Current status of g-2

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Outlook

- The muon magnetic anomaly
- Data on $e^+e^- \to$ hadrons
- Combination of all e^+e^- data (HVPTools)
- Results on a_μ
- Discussion and conclusion

Lepton Magnetic Anomaly: from Dirac to QED

$$\vec{\mu} = g\frac{e}{2m}\vec{s}, \qquad a = (g-2)/2$$

Dirac (1928) $g_e=2$ $a_e=0$

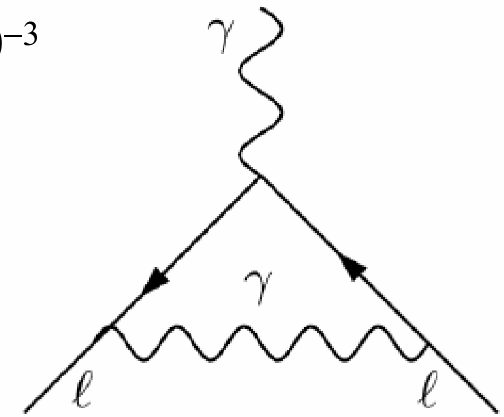
anomaly discovered:

Kusch-Foley (1948) $a_e = (1.19 \pm 0.05)\ 10^{-3}$

and explained by $O(\alpha)$ QED contribution:

Schwinger (1948) $a_e = \alpha/2\pi = 1.16\ 10^{-3}$

first triumph of QED



$\Rightarrow$ a_e sensitive to quantum fluctuations of fields

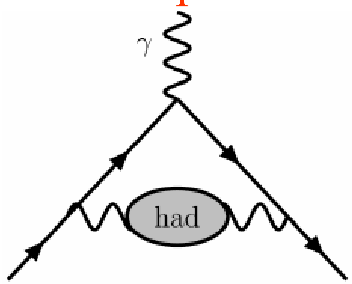
More Quantum Fluctuations

$$a = a^{\rm QED} + a^{\rm had} + a^{\rm weak} \quad + \ ?\ a^{\text{new physics}}\ ?$$

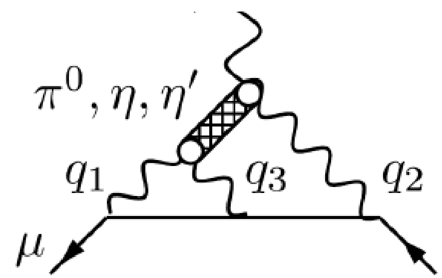
typical contributions:

QED up to $O(\alpha^4)$, α^5 in progress (Kinoshita et al.)

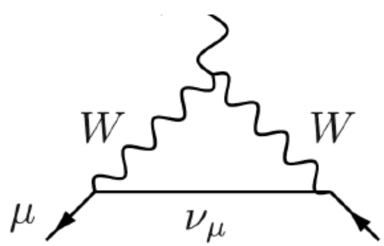
Hadrons vacuum polarization



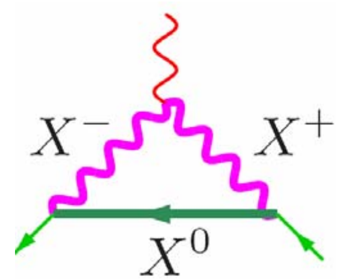
light-by-light (models)



Electroweak



new physics at high mass scale



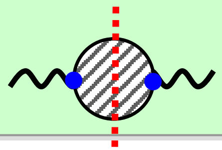
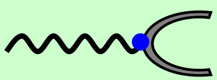
$$\delta a_\ell \propto \frac{m_\ell^2}{M^2} \quad \Rightarrow \ a_\mu \text{ much more sensitive to high scales}$$

Hadronic Vacuum Polarization and Muon $(g-2)_\mu$

Dominant uncertainty for the theoretical prediction: from lowest-order HVP piece
Cannot be calculated from QCD (low mass scale), but one can use experimental data on $e^+e^- \to$hadrons cross section

Born: $\sigma^{(0)}(s) = \sigma(s)\left(\alpha/\alpha(s)\right)^2$

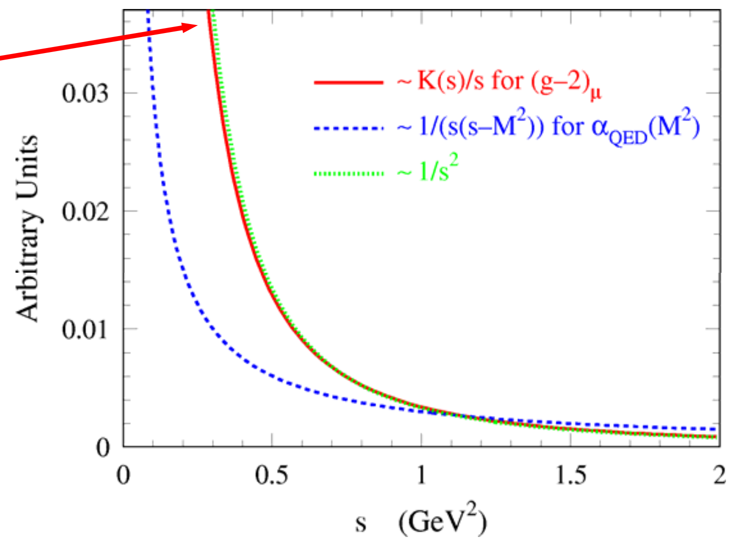
$$12\pi\, \mathrm{Im}\Pi_\gamma(s) = \frac{\sigma^0\,[e^+e^- \to \text{hadrons}\,(\gamma_{FSR})]}{\sigma_{pt}} \equiv R(s)$$

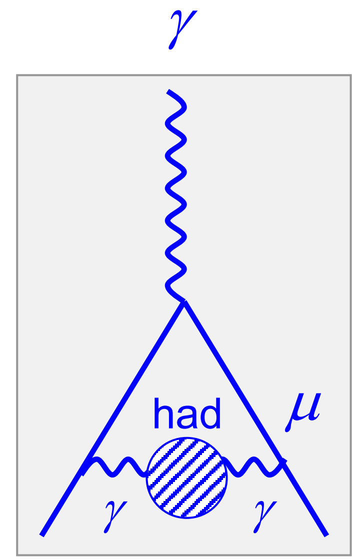
Im[] $\propto$ |  hadrons $|^2$

$$a_\mu^{had} = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^{\infty} ds\, \frac{K(s)}{s}\, R(s)$$

