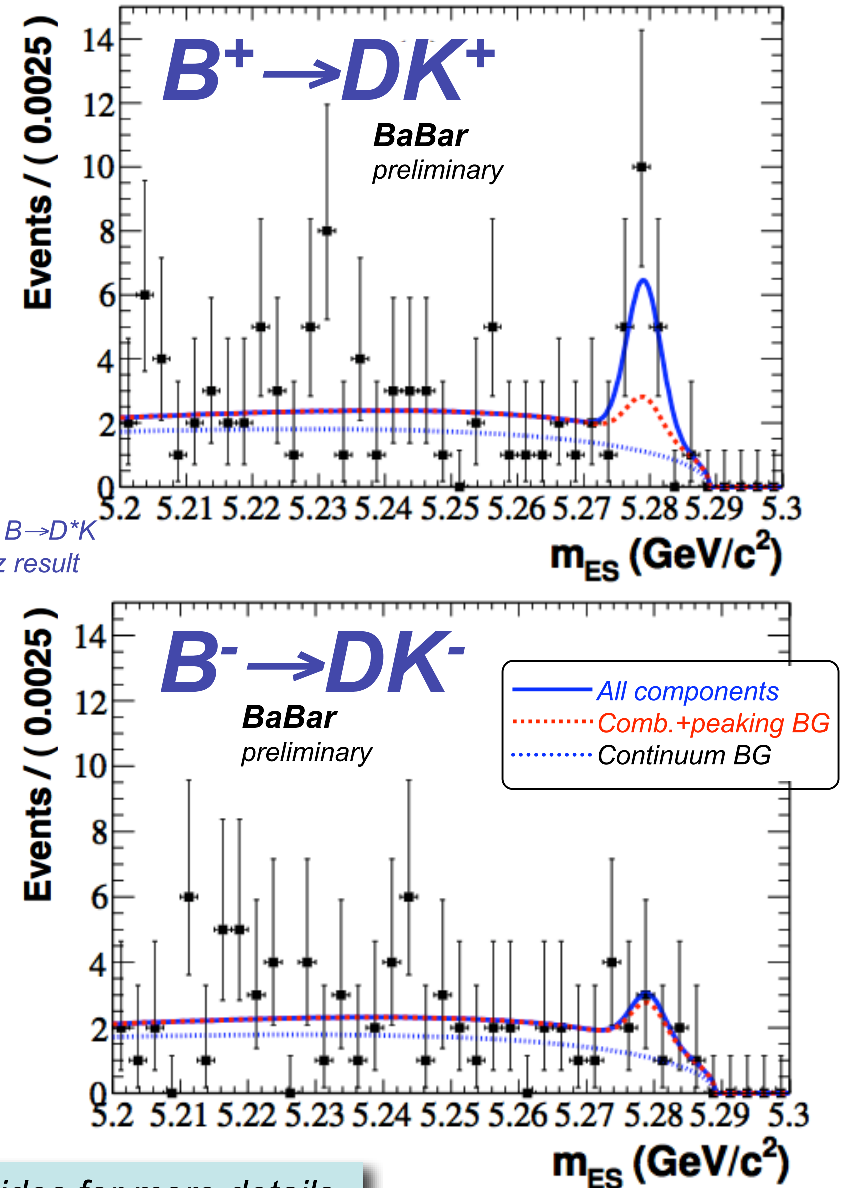








See talk by Neus Lopez March at EPS 2009 and backup slides for more details.

