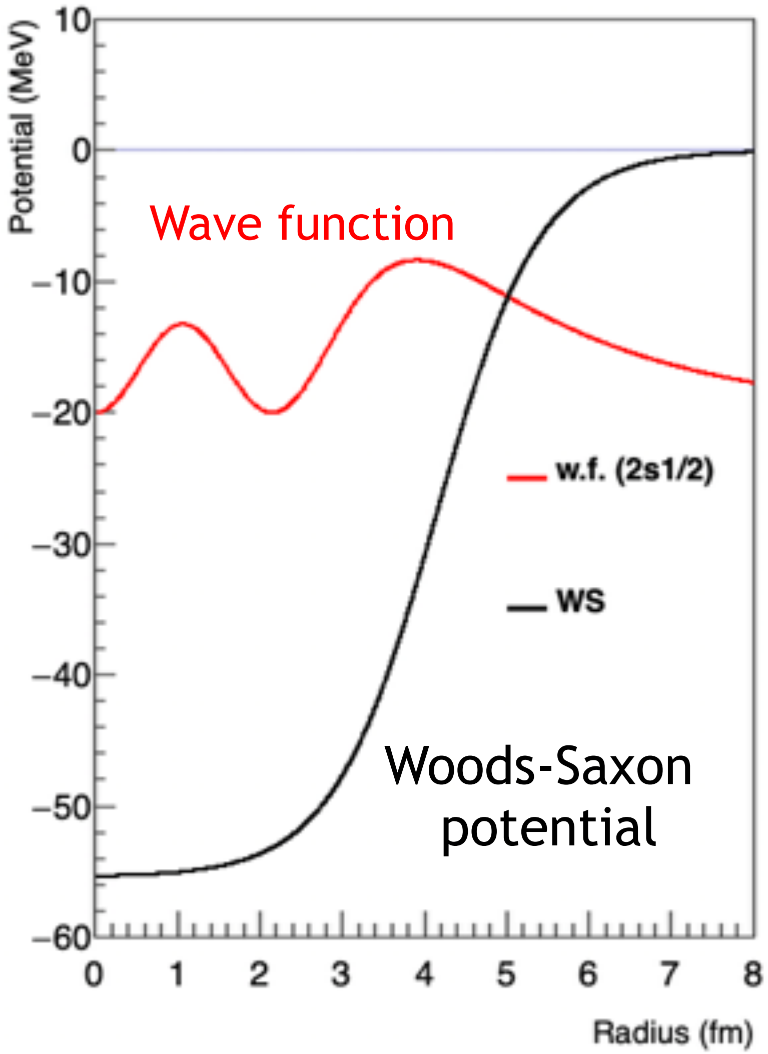
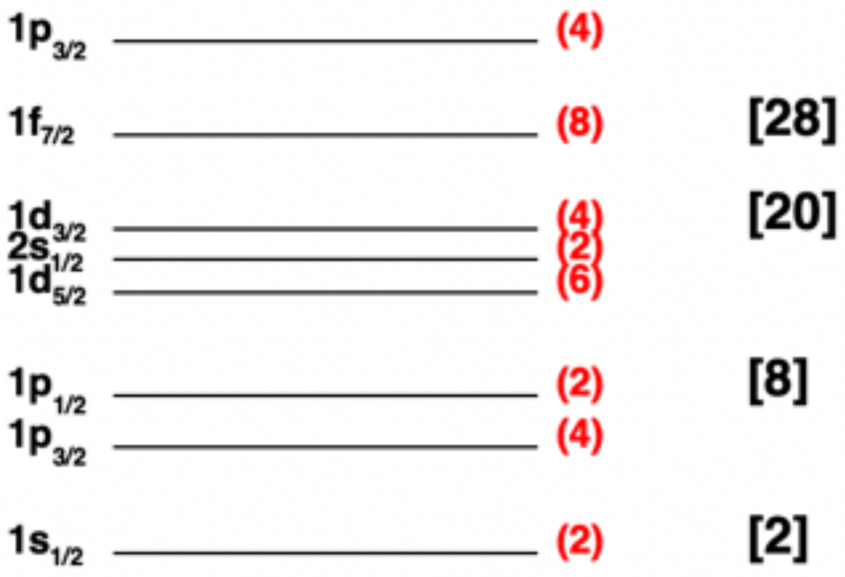
Breaking of **mirror symmetries** 2p **Clusters** in the **Ikeda conjecture** and Possible **astrophysical** implications

Nucleons in a Mean Field



Spin-orbit
+ Coulomb

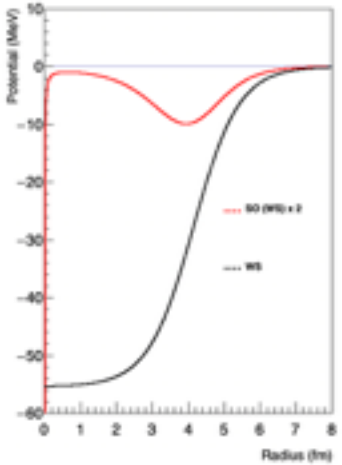
$$V_{so} = V_0 \times r_0^2(l.s)\frac{1}{r}\frac{d}{dr}V_{WS}(r)$$



Shell Model

Discrete states

Mirror symmetry



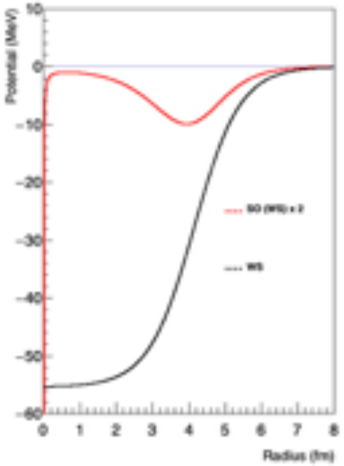
nn=pp

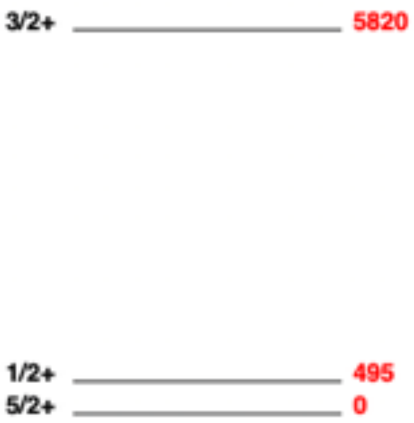
Charge symmetry

np =nn=pp

Charge independence

Mirror symmetry

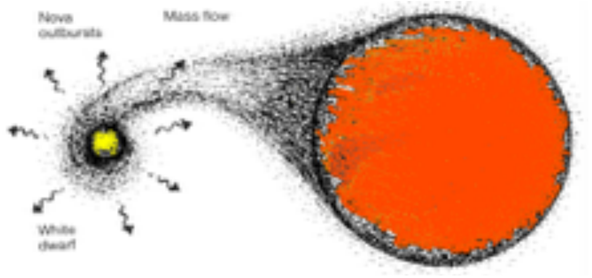


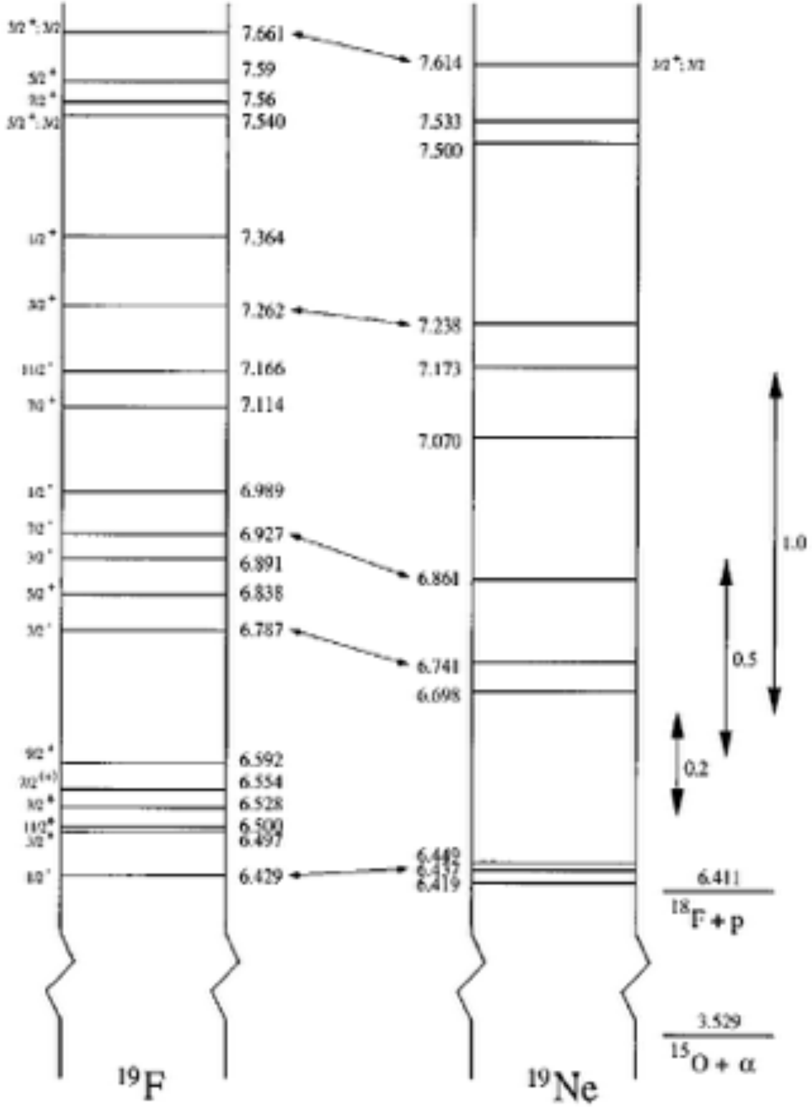


$^{17}_{8}O_9$ $^{17}_{9}F_8$

Generalized to A=17
Isobaric Analogues States

Astrophysical applications





From Utku et al. PRC 1998

Rate of $^{18}F(p,\alpha)^{15}O$

$^{18}F + p \rightarrow \ ^{19}Ne^*$ Breit-Wigner formula

$$\sigma(E) = \pi\lambda^2 \frac{2J+1}{(2J_1+1)(2J_2+1)} \frac{\Gamma_1\Gamma_2}{(E-E_r)^2 + \frac{\Gamma^2}{4}}$$

