Soft-walls: an alternative solution to the hierarchy problem

Moriond Electroweak
March 6-13th, 2010

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Based on: J. A. Cabrer, G. von Gersdorff and M.Q.
arXiv:0907.5361; arXiv:1003.nnnn

Outline

The outline of this talk is

Outline

- Introduction
- The soft-wall model
- Electroweak symmetry breaking (EWSB)
- Electroweak constraints (EWPT)
- Unitarity bounds
- Conclusion

Introduction

- At the start-up of the LHC it is important to study (revise) new solutions to old SM problems
- A typical SM problem is related to electroweak breaking: the Higgs sensitivity to UV physics or hierarchy problem
- Solutions to the hierarchy problem have motivated most of the BSM theoretical and experimental developments
- They can be classified in
  - 4D solutions: supersymmetry, technicolor, little Higgs,. . .
  - Extra dimensional solutions [1]: large extra dimensions (ADD), gauge-Higgs unification, Higgsless, warped extra dimension (RS1), holographic technicolor
- A warped extra dimension provides an explanation of the m_W/M_P hierarchy in terms of the brane distance

Geometry of extra dimension

$$m_W = e^{-ky_c} M_P \quad y_c = length \sim 30/M_P$$

[1] Reviews in Marc's and John's talks

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Stabilized RS (summary)

- The 5D theory is a slice of AdS [a] with a brane at the UV ($y = 0$) [a UV cutoff at M_P] and an IR brane ($y = y_c$)
- Brane spacing is a flat direction and there is a massless scalar (radion): it requires stabilization to give a mass to the radion
- It can be stabilized by the GW mechanism [b]
- It requires a bulk scalar with a particular 5D potential V (superpotential W)

$$V(\phi) = 1/2(\partial W/\partial \phi)^2 - 1/3W^2$$

$$W(\phi) = k(6 - u\phi^2), \quad \phi(y) = \phi_{UV} e^{-u\, ky}$$

- The hierarchy is generated as

$$ky_c = \frac{1}{u} \log \frac{\phi_{UV}}{\phi_{IR}}$$

- EW hierarchy requires $\log(\phi_{UV}/\phi_{IR})/u \simeq 30$

[a] L. Randall and R. Sundrum, hep-ph/9905221

[b] Goldberger and M. Wise, hep-ph/9907447

With a more *generic* potential of ϕ one obtains a

Soft wall (SW)

- There is no IR brane and the extra dimension is non-compact but of finite length
- This implies that the IR brane is replaced by a

Naked curvature singularity at $y = y_s$

- Soft-walls are
  - Generalizations of RS2 [a] with finite length
  - A general feature of "self-tuned " CC models [b]
  - Introduced for AdS/QCD studies [c]
  - Introduced to describe unparticles as fields in the bulk [d]
  - As alternatives to RS1 for solving the EW hierarchy [e]

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[a] L. Randall and R. Sundrum, hep-th/9906064

[b] N. Arkani-Hamed et al., hep-th/0001197; S. Kachru et al., hep-th/0002121

[c] A. Karch, E. Katz, D.T. Son and M.A. Stephanov, hep-ph/0602229

[d] G. Cacciapaglia, G. Marandella and J. Terning, arXiv:0804.0424; A. Falkowski and M. Perez-Victoria, arXiv:0806.1737

[e] B. Batell, T. Gherghetta and D. Sword, arXiv:0808.3977

Soft-wall model

We are seeking a class of models where:

- The metric is nearly AdS at the UV brane to generate the hierarchy by the warping
- There is a bulk scalar ϕ fixing dynamically y_s in a natural way: e.g. $k y_s \sim e^{\phi_0/k}$, with $\phi_0 \equiv \phi(0) = \mathcal{O}(1) k$
- The SM Higgs H is a bulk scalar triggering EWSB
- ϕ and H back react on the metric in a controlable way
- EWPT are satisfied leading to lower bounds on m_{KK}
- The Higgs and KK-gauge boson sector unitarize the theory at $s \gg m_W^2$
- In some limit the model describes unparticles with a mass gap
- Near the unparticle limit KK-modes are very close to each other and a few (many?) of them could be produced at LHC

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The proposed model is defined by the

