Pairing in nuclei:
some experimental aspects

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Agenda

I. Introduction: generalities on pairing.
II. Pairing at high spin.
III. Pairing in N~Z nuclei.

I: Generalities on pairing

Introduction

- Suppose that nuclei are built from fermions interacting with ***~charge independent*** NN force
- The ***independent particle*** shell model of Mayer-Jensen explains many data: g.s. spins, E* of first excited states, magnetic moments of s.p. states, shell gaps,…
- But there are some important deficiencies which cannot be understood at all considering the nucleus as an ensemble of protons and neutrons with independent motion

1) g.s. spin of e-e nuclei is 0^+

- The g.s. spin of **all** even-even nuclei always 0^+
- But, with the independent particle shell model, this is **only** achieved when the shell is full (^{16}O, ^{48}Ca, ^{208}Pb,… i.e. only for doubly-magic nuclei).
- Indeed: an (n,l,j) shell contains 2j+1 nucleons ➔ all m_i states (-j to j) occupied ➔ $M=\Sigma\, m_i = 0$ ➔ J=0

1) g.s. spin of e-e nuclei is 0^+

- What is the prediction for 2 nucleons in an incomplete shell?

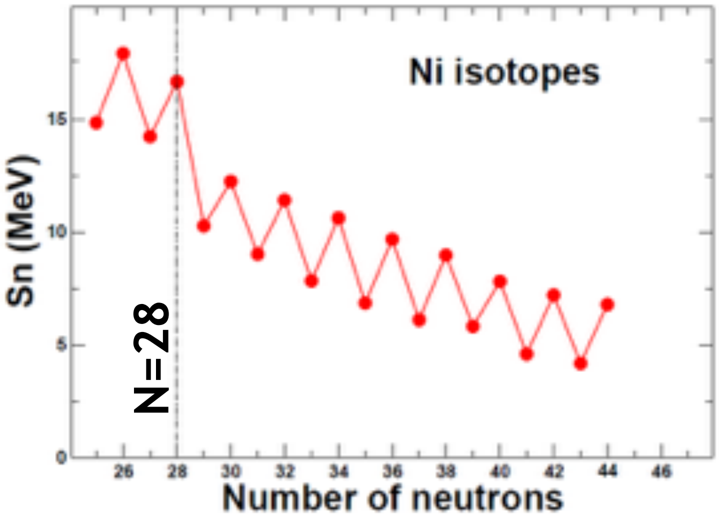
- Suppose **2 nucleons** in $g_{9/2}$ (case of ^{210}Po vs ^{208}Pb core): we expect 5 degenerate states with I=0, 2, 4, 6, 8

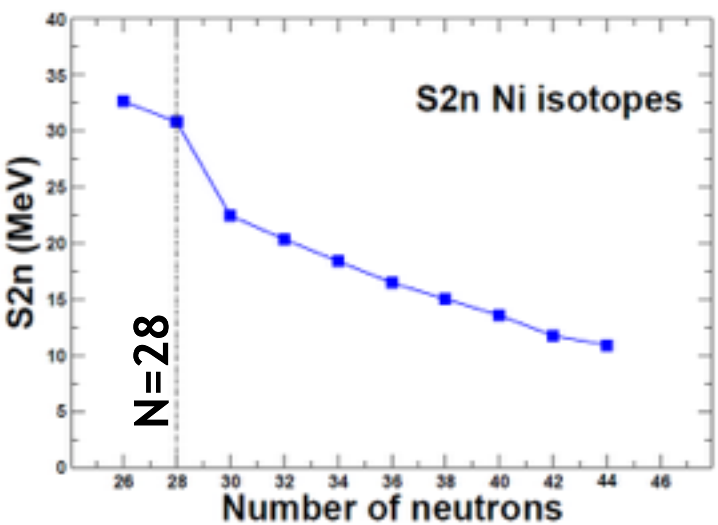
Since we observe g.s with I=0 (the other members of the multiplet are excited states):

1) an additional interaction lowers down in energy **only** the 0 spin state (saying this means also means that the two nucleons are **not anymore independent**)

2) M=0 → $m_1=-m_2$; classical picture is rotation on the same orbit in opposite way

2) Binding energy of e-e is larger than in the odd neighbours





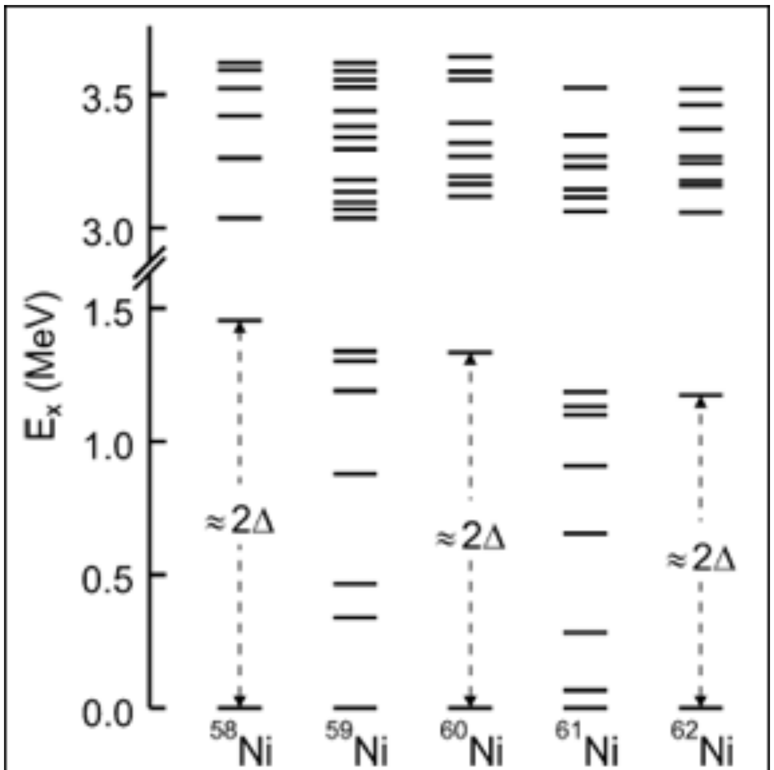
One nucleon separation energy:

- $S_n = B(N,Z) - B(N-1,Z)$
- $S_p = B(N,Z) - B(N,Z-1)$

- Oscillation pattern for both Sn and Sp (« o-e staggering »)
- Gain in energy ~1-3 MeV
- S_{2n} (S_{2p}) smooth
- points to the formation of pairs of nucleons

3) Low energy spectrum

- Energy spectra for odd-A nuclei are very different from even-even neighbours:
<u>Example of Ni isotopes</u>

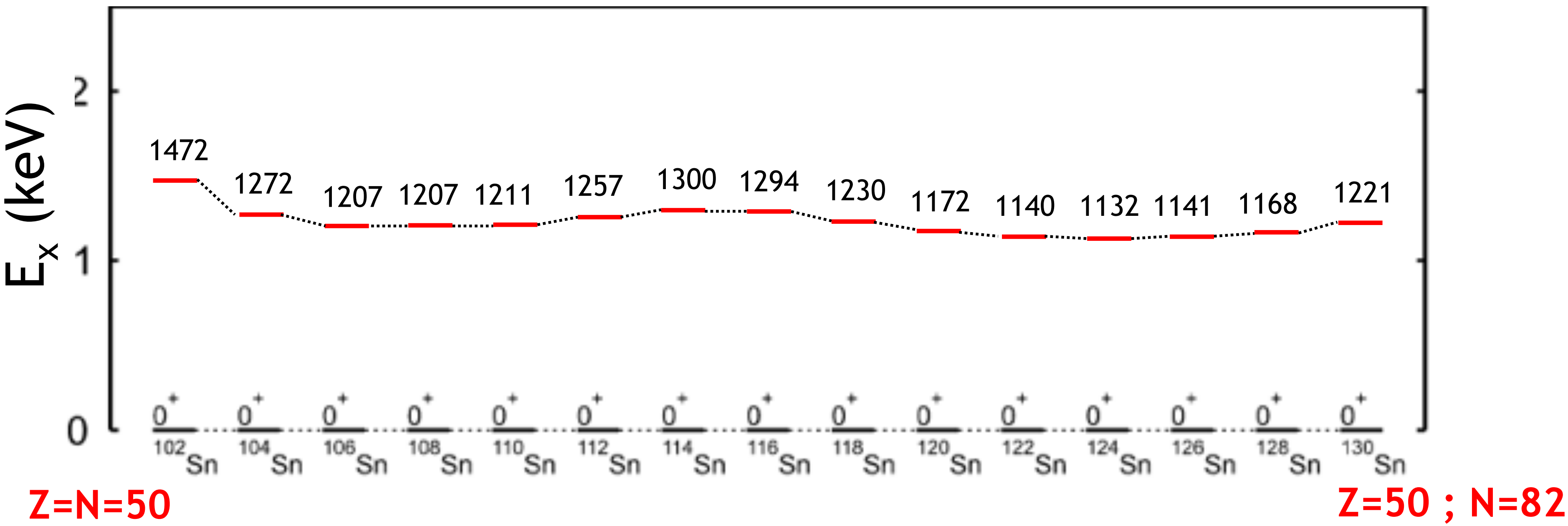


- Very few states (rotation or vibration at most) appear below a « certain » level at ~2Δ in e-e nuclei
- Much more in the odd neighbour (s.p. + collective states)
- Spectra much more similar above 2Δ: level density ρ depends on Δ

3) Low energy spectrum

- $E(2_1^+)$ states in e-e nuclei are remarkably constant

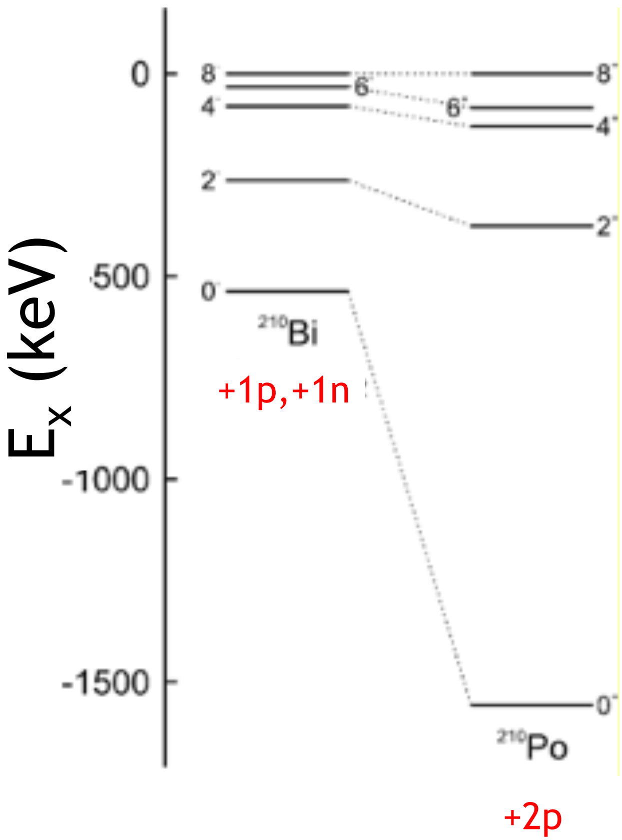
Example of Sn isotopes with neutrons successively filling $2d_{5/2}$, $1g_{7/2}$, $3s_{1/2}$, $2d_{3/2}$, $1h_{11/2}$:



- connected to neutron condensates: energy gap 2Δ

3) Low energy spectrum

- Energy spectra for nuclei near the closed shells: 0^+ g.s. in (A±2, A±4) show a pronounced gap compare to magic neighbour



- e.g. ^{210}Po = two protons in $1h_{9/2}$ compare to ^{208}Pb
- If no interaction between protons (i.e. independent particles) ➔ various spin couplings of $(1h_{9/2})^2$ would lead to degenerate states

⇩

There must exist correlations between the two protons

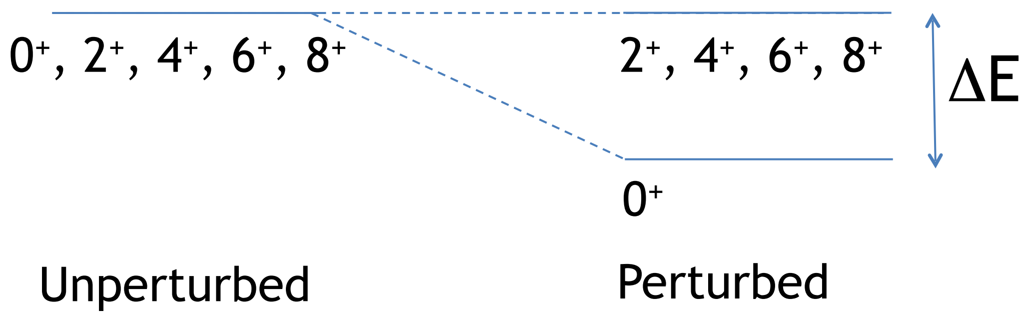
The pairing correlation

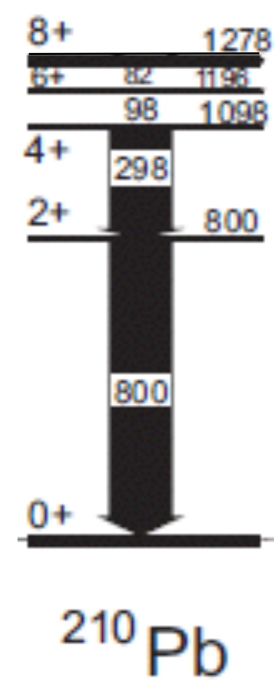
- simplest form of pairing:

$\langle j^2 JM | V_p | j^2 JM \rangle = -G\ (2j+1)\ \delta_{J0}$, G = pairing strength

  - This lowers the I=0 state by $\Delta E = G(2j+1)$
  - The lowering of I=0 is larger for large j
  - Other states not affected

e.g.: $(g_{9/2})^2$



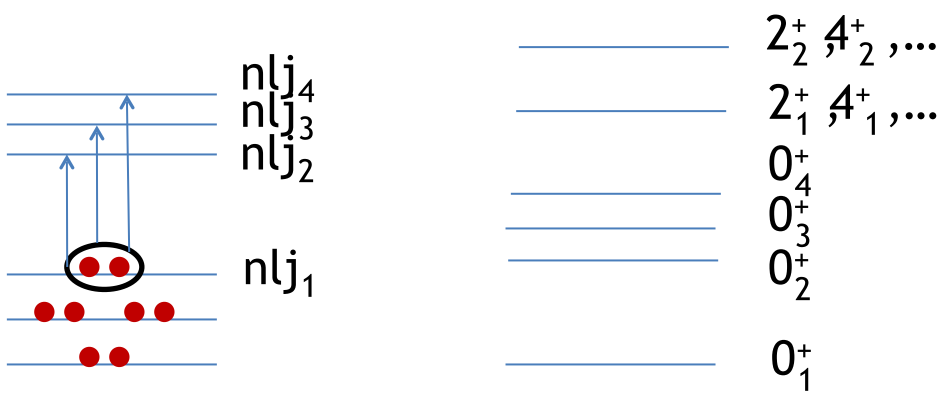


- Not realistic: experimentally $2^+, 4^+, 6^+, 8^+$ not degenerate (e.g. ^{210}Pb)…

The pairing correlation

Another problem:

- Suppose an e-e nucleus and pairing. A pair can scatter into the N available orbits ➔ N 0^+ states expected



- But in e-e nuclei we observe only one 0^+ state at low energy ➔ need to have an interaction between these N states. Then quantum mechanics mixes them all and gives one coherent mixing of all 0^+ states (the g.s.) for which all the amplitudes are positive

The pairing correlation

- To do that and reproduce the observed features, modify a bit the previous version:

$$\langle j^2 JM | V_p | j'^2 JM \rangle = -G\sqrt{2j+1}\,\sqrt{2j'+1}\,\delta_{J0}$$

  - diagonal terms lower state $(nlj)^2$ with J=0 (g.s.)
  - non-diagonal terms generate interaction between 0^+ states.

- Imagine a magic nucleus + 2 nucleons with n degenerate states where the pairs can scatter into…

A (too?) simple case

- H_0 = hamiltonian of the unperturbed system (2 nucleons without interaction). Schrödinger equation for this pair:

$$H_0\Psi = \varepsilon\Psi_i \ , \ i=1,\ldots,n$$

- V_p the pairing interaction. Suppose that all matrix elements of V_p are identical:

$$\langle\Psi_i|\ V_p\ |\Psi_j\rangle = -V \ , \ i,j=1,\ldots,n$$

- $H= H_0+V_p \rightarrow (H_0+V_p)\Psi = E\Psi$ Developing Ψ functions on the basis of the $\Psi_{i:}$

$$\begin{bmatrix} \varepsilon-V & -V & -V\ldots \\ -V & \varepsilon-V & -V\ldots \\ -V & -V & \varepsilon-V\ldots \\ . & & \\ . & & \\ . & & \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = E \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \Rightarrow \begin{vmatrix} \varepsilon-V-E & -V & -V\ldots \\ -V & \varepsilon-V-E & -V\ldots \\ -V & -V & \varepsilon-V-E\ldots \\ . & & \\ . & & \\ . & & \end{vmatrix} = 0$$

The pairing correlation

- Solution: $(-1)^n(E-\varepsilon)^{n-1}(E-\varepsilon+nV) = 0$
  - (n-1) states with energy $E=\varepsilon$
  - one state with energy $E=\varepsilon-nV$
- THE important result: the more 0^+ states (n), the larger the shift on the coherent 0^+ state (= the larger the gap)

- Limitations when the number of states around the Fermi energy is large ➔BCS and the concept of partial filling of levels around the Fermi energy

$\Delta = G\Sigma U_i V_i$: Δ gap parameter ; U_i and V_i are « emptiness » and « fullness » factors:

$$U_i = \frac{1}{\sqrt{2}}\left[1 + \frac{(\varepsilon_i - \lambda)}{\sqrt{(\varepsilon_i - \lambda)^2 + \Delta^2}}\right]^{\frac{1}{2}} \qquad V_i = \frac{1}{\sqrt{2}}\left[1 - \frac{(\varepsilon_i - \lambda)}{\sqrt{(\varepsilon_i - \lambda)^2 + \Delta^2}}\right]^{\frac{1}{2}}$$

II: Pairing at high spin

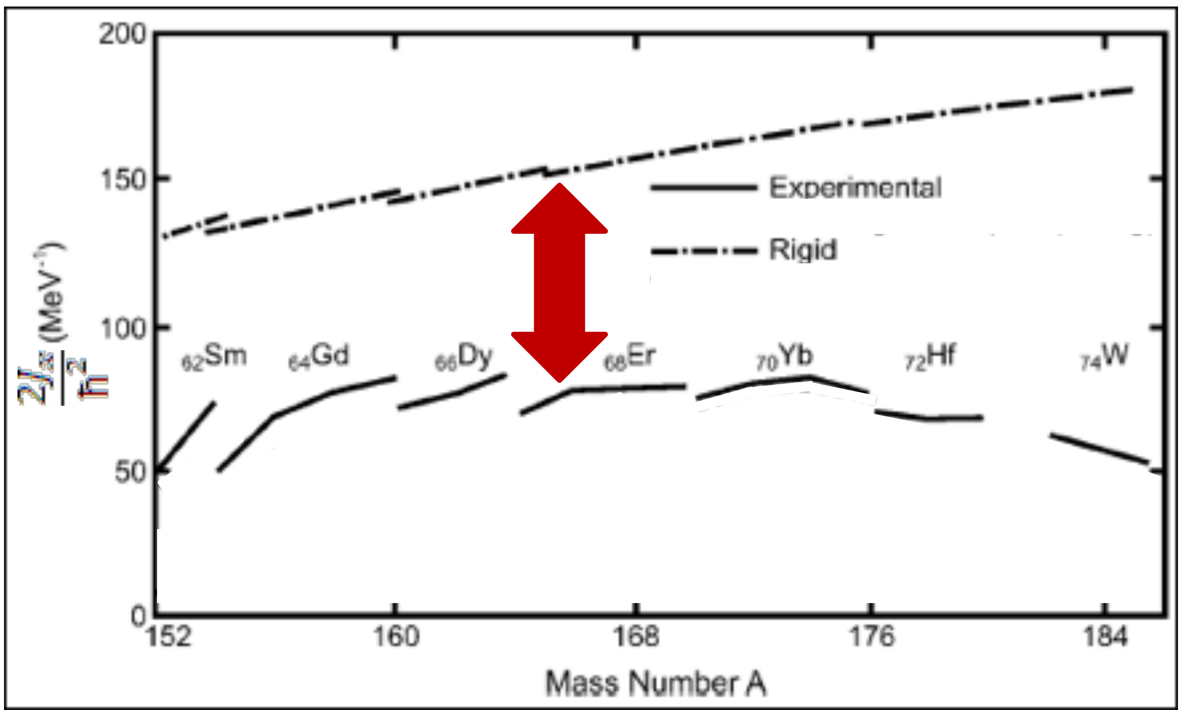
MoI and pairing

- Energy spectrum for a rotor:
- Moments of inertia of a rigid rotor from energy differences
- $J^{(2)}$ very sensitive to irregularities of E(I)

$$E(I) = \frac{\hbar^2}{2J_x} I(I+1)$$

$$\hbar\omega = \frac{dE}{dI_x} \simeq \frac{dE}{dI}$$

$$J^{(2)} = \frac{1}{\hbar^2}\left(\frac{d^2E}{dI^2}\right)^{-1}$$



- $J_{exp} << J_x$ (a factor of ~2-3)
- The collective modes are dramatically affected by pairing correlations

Quantal rotation: D. Inglis

- To determine the true MoI: cranking

$$E = E_0 + \frac{1}{2} J\omega^2$$

➔ MoI = slope of E=f(ω^2)

➔ Need to calculate the energy in the rotating frame to find J

➔ D Inglis formula: $$J = 2\hbar^2 \sum_{\Phi' \neq \Phi_0} \frac{|\langle \Phi' | J_1 | \Phi_0 \rangle|^2}{E' - E_0}$$

Φ_0 and Φ' = g.s and excited states; E' and E_0 associated s.p. energies

Now need to integrate pairing

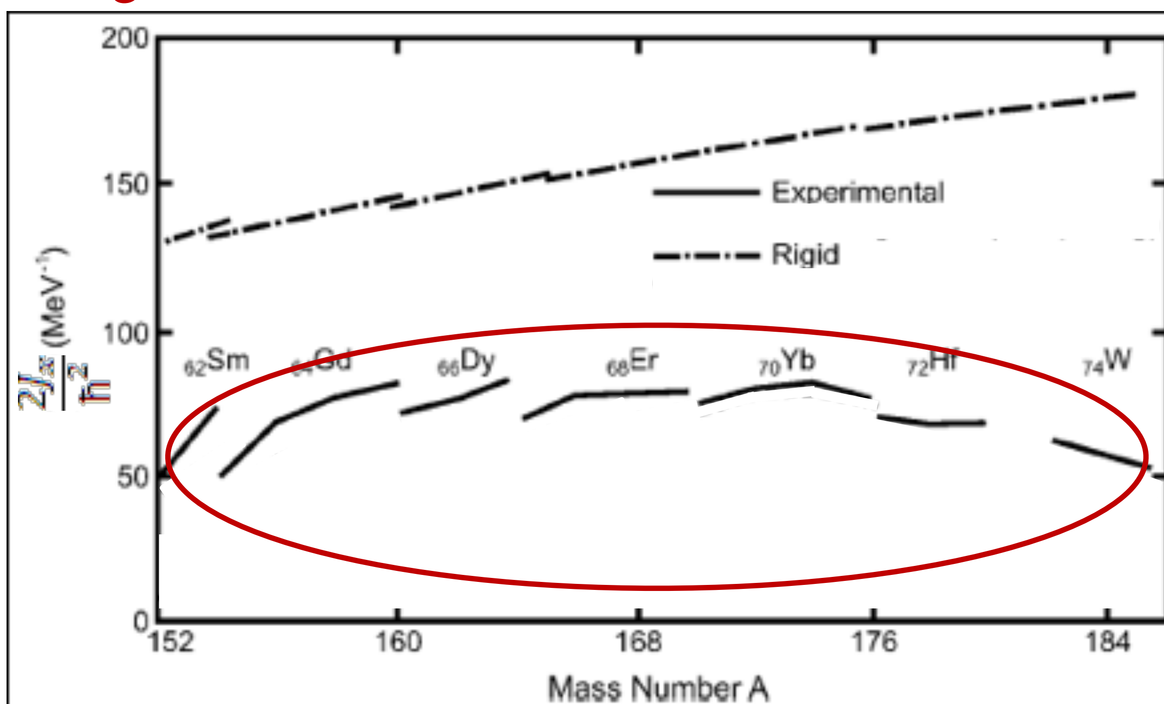
Inglis + pairing = Belyaev

$$\mathcal{J} = 2\hbar^2 \sum_{k',k>0} \frac{|\langle k|J_x|k'\rangle|^2}{E_k + E_{k'}} (u_k v_{k'} - v_k u_{k'})^2$$

- Note:
  - pairing factor $(u_k v_{k'} - u_k v_{k'})^2 < 1$
  - in the Inglis formula $E'-E_0$ is a *s.p. energy difference*.

