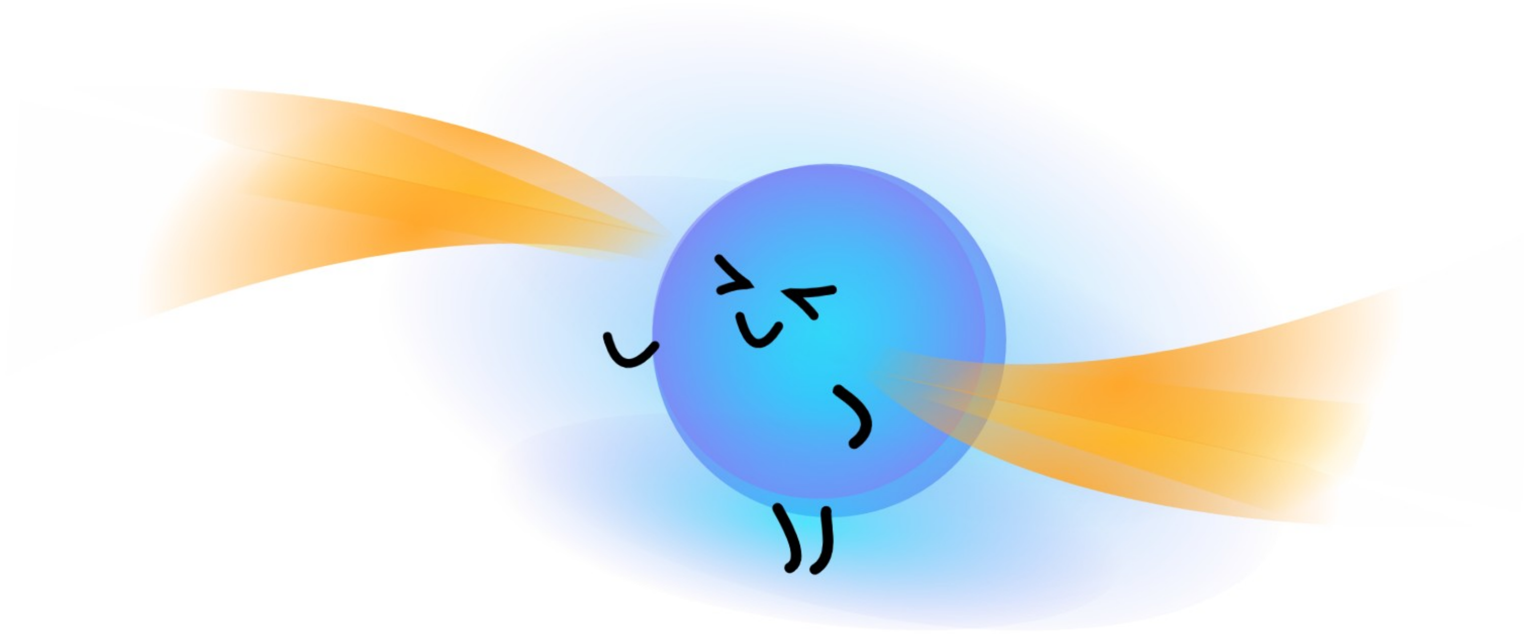
Dynamical aspects of superfluid neutron stars



Marco Antonelli

antonelli@lpccaen.in2p3.fr

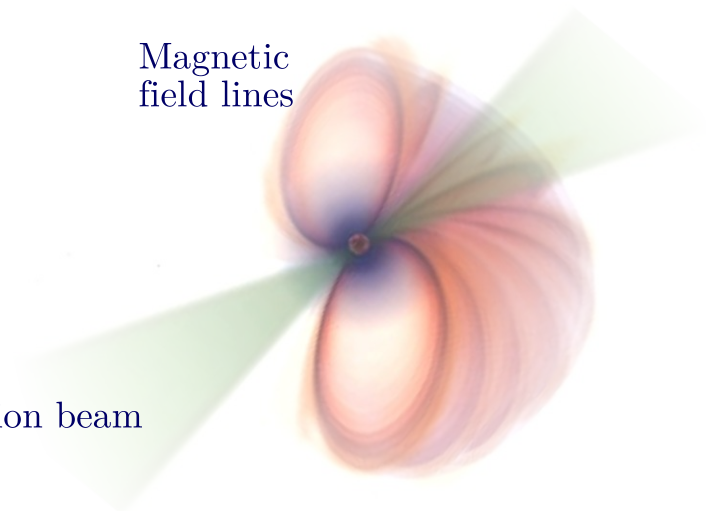
CNRS, Laboratoire de Physique Corpusculaire de Caen

Ecole Thématique PhyNuBe

Centre Paul Langevin, Aussois – December 7, 2021

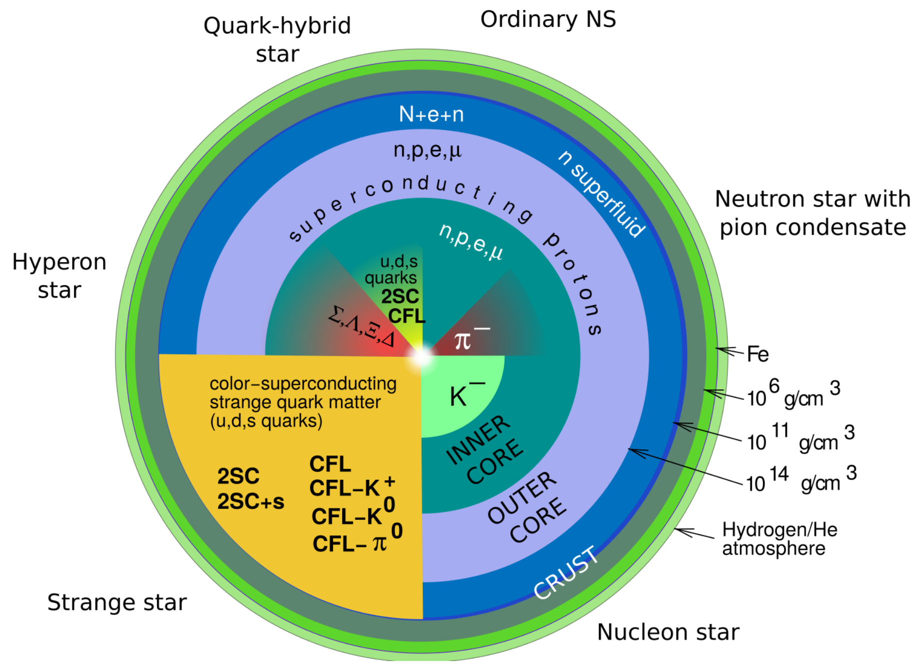
Neutron stars

What we can observe: lighthouse model



Open issue: precise description of the beamed emission mechanism

What we can not observe: internal structure



Compressing matter liberates degrees of freedom. Open issue: composition of the inner crust and transitions in the core.

Gravity: holds the star together (a NS is not a giant nucleus)

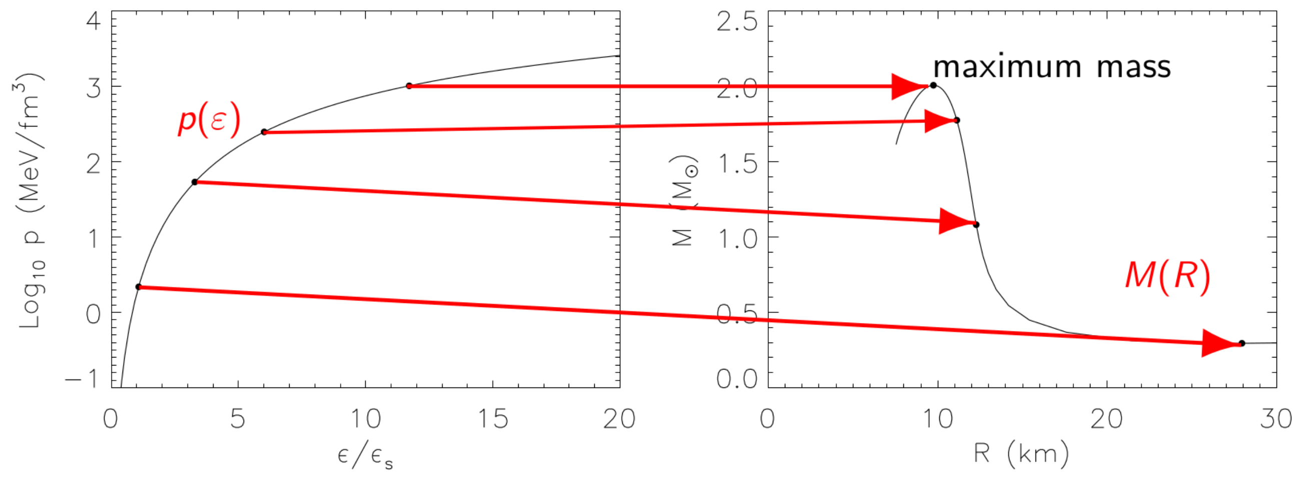
Electromagnetism: makes pulsars pulse and magnetars flare

Strong interaction: determines the internal composition and prevents gravitational collapse

Weak interaction: determines composition and reaction rates (chemical equilibrium, neutrino cooling, viscosity)

Static structure: EOS and M-R relation

Pressure – Energy relation (equation of state)



Mass - Radius relation

TOV equations (1934-39): hydrostatic equilibrium in GR

$$\frac{dm}{dr} = 4\pi r^2 \mathcal{E}$$

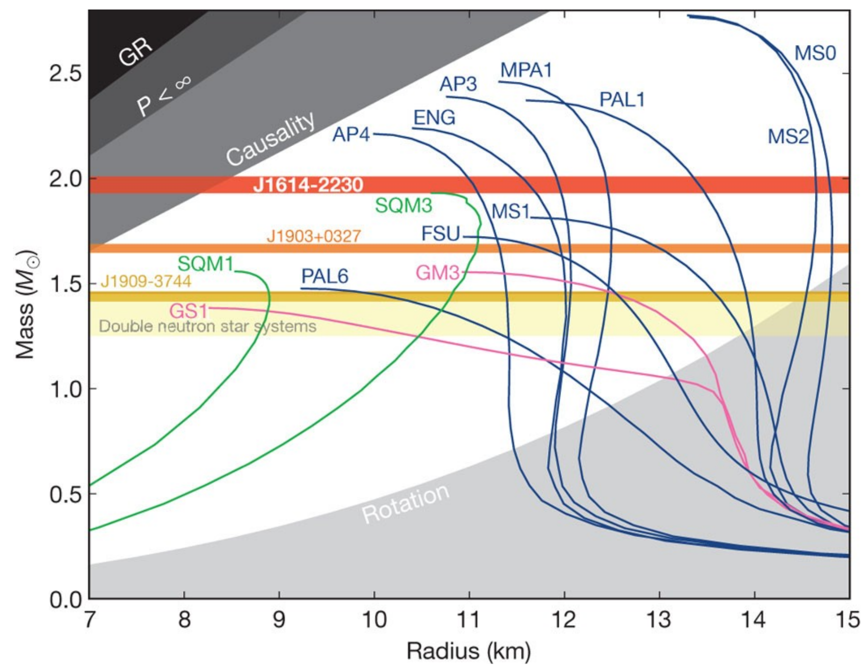
$$\frac{dP}{dr} = -\mathcal{E}\frac{d\Phi^*}{dr}\left(1+\frac{P}{\mathcal{E}}\right)$$

$$\frac{d\Phi^*}{dr} = \frac{m}{r^2}\left(1+\frac{4\pi r^3 P}{m}\right)\left(1-\frac{2m}{r}\right)^{-1}$$

TOV inversion: Lindblom, ApJ 398, 569 (1992)

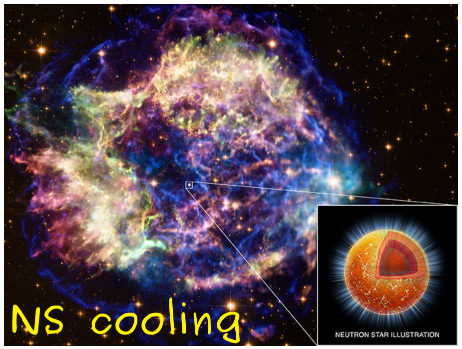
EOS lines not intersecting the J1614-2230 band are ruled out.

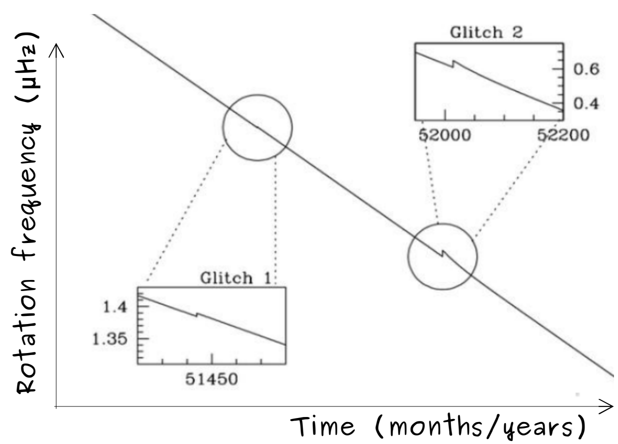
Rotation increases the maximum possible mass for each EOS: $\lesssim$2% correction for P~3 ms

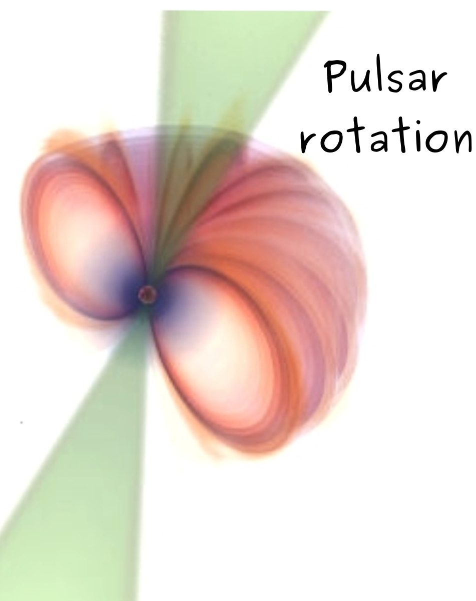


Demorest et al, Nature 467 (2010)

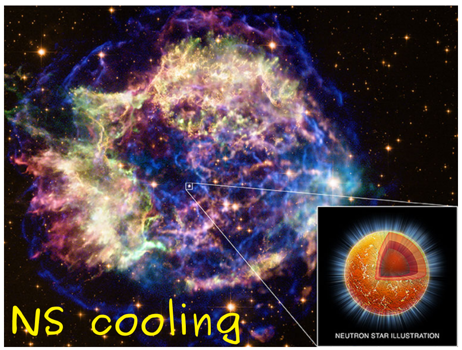
Dynamics of neutron stars (no B field)







Dynamics: superfluidity is important

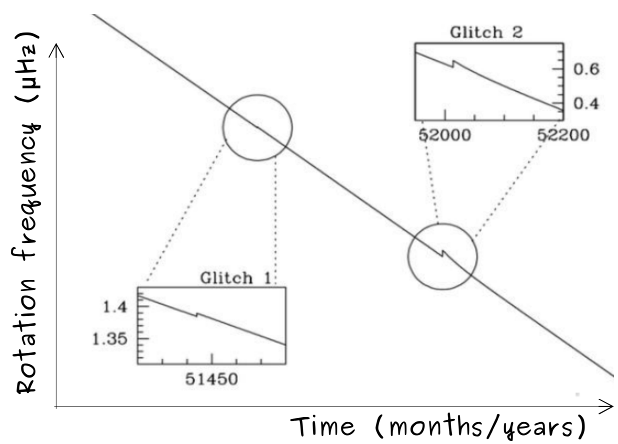


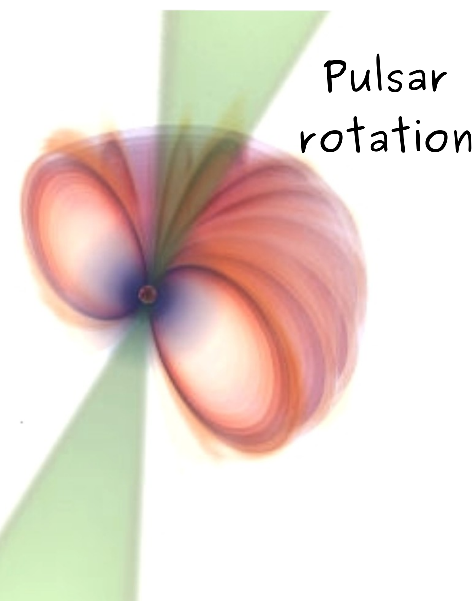
Superfluidity affects neutrino emission and heat capacity
(because of nn and pp pairing in the inner crust and core)

Superfluidity allows for more and different oscillation modes
(like second sound in Helium-II)

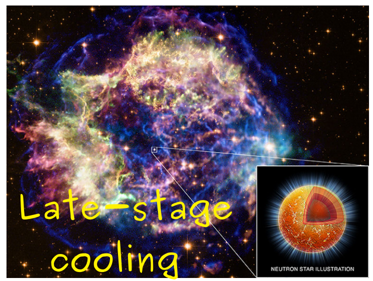
Pulsar glitches are direct manifestations of superfluidity

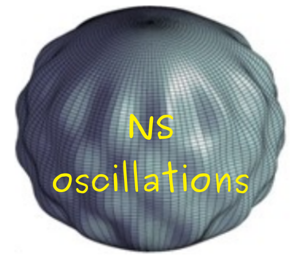
Pulsars are isolated → can spin up only if there is an internal superfluid that rotates faster than the observed component

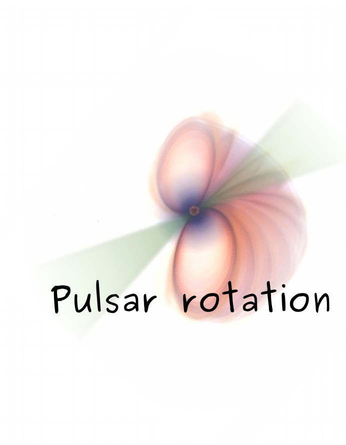




Dissipation in Neutron Stars



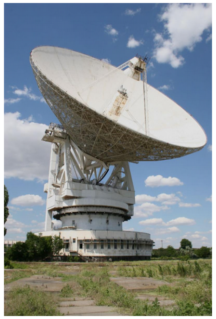




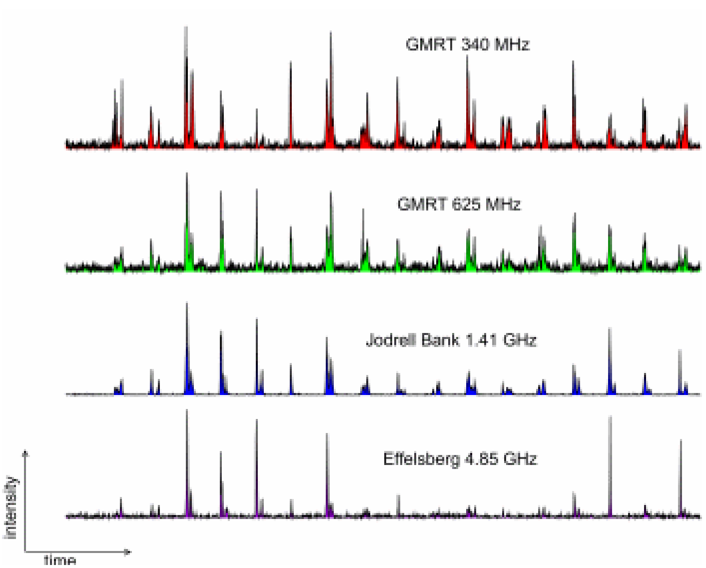
	Late-stage cooling	NS oscillations	Pulsar rotation	Merger
Shear viscosity: (out of equilibrium distribution) (electron VS nuclei, protons, impurities) (binary collisions of phonons)		✓		✓
Bulk viscosity: (out of equilibrium distribution) (nuclear reactions) (phonon-phonon collisions)		✓		✓
Vortex mediated friction: (vortex motion in the superfluid)	✓	✓	✓	

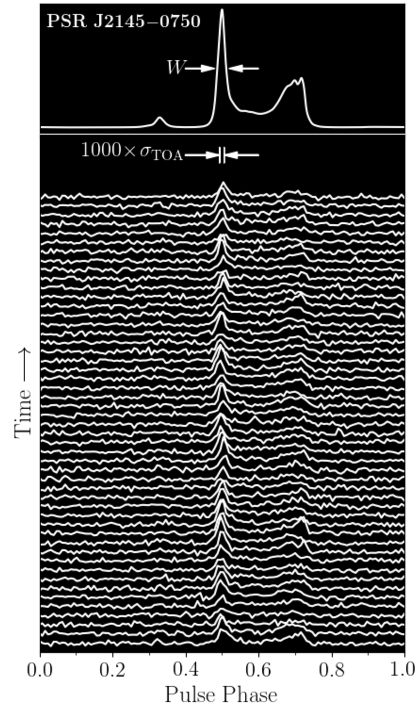
...not to mention the phenomena where the magnetic field is fundamental (magnetar flares).