$$^{19}Ne^* \ | \ ^{19}F^*$$

1) Missing states?
2) J^π not known?
3) Widths not known? => but not equal

Spectroscopic factors should be equal

$$<^{18}F + p|^{19}Ne^* > \ = \ <^{18}F + n|^{19}F^* >$$

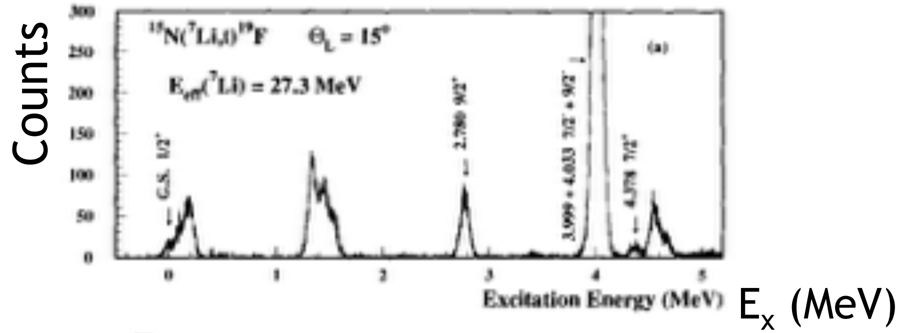
Spectroscopic factors

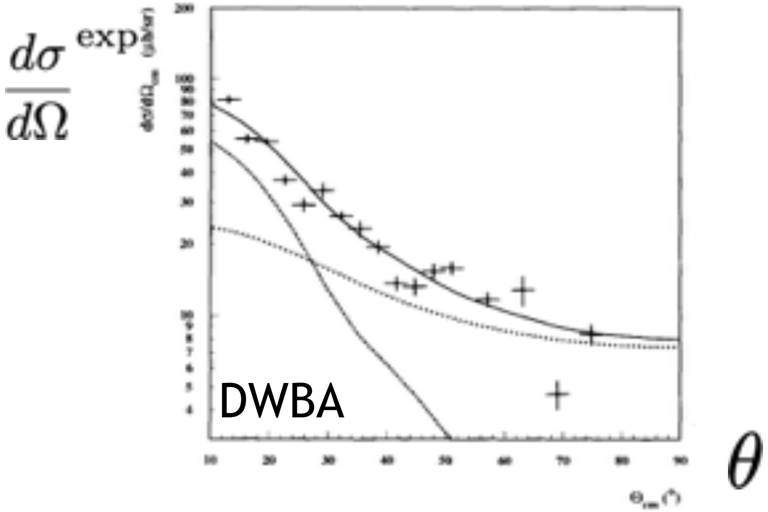
Instead of measuring

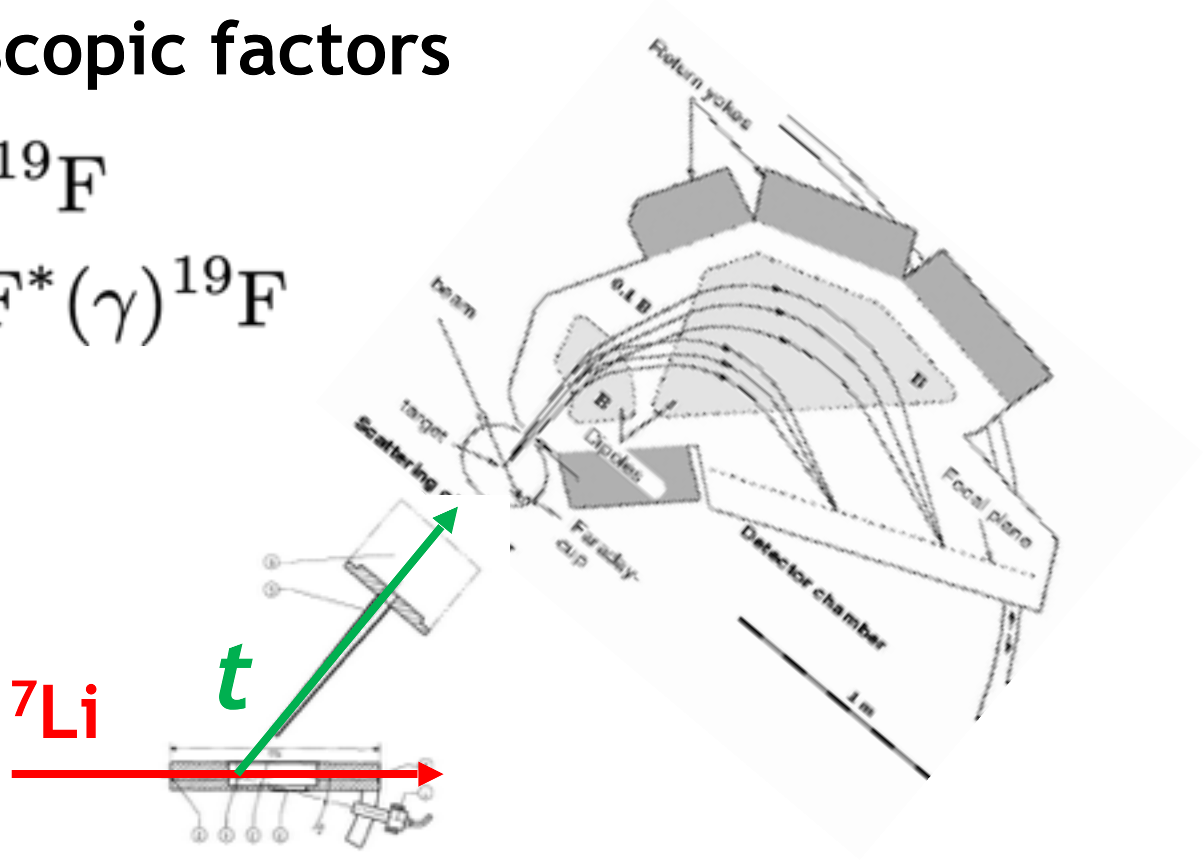
$$^{15}\mathrm{N}(\alpha,\gamma)^{19}\mathrm{F}$$

$$^{15}\mathrm{N}(^{7}\mathrm{Li},t)^{19}\mathrm{F}^{*}(\gamma)^{19}\mathrm{F}$$

We can measure



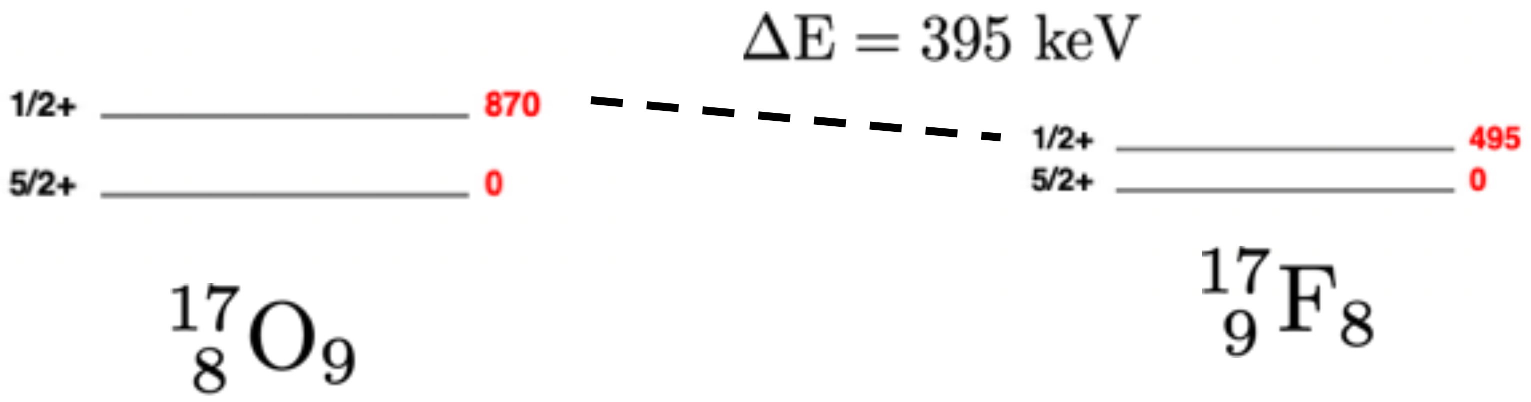




$$\frac{d\sigma}{d\Omega}^{\mathrm{exp}} = \mathrm{S}\,\frac{d\sigma}{d\Omega}^{\mathrm{theory}}$$

