One-loop Corrections to WIMP Annihilation Mediated by Massive Boson

Keiko I. Nagao
Department of Physics, Nagoya University, Nagoya 464-8602, Japan
&
Department of Physics, National Tsing Hua University, 101 Section 2 Kuang Fu Road, Hsinchu, Taiwan 30013, Taiwan

The loop correction to the annihilation cross section of weakly interacting massive particle (WIMP) is possible to be enhanced if the intermediate particle is lighter than WIMP mass. We present a formula with which we can include that one-loop effect easily in the calculation of relic abundance.

1 Introduction

Weakly interacting massive particle (WIMP) χ is a good candidate for dark matter. In order to calculate the relic abundance of WIMP, the cross section of WIMP annihilation is required. If a boson φ mediates annihilating WIMPs, the cross section can be enhanced. In particular, when the mass of the intermediate particle m_φ is much smaller than WIMP mass m_χ (i.e. $m_\varphi \ll m_\chi$), the loop correction to the annihilation cross section is enhanced as large as tree level cross section, therefore the loop corrections of all orders should be included in the calculation [2]. Even if the intermediate particle is lighter than WIMP mass ($m_\varphi \lesssim m_\chi$), the correction is still possible to be enhanced, though it is smaller than the correction in the case of $m_\varphi \ll m_\chi$. In that case, we can treat the correction perturbative, thus including one-loop correction is sufficient. In this work we calculate the one-loop correction to the WIMP annihilation amplitude and cross section. We also calculate the thermally averaged one-loop correction in order to apply our results to the calculation of relic density.

2 Correction to the WIMP Annihilation

Let us consider the WIMP annihilation $\chi(p_1) + \chi(p_2) \to X_1(p_1') + X_2(p_2')$, where $X_{1,2}$ and p in parentheses are possible final states and momentum of particles, respectively. Fig.1 shows a generic WIMP annihilation in tree level. We are interested in one-loop amplitude shown in Fig.2, in which a boson φ mediates two annihilating particles. In these figures, gray circles represent common structure for both of diagrams.

Afterward, we adopt the center of mass (COM) frame and follow the reference[3]. We perform the calculation in the case of Majorana fermion, however, our results in the non-relativistic limit can be used in the Dirac fermion and scalar WIMPs. We take parameters $P = (p_1 + p_2)/2 = (p_1' + p_2')/2$, $p = (p_1 - p_2)/2$ and $p' = (p_1' - p_2')/2$. The one-loop corrected amplitude is written as

$$A_L(|\vec{p}|, p') = A_{L,0}(|\vec{p}|, p') + \delta A_L(|\vec{p}|, p') \tag{1}$$

where L is the partial wave, $L = 0(1)$ for S (P)-wave. Notice that the amplitude is a function of $\vec{p}$, since in COM frame, $P_0 = \sqrt{\vec{p}^2 + m_\chi^2}$, $\vec{P} = 0$ and $p_0 = 0$ are satisfied.

The one-loop correction is represented with the tree level contribution $\tilde{A}_{0,L}$, which is shown

This article is based on a work with M. Drees and J. M. Kim [1].

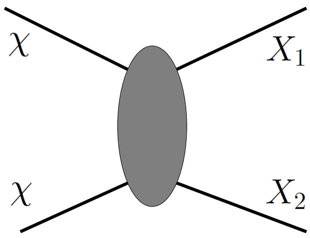

Figure 1: WIMP annihilation in tree-level.