Superpotential

$$W(\phi, H) = k\left(6 + e^{\nu\phi/\sqrt{6}}\right) + a|H|^2$$

leading to the

Background configurations

$$\phi(y) = -\frac{\sqrt{6}}{\nu}\log[\nu^2 k(y_s - y)/6]$$

$$h(y) = h_0 e^{aky} \Rightarrow \textit{EWSB}$$

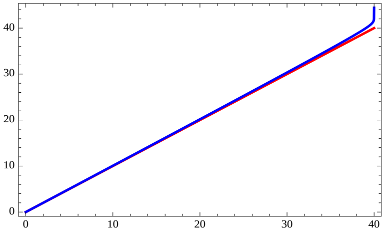
which back-react on the metric as

The metric: $a(y) = e^{-A}$

$$ds^2 = e^{-2A}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$$

$$A(y) = ky - \frac{1}{\nu^2}\log\left(1 - \frac{y}{y_s}\right) + \frac{1}{24}h^2(y) - \frac{1}{24}h_0^2$$

- The metric $A(y)$ separates from the AdS metric only near the singularity. E.g. for $\nu = 2, \; ky_s = 40$



- The relevant mass scale for 4D spectra is

The IR scale

$$\rho = k(ky_s)^{-1/\nu^2} e^{\frac{1}{\nu^2} e^{-\nu\phi_0/\sqrt{6}}}$$

- For $\phi_0 \sim -\mathcal{O}(few)$

The hierarchy

$$k \sim M_P \Rightarrow \rho \sim TeV$$

EWSB

- The Higgs background in the bulk triggers EWSB in the 5D theory leading to EWSB in 4D
- One makes a 4D mode decomposition of the gauge fields

Mode decomposition

$$A_\mu(x,y) = \sum_n \frac{a_\mu^{(n)}(x)\cdot f_n(y)}{\sqrt{y_s}}$$

with profiles

$$m_f^2 f + (e^{-2A} f')' - m_A^2(y) f = 0 \qquad \textit{Neumann B.C.}$$

where $m_A(y)$ is the 5D generalization of 4D gauge boson masses

$$m_W^2(y) = \frac{1}{4} g_5^2 h^2(y)\, e^{-2A}$$

- To fix the Higgs field h_0 at the UV brane we can use the 4D potential

$$V_4(h) = \gamma_H(|h| - h_0)^2$$

where $|h| = \sqrt{|H|^2}$ and $\gamma_H \sim 1$, which satisfies the boundary conditions at the UV brane
- Another possibility is using a stiff potential for the Higgs at the UV brane as

$$V_4(h) = \gamma_H(|H|^2 - h_0^2)^2$$

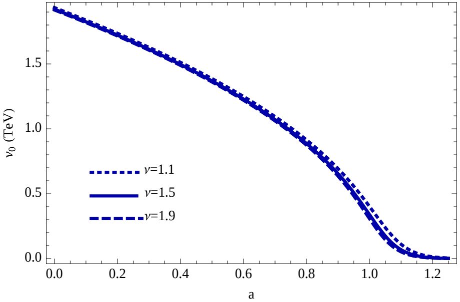
with $\gamma_H \gg 1$.
- The value of h_0 is not destabilized by radiative corrections. The reason is that the Higgs mass at the origin is $\sim \Lambda^2$, where Λ is the 4D cutoff of the theory, so that quadratic divergences, suppressed by loop factors, will not destabilize the hierarchy

Electroweak constraints

- In our 5D model (for fixed values of the parameters $\nu,\ y_s,\dots$) we have the free parameters (g_5, g_5', h_0, a) which fix the physical spectrum of zero mode masses
- Once we have fixed the condition

$$m_{f_Z} = m_Z$$

there appears relation $h_0(a)$



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- We will be assuming here (not necessary an assumption) that fermions are localized on the UV brane (fermiophobic Higgs and KK-modes) in which case

$$g_V = g_V^{SM} f_V(0) \equiv g_V[1 - \delta_V(a, m_{KK})]$$

- The latter changes the definition of the Fermi constant measured in the μ-decay

$$G_F = G_F^{SM}(1 - 2\delta_W)$$

- Z widths are modified by $1 - 2\delta_Z$
- We have taken the observables

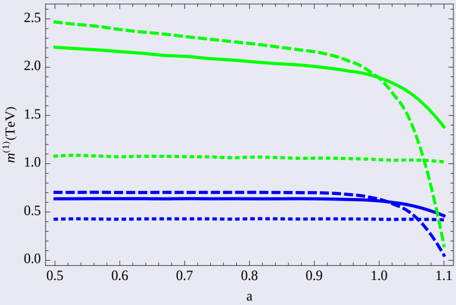
Obs	m_W [GeV]	$\bar{s}_\ell^2$	$\Gamma_{\ell^+\ell^-}$ [MeV]
Exp	80.398(25)	0.2324(12)	83.984(86)
SM	80.375(15)	0.23149(13)	83.988(16)