(see Didier Beaumel “Study of alpha clusters in KO experiments”)

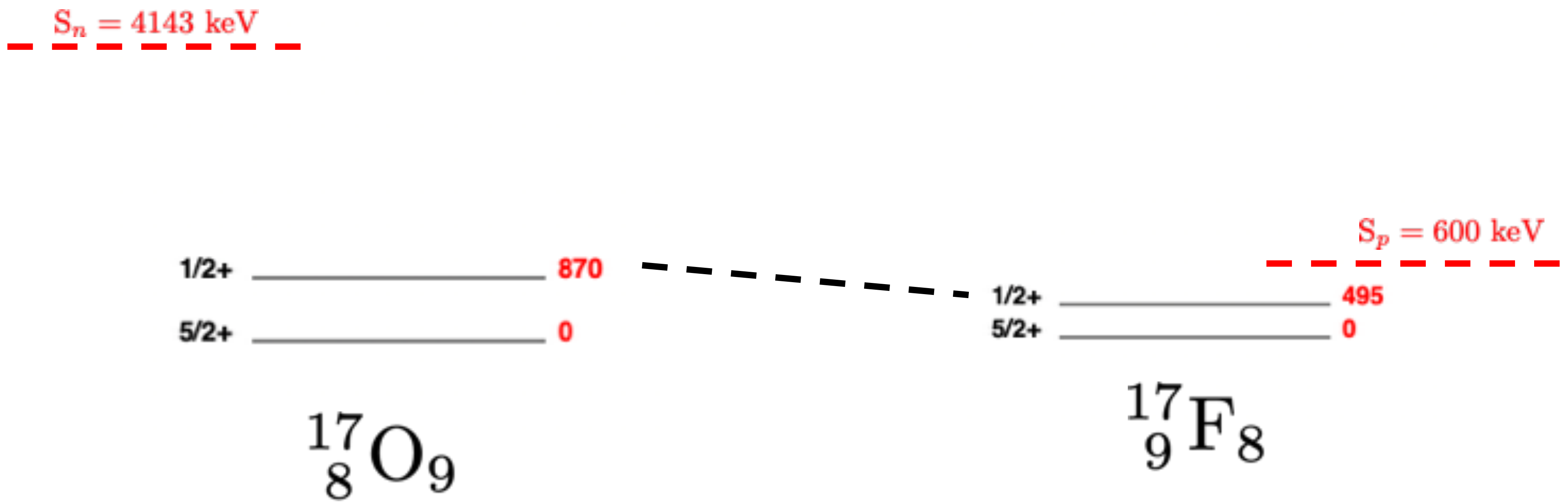
Asymmetry: Energies



$$E_x \approx \bar{E}_x \pm 1\text{MeV}$$

(One can expect al least the same difference with the predictions of the classical shell model)

Asymmetry: Energies



Not far from proton emission threshold!!

Asymmetry: Spectroscopic factors

$^{19}\mathrm{Ne}^* \mid {}^{19}\mathrm{F}^*$

Spectroscopic factors

TABLE II. Properties of some mirror levels in ^{19}F and ^{19}Ne corresponding to resonances in $^{15}\mathrm{N}(\alpha,\gamma)^{19}\mathrm{F}$ and $^{15}\mathrm{O}(\alpha,\gamma)^{19}\mathrm{Ne}$.

$E_x(^{19}\mathrm{F})$ (MeV)	$E_x(^{19}\mathrm{Ne})$ (MeV)	J^π	Γ_γ [a] (meV)	$B_\alpha(^{19}\mathrm{Ne})$ [b]	$\Gamma_\alpha(^{19}\mathrm{Ne})$ (meV)	$\theta_\alpha^2(^{19}\mathrm{Ne})$ [c] $(\times 10^{-2})$	$\theta_\alpha^2(^{19}\mathrm{F})$ [d] $(\times 10^{-2})$
4.378	4.379	$(7/2)^+$	> 60	0.044 ± 0.032	> 2.8	> 7.8	0.56
4.550	4.600	$(5/2)^+$	101 ± 55	0.25 ± 0.04	33 ± 18	3.2	4–8
4.556	4.549	$(3/2)^-$	38^{+23}_{-19}	0.07 ± 0.03	$2.9^{+1.7}_{-1.4}$	0.06	0.84
4.683	4.712	$(5/2)^-$	43 ± 8	0.82 ± 0.15	195 ± 36	0.67	1.5–2.4
5.107	5.092	$(5/2)^+$	> 22	0.90 ± 0.09	> 200	> 0.19	0.033–0.33

[a]Assuming $\Gamma_\gamma(^{19}\mathrm{Ne}) = \Gamma_\gamma(^{19}\mathrm{F}) = \Gamma(^{19}\mathrm{F})$ because $\Gamma_\gamma/\Gamma(^{19}\mathrm{F}) \approx 1$ (Ref. [9]).

More than a factor 10 difference!!

$$\theta^2 \approx \bar{\theta}^2 * (0.1 - 10)$$

(one can expect al least the same difference with the predictions of the shell model)

De Oliveira, F., Coc, A., Aguer, P., Bogaert, G., Kiener, J., Lefebvre, A., ... & Staudt, G. (1997). Comparison of low-energy resonances in 15 N (α, γ) 19 F and 15 O (α, γ) 19 Ne and related uncertainties. *Physical Review C, 55*(6), 3149.

De Oliveira, F., Coc, A., Aguer, P., Bogaert, G., Kiener, J., Lefebvre, A., ... & Staudt, G. (1997). Comparison of low-energy resonances in 15 N (α, γ) 19 F and 15 O (α, γ) 19 Ne and related uncertainties. *Physical Review C*, *55*(6), 3149.

Asymmetry: Spectroscopic factors

$^{19}\mathrm{Ne}^* \mid {}^{19}\mathrm{F}^*$

Spectroscopic factors

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4.[illegible]83	4.[illegible]12	$(5/2)^-$	43 ± 8	0.82 ± 0.15	195 ± 36	0.67	1.5–2.4
5.[illegible]07	5.[illegible]92	$(5/2)^+$	> 22	0.90 ± 0.09	> 200	> 0.19	0.033–0.33

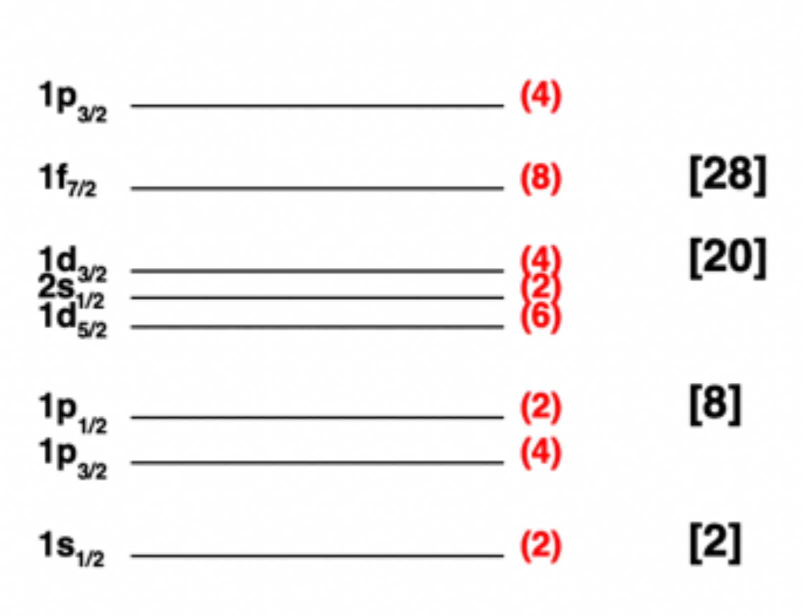
[a]Assuming $\Gamma_\gamma(^{19}\mathrm{Ne}) = \Gamma_\gamma(^{19}\mathrm{F}) = \Gamma(^{19}\mathrm{F})$ because $\Gamma_\gamma/\Gamma(^{19}\mathrm{F}) \approx 1$ (Ref. [9]).

