

Lepton and Slepton mass matrices from $\Delta(54)$ symmetry

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We present the lepton flavor model with $\Delta(54)$. Our model reproduces the tri-bimaximal mixing in the parameter region around degenerate neutrino masses or two massless neutrinos. We also study SUSY breaking terms in the slepton sector. Three families of left-handed and right-handed slepton masses are almost degenerate. Our model leads to smaller values of flavor changing neutral currents than the present experimental bounds.

1 Introduction

Recent experimental data of the neutrino oscillation indicate the tri-bimaximal form¹ of mixing angles in the lepton sector within a good accuracy². Thus, it is a promising step to study how to realize the tri-bimaximal mixing matrix, in order to understand the origin of the lepton flavor. Many authors have been attempting it by using various scenarios. Non-Abelian discrete flavor symmetries can provide a natural guidance to constrain many free parameters in the Yukawa sector. Actually, several types of models with various non-Abelian discrete flavor symmetries have been proposed, such as S_3 , D_4 , Q_4 , Q_6 , A_4 , T' , S_4 , $\Delta(27)$. In addition to the above (rather) bottom-up motivation, we also have a top-down motivation. Certain classes of non-Abelian flavor symmetries can be derived from superstring theories. For example, D_4 and $\Delta(54)$ flavor symmetries can be obtained in heterotic orbifold models³. In addition to these flavor symmetries, the $\Delta(27)$ flavor symmetry can be derived from magnetized/intersecting D-brane models⁴. Thus, it is quite important to study phenomenological aspects of these non-Abelian flavor symmetries.

Here, we focus on the $\Delta(54)$ discrete symmetry^{5,6}. Although it includes several interesting aspects, few authors have considered up to now. The first aspect is that it consists of two types of Z_3 subgroups and an S_3 subgroup. The S_3 group is known as the minimal non-Abelian discrete symmetry, and the semi-direct product structure of $\Delta(54)$ between Z_3 and S_3 induces triplet irreducible representations. That suggests that the $\Delta(54)$ symmetry could lead to interesting models.

2 $\Delta(54)$ flavor model for leptons

The group $\Delta(54)$ has irreducible representations 1_1 , 1_2 , 2_1 , 2_2 , 2_3 , 2_4 , $3_1^{(1)}$, $3_1^{(2)}$, $3_2^{(1)}$, and $3_2^{(2)}$. There are four triplets and products of $3_1^{(1)} \times 3_1^{(2)}$ and $3_2^{(1)} \times 3_2^{(2)}$ lead to the trivial singlet. The relevant multiplication rules are shown in⁵.

We present the model of the lepton flavor with the $\Delta(54)$ group. The triplet representations of the group correspond to the three generations of leptons. The left-handed leptons (l_e, l_μ, l_τ),

	(l_e, l_μ, l_τ)	(e^c, μ^c, τ^c)	$(N_e^c, N_\mu^c, N_\tau^c)$	$h_{u(d)}$	χ_1	(χ_2, χ_3)	(χ_4, χ_5, χ_6)
$\Delta(54)$	$3_1^{(1)}$	$3_2^{(2)}$	$3_1^{(2)}$	1_1	1_2	2_1	$3_1^{(2)}$

Table 1: Assignments of $\Delta(54)$ representations

the right-handed charged leptons (e^c, μ^c, τ^c) and the right-handed neutrinos $(N_e^c, N_\mu^c, N_\tau^c)$ are assigned to $3_1^{(1)}$, $3_2^{(2)}$, and $3_1^{(2)}$, respectively. New scalars are supposed to be $SU(2)$ gauge singlets. χ_1 , (χ_2, χ_3) and (χ_4, χ_5, χ_6) are assigned to 1_2 , 2_1 , and $3_1^{(2)}$ of the $\Delta(54)$ representations, respectively. The particle assignments of $\Delta(54)$ are summarized in Table 1. The usual Higgs doublets h_u and h_d are assigned to the trivial singlet 1_1 of $\Delta(54)$.

We assume that the scalar fields, $h_{u,d}$ and χ_i , develop their vacuum expectation values (VEVs) as follows:

$$\langle h_u \rangle = v_u, \quad \langle h_d \rangle = v_d, \quad \langle \chi_i \rangle = \alpha_i \Lambda, \quad (1)$$

where $i = 1, \dots, 6$ and Λ is the cutoff scale. We obtain the diagonal matrix for charged leptons

$$M_l = y_1^l v_d \begin{pmatrix} \alpha_1 & 0 & 0 \\ 0 & \alpha_1 & 0 \\ 0 & 0 & \alpha_1 \end{pmatrix} + y_2^l v_d \begin{pmatrix} \omega \alpha_2 - \alpha_3 & 0 & 0 \\ 0 & \omega^2 \alpha_2 - \omega^2 \alpha_3 & 0 \\ 0 & 0 & \alpha_2 - \omega \alpha_3 \end{pmatrix}, \quad (2)$$

