BAYESIAN STUDY AND NATURALNESS IN MSSM FORECAST FOR THE LHC

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We perform a forecast of the CMSSM for the LHC based in an improved Bayesian analysis taking into account the present theoretical and experimental wisdom about the model [1]. In this way we obtain a map of the preferred regions of the CMSSM parameter space and show that fine-tuning penalization arises from the Bayesian analysis itself when the experimental value of M_Z is considered. The results are remarkable stable when using different priors.

The start of the LHC has motivated a lot of effort to try to anticipate which kind of physics beyond the Standard Model is more likely to be there. Since the present experimental data are not powerful enough to select a small region of the parameter space of SUSY models, Bayesian Statistics becomes a very powerful tool to try to make an inference of the probability of certain regions of parameters of these models, where the choice of judicious prior probability for the parameters becomes more relevant.

1 Bayesian Statistics

The probability density of a particular point $\{p_i^0\}$ in the parameter space given a certain set of *data* is the posterior probability density function (pdf), given by

$$p(p_i^0|\mathrm{data}) = \frac{p(\mathrm{data}|p_i^0)\, p(p_i^0)}{p(\mathrm{data})}, \tag{1}$$

where $p(\mathrm{data}|p_i^0)$ is the likelihood, the probability density of measuring the given data for the chosen point in the parameter space. $p(p_i^0)$ is the prior, the "theoretical" probability density that we assign a priory to the point in the parameter space. $p(\mathrm{data})$ is the evidence. If one is interested in comparing regions of the parameter space of a given model, $p(\mathrm{data})$ is just a normalization constant.

2 Bayesian approach and Naturalness

The parameters of the MSSM should not be far from the electroweak scale in order to avoid unnatural fine-tunings to obtain the correct scale of the electroweak breaking. From the minimization of the tree-level form of the scalar potential,

$$M_Z^2 \;=\; 2\,\frac{m_{H_1}^2 - m_{H_2}^2 \tan^2\beta}{\tan^2\beta - 1} - 2\mu_{low}^2\ , \tag{2}$$

we can see that if μ and the soft masses $m_{H_{1,2}}$ are not close to the electroweak scale, a big cancellation is necessary in order to obtain the right value of M_Z. A conventional measure of this cancellation are the fine-tuning parameters [2,3],

$$c_i = \left| \frac{\partial \ln M_Z^2}{\partial \ln p_i} \right| . \tag{3}$$

Since naturalness arguments are actually statistical, one may expect that a penalization of fine-tunings should arise from the Bayesian analysis itself. Let us see how this comes about. Let us consider M_Z on a similar foot to the rest of experimental data,

$$p(\text{data}|s, m, M, A, B, \mu) \ = \mathcal{L}_{M_Z} \ \mathcal{L}_{\text{rest}} \ , \tag{4}$$

where s represents the SM parameters, $\mathcal{L}_{\text{rest}}$ is the likelihood of the all physical observables except M_Z, and $\mathcal{L}_{M_Z}$ is the likelihood of M_Z. Let us now use the sharpness of the likelihood of M_Z to approximate $\mathcal{L}_{M_Z} \simeq \delta(M_z - M_z^{exp})$ and marginalise the pdf in the $\mu-$parameter, performing a change of variable $\mu \to M_Z$:

$$\begin{aligned} p(s, m, M, A, B| \text{ data}) \ &\simeq \ \int dM_Z \left[\frac{d\mu}{dM_Z}\right] \delta(M_z - M_z^{exp}) \, \mathcal{L}_{\text{rest}} \ p(s, m, M, A, B, \mu) \\ &= \ \left[\frac{d\mu}{dM_Z}\right]_{\mu_0} \mathcal{L}_{\text{rest}} \ p(s, m, M, A, B, \mu_0) \ , \end{aligned} \tag{5}$$

where μ_0 is the one which reproduce the experimental value of M_Z as a function of the rest of the parameters [a]. Now, comparing this to the definition of fine-tuning parameters one gets,

$$p(s, m, M, A, B| \text{ data}) \ = \ 2\frac{\mu_0}{M_Z} \ \frac{1}{c_\mu} \ \mathcal{L}_{\text{rest}} \ p(s, m, M, A, B, \mu_0), \tag{6}$$

where the presence of the fine-tuning parameter does indeed penalize regions of the parameter space with large fine-tunings. As we will see, this is enough to make the high-energy region of the parameter space statistically insignificant.