Pulsars

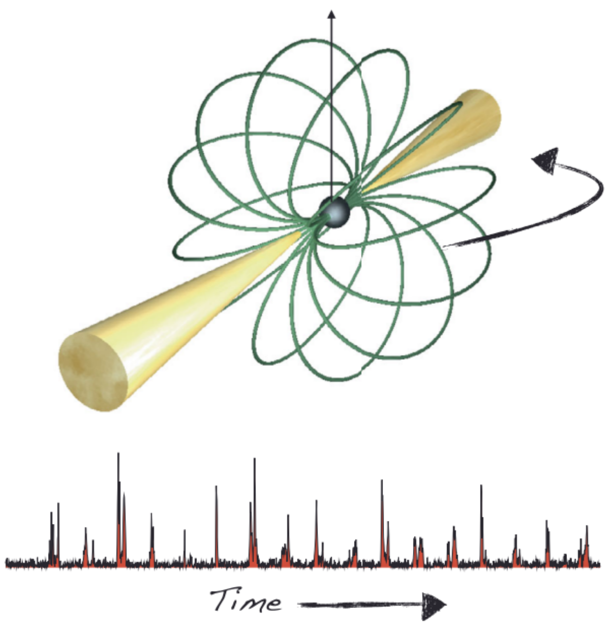


What we observe (first pulsar discovered in 1967):

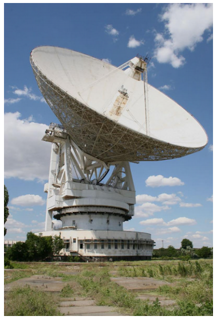




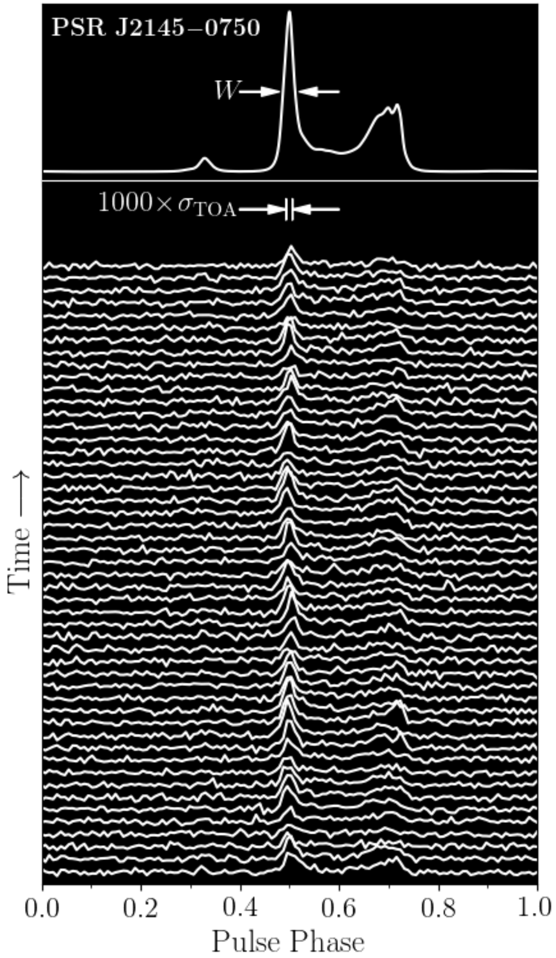
What we think it is: pulsars are the astronomical manifestation of rotating, magnetized neutron stars



Pulsar timing



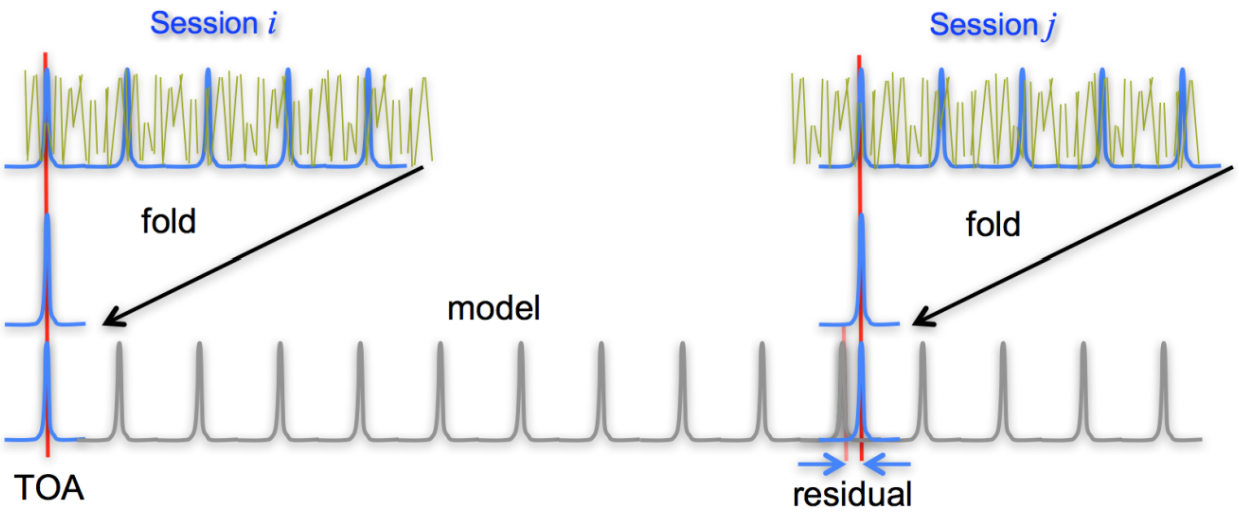
Fold the signal to obtain the period.



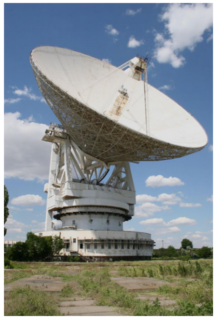
Obtain the best spin-down model for a certain session.

Compare TOA of new session with TOA expected from the previous one.

Timing irregularities: mismatch between expected and observed TOA.



Pulsar glitches

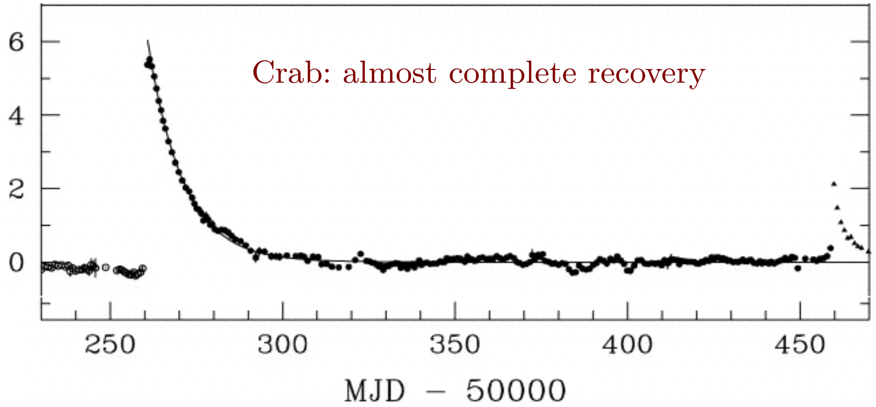


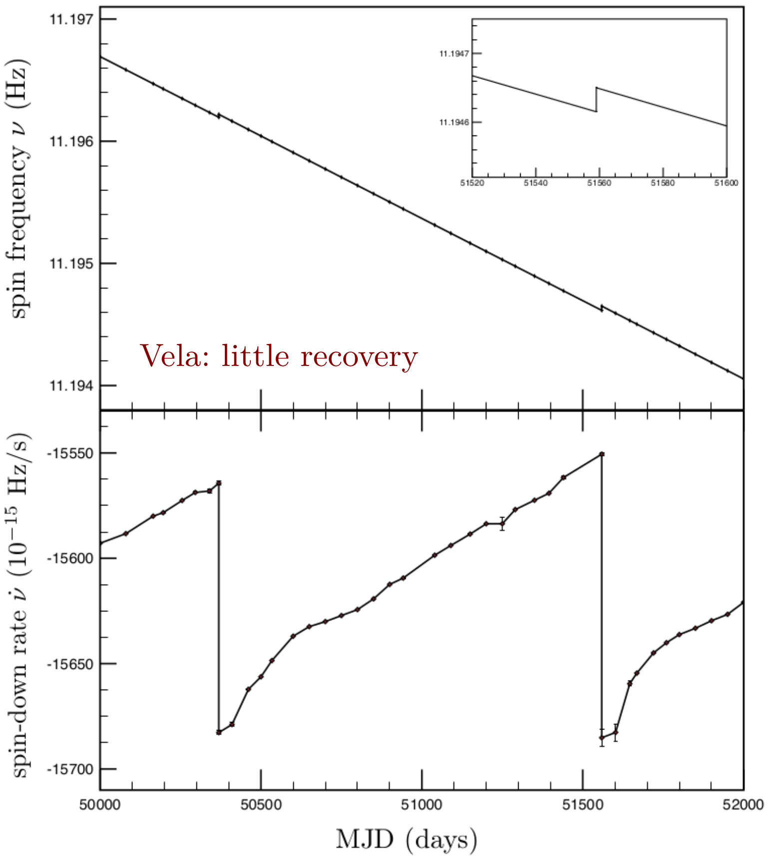
- First detected in the Vela and Crab in 1969

- Diverse phenomenology: probably due to different age (temperature), mass, rotational parameters, magnetic field

- Detected in isolated objects: conservation of the total angular momentum must be satisfied

- Typical amplitudes: $\Delta\Omega \sim 10^{-7} - 10^{-4}$ rad/s





Pulsar glitches: two-component model

Baym et al. *Spin Up in Neutron Stars: The Future of the Vela Pulsar*, Nature (1969)

The long recovery time-scales (of order months) observed after the first Vela glitch were considered to be evidence for a weakly coupled superfluid component in the stellar interior.

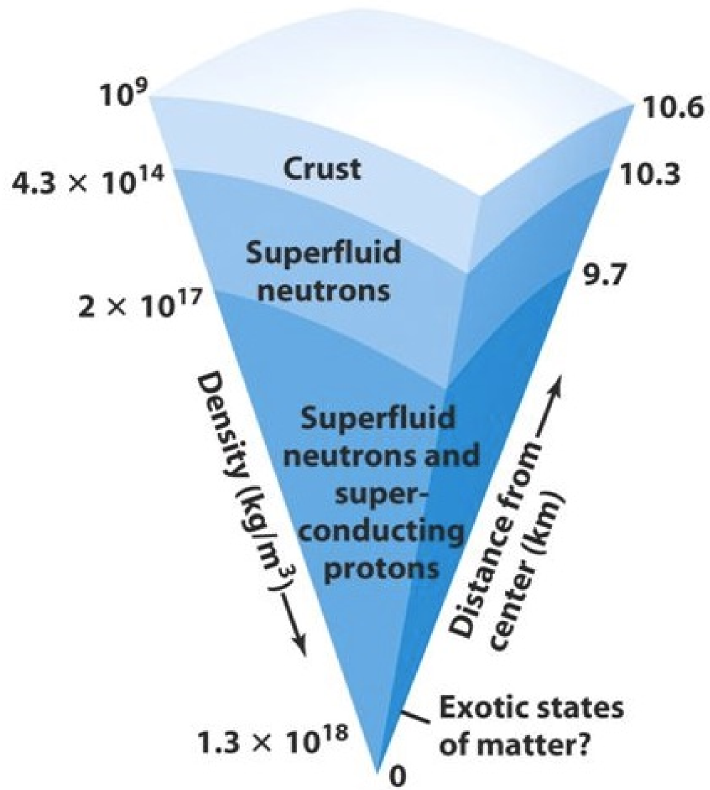
Superfluidity → "no viscosity" → long relaxation timescale

"crust" — braking torque

$$I_c \dot{\Omega}_c = -\alpha - \frac{I_c}{\tau_c}(\Omega_c - \Omega_n)$$

"neutrons"

$$I_n \dot{\Omega}_n = \underbrace{\frac{I_c}{\tau_c}(\Omega_c - \Omega_n)}_{\text{Internal torque ("mutual friction")}},$$



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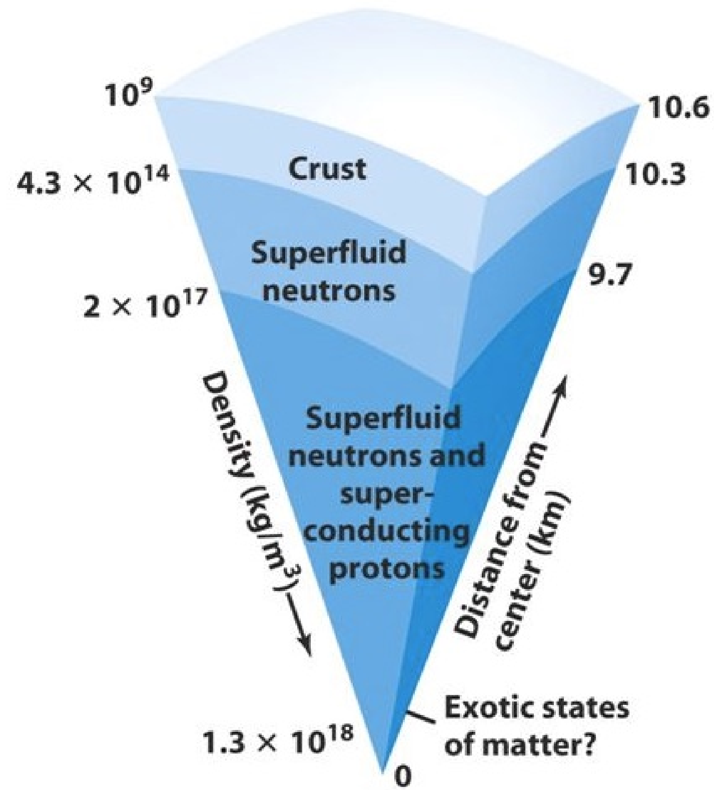
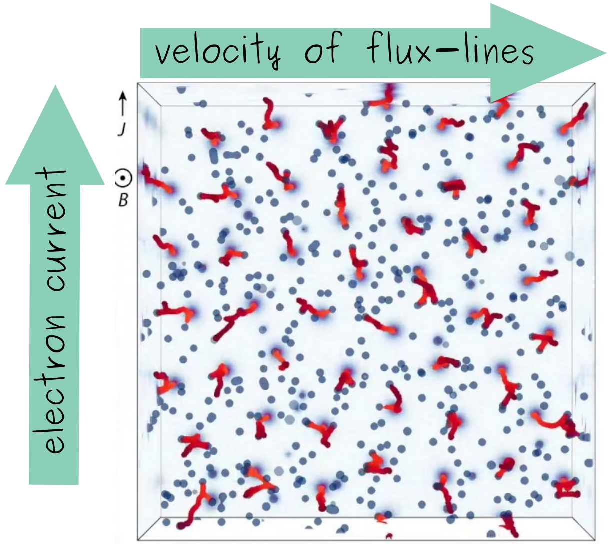
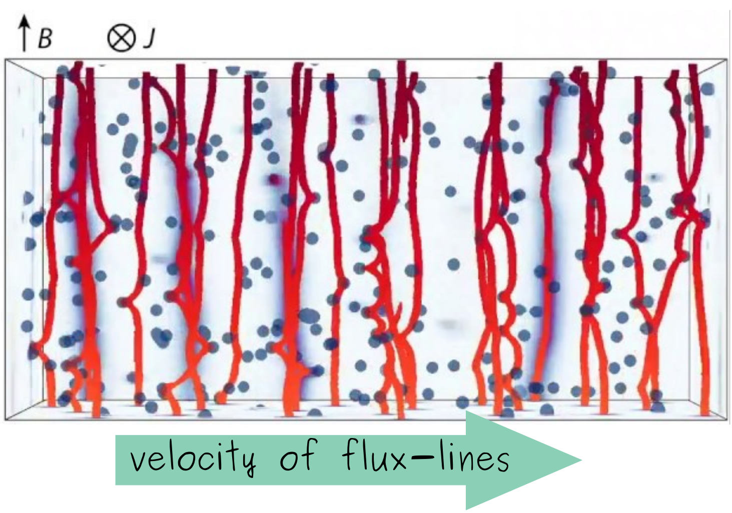
Internal torque ("mutual friction")



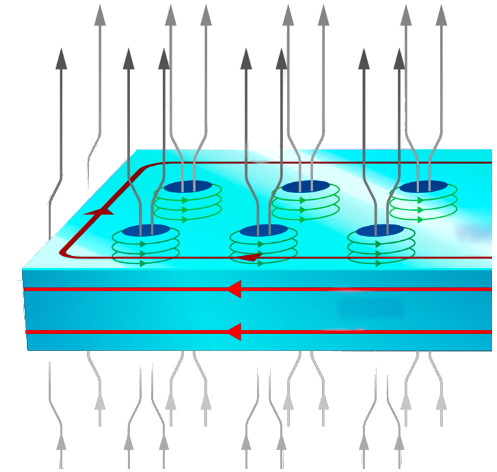
Question: how to "recharge" the velocity difference for the next glitch?

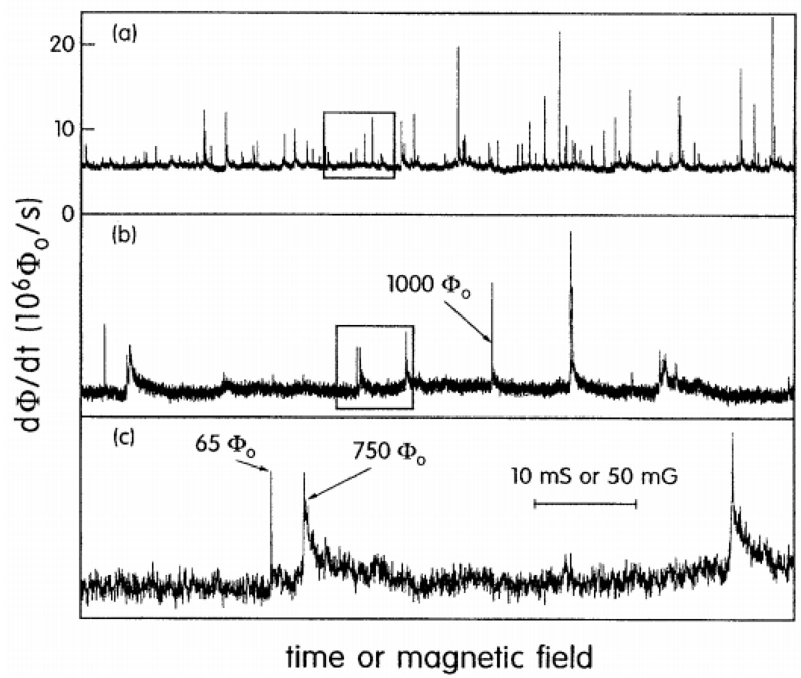
Vortices in type-II superconductors





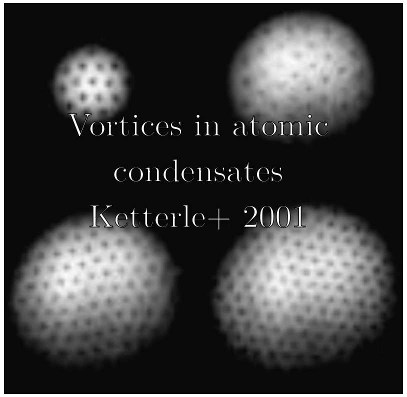
Willa et al. *Strong pinning regimes by spherical inclusions in anisotropic type-II superconductors* (2018)

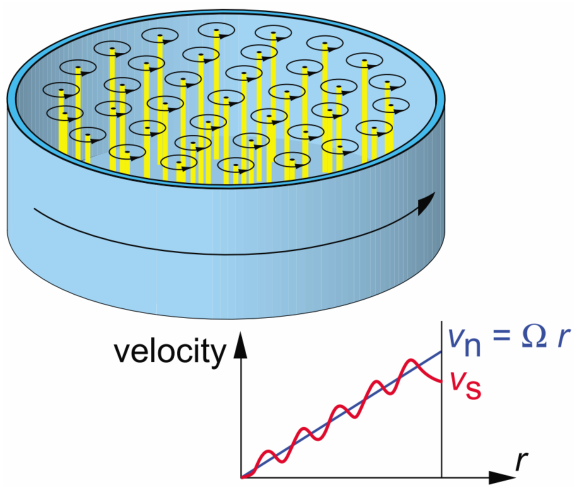




Field et al. *Superconducting Vortex Avalanches* (1995)

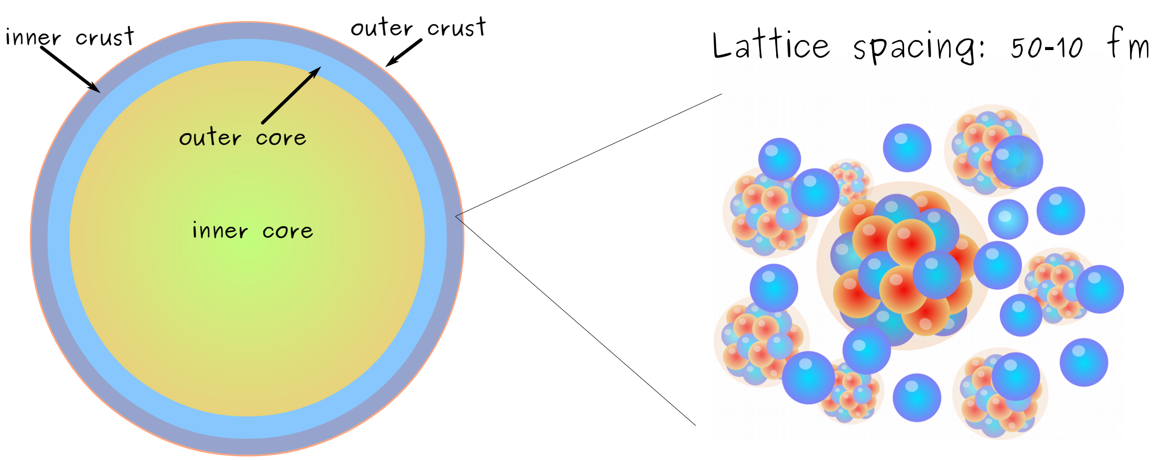
Quantized vortices in superfluids

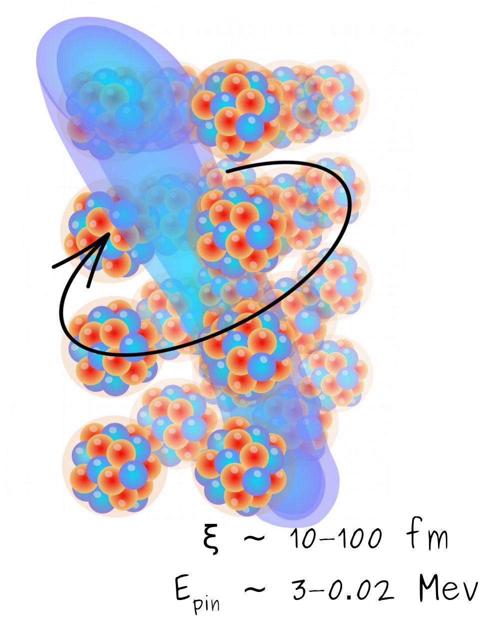




The angular velocity of a superfluid is determined by its vortex configuration
Feynman, Prog. Low Temp. Phys. 1955