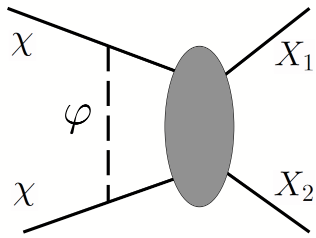

Figure 2: One-loop correction to Fig.1.

as gray circles in Fig.1, 2. The correction for the amplitude in case of scalar boson exchange is

$$\begin{aligned}\delta A_L(|\vec{p}|,p') &= ig^2\bar{v}(p_2)\int\frac{d^4q}{(2\pi)^4}\frac{\not{q}-\not{P}+m_\chi}{(q-P)^2-m_\chi^2+i\epsilon}(\gamma_5)^{n_L}\frac{\not{q}+\not{P}+m_\chi}{(q+P)^2-m_\chi^2+i\epsilon}\\ &\quad\times\frac{1}{(p-q)^2-\mu^2+i\epsilon}\tilde{A}_{0,L}(|\vec{p}|,p')u(p_1). \end{aligned}\tag{2}$$

Here g is the strength of the coupling between the boson φ and two WIMPs χ, and $\bar{v}$ and u are spinors, and $n_L = 0(1)$ for S (P)-wave. The fraction in Eq.(2) is rewritten with $\Lambda^{\pm}(\vec{q}) = \frac{\omega\pm(\gamma^0\vec{\gamma}\cdot\vec{q}+\gamma^0 m_\chi)}{2\omega}$ where $\omega^2 = \vec{q}^2 + m_\chi^2$, as

$$\frac{\not{q}+\not{P}+m_\chi}{(q+P)^2-m_\chi^2+i\epsilon} = \left(\frac{\Lambda^+(\vec{q})}{q_0+P_0-\omega+i\epsilon}+\frac{\Lambda^-(\vec{q})}{q_0+P_0+\omega+i\epsilon}\right)\gamma^0. \tag{3}$$

Another similar fraction can be also rewritten with $\Lambda^{\pm}(\vec{q})$. After integrating on q_0 and taking the non-relativistic limit, Eq.(2) becomes

$$\begin{aligned}\delta A_L(|\vec{p}|,p') &= ig^2\bar{v}(p_2)\int\frac{d^3q}{(2\pi)^3}\frac{\Lambda^-(\vec{q})}{2(\omega-P_0)}\gamma^0(\gamma_5)^{n_L}\Lambda^+(\vec{q})\gamma^0\frac{-1}{(\vec{p}-\vec{q})^2+\mu^2}\tilde{A}_{0,L}(p')u(p_1)\\ &= g^2\bar{v}(p_2)\int\frac{d^3q}{(2\pi)^3}\frac{(\not{q}-\not{P}+m_\chi)(\gamma_5)^{n_L}(\not{q}+\not{P}+m_\chi)}{8\omega^2(\omega-P_0)\left((\vec{p}-\vec{q})^2+\mu^2\right)}\tilde{A}_{0,L}(p')u(p_1). \end{aligned}\tag{4}$$

In the first line, we took only the residue at $q_0 = \omega - P_0$, since it becomes largest among other terms in the non-relativistic limit. After Performing the integration about angular variables, the corrections to the amplitude are represented with the integration about $|\vec{q}|$. The integral variable $|\vec{q}|$ can be replaced to $x = |\vec{q}|/|\vec{p}|$. Now the correction is represented as a function of a parameter $r = \mu^2/\vec{p}^2$, as

$$\delta A_L(|\vec{p}|,p') = \frac{g^2}{4\pi^2}\frac{1}{v}A_0(|\vec{p}|,p')I_L(r) \tag{5}$$

$$I_S(r) = \Re e\left[\int_0^\infty dx\frac{x}{x^2-1}\ln\frac{(1+x)^2+r}{(1-x)^2+r}\right] \tag{6}$$

$$I_P(r) = \Re e\left[\int_0^\infty dx\frac{2x^2}{x^2-1}\left(-1+\frac{x^2+1+r}{4x}\ln\frac{(1+x)^2+r}{(1-x)^2+r}\right)\right]. \tag{7}$$

Here, v represents the relative velocity of WIMP in COM frame, such as $|\vec{p}| = m_\chi v/2$. In Fig.3, $I(r)$ for S- and P-waves are shown. The correction for S-wave is always larger than that for

