Potentially Large One-loop Corrections to WIMP Annihilation

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Keiko I. Nagao (Nagoya University)

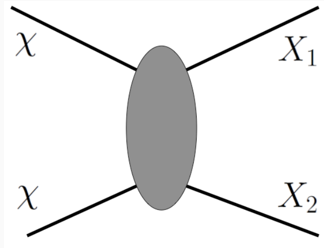
based on a work with

M. Drees and J. M. Kim

(University of Bonn)

WIMP abundance and annihilation

- Calculation of WIMP abundance requires the annihilation cross section



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< >:thermal average

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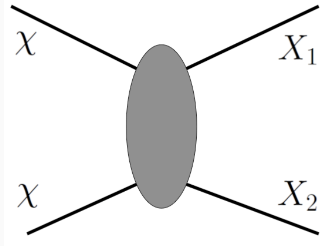
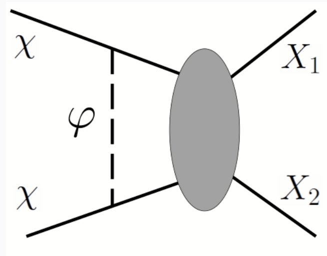
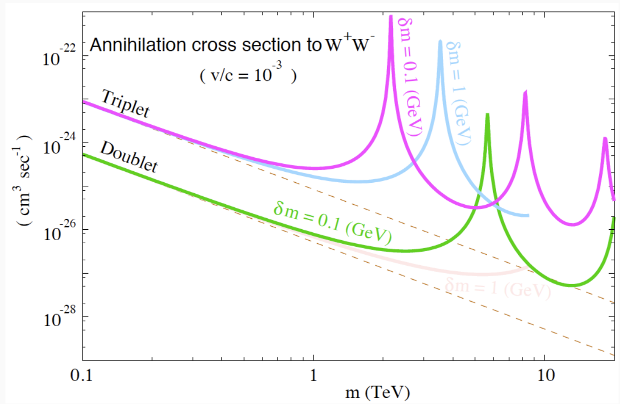


- Large loop correction

  If $\mu(\text{intermediate particle}) \ll m(\text{WIMP})$, the loop correction can be enhanced by several orders of magnitude.





Hisano, Matsumoto and Nojiri (2003)

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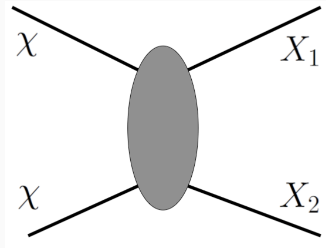

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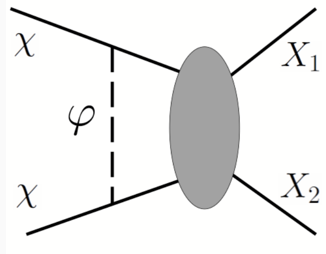
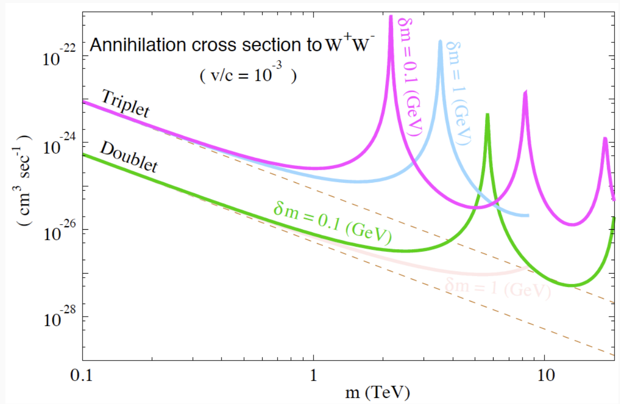


If $\mu(\mathrm{intermediate\ particle}) \ll m(\mathrm{WIMP})$,
the loop correction can be enhanced by several orders of magnitude.

- What if $\mu \lesssim m$?

-The correction becomes smaller, but still large!
- e.g. neutralino annihilation mediated by Higgs in SUSY models.



