Universality in many-body systems

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On the point

What are we talking about:

+ Structure of “many”-particle non relativistic systems (e.g., nucleons).
+ Build a interaction that describe such systems.
+ Make ab inito calculations (numerical problem)

The goals

+ Understanding of the **mechanisms** of nuclear proprieties;
+ Support to **experiments**;
+ **Precision description** of nuclear observables;

What is universality? – unitarity (2-body only)

*nonrelativistic & quantum

Size of the **two-body*** system

>>

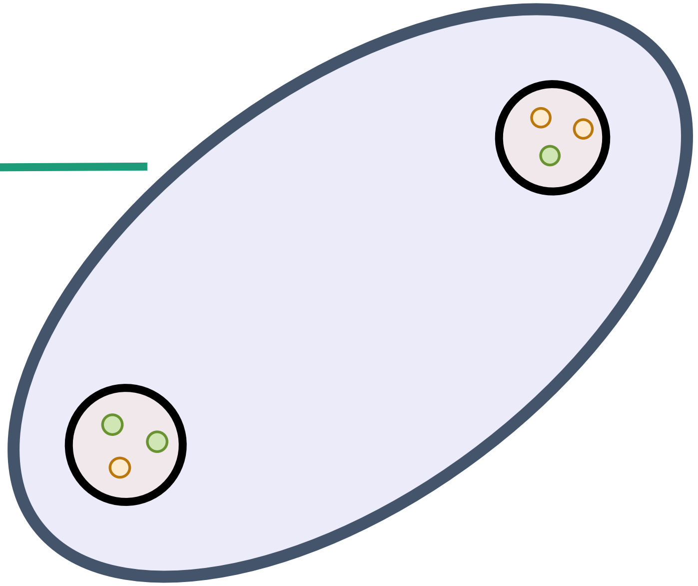
range of the **interaction**

and **particle size**.

Ideally:

$a_0 \to \infty$ (Scattering length)

$r_0 \to 0$ (Effective range)



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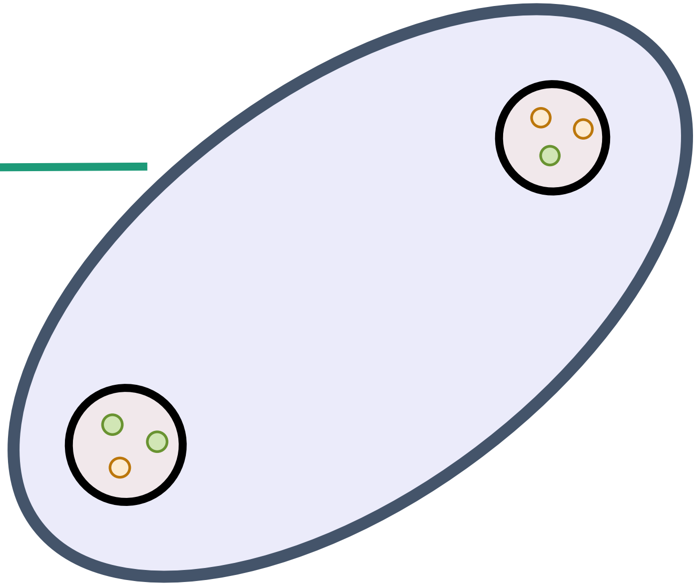

Systems close to the **Unitary limit** can be found in

- **Atomic physics** (Feshbach resonances, ${}^6\mathrm{Li}-{}^6\mathrm{Li}$, ${}^{40}\mathrm{K}-{}^{40}\mathrm{K}$ atoms)
- **Nuclear physics** ($n-p$ interaction)
- **Lattice nuclei** (Unphysically large m_π)
- **Hypernuclei** ($\Lambda - n$ interaction)
- **Hadronic physics** ($X(3872)$ Particles)
- **Cold Atom Physics** (Unitary two-specie fermions)

Atoms (experiments):
C.A. Regal (2003)
M.W. Zwierlein (2003)
M. E. Gehm (2003)
J. T. Stewart (2007)

Nuclei (theory):
U. van Kolck (1999)
S. König (2017)

Hypernuclei (theory):
H.-W. Hammer (2001)
L.C. (2018)

Hadrons (theory):
E. Braaten et al (2003)

Lattice Nuclei (theory):
N. Barnea et al (2015)
L.C. et al (2017)

Universality

We observe very different systems to have **similar few-body** proprieties

↓

2B **scattering parameters**;
Few-body **states**;

⟷

We can use a **similar theory**

Universality

Same microscopic **symmetries; Separation of scales**

We observe very different systems to have **similar few-body** proprieties

Same **many-body** phenomena

The Idea

If you know that **two systems** with the same fundamental symmetries or the same few-body proprieties, you can design a **common theory**.

2B **scattering parameters**; Few-body **states**;

We can use a **similar theory**

Simple and intuitive: Contact theory

- Treat **particles as degrees of freedom** (elementary particles)
- They can interact only **short-range**

  (Short range structure is irrelevant: no quark structure)

  (Long range interactions are negligible: no pion exchange)

- Works for a limited set of energies

- Easy to understand
- Clear limitations
- Expandible
- Minimal inputs required
- Universally transposable

Simple and intuitive: Contact theory

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  (Short range structure is irrelevant: no quark structure)

  (Long range interactions are negligible: no pion exchange)

- Works for a limited set of energies
- Tricky to be properly implemented
- Clear limitations only in the known cases
- Not trivial to be practically expanded beyond 1st order
- Minimal inputs required at the first orders

- Easy to understand
- Clear limitations
- Expandible
- Minimal inputs required
- Universally transposable

A complete theory

$r_{ij} = r_i - r_j$

$$V(r_{ij}) = \delta(r_{ij})$$

Contact theory formally:

$$L = N^{\dagger}\left(i\partial_0 + \frac{\hbar^2}{2m}\,\nabla^2\right)N \; - C_0\, N^{\dagger}N^{\dagger}NN$$

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$$L^{N^{>0}LO} = \quad C_2 \left(N^\dagger \nabla^2 N \, N^\dagger N + h.c. \right) + C_{11} \left(N^\dagger \vec{\nabla} \, N \, N^\dagger \vec{\nabla} N \right) +$$
$$C_4 \left(N^\dagger \nabla^4 N \, N^\dagger N + h.c. \right) + \; \ldots$$
$$D_0 \left(N^\dagger N^\dagger N^\dagger N N N \right) + E_0 \left(N^\dagger N^\dagger N^\dagger N^\dagger N N N N \right) + \; \ldots$$

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Including all the derivative/many-body operators one can **express any interaction**

A complete theory

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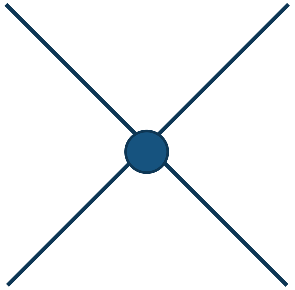

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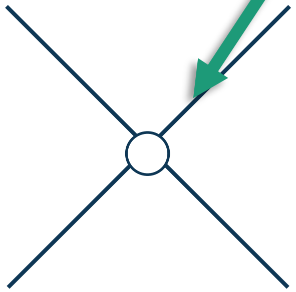
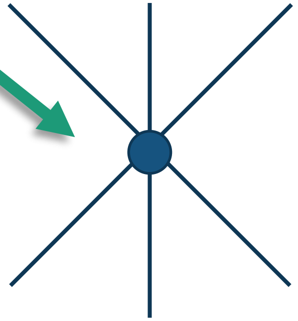
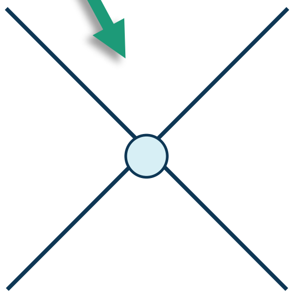
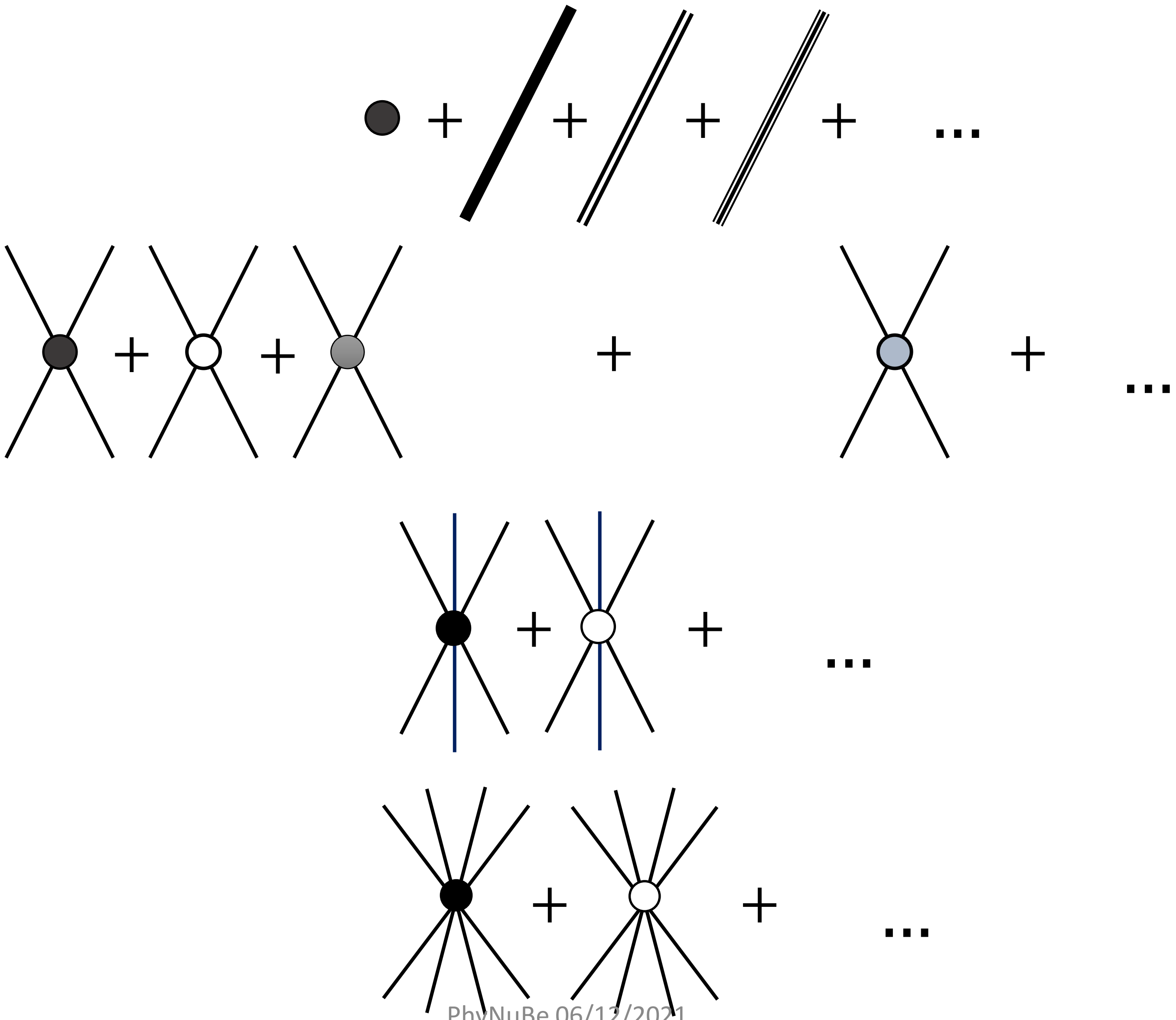