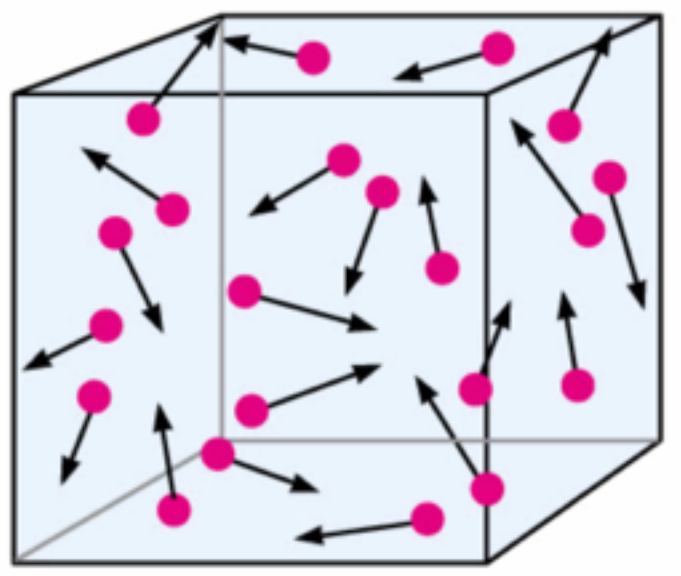
$$S_\alpha(^{19}\mathrm{F}) = 4.013\mathrm{MeV}$$

Not far from alpha emission threshold!!

$$S_\alpha(^{19}\mathrm{Ne}) = 3.528\mathrm{MeV}$$

But the world is also a continuum





Discrete states

Continuum

i.e. $^{15}\mathrm{F}^* \rightarrow \; ^{14}\mathrm{O} + p$

https://socratic.org/questions/how-does-gas-exerts-pressure-on-its-container

Effects of the continuum coupling: Broadening of the states

Quantum Mechanics exercise

Discrete state

$$H_0|\phi_i >= E_i|\phi_i >$$

Continuum

$$H_0|\alpha >= E|\alpha >$$

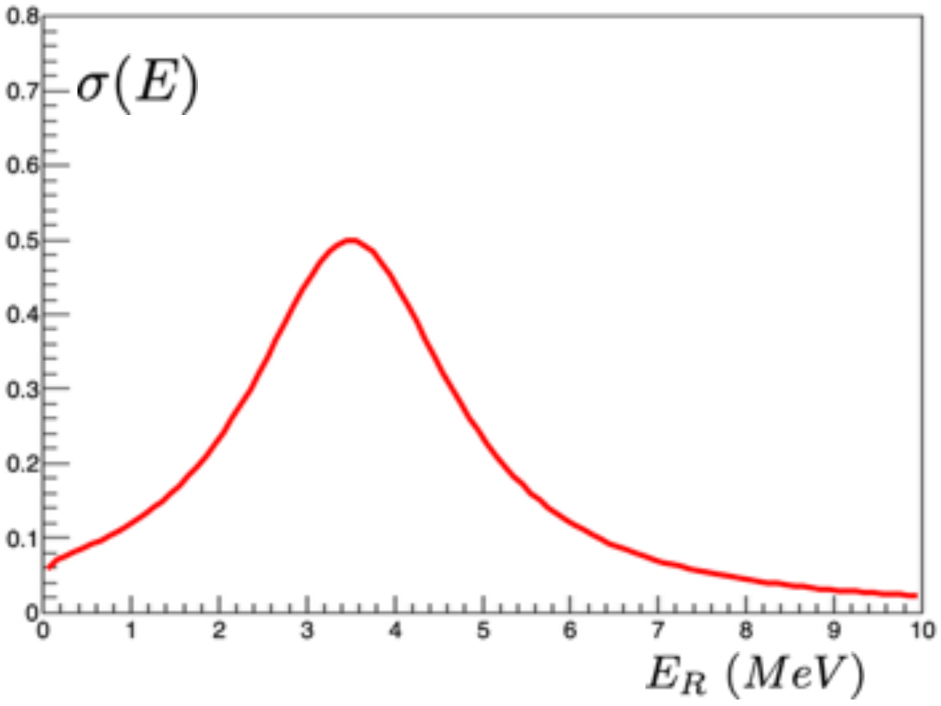
Breit-Wigner formula

$$\frac{dP(E)}{dE} \propto \frac{1}{(E - Ei - \delta E)^2 + \frac{\Gamma^2}{4}}$$

