

The effect of the energy functional on the pasta-phase properties of catalysed neutron stars

Hoa DINH THI, Anthea FANTINA, Francesca GULMINELLI

Aussois, December 9, 2021

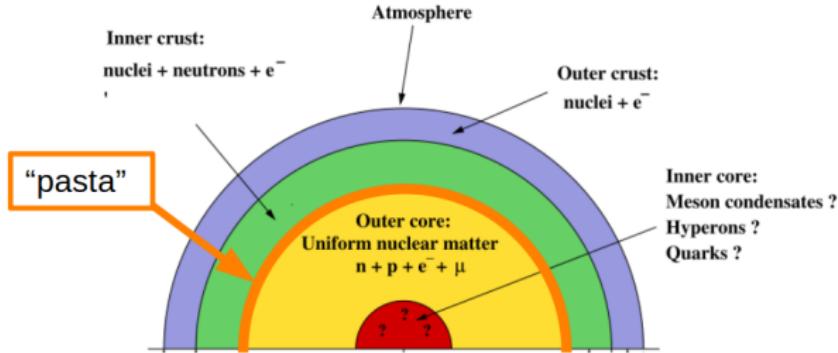
Ecole Thématique PhyNuBE : Première rencontre de Physique Nucléaire de Basse Energie 2021



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NORMANDIE



1. Pasta phase in neutrons stars

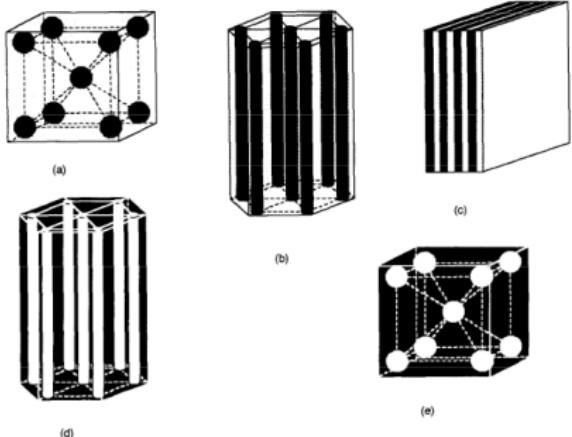


→ **Pasta phase** is expected to exist at the bottom of the **inner crust**, before the transition to the **core**.

Original figure taken from **Fiorella Burgio, G.; Vidana, I. Universe 2020, 6(8), 119**

2. Different shapes in the pasta phase

- 5 phases: spheres, rods, slabs, tubes, bubbles
 - “inverted” configurations



Ravenhall et al., Phys. Rev. Lett. 27, 2066 (1983)

Hashimoto et al., Prog. Theor. Phys. 71, 320 (1984)

K. Oyamatsu, Nucl. Phys. A561, 431 (1993)

3. Formalism

Thermodynamic potential in a **Compressible Liquid Drop Model**:

$$\begin{aligned}\Omega = & n_p m_p c^2 + (n_B - n_p) m_n c^2 \\ & + n_0 e_{HM}(n_0, I) \mathbf{f} + \epsilon_{surf} + \epsilon_{curv} + \epsilon_{Coul} \\ & + n_g e_{HM}(n_g, 1) (1 - \mathbf{f}) + \epsilon_e - \mu n_B.\end{aligned}$$

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Margueron et al. Phys. Rev. C, 97:025806, 2018

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- Surface, curvature, and Coulomb energies depend on nuclear shape. → 5 surface parameters: σ_0 , $\sigma_{0,c}$, β , b_s , p

Equilibrium configuration

- **Step 1:** Minimizing Ω for each geometry

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Equilibrium configuration

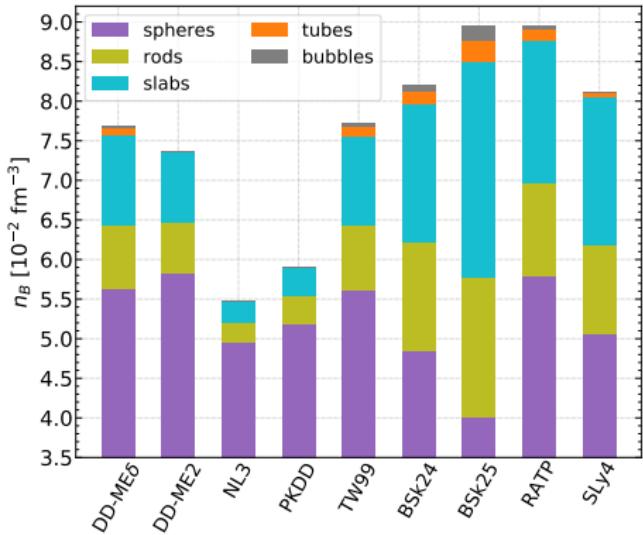
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→ composition (n_p, I, n_0, n_g, r_N) + energy density.
- **Step 2:** Comparing energy densities among all geometries
→ equilibrium geometry.

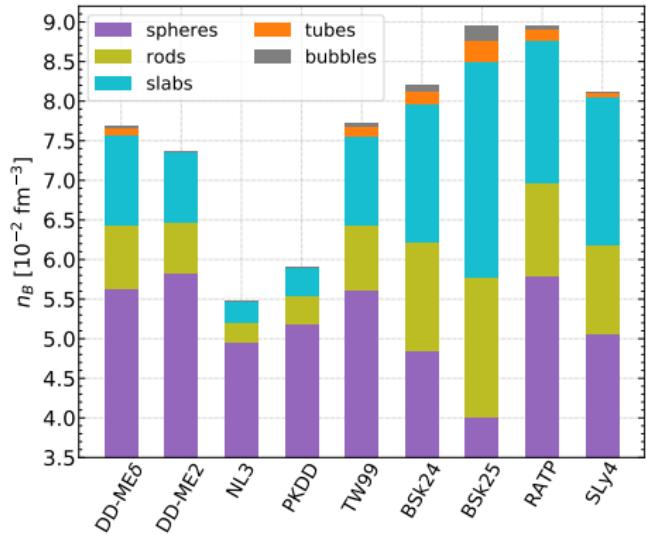
4. Model dependence of pasta-phase properties