Including all the derivative/many-body operators one can **express any interaction**

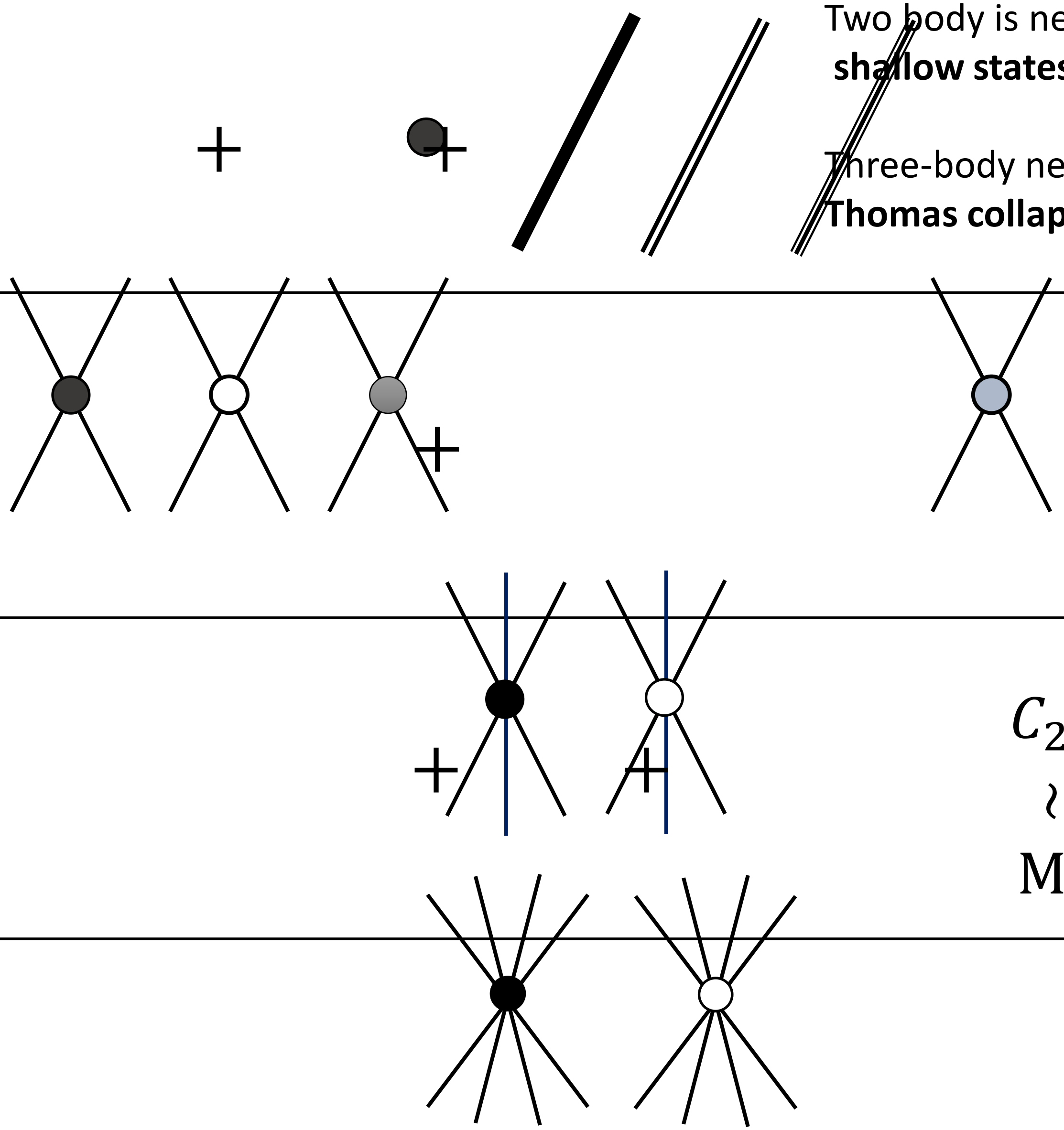
Pionless EFT powercounting



U. Van Kolck - Progress in Particle and Nuclear Physics 43, 1999, 337-418

Pionless EFT powercounting

LO	Two body is needed to provide **shallow states**. Three-body needed to avoid **Thomas collapse**.
NLO	
N^2LO	$C_2 \overleftarrow{\nabla} \cdot \overrightarrow{\nabla} \sim c_2 \frac{Q^2}{M_{hi}}$ $\wr$ M_{hi}

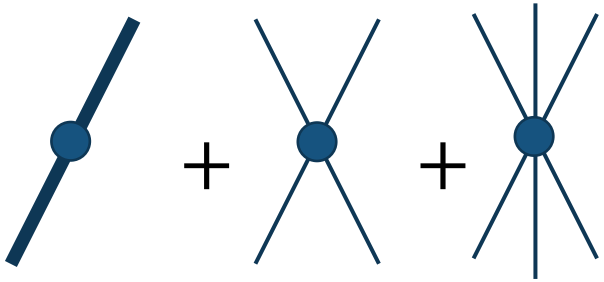


Pionless EFT powercounting

In the nuclear case: $\Gamma_{NN} = \frac{Q}{m_\pi} = 0.5 \sim 0.8$

Momentumless 2-3 body

LO



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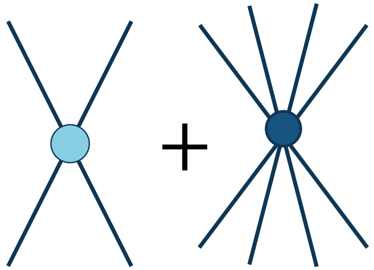
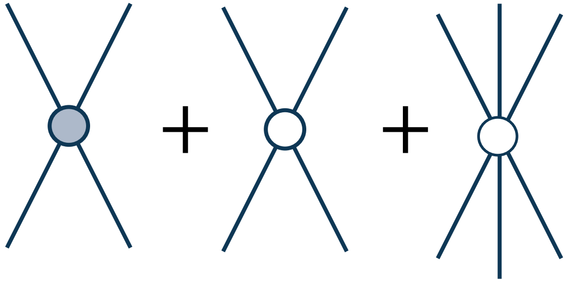
Momentum dependent / 4-body

NLO



$O(\Gamma)$

N^{2}LO



$O(\Gamma^2)$

$O(\Gamma^{\geq 3})$

G.P. Lepage, How to renormalize the Schrödinger equation (1997)
U. van Kolck, Nucl.Phys. A645 273-302 (1999)
J.-W. Chen, et al. Nucl.Phys. A653 (1999)
S. König, H. W. Grießhammer, H. W. Hammer, and U. van Kolck, J. Phys. G43, 055106 (2016)
B. Bazak, PRL 122.143001 (2019)

Duality

universality

Unitary limit: $a_0 = \infty$

$r_0 = 0$

Finite three-body scale: $0 > E_3 > -\infty$

(contact) EFT

(nonrelativistic)

LO

$$\mathcal{L} = N^\dagger \left(\partial_0 + \frac{\nabla^2}{2m} \right) N + $$

$$+ C_0 N^\dagger N^\dagger N N + D_0 N^\dagger N^\dagger N^\dagger N N N$$

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$$+ C_0 N^\dagger N^\dagger N N + D_0 N^\dagger N^\dagger N^\dagger N N N$$