By using the seesaw mechanism $M_\nu = M_D^T M_N^{-1} M_D$, the neutrino mass matrix can be obtained. In our model, the lepton mixing comes from the structure of the neutrino mass matrix. In order to reproduce the maximal mixing between ν_μ and ν_τ , we take $\alpha_5 = \alpha_6$, and then we have

$$M_\nu = \frac{y_D^2 v_u^2}{\Lambda d} \begin{pmatrix} y_1^2 \alpha_5^2 - y_2^2 \alpha_4^2 & -y_1 y_2 \alpha_5^2 + y_2^2 \alpha_4 \alpha_5 & -y_1 y_2 \alpha_5^2 + y_2^2 \alpha_4 \alpha_5 \\ -y_1 y_2 \alpha_5^2 + y_2^2 \alpha_4 \alpha_5 & y_1^2 \alpha_4 \alpha_5 - y_2^2 \alpha_5^2 & -y_1 y_2 \alpha_4^2 + y_2^2 \alpha_5^2 \\ -y_1 y_2 \alpha_5^2 + y_2^2 \alpha_4 \alpha_5 & -y_1 y_2 \alpha_4^2 + y_2^2 \alpha_5^2 & y_1^2 \alpha_4 \alpha_5 - y_2^2 \alpha_5^2 \end{pmatrix}, \quad (3)$$

where $d = y_1^3 \alpha_4 \alpha_5 \alpha_6 - y_1 y_2^2 \alpha_4^3 - y_1 y_2^2 \alpha_5^3 - y_1 y_2^2 \alpha_6^3 + 2 y_2^3 \alpha_4 \alpha_5 \alpha_6$. From now on, we denote y_D as Yukawa coupling of Dirac neutrino and y_1, y_2 of Majorana neutrino. Above mass matrix indicates $\theta_{23} = 45^\circ$, $\theta_{13} = 0$ and

$$\theta_{12} = \frac{1}{2} \arctan \frac{2\sqrt{2} y_2 \alpha_5}{y_1 \alpha_5 + y_2 \alpha_4 - y_1 \alpha_4} \quad (y_2 \alpha_4 \neq y_1 \alpha_5). \quad (4)$$

Neutrino masses are given by

$$\begin{aligned} m_1 &= \frac{y_D^2 v_u^2}{\Lambda d} [y_1^2 \alpha_5^2 - y_2^2 \alpha_4^2 - \sqrt{2}(-y_1 y_2 \alpha_5^2 + y_2^2 \alpha_4 \alpha_5) \tan \theta_{12}], \\ m_2 &= \frac{y_D^2 v_u^2}{\Lambda d} [y_1^2 \alpha_4 \alpha_5 - y_1 y_2 \alpha_4^2 + \sqrt{2}(-y_1 y_2 \alpha_5^2 + y_2^2 \alpha_4 \alpha_5) \tan \theta_{12}], \\ m_3 &= \frac{y_D^2 v_u^2}{\Lambda d} [y_1^2 \alpha_4 \alpha_5 + y_1 y_2 \alpha_4^2 - 2 y_2^2 \alpha_5^2], \end{aligned} \quad (5)$$

which are reconciled with the normal hierarchy of neutrino masses in the case of $y_1 \alpha_5 \simeq y_2 \alpha_4$.

Now, we can estimate magnitudes of α_i ($i = 4, 5, 6$) by using Eq.(5) and assuming $\alpha_4 \simeq \alpha_5 = \alpha_6$. If we take all Yukawa couplings to be order one, Eq.(5) turns to be $v_u^2 \sim \Lambda \alpha_4 m_3$ because of $d \sim \alpha_4^3$. Putting $v_u \simeq 165 \text{ GeV}$ ($\tan \beta = 3$), $m_3 \simeq \sqrt{\Delta m_{\text{atm}}^2} \simeq 0.05 \text{ eV}$, and $\Lambda = 2.43 \times 10^{18} \text{ GeV}$, we obtain $\alpha_4 = \mathcal{O}(10^{-4} - 10^{-3})$. Thus, values of α_i ($i = 4, 5, 6$) are enough suppressed to discuss perturbative series of higher mass operators.

3 Numerical result

We show our numerical analysis of neutrino masses and mixing angles in the normal mass hierarchy. Neglecting higher order corrections of mass matrices, we obtain the allowed region of parameters and predictions of neutrino masses and mixing angles. Here, we neglect the renormalization effect of the neutrino mass matrix because we suppose the normal hierarchy of neutrino masses and take $\tan \beta = 3$.