Equilibrium geometries

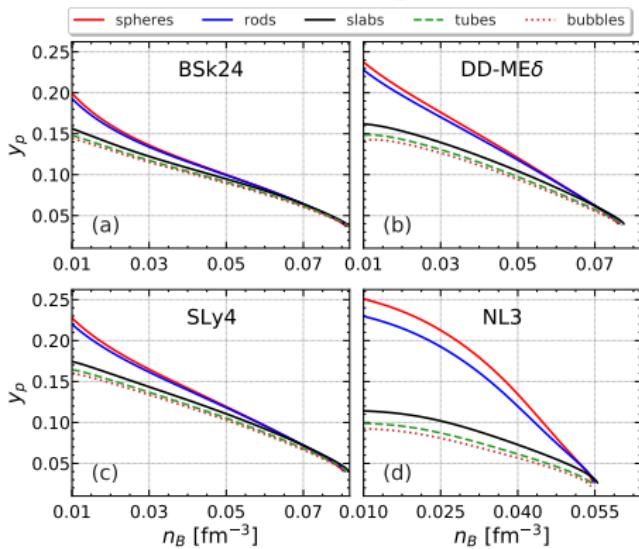


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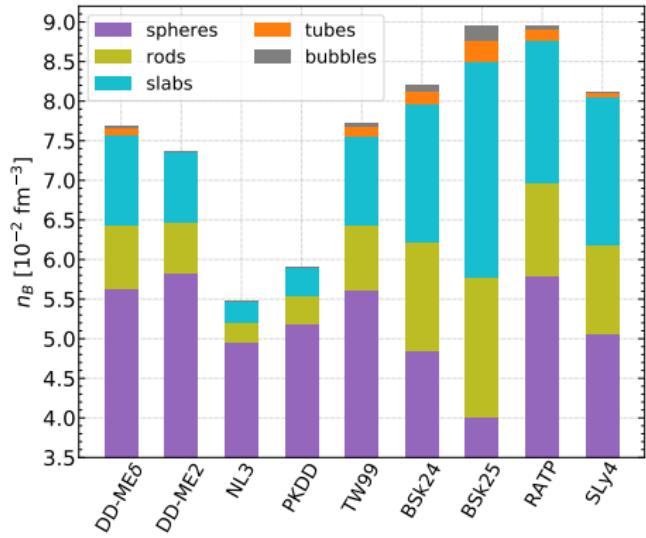


Proton fraction $y_p = Z/A$

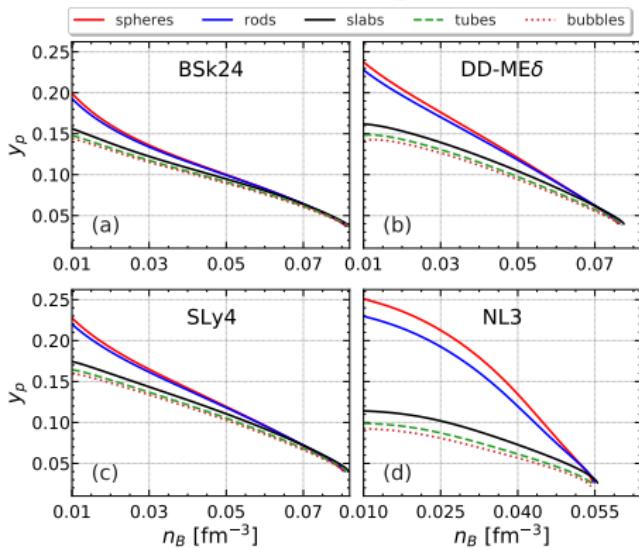


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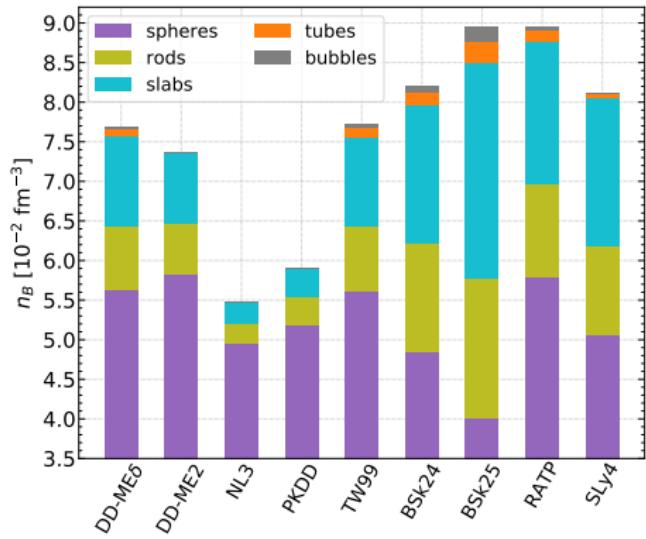
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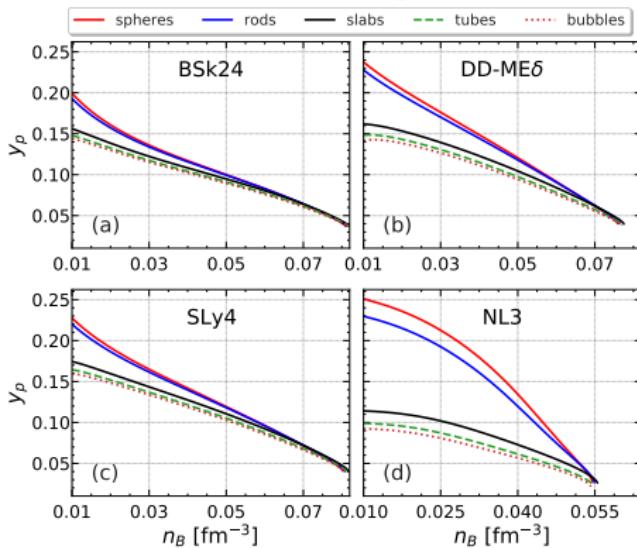
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4. Model dependence of pasta-phase properties

Equilibrium geometries



Proton fraction $y_p = Z/A$



- Properties of pasta phases are model dependent.
- Use Bayesian analysis to study the influence of energy functional on the uncertainties of pasta-phase properties.

