

Lepton and Slepton masses $\Delta(54)$ symmetry

Hajime Ishimori

Niigata University

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Collaborator : T. Kobayashi, H. Okada,
Y. Shimizu, M. Tanimoto,

JHEP 0904: 011, 2009 JHEP 0912: 054, 2009

Motivation

Experiments of neutrino oscillation indicate:

$$\sin^2 \theta_{12} \approx 0.32^{+0.046}_{-0.053}, \quad \sin^2 \theta_{23} \approx 0.45^{+0.12}_{-0.20}, \quad \sin^2 \theta_{13} \approx 0.014^{<0.052},$$

M.C. G-Garcia, M. Maltoni, J. Salvado, arXiv:1001.4524

It indicates that Lepton mixing is tri-bimaximal:

$$U_{\text{MNS}} \approx \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix},$$

$$M_\nu = \frac{m_1+m_3}{2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \frac{m_2-m_1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_1-m_3}{2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

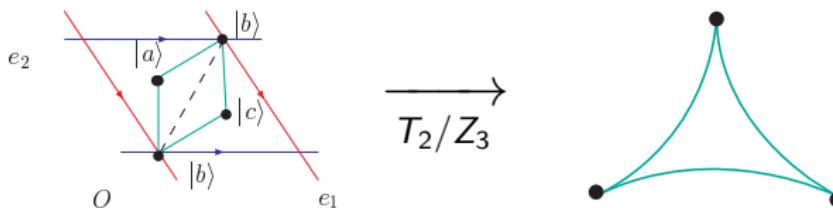
Flavor symmetry restricts lepton and slepton mass matrices.
Then the slepton mass spectrum can be predicted.

$\Delta(54)$ is a nonAbelian discrete symmetry from stringy origin.

T. Kobayashi, H. P. Nilles, F. Ploger, S. Raby, M. Ratz,

Nucl. Phys. B768: 135, 2007

We put particles on fixed points with states of $|a\rangle$, $|b\rangle$, and $|c\rangle$.



By considering relabeling and selection rule, it leads $\Delta(54)$ symmetry.

$$Z_3 : \begin{pmatrix} |a\rangle \\ |b\rangle \\ |c\rangle \end{pmatrix} \rightarrow \begin{pmatrix} \omega & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega \end{pmatrix} \begin{pmatrix} |a\rangle \\ |b\rangle \\ |c\rangle \end{pmatrix}, \quad \begin{pmatrix} |a\rangle \\ |b\rangle \\ |c\rangle \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix} \begin{pmatrix} |a\rangle \\ |b\rangle \\ |c\rangle \end{pmatrix},$$

$$S_3 : \begin{pmatrix} |a\rangle \\ |b\rangle \\ |c\rangle \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} |a\rangle \\ |b\rangle \\ |c\rangle \end{pmatrix}, \quad \Delta(54) = S_3 \ltimes (Z_3 \times Z_3),$$

Fundamental representations of $\Delta(54)$ are

$3_1^{(1)}$, $3_1^{(2)}$, $3_2^{(1)}$, $3_2^{(2)}$, 2_1 , 2_2 , 2_3 , 2_4 , 1_1 , 1_2 .

Our $\Delta(54)$ model (JHEP 0904: 011, 2009)

	lepton	right-handed neutrino
	(L_e, L_μ, L_τ)	$(R_e^c, R_\mu^c, R_\tau^c)$
$\Delta(54)$	$3_1^{(1)}$	$3_2^{(2)}$

	Higgs	new scalars (gauge singlet)		
	$H_{u,d}$	χ_ℓ	χ'_ℓ	χ_N
$\Delta(54)$	1_1	1_2	2_1	$3_1^{(2)}$

- $R^c L H_u \chi_\ell / \Lambda$,

	R^c	L	χ_ℓ
$\Delta(54)$	$3_2^{(2)}$	$3_1^{(1)}$	1_2

$$M_\ell = y_1^\ell v_d \begin{pmatrix} \alpha_\ell & 0 & 0 \\ 0 & \alpha_\ell & 0 \\ 0 & 0 & \alpha_\ell \end{pmatrix}$$