Other hadronic parameters relevant for γ/ϕ_3

- Updated search for $B^+ \to D^+ K^0$ and $B^+ \to D^+ K^{*0}$
  - Can only proceed through annihilation or rescattering.
  - Allows an estimation of r_B^0 **for B^0** from r_B for B^+ needed for
    - $B^0 \to D^0 K^0$ measures $\boldsymbol{r_B^0}$ $\sin(2\beta+\gamma)$ in time-dependent asymmetry.
    - $B^0 \to D^0 K^{*0}$ measures gamma in direct CPV (similar to B^+).
  - ***BaBar*** analysis with full dataset: 468 $\mathrm{MB\overline{B}}$ *(new and preliminary)*
    - No evidence for signals: $\mathcal{B}(B^+ \to D^+ K^0) < 2.9 \times 10^{-6},\ 90\%\mathrm{C.L.}$

$$\mathcal{B}(B^+ \to D^+ K^{*0}) < 3.0 \times 10^{-6},\ 90\%\mathrm{C.L.}$$

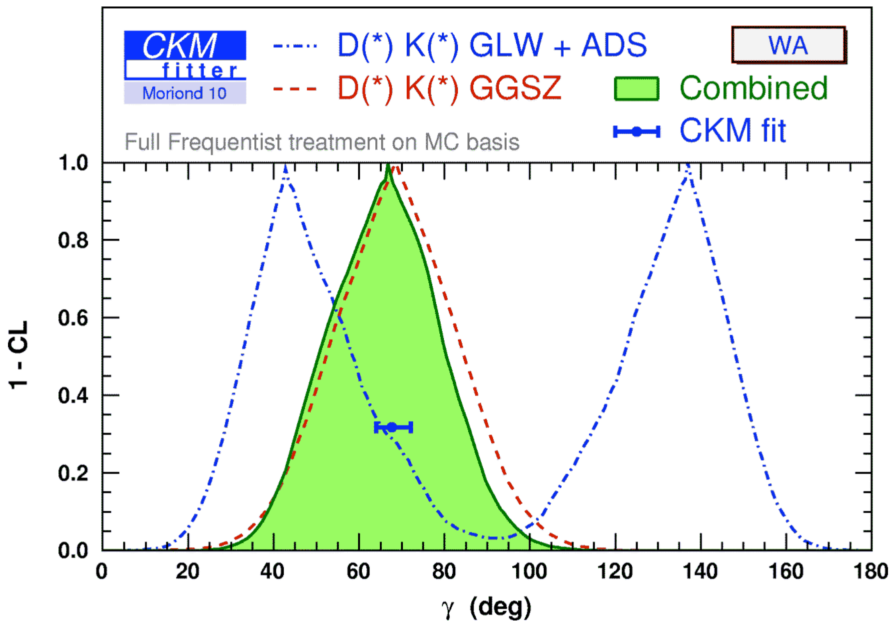
- Improved BF measurements: $B^0 \to D_s^{*+} \pi^-$ and $B^0 \to D_s^{*-} K^+$
  - Compute $R_{D^*\pi}$ from SU(3) for TD analysis of $B^0 \to D^{*+} \pi^-$ which measures $R_{D^*\pi} \sin(2\beta+\gamma)$.
  - $B^0 \to D_s^{*-} K^+$ proceeds only through W-exchange or rescattering.
  - ***BELLE*** analysis of 657 $\mathrm{MB\overline{B}}$: *PRD 81, 031101(R) (2010).*

$$R_{D^*\pi} = (1.58 \pm 0.15(\mathrm{stat}) \pm 0.10(\mathrm{syst}) \pm 0.03(\mathrm{th}))\%$$