However, no physical system is perfectly in the unitary limit

S. König (2016)

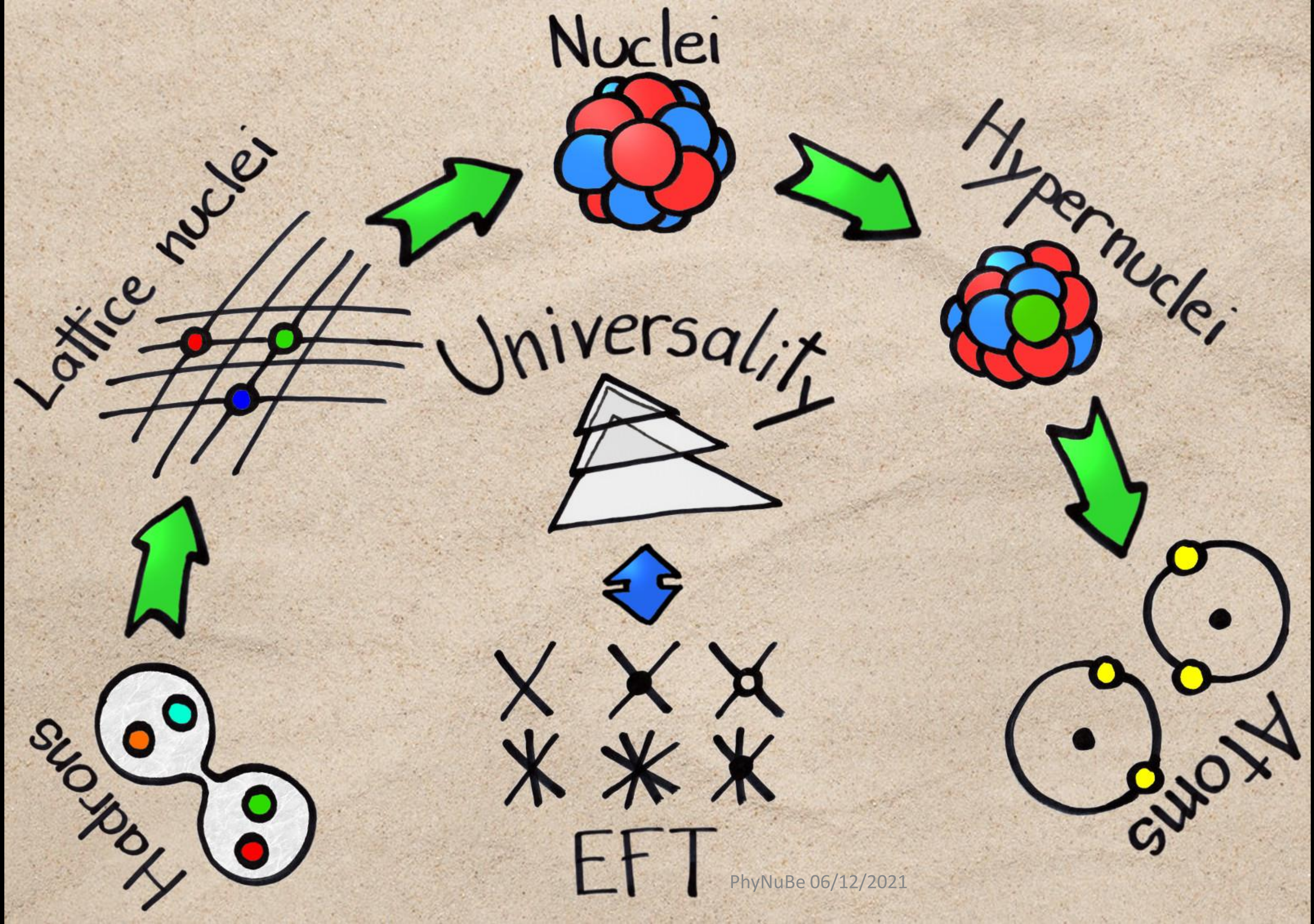
$\mathrm{N}^n\mathrm{LO}$

Physical systems can be close to the limit:
e.g. $|a_{n-n}| = (|-23.| \text{ fm}) \gg (\text{r}_0 \sim 2.7 \text{ fm})$

Deviation from the universal limit
are needed to predict physical phenomena.

Effective field theory **powercounting**

i.e. subleading perturbative corrections
define the specific physical system.



Interested in any of these systems?

Come for a chat!

Other examples are:
+ Alpha clusters
+ Condensed matter
+ Neutron dorps
& neutron matter

Hadrons

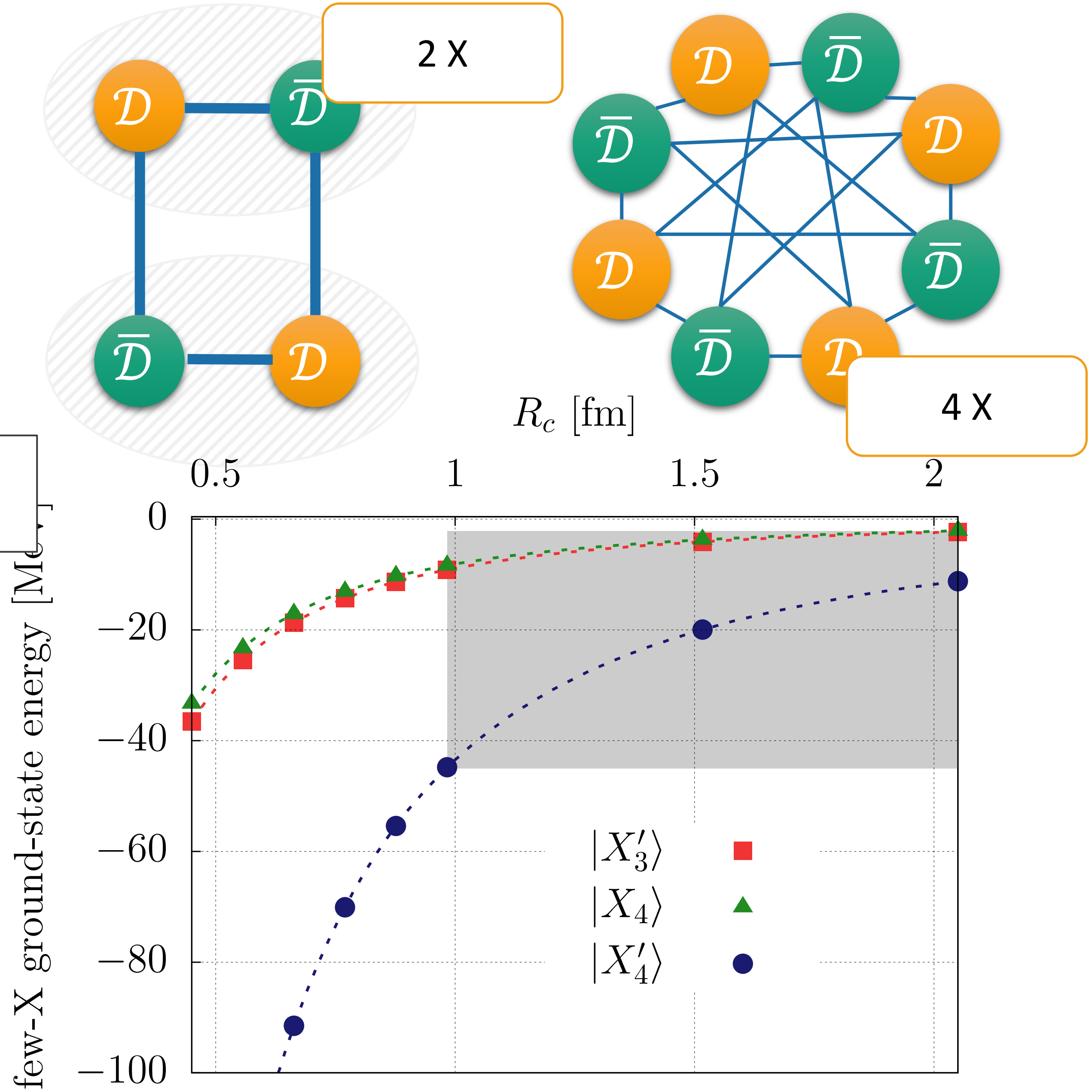
With a simple theory and only knowing
That $\mathcal{D} - \overline{\mathcal{D}}$ interaction is **unitary**
The **range** of such interaction

A LO theory can be fitted

This might describe a Brunnian system!

Predict bound $3X$, $4X$ (qualitative prediction)

$2X$ is uncertain (bound only for certain cut-offs)

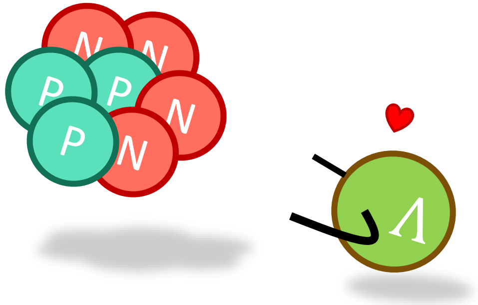


L.C., J. Kirscher, M Pavon Valderrama Phys. Rev. D., e-Print: 2008.12268

Minimal input parameters + underlying symmetries

Allow to predict effects unknown experimentally!

Hyperons ($\Lambda - $ Hypernuclei)



$\Lambda - N$ interaction is short range
$\Lambda - \#_{\#}$ is a shallowly bound system

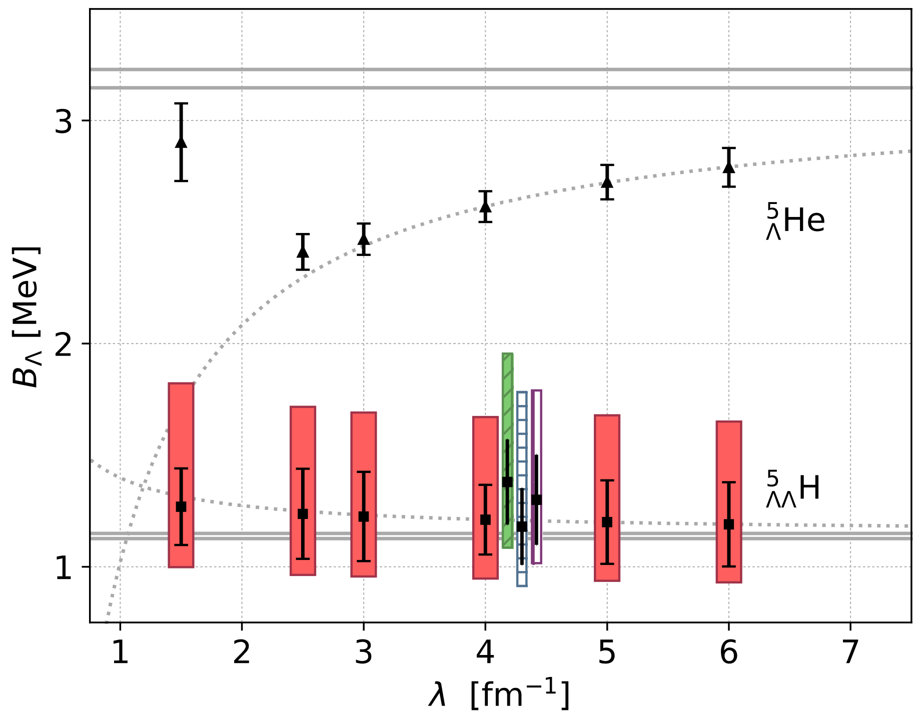
} Perfect example of interaction close to unitary limit