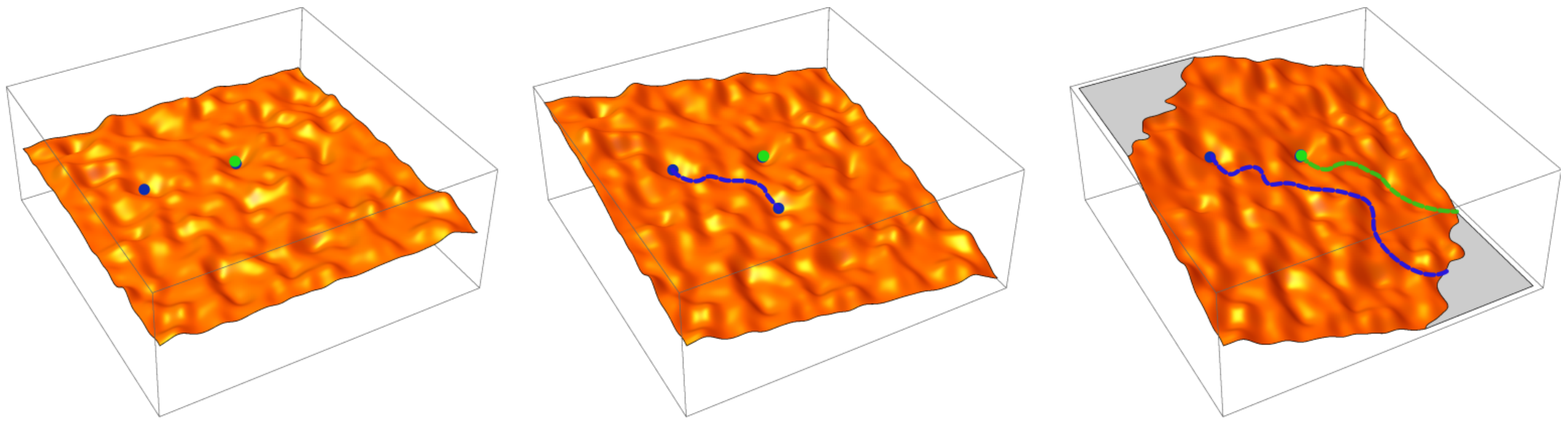
If vortices **"pin"** to the ions, the superfluid can not follow the crustal lattice during the spin down
Anderson & Itoh, Nature 1975





Inner crust: solid of heavy ions with "dripped" superfluid neutrons
(similar to a metal, but with a sea of relativistic electrons and paired neutrons)

Vortex motion in a pinning landscape



Vortices are immersed in a complex pinning landscape.

They tend to sit in the wells of the landscape:
→ superfluid can not spin down
→ the normal component spins down
→ a “lag” is created (i.e. a superfluid current in the frame of the normal component)

The “lag” slowly increases in time (because of the steady spin-down)
→ the pinning landscape is continuously tilted, till the vortex breaks free
→ possible to trigger a catastrophic unpinning event? (probably vortex-vortex-interactions needed)

Attractive features: complex evolution with possible avalanche-like dynamics, self-organized-criticality.

Superfluid hydrodynamics

Vortex core scale: “trunk”
~ 10 fm in a NS
(microscopic models)

Inter-vortex scale: “trees”
~ 10^{-3} cm in a NS
(vortex filament model)

Fluid element: “forest”
from mm to km in a NS
(macroscopic hydrodynamics)

We can not take into account each vortex (~10^{18} in a pulsar) → **“two-fluid”** smooth hydrodynamics

2 Euler-like equations + **entrainment** + **mutual friction**

$$\partial_t \rho_{\mathrm{x}} + \nabla_i(\rho_{\mathrm{x}} v^i_{\mathrm{x}}) = 0$$

$$(\partial_t + v^j_{\mathrm{x}}\nabla_j)(v^{\mathrm{x}}_i + \varepsilon_{\mathrm{x}} w^{\mathrm{yx}}_i) + \nabla_i(\tilde{\mu}_{\mathrm{x}} + \Phi) + \varepsilon_{\mathrm{x}} w^j_{\mathrm{yx}} \nabla_i v^{\mathrm{x}}_j = f^{\mathrm{x}}_i / \rho_{\mathrm{x}}$$

The **dynamics of vortices** in a fluid element gives the form and strength of the macroscopic **“mutual friction”**

Chemical label **X = n,p** **n**→superfluid neutrons **p**→normal component

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$$\rho_n D_t \mathbf{v}_n + \ldots = \mathbf{F}_{MF}$$
$$\rho_p D_t \mathbf{v}_p + \ldots = -\mathbf{F}_{MF}$$

The **dynamics of vortices** in a fluid element gives the form and strength of the macroscopic **“mutual friction”**

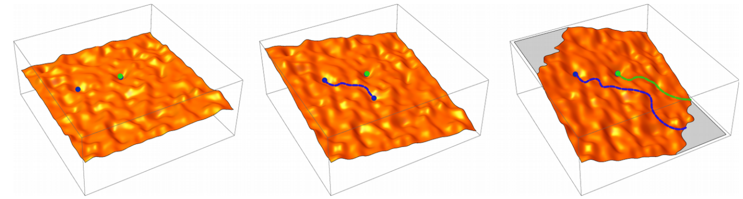
$$\boldsymbol{F}_n = -\kappa\, n_v\, \hat{\boldsymbol{\kappa}} \times \left(\langle \dot{\boldsymbol{x}} \rangle - \boldsymbol{v}_{np} \right)$$

Chemical label **X = n,p** **n**→superfluid neutrons **p**→normal component

Average vortex motion → Mutual friction

Antonelli & Haskell, MNRAS (2020)

It's a "kinetic approach" but with point vortices instead of particles



- Fix a background "lag" (background current of superfluid neutrons)
- Assign random position of a vortex in the pinning landscape and solve the trajectory

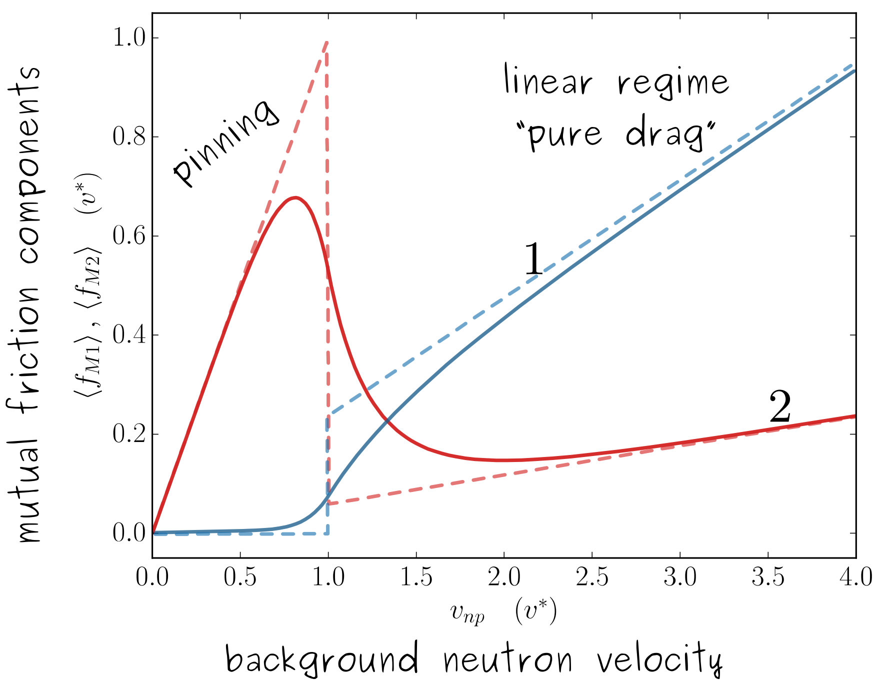
disordered pinning force field

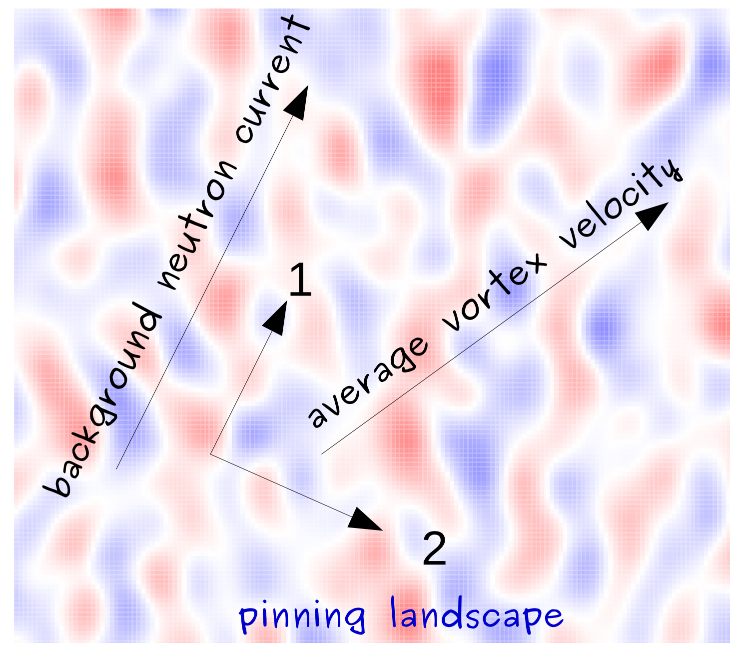
$$\hat{\boldsymbol{\kappa}} \times (\dot{\boldsymbol{x}}(t) - \boldsymbol{v}_{np}) - \mathcal{R}\,\dot{\boldsymbol{x}}(t) + \boldsymbol{f} = 0$$

- Repeat many times and find the average vortex velocity for the given "lag"
- The mutual friction is given by $\boldsymbol{F}_n = -\kappa\, n_v\, \hat{\boldsymbol{\kappa}} \times (\langle \dot{\boldsymbol{x}} \rangle - \boldsymbol{v}_{np})$

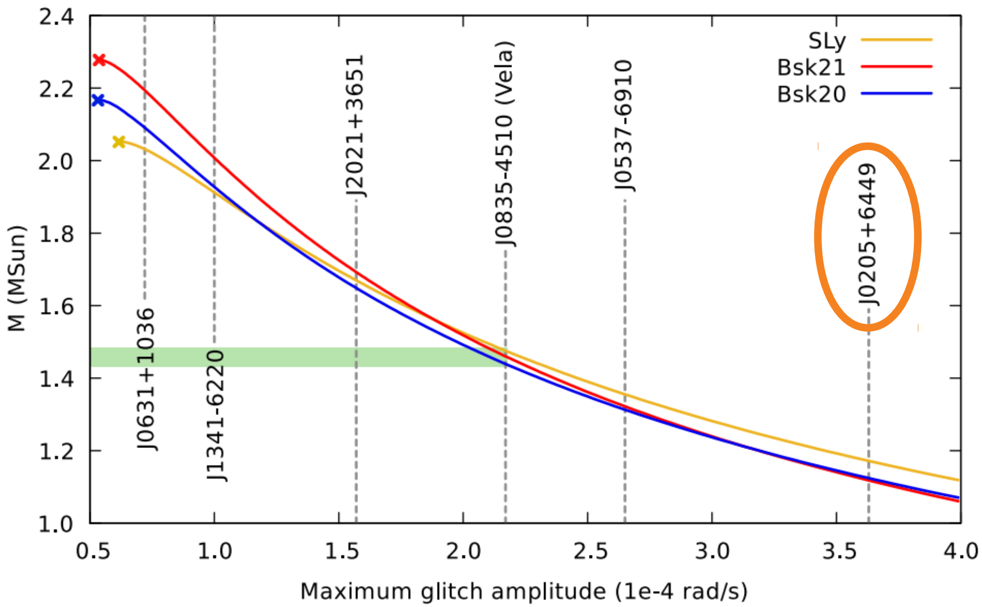
average vortex velocity

assigned "lag"





Static constraints from glitches



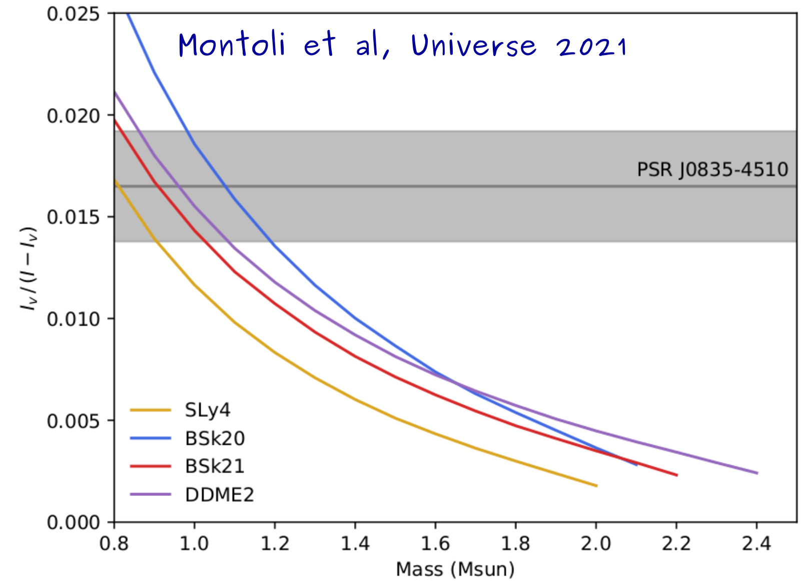
The largest **glitch size** constraints the **pinning forces**

$$\Delta\Omega_{\rm abs} = \frac{\pi^2}{I\kappa}\int_0^{R_d} dr\, r^3\, e^{\Lambda(r)}\, \frac{\mathcal{E}(r)+P(r)}{m_n\, n_B(r)\, c^2}\, f_P(r)$$

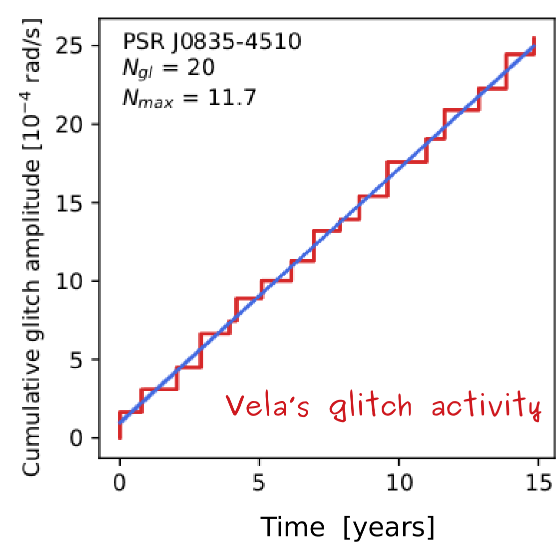
Minimum masses:

Observed: M = 1.174 ± 0.004 M⊙ Martinez et al, ApJ 2015

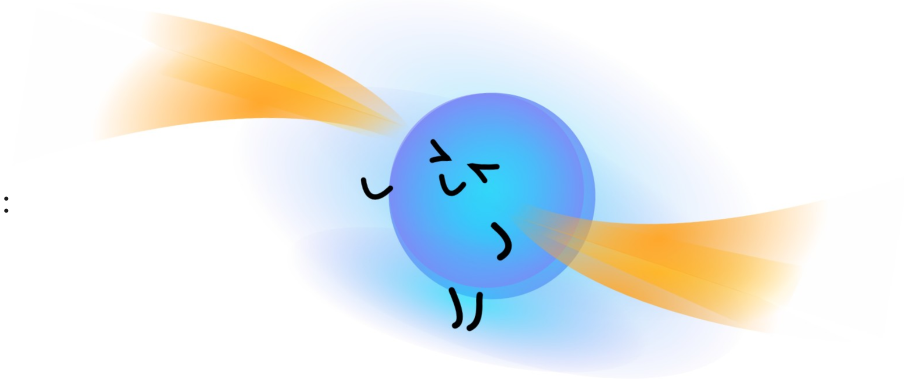
From CCS simulations: M ≈ 1.15 M⊙ Lattimer, Ann.Rev.Nucl.Part.Sci. 2015



Vela's **glitch activity** constraints the **entrainment** in the crust



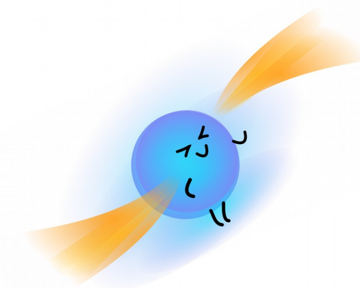
Some considerations



Glitches provide us with some interesting theoretical challenges:

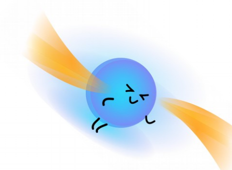
...thank you spinning pulsar!

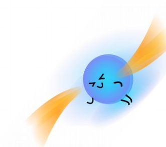
→ single vortex dynamics in non-homogeneous environments
→ collective avalanche dynamics
→ how to formulate superfluid hydro in GR?
→ how to describe pinning at the microscopic scale?



Cross contamination between different fields is necessary. Some open questions:

→ glitch trigger: role of starquakes? (can we really have quakes in a NS?)
→ role of entrainment (strong/weak? affected by disorder?)
→ better understanding of dissipation at micro/meso scale
→ collective aspects of vortex dynamics (rigidity? viscoelasticity?)





The most important thing: more and better observations!

Improved timing techniques (and more observation time) → falsify current spin-up models

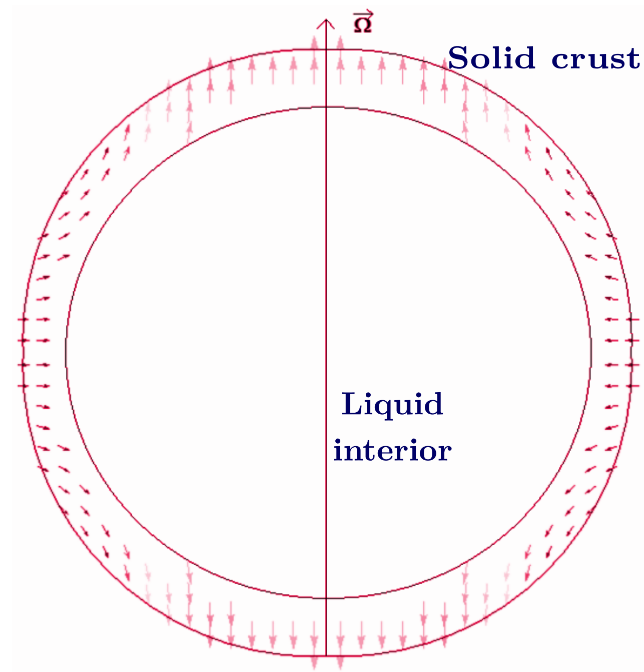
Pulsar glitches: starquake model

Neutron Starquakes and Pulsar Periods

M. RUDERMAN

Nature **223**, 597–598 (09 August 1969)

THE outer layers of neutron stars form a solid crust with a calculable rigidity (shear modulus) very soon after the stars are born. Subsequent changes in stellar shape from oblate toward spherical, as the neutron star angular velocity decreases, will induce stresses in the crust until the maximum shear strain which the solid can support is reached. Beyond this yield point there will be a sudden relaxation of the stress, and a very slight change in stellar shape and moment of inertia. The calculated accompanying jump in angular velocity is close to that which has been observed in a pulsar.