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5. Bayesian analysis

- Prior distribution: generated based on the current nuclear physics knowledge. (See Margueron et al. Phys. Rev. C, 97:025806, 2018)

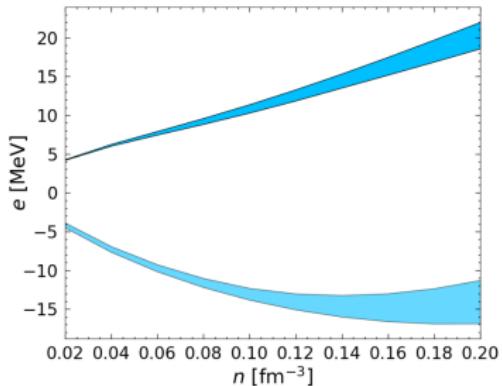
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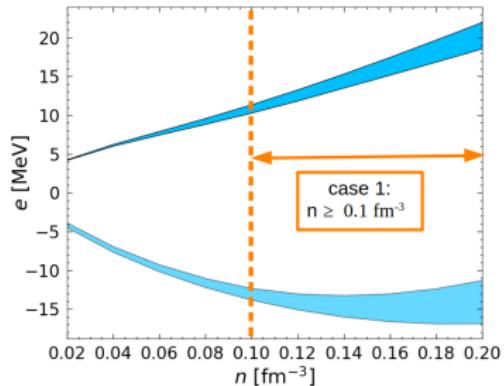
1. Low density (LD) filter (Drischler et al., Phys. Rev. C, 93, 054314, 2016):



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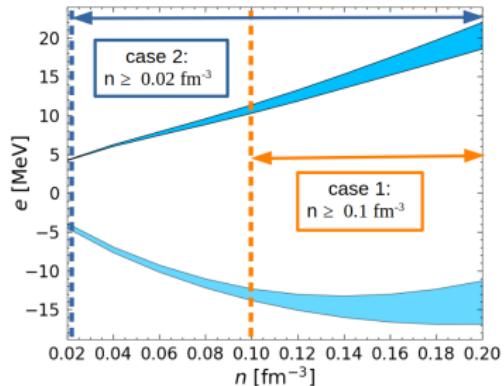
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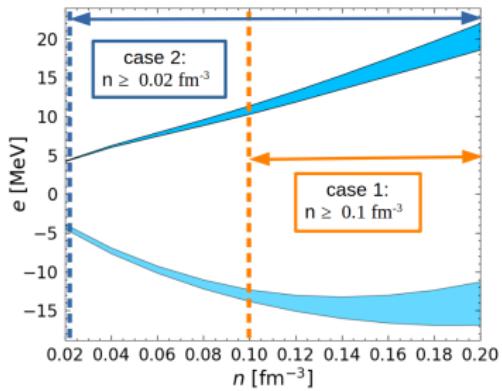
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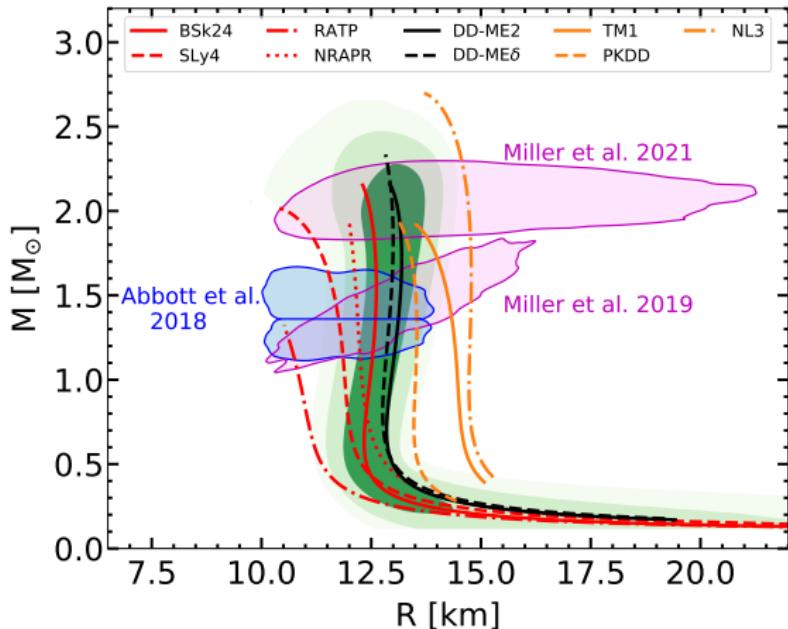
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2. High density (HD) filter: $c_s/c < 1$; $dP/d\rho > 0$; $e_{sym} \geq 0$; and $M_{max} \geq 1.97M_\odot$.

5.1. Compatibility of the posterior and observations

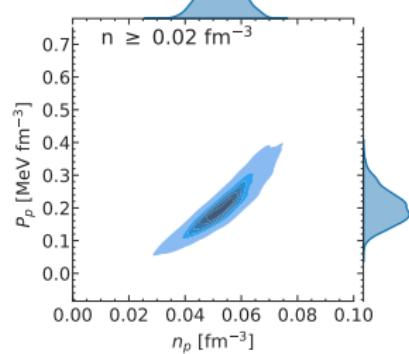
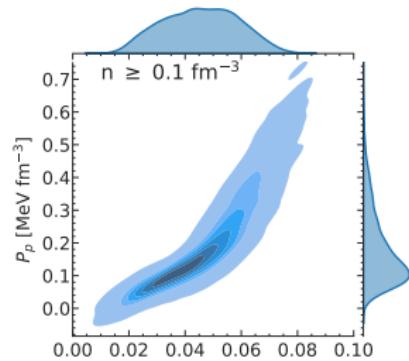


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→ Posterior M-R distribution is **in good agreement** with results from NICER and LIGO/Virgo.