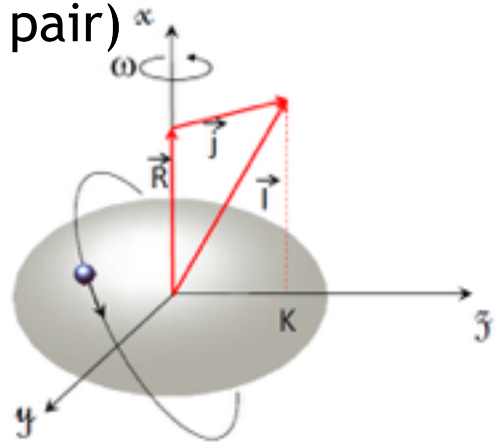
  Here, it is a *sum of qp energies* taken relative to Fermi energy. $E_k+E_{k'} > E'-E_0$

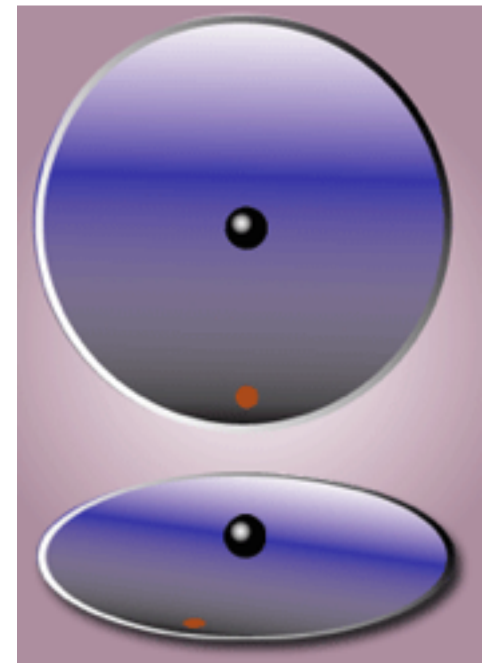
- $\Rightarrow$ $\mathcal{J} < \mathcal{J}_{inglis}$
- MoI nicely reproduced when pairing is included:



Effect of rotation on pairing: Coriolis

- Coriolis: deflection of moving objects when they are viewed in a rotating reference frame
- This is the case when we have a system consisting of a core + 1 nucleon (or a pair)





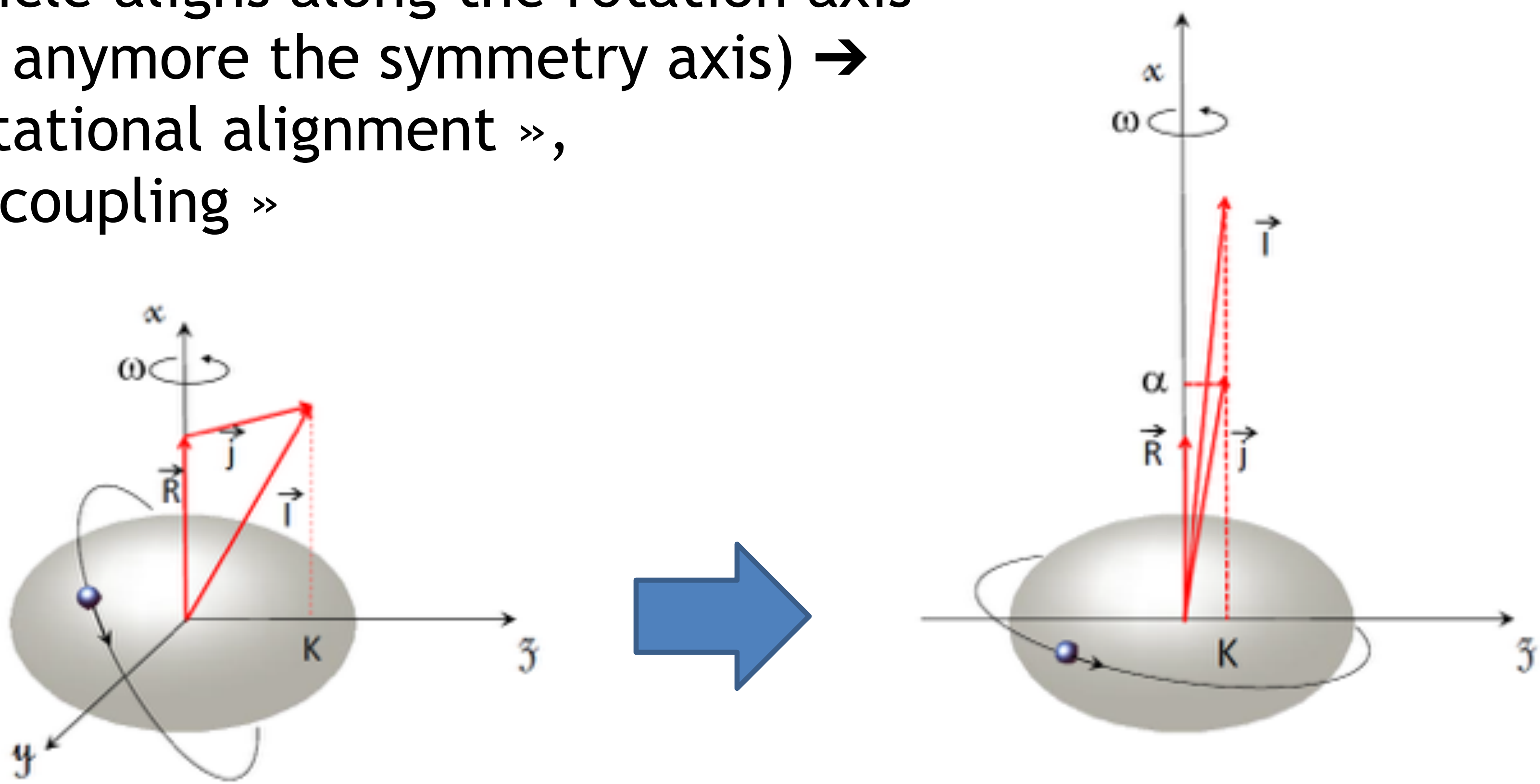
From Wikipedia

- Vcor: Coriolis coupling between the angular momentum ***j*** and ***I***); effect of Coriolis depends on: J, K and Vcor; strong and weak coupling limits, high spin limit…

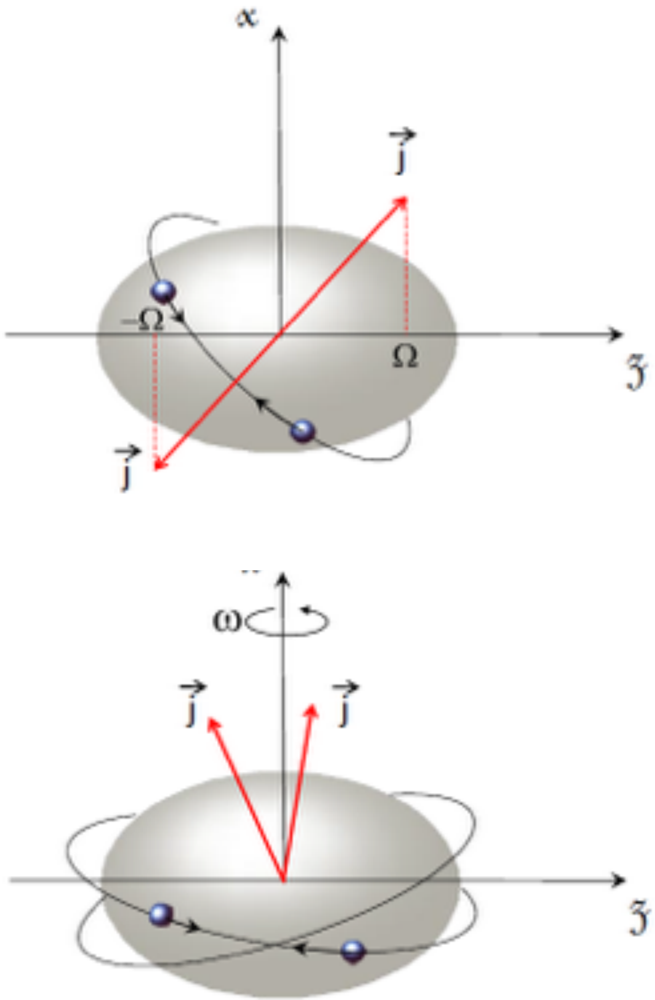
$$E(I) = \frac{\hbar^2}{2J}\left[I(I+1) - 2K^2 + \langle j^2 \rangle\right] \underbrace{- \frac{\hbar^2}{2J}(\boldsymbol{I}_+ \boldsymbol{j}_- + \boldsymbol{I}_- \boldsymbol{j}_+)}_{V_{cor}}$$

Coriolis strength: the high spin case

- The angular momentum of the particle aligns along the rotation axis (not anymore the symmetry axis) ➔ « rotational alignment », « decoupling »



Effect of Coriolis (rotation) on pairing



- <u>No rotation</u>: particles coupled to J=0 on time reversed orbit

- <u>With rotation</u>:
  - E*< Epairing: the angular momentum increase smoothly with rotation
  - E*>Epairing: with opposite spins, the two nucleons feel rotation in an oposite way (Mottelson-Valatin effect*) ➔ the pair is broken and alignment of s.p. angular momenta along rotation axis ➔ backbending

B.M. Mottelson and J.G. Valatin, PRL 5 (1960) 511

Backbending

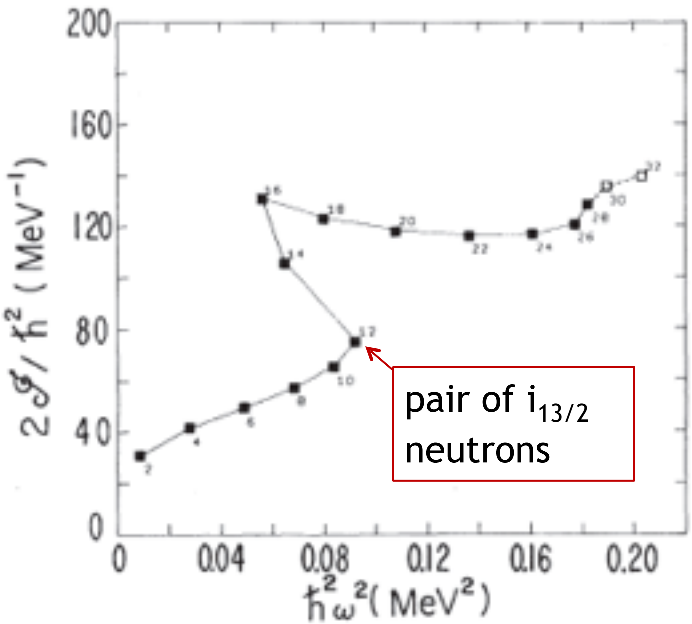
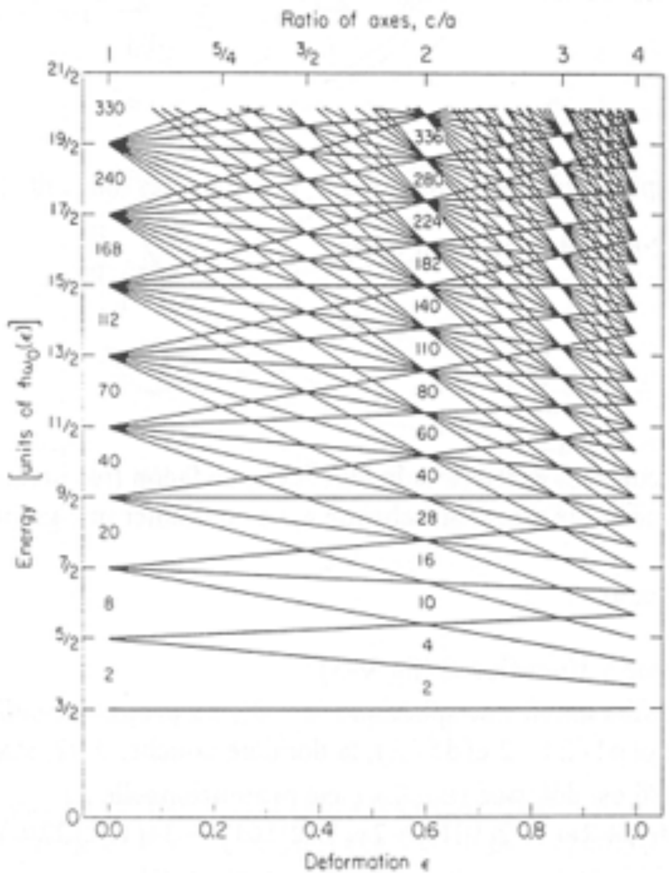


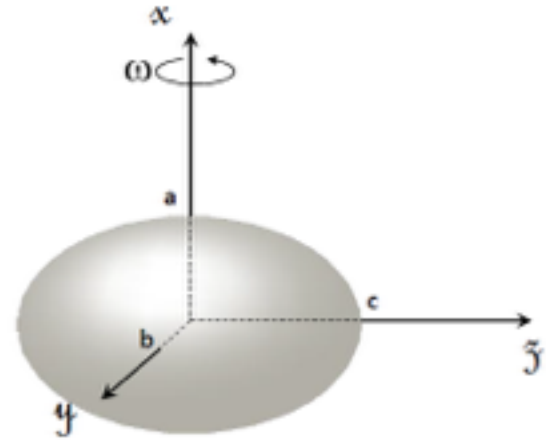
FIG. 2. Plot of the moment of inertia vs the square of the angular velocity for ^{158}Er.