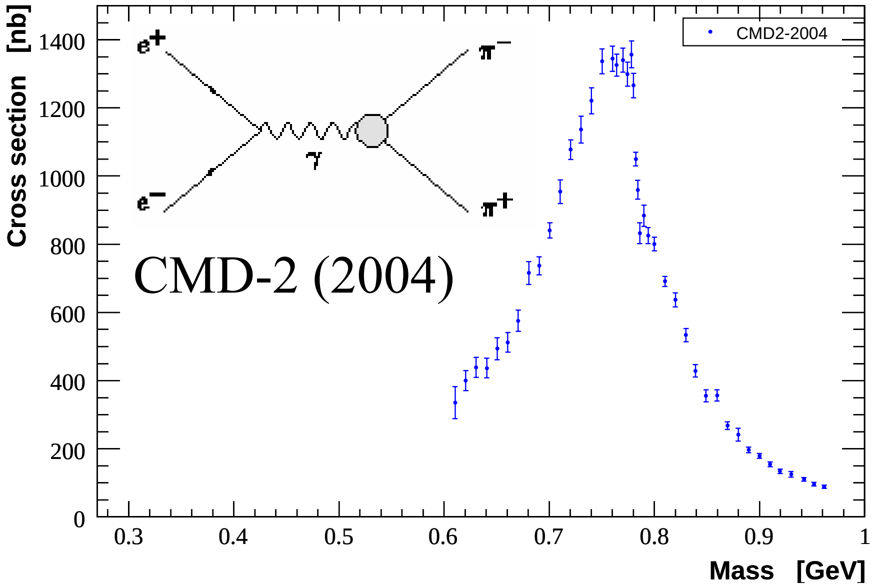
Dispersion relation

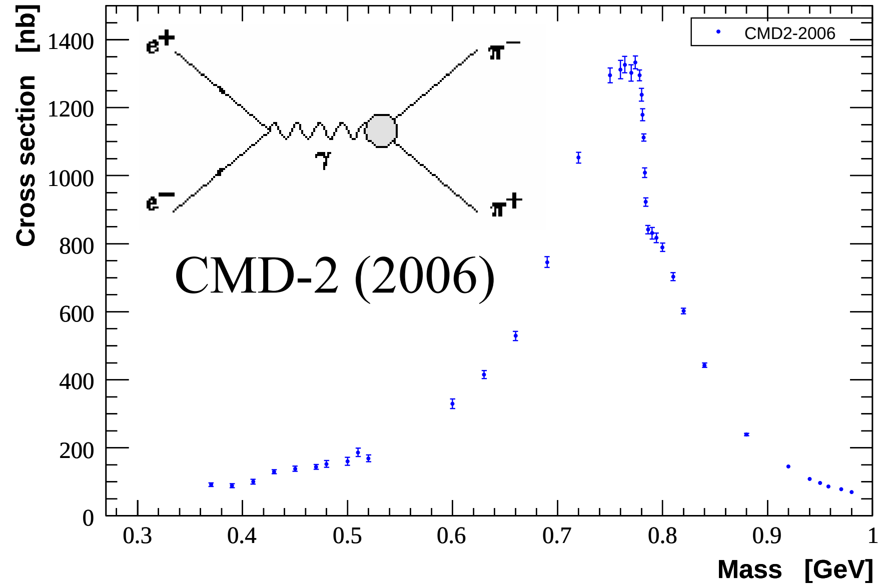
Bouchiat and Michel, 1961

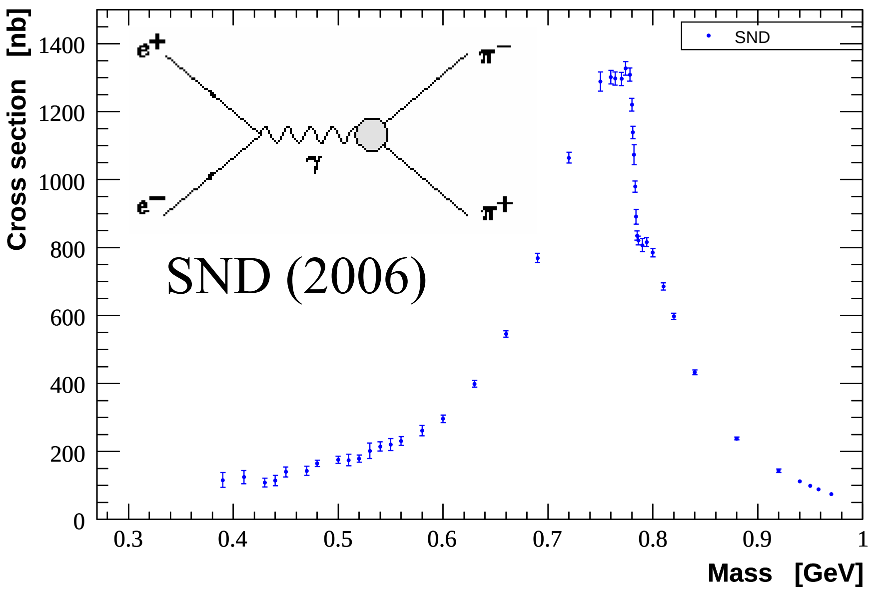


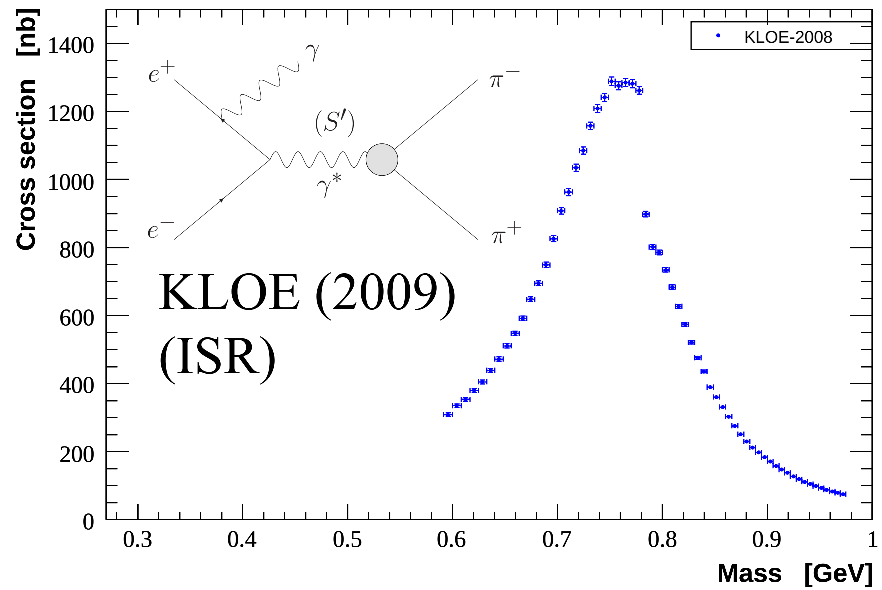


HVP: Data on $e^+e^- \to$ hadrons

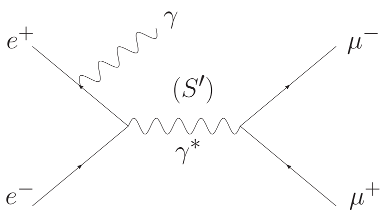


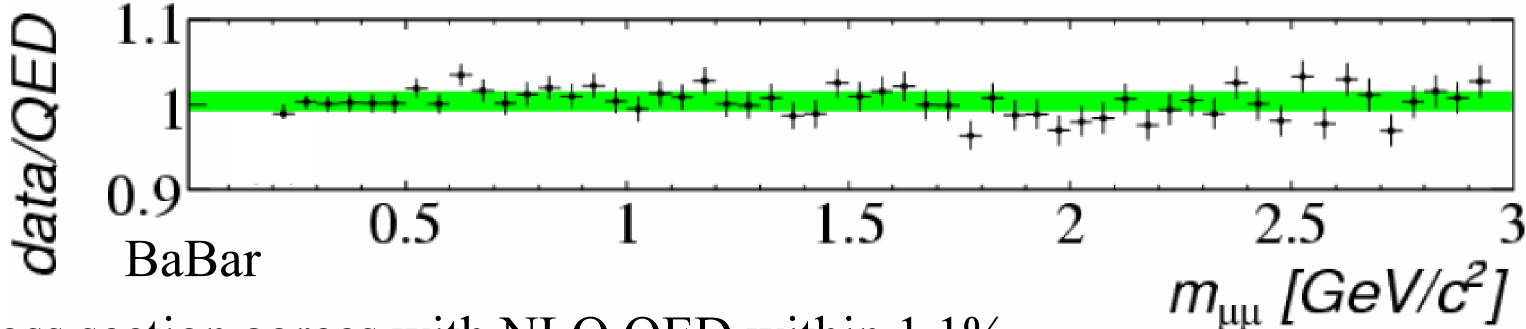




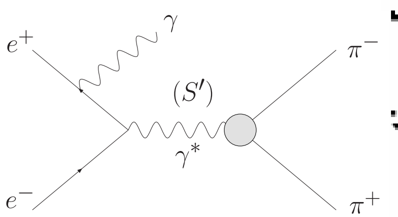


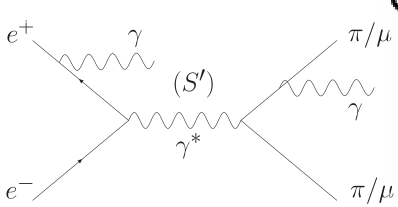
BaBar results (arXiv:0908.3589, PRL 103, 231801 (2009))