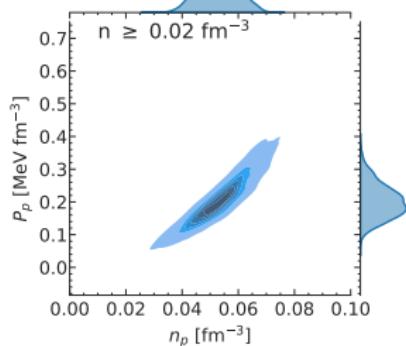
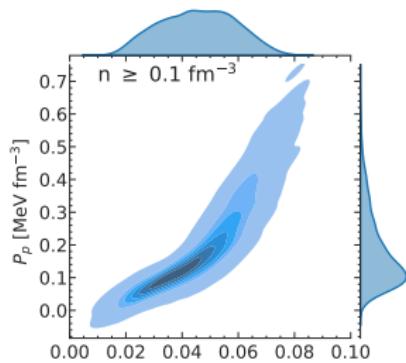
5.2. Uncertainties in pasta-phase properties

Sphere-pasta transition:

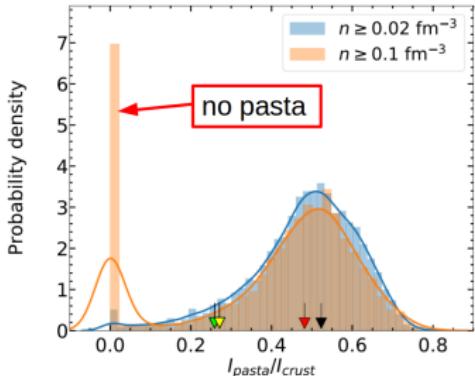
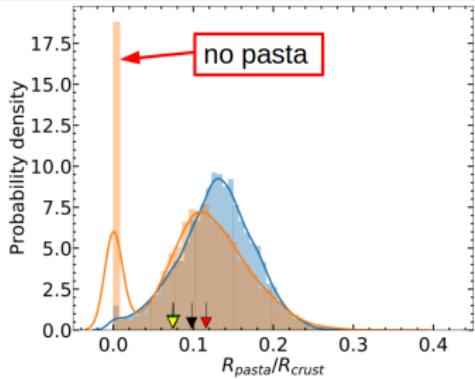


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Thickness&moment of inertia:

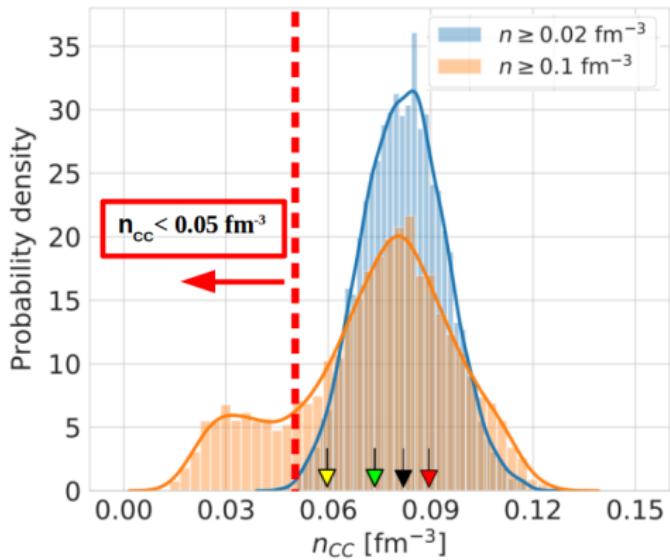


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5.3. Crust-core transition density

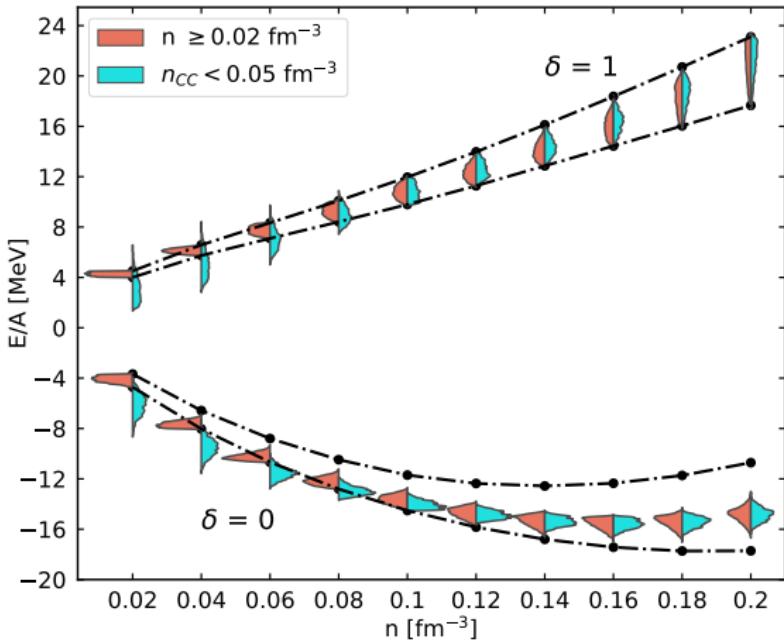
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- Models resulting in $n_{CC} < 0.05 \text{ fm}^{-3}$ are eliminated if the LD filter is applied from 0.02 fm^{-3} .

5.4. Nuclear matter energy

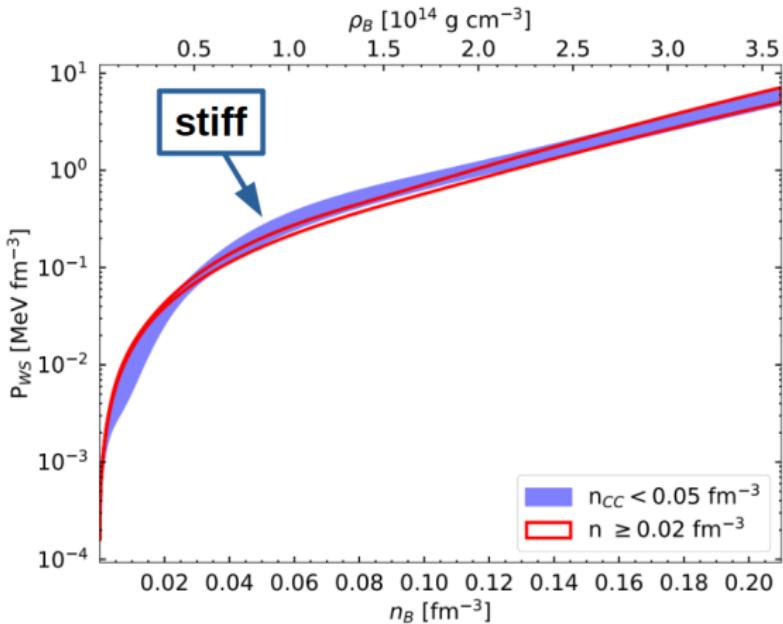
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- Models satisfying the LD filter at $n \geq 0.1 \text{ fm}^{-3}$ and associated $n_{CC} < 0.05 \text{ fm}^{-3}$ result in lower E/A at density $n < 0.1 \text{ fm}^{-3}$.