Overall γ (or ϕ_3) results

Frequentist interpretation

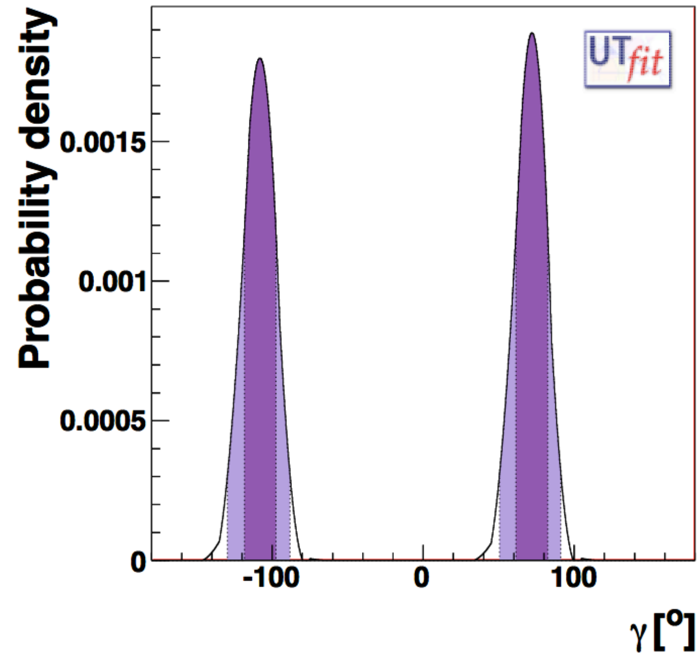
http://ckmfitter.in2p3.fr



$$\gamma = (69\ {}^{+19}_{-21})^\circ$$

Bayesian interpretation

http://www.utfit.org

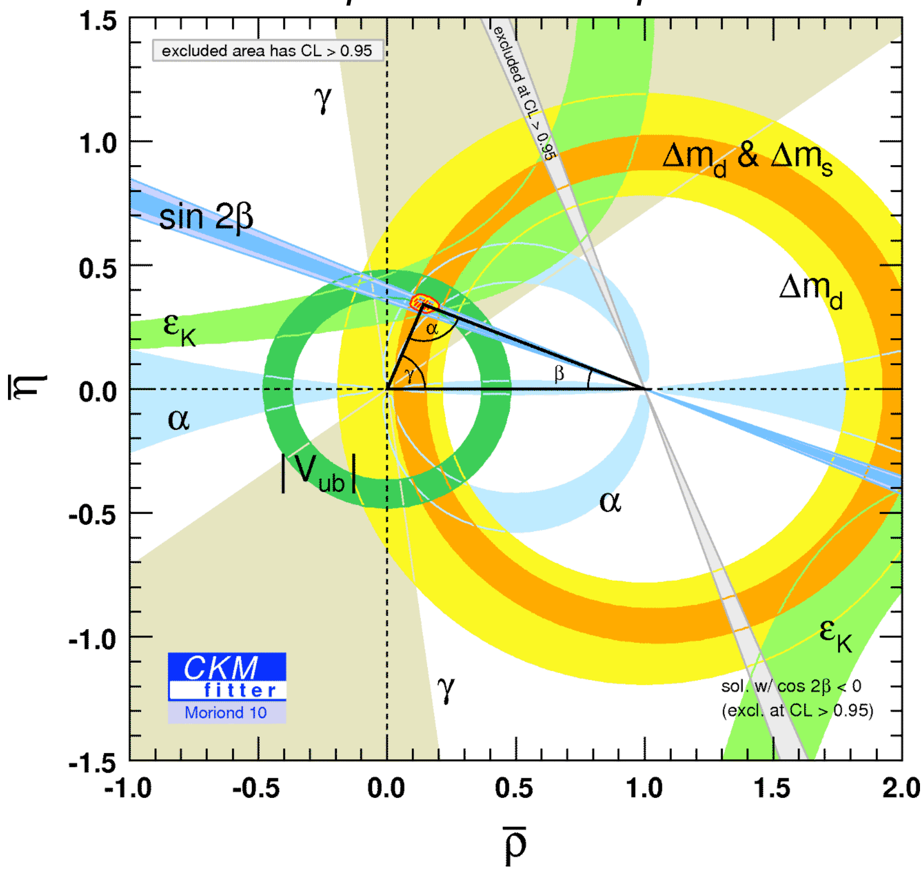


$$\gamma = (72 \pm 11)^\circ$$

μ supremum method used to combine HFAG averages of experimental inputs (conservative, but guarantees coverage).
See Karim Trabelsi's talk at CKM 2008 for details.