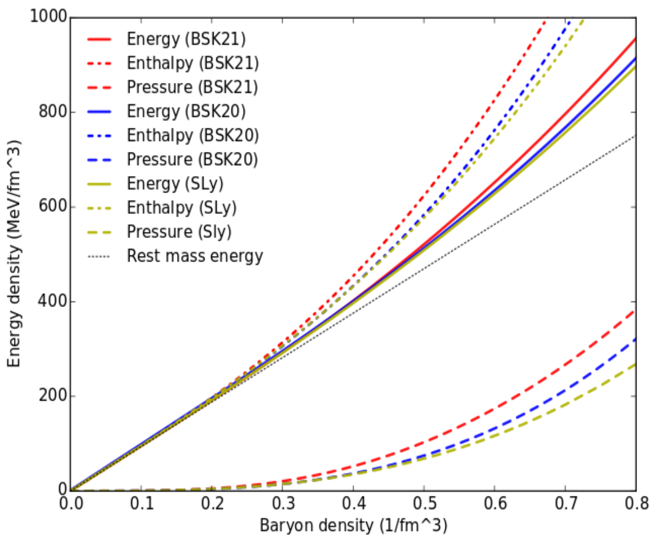
$$\frac{\Delta\dot{\Omega}}{\dot{\Omega}} \approx -\frac{\Delta I}{I} \approx -\frac{\Delta\Omega}{\Omega}$$

Detected pulsar glitches: $\Delta\Omega/\Omega \sim 10^{-9} - 10^{-6}$

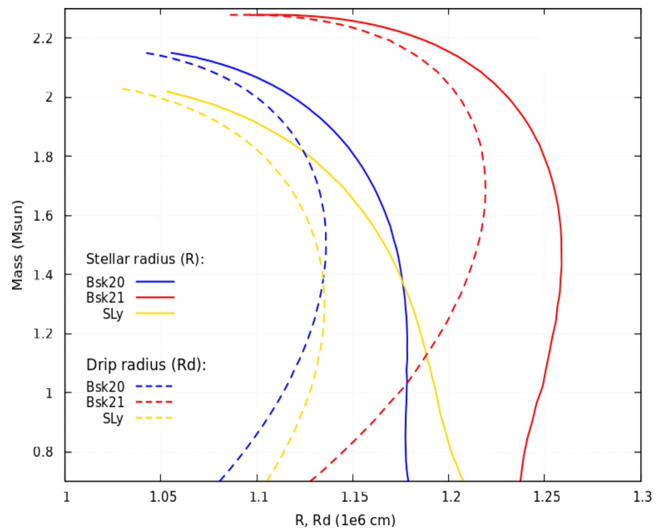
Comparison with earthquakes: the Sumatra earthquake in December 2004 (magnitude 9) shortened the length of a day by ~7 millionths of a second, $\Delta\Omega/\Omega \sim 10^{-10}$
(we can currently measure the length of an Earth day with an accuracy of ~20 millionths of a second, so the shortened day caused by earthquakes can be estimated but not measured)

Unified EOS

Astrophysical implications: Fantina et al, A&A 2013



EOS: E(nB)



M-R relation

Crust-core transition at ~0.5 of nuclear saturation density: 0.16 fm^{-3}

EoS	n_{edge} $[\mathrm{fm}^{-3}]$	$M_{\max}$
SLy4	0.076÷0.077	2.05 $M_\odot$
BSk20	0.0854	2.16 $M_\odot$
BSk21	0.0809	2.28 $M_\odot$

Unified EOSs of catalysed matter for application to non-accreting and non-magnetised cold Nss

- Outer crust: based on the seminal BPS model (Baym,1971). Assumption: BCC and full ionization.

- Semiclassical approach: BSk20, BSk21: ETF + Strutinski integral + Eff. Skyrme force
  Based on effective density-dependent NN force with parameters fitted on nuclei properties
  Goriely et al, PRC 2010

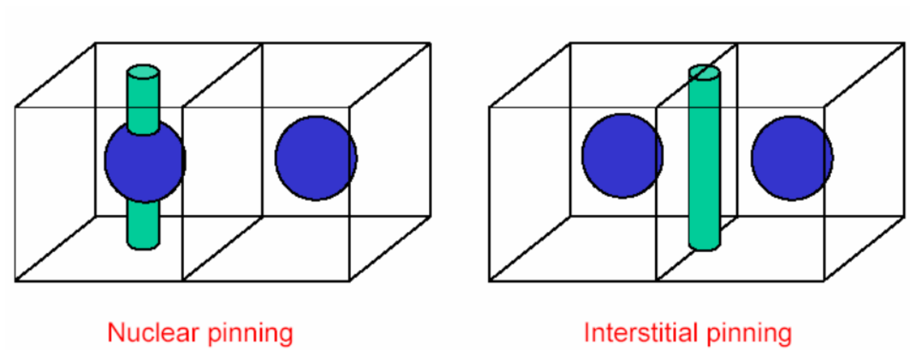
- Classical approach (compressible liquid drop model): SLy: based on the NN interaction SLy4
  Douchin & Haensel, A&A 2001

Pinning energies

Donati & Pizzochero, Nuclear Physics A, 724 (2004)

vortex-nucleus configurations in nuclear lattice

↓ ↓



↓ E_{NP} ↓ E_{IP}

The inning energy is: $\Delta E_{pin} = E_{NP} - E_{IP}$

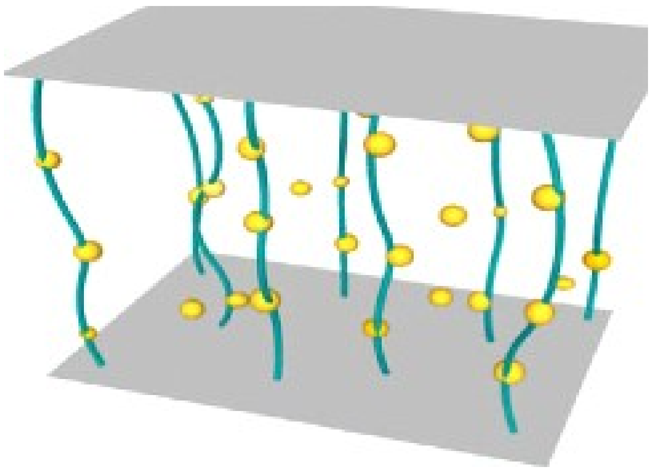
Semiclassical approach: static LDA calculation (i.e. the local Fermi momentum is a function of the neutron number density)
Recent improvement: TDLDA, Wlazlowski et al (2016)

Energy contributions to pinning:
→ negative condensation energy of the order of Δ^2 / E_F
→ kinetic energy of the irrotational vortex–induced flow
→ Fermi energy E_F of neutrons
→ nuclear cluster energy (Woods–Saxon potential)

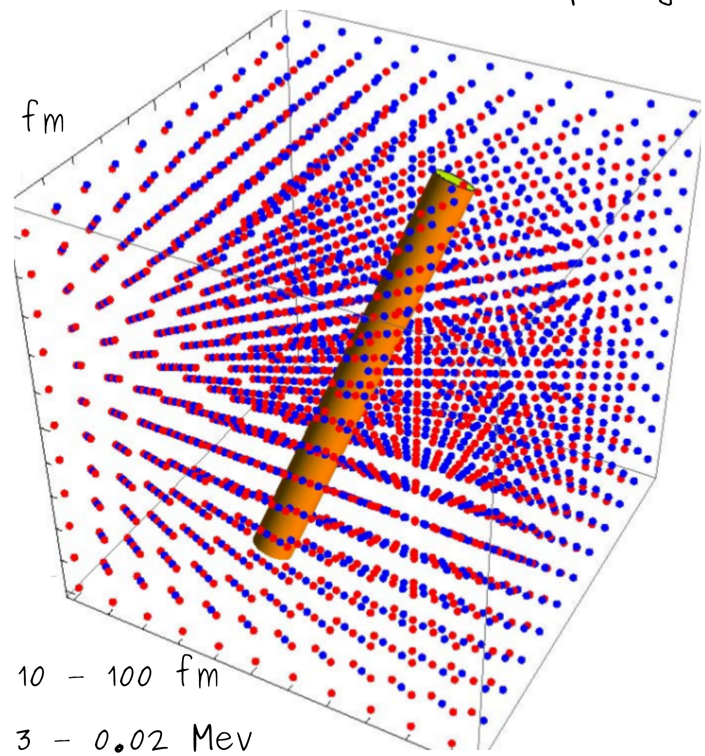
Uncertain pairing gap Δ: modifies the strength and location of the pinning energies
Maximum pinning energies < 3.5 MeV
Significant pinning occurs only in a restricted range: $0.07\ n_0 < n_B < 0.2\ n_0$ Donati & Pizzochero, Phys Lett B, 640 (2006)

Pinning forces (inner crust)

Seveso et al, MNRAS 2016



Lattice spacing: 50-10 fm



Lattice spacing: 50—10 fm

ξ = 10 – 100 fm

E_{pin} = 3 – 0.02 Mev

Qualitatively:

Coherence length ξ = vortex core radius

Strong pinning when ξ < lattice spacing

Pinning to **single defects** VS **"collective pinning"**:

Rigid (straight) vortices are "less pinned"

Coherence length ξ estimates: Mendell, ApJ 38 1991

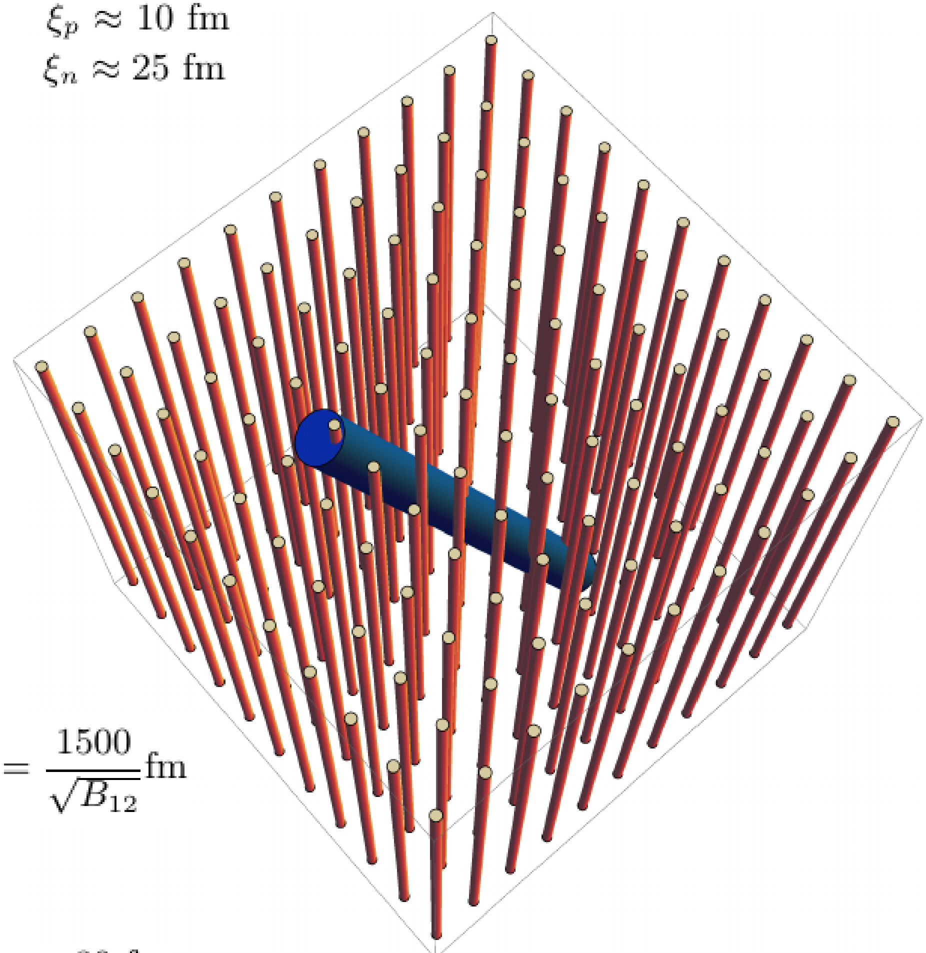
Inner crust:

Problem: how to calculate the "vortex—lattice" interaction from the "vortex—nucleus" interaction ?

IDEA: consider a segment of vortex line (the length L is given by the tension) and average over translations and rotations of the total pinning force divided by L

Pinning forces - core

$\xi_p \approx 10$ fm
$\xi_n \approx 25$ fm



$$R_{ws} = \frac{1500}{\sqrt{B_{12}}}\text{fm}$$

$\Lambda_p \approx 80$ fm

$$T_n \approx \frac{\kappa \rho_n}{4\pi} \ln(l_v/\xi_n)$$

$$T_p \approx \frac{\kappa \rho_p}{4\pi} \ln(\Lambda_p/\xi_p)$$

NOT Vortex–flux tube interaction...
...BUT vortex–array interaction
Result: pinning to flux–tubes negligible for normal pulsars

$$\xi_p \approx 16\, x_p^{1/3} \rho_{14}^{1/3} \Delta_p(\text{MeV})^{-1}\text{fm}$$

$$\xi_n \approx 16\, x_n^{1/3} \rho_{14}^{1/3} \Delta_n(\text{MeV})^{-1}\text{fm}$$

Coherence length estimates: Mendell, ApJ, 380 (1991)

Overlap of vortex line and flux tube is energetically favored because the volume of non–condensed fluid is **minimized** by such overlap (Srinivasan et al. 1990)

$$E_{\text{int}} \sim n_n \frac{\Delta_p^2}{E_{F_p}^2} \frac{\Delta_n^2}{E_{F_n}} (\xi_n^2\, \xi_p) \simeq 0.1 \text{ MeV}$$

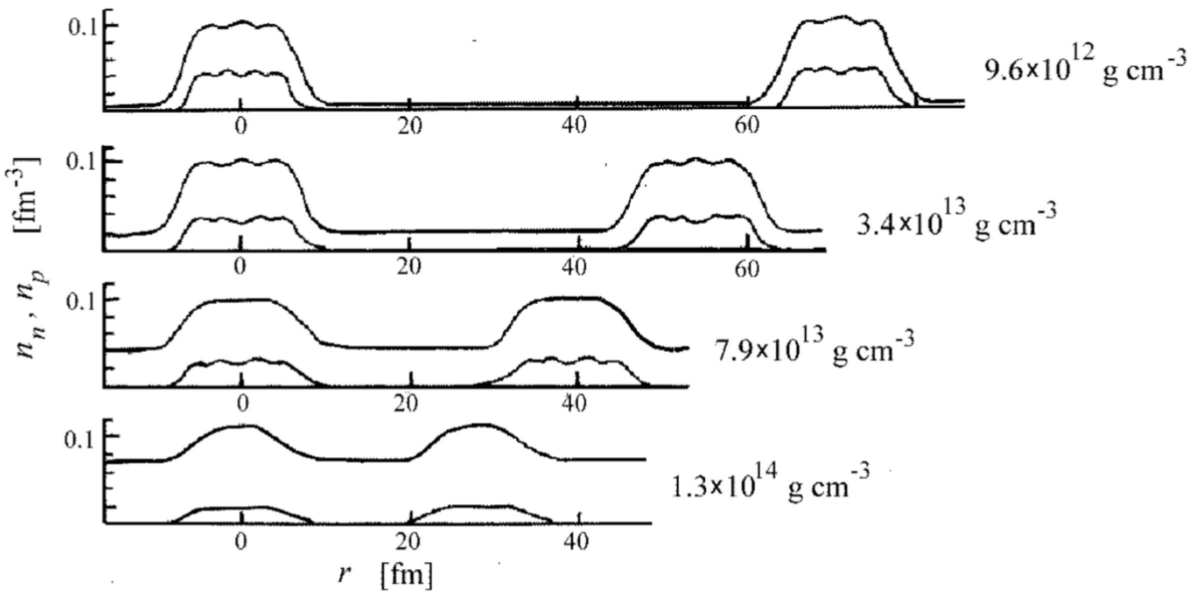
A larger contribution to the interaction energy is the **magnetic interaction** between the vortex and a flux–tube. The magnetic field in a flux tube is $B \sim 10^{15}$ G

$$E_p(\theta) \simeq l\, \frac{E_0}{L} = \frac{\pi}{8} \boldsymbol{B}_v \cdot \boldsymbol{B}_\Phi\, (\Lambda_*^2 l) \ln\left(\frac{\Lambda_*}{\xi_n}\right)$$

$$E_p \approx 5\,\text{MeV},$$

(Alpar et al 1984, Jones 1991, Link 2012)

Inner crust structure



Density profiles of neutron and protons, at several average densities, along a line joining the centers of two adjacent unit cells (HF calculation of the GS in the **inner crust** with effective NN interaction, **no pairing correlations**)

Negele & Vautherin, *Neutron star matter at sub-nuclear densities* (1973)

Include **pairing correlations**: Baldo et al, *The role of superfluidity in the structure of the neutron star inner crust* (2005)

Band theory of solids: Carter et al, *Entrainment Coefficient and Effective Mass for Conduction Neutrons in Neutron Star Crust* (2006)

Negele & Vautherin (1973) zones

Zone	1	2	3	4	5
ρ_B	1.5×10^{12}	9.6×10^{12}	3.4×10^{13}	7.8×10^{13}	1.3×10^{14}
n_G	4.8×10^{-4}	4.7×10^{-3}	1.8×10^{-2}	4.4×10^{-2}	7.4×10^{-2}
R_{WS}	44.0	35.5	27.0	19.4	13.77
R_N	6.0	6.7	7.3	6.7	5.2
a	0.77	0.83	0.94	1.12	1.25
N	280	1050	1750	1460	950
Z	40	50	50	40	32
N_{bound}	110	110	110	70	40
N'_{free}	324	1795	3132	2654	1738

Negele & Vautherin, *Neutron star matter at sub-nuclear densities* (1973)

#	ρ [g cm^{-3}]	Element	R_{ws} [fm]	R_N [fm]	ξ [fm]		E_p [MeV]	
					$\beta = 1$	$\beta = 3$	$\beta = 1$	$\beta = 3$
1	1.5×10^{12}	$^{320}_{40}$Zr	44.0	6.0	6.7	20.0	2.63	0.21
2	9.6×10^{12}	$^{1100}_{50}$Sn	35.5	6.7	4.4	13.0	1.55	0.29
3	3.4×10^{13}	$^{1800}_{50}$Sn	27.0	7.3	5.2	15.4	-5.21	-2.74
4	7.8×10^{13}	$^{1500}_{40}$Zr	19.4	6.7	11.3	33.5	-5.06	-0.72
5	1.3×10^{14}	$^{982}_{32}$Ge	13.8	5.2	38.8	116.4	-0.35	-0.02

Parameters used by: Seveso et al, *Mesoscopic pinning forces in neutron star crusts* (2016)

Last two columns, **pinning energies** from: Donati & Pizzochero, *Realistic energies for vortex pinning in intermediate-density neutron star matter* (2006)

Physical scales

Different scales are involved in glitch modelling:

$$\int_C d\mathbf{x} \cdot \left((2m_p)\mathbf{v} + \frac{(2e)}{c}\mathbf{A} \right) = h\mathcal{N}_C$$

Core → "Abrikosov lattice" spacing between flux-tubes ~ 1000 fm

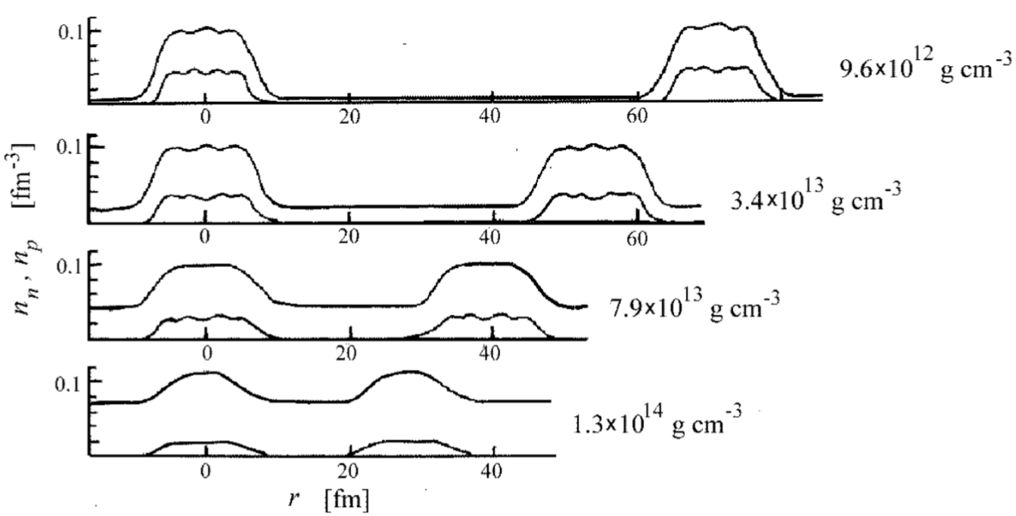
$$l_v = \frac{\sqrt{\kappa P}}{2\pi} \approx 7 \times 10^{-3} \sqrt{P} \text{ cm}$$

Inter-vortex spacing

Crust → crustal lattice spacing ~ 100 - 20 fm

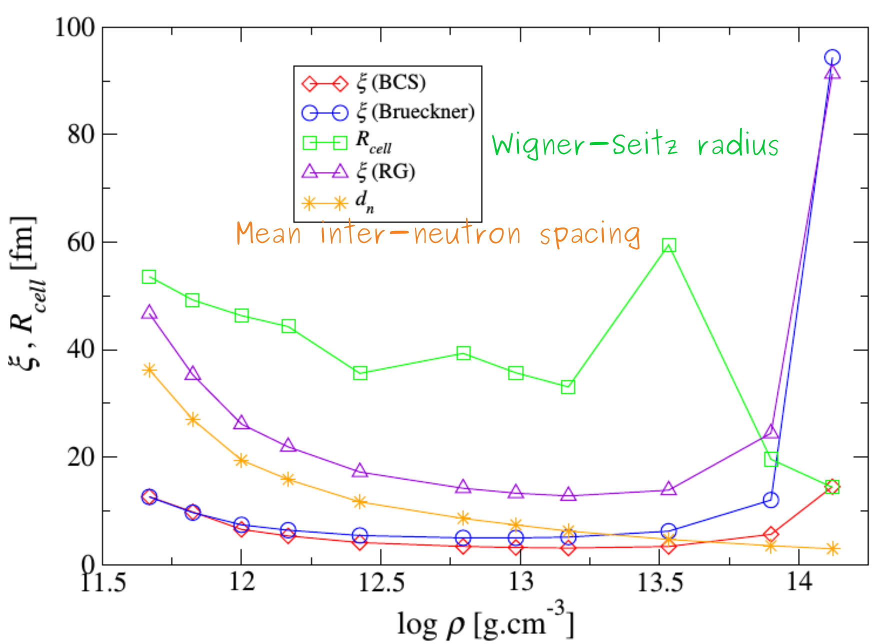
Vortex-nucleus interaction → coherence length ~ 10 - 100 fm