5.5. Equation of state

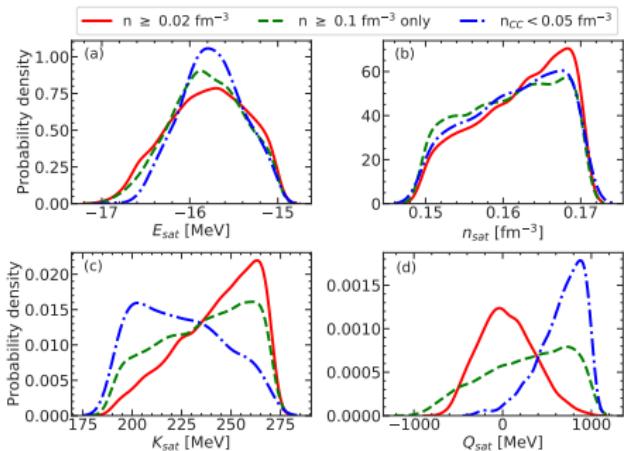
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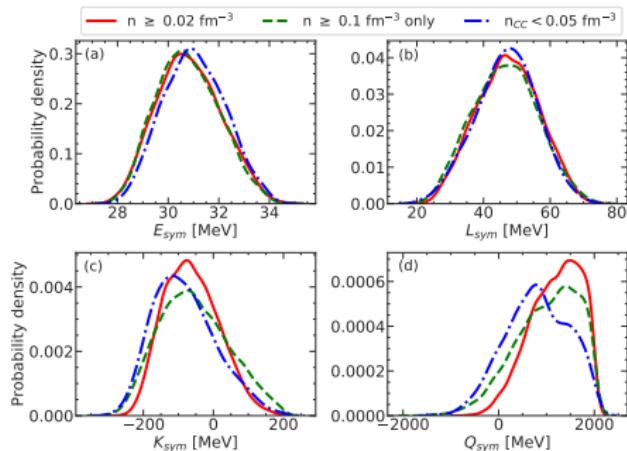
- Models satisfying the LD filter at $n \geq 0.1 \text{ fm}^{-3}$ and associated $n_{CC} < 0.05 \text{ fm}^{-3}$ result in **stiffer EoS** at density $n < 0.1 \text{ fm}^{-3}$.

5.6. Empirical parameters

Isoscalar parameters

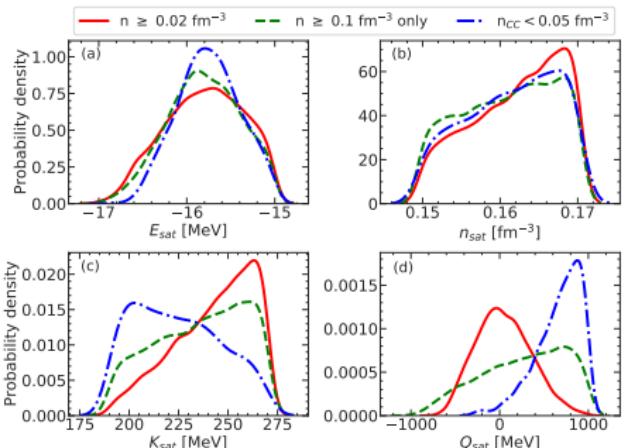


Isovector parameters

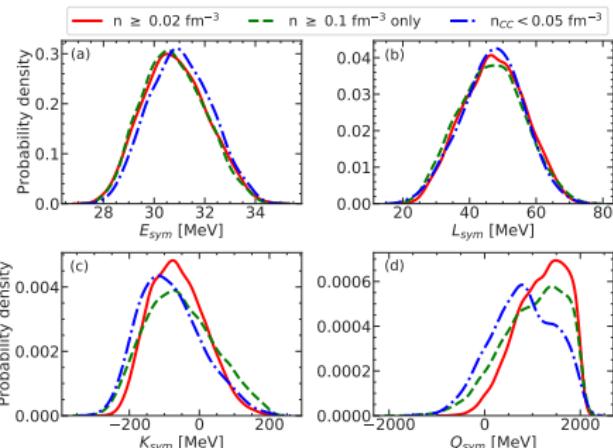


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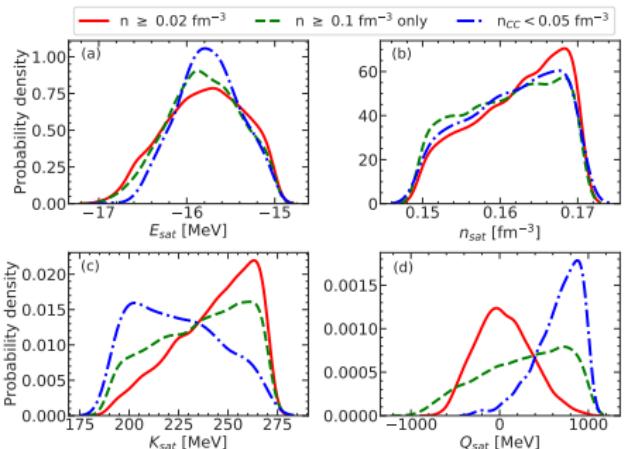
- Taylor expansions ($x = \frac{n - n_{\text{sat}}}{3n_{\text{sat}}}$, where $n_{\text{sat}} \approx 0.15 \text{ fm}^{-3}$):

$$e(n, \delta) \approx E_{\text{sat}} + \frac{1}{2} K_{\text{sat}} x^2 + \frac{1}{6} Q_{\text{sat}} x^3 + \delta^2 \left(E_{\text{sym}} + L_{\text{sym}} x + \frac{1}{2} K_{\text{sym}} x^2 + \frac{1}{6} Q_{\text{sym}} x^3 \right),$$