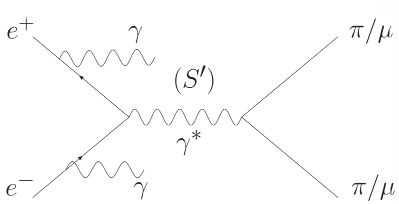


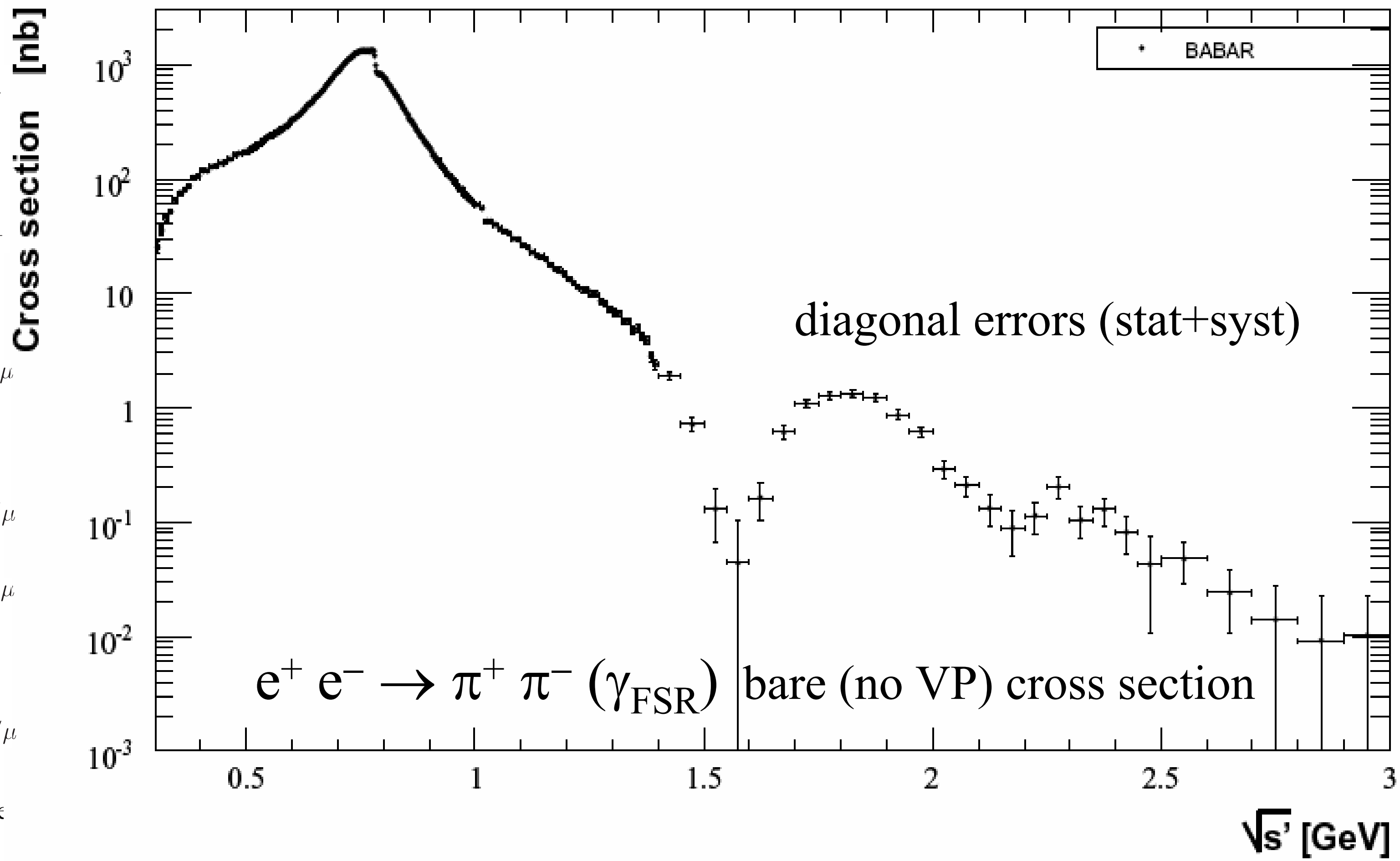


Absolute $\mu^+\mu^-$ cross section agrees with NLO QED within 1.1%



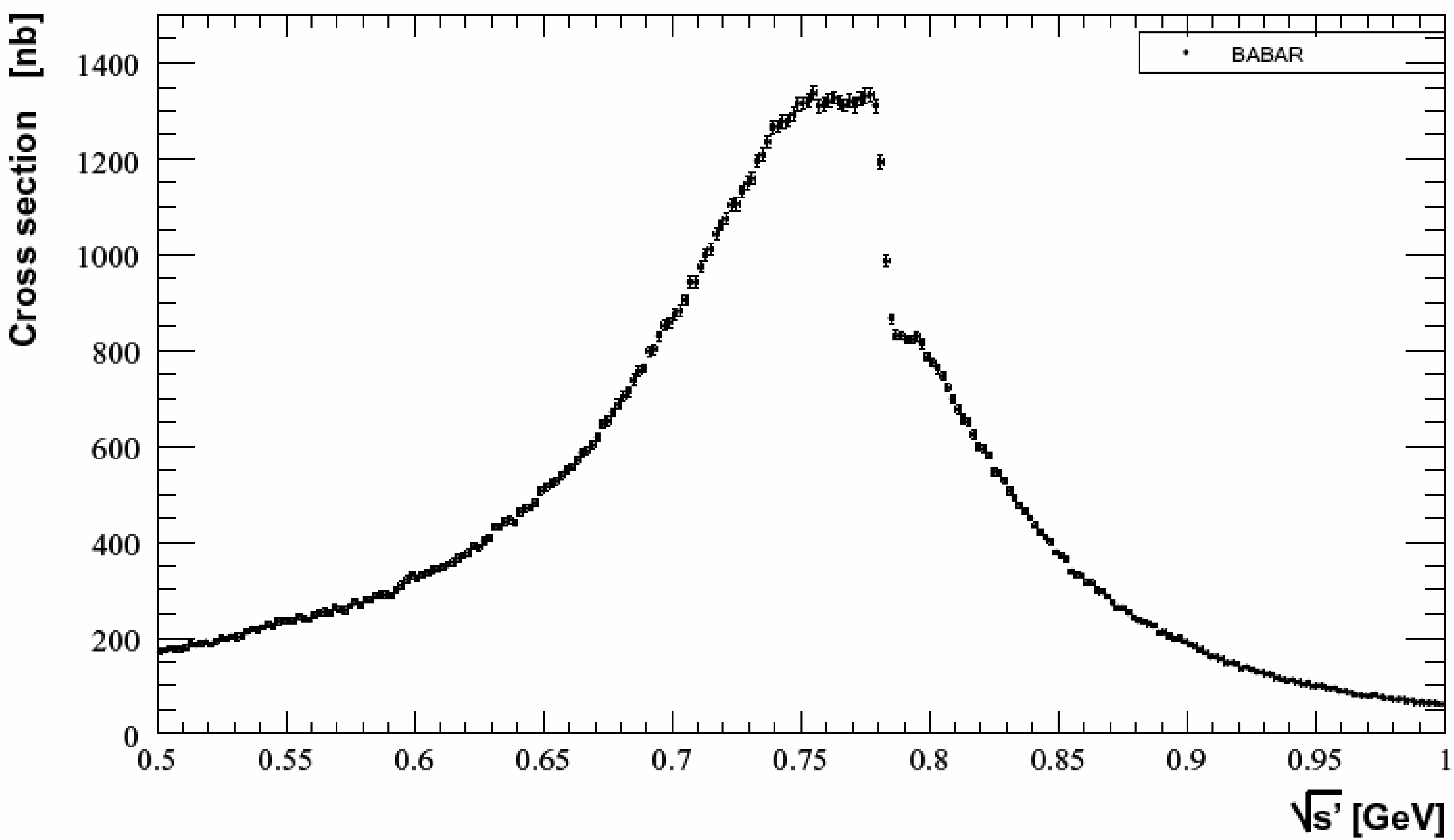






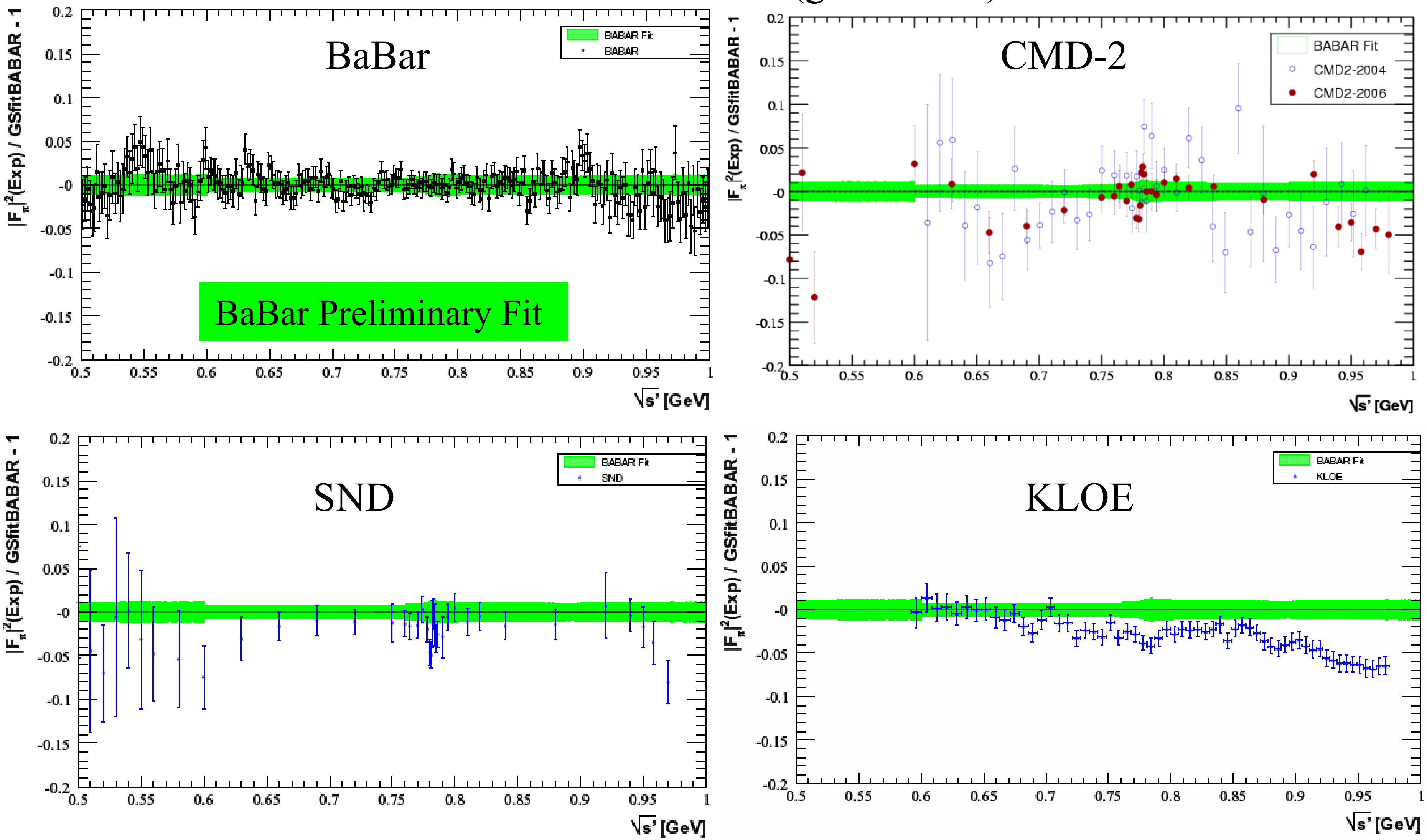
BaBar results in ρ region

2-MeV energy intervals

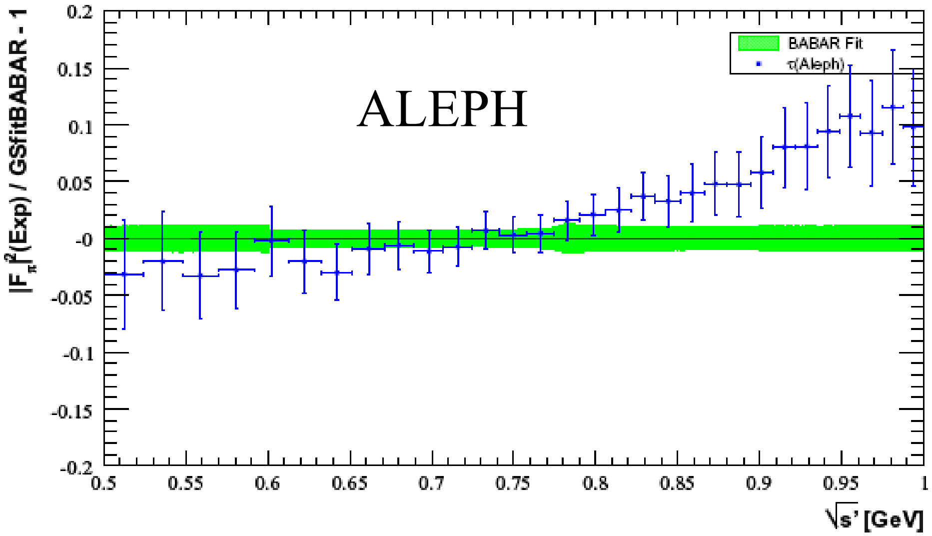


BaBar vs. other e^+e^- data (0.5-1.0 GeV)

direct relative comparison of cross sections with BaBar fit (stat + syst errors included) (green band)



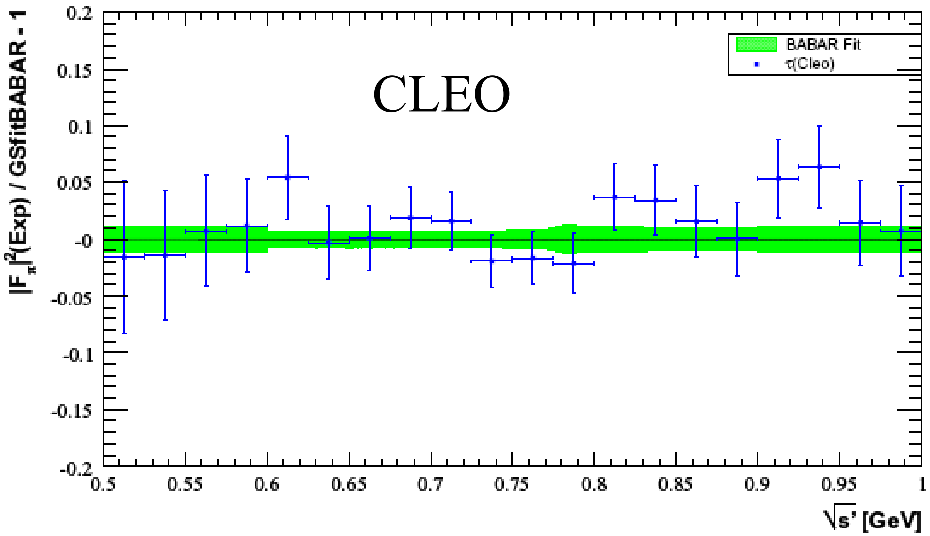
BaBar vs. IB-corrected τ data (0.5-1.0 GeV)

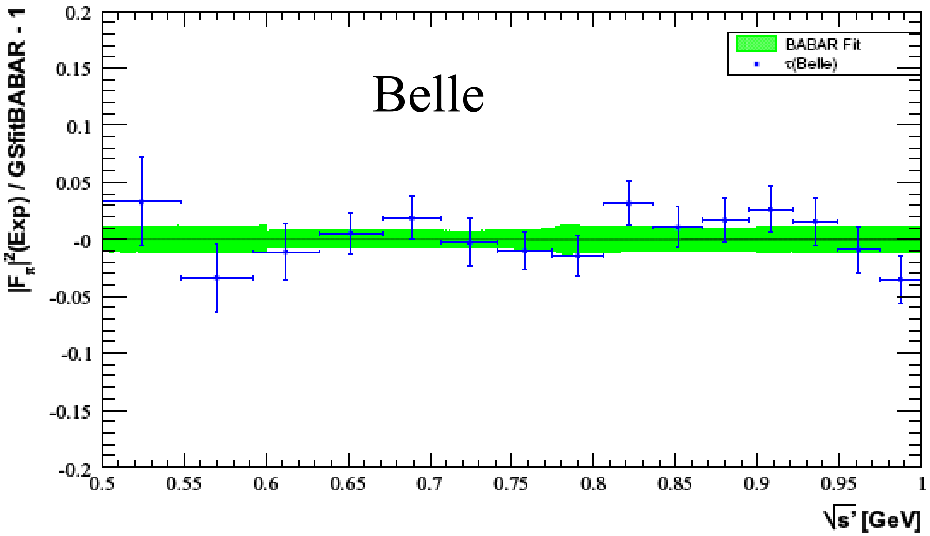


relative comparison w.r.t. BaBar of τ spectral functions corrected for isospin-breaking(IB)

IB corrections: radiative corr., π masses, ρ-ω interference, ρ masses/widths

each τ data normalized to its own BR





Computing $a_\mu^{\pi\pi}$

$$a_\mu^{\pi\pi(\gamma),LO} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} ds\, K(s)\, \sigma^0_{\pi\pi(\gamma)}(s) \;,$$

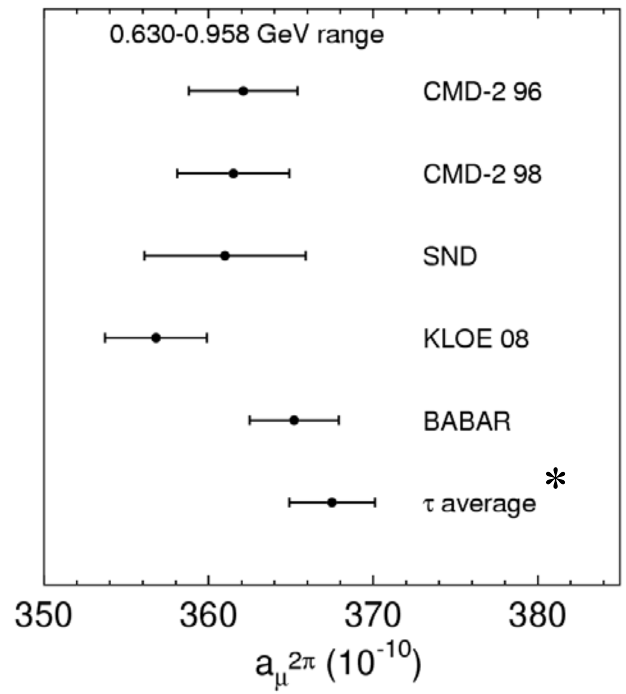
where $K(s)$ is the QED kernel,

$$K(s) = x^2\left(1-\frac{x^2}{2}\right) + (1+x)^2\left(1+\frac{1}{x^2}\right)\left[\ln(1+x) - x + \frac{x^2}{2}\right] + x^2\frac{1+x}{1-x}\ln x \;,$$

with $x = (1-\beta_\mu)/(1+\beta_\mu)$ and $\beta_\mu = (1-4m_\mu^2/s)^{1/2}$.

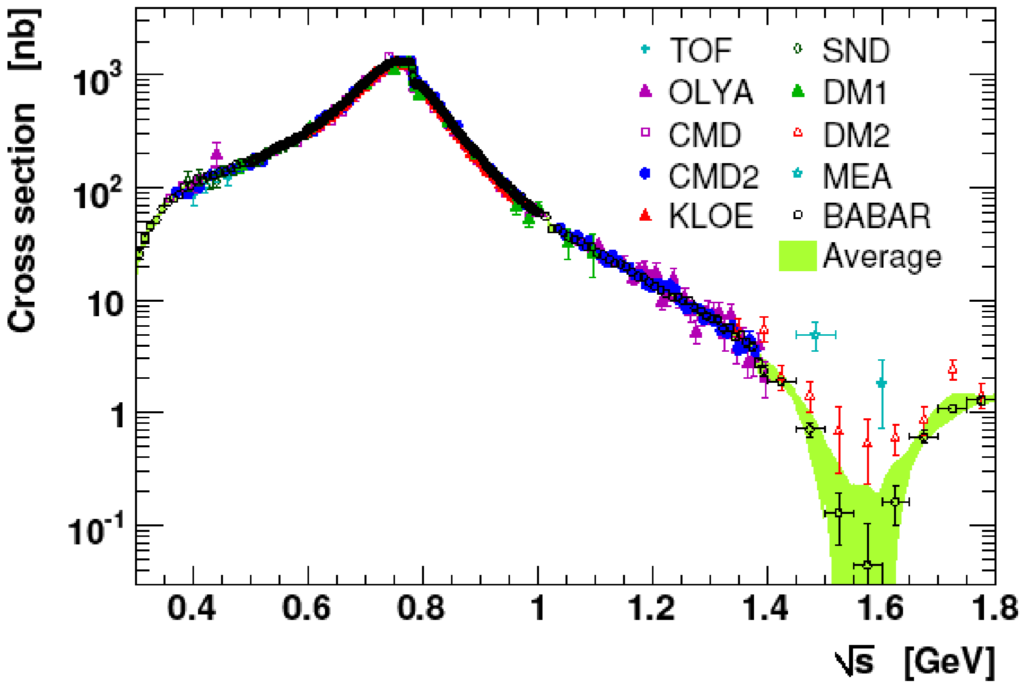
0.28–1.8 (GeV)

BABAR	514.1 ± 3.8	
previous e^+e^- combined τ combined	503.5 ± 3.5 * 515.2 ± 3.5 *	2.4σ



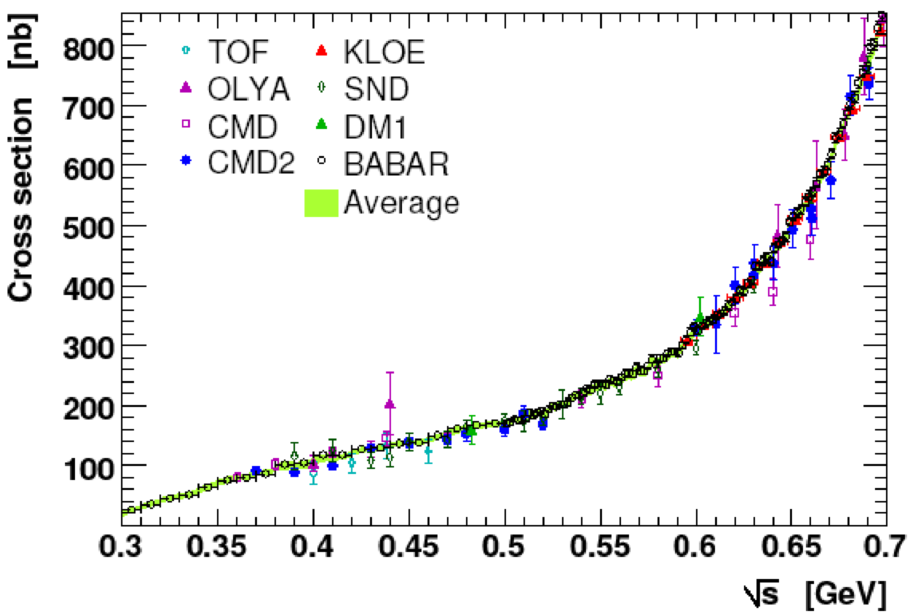
•arXiv:0906.5443 (EPJ C)
Davier-Hoecker-Lopez Castro-BM-Mo-Toledo Sanchez-Wang-Yuan-Zhang