We have performed a Bayesian analysis of the MSSM with the following improvements [4]: as we show above, the fine-tuning penalization arises from the Bayesian analysis itself; we have made a rigorous treatment of the nuisance variables where Yukawa couplings are fundamental parameters in contrast with previous analysis; and, finally, we have used an efficient set of variables to scan the MSSM, $\{m, M, A, B, mu, y_t\} \to \{m, M, A, \tan\beta, M_z, m_t\}$. The last change of variables introduces a jacobian factor,

$$p(g_i, m_t, m, M, A, \tan\beta|\text{data}) = \mathcal{L}_{\text{rest}} \ J|_{\mu=\mu_0} \ p(g_i, y_t, m, M, A, B, \mu = \mu_0)$$

where,

$$J = \frac{\partial \mu}{\partial M_Z} \frac{\partial y_t}{\partial m_t} \frac{\partial B}{\partial t\beta} \simeq \frac{1}{4}(g^2 + g'^2)^{1/2} \left[\frac{E}{R_\mu^2}\right] \frac{B_{low}}{\mu} \frac{t^2 - 1}{t(1 + t^2)} \left(\frac{y_t}{y_t^{low}}\right)^2 s_\beta^{-1}. \tag{7}$$

In addition, we have developed sensible priors which assume that soft-breaking term share a common origin. This analysis have been implemented using `MultiNest` [5] algorithm as implemented in the *SuperBayeS* [6] code which incorporate `SoftSusy` [7], `SusyBSG` [8], `SuperIso` [9], `MicrOMEGAs` [10] codes.

As can be seen in Fig 1, besides making the high-energy parameter space quite irrelevant, the EW breaking has a another remarkable effect, the probability distribution (pdfs) based on logarithmic or flat prior are quite similar after the incorporation of the EW scale.

[a] We have ignored here the nomalization factor. See eq. [1]

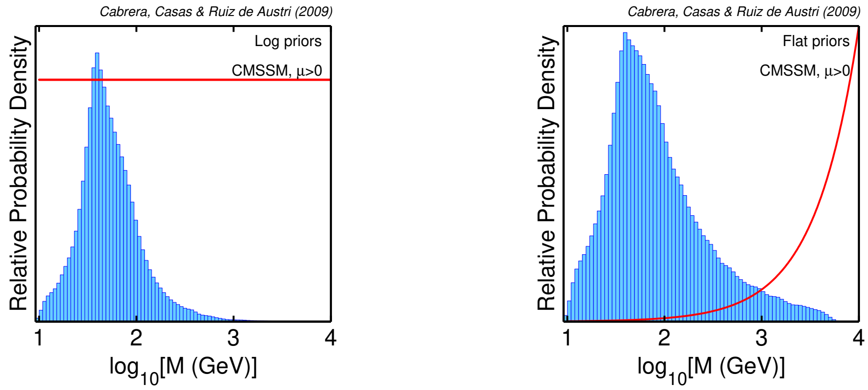

Figure 1: 1D marginalized posterior probability distribution of the M for logarithmic (left panels) and flat (right panel) priors in the $\mu > 0$ case, for a scan including the EW breaking information ($M_z^{\rm exp}$). The red lines represents the marginalized prior.

3 Experimental Constraints

In this section we incorporate all the relevant experimental information to the likelihood piece of the probability distribution. We start by considering the most reliable and robust pieces of experimental information: EW and B(D)-physics observables and lower bounds on the masses of supersymmetric particles and the Higgs mass. The left panel of Fig. 2 shows the pdf of $\{M-m\}$ plane once this experimental information is incorporated. Clearly the bulk of the probability is now pushed into higher energy. This effect is basically due to the Higgs mass bound. It is well known that the tree-level Higgs mass is bounded from above by M_Z, so radiative corrections are needed. Concerning the other observables, everything works fine as long as SUSY is not at too low scale. We also show the discovery reach of LHC [11] for 1 fb^{-1} and 100 fb^{-1}. These lines correspond to $A = 0$ and $\tan\beta = 45$, but they provide a good indication for the LHC discovery potential.

Next, we have added information about anomalous magnetic momentum of the muon, a_μ. Taking $e^+e^- \to$ hadrons data, there is a 3.3σ discrepancy between the experimental and SM theoretical prediction, which has been often claimed as a signal of new physics. If one accept this, the supersymmetric masses should be brought to quite small values in order to produce a large enough contribution $\delta^{\rm MSSM} a_\mu$ to reconcile theory and experiment, as we can see in the center panel of Fig. 2. If we instead use $\tau-$decay data there is no big discrepancy, SUSY contribution does not need to be large and the pdfs are essentially unchanged by its inclusion.

Supersymmetry offers good candidates for Cold Dark Matter (CDM), the most popular and natural one is the lightest neutralino. The right panel of Fig. 2 shows the pdfs after the assumption of CDM is made of neutralinos and without the information from a_μ.

We have repeated the same analysis for $\mu < 0$. We have evaluated the ratio of the evidence of both cases, the Bayes factor, in order to compare the relative probability of the $\mu > 0$ and $\mu < 0$ branches. The $\mu < 0$ is slightly favoured, due to its capability to reproduce the central value of $b \to s\,\gamma$, but the effect is not really significant. On the other hand a_μ favours $\mu > 0$ branch, this effect is stronger when Ω_{DM} is included. This is because Ω_{DM} constraints favours the low-energy region of the parameter space, and this is strongly preferred by a_μ.