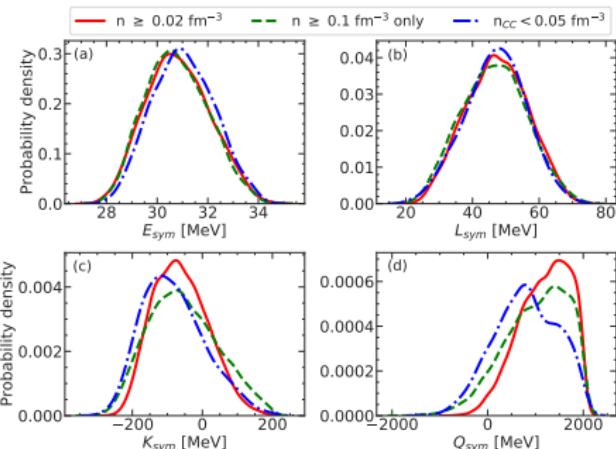
$$P(n, \delta) \approx \frac{n_{\text{sat}}}{3} (1 + 3x)^2 \left[K_{\text{sat}} x + \frac{1}{2} Q_{\text{sat}} x^2 + \delta^2 \left(L_{\text{sym}} + K_{\text{sym}} x + \frac{1}{2} Q_{\text{sym}} x^2 \right) \right].$$

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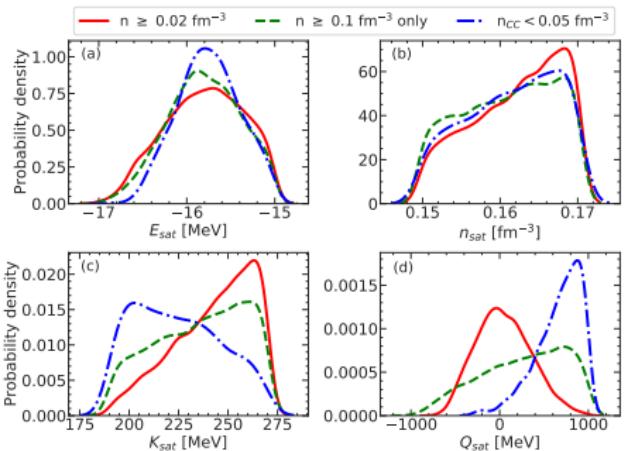
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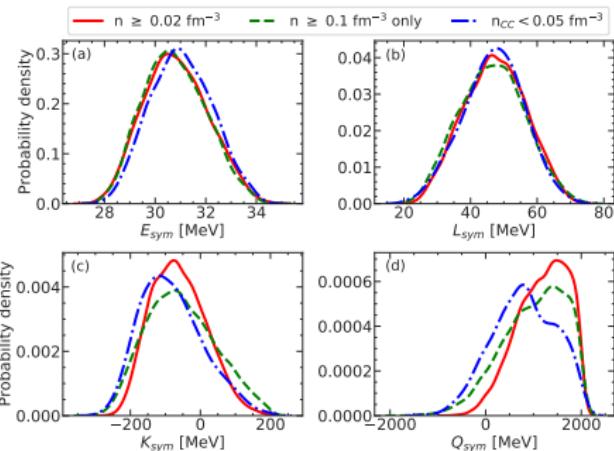
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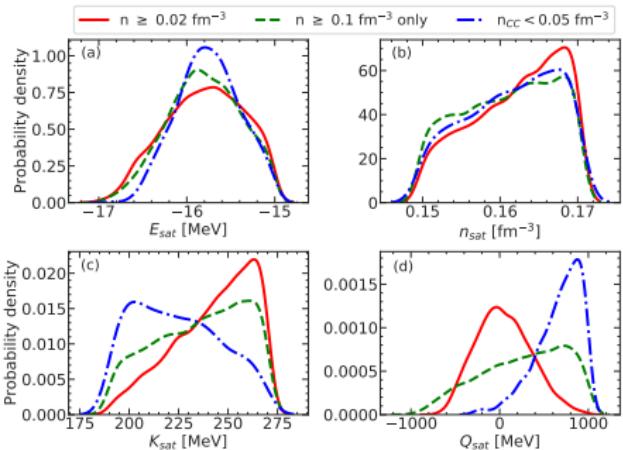
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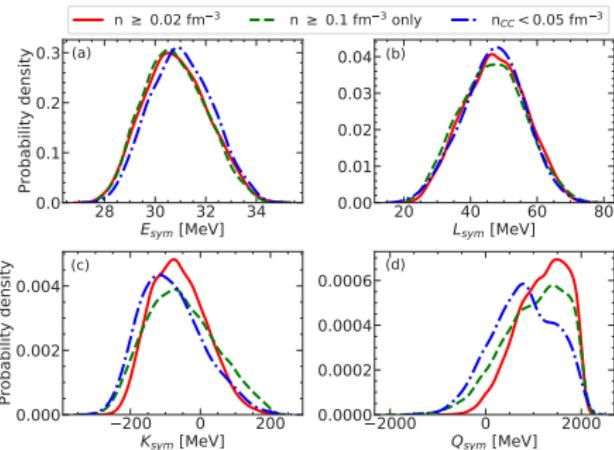
- $n < n_{\text{sat}} \rightarrow x < 0$

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- $n < n_{\text{sat}} \rightarrow x < 0 \rightarrow$ lower K_{sat} and higher Q_{sat} lead to lower energy and stiffer EoS.

5.7. Correlations

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	n_p														
	E_{sat}	n_{sat}	K_{sat}	Q_{sat}	Z_{sat}	E_{sym}	L_{sym}	K_{sym}	Q_{sym}	Z_{sym}	σ_0	b_s	σ_{0c}	β	p
LD+HD ($n \geq 0.02 \text{ fm}^{-3}$)	-0.86	0.20	0.32	-0.27	0.02	0.32	-0.20	-0.25	0.22	-0.00	0.87	0.08	-0.73	-0.84	-0.09
LD+HD ($n \geq 0.1 \text{ fm}^{-3}$)	-0.44	0.03	0.39	-0.42	0.14	0.18	-0.19	-0.40	0.45	-0.14	0.44	0.02	-0.36	-0.47	-0.04
Prior	-0.28	0.01	0.09	-0.11	0.03	-0.04	0.06	-0.49	0.44	-0.06	0.29	0.06	-0.26	-0.22	-0.04

	n_{CC}														
	E_{sat}	n_{sat}	K_{sat}	Q_{sat}	Z_{sat}	E_{sym}	L_{sym}	K_{sym}	Q_{sym}	Z_{sym}	σ_0	b_s	σ_{0c}	β	p
LD+HD ($n \geq 0.02 \text{ fm}^{-3}$)	-0.04	-0.07	0.11	-0.05	-0.02	-0.30	-0.57	-0.15	0.45	-0.15	0.05	0.52	-0.15	-0.04	0.51
LD+HD ($n \geq 0.1 \text{ fm}^{-3}$)	-0.06	-0.06	0.33	-0.46	0.17	-0.15	-0.29	-0.10	0.39	-0.16	0.06	0.34	-0.11	-0.08	0.33
Prior	0.14	0.09	0.13	-0.18	0.02	0.08	-0.56	0.11	0.20	-0.05	-0.17	0.07	0.29	0.18	0.18

- Sphere-pasta transition density: isoscalar bulk parameter (E_{sat}) and **surface parameters** ($\sigma_0, \sigma_{0c}, \beta$) are most influential.
- Crust-core transition density: isovector bulk parameter (L_{sym}) and **surface parameters** (b_s, p) parameters are most influential.