I.Y. Lee et al, PRL 38 (1977) 1454

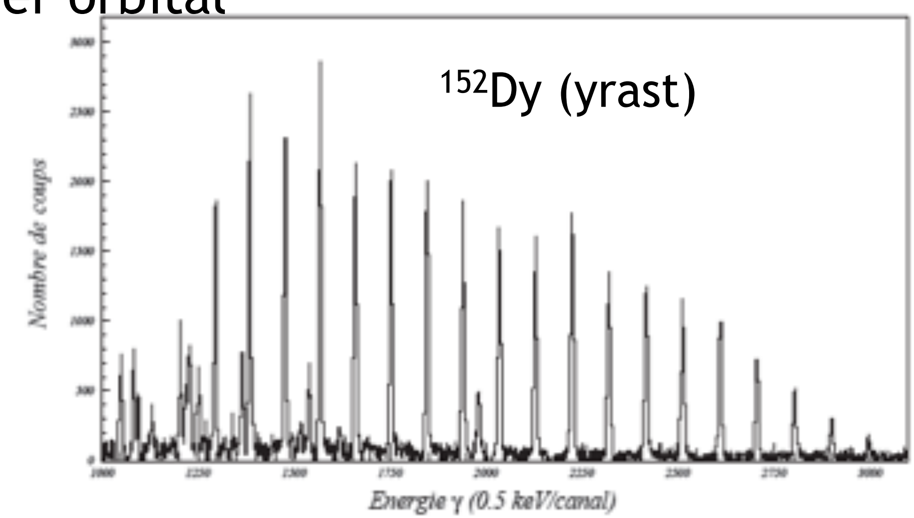
- « Coriolis anti-pairing » or « rotational alignment »
- Example of ^{158}Er: alignment of two $i_{13/2}$neutrons + two $h_{11/2}$ protons
- The abrupt alignment of high-j particles increase the angular momentum which is converted in inertia
- Pairing collapse at the highest spins when all the pairs are aligned? -> SD

Pairing at the highest spin: superdeformation





- Deformed harmonic oscillator (ellipsoid; no s.o.)
- Very low level density in some zones (c/a=2, 3)
- Intruder orbital



- Experimental signature

^{152}Dy

A superdeformed rotor

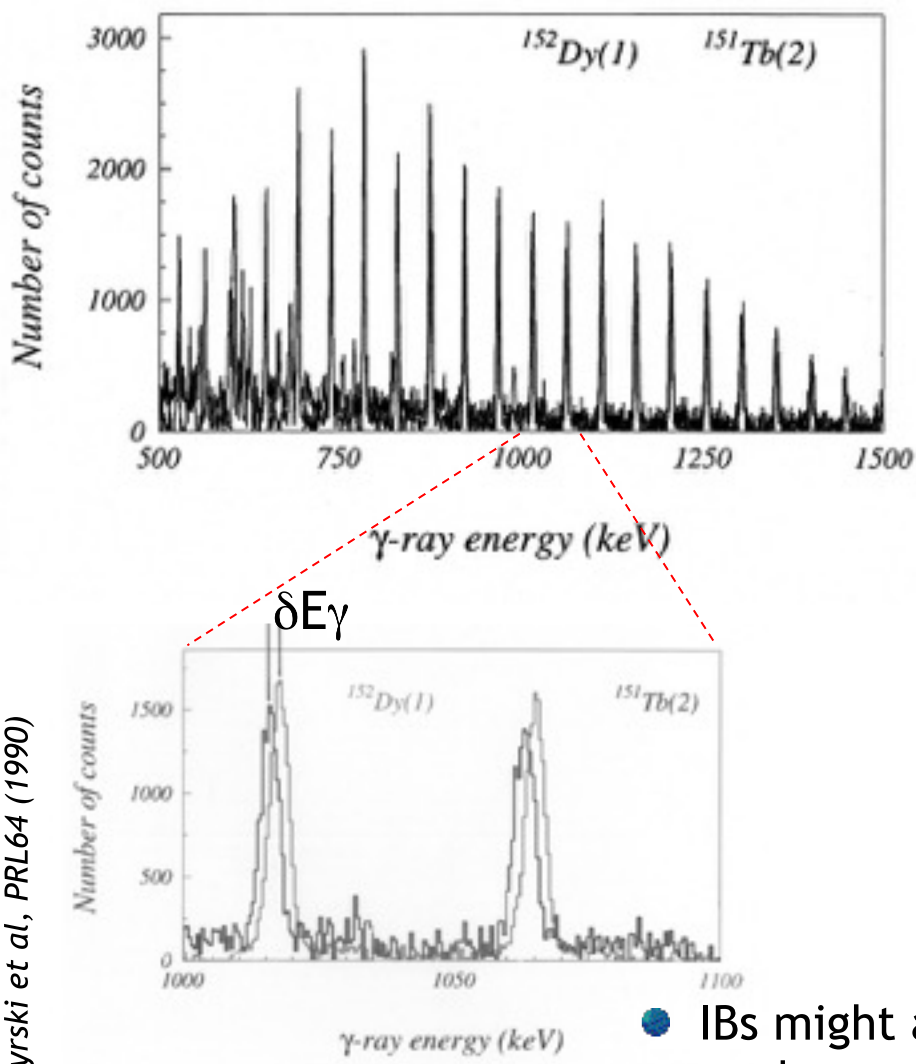
Energy of excited states E(I) : $$E(I) = \frac{\hbar^2}{2J_x} I(I+1)$$

Rotational frequency : $$\hbar\omega = \frac{dE}{dI_x} \simeq \frac{dE}{dI}$$

Dynamical MoI : $$J^{(2)} = \frac{1}{\hbar^2}\left(\frac{d^2E}{dI^2}\right)^{-1}$$

- $\Delta I = 2\hbar \rightarrow \hbar\omega = \Delta E/2 = E_\gamma/2$
- $E_\gamma = E(I+2) - E(I) \rightarrow E_\gamma = \frac{\hbar^2}{2J_x}(4I+6)$
- $J^{(2)} = 4/\Delta E_\gamma$, unit: $\hbar^2.MeV^{-1}$

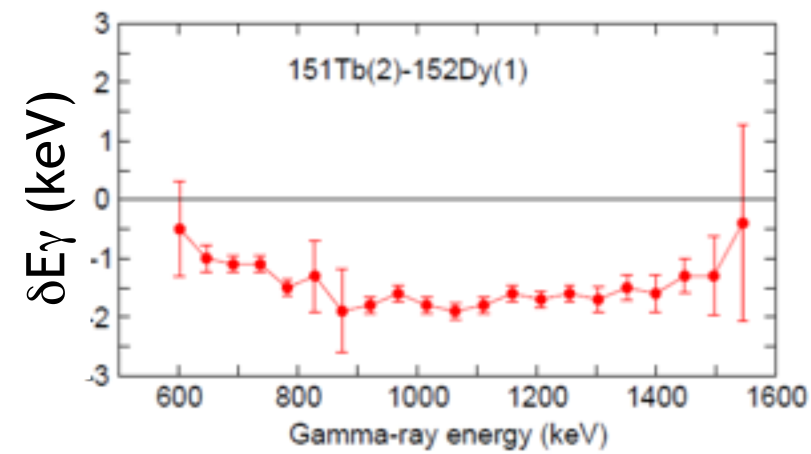
Pairing in SD nuclei: the IB probe



Th. Byrski et al, PRL64 (1990) 1650

Identical SD bands:

- ^{152}Dy(1)/ ^{151}Tb(2)



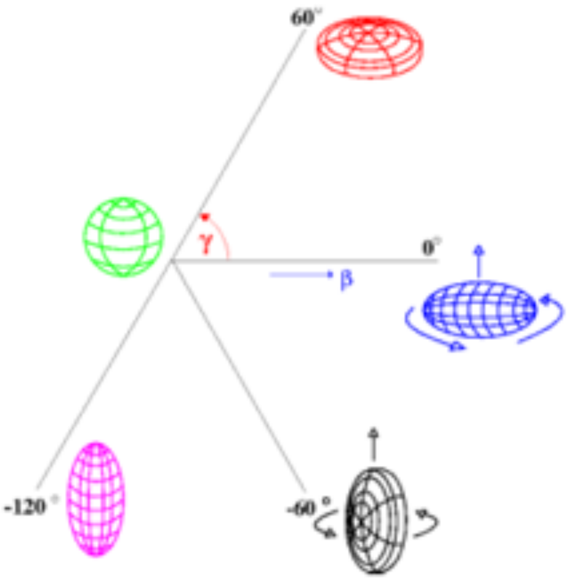
$$\frac{\Delta E_\gamma}{E_\gamma} = \frac{\Delta J_x}{J_x} \sim 0.001$$

- IBs might appear for bands differing by orbital that are only spectator (from the point of view of MoI: zero alignment)

Pairing in SD nuclei: the IB probe

- MoI around axis k:

$$J_k = \frac{2}{5} A R_0^{\,2} \{1 - \sqrt{\frac{5}{4\pi}} \beta \cos\left(\gamma - \frac{2k\pi}{3}\right)\}$$



$R_0 \sim A^{1/3}$ ⇨ $J_x \sim A^{\frac{5}{3}}$

- for two neighbouring nuclei:

$$\frac{\Delta J_x}{J_x} = \frac{J_x(A+1) - J_x(A)}{J_x(A)} = (1 + \frac{1}{A})^{\frac{5}{3}} - 1$$

⇨ $\frac{\Delta J_x}{J_x} \sim 0.01$

Pairing in SD nuclei: the IB probe

Statistical approach

- Large number of SD bands in several mass regions
- The fractional change to quantitatively « measure » identicality:

$$FC_{X(n),Y(m)} = \frac{J^{(2)}_{X(n)} - J^{(2)}_{Y(m)}}{J^{(2)}_{X(n)}} = \Delta J^{(2)} / J^{(2)}_{X(n)}$$

With: $$J^{(2)}_{X(n)} - J^{(2)}_{Y(m)} = d(I^{X(n)} - I^{Y(m)})/d\omega$$

We obtain: $$FC_{X(n),Y(m)} = di_{eff}/dI_{X(n)}$$

(Note: bands ordered in mass ➔ sign of FC has a physical meaning)

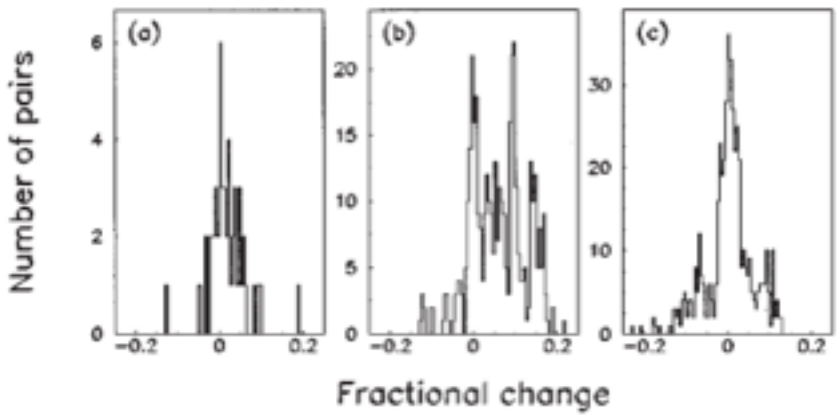
- In practice: if *i* is a linear fit of *I* ➔ we obtain FC

If : $$\overline{FC} < \frac{1}{2}\frac{\Delta A^{5/3}}{A_X^{5/3}}$$ ➔ IBs

Pairing in SD nuclei: the IB probe

Statistical approach

- FC distribution for SD bands in various mass region



- Whatever the region: peak around FC=0
- Statistics too low for A~130
- For A~150:
  - only positive FC values➔FC grows with mass as expected
  - Inspection of the peaks content indicate differences in intruder content:

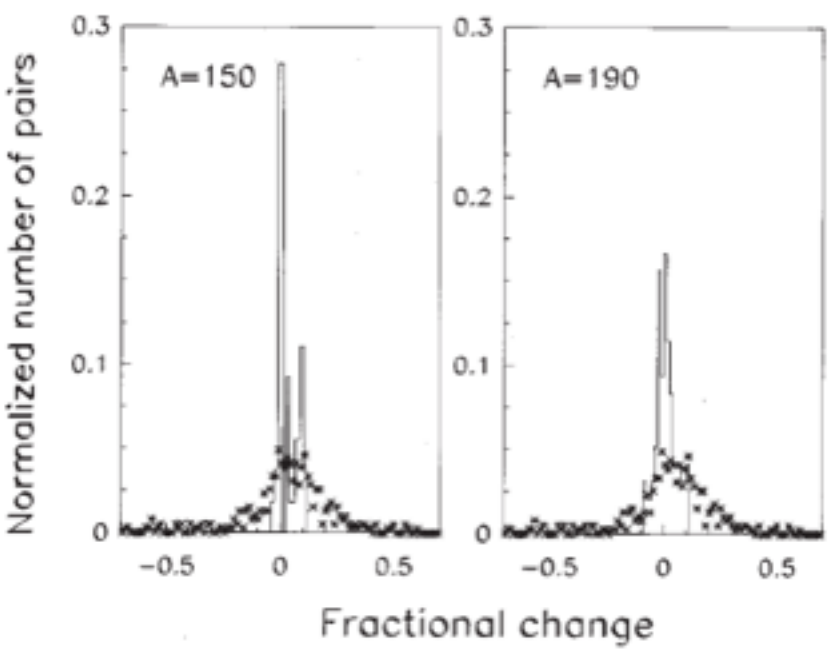
FC value	Difference in intruder content
~0	none
~0.05	1
~0.1	2
~0.16	3

- Several peaks in A~150 vs a single broad peak in A~190 with negative values
- x2 more IBs in A~190 compared to A~150

Pairing in SD nuclei: the IB probe

Statistical approach

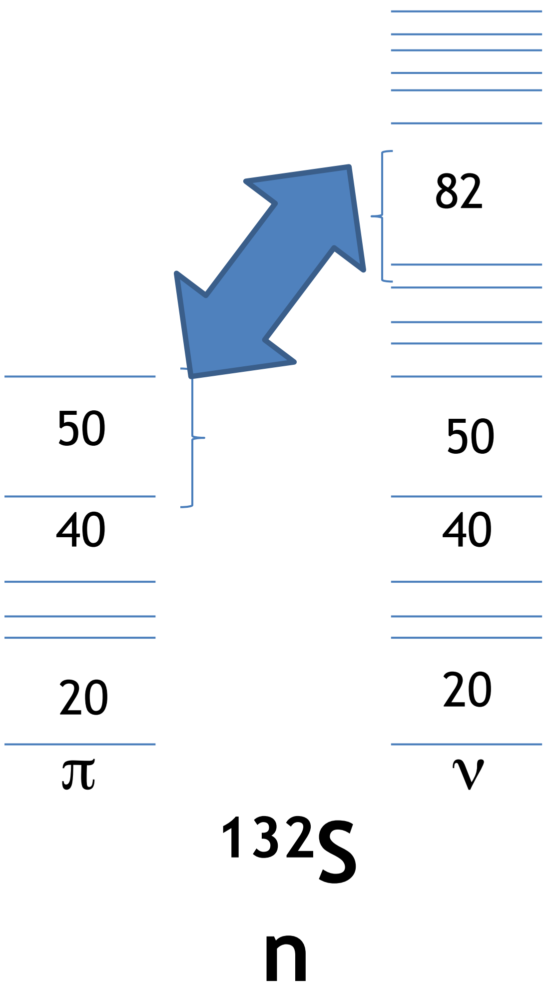
- How does it compare with ND nuclei?
  - same procedure with ND nuclei of the rare earth region



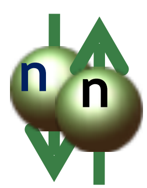
- How does it compare with ND nuclei?
  - more IBs in SD nuclei/ND ones
  - the FWHM of the FC distribution reflects the pairing strength
  - the clear identification of the high-j content in FC distribution of A~150 confirms that pairing is greatly reduced (if not completely collapsed)
  - a contrario: pairing is still active in the lead region and induce level mixing at the Fermi surface which smears out the FC distribution

III: Pairing in N~Z nuclei

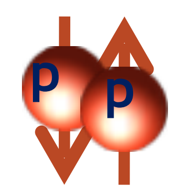
Different kind of pairing



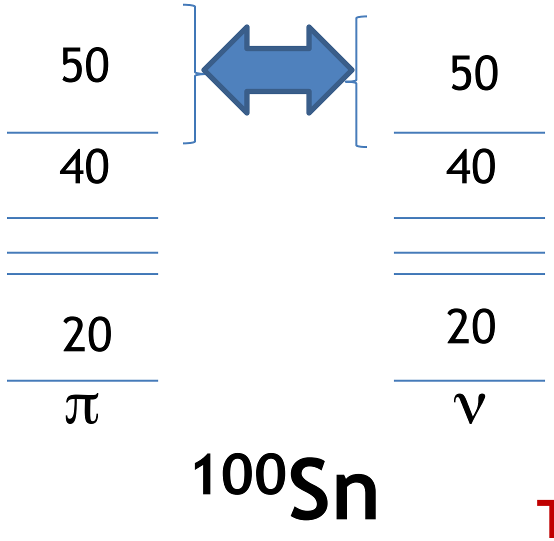
- In nuclei far from N=Z, protons and neutrons occupy very different orbitals
- No overlap, hence the valence nucleons do not interact
- In the isospin formalism: the nucleon has t=1/2 and t_z=-1/2 (proton); 1/2 (neutron)
- For a pair: T≤A/2=1 ➔ T=0,1
- Only nn and pp pairing: **identical** particles in **time reversed orbits (J=0)** ➔ **T=1 (Pauli)**. This is the **<u>isovector</u>** nn and pp Cooper pairs



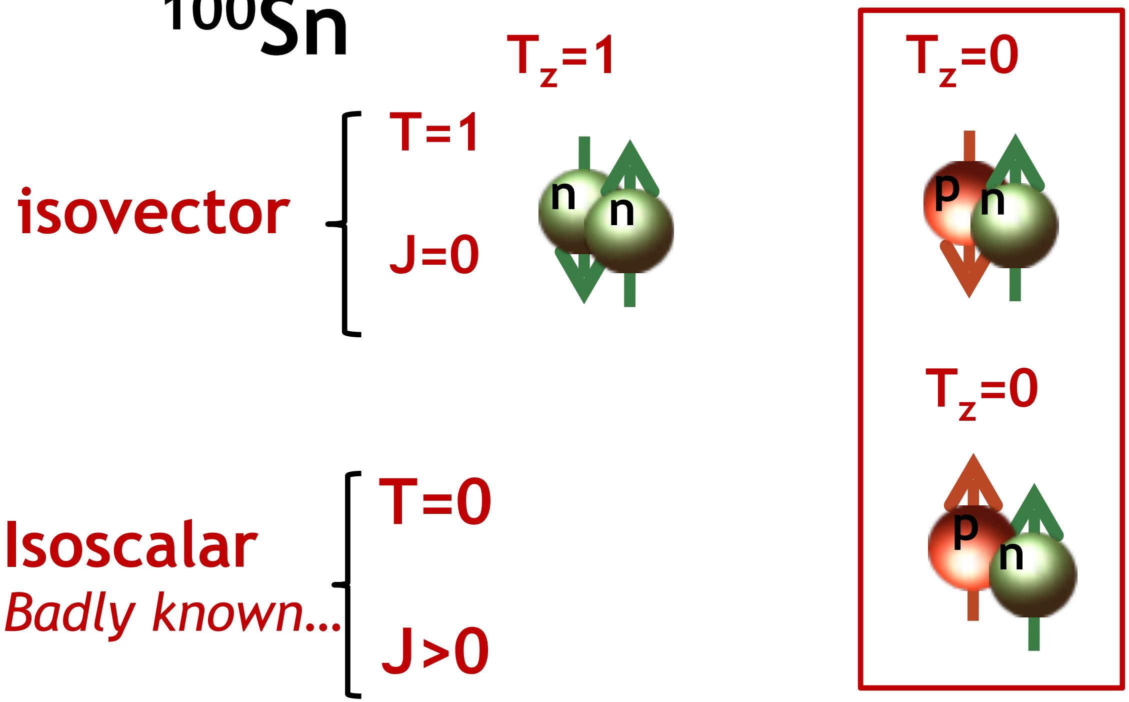
Like nucleon pairs:
T=1, J=0



Different kind of pairing: along N=Z



- Not anymore true along the N=Z line: protons and neutrons occupy the same orbitals
- np pairs with Pauli principle:

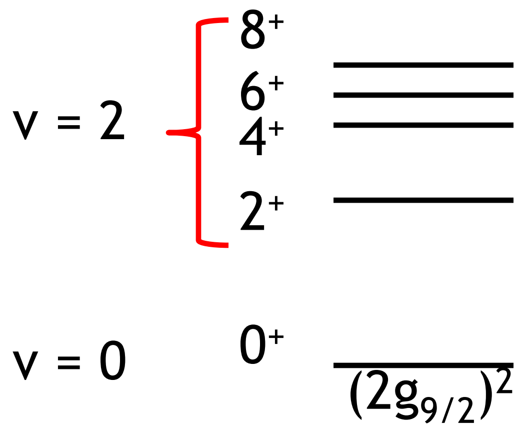


Our way to probe:

- Seniority scheme
- Rotational properties

The seniority scheme

- The seniority ν is the **number of nucleons that are not in pairs coupled to 0**
- In a valence configuration j^n, the states can be labelled according ν:

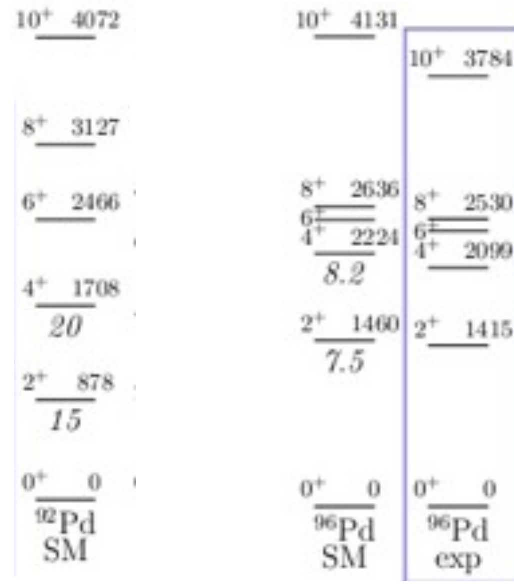


- The **seniority scheme** is revealed by the **energy spacing** between excited states

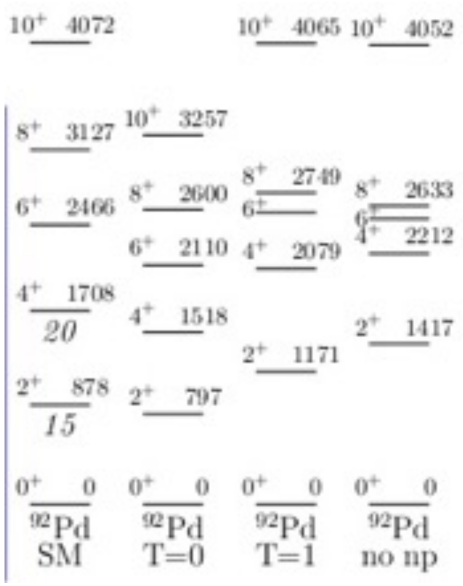
SM evolution

- Profound modification of the level scheme:

Regular spacing →



→ **Seniority type**

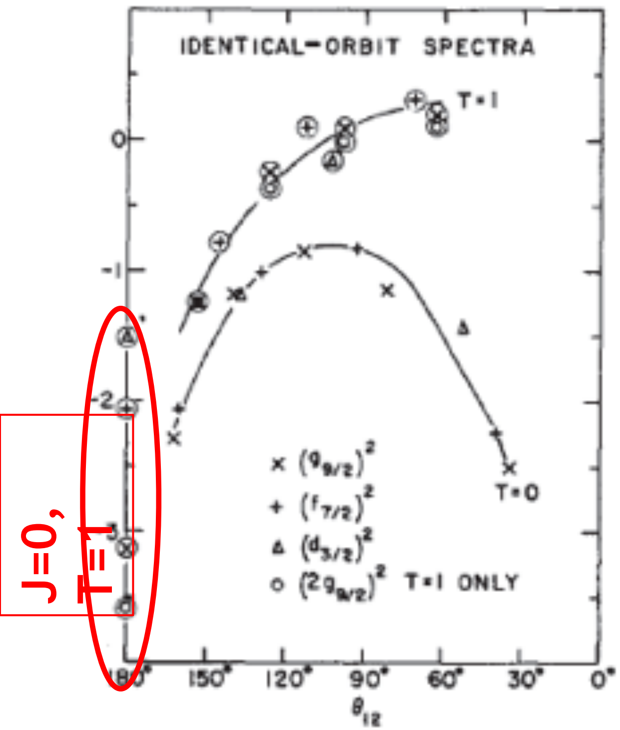


- Predicted effect of the T=0 and T=1 channels
- **The major influence of T=0**

J. Blomqvist et al

Rotational properties: T=0 vs T=1 strength

Matrix elements particle-particle of magic nuclei+2 nucleons in the same orbit (from E*) as a function of coupling angle (➔ independent of the considered orbit)