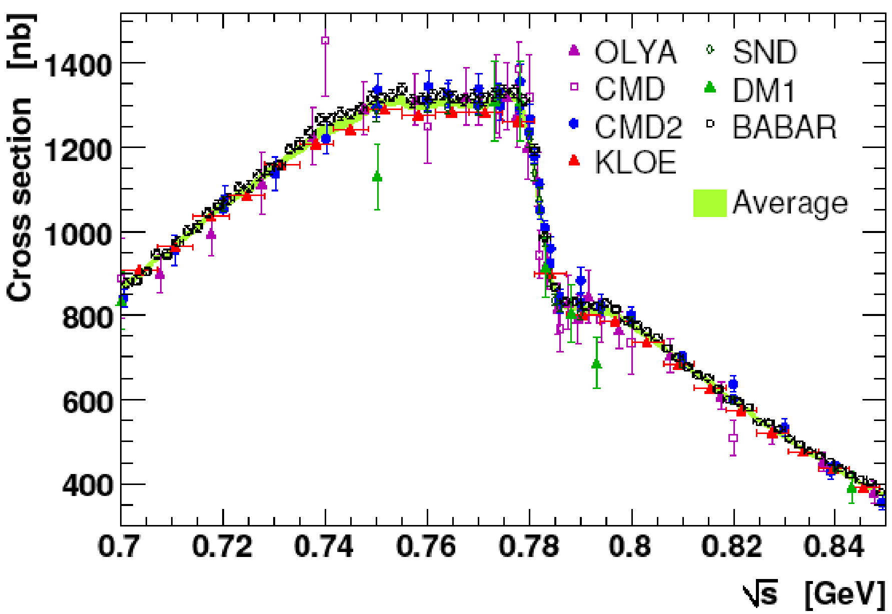
Including BaBar in the e$^+$e$^-$ Combination



arXiv: 0908.4300 (EPJ C)
Davier-Hoecker-BM-Yuan-Zhang

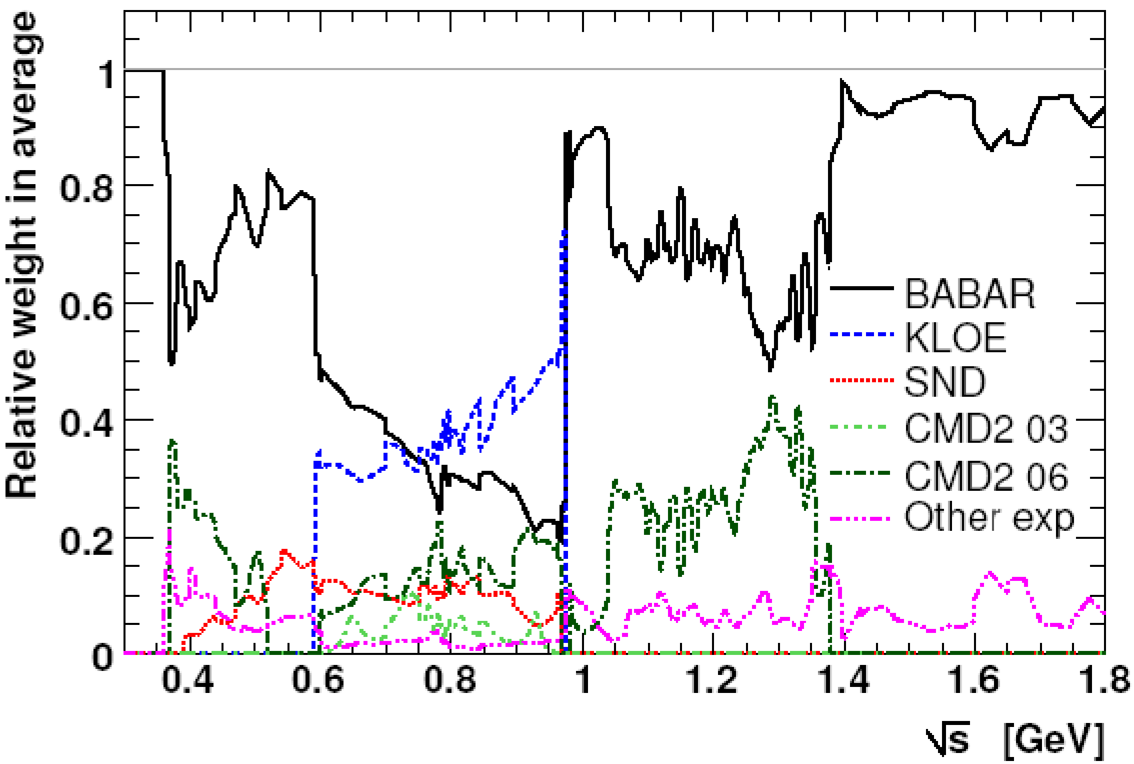
Improved procedure and software (HVPTools) for combining cross section data with arbitrary point spacing/binning





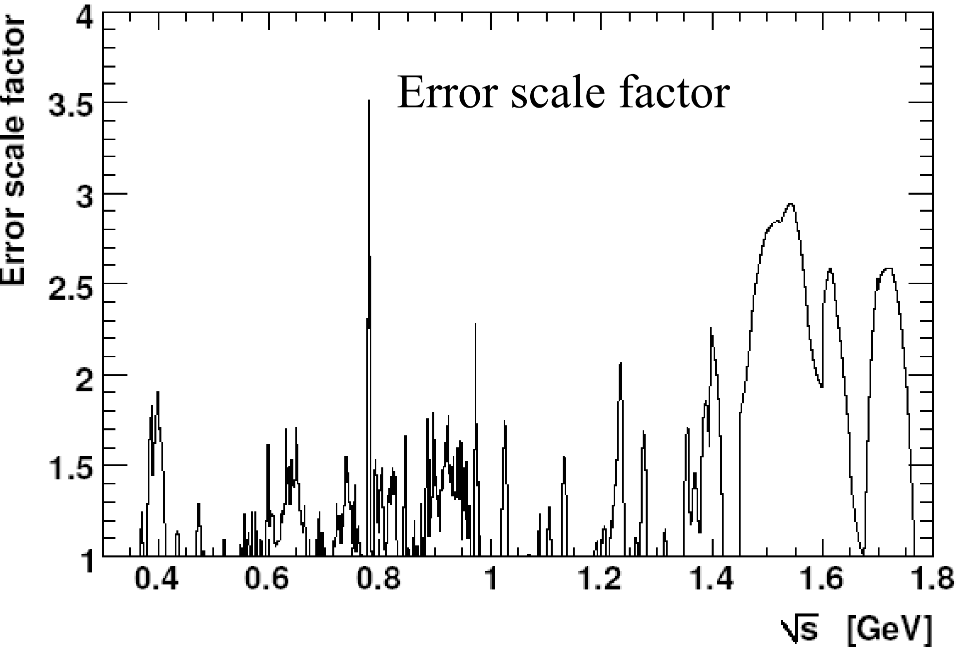
Obtaining the average cross section

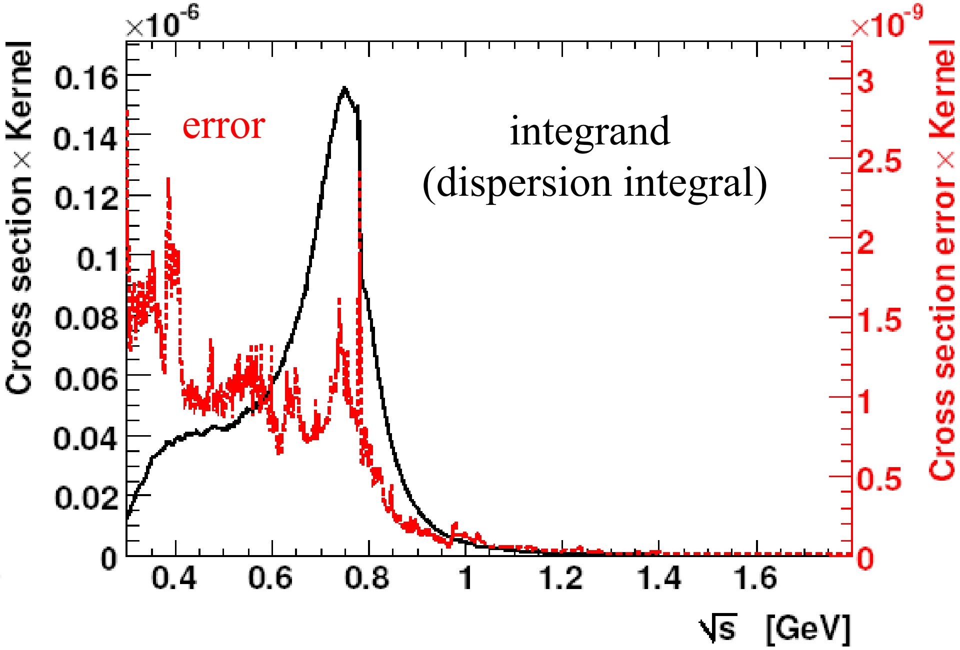
- quadratic interpolation of the data points/bins for each experiment
- local weighted average performed (account for different point spacing/binning)
- possible bias tested using a GS model: negligible for quadratic interpolation
- full covariance matrices: correlations between the data points/bins of an experiment (corrections & systematic errors) and between experiments (ex: VP)
- error propagation using toy simulations
- average dominated by BaBar and KLOE, BaBar covering full range



Obtaining the average cross section

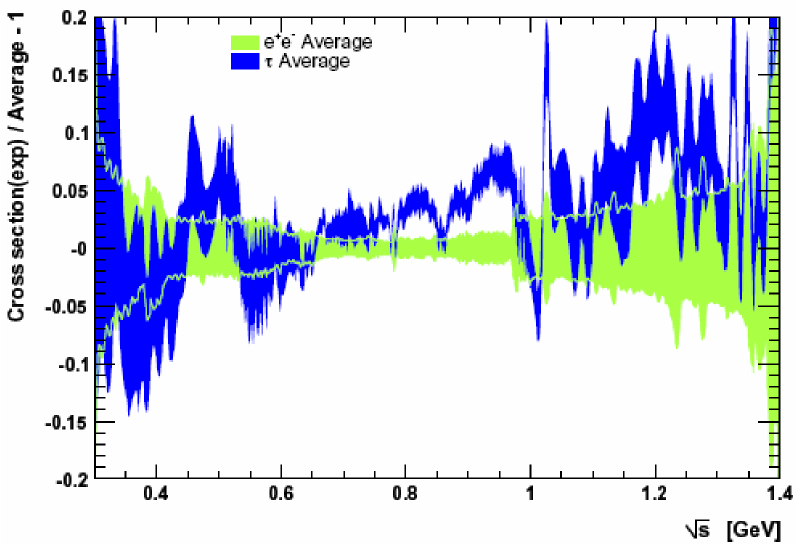
local χ^2 used for error rescaling





g-2 Values for Various Mass Intervals

Energy range (GeV)	Experiment	$a_\mu^{\mathrm{had,LO}}[\pi\pi]$ (10^{-10})
$2m_{\pi^\pm} - 0.3$	Combined e^+e^- (fit)	0.55 ± 0.01
$0.30 - 0.63$	Combined e^+e^-	$132.6 \pm 0.8 \pm 1.0$ (1.3_{tot})
$0.63 - 0.958$	CMD2 03	$361.8 \pm 2.4 \pm 2.1$ (3.2_{tot})
	CMD2 06	$360.2 \pm 1.8 \pm 2.8$ (3.3_{tot})
	SND 06	$360.7 \pm 1.4 \pm 4.7$ (4.9_{tot})
	KLOE 08	$356.8 \pm 0.4 \pm 3.1$ (3.1_{tot})
	BABAR 09	$365.2 \pm 1.9 \pm 1.9$ (2.7_{tot})
	Combined e^+e^-	$360.8 \pm 0.9 \pm 1.8$ (2.0_{tot})
$0.958 - 1.8$	Combined e^+e^-	$14.4 \pm 0.1 \pm 0.1$ (0.2_{tot})
Total	Combined e^+e^-	$508.4 \pm 1.3 \pm 2.6$ (2.9_{tot})
Total	Combined τ [1]	$515.2 \pm 2.0_{\mathrm{exp}} \pm 2.2_{\mathcal{B}} \pm 1.9_{\mathrm{IB}}$ (3.5_{tot})



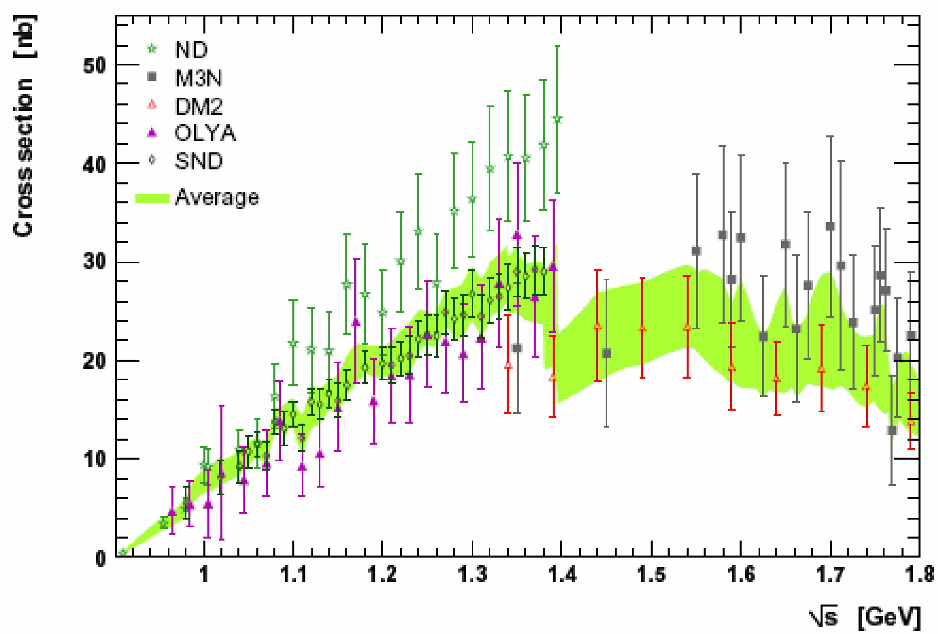
1.5σ

Still difference in shape (KLOE)