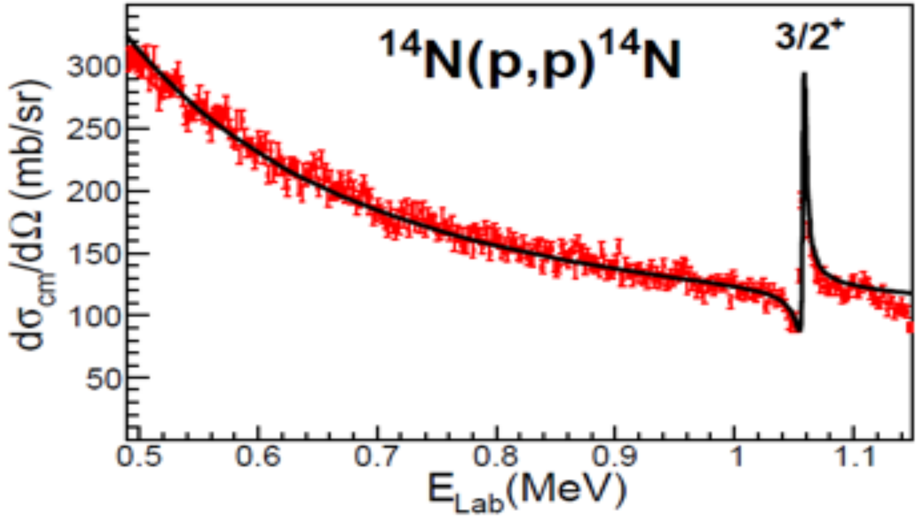
Shell-model energy

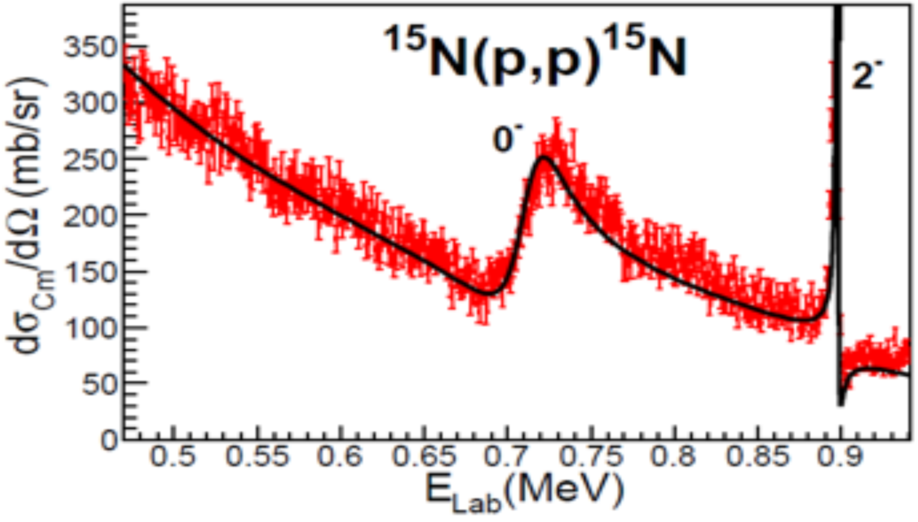
shift

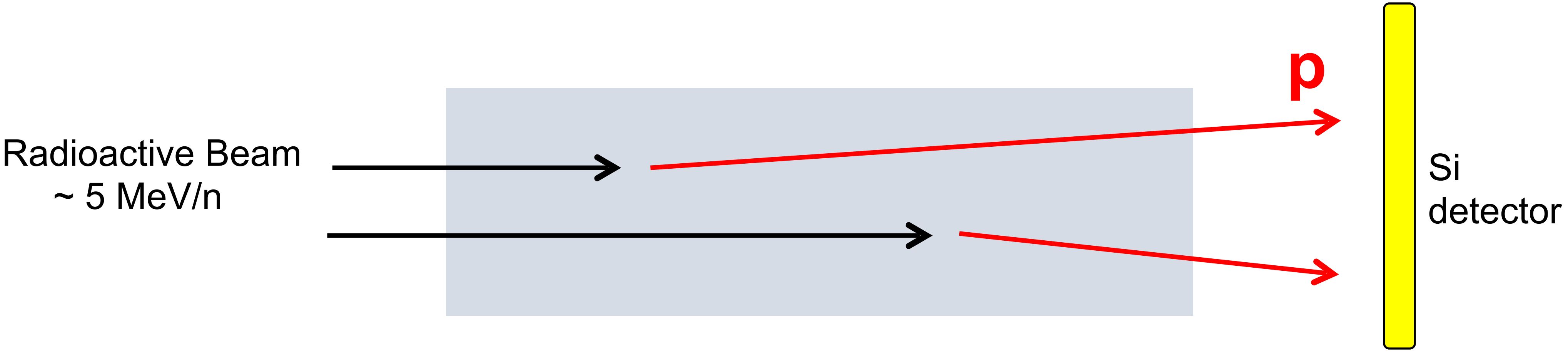
width



Resonant Elastic Scattering





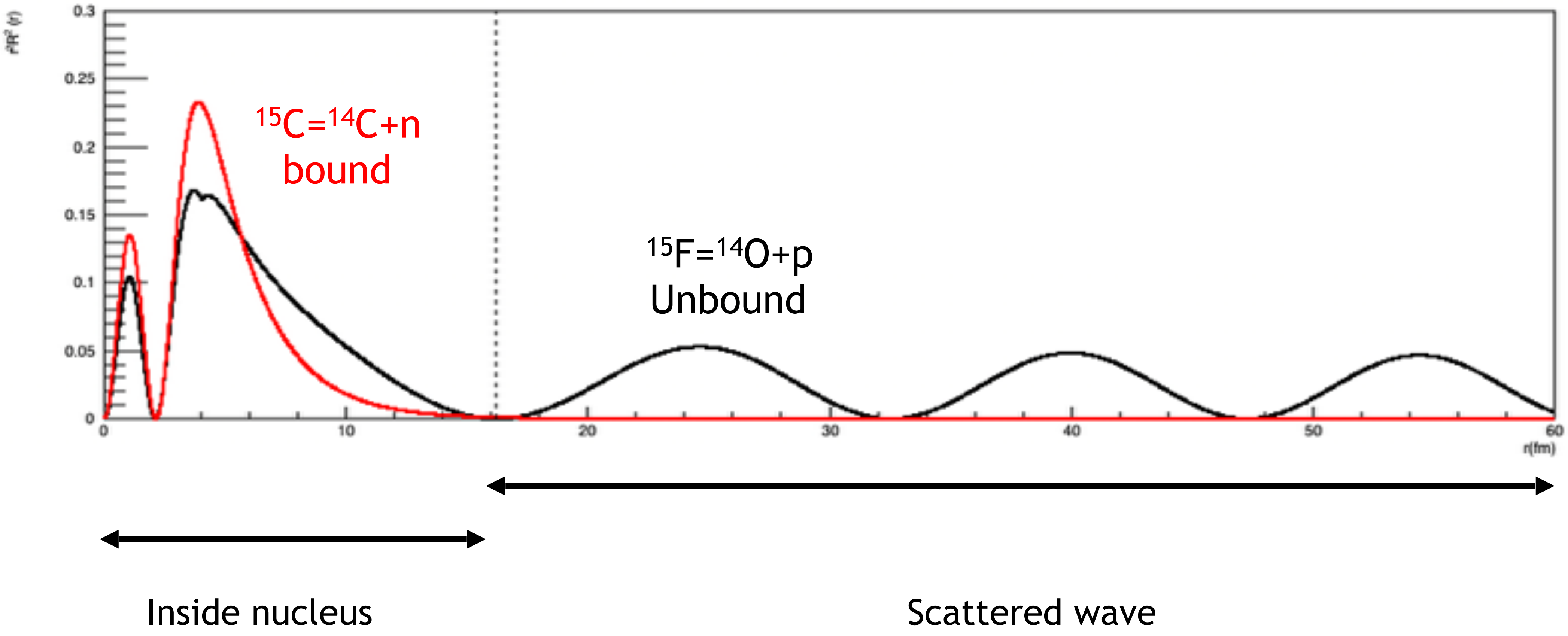


The Inverse Kinematics Thick Target scattering method

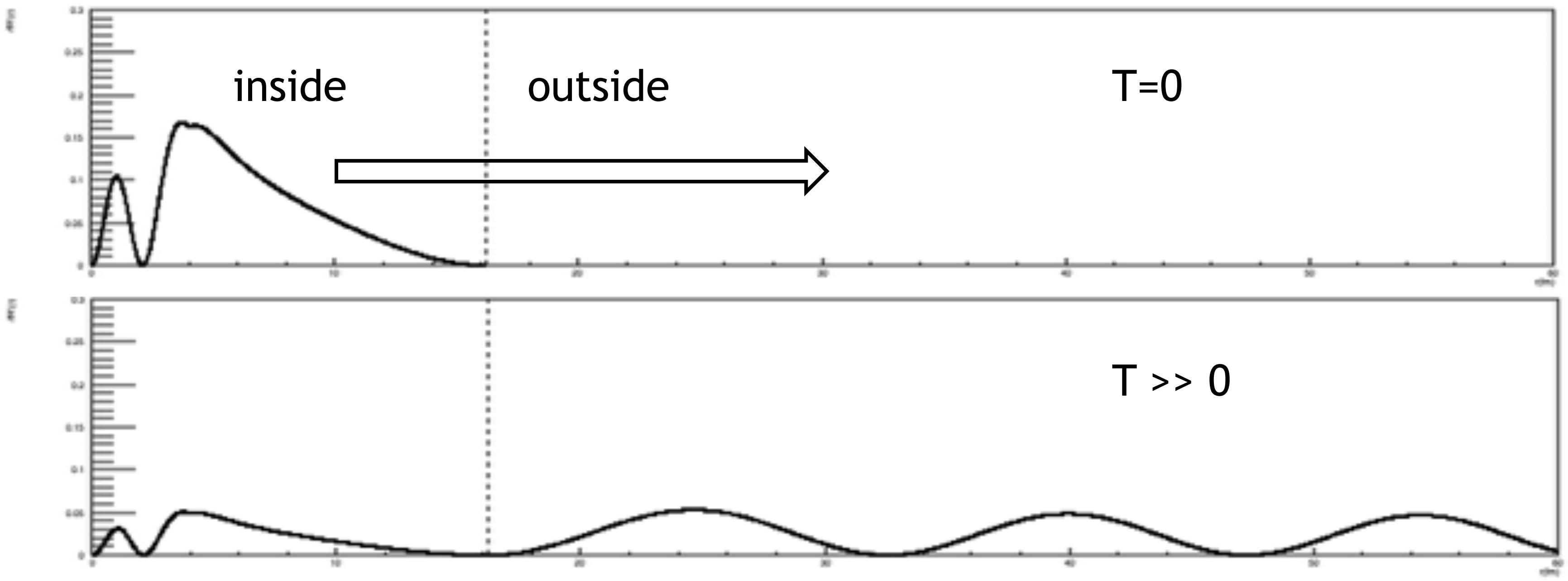
Resolution:
σ_{CM} = 3 keV

Coupling with continuum

Single-particle wave functions with the same quantum numbers (2s1/2)

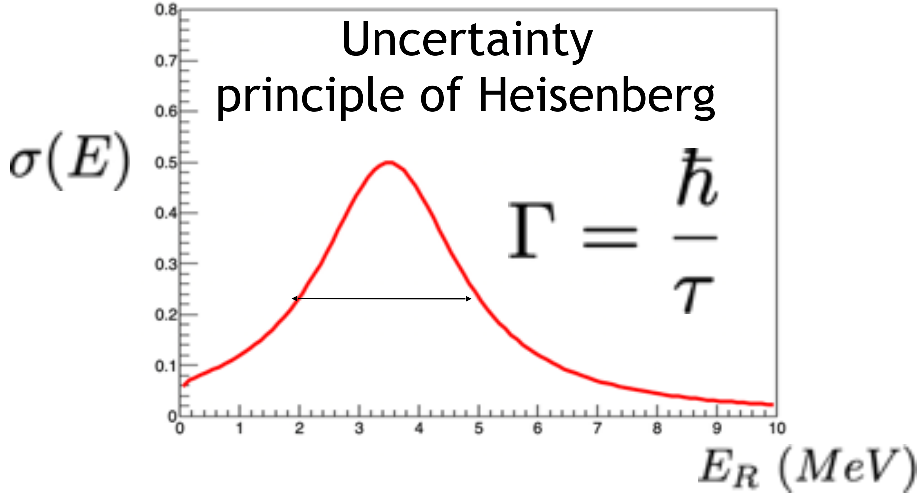


We should use the time-dependent Schrödinger equation



First order perturbation theory
(Fermi's golden rule)

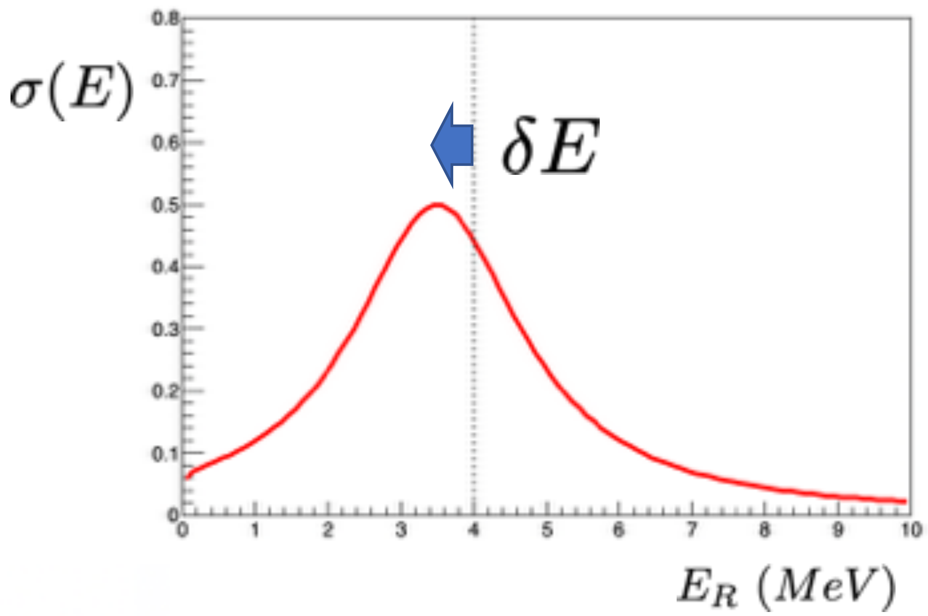
$$P_f(E) = \Gamma t$$



This is a technique to measure lifetimes of the order of 10^{-21} second!!