Summary and outlook

http://ckmfitter.in2p3.fr



See also UTfit analysis at
http://www.utfit.org/

- Recent progress on γ or ϕ_3 (the hardest UT angle to measure).
  - We are still statistics limited, even with full datasets.
- ***BaBar*** *$B \rightarrow D^{(*)}K^{(*)}$; $D \rightarrow K_s h^+ h^-$* analysis benefited from overlap with *D^0* mixing analysis (*Jordi Garra Tico* in next talk).
  - Dalitz model systematic on gamma reduced from 5^o to 3^o.
- Projections for analysis at LHCb look very good
  - Much higher signal statistics
  - Model independent approach viable using input from CLEOc.
- Super e^+e^- flavor factory would also do very well.

BACKUP SLIDES

$B^{\pm} \to D^{(*)}K^{(*)\pm}$ Dalitz: HFAG averages

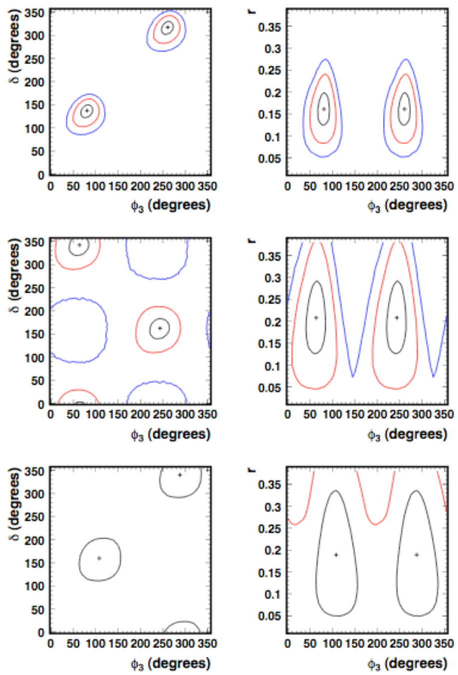
$$x_{\pm} = r_B \cos(\delta_B \pm \gamma) \qquad y_{\pm} = r_B \sin(\delta_B \pm \gamma)$$

Mode	Experiment	x+	y+	x-	y-	Correlation	Reference
DK^-	BaBar N(BB)=468M	$-0.102 \pm 0.037 \pm 0.006 \pm 0.007$	$-0.018 \pm 0.047 \pm 0.004 \pm 0.009$	$0.057 \pm 0.038 \pm 0.007 \pm 0.006$	$0.059 \pm 0.045 \pm 0.004 \pm 0.006$	(stat) (syst) (model)	Moriond 2010 preliminary
	Belle N(BB)=657M	$-0.107 \pm 0.043 \pm 0.011 \pm 0.055$	$-0.067 \pm 0.059 \pm 0.018 \pm 0.063$	$0.105 \pm 0.047 \pm 0.011 \pm 0.064$	$0.177 \pm 0.060 \pm 0.018 \pm 0.054$	(stat) (model)	arXiv:0803.3375
	Average No model error	-0.104 ± 0.029	-0.036 ± 0.037	0.082 ± 0.030	0.104 ± 0.036	(stat+syst)	HFAG correlated average χ^2 = 3.9/4 dof (CL=0.42 ⇒ 0.8σ)
D^*K^-	BaBar N(BB)=468M	$0.149 \pm 0.053 \pm 0.017 \pm 0.003$	$-0.029 \pm 0.078 \pm 0.008 \pm 0.006$	$-0.109 \pm 0.052 \pm 0.019 \pm 0.002$	$-0.052 \pm 0.064 \pm 0.008 \pm 0.007$	(stat) (syst) (model)	Moriond 2010 preliminary
	Belle (*) N(BB)=657M	$0.083 \pm 0.092 \pm 0.081$	$0.157 \pm 0.109 \pm 0.063$	$-0.036 \pm 0.127 \pm 0.090$	$-0.249 \pm 0.118 \pm 0.049$	(stat) (model)	arXiv:0803.3375 EPS2009 preliminary
	Average No model error	0.132 ± 0.048	0.035 ± 0.064	-0.094 ± 0.051	-0.100 ± 0.056	(stat+syst)	HFAG correlated average χ^2 = 5.0/4 dof (CL=0.29 ⇒ 1.1σ)
DK^{*-}	BaBar N(BB)=468M	$-0.151 \pm 0.083 \pm 0.028 \pm 0.006$	$0.045 \pm 0.106 \pm 0.036 \pm 0.008$	$0.075 \pm 0.096 \pm 0.029 \pm 0.007$	$0.127 \pm 0.095 \pm 0.027 \pm 0.006$	(stat) (syst) (model)	Moriond 2010 preliminary
	Belle N(BB)=386M	$-0.105^{+0.177}_{-0.167} \pm 0.006 \pm 0.088$	$-0.004^{+0.164}_{-0.156} \pm 0.013 \pm 0.095$	$-0.784^{+0.249}_{-0.295} \pm 0.029 \pm 0.097$	$-0.281^{+0.440}_{-0.335} \pm 0.046 \pm 0.086$	(stat) (model)	PRD 73, 112009 (2006)
	Average No model error	-0.151 ± 0.077	0.024 ± 0.091	-0.043 ± 0.094	0.091 ± 0.096	(stat+syst)	HFAG correlated average χ^2 = 13/4 dof (CL=0.011 ⇒ 2.5σ)