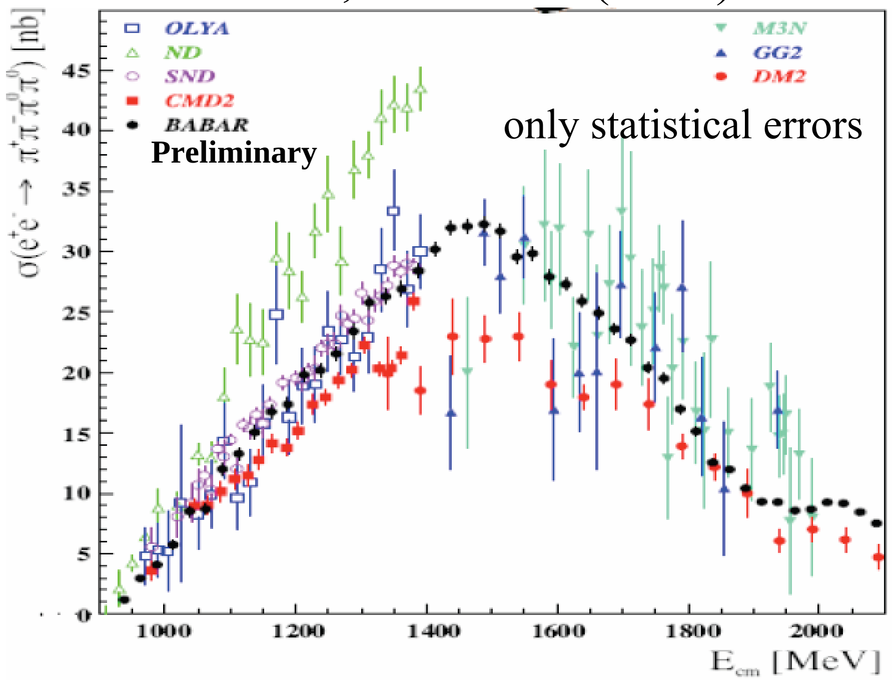
The Problematic $\pi^+\pi^-2\pi^0$ Contribution

old contribution	16.8 ± 1.3	(Davier-Eidelman-Hoecker-Zhang, 2006)
update	17.6 ± 1.7	probably still underestimated (BaBar)
τ	21.4 ± 1.4	

$e^+e^- \rightarrow \pi^+\pi^-2\pi^0$ data (CMD2 discarded)



preliminary ISR BaBar data:
A. Petzold, EPS-HEP (2007)



Where are we?

- including BaBar 2π results in the e+e– combination + estimate of hadronic LBL contribution (Prades-de Rafael-Vainhstein, 2009) yields

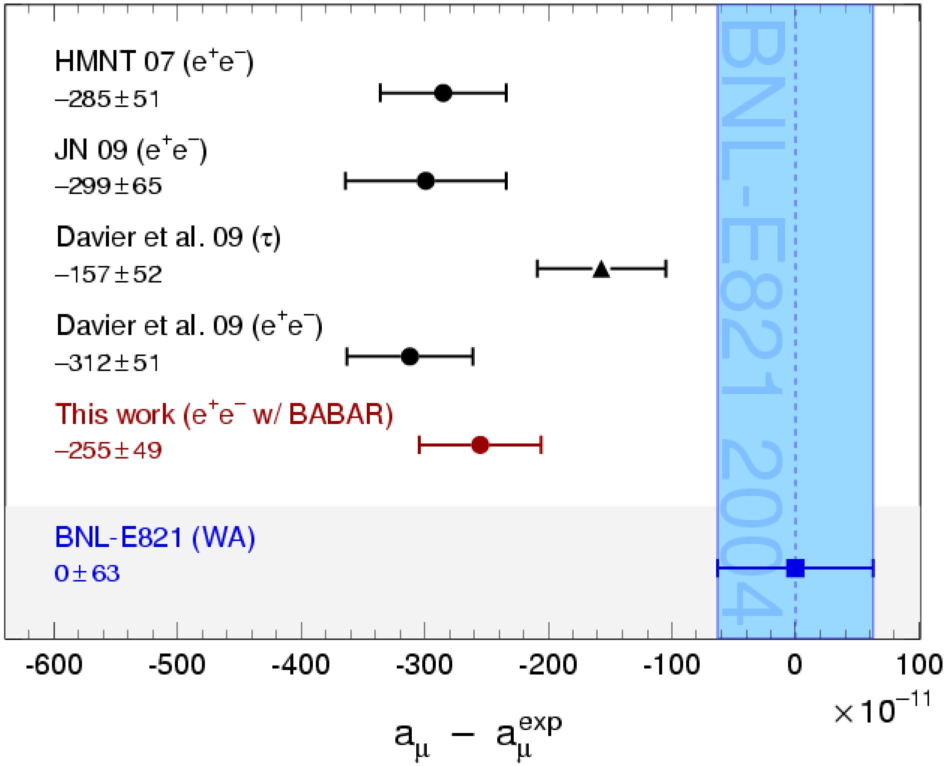
$a_\mu^{SM}[e+e-] = (11\ 659\ 183.4 \pm 4.1 \pm 2.6 \pm 0.2)\ 10^{-10}$

HVP LBL EW (±4.9)

- E-821 updated result* $(11\ 659\ 208.9 \pm 6.3)\ 10^{-10}$

- deviation (ee) $(25.5 \pm 8.0)\ 10^{-10}$ (3.2 σ)

- updated τ analysis +Belle +revisited IB corrections
- deviation (τ) $(15.7 \pm 8.2)\ 10^{-10}$ (1.9 σ)



*new proposal submitted to Fermilab to improve accuracy by a factor 4

Conclusions

- BaBar 2π data are published: the most accurate, but expected precision improvement on the average not reached because of discrepancy with KLOE
- BaBar: absolute $\mu^+\mu^-$ cross section agrees with NLO QED within 1.1%
- previous τ/ee disagreement strongly reduced

  2.9σ (2006) $\rightarrow$ 2.4σ (τ update) $\rightarrow$ 1.5σ (including BaBar)
- a range of values for the deviation from the SM can be obtained, depending on the 2π data used:

BaBar	2.4σ
all ee	3.2σ
all ee –BaBar	3.7σ
all ee –KLOE	2.9σ
τ	1.9σ

- all approaches yield a deviation, but SM test limited by systematic effects not accounted for in the experimental analyses (ee) and/or the corrections to τ data

- at the moment some evidence for a deviation (2–4σ), but not sufficient to establish a contribution from new physics

Backup Slides

Situation at ICHEP'06 / 08

$a_\mu^{had}\,[ee] = (690.9 \pm 4.4) \times 10^{-10}$

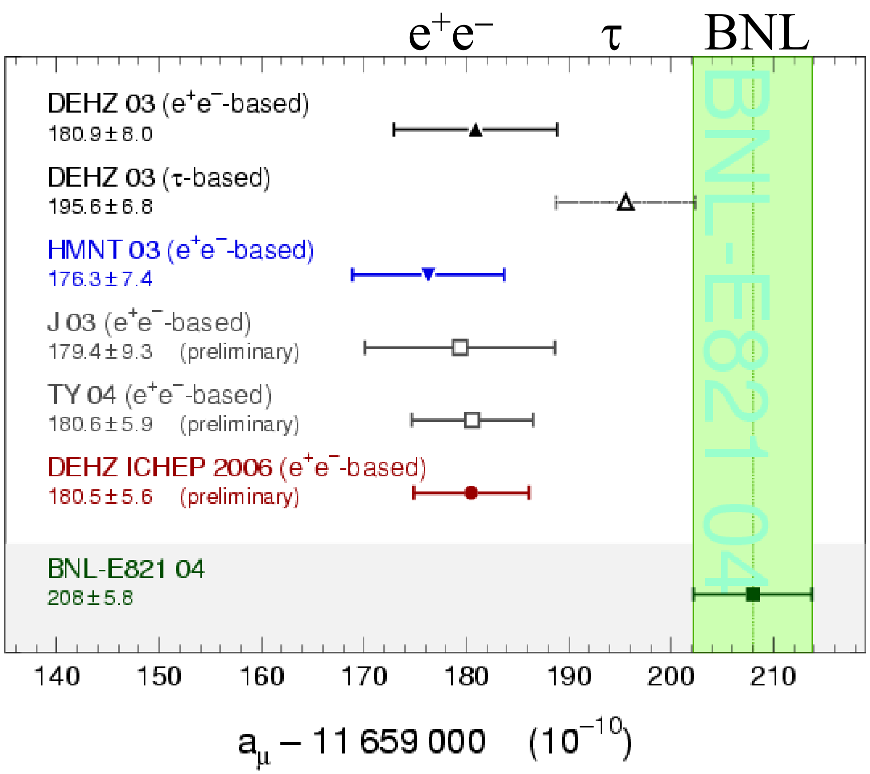
$a_\mu\,[ee] = (11\,659\,180.5 \pm 4.4_{had} \pm 3.5_{LBL} \pm 0.2_{QED+EW}) \times 10^{-10}$

Hadronic HO	$-(9.8 \pm 0.1) \times 10^{-10}$
Hadronic LBL	$+(12.0 \pm 3.5) \times 10^{-10}$
Electroweak	$(15.4 \pm 0.2) \times 10^{-10}$
QED	$(11\,658\,471.9 \pm 0.1) \times 10^{-10}$

Knecht-Nyffeler (2002), Melnikov-Vainhstein (2003)

Davier-Marciano (2004)

Kinoshita-Nio (2006)



Observed Difference with BNL using e^+e^-:

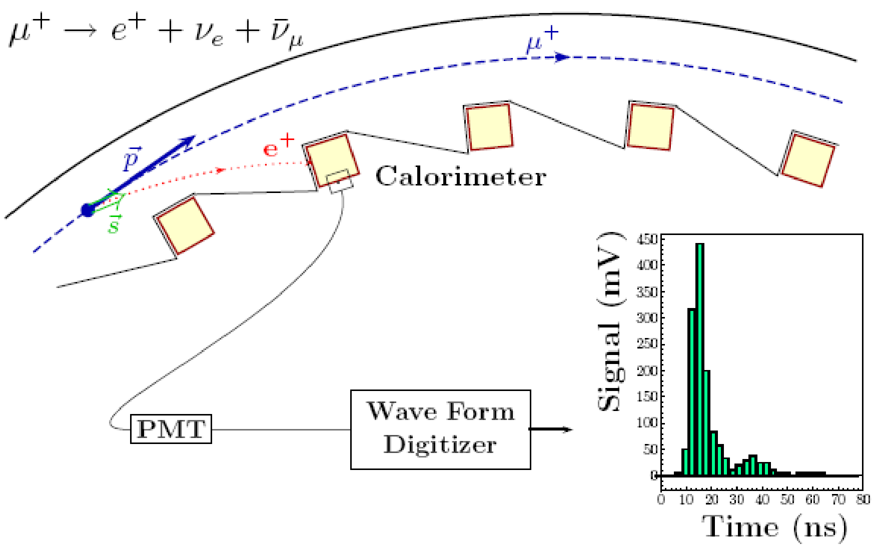
$a_\mu\,[\text{exp}] - a_\mu\,[\text{SM}] = (27.5 \pm 8.4) \times 10^{-10}$

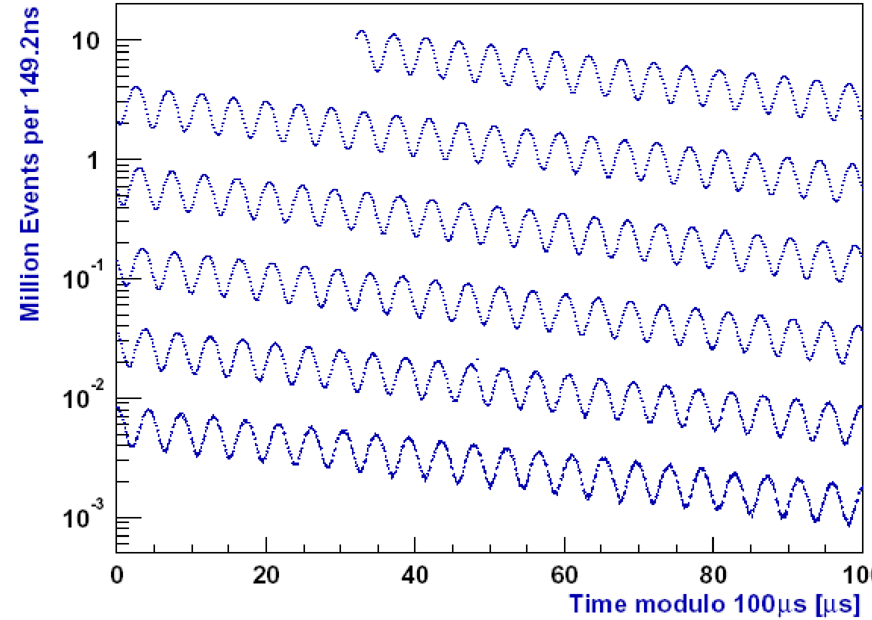
➡ 3.3 „standard deviations"

Estimate using τ data consistent with E-821

The E-821 Direct a_μ Measurement at BNL*

Storage ring technique pioneered at CERN (Farley-Picasso…, 1970s), with μ^+ and μ^- data





$$\omega_a = a_\mu \frac{eB}{m_\mu}$$

$\omega_{precession} - \omega_{rotation}$

a_μ obtained from a ratio of frequencies

result updated with new value for μ_μ/μ_p $(+0.9\ 10^{-10})$

(see new review in RPP2009 (Hoecker-Marciano))