Vortex dynamics and vortex-lattice interaction → "mesoscale" (inter-vortex spacing)



Negele & Vautherin (1973)

Neutron star matter at sub-nuclear densities



Chamel & Haensel, Living Rev.Rel. 11 (2008)

Superfluidity in NS

Neutron stars are "cold":
($T = 10^8$ K = 0.01 MeV) << (E_F = 10 - 100 MeV)

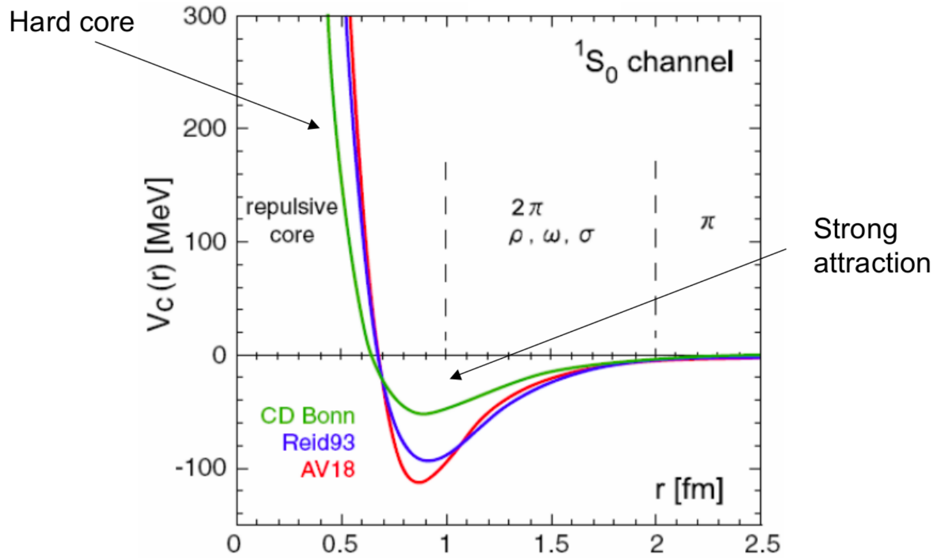
Fermi surface is "unstable" against pairing:

Neutrons in the **crust** feel **attractive** components of the NN potential in the S-wave

Core: $^0S_1$ NN force is repulsive above 0.16 fm^{-3}

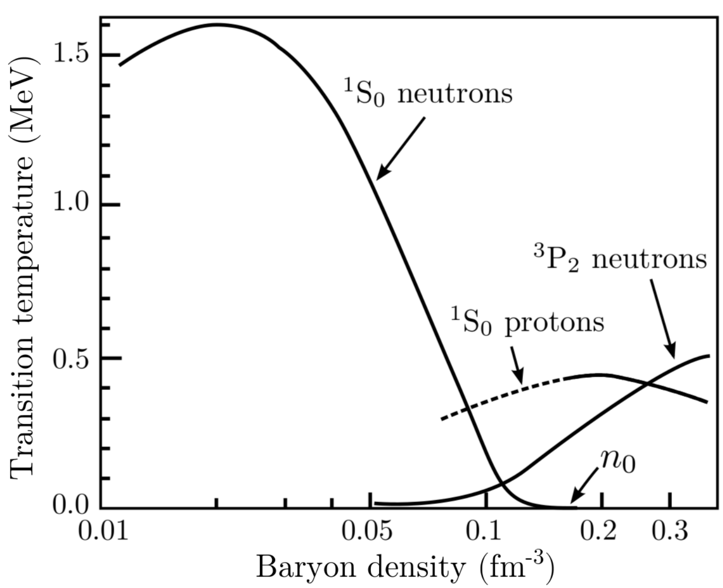
$^3S_1$-$^3D_1$ binds the deuteron: but in NS **n** and **p** have very different Fermi surfaces → **no n-p superfluid**

NN potential for S=0, L=0, J=0

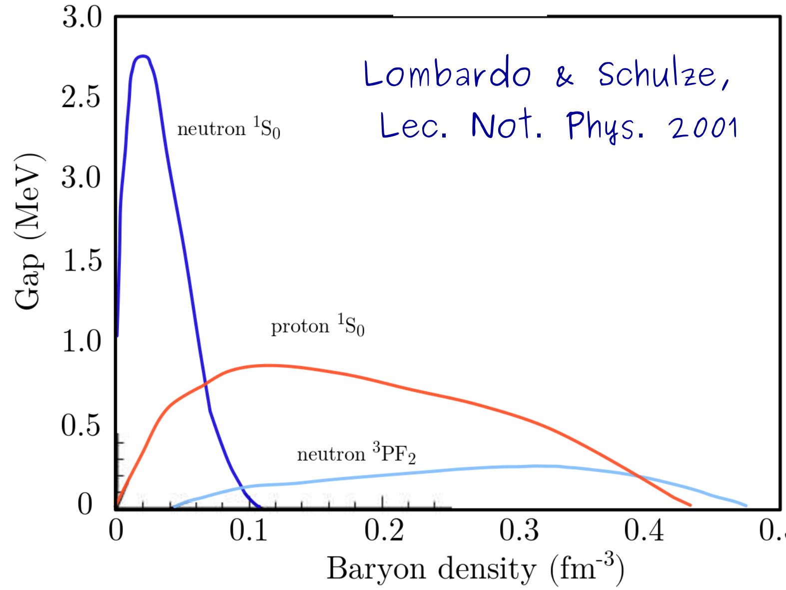


Ishii et al, PRL 2007

Transition temperature



Pairing gap Δ ~ 1 MeV ~ 10^{10} K



Pairing channels

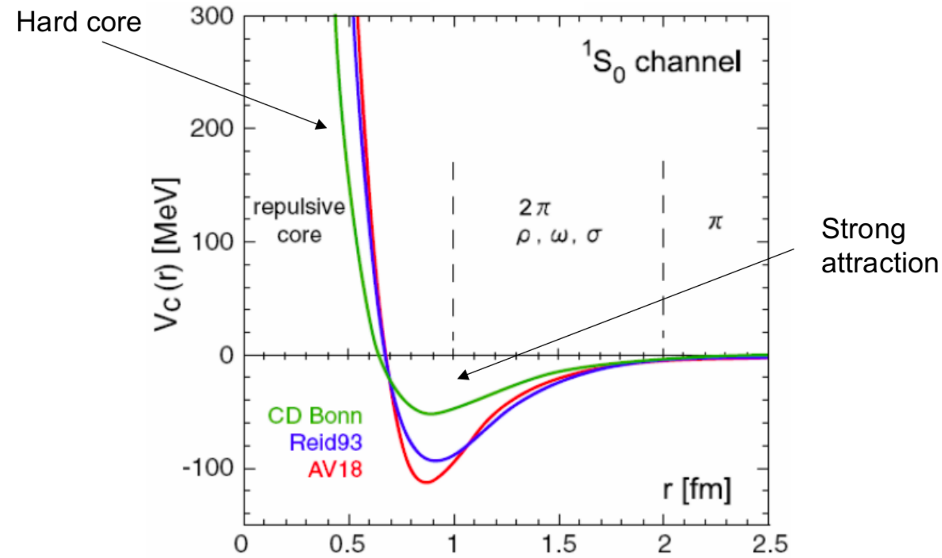
Total angular momentum operator: $\mathbf{J} = \mathbf{L} + \mathbf{S}$
Usual notation $^{2S+1}L_J$ (L=0,1,2,3... → S,P,D,F...)

$^1S_0$ isotropic pairing: Δ = "energy gap" ~ 0.57 T_c

$^3S_1$–$^3D_1$ binds the deuteron: but in NS **n** and **p** have very different Fermi surfaces → **no n-p pairing**

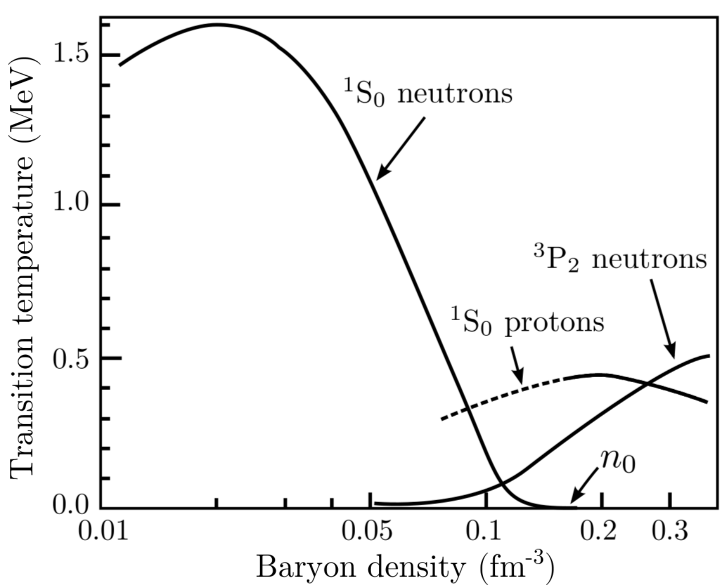
$^3PF_2$ partial–wave channel (Δ has contributions from both L=1,3) is preferred at larger Fermi momenta ($^1S_0$ becomes repulsive). Huge uncertainties: usually treated as free **parameter** in cooling simulations.

NN potential for S=0, L=0, J=0

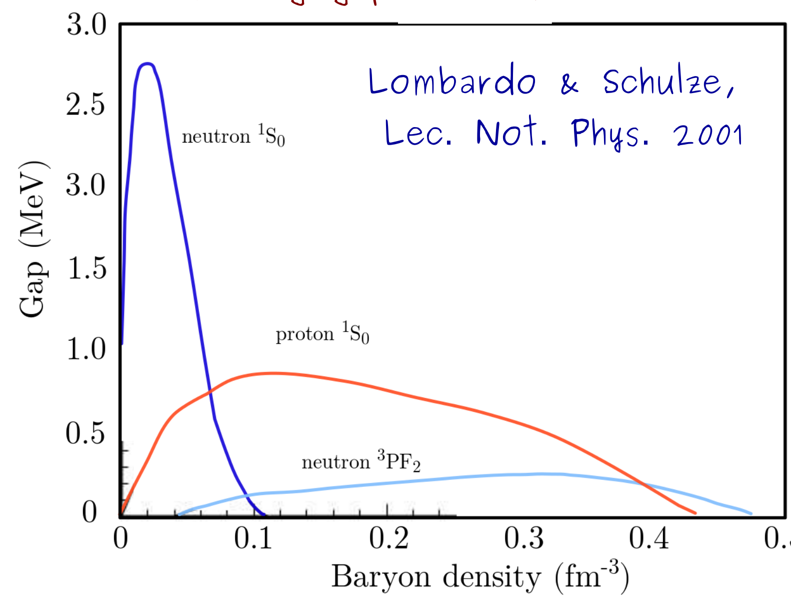


Ishii et al, PRL 2007

Transition temperature



Pairing gap Δ ~ 1 MeV ~ 10^{10} K



$\Omega = 2\pi/P$

Vacuum dipole model (Deutsch 1965, Pacini 1968)

- Kinetic rotational energy loss = energy loss from rotating dipole (non rel. Larmor formula)

$$\dot{E} = I\Omega\dot{\Omega} = -\frac{4\pi^2 I\dot{P}}{P^3} = -2|\ddot{\mathbf{m}}|^2/3c^3 = -\frac{2}{3c^3}m^2\Omega^4\sin^2\alpha = -\frac{2B_p^2R^6\Omega^4\sin^2\alpha}{3c^3}$$

- Another way:

$$\dot{E} = -4\pi R_{LC}^2 S \sim -\frac{B_p^2R^6\Omega^4}{c^3}$$

$$\begin{cases} S \sim cB^2/4\pi \\ R_{LC} = \dfrac{c}{\Omega} \sim 5\times 10^9 P \text{ cm s}^{-1} \\ B \sim B_p\left(\dfrac{R}{r}\right)^3 \quad r < R_{LC} \end{cases}$$

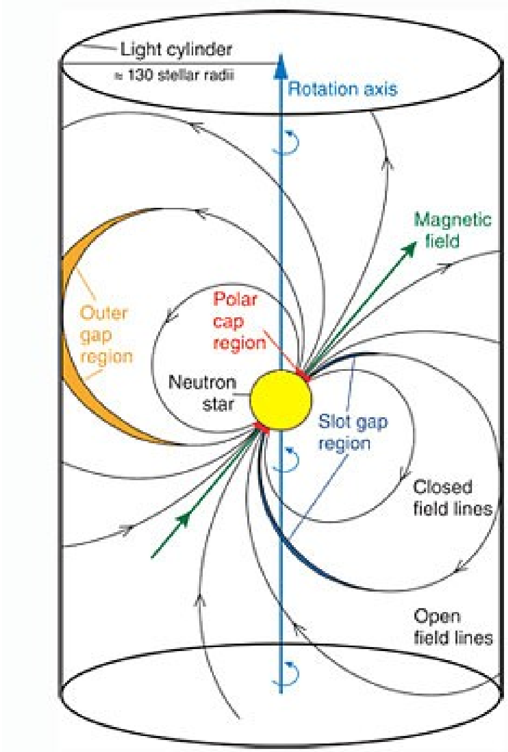
- Braking Index: n=3 for pure dipole model
  n=3 also with magnetospheric effects

- "Age" of the object:
  – assume n=3

$$\dot{\Omega} = -K\Omega^n \qquad n = \frac{\Omega\ddot{\Omega}}{\dot{\Omega}^2}$$

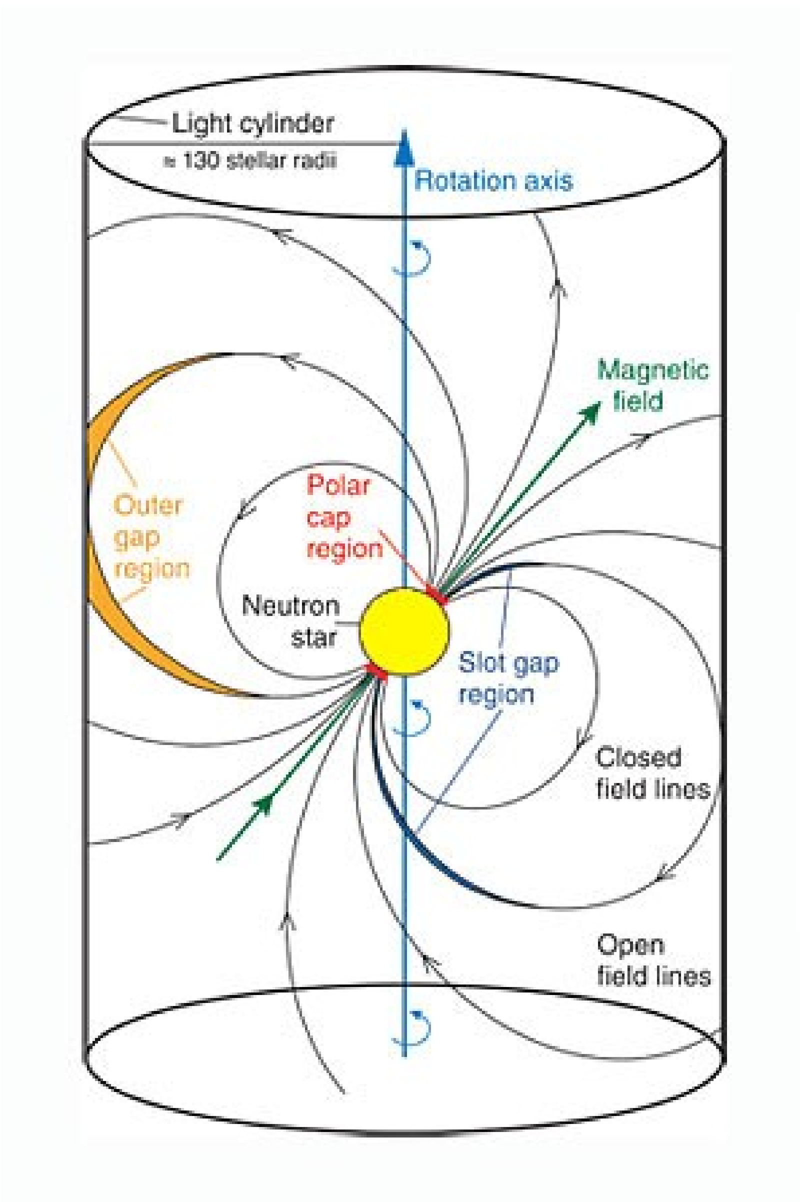
$$\tau = \frac{P}{(n-1)\dot{P}}\left[1 - \left(\frac{P_o}{P}\right)^{n-1}\right] \qquad \tau_c = \frac{P}{2\dot{P}}$$

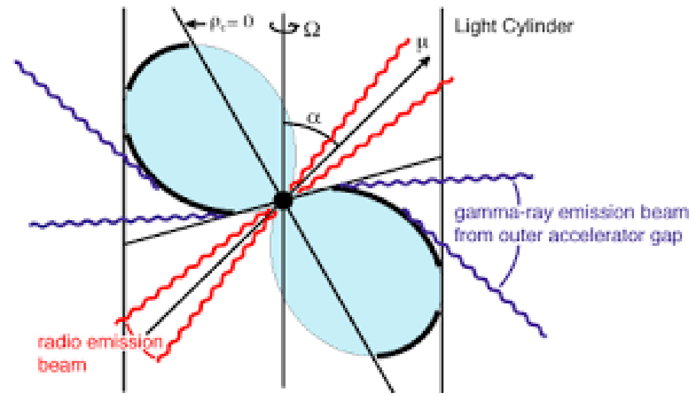
$P_o \simeq 19$ ms $\qquad P \gg P_0$

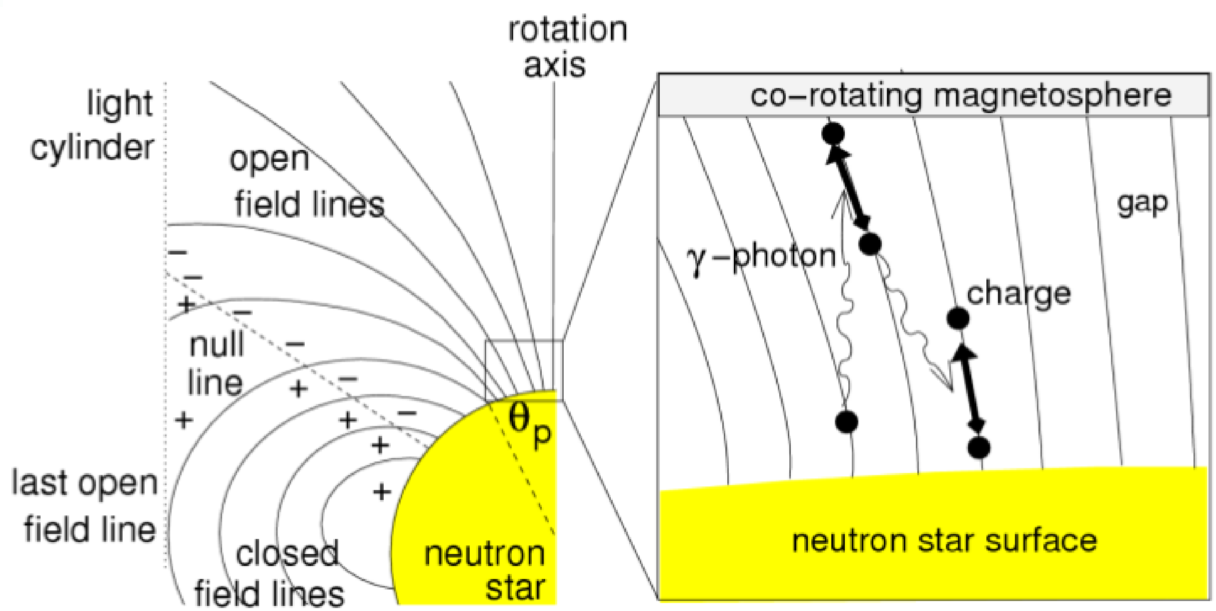


The magnetosphere

Radio emission mechanism?







