Recent applications of Gamow Shell Model in structure and reactions

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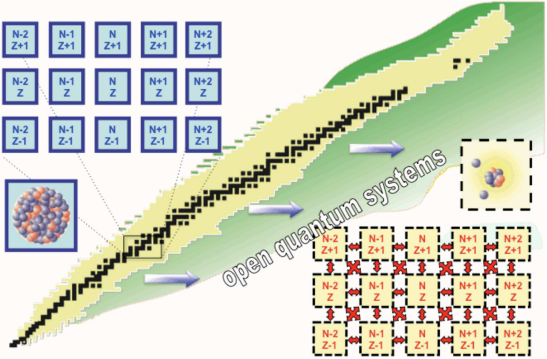
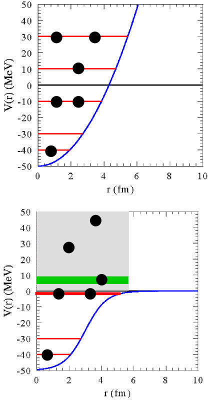
Overview of this presentation

1. Nucleus as an open quantum system
2. Gamow Shell Model
3. Near-threshold behavior of cross-sections, spectroscopic factors, and energies
4. Description of low energy reactions with complex projectiles

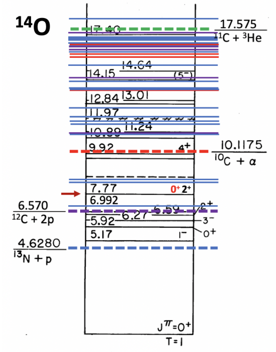
Open quantum system perspective

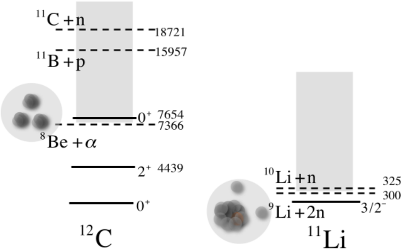




- Drip line studies
- Multidimentional network of correlated states

Open quantum system perspective

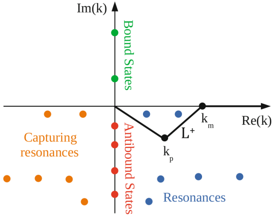


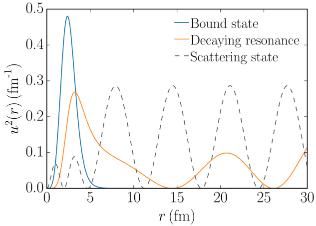


- Shell model state are embedded in the continuum
- Couplings to decay channels
- Formation of collective eigenstates

- Near-threshold clustering
- The standard shell model fails to describe weakly bound or resonance states

Gamow states



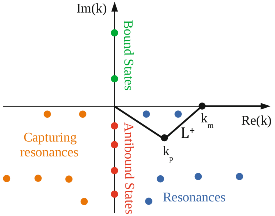


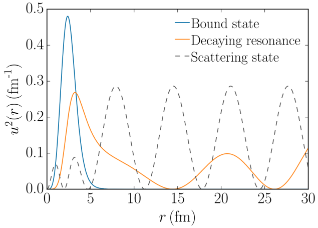
- Bound states. $k \to i\kappa$
- Resonant/Gamow states: poles of the S-matrix. $k \to \kappa_1 + i\kappa_2$
- Scattering states: nonresonant continuum states.

$$\sum_n u_n(r)u_n(r') + \int_{\mathcal{L}^+} u_k(r)u_k(r')dk = \delta(r - r') \qquad u(r) \to e^{ikr}$$

Berggren 1968

Gamow states





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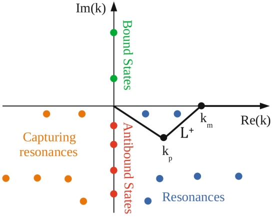
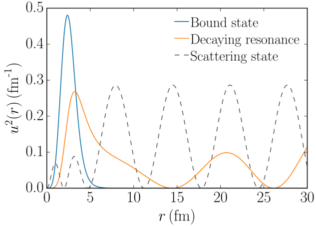
Normalization:
$\int_0^\infty u_k(r)u_k(r')dk$.