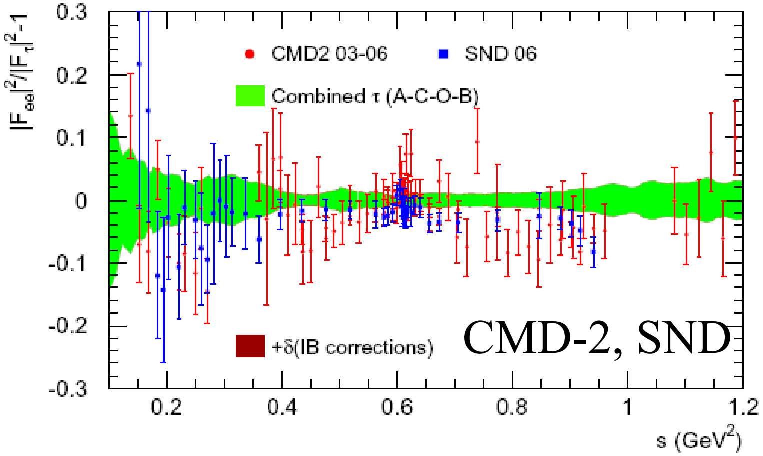
$a_\mu^{exp} = (11\ 659\ 208.9 \pm 5.4 \pm 3.3)\ 10^{-10}$

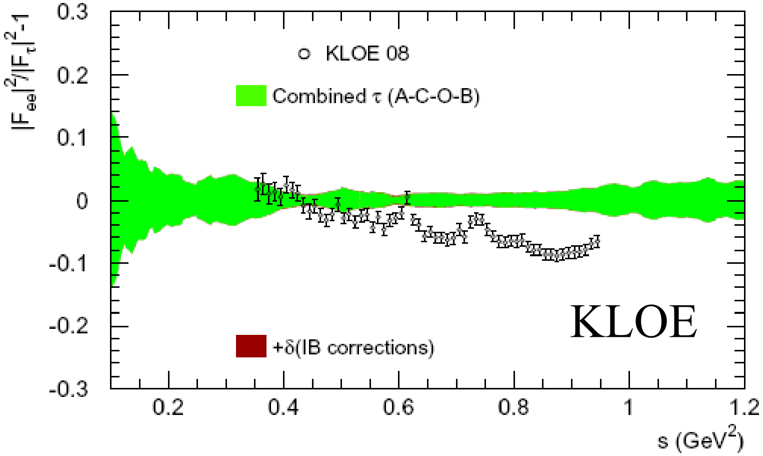
$(\pm\ 6.3)$ (0.54 ppm)

*Bennet et al., 2006, Phys. Rev. D 73, 072003

Revisited Analysis using τ Data: Belle + new IB

Relative comparison of τ and ee spectral functions (τ green band) arXiv:0906.5443 M.Davier et al.



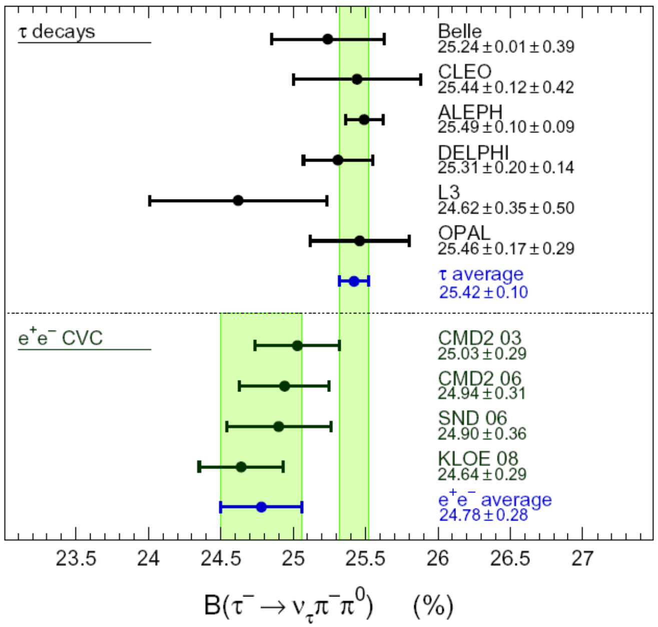


slope…

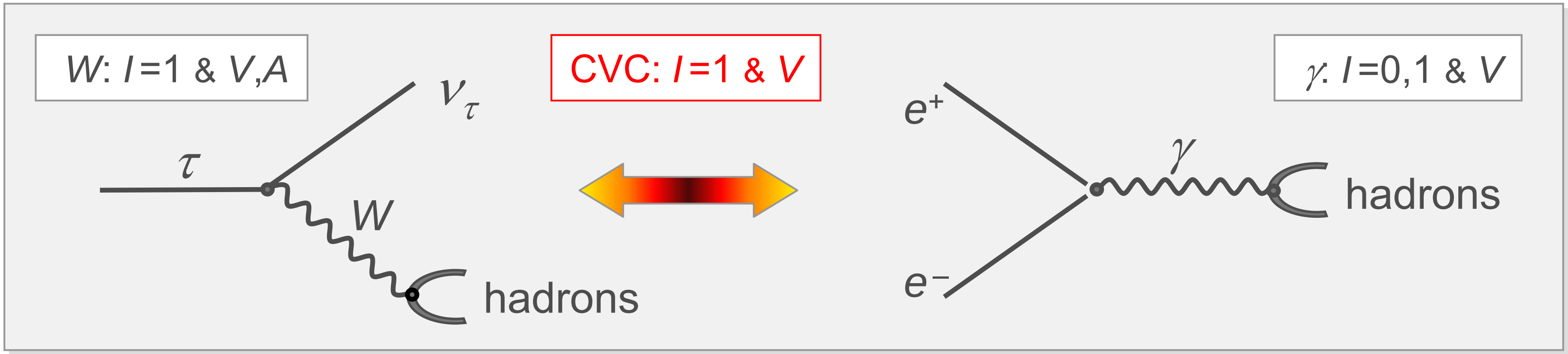
Global test of spectral functions:
prediction of τ BR using ee data

$$\mathcal{B}_X^{\mathrm{CVC}} = \frac{3}{2}\frac{\mathcal{B}_e|V_{ud}|^2}{\pi\alpha^2 m_\tau^2}\int_{s_{\min}}^{m_\tau^2} ds\, s\, \sigma_{X^0}^I \left(1-\frac{s}{m_\tau^2}\right)^2\left(1+\frac{2s}{m_\tau^2}\right)$$

⇒ larger disagreement with KLOE



HVP: τ Data through CVC – SU(2)



Hadronic physics factorizes (spectral Functions)

$$\sigma^{(I=1)}\left[e^+e^- \to \pi^+\pi^-\right] = \frac{4\pi\alpha^2}{s}\upsilon\left[\tau^- \to \pi^-\pi^0\nu_\tau\right]$$

$$\upsilon\left[\tau^- \to \pi^-\pi^0\nu_\tau\right] \propto \frac{BR\left[\tau^- \to \pi^-\pi^0\nu_\tau\right]}{BR\left[\tau^- \to e^-\bar{\nu}_e\nu_\tau\right]} \frac{1}{N_{\pi\pi^0}}\frac{dN_{\pi\pi^0}}{ds} \frac{m_\tau^2}{\left(1-s/m_\tau^2\right)^2\left(1+s/m_\tau^2\right)}$$

branching fractions — mass spectrum — kinematic factor (PS)

SU(2) Breaking

Corrections for SU(2) breaking applied to τ data for dominant $\pi^-\pi^+$ contrib.:

- Electroweak radiative corrections:
  - dominant contribution from short distance correction S_{EW}
  - subleading corrections (small)
  - long distance radiative correction $G_{EM}(s)$
- Charged/neutral mass splitting:
  - $m_{\pi-} \neq m_{\pi 0}$ leads to phase space (cross sec.) and width (FF) corrections
  - ρ - ω mixing (EM $\omega \to \pi^-\pi^+$ decay) corrected using FF model
  - $m_{\rho-} \neq m_{\rho 0}$ *** and $\Gamma_{\rho-} \neq \Gamma_{\rho 0}$ ***
- Electromagnetic decays: $\rho \to \pi\pi\gamma$***, $\rho \to \pi\gamma$, $\rho \to \eta\gamma$, $\rho \to l^+l^-$
- Quark mass difference $m_u \neq m_d$ (negligible)

Marciano-Sirlin' 88

Braaten-Li' 90

Cirigliano-Ecker-Neufeld' 02
Lopez Castro et al.' 06

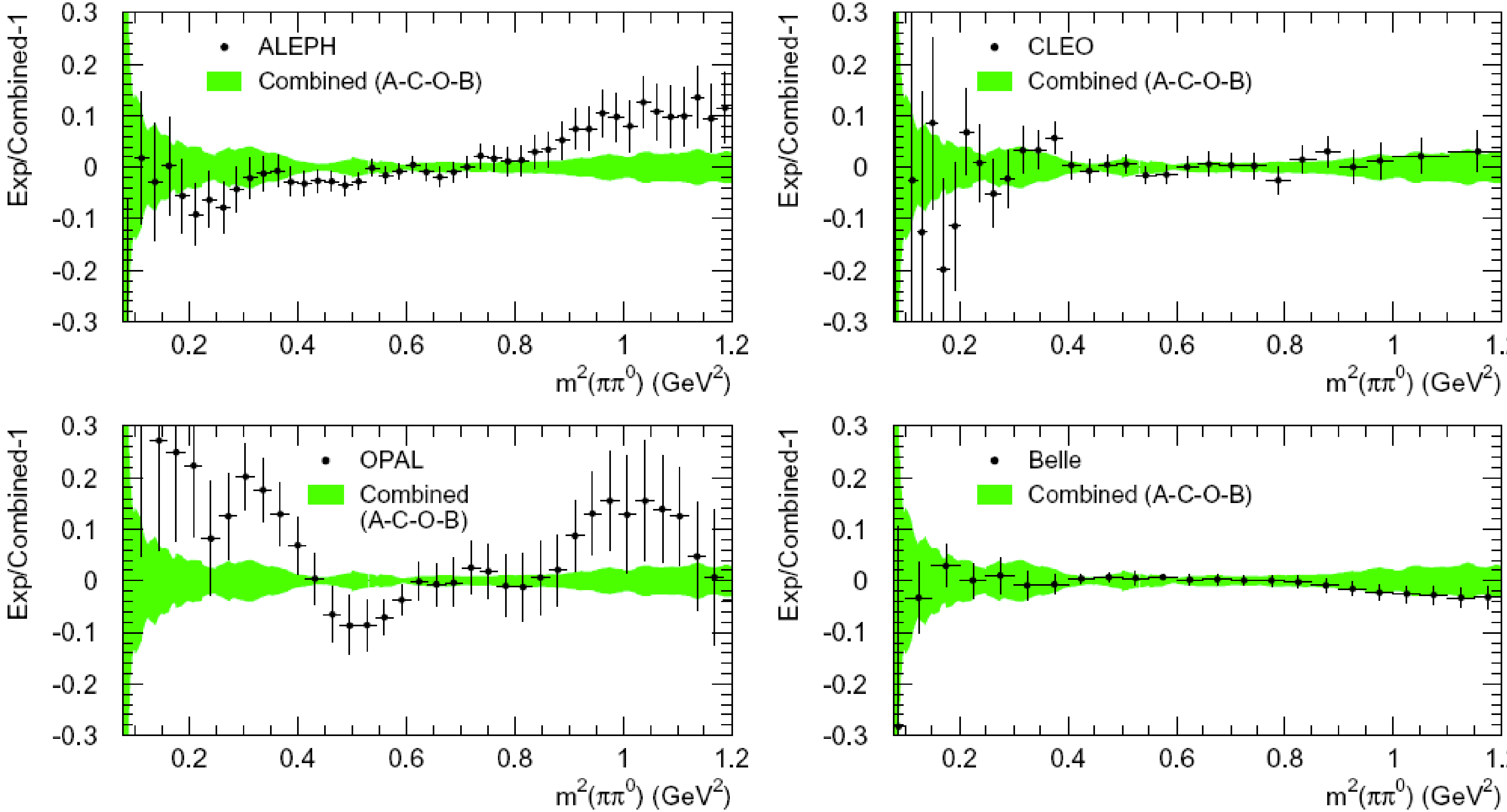
Alemany-Davier-Höcker' 97, Czyż-Kühn' 01

Flores-Baez-Lopez Castro' 08
Davier et al.'09

Revisited Analysis τ Data: new IB corrections

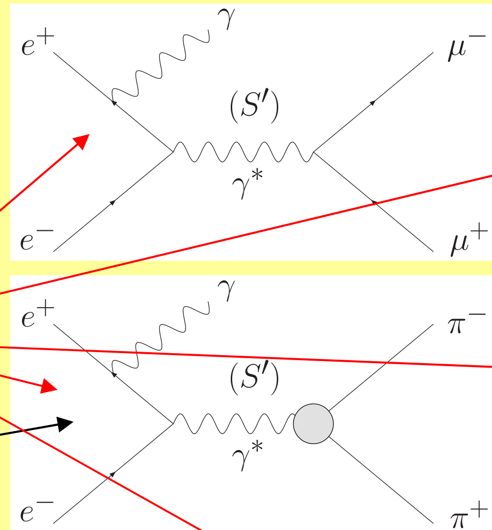
Source	$\Delta a_\mu^{\rm had,LO}[\pi\pi,\tau]$ (10^{-10}) GS model	KS model
$S_{\rm EW}$	-12.21 ± 0.15	
$G_{\rm EM}$	-1.92 ± 0.90	
FSR	$+4.67 \pm 0.47$	
ρ–ω interference	$+2.80 \pm 0.19$	$+2.80 \pm 0.15$
$m_{\pi^\pm} - m_{\pi^0}$ effect on σ	-7.88	
$m_{\pi^\pm} - m_{\pi^0}$ effect on Γ_ρ	$+4.09$	$+4.02$
$m_{\rho^\pm} - m_{\rho^0_{\rm bare}}$	$0.20^{+0.27}_{-0.19}$	$0.11^{+0.19}_{-0.11}$
$\pi\pi\gamma$, electrom. decays	-5.91 ± 0.59	-6.39 ± 0.64
Total	-16.07 ± 1.22	-16.70 ± 1.23
	-16.07 ± 1.85	