Single Λ: very few experimental data
Double Λ: hard to be created experimentally
Few-body hypernuclei: theoretically hard to be described all together

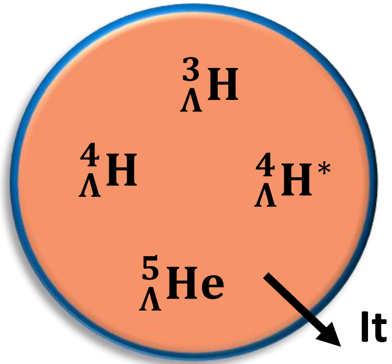
LO EFT has few input : can be fitted from existing experiments;
Theoretical errors are relatively small;
The powercounting guides in the choice of the relevant operators:
solves the overbending problem;

Is proved only for shallowly bound (**few-body**) hypernuclei

$$B_\Lambda({}^{5}_{\Lambda\Lambda}\mathrm{He}) = 1.14(1)^{+(44)}_{-(26)}$$



L. C. , M. Schäfer, N. Barnea, A. Gal, J. Mareš
Phys. Lett. B 797 (2019) 134893



Solidly bound!

It is possible to describe them all together.
(No overbinding problem!)

Nuclei

Nuclear physics is "**almost**" at the unitary limit

$a_{np} \sim 5.5$ fm $r_{np} \sim 1.5$ fm

$a_{nn} \sim 20$ fm $r_{nn} \sim 2.5$ fm

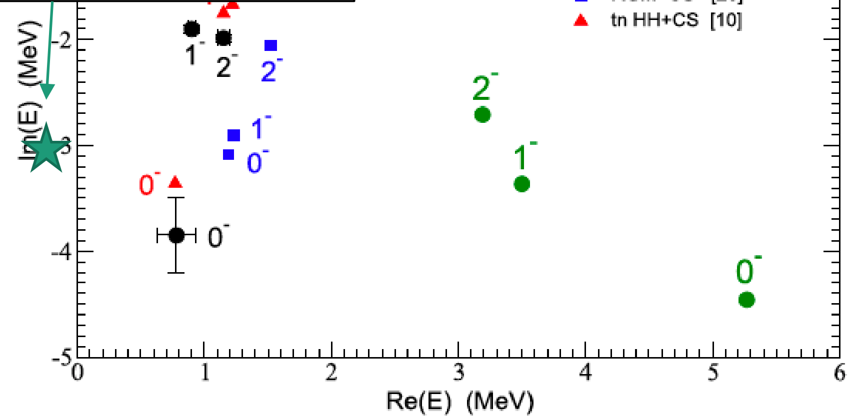
Deviations may play an **important role**!

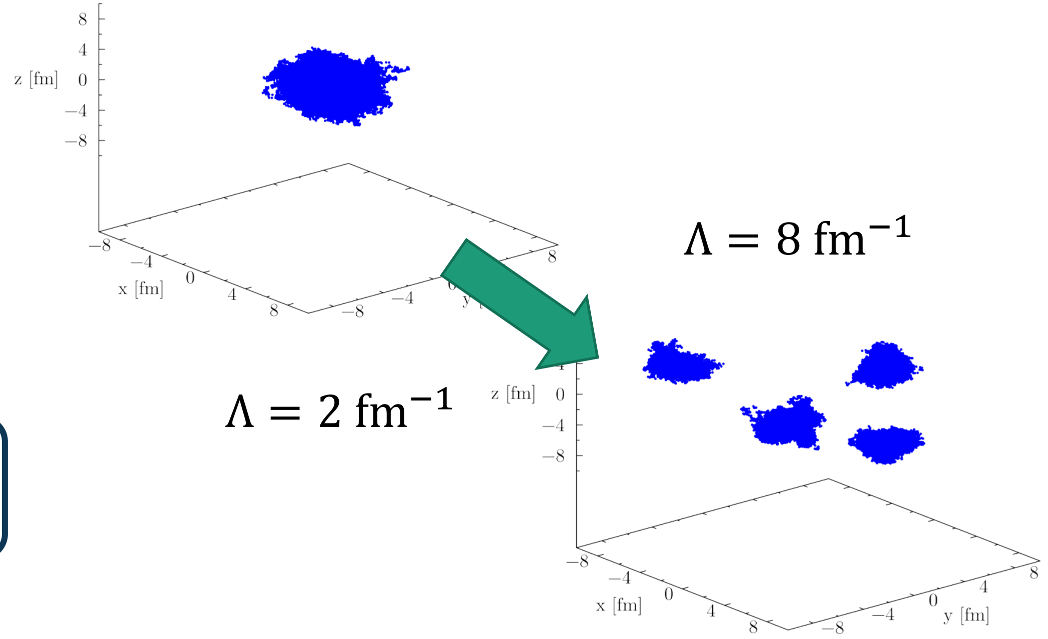
$\mathrm{E}^{\mathrm{LO}}(\,^4\mathrm{He}) = 29.2 \pm 0.5$ MeV
Is **close** to the experimental data
$\mathrm{E}^{Exp}(\,^4\mathrm{He})$ 28.3 MeV.

But larger nuclei (systems with mixed symmetry)
Appear to be **never stable in the universal regime**!

What kind of **deviations** should we include to stabilize them?

The theory predicts resonant states but not stable ones.
Here is our prediction.





Λ is the cut-off of the interaction
($\Lambda \to \infty$ is the universal limity)

M. Schäfer, L.C., J. Kirscher, J. Mareš Phys.Lett.B 816 (2021) 136194

Lattice nuclei

L. C., A. Lovato, F. Pederiva, A. Roggero, J. Kirscher, U. van Kolck Phys. Lett. B **772**, 839-848 (2017)

With Lattice QCD you can access **high pion masses** physics.

Completely **ab initio** calculations.

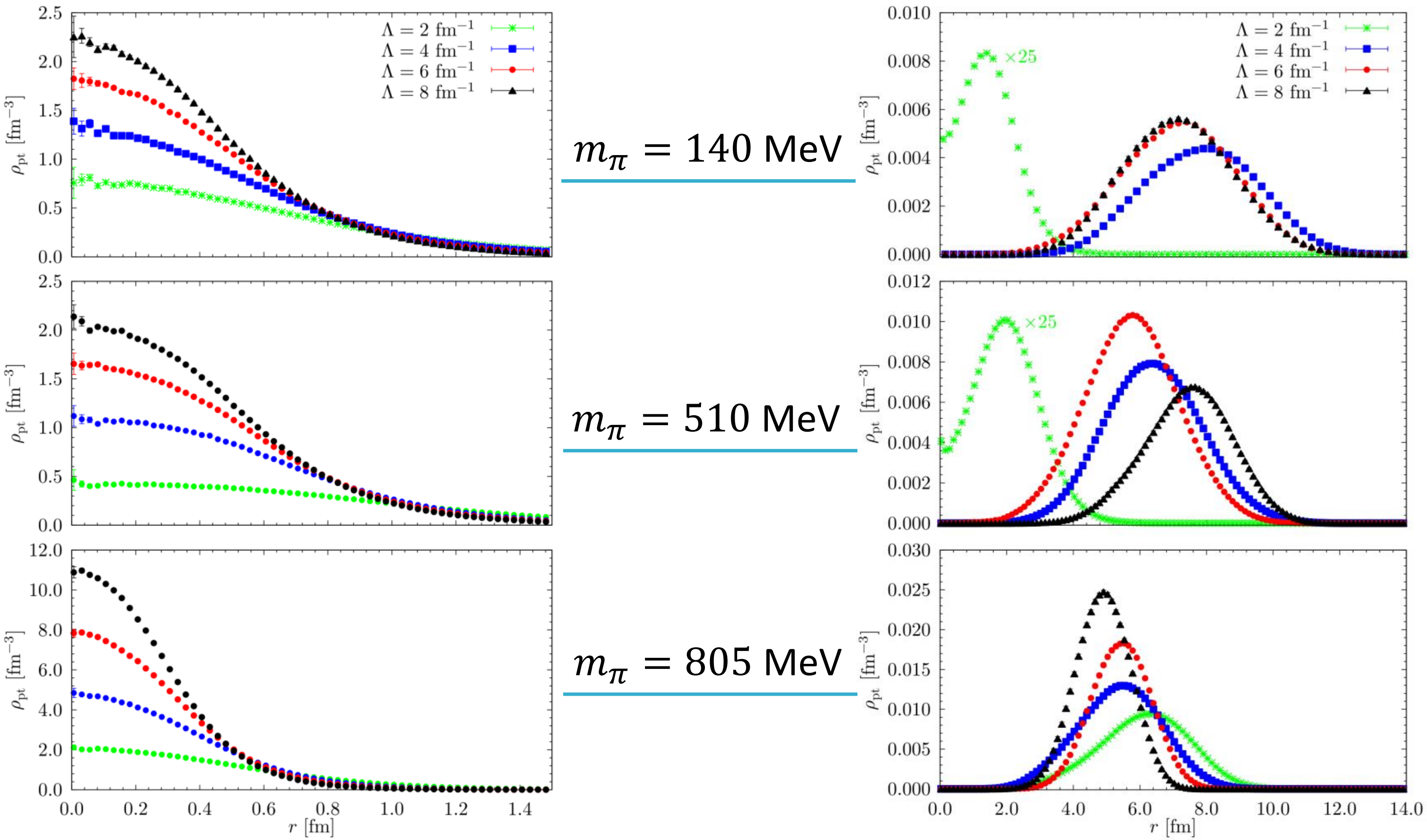
Would be the physics as we know it?