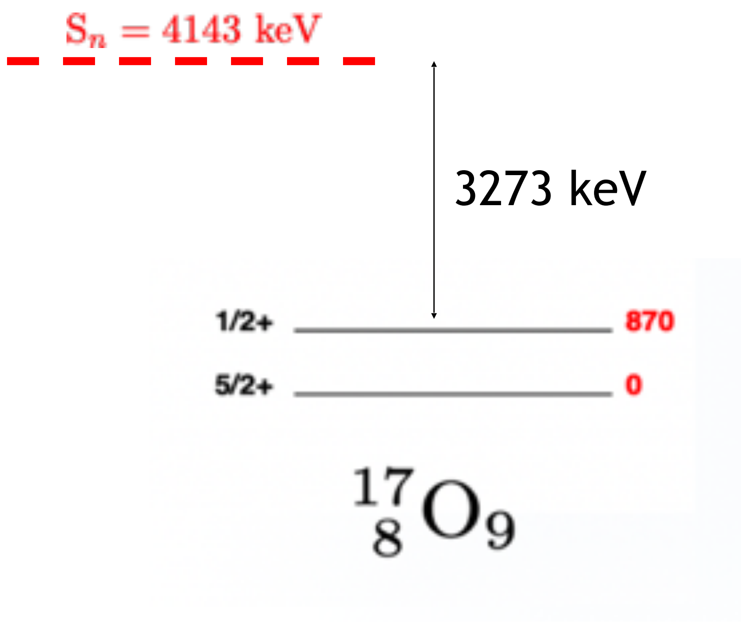
Effects of the continuum coupling: Thomas Ehrmann shift

$$\frac{dP(E)}{dE} \propto \frac{1}{(E - Ei - \delta E)^2 + \frac{\Gamma^2}{4}}$$



$$\delta E^{exp} = 395\ \text{keV}$$

Effects of the continuum coupling: Thomas Ehrmann shift



1) Adjust a Woods-Saxon well to fit the binding energy
   Suppose: $^{17}O = {}^{16}O + 1\ n(2s1/2)$

2) Use the same potential for the mirror nucleus ^{17}F
   Suppose: $^{17}O = {}^{16}O + 1\ p(2s1/2)$

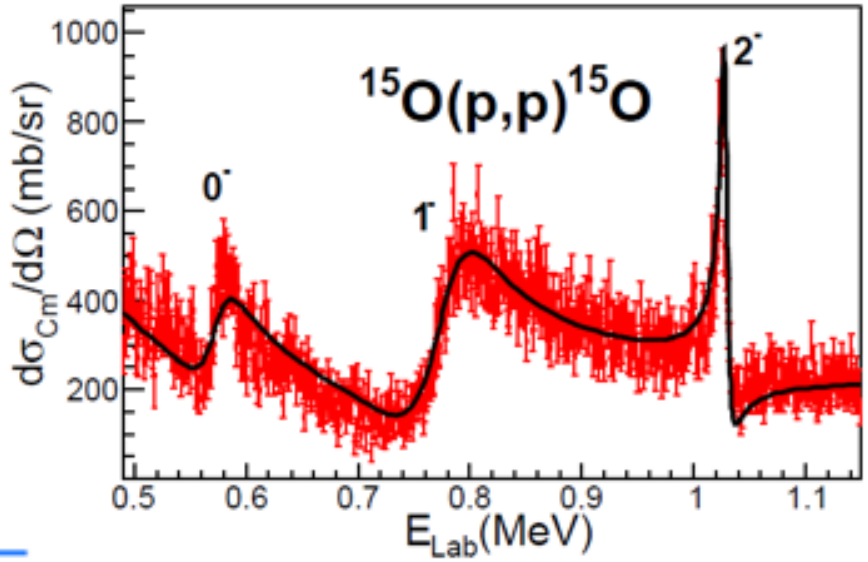
$$\delta E^{theo} = 386 \text{ keV}$$
$$\delta E^{exp} = 395 \text{ keV}$$

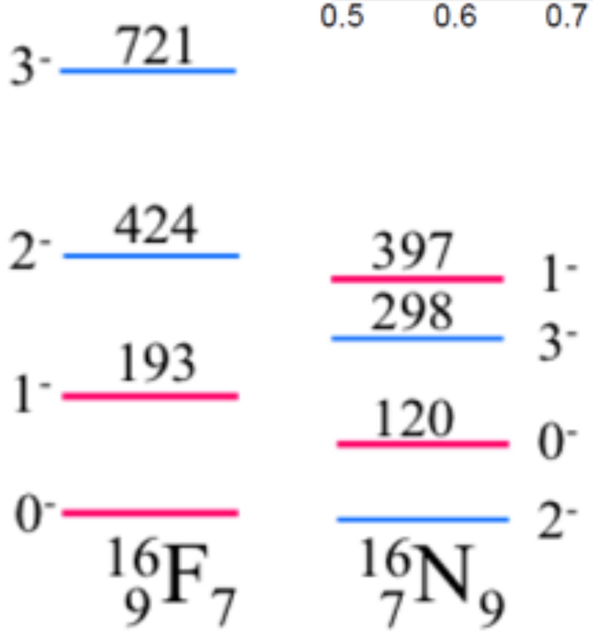
Only 9 keV difference!

Thomas Ehrmann shift is a function of the structure of the state

Suppose: $^{17}O = {}^{16}O + 1\ n(1d5/2)$ $\delta E^{theo} = 70 \text{ keV}$

“A core + two nucleons”

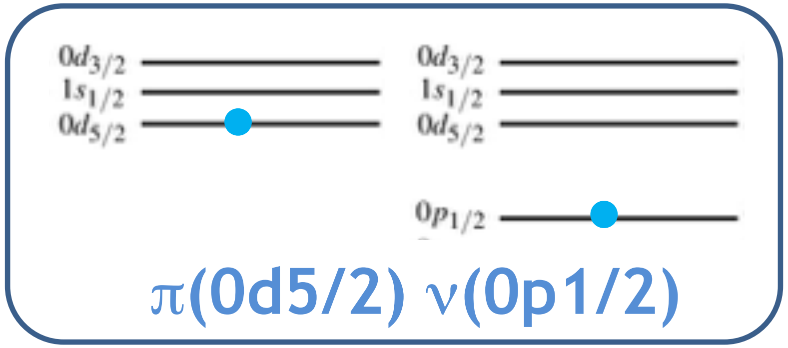


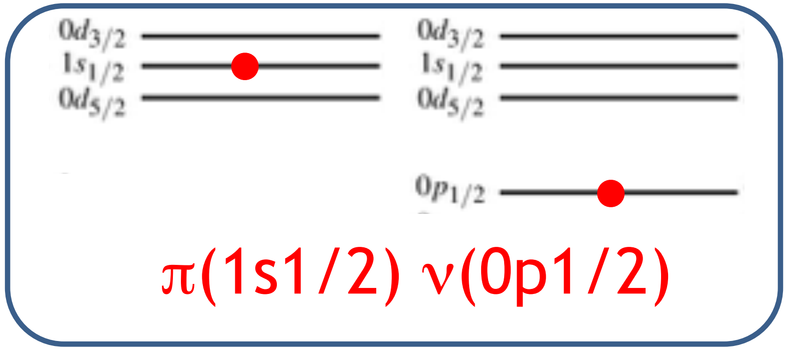


Both case well described as

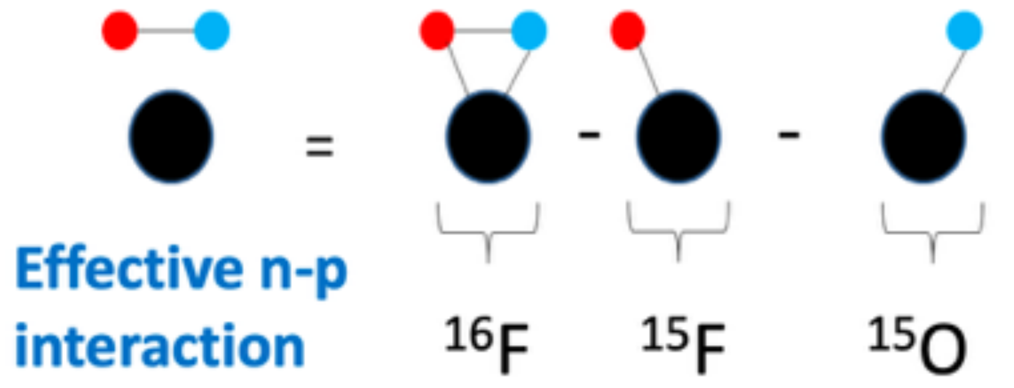
Core + n + p

Ex: $^{16}F = {}^{14}O + n + p$





Effective *n-p force*

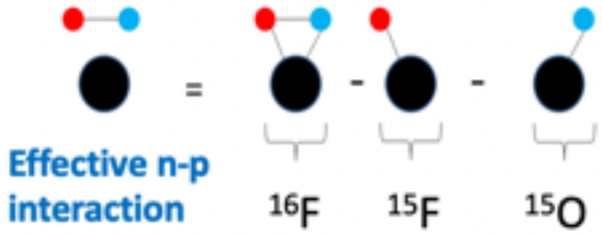


State (J)	^{16}N	^{16}F
0^-	-1.151	-0.775
1^-	-0.874	-0.577
2^-	-2.011	-1.829
3^-	-1.713	-1.523