Revisited Analysis using τ Data: including Belle

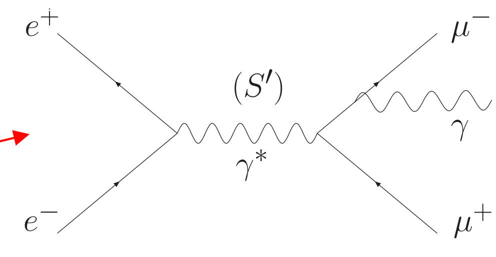


The Relevant Processes in ISR Measurements

ISR



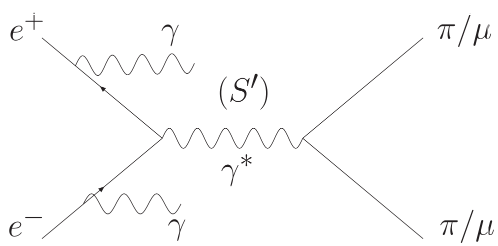
FSR



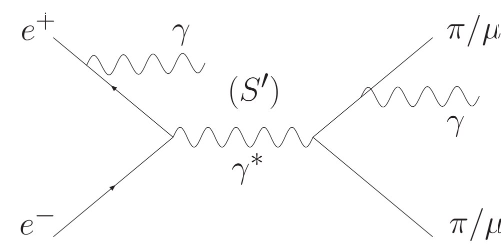
BABAR (tagged ISR)
&
KLOE (fitted ISR)

LO FSR negligible for ππ
at s~(10.6 GeV)^2

ISR + add. ISR



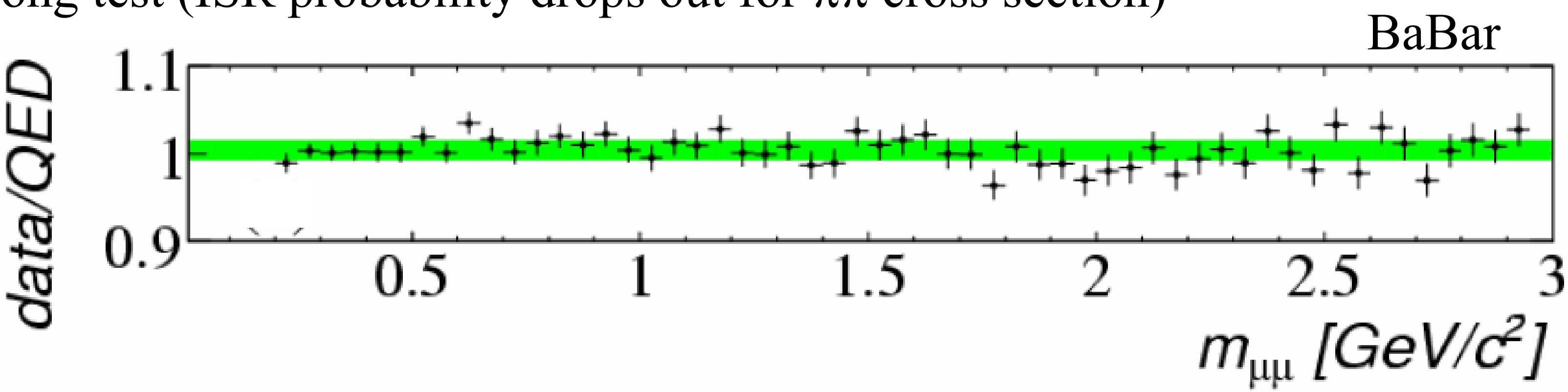
ISR + add. FSR



$e^+ e^- \to \mu^+ \mu^- \gamma_{ISR} (\gamma_{add.})$ and $\pi^+ \pi^- \gamma_{ISR} (\gamma_{add.})$ measured simultaneously in BABAR

BABAR QED Test with μμγ sample

- absolute comparison of μμ mass spectra in data and in simulation
- simulation corrected for data/MC efficiencies
- AfkQed corrected for incomplete NLO using Phokhara
- strong test (ISR probability drops out for ππ cross section)



$$\frac{\sigma^{data}_{\mu\mu\gamma(\gamma)}}{\sigma^{NLO\ QED}_{\mu\mu\gamma(\gamma)}} = 1 + (4.0 \pm 1.9 \pm 5.5 \pm 9.4)\ 10^{-3}$$

(0.2 – 3 GeV)

ISR γ efficiency 3.4 syst.
trig/track/PID 4.0

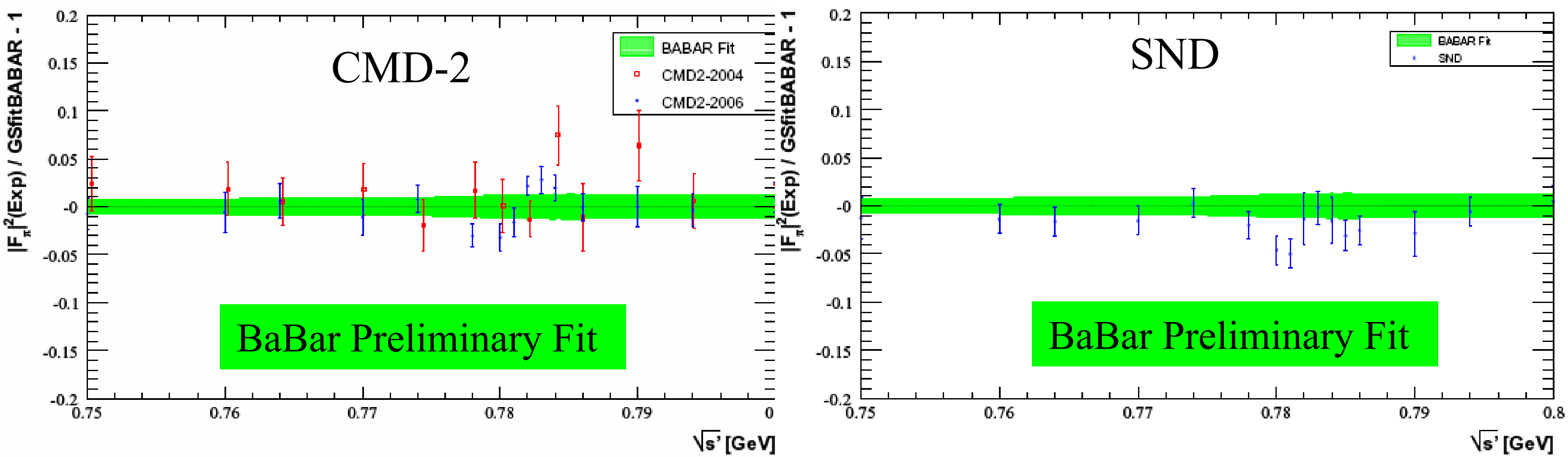
BaBar ee luminosity

BaBar vs.other ee data (ρ–ω interference region)

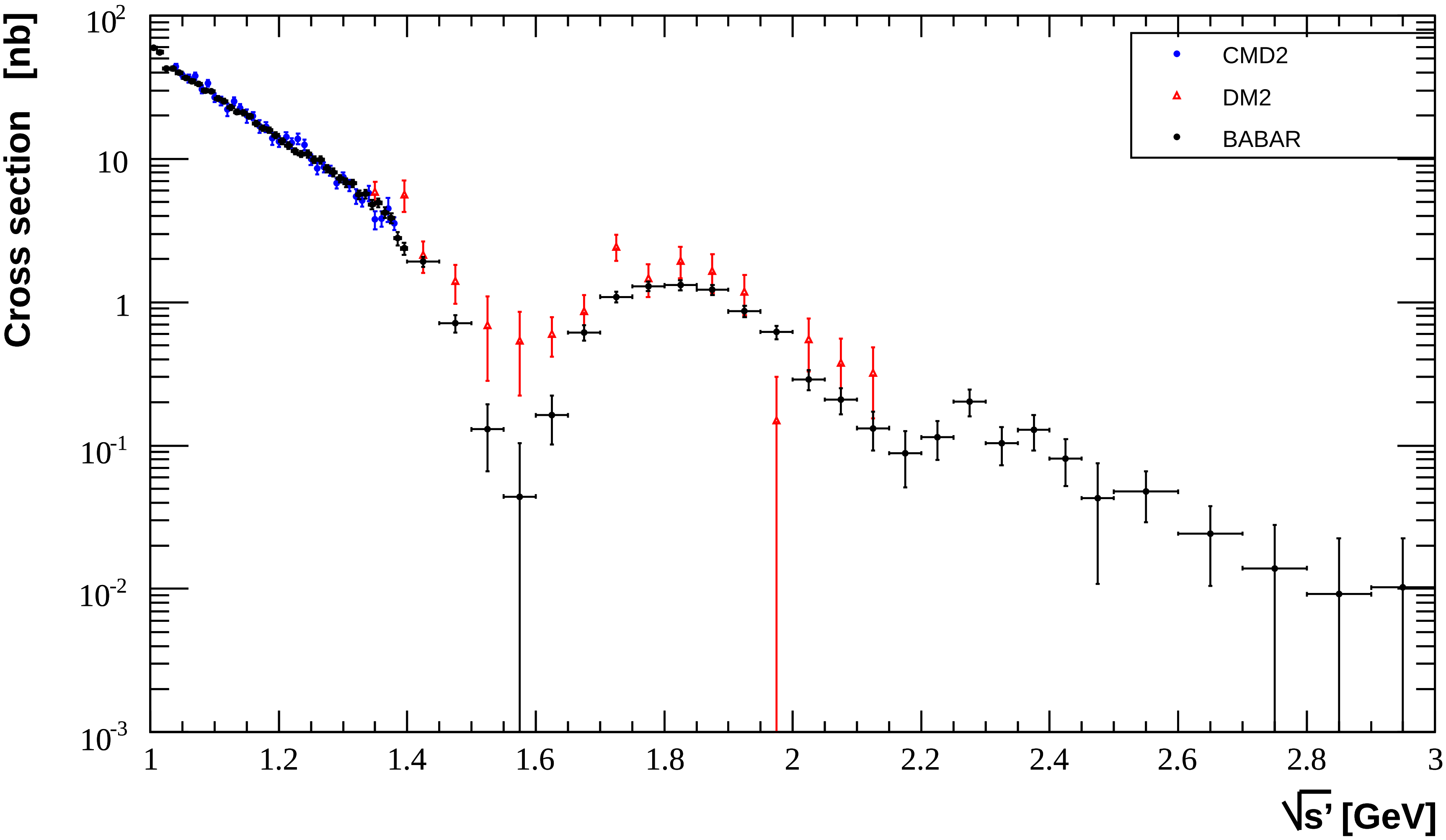
- mass calibration of BaBar checked with ISR-produced $J/\psi \rightarrow \mu\mu$
- expect $-(0.16 \pm 0.16)$ MeV at ρ peak
- ω mass determined through VDM mass fit

$$m_\omega^{fit} - m_\omega^{PDG} = -(0.12 \pm 0.29)\ \text{MeV}$$

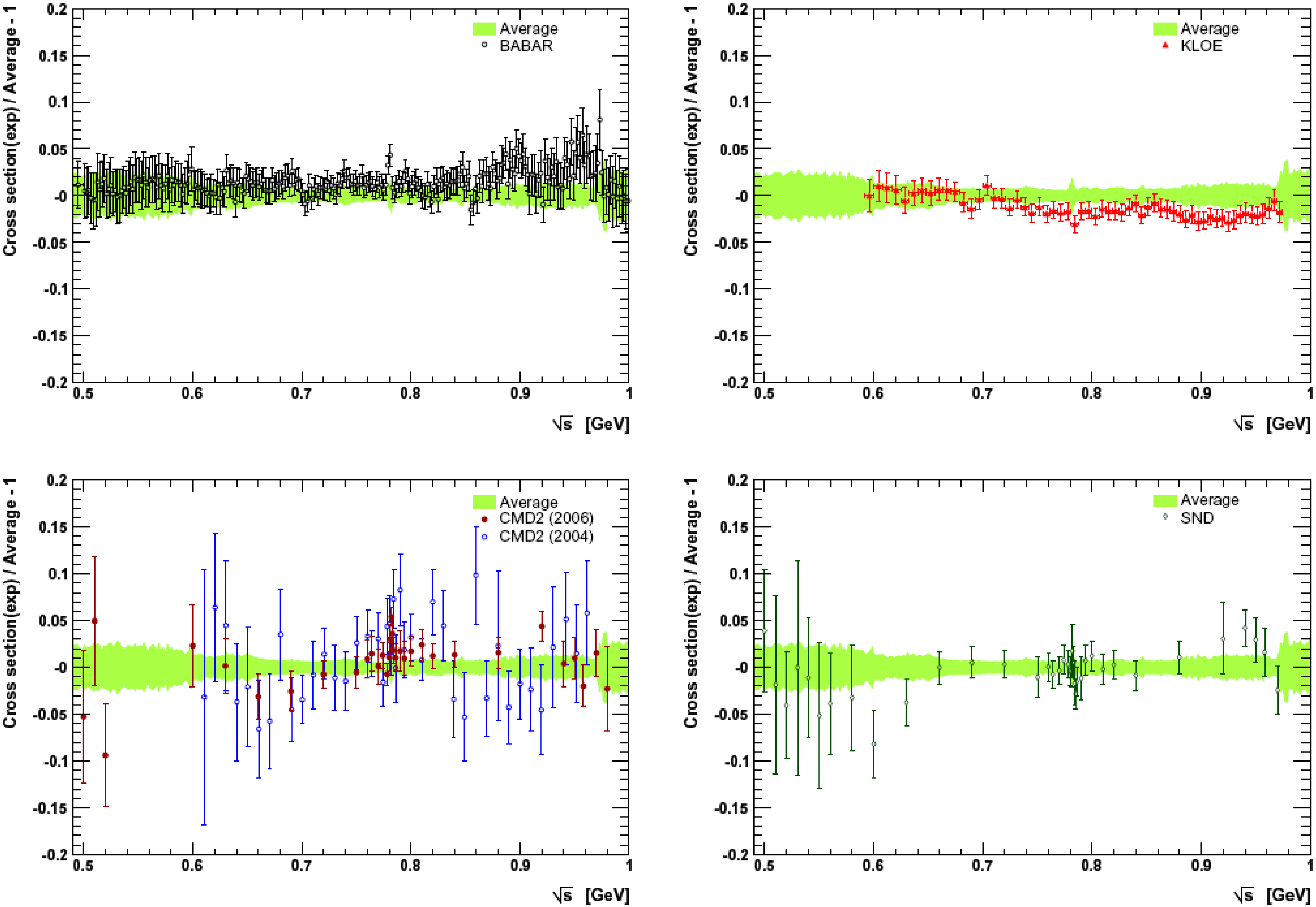
- Novosibirsk data precisely calibrated using resonant depolarization
- comparison BaBar/CMD-2/SND in ρ-ω interference region shows no evidence for a mass shift