- $R^c L H_u \chi'_\ell / \Lambda$,

	R^c	L	χ'_ℓ
$\Delta(54)$	$3_2^{(2)}$	$3_1^{(1)}$	2_1

$$+ y_2^\ell v_d \begin{pmatrix} \omega \alpha_{\ell_1} - \alpha_{\ell_2} & 0 & 0 \\ 0 & \omega^2 (\alpha_{\ell_1} - \alpha_{\ell_2}) & 0 \\ 0 & 0 & \omega \alpha_{\ell_1} - \omega \alpha_{\ell_2} \end{pmatrix}$$

where $\alpha_i = \langle \chi_i \rangle / \Lambda$ and Λ is the cut-off scale.

Dirac neutrino

- $N^c LH_u$,

	N^c	L
$\Delta(54)$	$\mathbf{3}_1^{(2)}$	$\mathbf{3}_1^{(1)}$

Majorana neutrino

- $\bar{N}^c N \chi_N$

	N	χ_N
S_4	$\mathbf{3}_1^{(2)}$	$\mathbf{3}_1^{(2)}$

We suppose a vacuum alignment $\langle \chi_N \rangle = (\alpha_{N_1}, \alpha_{N_2}, \alpha_{N_2}) \Lambda$.

$$M_D = y_D v_u \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad M_N = \Lambda \begin{pmatrix} y_1 \alpha_{N_1} & y_2 \alpha_{N_2} & y_2 \alpha_{N_2} \\ y_2 \alpha_{N_2} & y_1 \alpha_{N_2} & y_2 \alpha_{N_1} \\ y_2 \alpha_{N_2} & y_2 \alpha_{N_1} & y_1 \alpha_{N_2} \end{pmatrix},$$

Neutrino mass matrix is obtained by see-saw mechanism:

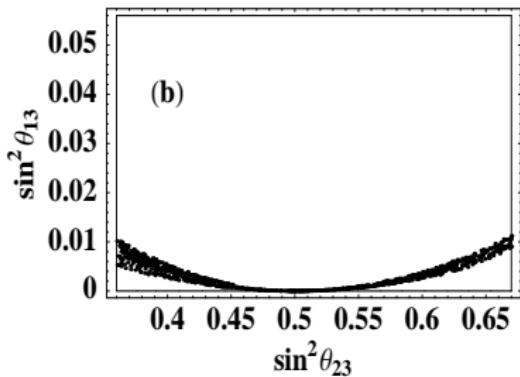
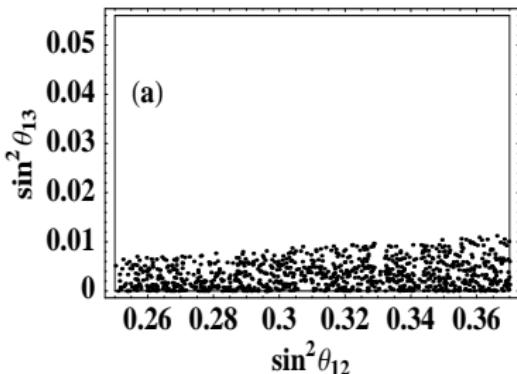
$$M_\nu = M_D^T M_N^{-1} M_D$$

There are two cases to realize tri-bimaximal mixing.

$$\begin{aligned}\alpha_{N_1} &\approx \alpha_{N_2} \\ U_{\text{MNS}} &\approx U_{\text{tri-bi}} \\ m_1 &\approx m_2 \approx m_3\end{aligned}$$

$$\begin{aligned}y_1 \alpha_{N_2} &\approx y_2 \alpha_{N_1} \\ U_{\text{MNS}} &\approx U_{\text{tri-bi}} \\ m_1 &\approx m_2 \ll m_3\end{aligned}$$

We analyze the second case, $\alpha_{N_1}, \alpha_{N_2} \sim O(10^{-4} - 10^{-3})$, $y_1, y_2 \sim O(1)$:



$$\sin^2 \theta_{13} < 0.01$$

θ_{13} and θ_{23} are correlated.