5.8. Surface parameters

- Mass of a spherical nucleus of charge Z and mass number A in vacuum:

$$M(A, Z)c^2 = m_p c^2 Z + m_n c^2 (A - Z)$$

$$+ \underbrace{Ae_{HM}(n_0, I)}_{\text{bulk energy}} + \underbrace{4\pi r_N^2 \left(\sigma_s + \frac{2\sigma_c}{r_N} \right)}_{\text{surface + curvature energies}} + \underbrace{\frac{3}{5} \frac{e^2 Z^2}{r_N}}_{\text{Coulomb energy}}$$

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- Surface and curvature tensions depend on 5 surface parameters:

$$\sigma_s = \sigma_0 \frac{2^{p+1} + b_s}{y_p^{-p} + b_s + (1 - y_p)^{-p}}$$

$$\sigma_c = 5.5 \sigma_s \frac{\sigma_{0,c}}{\sigma_0} (\beta - y_p),$$

5.8. Surface parameters

- Mass of a spherical nucleus of charge Z and mass number A in vacuum:

$$M(A, Z)c^2 = m_p c^2 Z + m_n c^2 (A - Z) + \underbrace{A e_{HM}(n_0, I)}_{\text{bulk energy}} + \underbrace{4\pi r_N^2 \left(\sigma_s + \frac{2\sigma_c}{r_N} \right)}_{\text{surface + curvature energies}} + \underbrace{\frac{3}{5} \frac{e^2 Z^2}{r_N}}_{\text{Coulomb energy}}$$

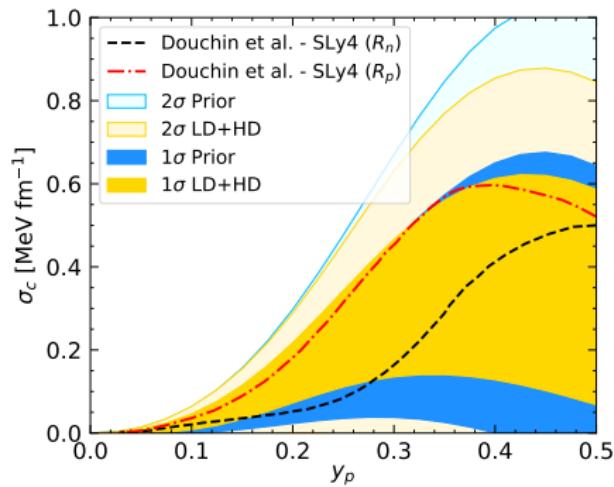
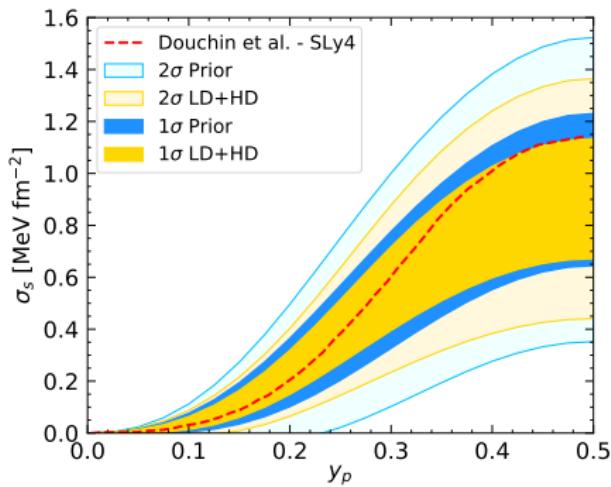
- Surface and curvature tensions depend on 5 surface parameters:

$$\sigma_s = \sigma_0 \frac{2^{p+1} + b_s}{y_p^{-p} + b_s + (1 - y_p)^{-p}}$$

$$\sigma_c = 5.5 \sigma_s \frac{\sigma_{0,c}}{\sigma_0} (\beta - y_p),$$

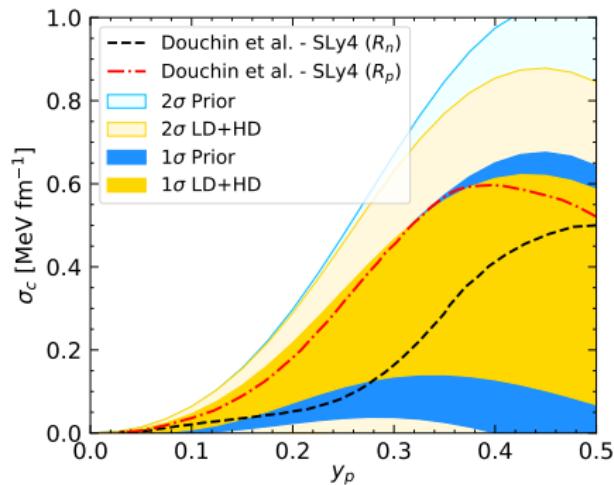
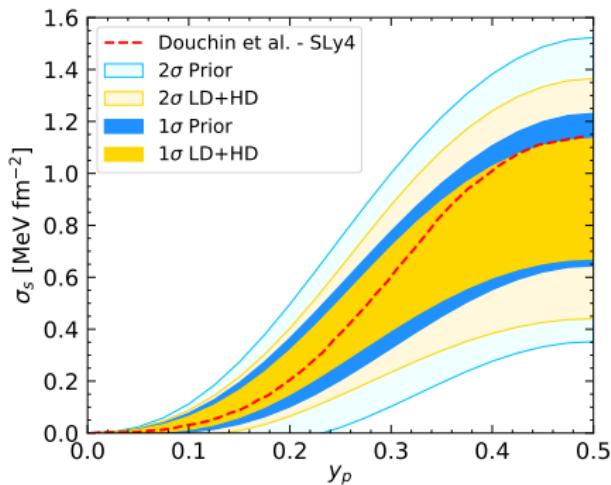
- The 5 surface parameters are obtained by fitting $M(A, Z)$ to experimental nuclear mass table.