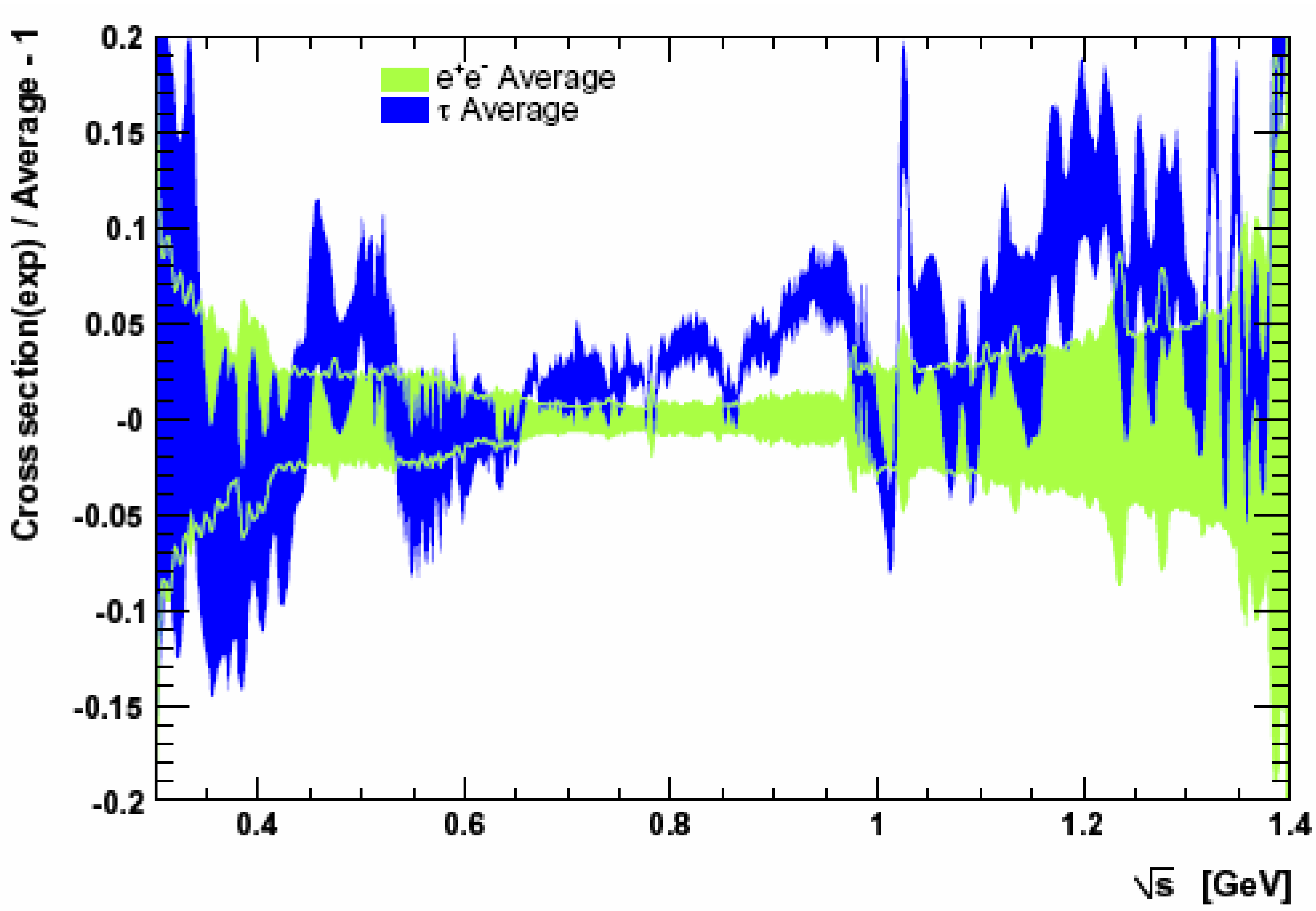
BaBar vs. other experiments at larger mass



Consistency of Experiments with Average

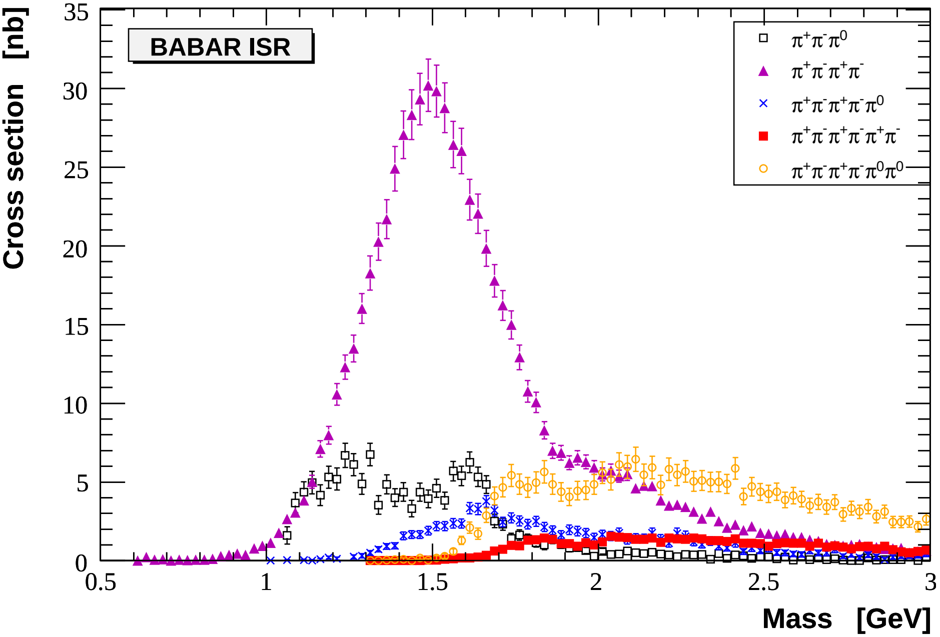


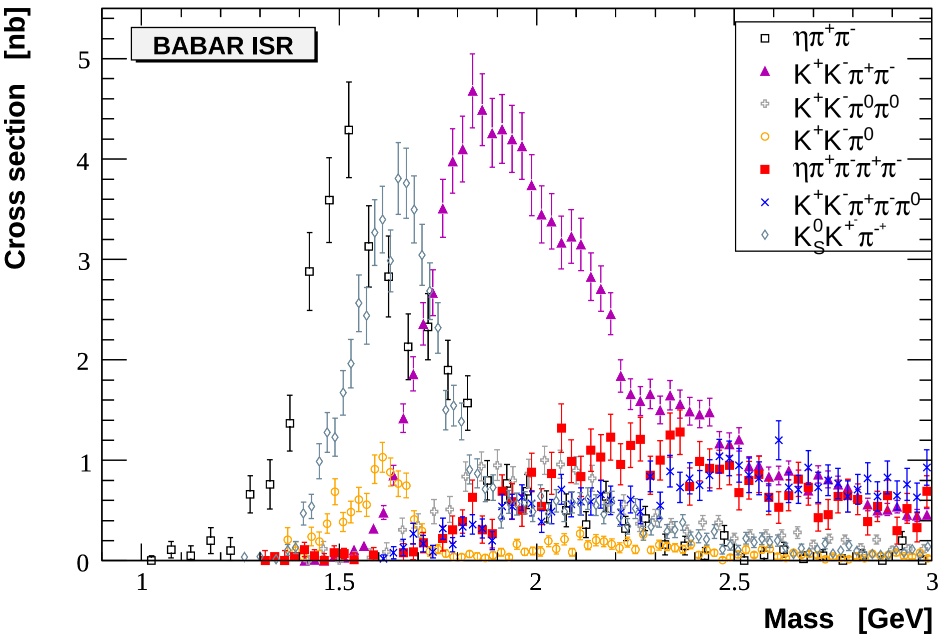
Combined Spectral Functions Comparison



BaBar Multi-hadronic Published Results

Statistical + systematic errors





Still more channels under analysis: K^+K^-, $KK\pi\pi$ with K^0

Other hadronic contributions

from Davier-Eidelman-Hoecker-Zhang (2006)

Modes	Energy [GeV]	e^+e^-	τ
$\pi^+\pi^- 2\pi^0$	$4m_\pi$ – 1.8	$16.8 \pm 1.3 \pm 0.2_{rad}$	$21.4 \pm 1.3 \pm 0.6_{SU(2)}$
$2\pi^+ 2\pi^-$ (+BaBar)	$4m_\pi$ – 1.8	$13.1 \pm 0.4 \pm 0.0_{rad}$	$12.3 \pm 1.0 \pm 0.4_{SU(2)}$
ω (782)	0.3 – 0.81	$38.0 \pm 1.0 \pm 0.3_{rad}$	–
ϕ (1020)	1.0 – 1.055	$35.7 \pm 0.8 \pm 0.2_{rad}$	–
Other excl. (+BaBar)	$2m_\pi$ – 1.8	$24.3 \pm 1.3 \pm 0.2_{rad}$	–
J/ψ, ψ(2S)	3.08 – 3.11	$7.4 \pm 0.4 \pm 0.0_{rad}$	–
R [QCD]	1.8 – 3.7	$33.9 \pm 0.5_{theo}$	–
R [data]	3.7 – 5.0	$7.2 \pm 0.3 \pm 0.0_{rad}$	–
R [QCD]	5.0 – ∞	$9.9 \pm 0.2_{theo}$	–

$\Rightarrow$ another large long-standing discrepancy in the $\pi^+\pi^- 2\pi^0$ channel !

Discussion

- BaBar analysis of $\pi^+\pi^-$ and $\mu^+\mu^-$ ISR processes completed, precision: 0.5% in ρ region (0.6-0.9 GeV)
- Absolute $\mu^+\mu^-$ cross section agrees with NLO QED within 1.1%
- $ee\rightarrow\pi^+\pi^-(\gamma)$ cross section very insensitive to MC generator
- full range of interest covered from 0.3 to 3 GeV
- Comparison with data from earlier experiments
  - fair agreement with CMD-2 and SND, poor with KLOE
  - agreement with τ data
- Contribution to a_μ from BaBar is $(514.1 \pm 2.2 \pm 3.1)\times 10^{-10}$ in 0.28-1.8 GeV
- BaBar result has an accuracy (0.7%) comparable to combined previous results
- Contribution from multi-hadronic channels will continue to be updated with more results forthcoming from BaBar, particularly $\pi^+\pi^-2\pi^0$
- Deviation between BNL measurement and theory prediction reduced using BaBar $\pi^+\pi^-$ data

$$\boldsymbol{a_\mu}[\text{exp}] - \boldsymbol{a_\mu}[\text{SM}] = (\mathbf{19.8} \pm \mathbf{8.4})\times 10^{-10} \quad \pi^+\pi^- \text{ from BaBar only}$$

$$25.5 \pm 8.0 \quad \text{combined } e^+e^- \text{ including BaBar}$$

Perspectives

- first priority is a clarification of the BaBar/KLOE discrepancy:
  - origin of the 'slope' (was very pronounced with the 2004 KLOE results, reduced now with the 2008 results)
  - normalization difference on ρ peak (most direct effect on a_μ)
  - Novosibirsk results in-between, closer to BaBar
  - slope also seen in KLOE/τ comparison; BaBar agrees with τ

- further checks of the KLOE results are possible: as method is based on MC simulation for ISR and additional ISR/FSR probabilities $\Rightarrow$ test with $\mu\mu\gamma$ analysis

- contribution from multi-hadronic channels will continue to be updated with more results forthcoming from BaBar, particularly $2\pi\ 2\pi^0$

- more ee data expected from VEPP-2000 in Novosibirsk

- experimental error of E-821 direct a_μ measurement is a limitation, already now
  - $\Rightarrow$ new proposal submitted to Fermilab to improve accuracy by a factor 4
  - $\Rightarrow$ project at JPARC