For more info, please see http://www.slac.stanford.edu/xorg/hfag/triangle/moriond2010

Fit results in (ϕ_3, r, δ)

Feldman-Cousins stat. treatment to translate $(x_\pm, y_\pm) \to (r, \phi_3, \delta)$ *preliminary*



$B^\pm \to DK^\pm$

ϕ_3	$81^{\circ}{}^{+13^\circ}_{-15^\circ}$
r_{D*K}	0.16 ± 0.04
δ_{D*K}	$137^{\circ}{}^{+13^\circ}_{-16^\circ}$

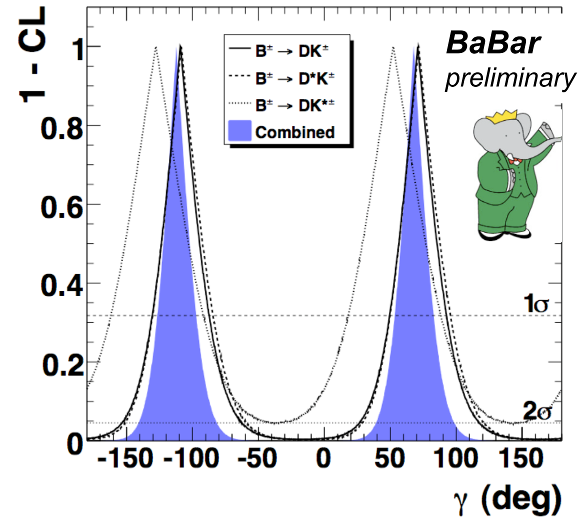
$B^\pm \to D^*K^\pm$, $D^* \to D\pi^0$

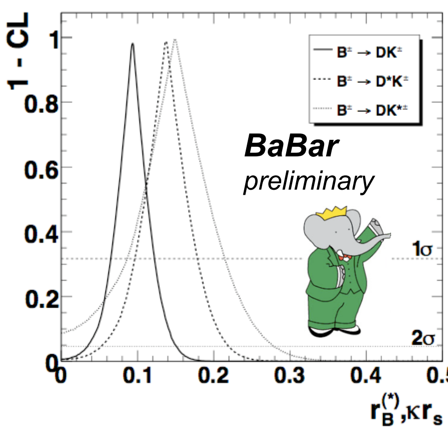
ϕ_3	$64^{\circ}{}^{+21^\circ}_{-23^\circ}$
r_{D*K}	$0.21^{+0.09}_{-0.08}$
δ_{D*K}	$342^{\circ}{}^{+21^\circ}_{-23^\circ}$