(MeV)

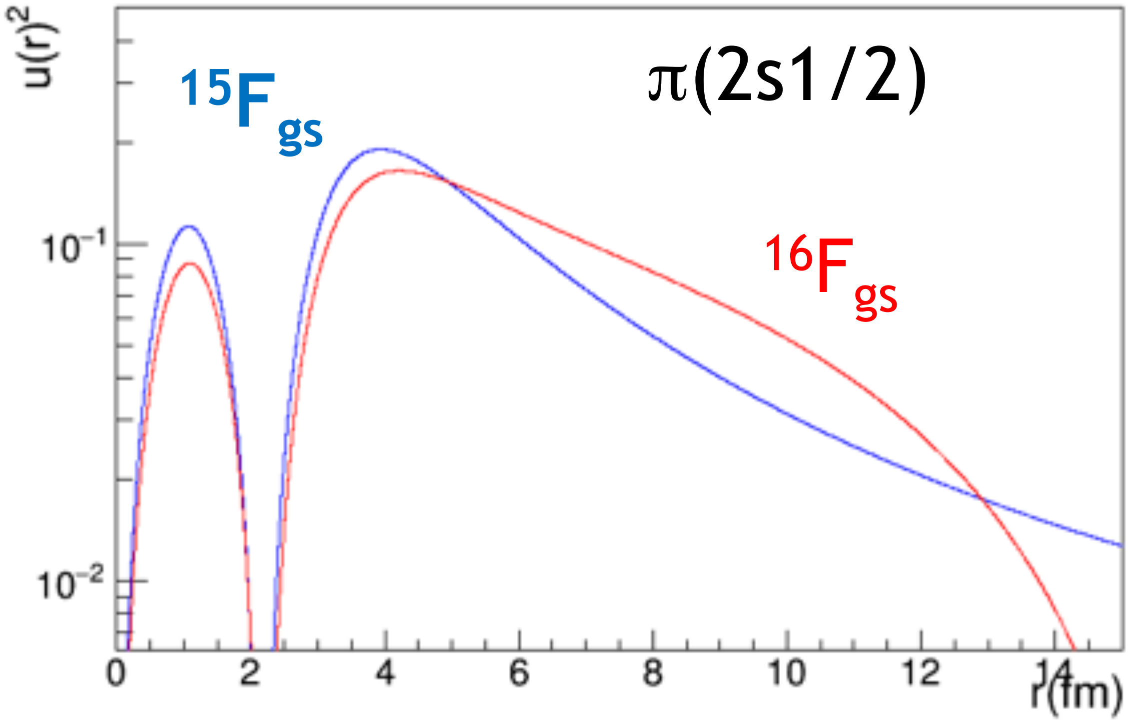
Up to 40% difference
Seems weaker in ^{16}F

Is it possible to understand this difference?



Coulomb energy $^{15}F \neq {}^{16}F$

$$E_c(J) = \frac{Z_{\text{core}}}{4\pi\epsilon_0}\int_0^\infty \frac{\rho(r)u_p(r,J)^2}{r}dr.$$



(Q_p=1.3 MeV / Q_p=0.535 MeV)

It reduce the difference in *n-p* force from

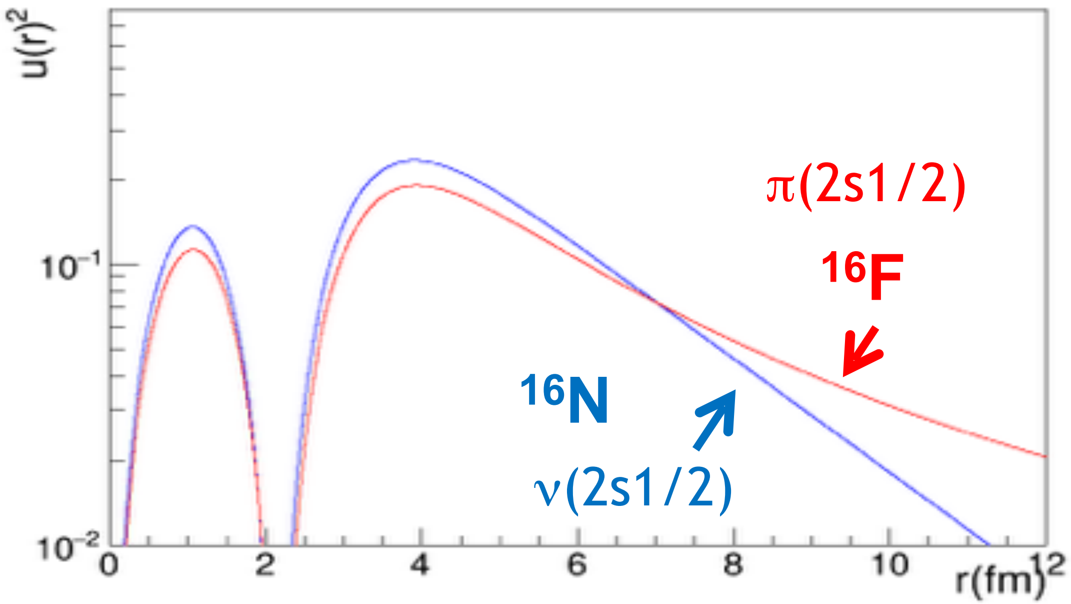
40 % to 20 %

But, still a breaking of the nuclear force symmetry?

Predicting Effective *n-p* interaction

Suppose *n-p* interaction $\mathbf{a} \times \delta(r_p - r_n)$

Effective *n-p* interaction $\mathbf{V}_{n\text{-}p} = \frac{a_J}{4\pi}\int_0^\infty \frac{1}{r^2}[u_p(r,J)u_n(r,J)]^2 dr$



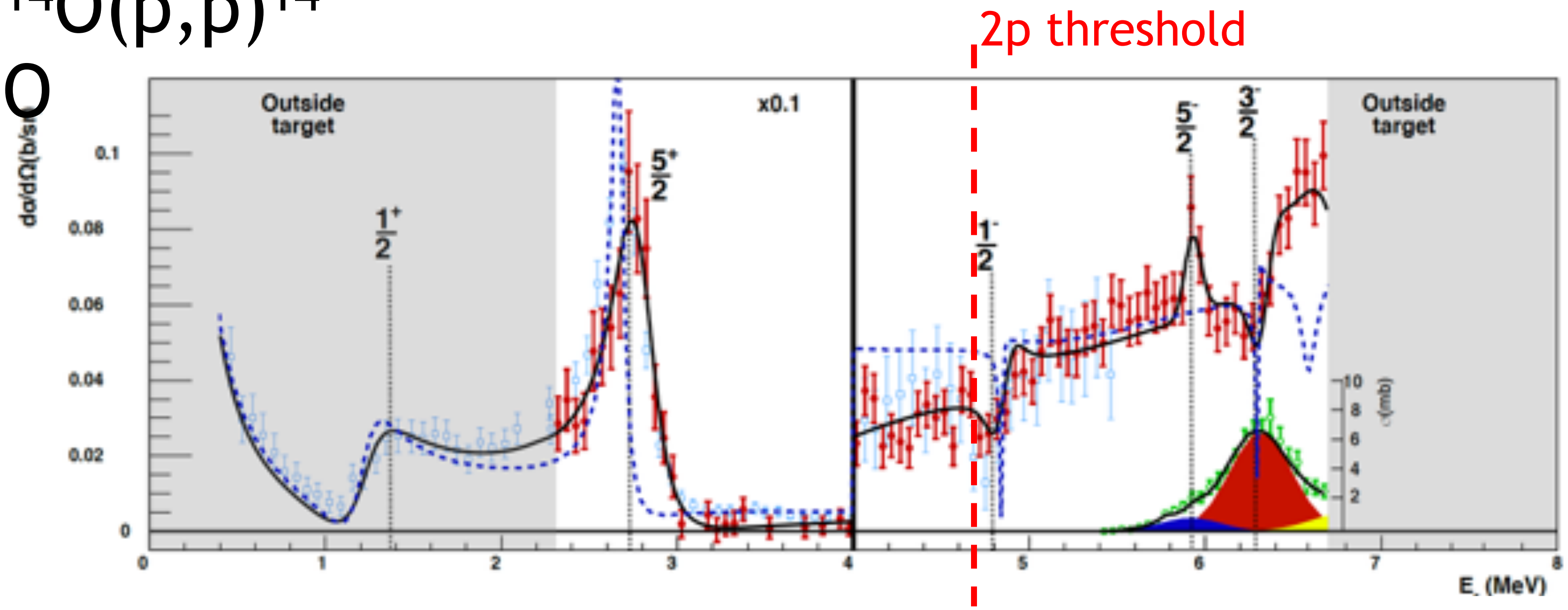
State (J)	$^{16}N_{cor}$	^{16}F
0^-	-0.775	-0.775
1^-	-0.581	-0.577
2^-	-1.842	-1.829
3^-	-1.574	-1.523

Almost no difference now!