Input data of masses and mixing angles are taken in the region of 3σ of the experimental data²:

$$\begin{aligned} \Delta m_{\text{atm}}^2 &= (2.07 \sim 2.75) \times 10^{-3} \text{eV}^2, & \Delta m_{\text{sol}}^2 &= (7.05 \sim 8.34) \times 10^{-5} \text{eV}^2, \\ \sin^2 \theta_{\text{atm}} &= 0.36 \sim 0.67, & \sin^2 \theta_{\text{sol}} &= 0.25 \sim 0.37, & \sin^2 \theta_{\text{reactor}} &\leq 0.056, \end{aligned} \quad (6)$$

and $\Lambda = 2.43 \times 10^{18} \text{GeV}$ is taken. We fix $y_D = y_1 = 1$ as a convention, and vary y_2/y_1 . The change of y_D and y_1 is absorbed into the change of $\alpha_i (i = 4, 5, 6)$. If we take a smaller value of y_1 , values of α_i scale up. On the other hand, if we take a smaller value of y_D , the magnitude of α_i scale down. As expected in the discussion of previous section, the experimentally allowed values are reproduced around $\alpha_4 = \alpha_5 = \alpha_6$.

We can predict the deviation from the tri-bimaximal mixing. The remarkable prediction is given in the magnitude of $\sin^2 \theta_{13}$. In Figures 1 (a) and (b), we plot the allowed region of mixing angles in planes of $\sin^2 \theta_{12} - \sin^2 \theta_{13}$ and $\sin^2 \theta_{23} - \sin^2 \theta_{13}$, respectively. It is found that the upper bound of $\sin^2 \theta_{13}$ is 0.01. It is also found the strong correlation between $\sin^2 \theta_{23}$ and $\sin^2 \theta_{13}$. Unless θ_{23} is deviated from the maximal mixing considerably, θ_{13} remains to be tiny. This is a testable relation in this model.

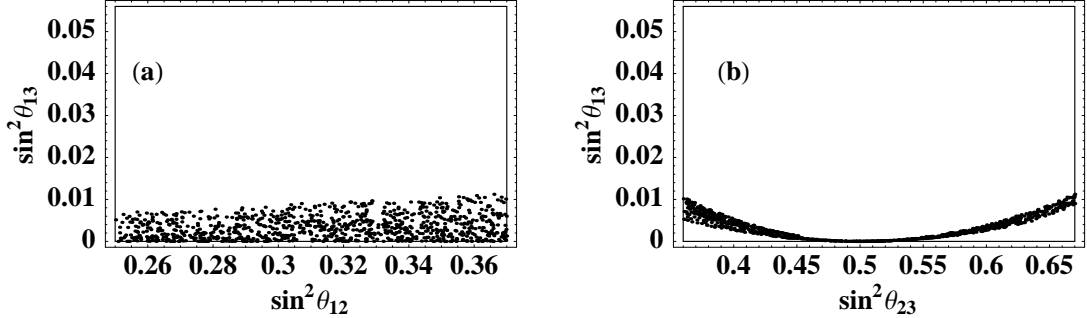


Figure 1: Prediction of the upper bound of $\sin^2 \theta_{13}$ on (a) $\sin^2 \theta_{12} - \sin^2 \theta_{13}$ and (b) $\sin^2 \theta_{23} - \sin^2 \theta_{13}$ planes.

4 SUSY breaking terms

In this section, we study SUSY breaking terms, which are predicted in our $\Delta(54)$ model. We consider the gravity mediation within the framework of supergravity theory. Let us study soft scalar masses. Within the framework of supergravity theory, soft scalar mass squared is obtained as⁷

$$m_{\bar{I}J}^2 K_{\bar{I}J} = m_{3/2}^2 K_{\bar{I}J} + |F^{\Phi_k}|^2 \partial_{\Phi_k} \partial_{\bar{\Phi}_k} K_{\bar{I}J} - |F^{\Phi_k}|^2 \partial_{\bar{\Phi}_k} K_{\bar{I}L} \partial_{\Phi_k} K_{\bar{M}J} K^{L\bar{M}}, \quad (7)$$

where K denotes the Kähler potential, $K_{\bar{I}J}$ denotes second derivatives by fields, i.e. $K_{\bar{I}J} = \partial_{\bar{I}} \partial_J K$ and $K^{\bar{I}J}$ is its inverse. The invariance under the $\Delta(54)$ flavor symmetry as well as the gauge invariance requires the following form of the Kähler potential of l_I and e_I ($I = e, \mu, \tau$)

$$K = Z^{(L)}(Z) \sum_{I=e,\mu,\tau} |l_I|^2 + Z^{(R)}(Z) \sum_{I=e,\mu,\tau} |e_I|^2, \quad (8)$$

at the lowest level, where $Z^{(L)}(Z)$ and $Z^{(R)}(Z)$ are arbitrary functions of the singlet fields Z . Both matrices are proportional to the (3×3) identity matrix. This form would be obvious because (l_e, l_μ, l_τ) and (e^c, μ^c, τ^c) are $\Delta(54)$ triplets. At any rate, it is the prediction of our model that three families of left-handed and right-handed masses are degenerate.