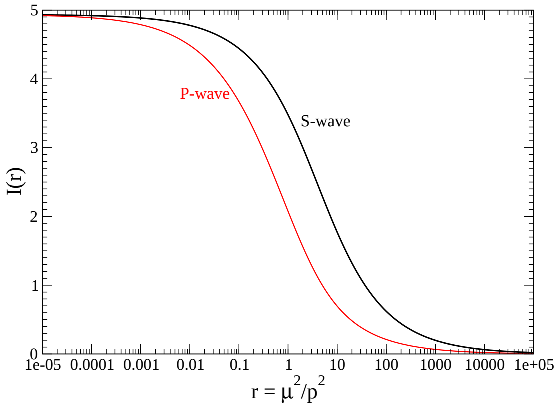

Figure 3: Integrals $I(r)$ for S- and P-waves.

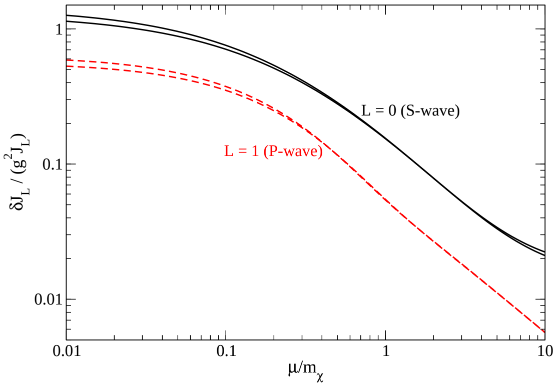

Figure 4: Correction to annihilation integrals δJ_L compared to the tree level contribution.

P-wave. We approximate the integral $I(r)$ for small and large r as

$$I_S(r) \simeq \begin{cases} \frac{2\pi}{\sqrt{r+1}}\left(1-\frac{1}{r+2}\right) & \text{(large } r) \\ \frac{\pi^2/2}{1+\frac{\sqrt{r}}{\pi}+\frac{r}{\pi^2}} & \text{(small } r) \end{cases}$$

$$I_P(r) \simeq \begin{cases} \frac{2\pi}{3\sqrt{r+1}}\left(1+\frac{1.3}{r+1}\right) & \text{(large } r) \\ \frac{\pi^2/2}{1+\frac{3\sqrt{r}}{\pi}+\frac{r}{\pi}} & \text{(small } r) \end{cases} .$$

3 Correction to the Relic Abundance

To obtain the accurate relic abundance of WIMP, we must solve the Boltzmann equation. Its solution is approximated very well as

$$\Omega_\chi h^2 = \frac{8.5\times 10^{-11} x_F \mathrm{GeV}^{-2}}{\sqrt{g_*(x_F)}J(x_F)} \tag{8}$$

where Ω_χ is the energy density of WIMP in the unit of critical density, h is the scaled Hubble parameter, $x_F = m_\chi/T_F$ (T_F is the freeze-out temperature), g_* is the number of freedom, and $J(x_F)$ is the annihilation integral which is defined as

$$J(x_F) = \int_{x_F}^{\infty} dx\, \frac{\langle\sigma v\rangle}{x^2}. \tag{9}$$

Here $\langle\sigma v\rangle$ is the thermal averaged cross section

$$\langle\sigma v\rangle = \frac{2x^{3/2}}{\sqrt{\pi}} \int_0^\infty dv\, \sigma v \frac{v^2}{4} e^{-xv^2/4}. \tag{10}$$

Since the freeze-out occurs when WIMP is the non-relativistic particle, it gives a good approximation to expand the tree-level averaged cross section with x

$$\langle\sigma_0 v\rangle(x) \simeq \mathscr{A} + \frac{6\mathscr{B}}{x} + \cdots . \tag{11}$$

In this expansion, $\mathscr{A}$ contains only the S-wave contribution and $\mathscr{B}$ contains both S- and P-waves contributions. However, we suppose that $\mathscr{B}$ contains only P-wave contribution for simplicity. Notice that this simplification does not make the accuracy of the calculation so much since the loop correction itself is smaller than the tree level cross section for $\mu \lesssim m_\chi$.