Conclusion: Coupling with continuum induces a spreading of the wave functions that is fully responsible of the observed difference energies between ^{16}N and ^{16}F.

Effective *p-p* interaction

$^{14}O(p,p)^{14}O$



$^{15}F = {}^{13}N + p + p$

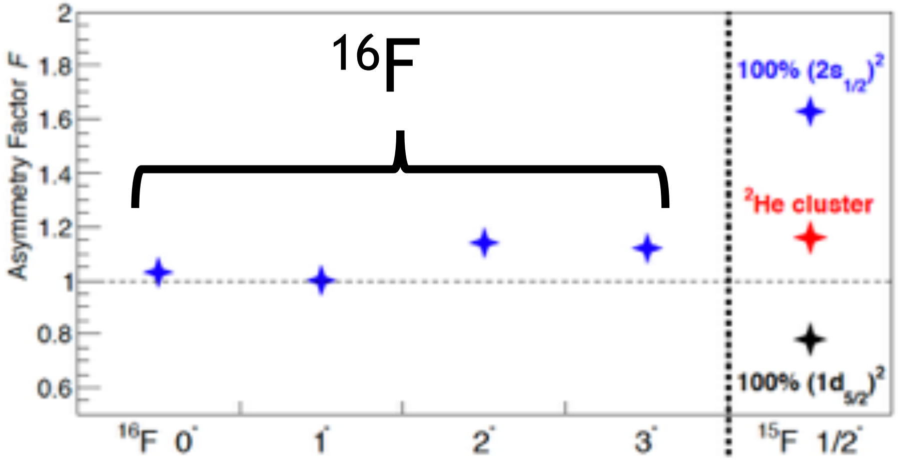
3 narrow resonances observed

V. Alcindor, A. Mercennes et al. Submitted (2021)

The Asymmetry Factor

$$F = \frac{\delta V^{\exp}(^{16}\mathrm{F})}{\delta V^{\mathrm{th}}(^{16}\mathrm{F})} \times \frac{\delta V^{\mathrm{th}}(^{16}\mathrm{N})}{\delta V^{\exp}(^{16}\mathrm{N})}$$

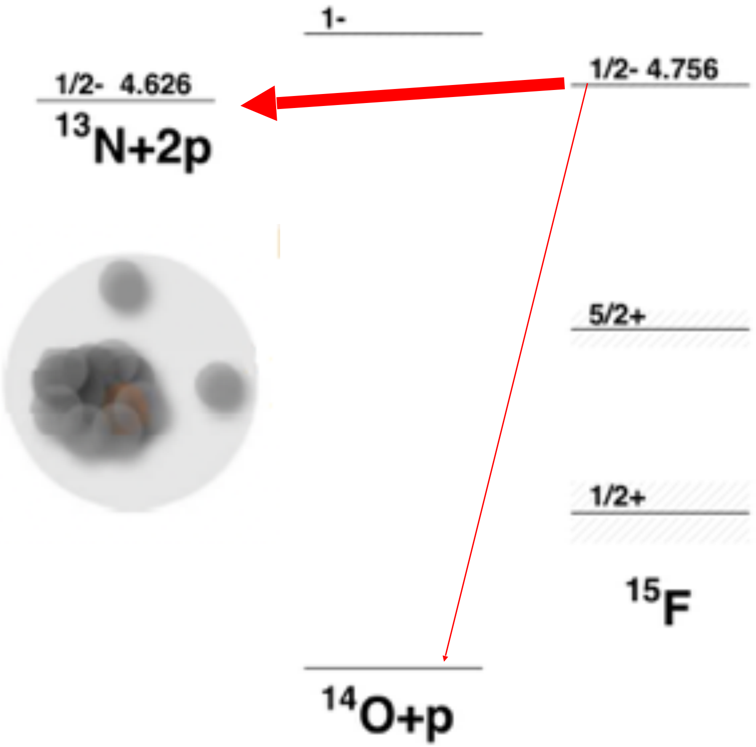
V. Girard-Alcindor, at al (2021). *EPJA*, *57*(3) 1-7



It looks like it is better described with a “^{2}He” cluster

Clustering effect ?

Effects of the continuum coupling: Clustering and correlations

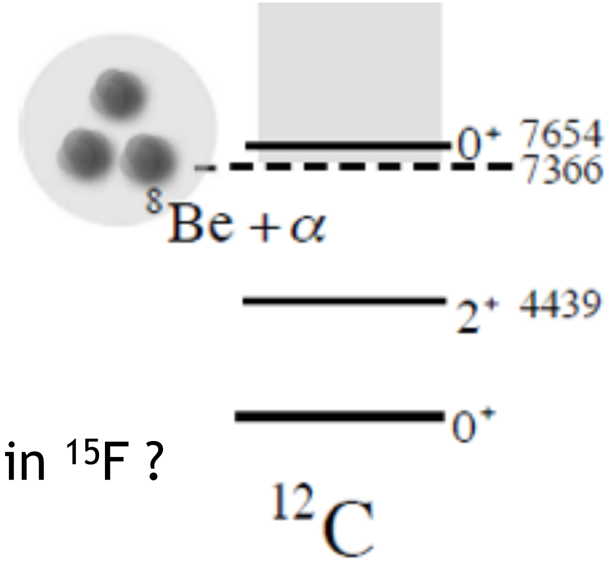


Confirmed by Gamow Shell Model
Structure of the 1/2- state is mainly ^{13}N+2p

$$< \Psi|0p_{1/2}[1]s_{1/2}[2] >^2 = 0.97$$

with 7% ^{2}He(0+)

De Grancey, F., Mercenne, A., et al. PLB, 758, 26-31. (2016)



Cousin of the « Hoyle » state in ^{15}F ?

Effects of the continuum coupling: Clustering and correlations

Ikeda conjecture = The cluster structures appears at each decay threshold

K. Ikeda et al., Prog. Theor. Phys. Suppl., Extra Number, 464(1968).

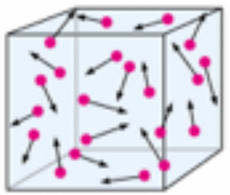
Generalized : "The clustering is a generic near-threshold phenomenon in open quantum system"

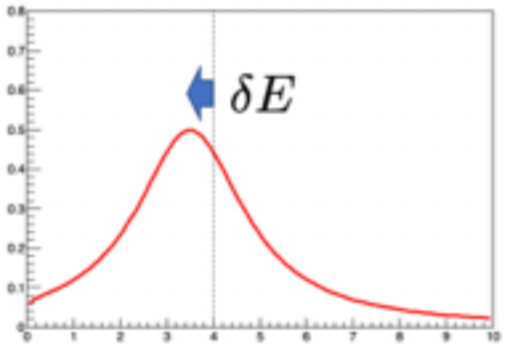
Increase of correlation
Change spectroscopic factor

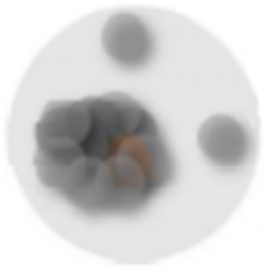
J. Okolowicz , M. Ploszajczak and W. Nazarewicz
Prog. Theor. Phys. Supplement 196 (2012) 230.
J.-P. Ebran, E. Khan, T. Niksic & D. Vretenar, Nature 487, 341, (2012)
M. Freer, Nature 487, 309 (2012)

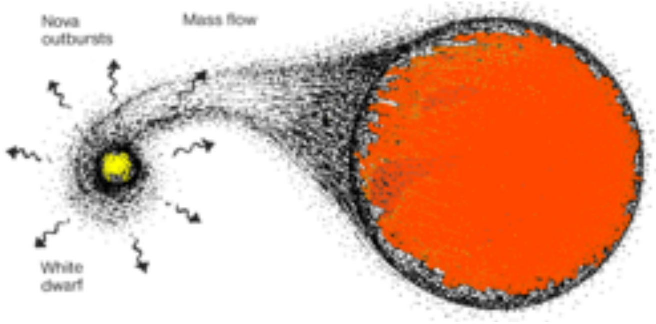
See:
Jose Pablo LINARES FERNANDEZ - Continuum coupling correction in **Gamow Shell Model**
Jean-Paul Ebran - Nuclear energy density functionals

Breaking of **mirror symmetries**
2p **Clusters** in the **Ikeda conjecture** and
Possible **astrophysical** implications









The end

Thank you