Let us estimate corrections including $\chi_i \chi_k$ as well as $\chi_i \chi_k^*$ for $i, k = 1, 2, 3, 4, 5, 6$. The $\Delta(54)$ flavor symmetric invariance allows only the terms such as $\chi_i \chi_k^*$ for $i, k = 4, 5, 6$ to appear in off-diagonal entries of the Kähler metric of (l_e, l_μ, l_τ) . When we take into account the corrections from $\chi_i \chi_k^*$ for $i, k = 4, 5, 6$ to the Kähler potential, the soft scalar masses squared for left-handed charged sleptons have the following corrections,

$$\begin{aligned} (m_L^2)_{IJ} &= m_L^2 \begin{pmatrix} 1 + \mathcal{O}(\tilde{\alpha}^2) & \mathcal{O}(\alpha_4^2) & \mathcal{O}(\alpha_4^2) \\ \mathcal{O}(\alpha_4^2) & 1 + \mathcal{O}(\tilde{\alpha}^2) & \mathcal{O}(\alpha_4^2) \\ \mathcal{O}(\alpha_4^2) & \mathcal{O}(\alpha_4^2) & 1 + \mathcal{O}(\tilde{\alpha}^2) \end{pmatrix}, \\ (m_R^2)_{IJ} &= m_R^2 \begin{pmatrix} 1 + \mathcal{O}(\tilde{\alpha}^2) & \mathcal{O}(\alpha_4^2) & \mathcal{O}(\alpha_4^2) \\ \mathcal{O}(\alpha_4^2) & 1 + \mathcal{O}(\tilde{\alpha}^2) & \mathcal{O}(\alpha_4^2) \\ \mathcal{O}(\alpha_4^2) & \mathcal{O}(\alpha_4^2) & 1 + \mathcal{O}(\tilde{\alpha}^2) \end{pmatrix}, \end{aligned} \quad (9)$$

where $\tilde{\alpha}$ is the averaged value of α_{1-6} . These deviations may not be important for direct measurement of slepton masses. However, the off-diagonal entries in the SCKM basis are constrained by the FCNC experiments. Our model predicts

$$(\Delta_{LL})_{12} = \frac{(m_L^2)_{12}^{(SCKM)}}{(m_L^2)_{11}} = \mathcal{O}(\alpha_4^2), \quad (\Delta_{RR})_{12} = \frac{(m_R^2)_{12}^{(SCKM)}}{(m_R^2)_{11}} = \mathcal{O}(\alpha_4^2). \quad (10)$$

Recall that the diagonalizing matrices of left-handed and right-handed fermions are almost the identity matrix. The $\mu \rightarrow e\gamma$ experiments constrain these values as $(\Delta_{LL,RR})_{12} \leq \mathcal{O}(10^{-3})$, when $m_{L,R} = 100$ GeV. On the other hand, the parameter space in the numerical result corresponds to $\alpha_4 \leq 10^{-2}$ and leads to $(\Delta_{LL,RR})_{12} \leq \mathcal{O}(10^{-4})$. Thus, our parameter region would be favorable also from the viewpoint of the FCNC constraints.

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References

1. P.F. Harrison, D.H. Perkins, and W.G. Scott, Phys. Lett. B **530**, 167 (2002);
P.F. Harrison and W.G. Scott, Phys. Lett. B **535**, 163 (2002).
2. T. Schwetz, M. Tortola, and J.W.F. Valle, arXiv:0808.2016;
G.L. Fogli, E. Lisi, A. Marrone, A. Palazzo, and A.M. Rotunno, Phys. Rev. Lett. **101** 141801 (2008), arXiv:0806.2649.
3. T. Kobayashi, H. P. Nilles, F. Ploger, S. Raby and M. Ratz, Nucl. Phys. B **768**, 135 (2007), arXiv:hep-ph/0611020.
4. H. Abe, K. S. Choi, T. Kobayashi and H. Ohki, Nucl. Phys. B **820**, 317 (2009) [arXiv:0904.2631 [hep-ph]].
5. H. Ishimori, T. Kobayashi, H. Okada, Y. Shimizu and M. Tanimoto, JHEP **0904**, 011 (2009) [arXiv:0811.4683 [hep-ph]].
6. H. Ishimori, T. Kobayashi, H. Okada, Y. Shimizu and M. Tanimoto, JHEP **0912**, 054 (2009) [arXiv:0907.2006 [hep-ph]].
7. V. S. Kaplunovsky and J. Louis, in Phys. Lett. B **306**, 269 (1993), arXiv:hep-th/9303040.