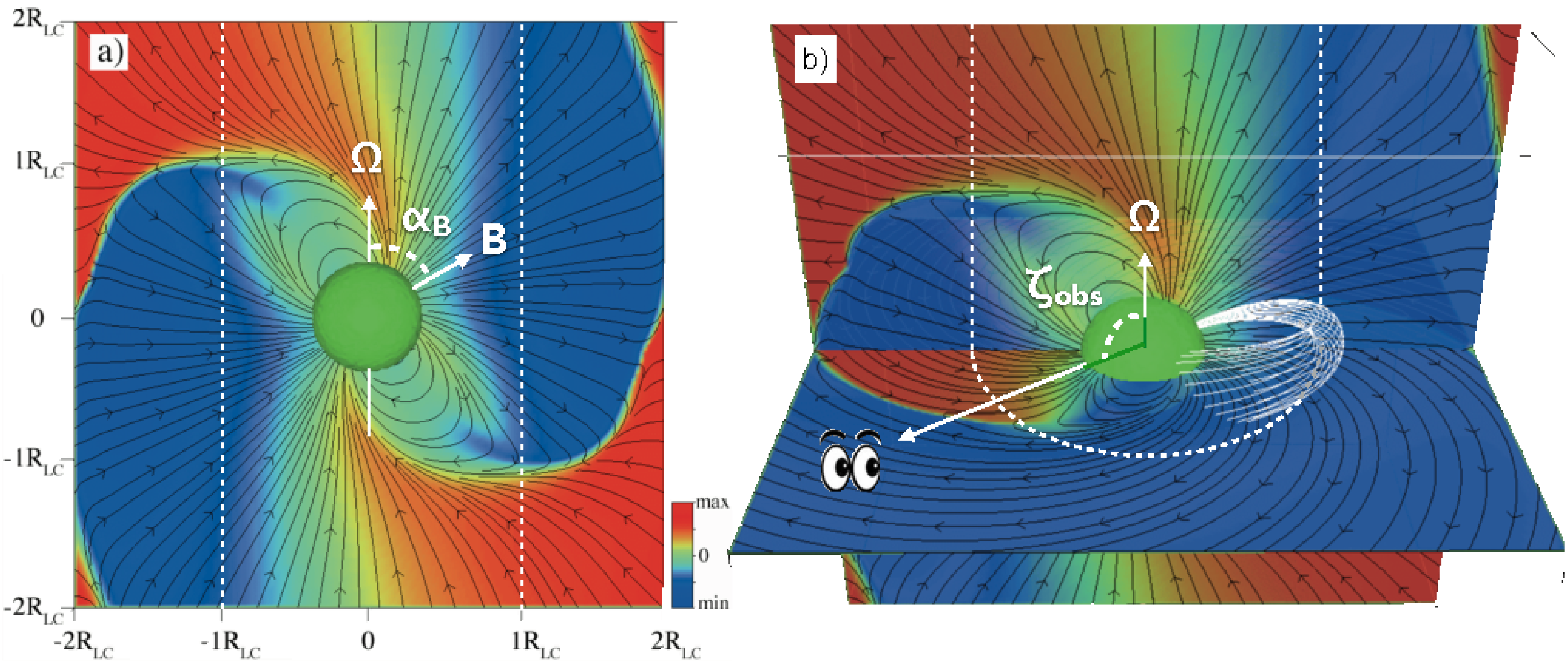
Caartoon of the electon-positron cascade which is required by many models of coherent pulsar radio emission

Dipolar pulsar magnetosphere (Spitkovsky06)

Ideal MHD/GRMHD in the magnetosphere works well, braking index n = 3

Color → B component perpendicular to the plane.
Closed field lines → inside light cylinder (corotating, force free).
Open field lines → current free region (but current sheets).

The boundary between open and closed field lines regions at the neutron star surface defines the **polar cap.**
A **current sheet** forms where the magnetic field reverses direction near and past the light cylinder (the boundary between the red and blue regions).



Starquake model: two issues

1 – Large glitches in the Vela occur every ~2.5 years.

Is it possible to reach the breaking strain so frequently just because of the spin-down?

Strain ~ $\Omega\ \Delta\Omega_{ab}$

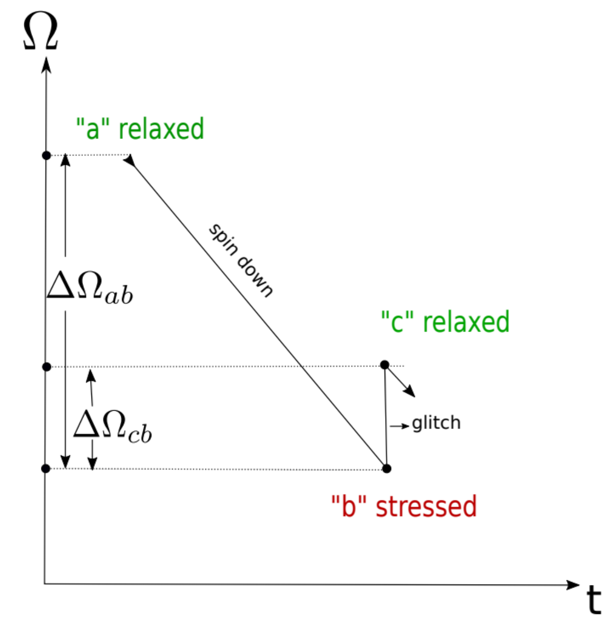
Estimated breaking strain is too high → crust is always close to the breaking strain and not all the stress is released in quakes Giliberti+ 2019, Giliberti+ 2020

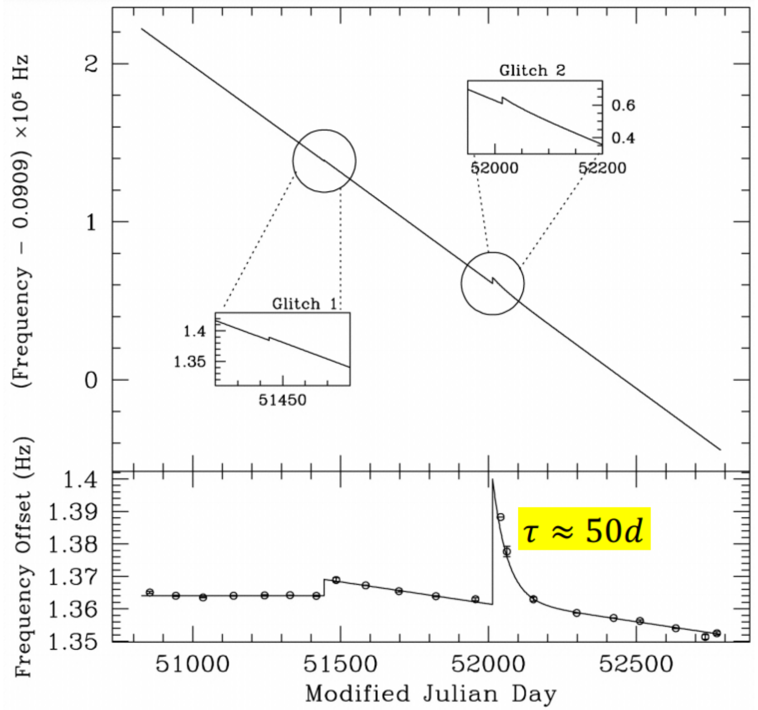
Reconret+ 2020: the glitch activity associated with quakes is far too small to explain even the subclass of small glitches, independently of the breaking strain.

2 – After the first glitches it was soon realized that the spin-up jump is not the whole story.

Why should the quake induce also a relaxation?

Relaxation is due to the superfluid: Baym et al, Nature (1969)





Vela 2016 glitch

- First case of a gltich detected in the act

- TOA of single pulses detected

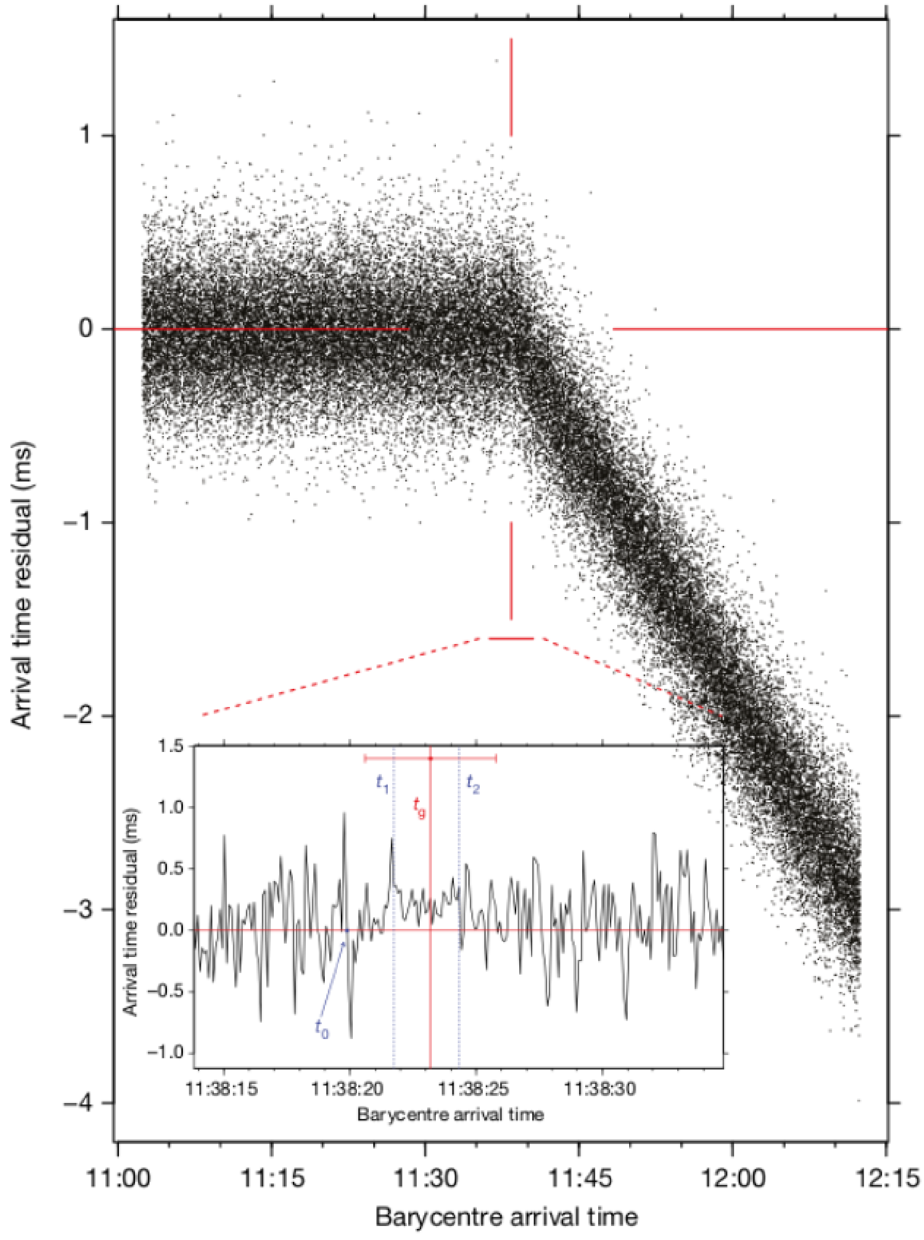
- Residual of TOA: tells us if the pulse arrives before or after the expected arrival time predicted by a spin-down model.

Palfreyman et al, Nature 2018

- We may apply the Baym's model to fit the spin up...

$$I_c\dot{\Omega}_c = -\alpha - \frac{I_c}{\tau_c}(\Omega_c - \Omega_n)$$

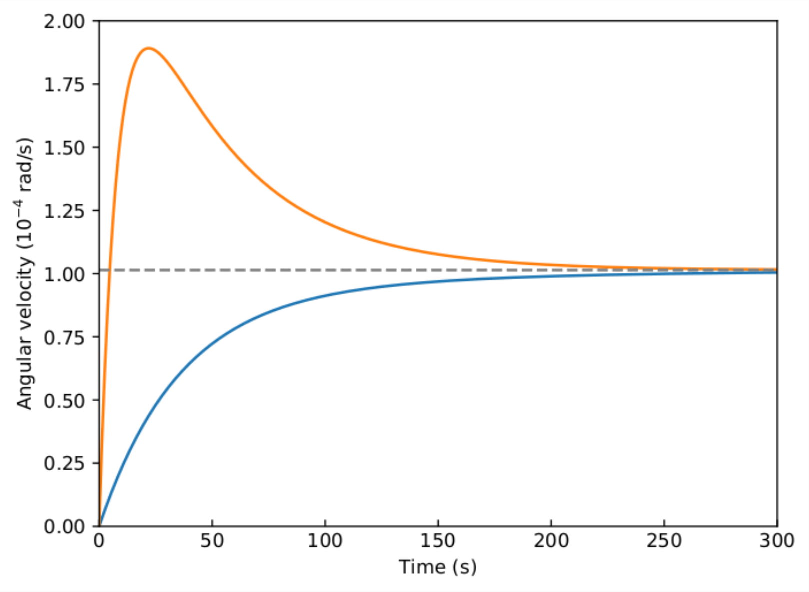
$$I_n\dot{\Omega}_n = \frac{I_c}{\tau_c}(\Omega_c - \Omega_n),$$



Vela 2016 glitch: phenomenological modelling

Instead of using the 2-component model of Baym we can try to fit the spin-up with a 3-component model:

- natural idea because there is superfluid in the crust but also in the core
- allows us to resolve a possible overshoot during the spin-up Graber et al. 2018, Ashton et al. 2019



$$\dot{\Omega}_p = -\frac{1}{x_p}\left(x_1\dot{\Omega}_1 + x_2\dot{\Omega}_2 + |\dot{\Omega}_\infty|\right)$$

$$\dot{\Omega}_1 = -b_1\left(\Omega_1 - \Omega_p\right)$$

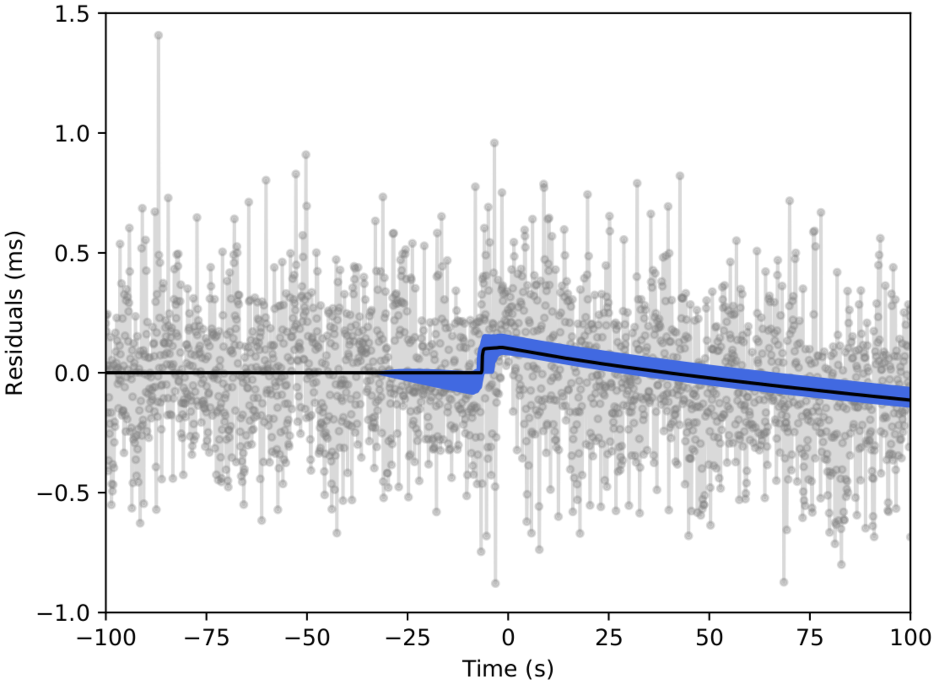
$$\dot{\Omega}_2 = -b_2\left(\Omega_2 - \Omega_p\right)$$

It is possible to solve analytically the system, in order to obtain the angular velocity of the normal component with respect to the spin down of the star.

$$\Delta\Omega_p(t) = \Delta\Omega_p^\infty\left[1 - \omega\, e^{-t\lambda_+} - (1-\omega)\, e^{-t\lambda_-}\right]$$

Pizzochero et al. A&A 2020

Bayesian fit of Vela 2016



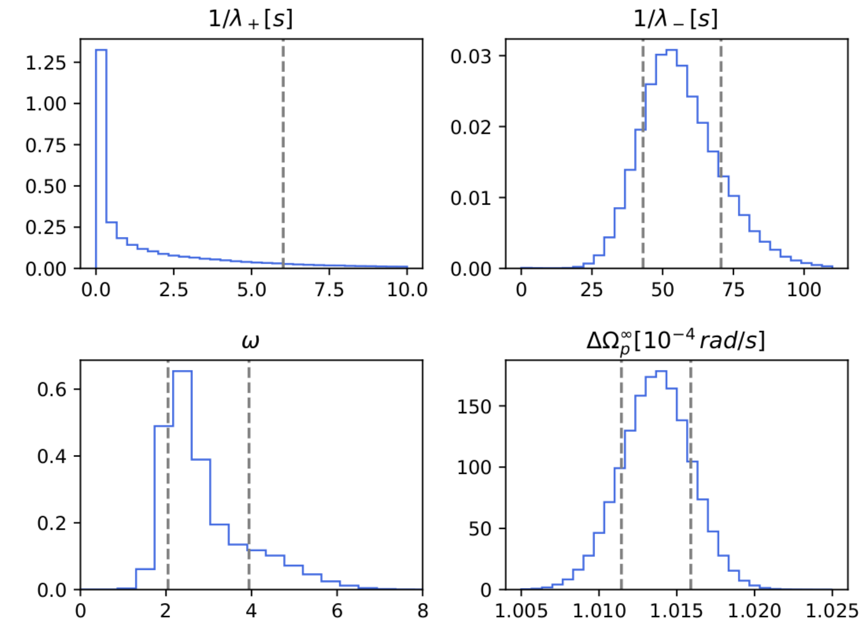
Fit of the TOA residuals of Palfreyman+ 2018 with a three compnent model (**Montoli+ 2020**)

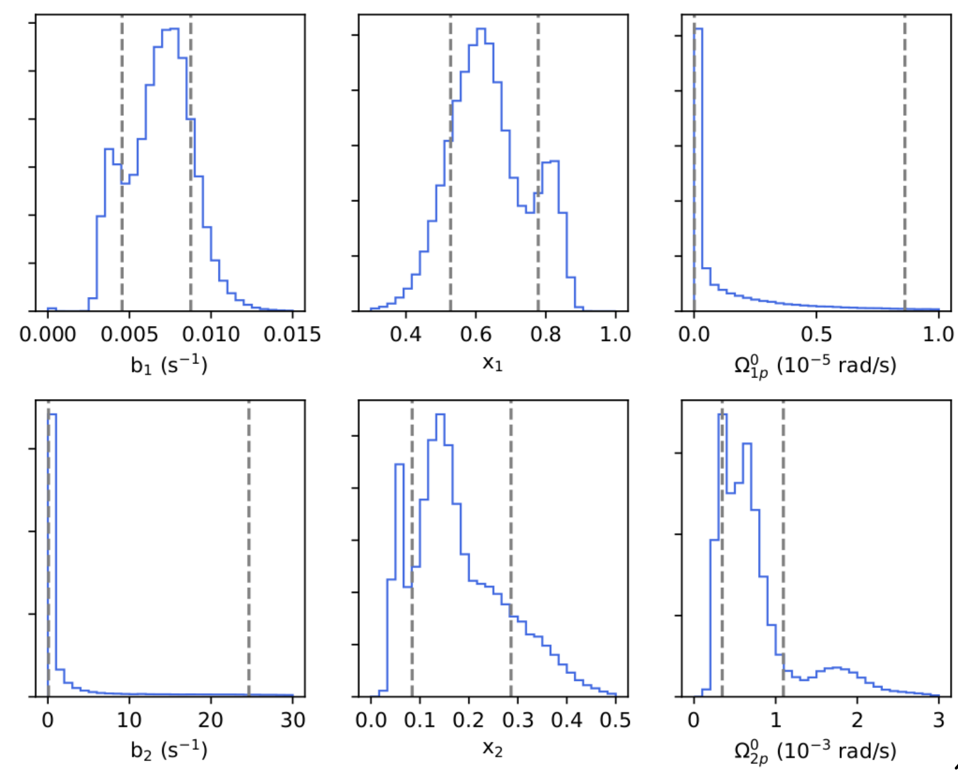
Estimated moment of inertia fractions of the two superfluid components:

"active" superfluid: $x_2 \sim 0.1 - 0.3$

"passive" superfluid: $x_1 \sim 0.5 - 0.7$

Likely occurrence of an "overshoot", in agreement with the analysis of Ashton+ 2020



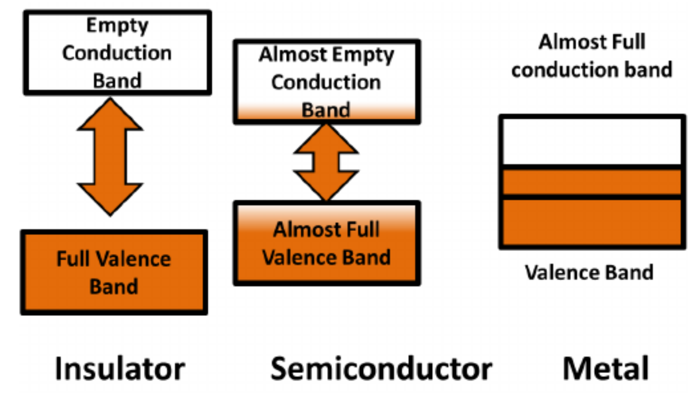


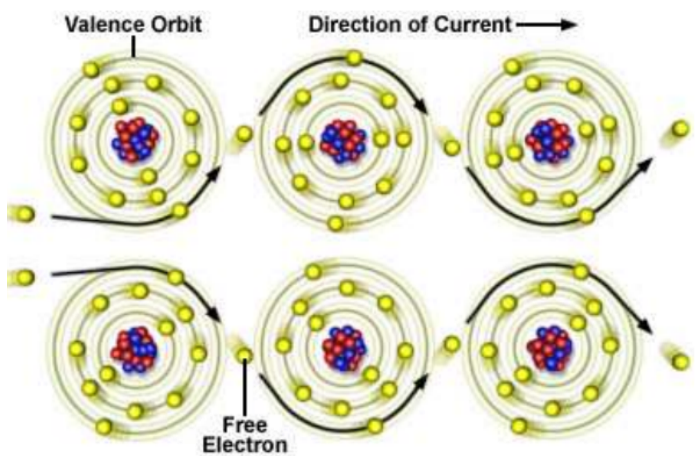
Band theory (inner crust VS “metal”)

Due to the interactions with the periodic lattice, neutrons move in the inner crust as if they had an effective mass m*.