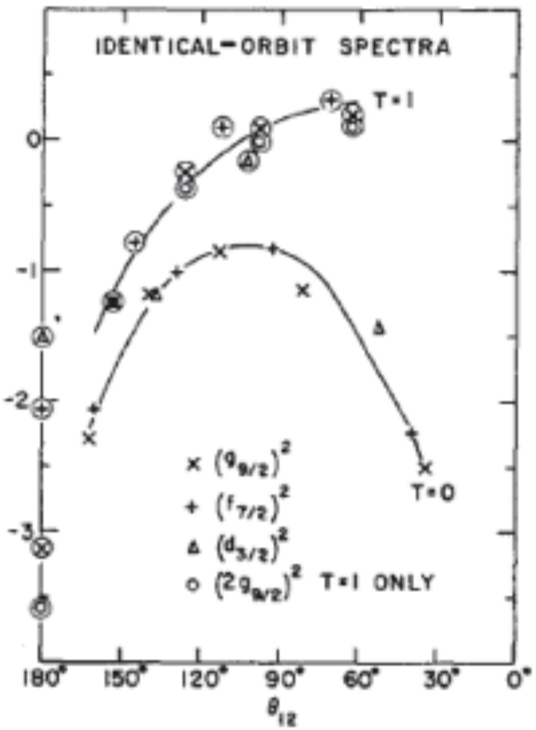


N. Anantaraman and J.P. Schiffer PLB37 (1971) 229

- **2 « universal » curves** for all the orbits: one for T=1 and one for T=0 (except for J=0, T=1)
- For T=1, strength concentrate in **J=0** i.e. (j,m)(j,-m)
- When spin increases: pairs are less bound; and less and less

  this justifies the description of like nucleon pairs (T=1) by a seniority pairing (i.e. considering only J=0)

Rotational properties: T=0 vs T=1 strength



- Except T=1, J=0, the **T=0 channel has a larger strength** compared to T=1 for two nucleons in the same orbit
  →No reason to neglect T=0 (a fortiori in N=Z nuclei)…
- Effect on **rotational properties**

EXOGAM-NWall-DIAMANT:

The power of the coupling

N=Z nuclei close to ^{100}Sn

^{36}Ar (111 MeV) + ^{58}Ni ⟶ ^{94}Pd* ⟶ ^{91}Tc + 3p

^{91}Ru + 2p1n

^{90}Mo + 4p

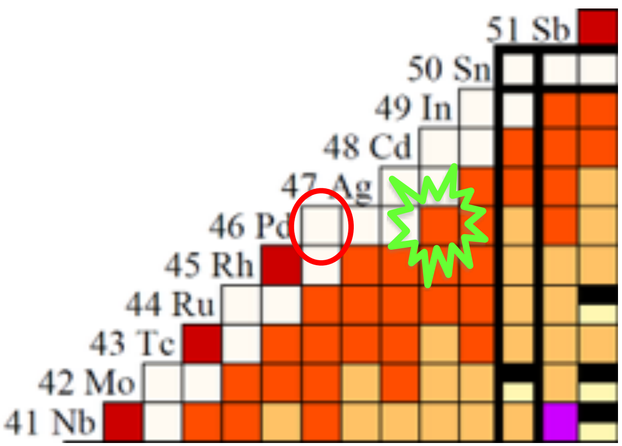
^{88}Mo + 4p2n

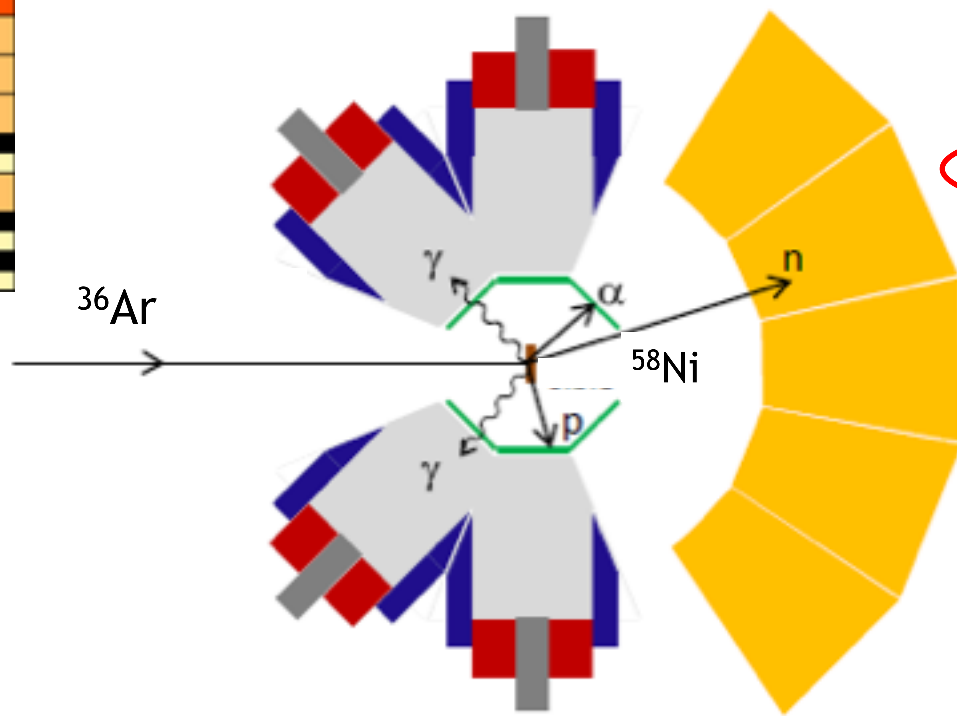
^{90}Tc + 3p1n

^{92}Rh + 2p

...

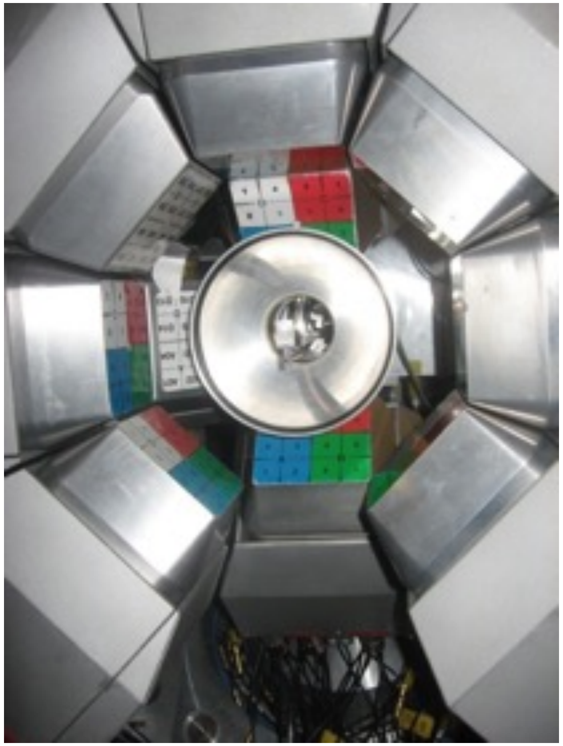
^{92}Pd + 2n





EXOGAM-NWall-DIAMANT:

The power of the coupling

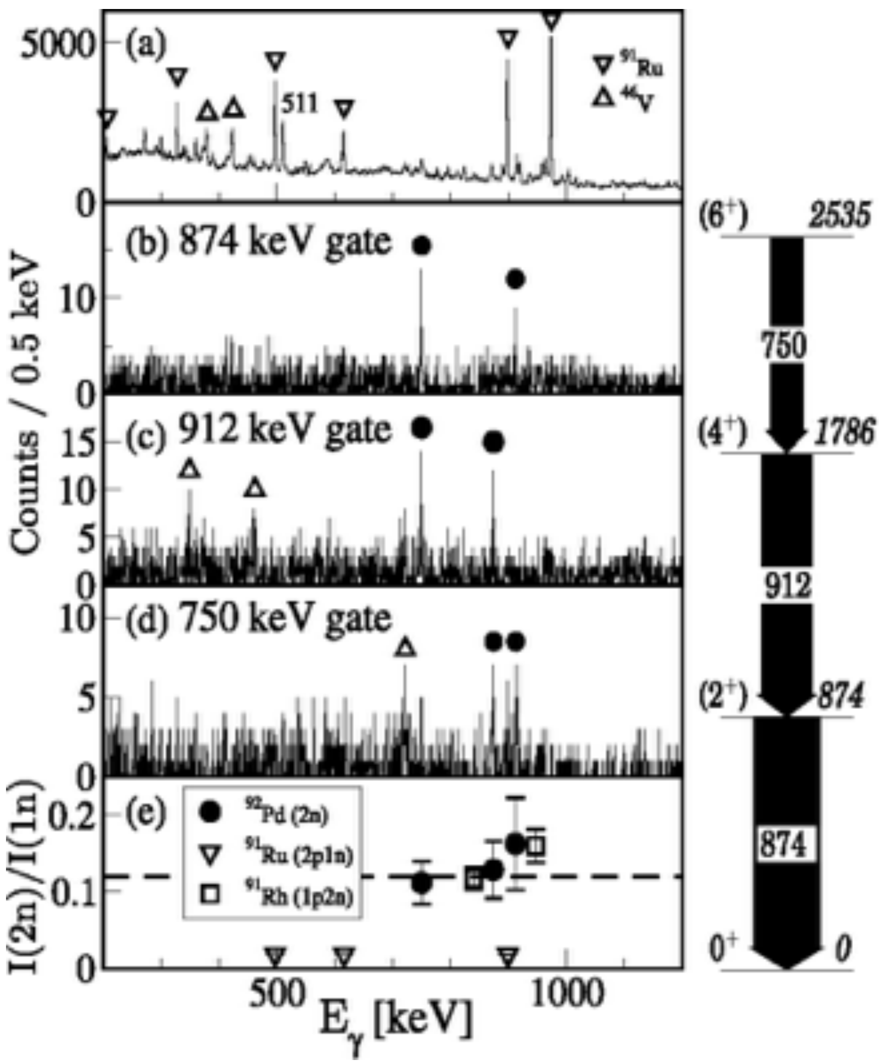


- EXOGAM: 11 Clovers with partial shield. $\varepsilon_p\omega$ ~ 10% for E_γ=1.3 MeV

- The Neutron Wall: 50 liquid scintillator detectors. ε_{1n} ~ 23%

- DIAMANT: 80 CsI(Tl) dets. $\varepsilon_{p\text{ or }\alpha}$ ~ 66%

EXOGAM:
First identification of γ-rays in ^{92}Pd



- Three γ-rays firmly identified
- In coincidence with 2n
- Not in coincidence with charged particles
- Mutually coincident
- All possible contaminants excluded