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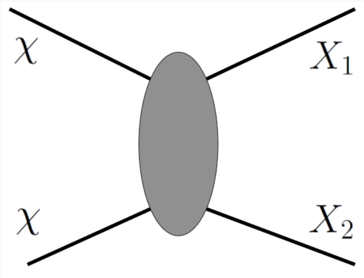
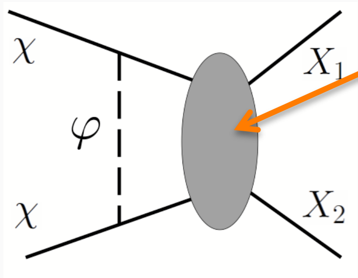
One-loop correction to amplitude

- For majorana WIMP,
  (final result in NR limit is valid for Dirac/scalar WIMP),

Tree : $A_{0,L}$



One-loop : δA_L



$\tilde{A}_{0,L}$

Common among tree and one-loop

n_L= 1(0) for S (P)-wave

$$\delta A_L(|\vec{p}|, p') \;=\; ig^2 \bar{v}(p_2) \int \frac{d^4 q}{(2\pi)^4} \frac{\not{q} - \not{P} + m_\chi}{(q-P)^2 - m_\chi^2 + i\epsilon} (\gamma_5)^{n_L} \frac{\not{q} + \not{P} + m_\chi}{(q+P)^2 - m_\chi^2 + i\epsilon}$$
$$\times \frac{1}{(p-q)^2 - \mu^2 + i\epsilon} \tilde{A}_{0,L}(|\vec{q}|, p')\, u(p_1)\,.$$

$$p = (p_1 - p_2)/2$$
$$P = (p_1 + p_2)/2 = (p_1' + p_2')/2$$

One-loop correction

- Correction in the non-relativistic limit

$$\delta A_S(|\vec{p}|, p')|_{1-\text{loop}} = \frac{g^2}{4\pi^2}\frac{1}{v}A_0(|\vec{p}|, p')I_L(r)$$

$$\begin{cases} I_S(r) = \Re\text{e}\left[\int_0^\infty dx \frac{x}{x^2-1}\ln\frac{(1+x)^2+r}{(1-x)^2+r}\right] \\ I_P = \Re\text{e}\left\{\int_0^\infty dx \frac{2x^2}{x^2-1}\left[-1+\frac{x^2+1+r}{4x}\ln\frac{(x+1)^2+r}{(x-1)^2+r}\right]\right\} \end{cases}$$

$$r = \mu^2/\vec{p}^2$$

- To apply these results to the calculation of WIMP abundance, let's consider the correction to the thermally averaged cross section.

Averaged cross section

- Relic abundance

$$\Omega_\chi h^2 = \frac{8.5 \times 10^{-11}\ x_F\ \mathrm{GeV}^{-2}}{\sqrt{g_*(x_F)}J(x_F)}$$

where $x \equiv m/T$

$$J(x_F) = \int_{x_F}^{\infty} dx \frac{\langle \sigma v \rangle}{x^2}$$

Tree level averaged cross section : $\langle \sigma_0 v \rangle (x) \simeq \mathscr{A} + \dfrac{6\mathscr{B}}{x}$

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- Correction to J(x)

$$J_{\mathrm{total}} = J_0 + \delta J_S + \delta J_P$$

We can easily calculate the correction with tree-level cross section!

$$\delta J_S(x_f) = \frac{g^2\mathscr{A}}{\pi^{5/2}}\frac{\mu}{m_\chi}\int_{z_F}^{\infty}\frac{dz}{z^2}\left(\frac{1}{a_S z^2 + b_S z + c_S} + d_S\right),$$

$$\delta J_P(x_f) = \frac{64 g^2\mathscr{B}}{\pi^{5/2}}\left(\frac{\mu}{m_\chi}\right)^3\int_{z_F}^{\infty}\frac{dz}{z^4}\left[\exp(-a_P z + b_P) + \frac{1}{c_P z + d_P}\right]$$

$$\begin{cases} a_S = 0.000593;\ b_S = 0.03417;\ c_S = 0.1015;\ d_S = 0.1182 \\ a_P = 0.318;\ b_P = 0.1226; c_P = 0.3309; d_P = 0.6306 \end{cases}$$

Summary

- If mass of intermediate particle is lighter than or same as WIMP mass, the one-loop correction is expected to be large. We formulate the correction to the thermally averaged cross section.

- With the formula, we can include one-loop effect easily if tree level cross section is already known.