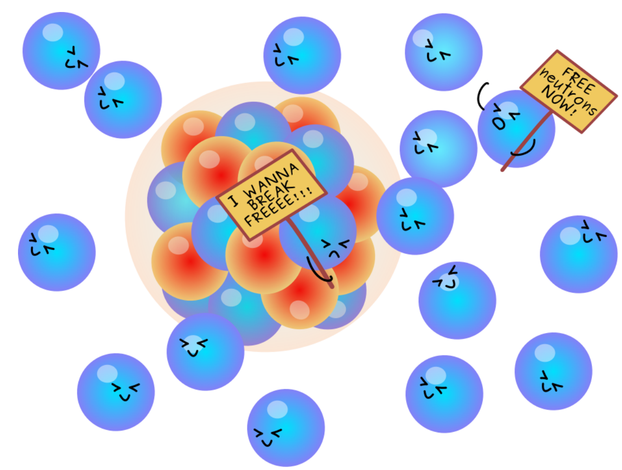
At the highest energies of the valence band (or at the lowest energies of the conduction band), the band structure E(K) of an electron can be approximated as a “free electron” but with an “effective mass”

$$m^* \leftrightarrow \text{crustal entrainment}$$





Usual metal: how to distinguish between a “conduction electron” and a “confined” one?



Neutron star inner crust: how to distinguish a “leaked neutron” from a “confined” one?

Entrainment coupling: crust and core

In the inner crust (lattice of ions & S-wave superfluid):

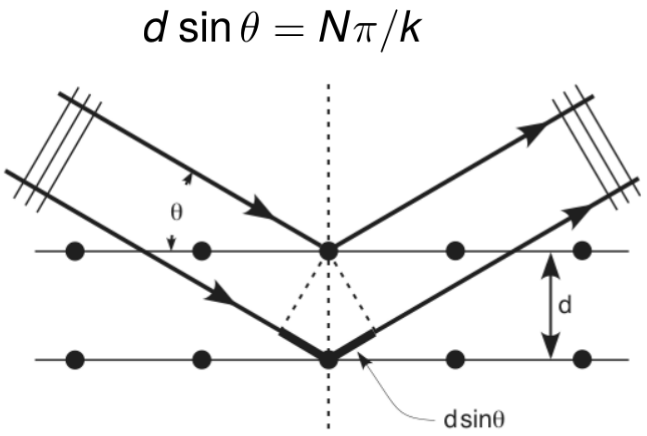
Chamel, PRC 2012

Bragg scattering by crustal lattice entrains the "free" neutrons.

Non-local effect: m* > 1

→ Consequence: the crustal superfluid is entrained by the normal component: **reduced mobility** of "free" neutrons is a potential problem for pulsar glitch theory.

Chamel PRL 2013, Montoli, Antonelli et al, Universe 2020



In the core (S-wave superconductor & P-wave superfluid):

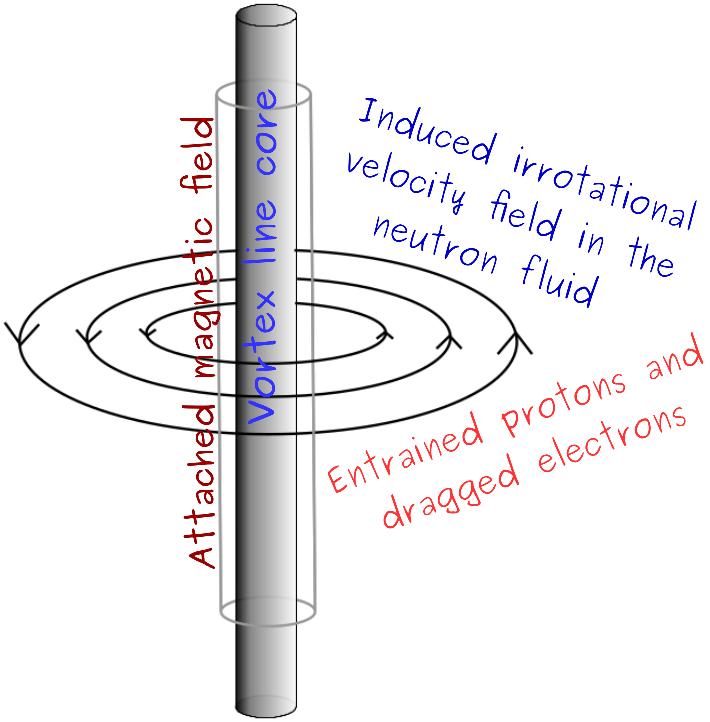
Chamel & Haensel PRC 2006

Entrainment is due to the strong interaction between protons and neutrons. Local effect: m*<1

−Consequence #1: Scattering of electrons off vortex cores: the core is coupled to the crust on the timescale of a second

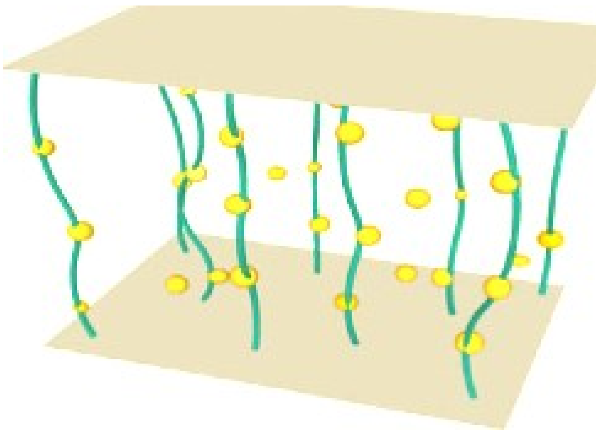
Alpar et al, ApJ 1984

−Consequence #2: Dipole−dipole interaction with flux−tubes (core pinning?)



Pinning forces in the inner crust

(Seveso et al. 2016)



Qualitatively:

Coherence length ξ ~ vortex core radius.

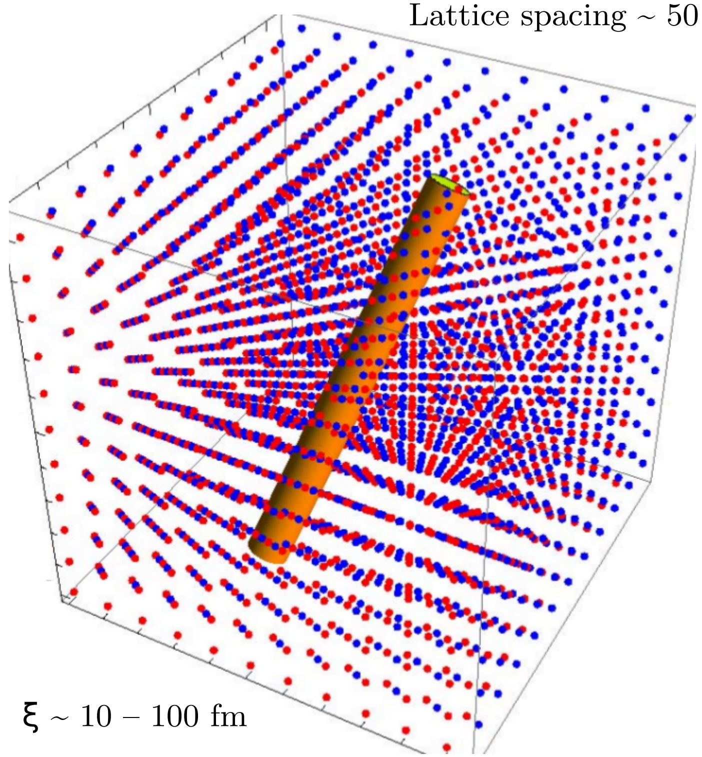
Strong pinning when ξ < lattice spacing.

Pinning to single defects VS “collective pinning”:

Rigid (straight) vortices are “less pinned”.

Coherence length estimates: Mendell, ApJ 380 (1991)

Lattice spacing ~ 50 – 10 fm



ξ ~ 10 – 100 fm

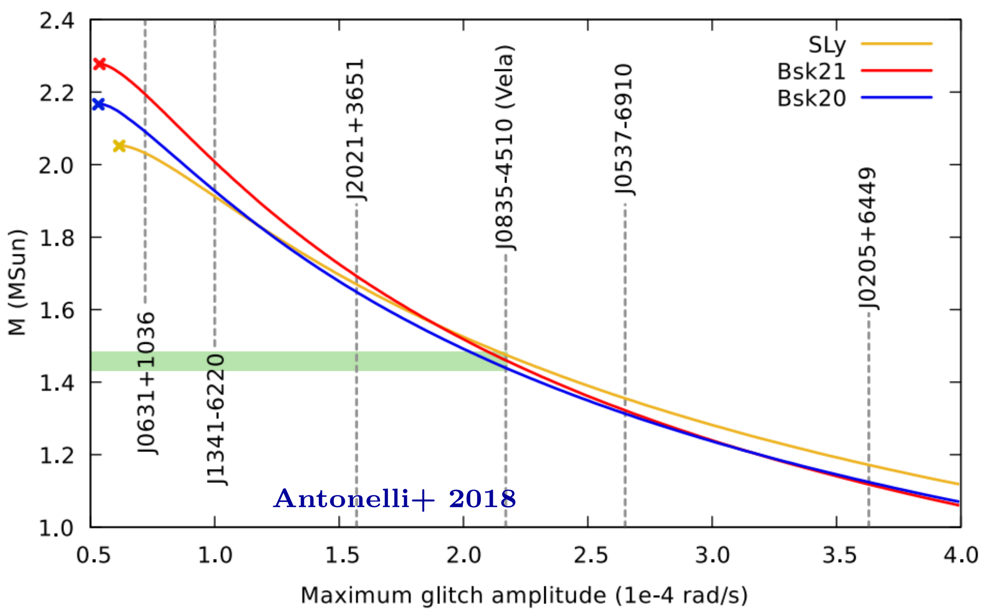
E_{pin} ~ 3 – 0.02 Mev

Inner crust:

Problem: how to calculate the “vortex-lattice” interaction from the “vortex-nucleus” interaction ?

Consider a segment of vortex line (the length is fixed by the tension) and average over translations and rotations of the total pinning force.

Static constraints from glitches



The pinning force defines the maximum neutron current that can develop in the crust before the vortices unpin and dissipate the current.

The observed largest **glitch size** constraints the **pinning forces:**

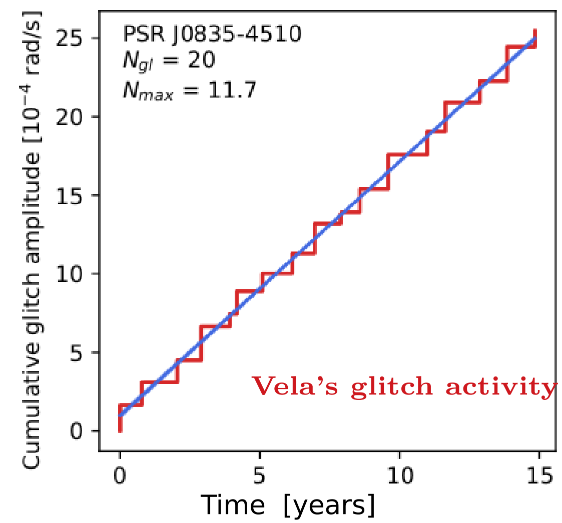
- fix the Eos
- fix the pinning forces as a function of the baryon density
- solve the TOV and calculate a simple integral over the crustal region

...a similar idea but with the glitch activity allows to constrain the entrainment

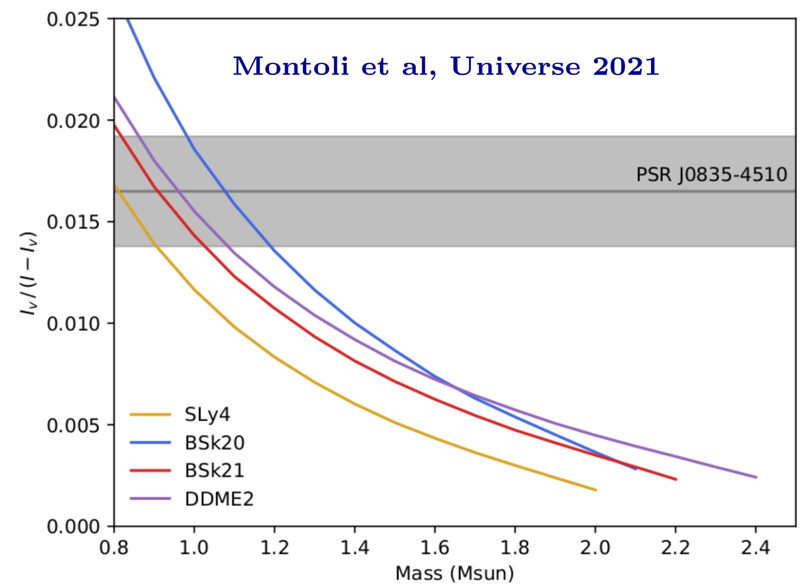
Minimum NS masses:

Observed: M = 1.174 ± 0.004 M⊙ Martinez+ 2015

From CCS simulations: M ≈ 1.15 M⊙ Lattimer+ 2015



The observed **glitch activity** can constrain the **entrainment** in the crust



Constraints from the largest glitch: results

Maximum glitch amplitude at corotation:

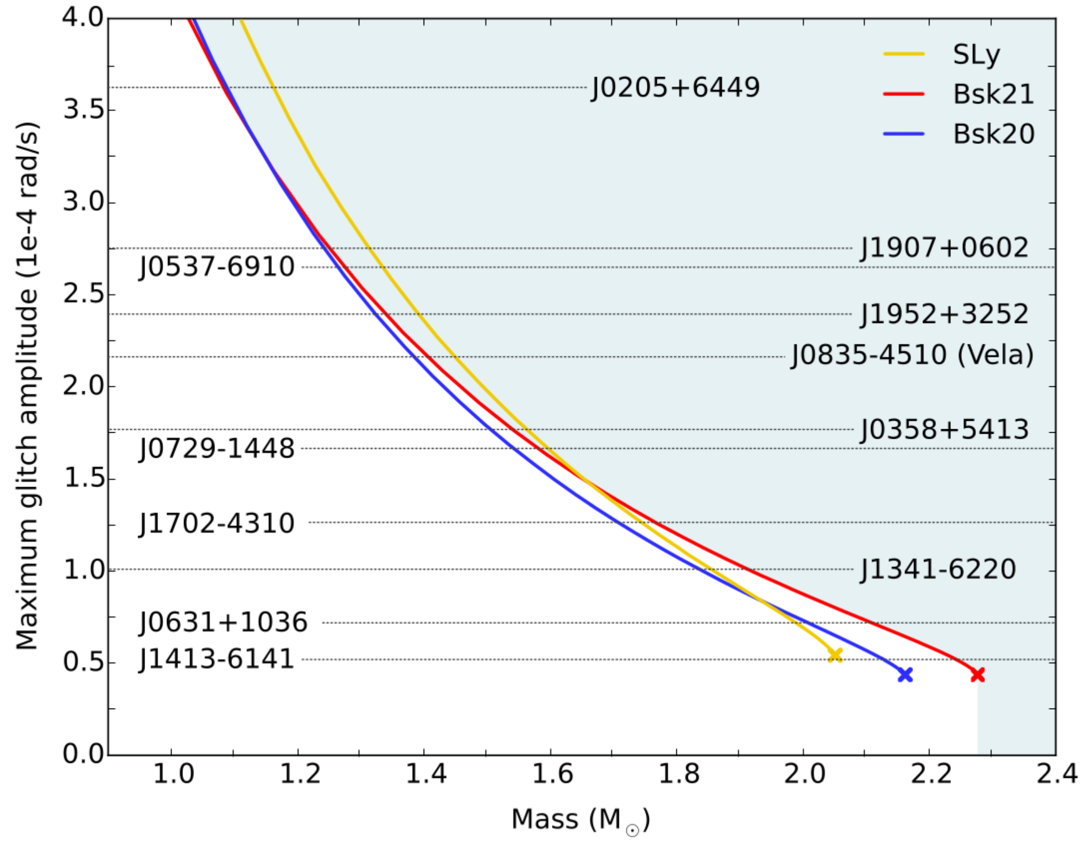
$$\Delta\Omega_{\max} = \frac{\pi^2}{\kappa I} \int dr\, r^3\, f_P(r)$$

→ Only dependent on pinning forces
and on the mass of the star

→ Entrainment independent

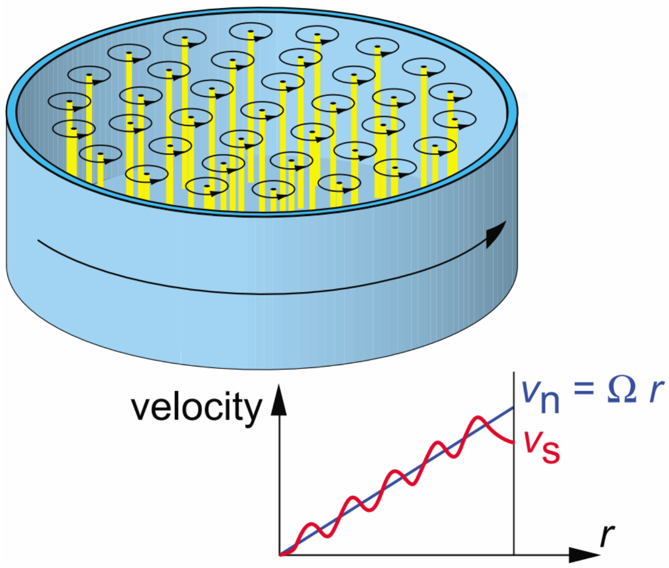
→ No need to consider straight
vortex lines

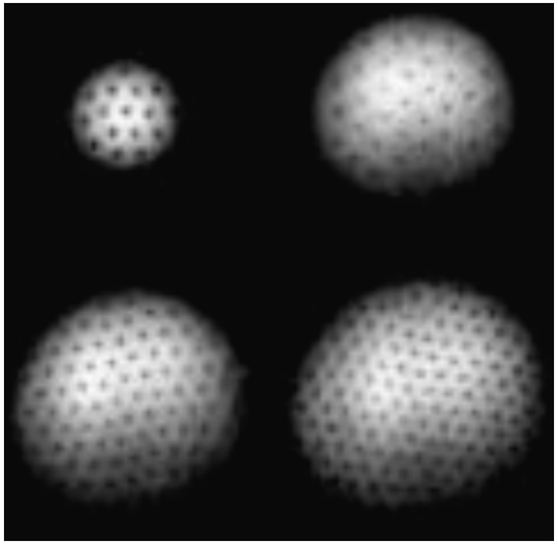
→ As long as pinning is crust-confined
the maximum glitch amplitude does not
depend on the extension of vortices in the
outer core



Pizzochero, Antonelli, Haskell, Seveso, *Constraints on pulsar masses from the maximum observed glitch* (2017)

Rotating superfluids





Observation of vortex lattice in Atomic BEC
Abo-Shaeer, Raman, Vogels, Ketterle
Science (2001)

Depending on the space of the order parameter, a superfluid rotates by means of quantized topological defects.

We assume vortex lines but in ^{3}He also vortex sheets are possible (vortex sheets are unstable in ^{4}He).