Table: *Observables used in the analysis*

- Fluctuations of the Higgs and the radion mix to each other
- Equations for mass eigenvalues and profiles need to be solved numerically

Lower bound in TeV on the first KK-mode

- preliminary

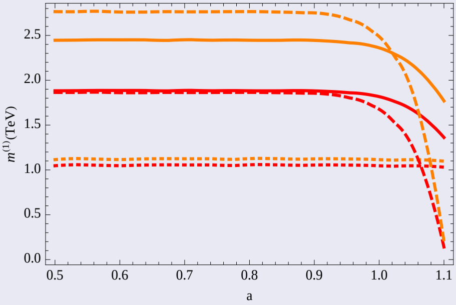


Gauge bosons and Higgs boson for different values of $\nu = 1.9$ (dashed), 1.5 (solid) and 1.1 (dotted)

- The bound on m_{KK} also propagates in the gravitational sector: graviton and radion KK-modes

Lower bound in TeV on the first KK-mode

- preliminary

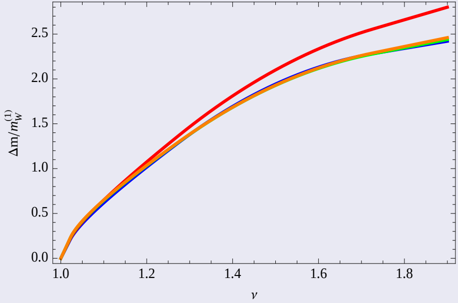


Gravitons and radion for different values of $\nu = 1.9$ (dashed), 1.5 (solid) and 1.1 (dotted)

- The mass differences for higher modes are important for their experimental detection
- Mass differences Δm depend on ν. In the unparticle limit $\nu \to 1$ $\Delta m \to 0$

$(m^{(2)} - m^{(1)})/m_W^{(1)}$

preliminary



Mass differences for Gauge bosons, Higgs, graviton and radion in units of $m_W^{(1)}$ as a function of ν

Unitarity bounds

- We will consider the unitarity bounds on $W_L W_L \to Z, \gamma, H \to W_L W_L$ scattering in the s and t channels
- One can expand the total amplitude

$$\mathcal{A} = \mathcal{A}^{(4)}(s/m_W^2)^2 + \mathcal{A}^{(2)}(s/m_W^2) + \mathcal{A}^{(0)}$$

- Using the 4D couplings

Relevant couplings

$$g_{0000} = g_5^2 \int f_0^4(y)$$

$$g_{00n} = g_5 \int f_0^2(y) f_n(y)$$

$$h_{00n} = \int e^{-A(y)} m_A(y) f_0^2(y) \xi_n(y), \quad \xi_n(y) \; Higgs\; KK-profile$$

- One can easily prove the relations

Relations

$$g_{0000} = \sum_n g_{00n}^2$$

$$\sum_n g_{00n}^2 m_{f,n}^2 = \frac{4}{3} m_W^2 g_{0000} - \frac{1}{3} \sum_n h_{00n}^2$$

- Leading to

Cancellations

$$\mathcal{A}^{(4)} = \mathcal{A}^{(2)} = 0$$

- Previous relations and cancellations are generalizations of those which appear in Higgsless theories
- Heavy KK-modes restore unitarity (as the Higgs in the SM) $\Rightarrow$ upper bound on $m_{KK}^{(1)}$

Conclusion

We have proposed

- A soft-wall model with a bulk scalar dynamically generating (double exponential) the IR/UV hierarchy
- EWSB is triggered by a Higgs doublet in the bulk which back react on the gravitational metric
- Near the UV cutoff the metric is an AdS one $\Rightarrow$ AdS/CFT interpretation
- Electroweak constraints lead to rather low (ν-dependent) bounds

EWPT (no local $SU(2)_R$ required)

$$m_W^{(1)} > \mathcal{O}(few)100\ GeV$$

- Unitarity bounds satisfied up to rather high scales (similar to Higgsless models)
- Higher modes can have $\Delta m \ll m_W^{(1)}$ which can be relevant for LHC detection