The interaction range is **shorter**
The binding momentum **is the same**

Contact interaction

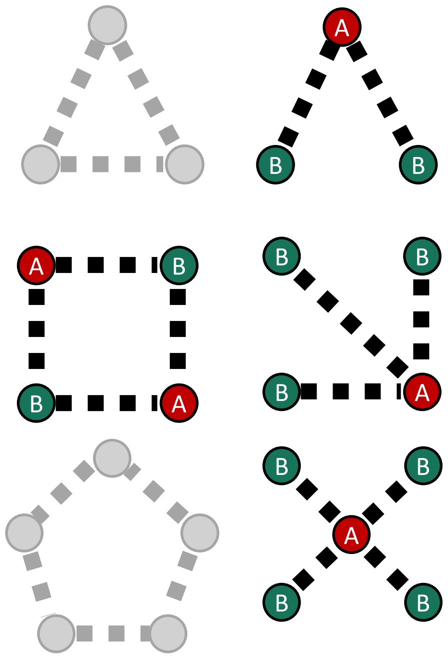
No stable P-shell nuclei
(as universality predicts)

The world would be very different!



Atoms (Bosons)

Systems with only a **subset of unitary interactions**
We see two different kind of behaviours



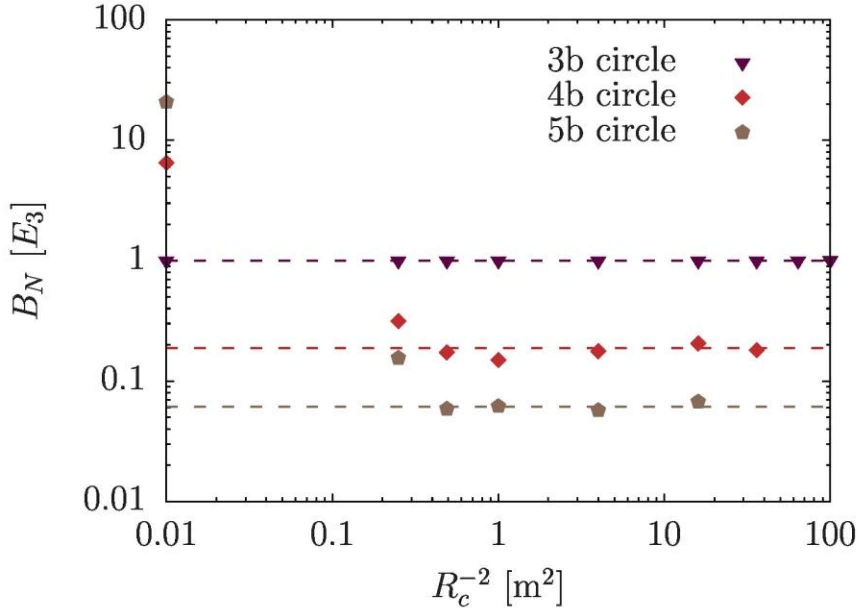
Circles Dandelions

The square system **behave differently** compared with the other systems composed by two-atom species.

Can this be seen in experiments?

The □ system energy depends from the △ but the △ is formed by different kind of particles

Circles



L.C., J. Kirscher, M Pavon Valderrama
Phys. Lett. A 408, 127479 (2021)

Numerics

Fit the theory:

- **two-body bound** state and scattering methods (Numerov, finite difference methods)
- **Few-body** ab-initio methods (e.g. stochastic variational method)

Predictions:

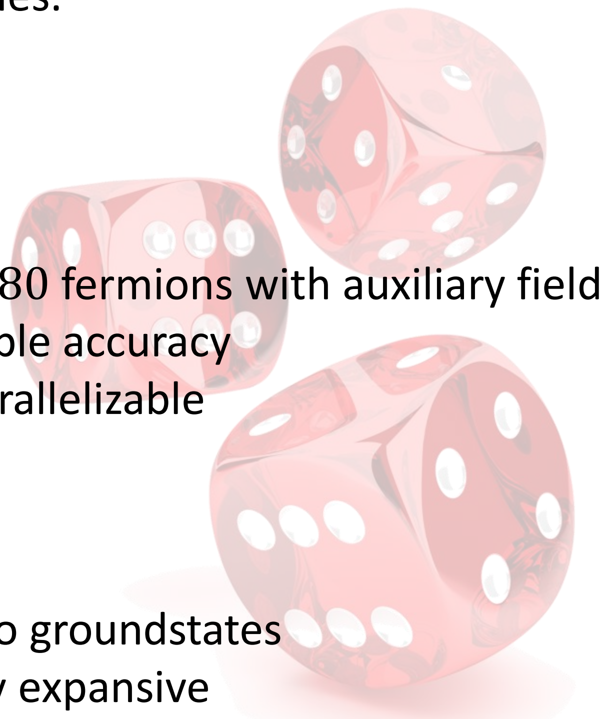
- **Few-body** ab-initio methods (e.g. stochastic variational method)
- **Many-body** ab-initio methods (e.g. quantum Monte Carlo method)

SVM:

Variational diagonalization method based on random gaussian basis expansion

Monte Carlo:

Quantum Monte Carlo is a class of ab initio, numerical, stochastic many-body methods able to solve the Schrödinger equation with improvable uncertainties.



Pros:

+ Up to $\sim$ 80 fermions with auxiliary field technique
+ Improvable accuracy
+ Easily parallelizable

Cons:

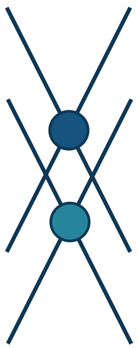
- Limited to groundstates
- Generally expansive
- Sign problem makes things hard

Nuclei: a personal story

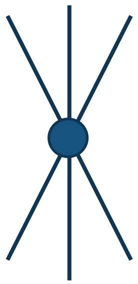
Leading order theory:

2-body + 3-body fitted to reproduce (n-n) scattering, deuterium, and triton binding energy

$$H^{LO} = -\frac{\hbar^2}{2m}\sum_i \nabla^2 +$$

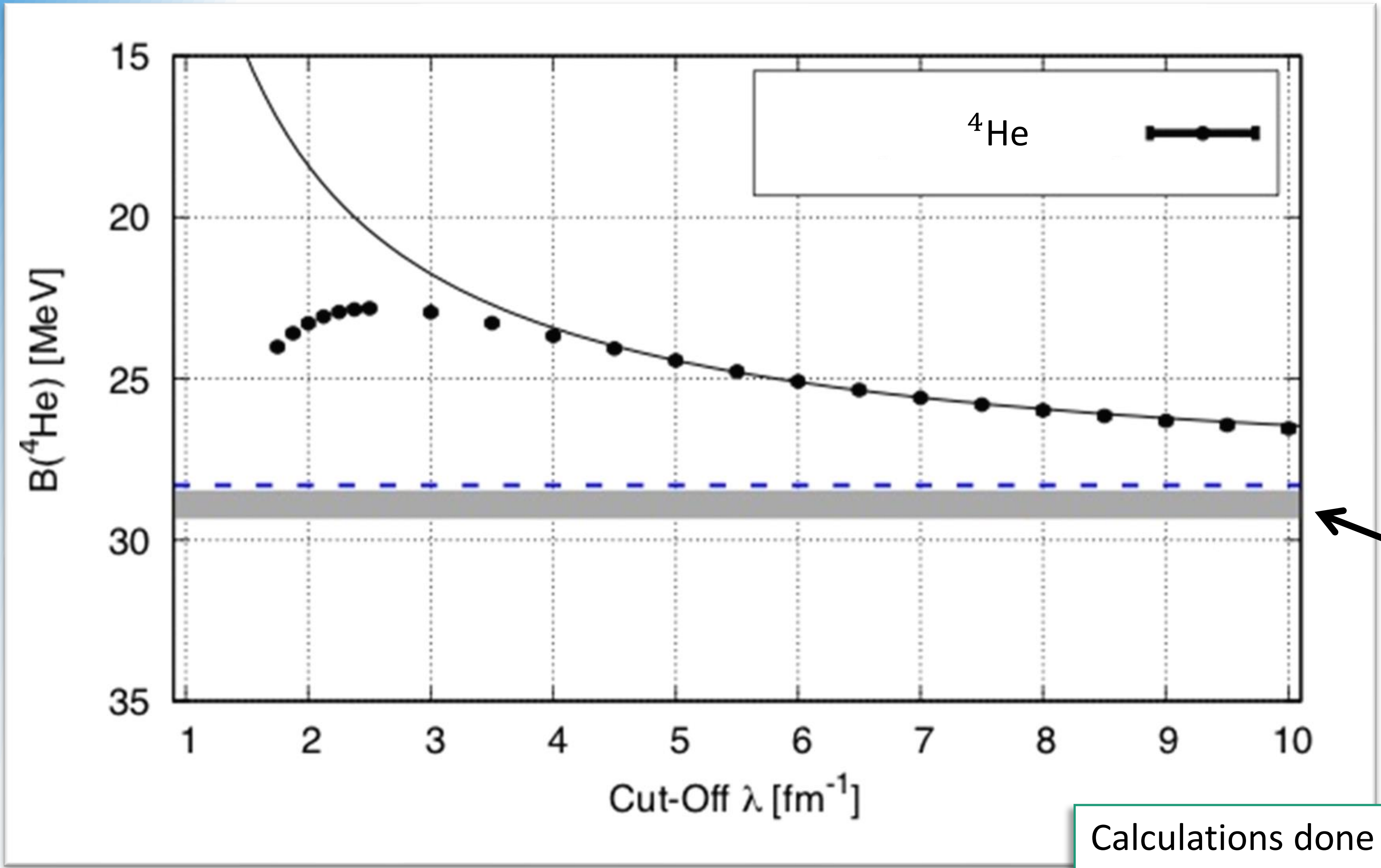


$$+$$



No coulomb, no tensor force, no spin orbit at LO

A practical example: few nucleons



L. C., N. Barnea, and A. Gal Phys. Rev. Lett. **121**, 102502

LO pionless EFT theory fitted on **two- and three-body** observables **predicts well ^{4}He energy**!