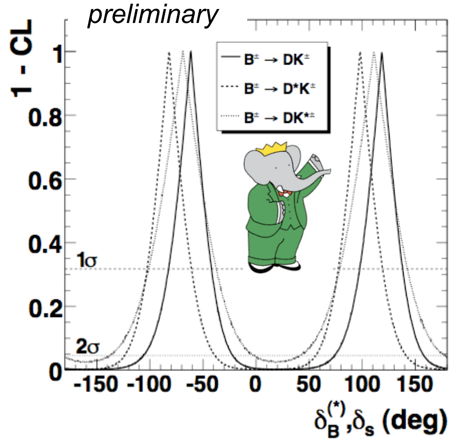
$B^\pm \to D^*K^\pm$, $D^* \to D\gamma$

ϕ_3	$108^{\circ}{}^{+53^\circ}_{-50^\circ}$
r_{D*K}	$0.19^{+0.14}_{-0.12}$
δ_{D*K}	$160^{\circ}{}^{+48^\circ}_{-55^\circ}$

Anton Poluektov | CPV in *B* decays at Belle | EPS-2009, Krakow, July 2009 | 12/17







BaBar *preliminary*

Parameter	68.3% CL	95.4% CL
γ (°)	68^{+15}_{-14} $\{4, 3\}$	$[39, 99]$
r_B (%)	$9.4^{+2.8}_{-2.9}$ $\{0.5, 0.4\}$	$[3.4, 15.1]$
r_B^* (%)	$13.7^{+4.2}_{-4.0}$ $\{1.4, 0.3\}$	$[5.2, 22.0]$
κr_s (%)	$14.9^{+6.6}_{-6.2}$ $\{2.6, 0.3\}$	< 27.9
δ_B (°)	118^{+20}_{-21} $\{3, 3\}$	$[74, 158]$
δ_B^* (°)	-82 ± 21 $\{5, 2\}$	$[-124, -39]$
δ_s (°)	111 ± 32 $\{12, 3\}$	$[42, 178]$

Value ± total error {± syst., ± model }

New BaBar ADS results

Preliminary, 486 MB$\overline{B}$.

$$\mathcal{R}^{\pm}_{DK} \equiv \frac{\Gamma([K^{\mp}\pi^{\pm}]_D K^{\pm})}{\Gamma([K^{\pm}\pi^{\mp}]_D K^{\pm})} = r_B^2 + r_D^2 + 2\, r_B r_D \cos(\pm\gamma + \delta)$$

$$\mathcal{R}_{DK} \equiv \frac{1}{2}(\mathcal{R}^{+}_{DK} + \mathcal{R}^{-}_{DK}) = r_B^2 + r_D^2 + 2\, r_B r_D \cos\gamma \cos\delta$$

$$\mathcal{R}_{DK} = (1.1 \pm 0.5 \pm 0.2) \times 10^{-2}$$

$$\mathcal{R}^{*}_{DK,D^0\pi^0} = (1.8 \pm 0.9 \pm 0.4) \times 10^{-2}$$

$$\mathcal{R}^{*}_{DK,D^0\gamma} = (1.3 \pm 1.4 \pm 0.7) \times 10^{-2}$$

$$\mathcal{A}_{DK} \equiv \frac{\mathcal{R}^{-}_{DK} - \mathcal{R}^{+}_{DK}}{\mathcal{R}^{-}_{DK} + \mathcal{R}^{+}_{DK}} = 2\, r_B r_D \sin\gamma \sin\delta / \mathcal{R}_{DK}$$

$$\mathcal{A}_{DK} = -0.86 \pm 0.47\ ^{+0.11}_{-0.15}$$

$$\mathcal{A}^{*}_{DK,D^0\pi^0} = +0.77 \pm 0.35 \pm 0.12$$

$$\mathcal{A}^{*}_{DK,D^0\gamma} = +0.36 \pm 0.94\ ^{+0.25}_{-0.41}$$

$$r_B = (9.0^{+5.6}_{-5.1})\%$$

$$r_B^{*} = (11.6^{+3.4}_{-5.0})\%$$