Carter and Khalatnikov, Phys.Rev. D45 (1992): the Feynman-Onsager quantization rule is nothing but the Bohr-Sommerfeld quantization

$$\int_C d\mathbf{x} \cdot \mathbf{p} = \frac{h}{2} \mathcal{N}_C \qquad\qquad \Omega_v(x) = \frac{\kappa\, N(x)}{2\pi\, x^2}$$

Shapiro delay for PSR J1614-2230

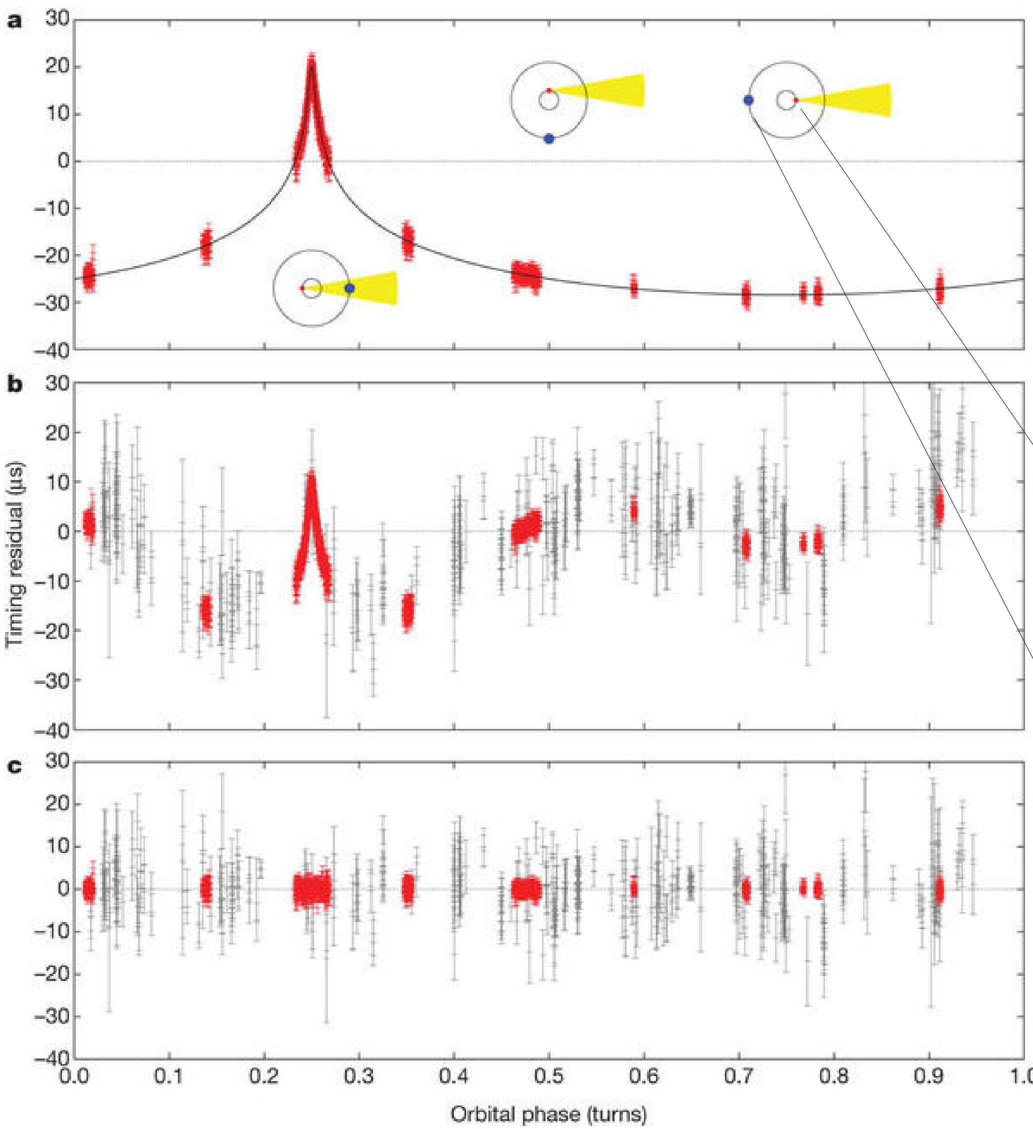
Demorest et al, Nature 2010

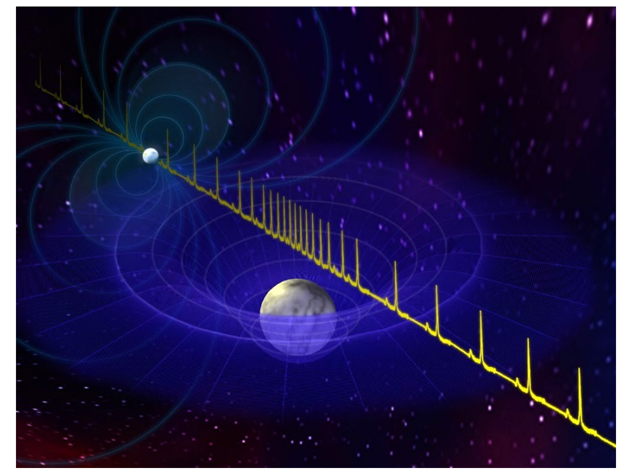
For nearly edge-on binary millisecond radio pulsar systems, Shapiro delay allows to infer the masses of both the neutron star and the companion with great precision

Shapiro delay functional form

Residuals "without GR"

Residuals "with GR"



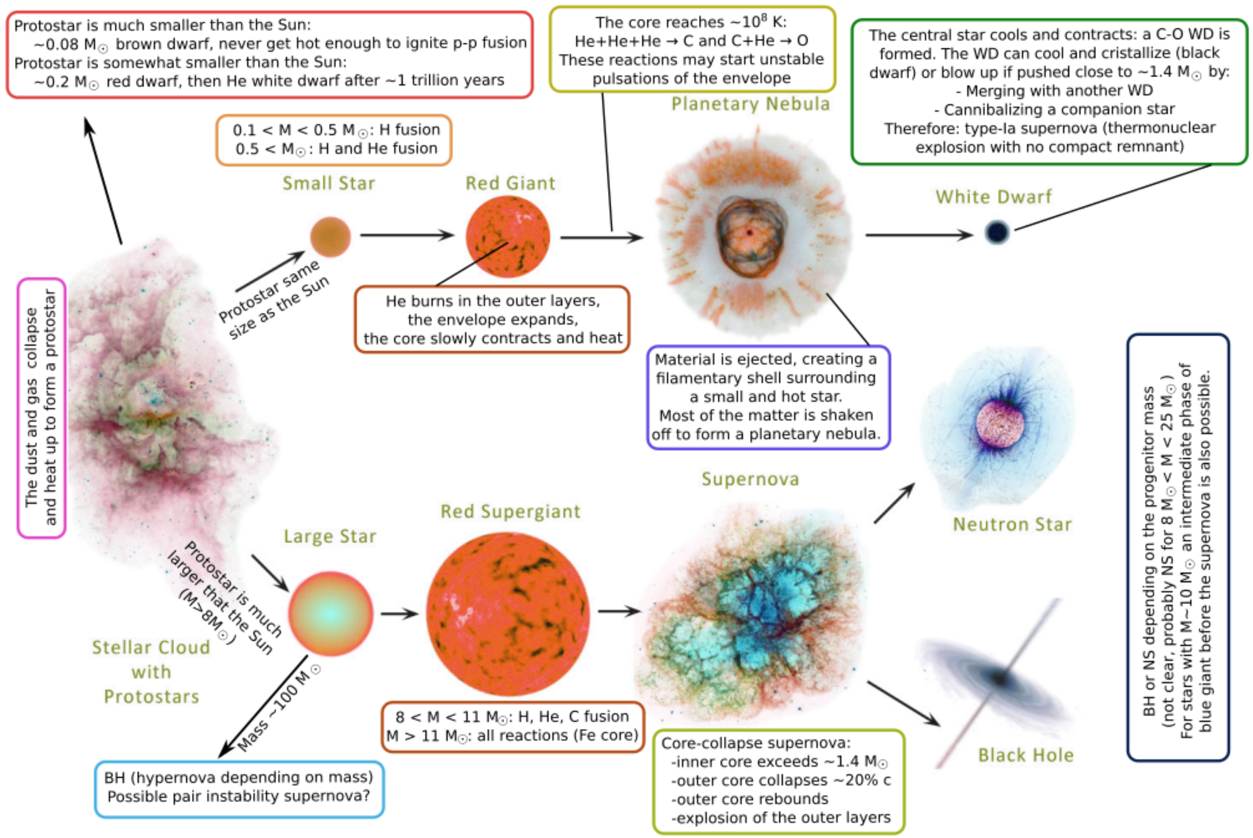


Millisecond pulsar
(1.97±0.04) M⊙

Companion:
helium-carbon-oxygen WD
(0.500±0.006) M⊙

In contrast with X-ray-based mass/radius measurements, Shapiro delay provides no information about the NS radius

Dead stars (compact objects)

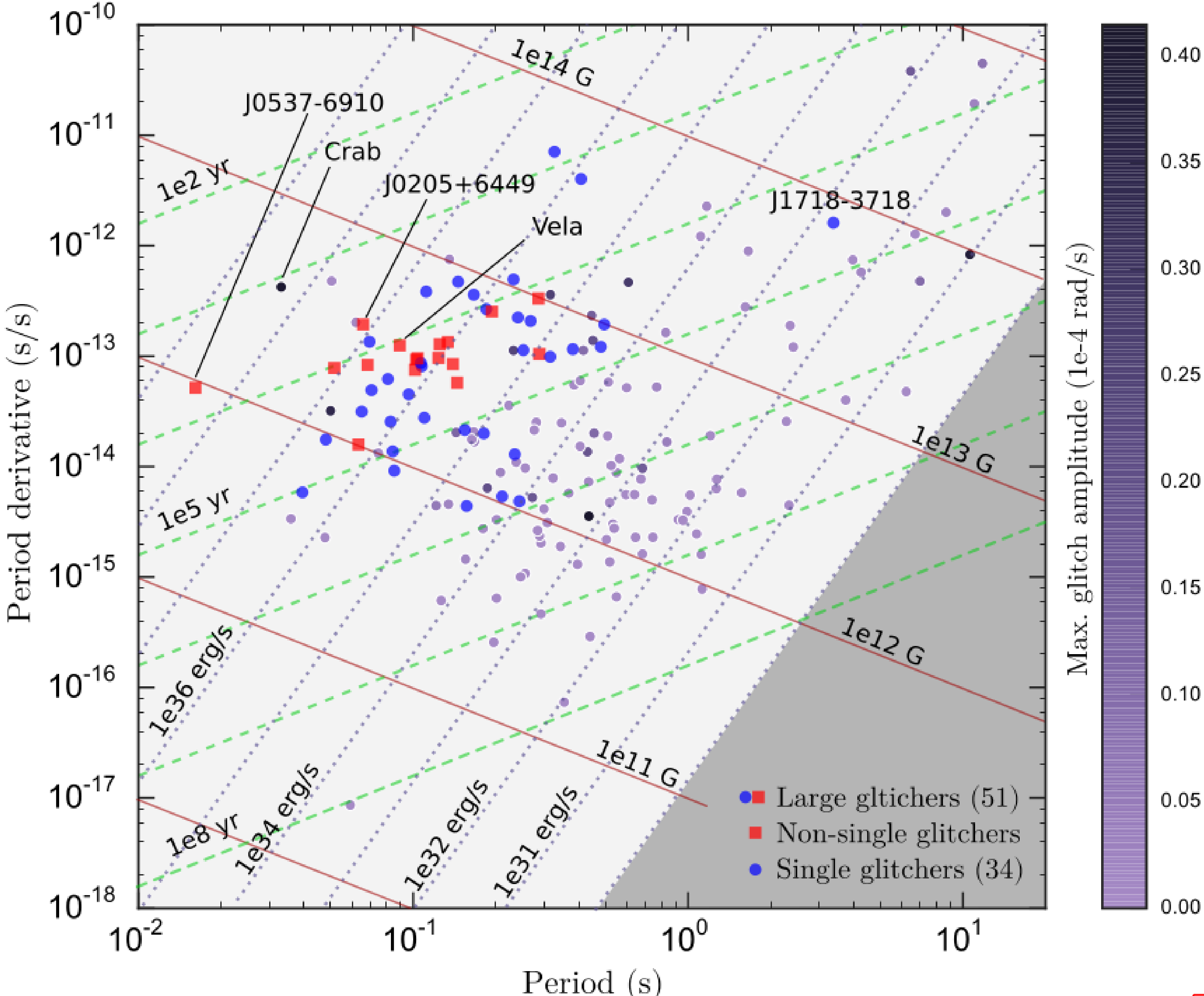


Early history in a nutshell:

1931 - Landau, Bohr, Rosenfeld: possible existence of compact stars dense as atomic nuclei
1932 - Chadwick “The Existence of a Neutron”
1932 - Baade & Zwicky predicted the existence of neutron stars as supernova remnants
1939 - Oppenheimer & Volkoff “On Massive Neutron Cores”

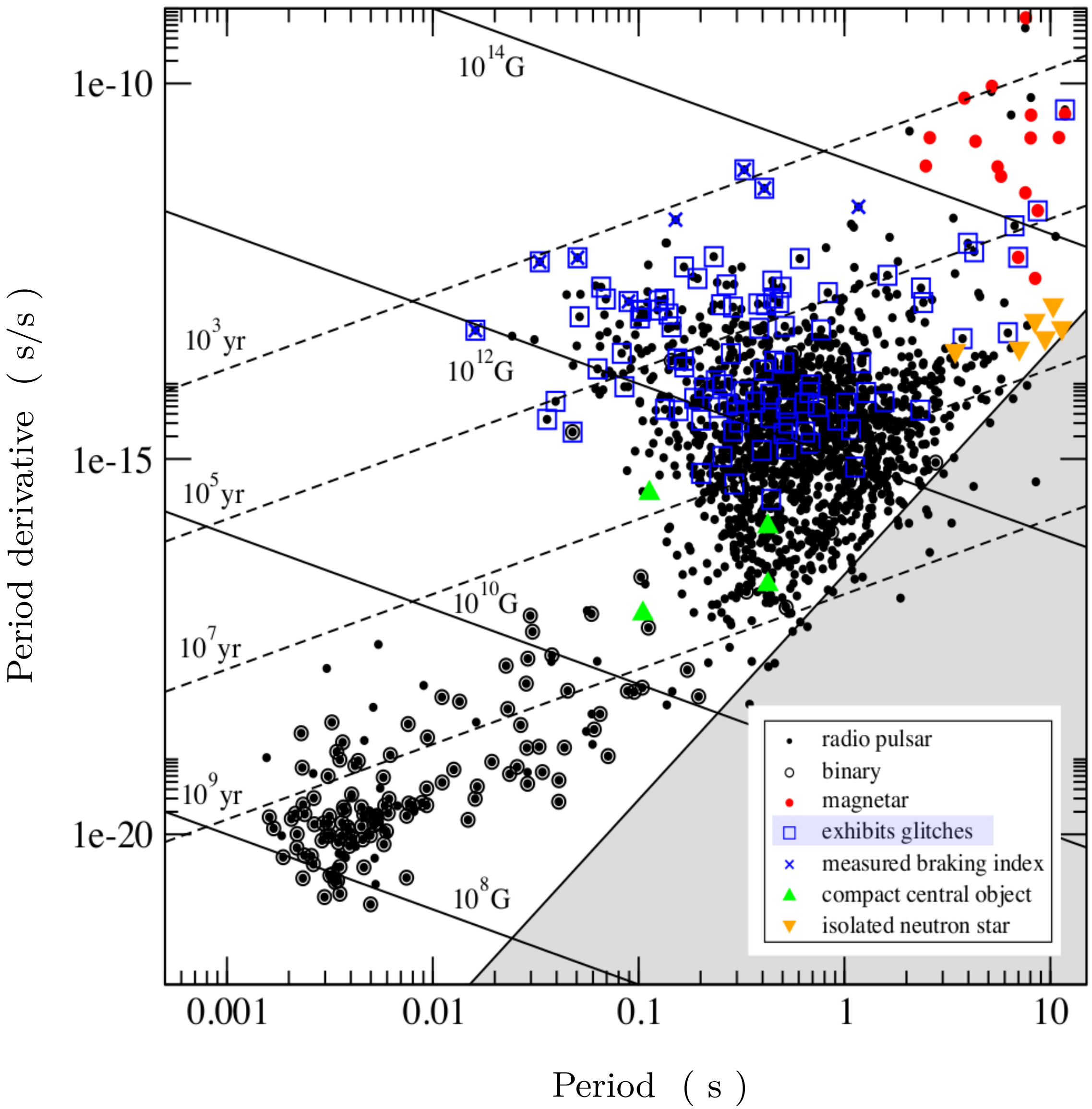
Then ~30 years of purely theoretical speculations (first “radio pulsar” detected in 1967)

Large glitchers



$B \propto \sqrt{P\dot{P}}$, $\tau \propto P/\dot{P}$

Glitches across the pulsar population



~2600 known pulsars

Different classes populate different regions (inferred age and magnetic field B). Sanity check from the braking index, but the second derivative of P is needed.

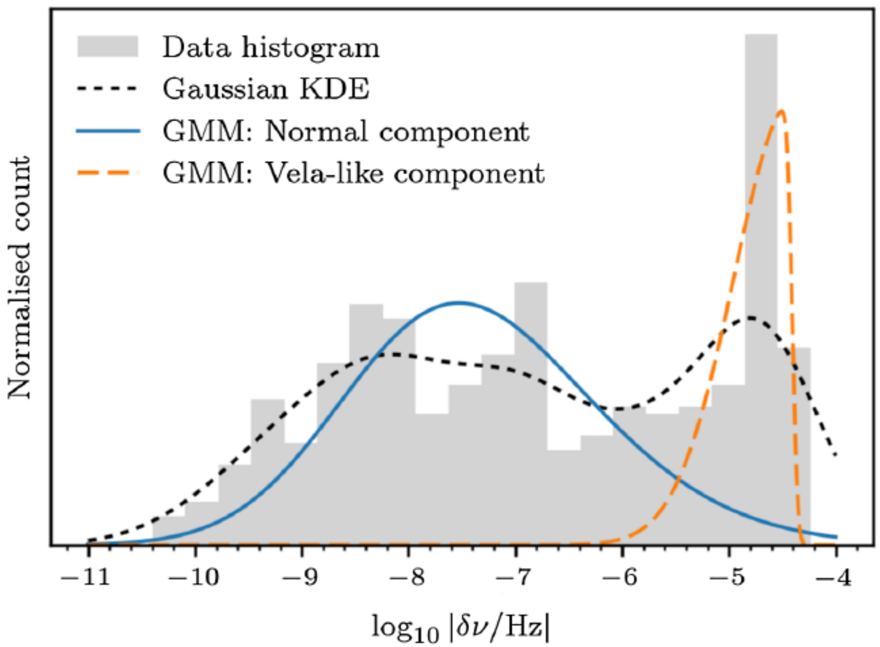
$$B \propto \sqrt{P\dot{P}},\ \tau \propto P/\dot{P}$$

Stable clocks with predictable spin-down, except for glitches and timing noise: ~500 glitche events detected in ~170 objects to date.

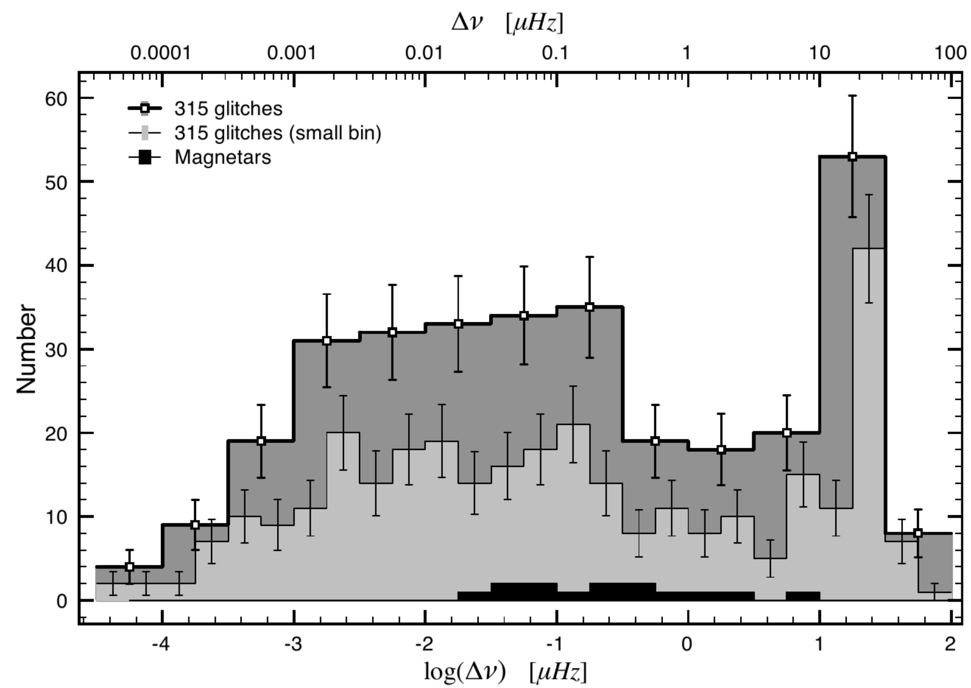
Glitch sizes

Sample of all known glitches: strong statistical evidence for bimodality of the distribution of glitch sizes.
This may underlie a bimodality in the pulsar population or a difference in the glitch mechanism.

Large glitches with $\Delta\Omega \gtrsim 0.5 \times 10^{-4}\,\mathrm{rad/s} \sim 10\,\mu\mathrm{Hz}$ can be used to test the pinning forces inside the crust.



Ashton et al. Pys Rev D (2017)



Espinoza et al. MNRAS (2011)

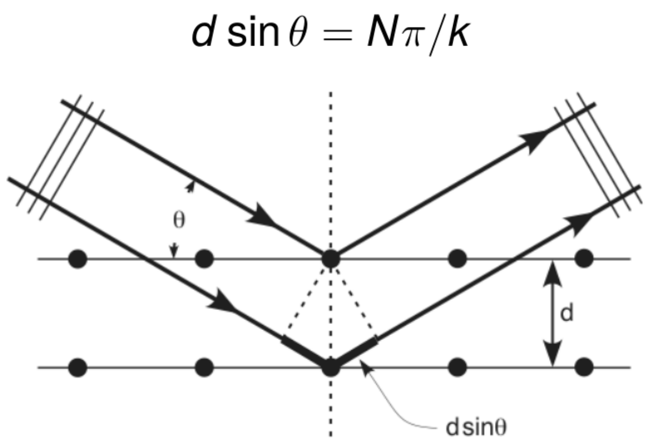
Entrainment coupling: crust and core

- In the crust:

Chamel N. *Neutron conduction in the inner crust of a neutron star in the framework of the band theory of solids,* Phys Rev C 85 (2012)

Bragg scattering by crustal lattice, non-local m* > 1

→ Consequence: the crustal superfluid is entrained by the normal component: reduced mobility of free neutrons is a potential problem for pulsar glitch theory.



- In the core:

Chamel N., Haensel P. *Entrainment parameters in a cold superfluid neutron star core,* Phys. Rev. C 73 (2006).

Entrainment is due to the strong interaction between protons and neutrons

Very different mechanism: actually more similar to the original A&B idea

Local effect, m*<1

→ Consequence #1: Scattering of electrons off vortex cores: the core is coupled to the crust on the timescale of a second.
Alpar et al. *Rapid postglitch spin-up of the superfluid core in pulsars(1984)*

→ Consequence #2: Dipole-dipole interaction with flux-tubes (core pinning?)