Fitted on: $a_{n-n} = -18.63$ fm

$B_d = -2.22$ MeV

$B_t = -8.48$ MeV

Experimental data: 28.3 MeV

Extrapolation: 29.2±0.5 MeV

Calculations done with **few-body stochastic variational diagonalization method**: Y. Suzuki, K. Varga (2003)

^{16}O - Monte Carlo calculation

Phys.Lett.B 772 (2017) 839-848

S-wave system

Λ $[fm^{-1}]$	^4He Energy [MeV]
2	-23.17(2)
4	-23.63(3)
6	-24.06(2)
8	-26.04(5)
∞	$-30^{0.3(sys)}_{2.0(stat)}$
Exp	-28.296

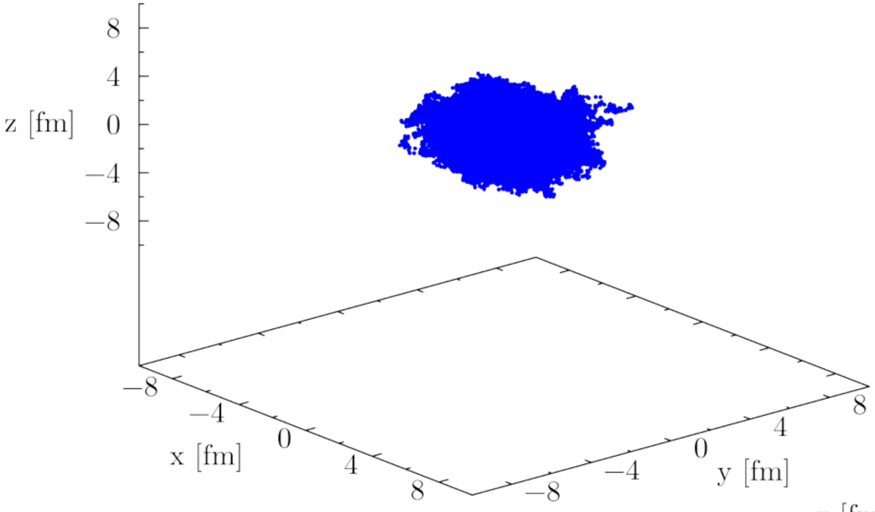
P-wave system

Λ $[fm^{-1}]$	^{16}O Energy [MeV]	4α treshold [MeV]
2	-97.19(6)	-92.68(8)
4	-92.23(14)	-94.52(9)
6	-97.51(14)	-100.24(8)
8	-100.97(20)	-104.2(2)
∞	$-115^{1(sys)}_{8(stat)}$	$-120^{1(sys)}_{8(stat)}$

- All the errors shown are statistical errors from Monte Carlo method.

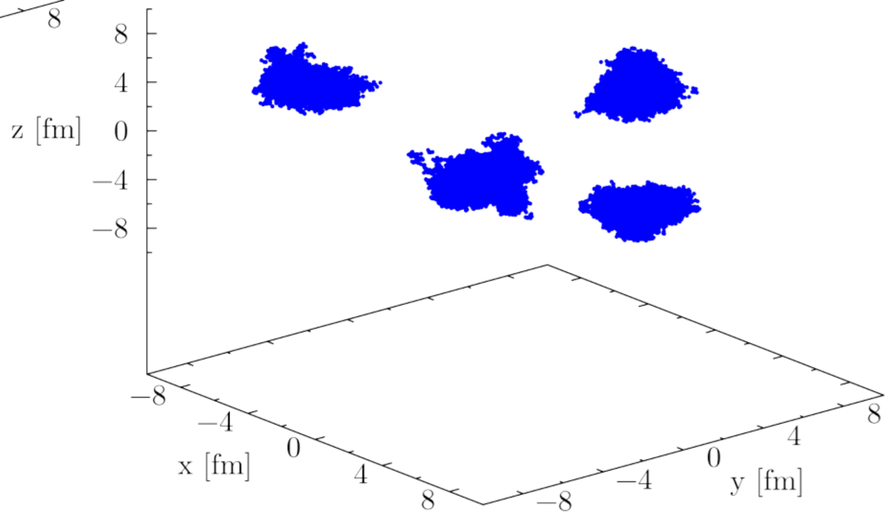
Be(^{16}O) ~ 127 MeV
Be(4α) ~ 113 MeV
It is only 10% difference!

Oxygen density ($m_\pi = 140$ MeV)



$\Lambda = 2\ \text{fm}^{-1}$

$\Lambda = 8\ \text{fm}^{-1}$



More evidences

5He 6He

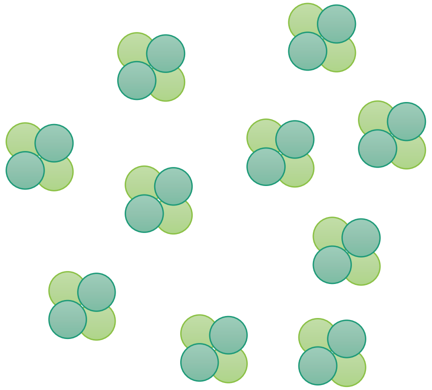
J. Kirscher, H. W. Grießhammer, D. Shukla, H. M. Hofmann: arXiv:0909.5606

Breaks in $\alpha + n$ and $\alpha + n + n$

40Ca

QMC calculation suggests the breaking in:

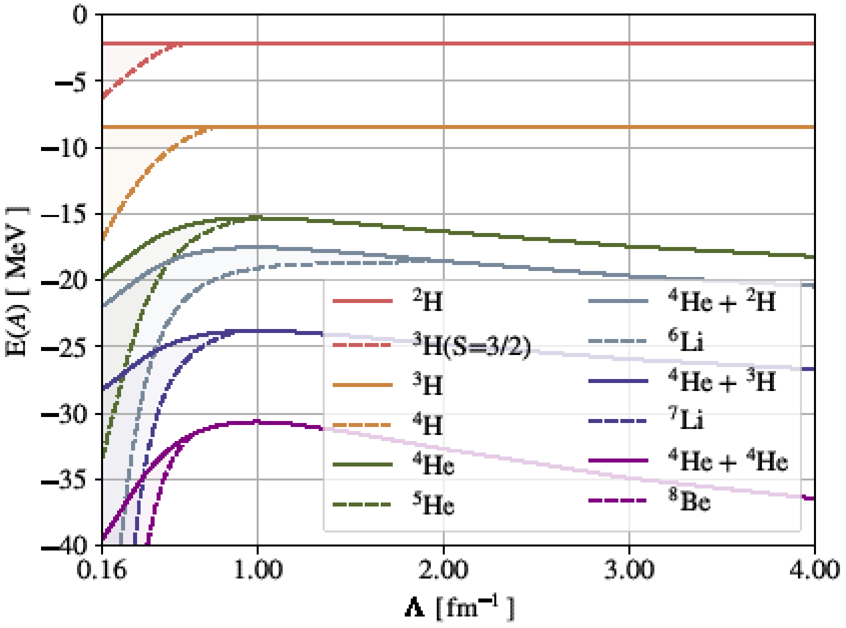
Breaks in $\alpha + \alpha + \alpha +$ …