In conclusion, LHC offers an exciting horizon for SUSY discovery, but there is still a possibility that this escapes detection, especially if the Higgs mass is not close to its present experimental bound.

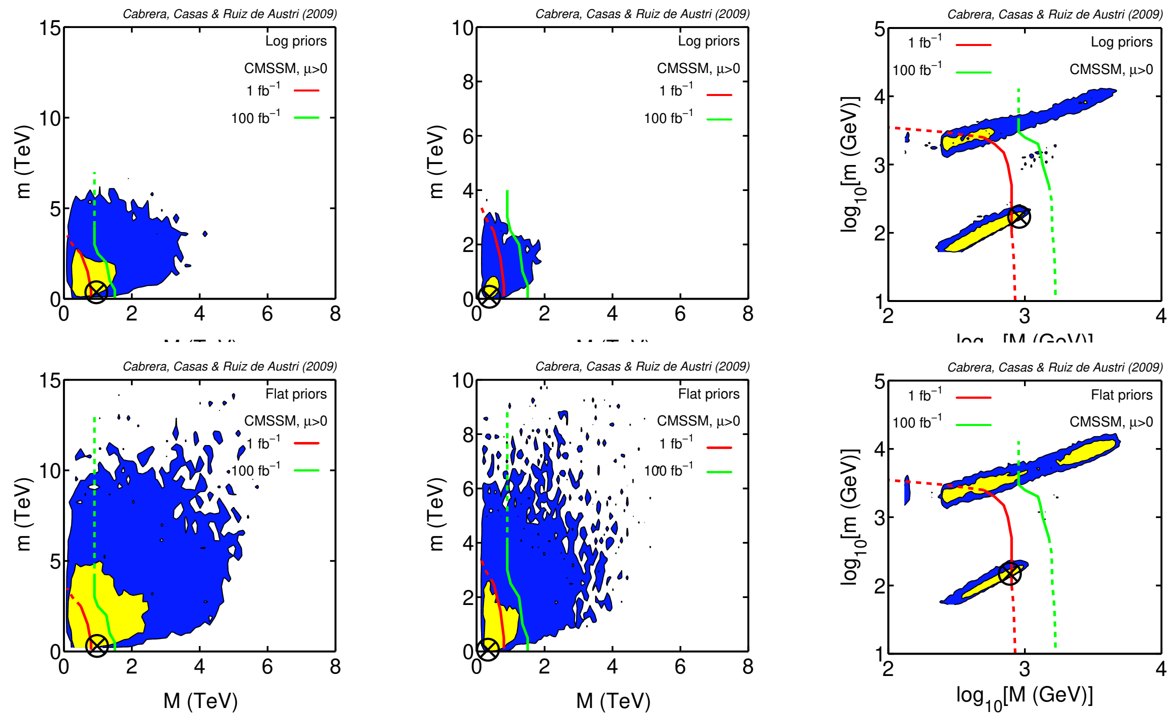

Figure 2: 2D marginalized posterior probability distribution for logarithmic (upper panel) and flat (lower panel) priors in the $\mu > 0$ case including: EW + B(D)-physics observables (left panel); + a_μ (center panel); + CDM (right panel). The inner and outer contours enclose respective 68% and 95% joint regions.

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References

1. M. E. Cabrera, A. Casas and R. R. de Austri, arXiv:0911.4686 [hep-ph]. To appear in JHEP
2. J. R. Ellis, K. Enqvist, D. V. Nanopoulos and F. Zwirner, Mod. Phys. Lett. A **1** (1986) 57.
3. R. Barbieri and G. F. Giudice, Nucl. Phys. B **306** (1988) 63.
4. M. E. Cabrera, J. A. Casas and R. Ruiz de Austri, JHEP **0903** (2009) 075 [arXiv:0812.0536 [hep-ph]].
5. F. Feroz and M. P. Hobson Mon. Not. Roy. Astron. Soc. **384** 449 (2008).
6. Available from: `http://superbayes.org`
7. B. C. Allanach, Comput. Phys. Commun. **143** (2002) 305 [arXiv:hep-ph/0104145].
8. G. Degrassi, P. Gambino and P. Slavich, Comput. Phys. Commun. **179** (2008) 759 [arXiv:0712.3265 [hep-ph]].
9. F. Mahmoudi, Comput. Phys. Commun. **180**, 1579 (2009) [arXiv:0808.3144 [hep-ph]].
10. G. Belanger, F. Boudjema, A. Pukhov and A. Semenov, Comput. Phys. Commun. **149** (2002) 103 [hep-ph/0112278]; Comput. Phys. Commun. **174**, 577 (2006) [hep-ph/0405253].
11. H. Baer, V. Barger, A. Lessa and X. Tata, JHEP **0909** (2009) 063 [arXiv:0907.1922 [hep-ph]].