5.9. Uncertainties in surface and curvature tensions



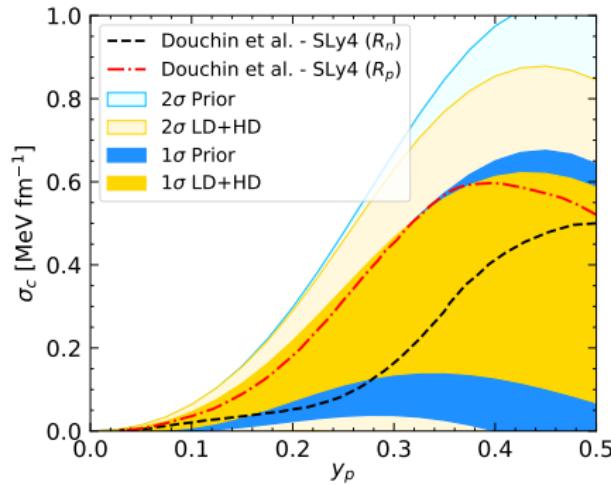
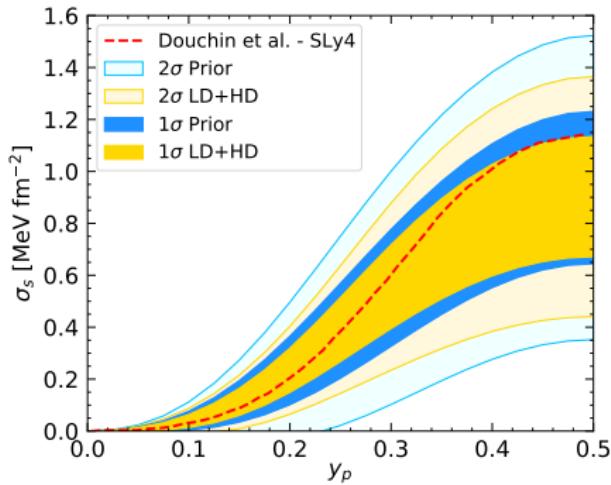
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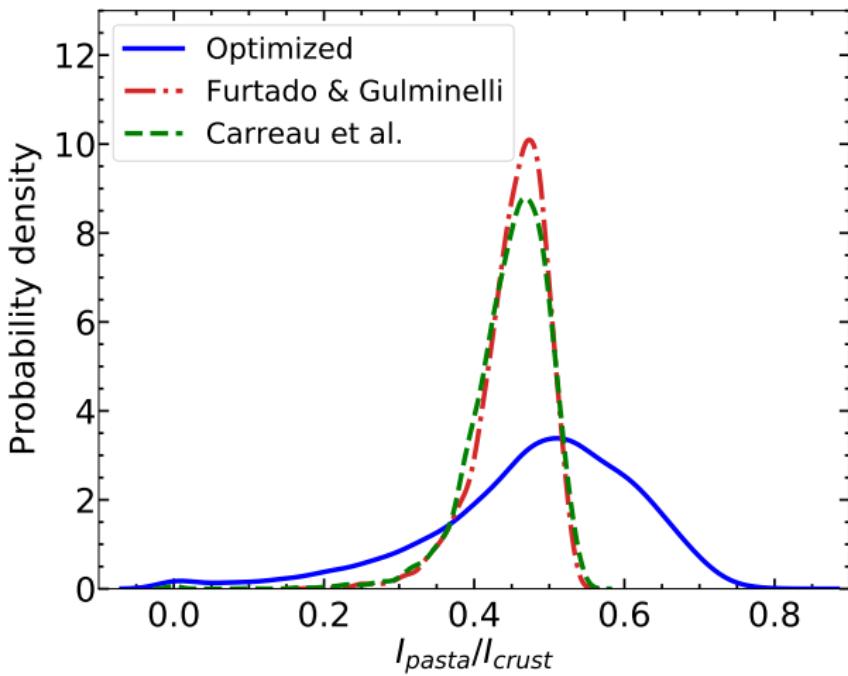
5.9. Uncertainties in surface and curvature tensions



- Absolute uncertainties in surface and curvature tensions increase with increasing proton fraction.
- Relative uncertainties in surface and curvature tensions decrease with increasing proton fraction.
- The posteriors encompass the results from Douchin et al. 2000 for the SLy4 functional.

5.10. Fixing surface parameters

H. Dinh Thi et al. A&A 654, A114 (2021)



→ Fixing surface parameters leads to an underestimation in the uncertainties of pasta-phase properties.

6. Conclusions

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- Properties of the pasta phase are **strongly model dependent**.
- The **low-density** part of the chiral EFT calculation is **crucial** in determining the pasta properties.
- Apart from the bulk parameters, **surface parameters** are also influential in the determination of pasta properties.

**THANK YOU VERY MUCH
FOR YOUR ATTENTION!**



Hoa's very first snowwoman
PhyNuBE-Aussois, 06/12/2021

BACKUP SLIDES

Prior distribution

Parameters	Min	Max
E_{sat} (MeV)	-17	-15
n_{sat} (fm^{-3})	0.15	0.17
K_{sat} (MeV)	190	270
Q_{sat} (MeV)	-1000	1000
Z_{sat} (MeV)	-3000	3000
E_{sym} (MeV)	26	38
L_{sym} (MeV)	10	80
K_{sym} (MeV)	-400	200
Q_{sym} (MeV)	-2000	2000
Z_{sym} (MeV)	-5000	5000
$m*_{sat}$	0.6	0.8
$\Delta m *_{sat} / m$	0.0	0.2
b	1	6

Shape dependence

- Expressions of **surface**, **curvature**, and **Coulomb** energy densities:

$$\epsilon_{surf} = \frac{ud\sigma_s}{r_n}, \quad (1)$$

$$\epsilon_{curv} = \frac{ud(d-1)\sigma_c}{r_n^2}, \quad (2)$$

$$\epsilon_{Coul} = 2\pi(eY_p n_0 r_n)^2 u \eta_{Coul,d}(u). \quad (3)$$

Nuclear matter energy

Dinh Thi, H., et al. . Eur. Phys. J. A 57, 296 (2021)