7Li 8Be

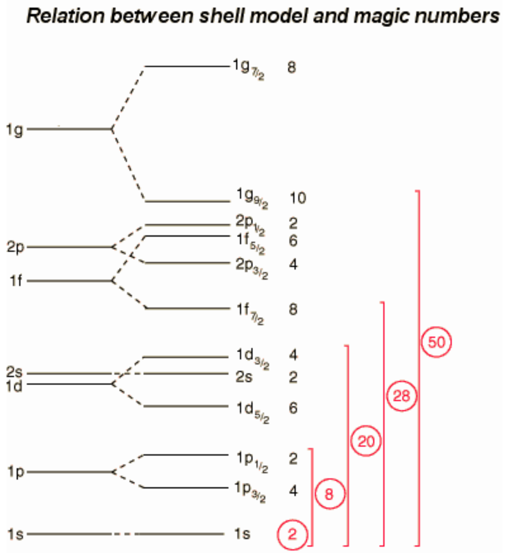
Our calculations in SU(4) symmetry

Breaks in $\alpha + d$ and $\alpha + \alpha$



P-wave systems

In a shell model representation



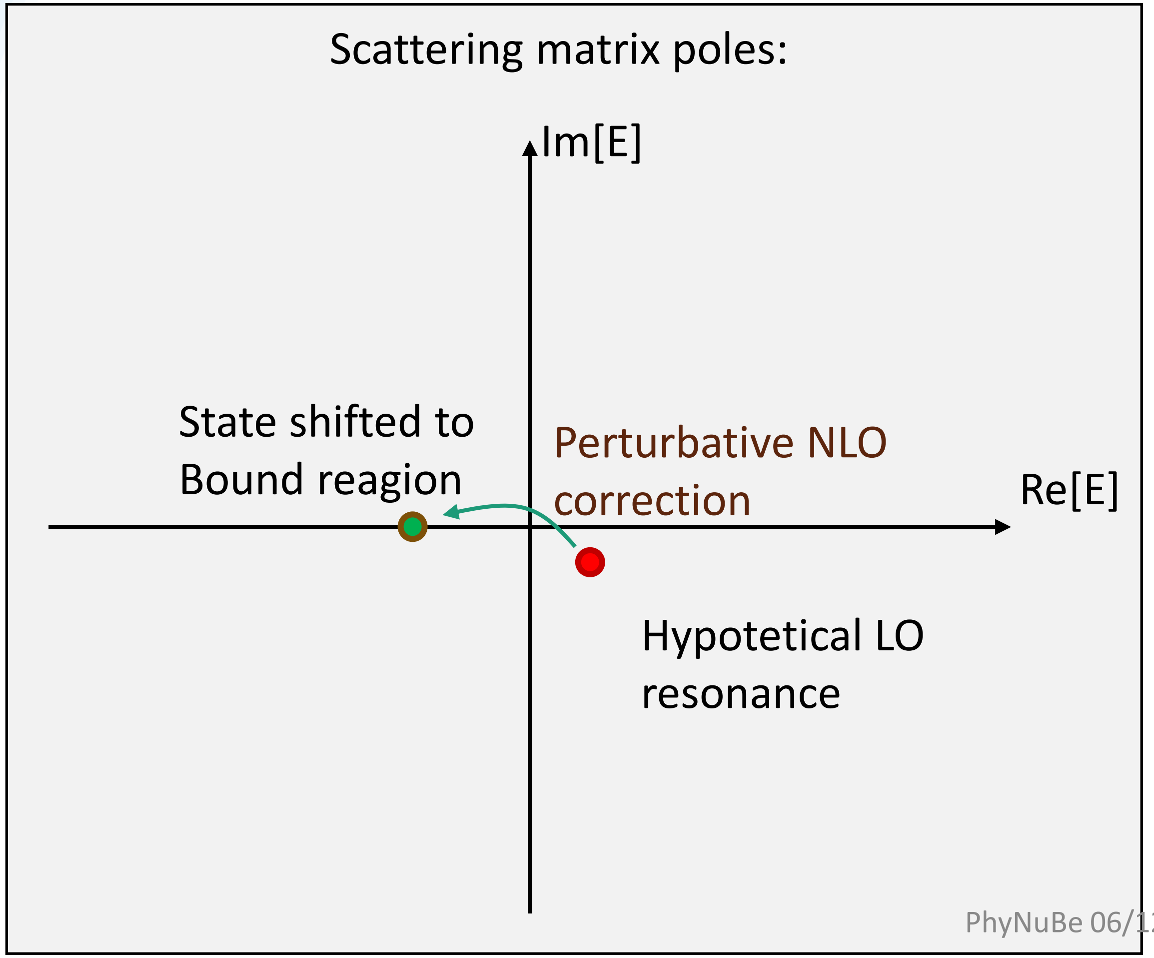
Will they ever bind?

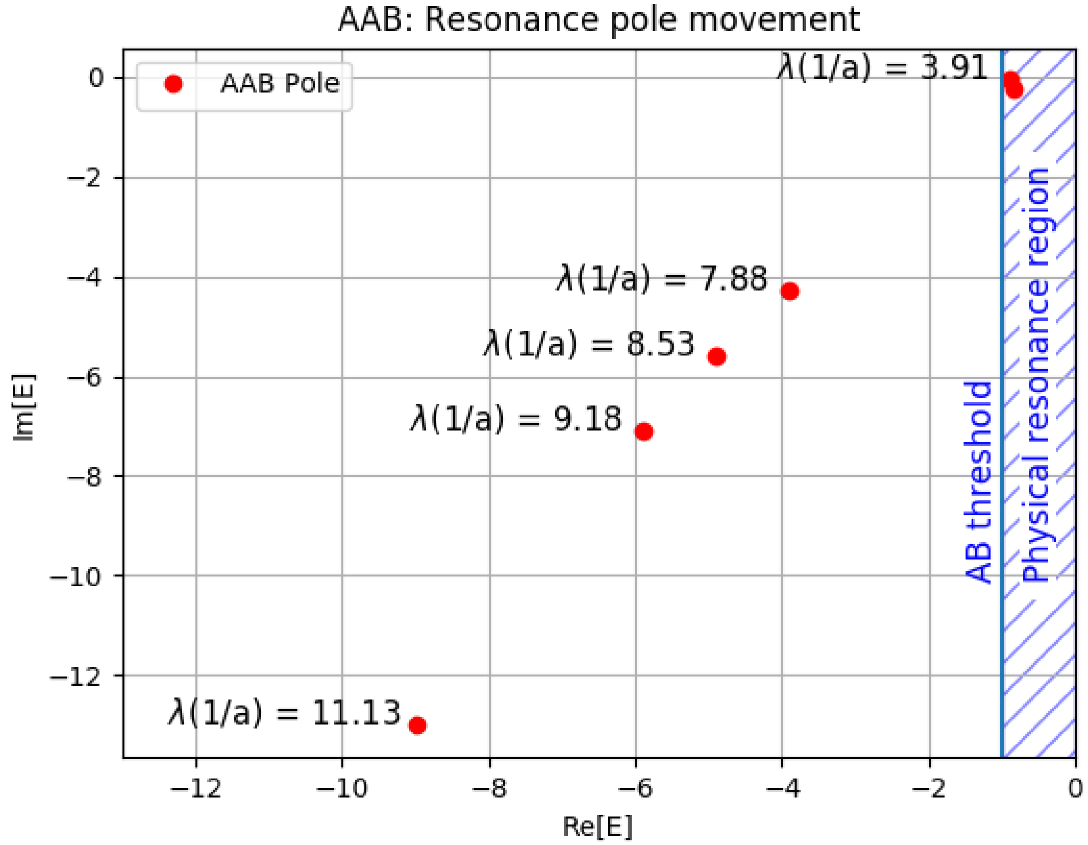
One little step further is necessary:

If a resonance is close to the **threshold**, it might be possible to move it with a **subleading correction**
(there is no proof this is possible, nor proof this is not possible)

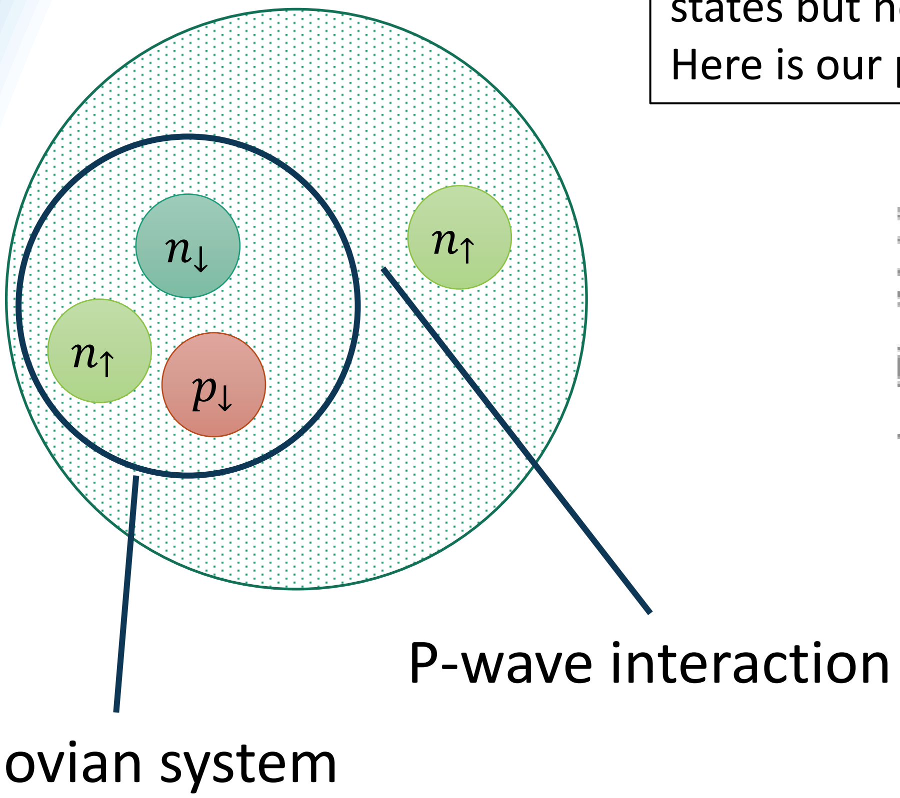
Known **three-fermion** case:
No physical resonance is found.

No scale invariance breaking,
Three-body force might change picture.