Let us expand the one-loop corrected annihilation cross section as well as the annihilation amplitude with the tree-level and one-loop contributions $\sigma_L = \sigma_{0,L} + \delta\sigma_L$. Then, the one-loop contribution $\delta\sigma_L$ is represented with the integration in Eq.(7)

$$\delta\sigma_L = \frac{g^2}{2\pi^2 v} I_S(r)\sigma_{0,L} \tag{12}$$

With Eq.(9), Eq.(10), Eq.(11) and Eq.(12), the one-loop corrected annihilation integrals can be calculated as a function of $z_F = (2\mu/m_\chi)\sqrt{x_F}$, as

$$\begin{aligned} \delta J_S(x_f) &= \frac{g^2\mathscr{A}}{\pi^{5/2}}\frac{\mu}{m_\chi}\int_{z_F}^{\infty}\frac{dz}{z^2}\left(\frac{1}{a_S z^2 + b_S z + c_S} + d_S\right), \\ \delta J_P(x_f) &= \frac{64 g^2\mathscr{B}}{\pi^{5/2}}\left(\frac{\mu}{m_\chi}\right)^3\int_{z_F}^{\infty}\frac{dz}{z^4}\left[\exp(-a_P z + b_P) + \frac{1}{c_P z + d_P}\right]. \end{aligned} \tag{13}$$

Here, parameters a_S, b_S, c_S, d_S and a_P, b_P, c_P, d_P are fitted by numerical calculation:

$$a_S = 0.000593;\ b_S = 0.03417;\ c_S = 0.1015;\ d_S = 0.1182$$
$$a_P = 0.318;\ b_P = 0.1226; c_P = 0.3309; d_P = 0.6306.$$

Notice that Eq.(13) can be calculated only if we know the parameters in tree level, $\mathscr{A}$ and $\mathscr{B}$.

In Fig.4, the one-loop correction to the annihilation integral δJ_L compared to the strength of the coupling times the tree level annihilation integral $g^2 J_L$. The upper (lower) curves are for $x_F = m_\chi/T_F = 25$ (20). As easily expected, the corrections becomes large as μ/m_χ becomes small. In the massless limit (i.e. $\mu/m_\chi \to 0$), correction for the S-wave becomes as large as tree level contribution.

4 Conclusion and Discussion

We study the one-loop correction to the WIMP annihilation cross section, in which a massive (i.e. $m_\varphi \lesssim m_\chi$) boson mediates the annihilating WIMPs. In order to apply the result to the calculation of WIMP abundance, we also formulate the one-loop correction to the annihilation integral J_L in Eq.(13). With the tree-level annihilation cross section (i.e. $\mathscr{A}$ and $\mathscr{B}$), we can include the one-loop effect which can be large in the calculation of relic density. Notice that even if we have only the exact annihilation cross section (not expanded by x), we can obtain parameters $\mathscr{A}$ and $\mathscr{B}$ by solving simultaneous equations using exact cross sections with slightly different x.

We also performed the calculation of the one-loop correction of neutralino annihilation in the minimal supersymmetric standard model (MSSM)[1]. In that case, the correction is up a few percent of the tree-level contribution because of small strength of the coupling.

References

1. M. Drees, J. M. Kim and K. I. Nagao, arXiv:0911.3795 [hep-ph].
2. J. Hisano, S. Matsumoto and M. M. Nojiri, Phys. Rev. Lett. **92**, 031303 (2004) [arXiv:hep-ph/0307216]. J. Hisano, S. Matsumoto, M. M. Nojiri and O. Saito, Phys. Rev. D **71**, 015007 (2005) [arXiv:hep-ph/0407168]. J. Hisano, S. Matsumoto, M. M. Nojiri and O. Saito, Phys. Rev. D **71**, 063528 (2005) [arXiv:hep-ph/0412403].
3. R. Iengo, JHEP **0905**, 024 (2009) [arXiv:0902.0688 [hep-ph]].