Slepton sector

Flavor symmetry $\Delta(54)$ restricts the slepton mass matrix.

Slepton mass matrices and A-term are predicted:

$$(\tilde{m}_L^2)_{IJ} = m_L^2 \begin{pmatrix} 1 + \mathcal{O}(\alpha_N^2) & \mathcal{O}(\alpha_N^2) & \mathcal{O}(\alpha_N^2) \\ \mathcal{O}(\alpha_N^2) & 1 + \mathcal{O}(\alpha_N^2) & \mathcal{O}(\alpha_N^2) \\ \mathcal{O}(\alpha_N^2) & \mathcal{O}(\alpha_N^2) & 1 + \mathcal{O}(\alpha_N^2) \end{pmatrix}.$$
$$(\tilde{m}_{\tilde{R}}^2)_{IJ} = m_R^2 \begin{pmatrix} 1 + \mathcal{O}(\alpha_N^2) & \mathcal{O}(\alpha_N^2) & \mathcal{O}(\alpha_N^2) \\ \mathcal{O}(\alpha_N^2) & 1 + \mathcal{O}(\alpha_N^2) & \mathcal{O}(\alpha_N^2) \\ \mathcal{O}(\alpha_N^2) & \mathcal{O}(\alpha_N^2) & 1 + \mathcal{O}(\alpha_N^2) \end{pmatrix}.$$
$$(\tilde{m}_{LR}^2)_{IJ} = m_{3/2} \begin{pmatrix} m_e & \mathcal{O}(m_\mu \alpha_N^2) & \mathcal{O}(m_\tau \alpha_N^2) \\ \mathcal{O}(m_\mu \alpha_N^2) & m_\mu & \mathcal{O}(m_\tau \alpha_N^2) \\ \mathcal{O}(m_\tau \alpha_N^2) & \mathcal{O}(m_\tau \alpha_N^2) & m_\tau \end{pmatrix}.$$

Slepton masses are almost **degenerate**.

FCNC ($\mu \rightarrow e\gamma$)

Slepton mass matrix is strongly constrained by $\mu \rightarrow e\gamma$.

Experimental bounds are

$$\frac{(\tilde{m}_L^2)_{12}}{m_{\text{SUSY}}^2} \leq O(10^{-3}), \quad \frac{(\tilde{m}_R^2)_{12}}{m_{\text{SUSY}}^2} \leq O(10^{-3}), \quad \frac{(\tilde{m}_{LR}^2)_{12}}{m_{\text{SUSY}}^2} \leq O(10^{-6}).$$

F. Gabbiani, et al, Nucl. Phys. B 477, 321 (1996).

Our results are consistent with them ($m_{\text{SUSY}} \sim 100\text{GeV}$)

$$\frac{(\tilde{m}_L^2)_{12}}{m_{\text{SUSY}}^2} \approx O(10^{-6}), \quad \frac{(\tilde{m}_R^2)_{12}}{m_{\text{SUSY}}^2} \approx O(10^{-6}), \quad \frac{(\tilde{m}_{LR}^2)_{12}}{m_{\text{SUSY}}^2} \approx O(10^{-8}),$$

Flavor symmetry can suppress lepton flavor violation.

Summary

We have considered a model of $\Delta(54)$ flavor symmetry from stringy origin.

- $\Delta(54)$ can lead nearly tri-bimaximal matrix.
- θ_{13} is less than 0.1 and correlated with θ_{23} .

We have also analyzed the slepton sector.

- Slepton masses are almost degenerate.
- Lepton flavor violation ($\mu \rightarrow e\gamma$) is suppressed.