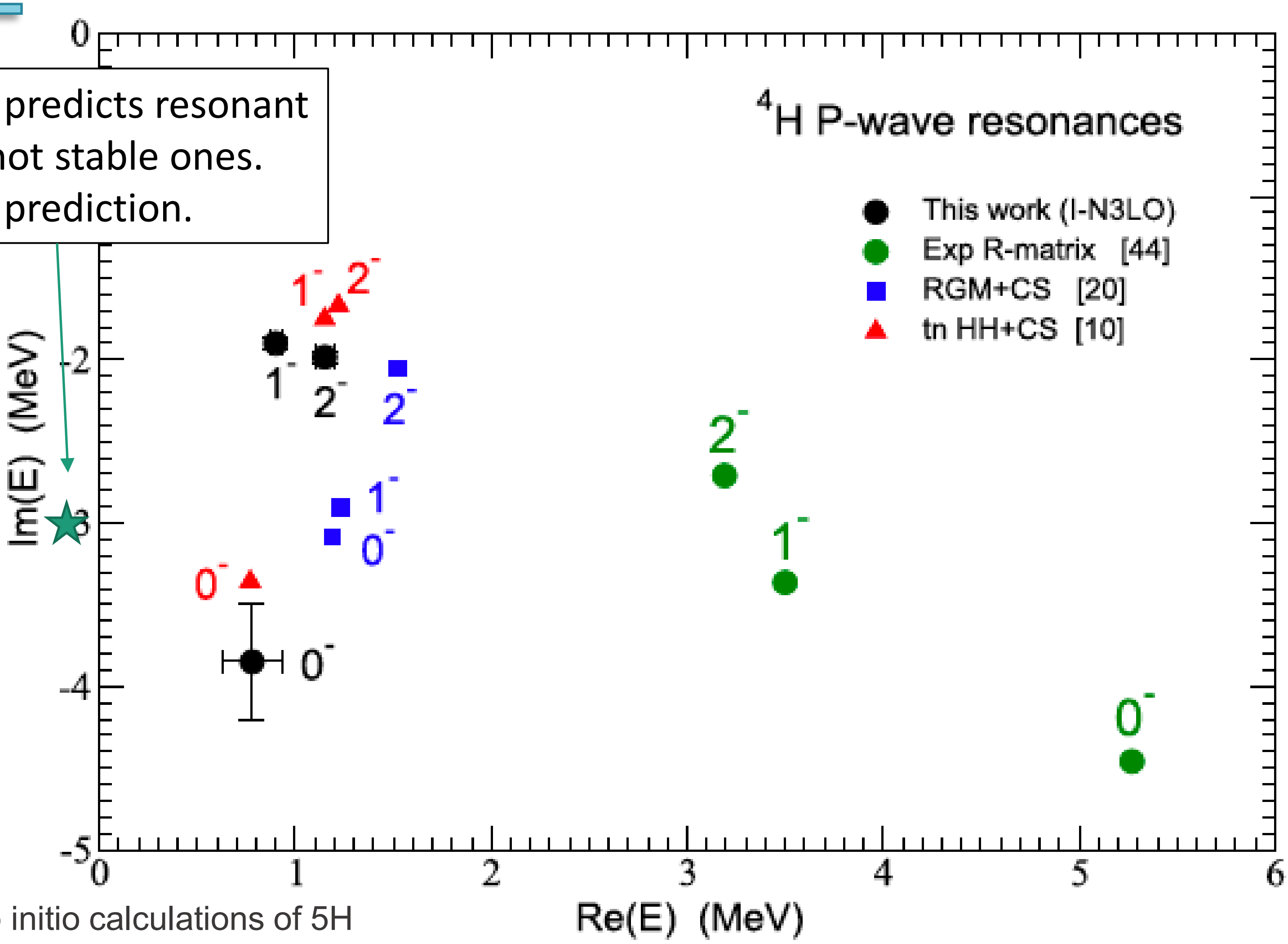




4H resonance:
the minimal nuclear system with an Efimovian component



The theory predicts resonant states but not stable ones. Here is our prediction.



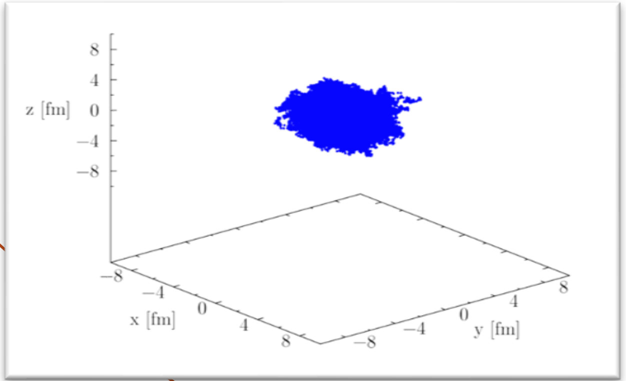
R. Lazauskas, E. Hiyama, and J. Carbonell, "Ab initio calculations of 5H resonant states,"Phys. Lett. B, vol. 791, pp. 335–341, 2019.

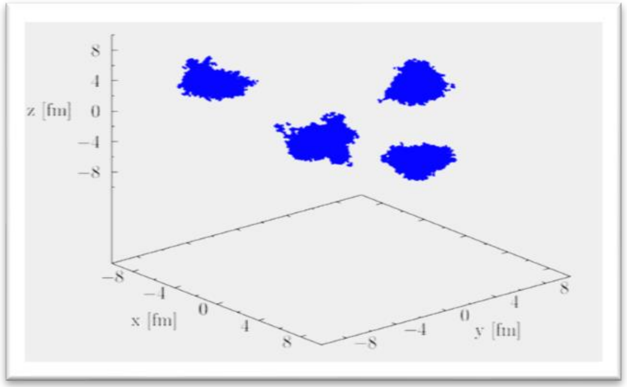
Contact EFT: a sub-threshold resonance is present

Contact theory → everything fine in **S-wave**
→ **no P-wave boundstates**

A resonance is found in ^{4}H → many-body **P-shell poles can be created**
(not in the correct physical position)

Can the resonant pole be moved to the bounded region with a **perturbative** NLO insertion?
Preliminary calculation showed that either $\boldsymbol{P^2}$ or $\boldsymbol{P \cdot P'}$ can do the job.



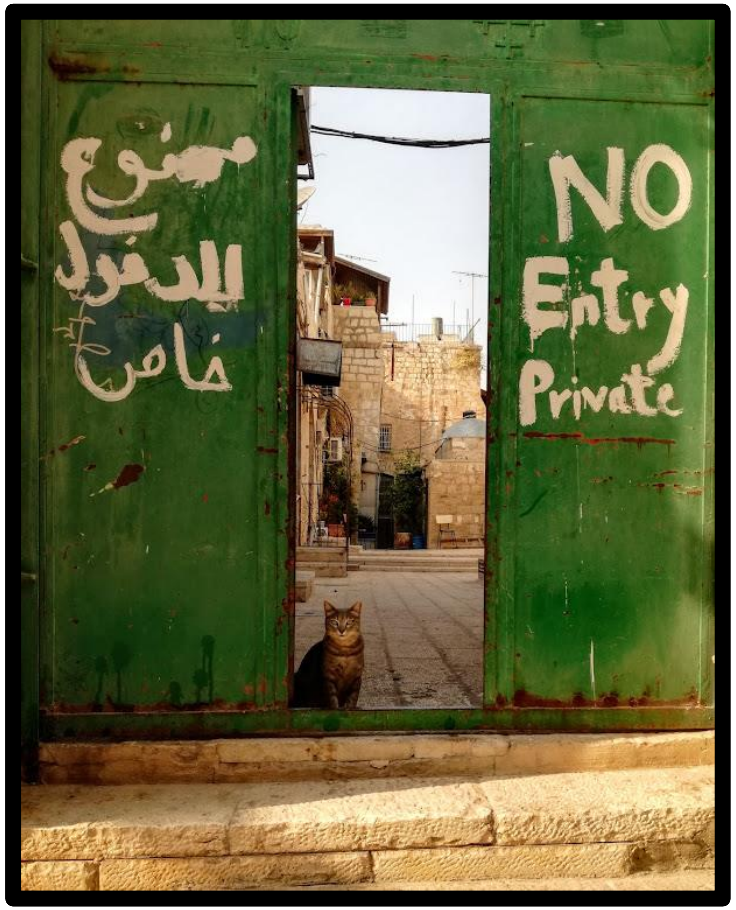


What can be learned

About universality:

- Many systems show a **universal behaviour**.
- **LO** of EFT takes care of **universality;**
  Sub-leading orders account for **deviations**.
- Similar systems share **common EFT** description and you can transfer the knowledge to **unknown physics**.

- Many other **universality classes** are out-there.
  What are? **Where** to find them? Can they be **useful**?



The guarding “Chatula”
Sept 2017 Jerusalem

A weird concept: scale invariance (this is quite theoretical)

Define the **typical momentum** you are interested
(probe momentum)

Two-body system is **infinitely large** with respect to this probe

The range of the interaction is **infinitesimal**

No scale in the system: everything should be $\mathbf{0}$ or ∞
[no measure units]

Problem:

In real physics the three body energy is not $\mathbf{0}$ nor ∞ …
Moreover, if you make the calculation in universality
$E_3 \to -\infty$

Discrete scale invariance: 3+ body

Unitary system with 3 particles in S-wave show:

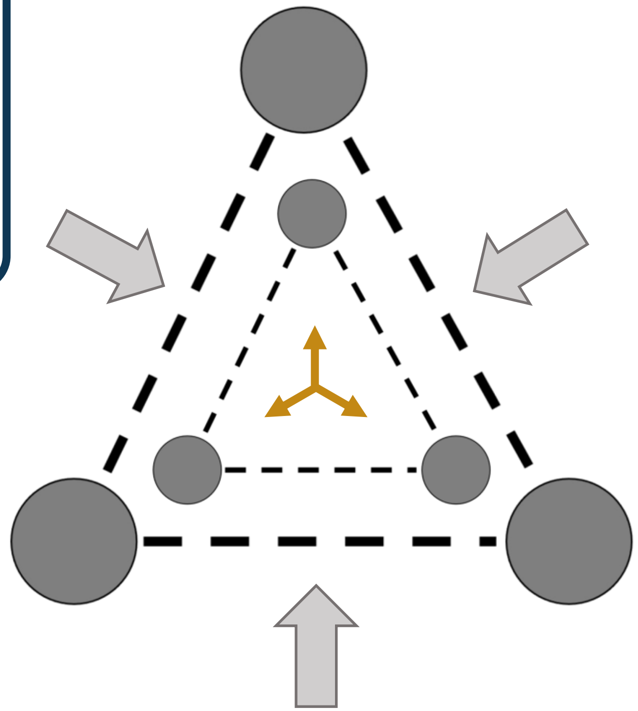
Thomas Collapse / Efimov Effect

In the **unitary limit** a system of **3 bosons/distinguishable particles collapses**

$$Unitarity \Longrightarrow E_3 \to -\infty$$

A **repulsion** is needed to **stabilize the system to a finite energy** $\boldsymbol{E_3}$.
$\boldsymbol{E_3}$ **breaks the scale invariance** of the system!

L. H. Thomas (1935)
G. Skorniakov and
K. Ter-Martirosian (1957)



Conditions to (Efimov) unitarity

Range of the interaction	$(r_0 \to 0)$
Size of the two body system	$(a_0 \to \infty)$
Size of the three body system	$(R_3 < \infty)$