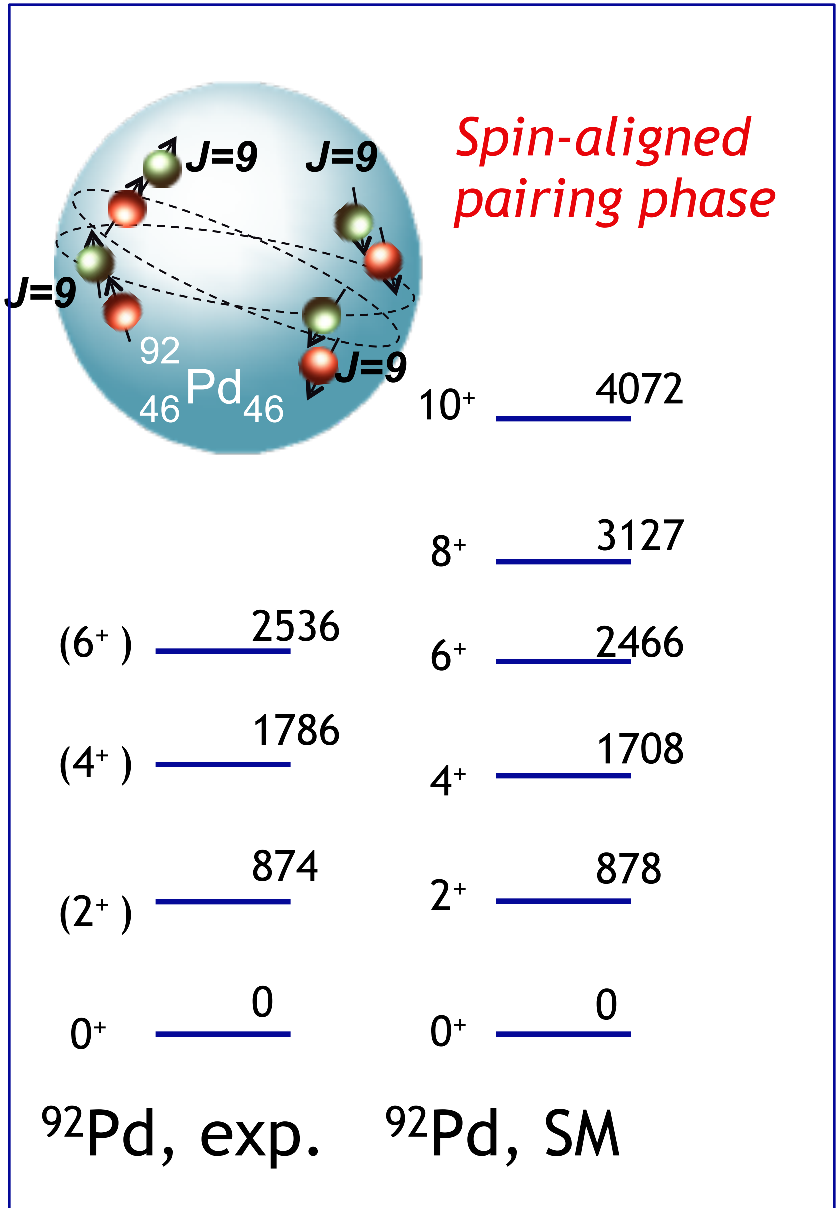
→ Unambiguously assigned to ^{92}Pd

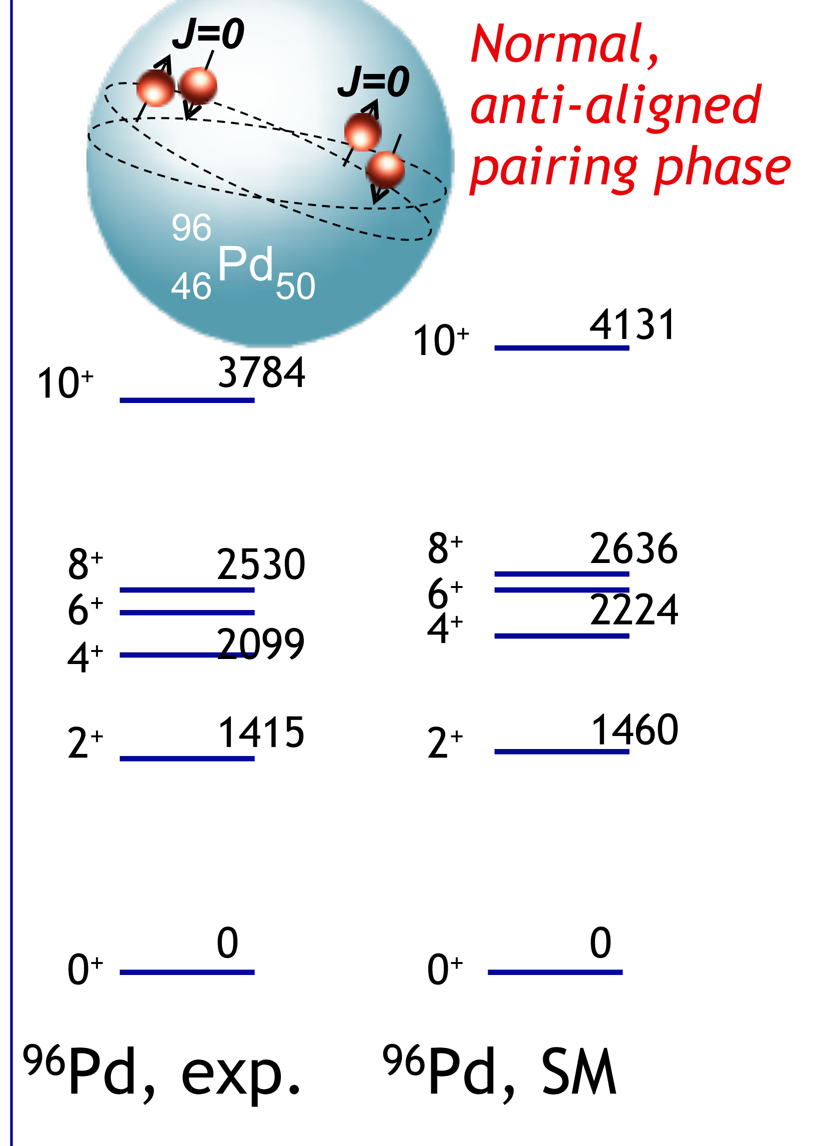
Production cross section ~ 0.5 μb

B Cederwall, F. Ghazi-Moradi, T Back, A Johnson, J. Blomqvist, E Clément, G. de France,
R Wadsworth et al,

Nature 469, 68-71 (2011)

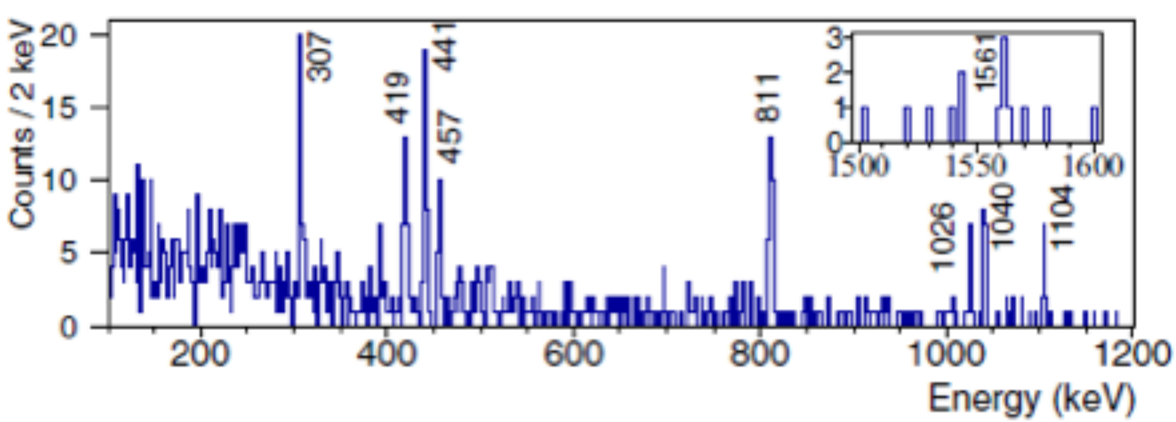
^{92}Pd: A new spin aligned np coupling scheme

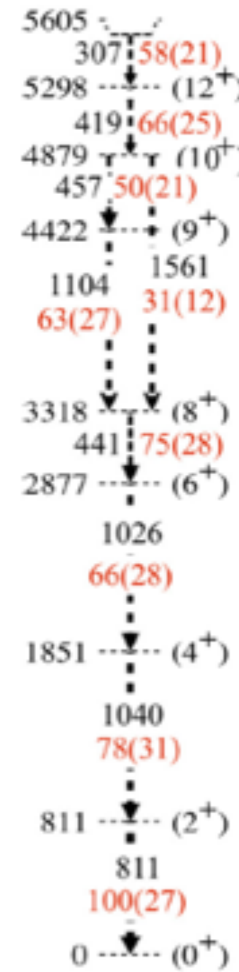




^{96}Cd RIKEN results

- RIKEN experiments
- Singles [50,1200] ns in EURICA (gate on identified ^{96}Cd implanted ions)
- Measured lifetime: decay from a single isomeric state $T_{1/2}=197^{+19}_{-17}$ ns
- Tentative level scheme

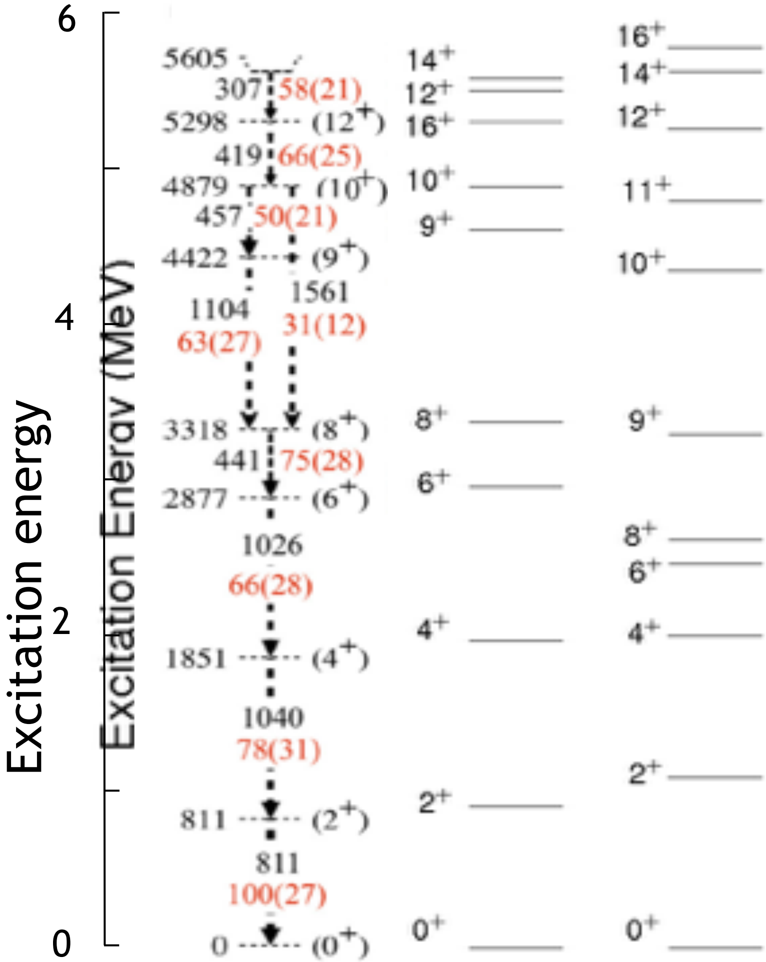




^{96}Cd, exp

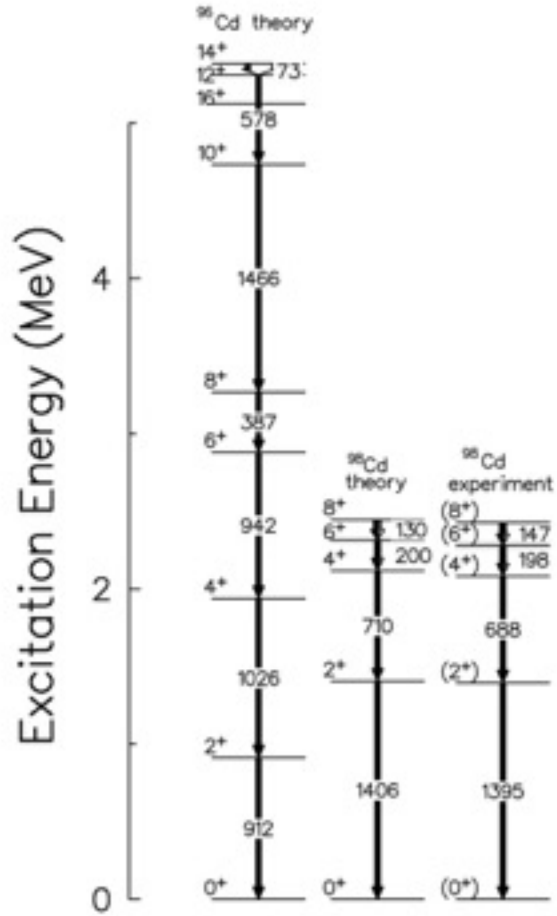
PJ Davies et al, PLB 767, 474 (2017)

^{96}Cd RIKEN results



^{96}Cd, exp

H. Grawe

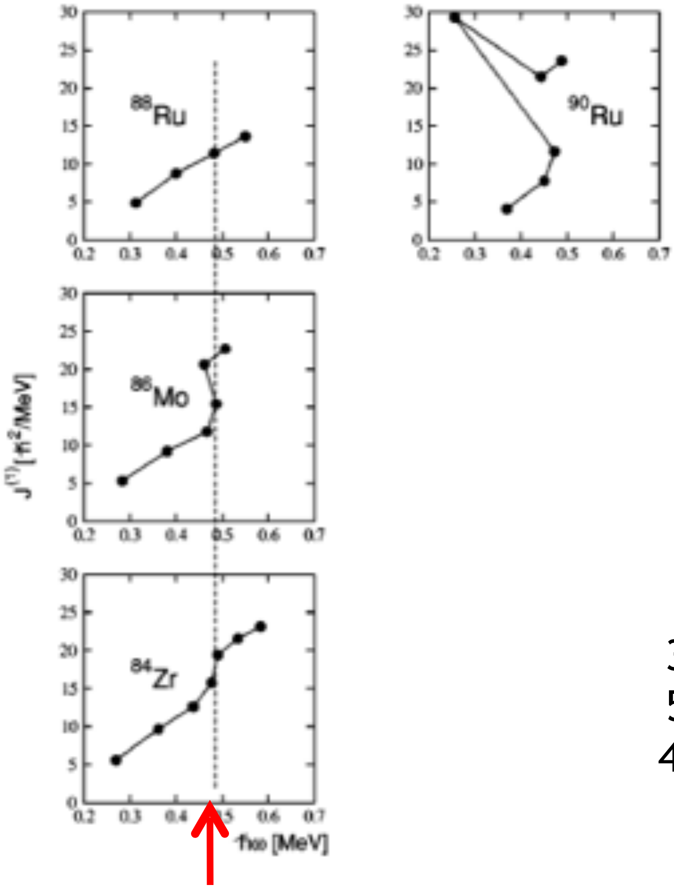


J. Blomqvist (2007)

PJ Davies et al, PLB 767, 474 (2017)

Rotational properties: delayed alignment

T=0 larger strength compared to T=1 for two nucleons in the same orbit/delayed alignment?

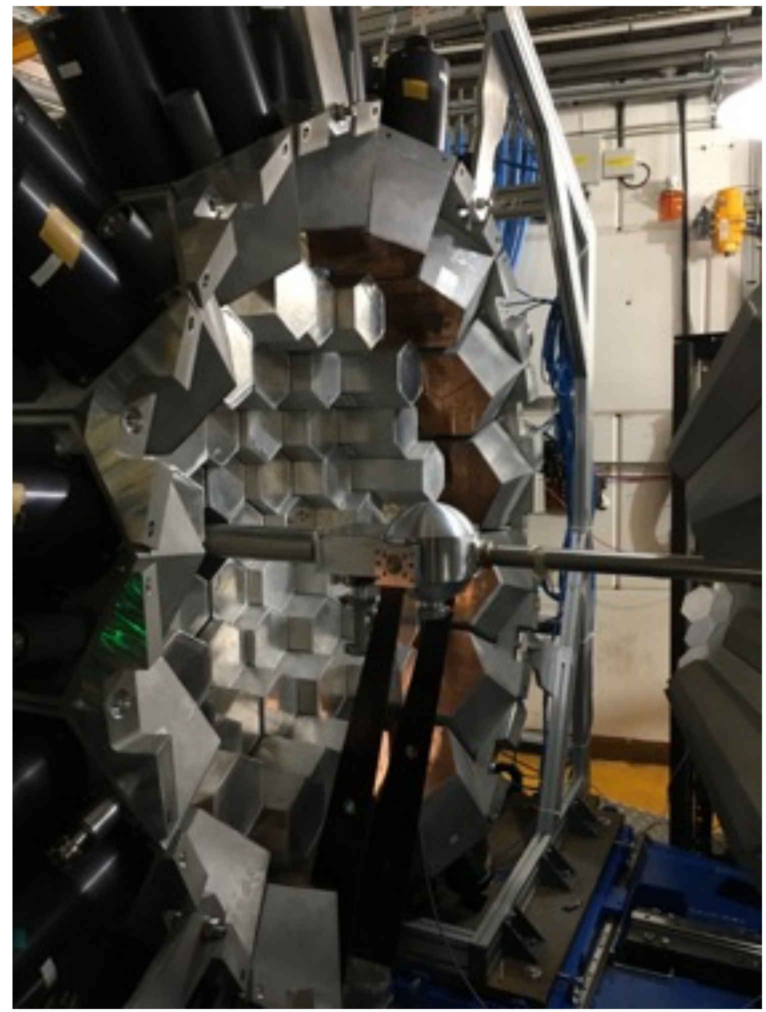


$g_{9/2}$ crossing freq from CSM

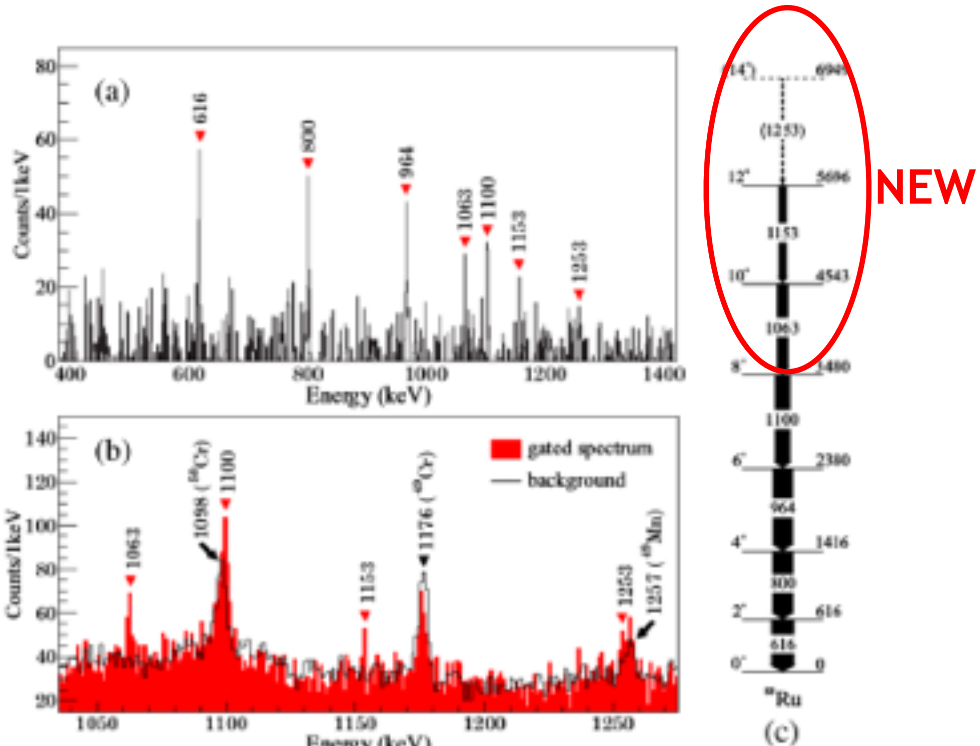
N. Marginean et al, PRC 63, 031303 (2001)

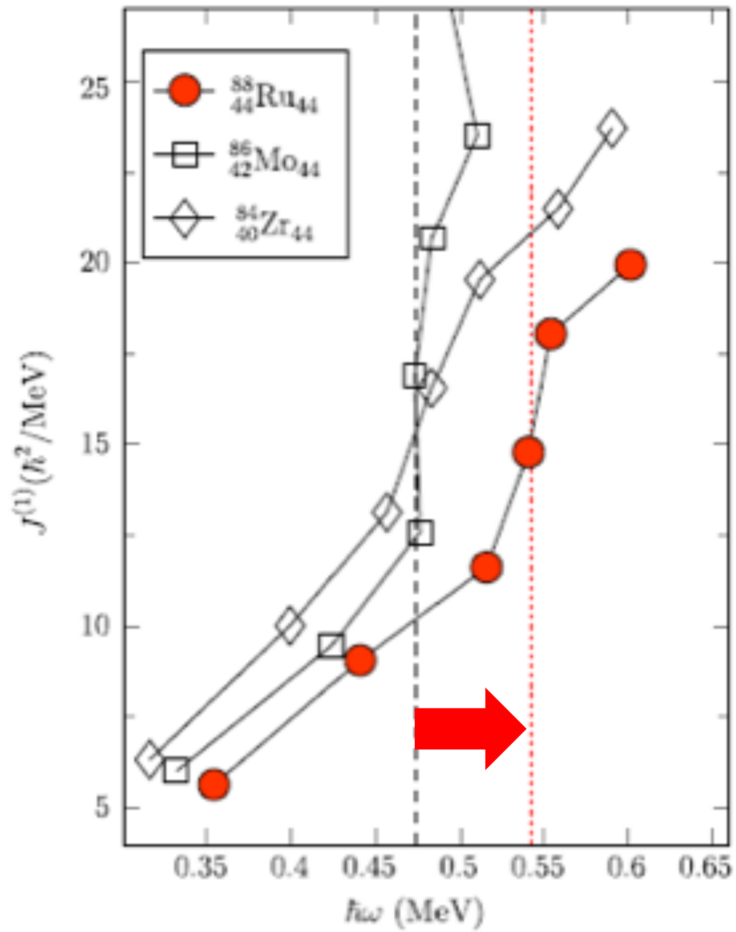
New experiment:
AGATA- NEDA-DIAMANT

35 AGATA Capsules
54 NEDA detectors
42 Nwall detectors
60 DIAMANT CsI



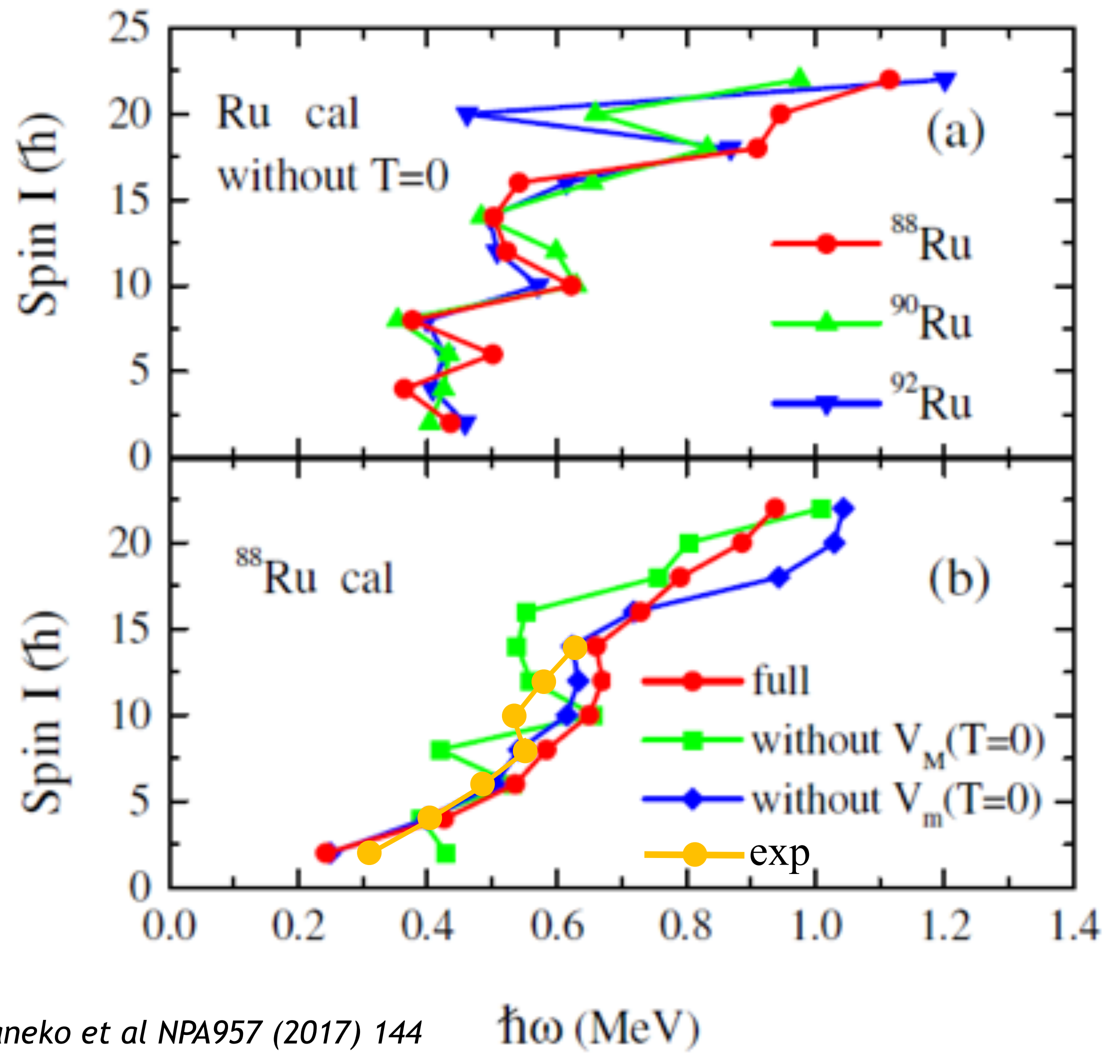
Rotational properties: delayed alignment





→ Alignment at a significant higher rotational frequency

Influence of QQ strength (V_M)



Kaneko et al NPA957 (2017) 144

- Removing the isoscalar monopole (V_m, T=0) in the T=0, np interaction has no effect
- Strong deviation appears when removing the multipole part (V_M, T=0)
- loose the smooth behaviour experimentally observed which became similar to that in ^{90,92}Ru
- Data follow calculations at low frequency
- Good overall fit when QQ strength increased by ~9%

Summary/perspectives

- **Importance of pairing up to the highest spins:** cranking+pairing, Coriolis
- **IBs are a very precise probe of the persistance of pairing** (FC distributions)

- **isovector and isoscalar pairing**
- **seniority violation** and **delayed alignment as signature of the role of T=0:**
  - **First identification of N=Z=46 ^{92}Pd**
  - Evidence for a **new spin-aligned pairing phase** due to the role of **T=0 isoscalar pairing** channel
  - Confirmed by **^{96}Cd** level scheme

  - Delayed alignment in **^{88}Ru** compatible with T=0 strength > T=1
  - Role of **QQ component** in the NN interaction

A more definite evidence for T=0 would be the measurement of **deuteron transfer** cross section between N=Z nuclei g.s. to g.s. (J=0+,T=1) and g.s. to first excited (J=1+,T=0) state