Hilbert space	Rigged Hilbert Space
$\int_0^\infty u_k^*(r)u_k(r')dk$	$\int_0^\infty \tilde{u}_k^*(r)u_k(r')dk$

Gamow states





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Resonant states regularized via complex scaling.

$$\int_0^\infty f(r)dr = \int_0^R f(r)dr + \int_0^\infty f(R + x \cdot e^{i\theta})e^{i\theta}dx$$

Many-body Berggren basis

How to use the Berggren basis in practical applications?

$$\int_{\mathcal{L}^+} u_k(r)u_k(r') \approx \sum_{i}^{N_d} u_i(r)u_i(r')$$

Normalized discrete single particle Berggren basis $\{\phi_{\ell,\eta}\}$,

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Normalized discrete single particle Berggren basis $\{\phi_{\ell,\eta}\}$, which can be used to build a many-body basis

$$\sum_n |SD_n\rangle\langle\widetilde{SD_n}| \approx 1 \qquad |SD\rangle = |\phi_1\phi_2...\phi_M\rangle$$

Many-body completeness relation

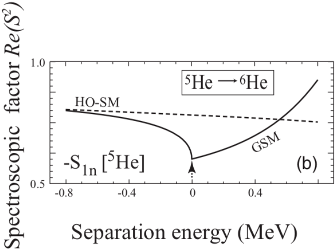
.

Near threshold effects in the spectroscopic factors and energies

Reactions for neutral particles

$$\sigma \sim \begin{cases} k^{2\ell-1} & k>0 \\ k^{2\ell+1} & k<0 \end{cases}$$ Wigner 1948

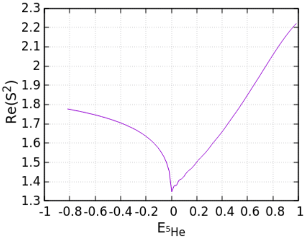
$$S^2 = \sum\!\!\!\!\!\!\int_{\mathcal{B}} \langle \widetilde{\Psi_A^{J_A}} || a^\dagger_{\ell j}(\mathcal{B}) || \Psi_{A-1}^{J_{A-1}} \rangle$$

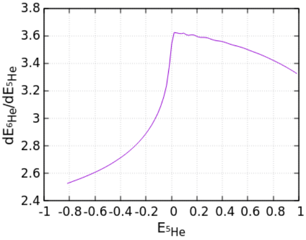


Michel, Nazarewicz, and Płoszajczak 2007

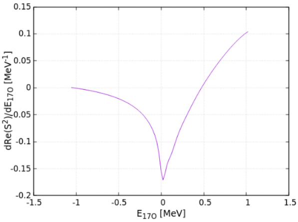
- Interference phenomenon between resonant and non-resonant states.
- Spectroscopic factor (SF) → measure of the occupancy of a single particle shell.
- Threshold effects in SF have the same dependence in k, ℓ as in the Wigner cusp law.

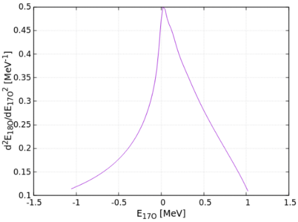
Near threshold effects in the spectroscopic factors and energies





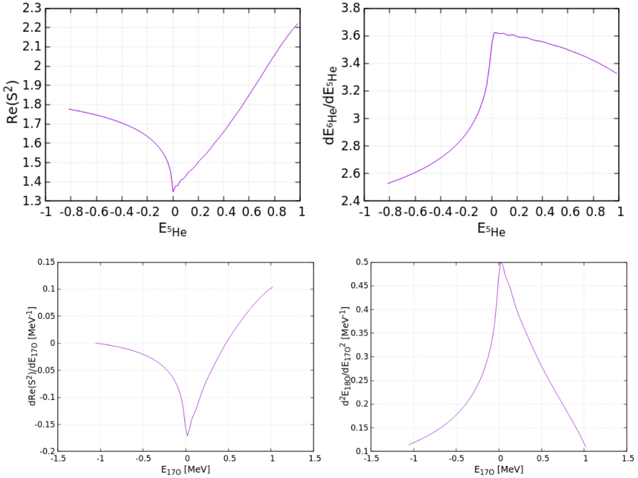
^{4}He core, $\ell = 1$





^{16}O core, $\ell = 2$

Near threshold effects in the spectroscopic factors and energies



^{4}He core, $\ell = 1$

^{16}O core, $\ell = 2$

GSM works in the Slater determinant representation $\rightarrow$ reaction channels cannot be correctly defined.

Coupled channel formulation of GSM (GSM-CC)

The channel wave-function is defined as

$$|(c,r)\rangle = \mathcal{A}\left\{|\Psi_T^{J_T}\rangle \otimes |\Psi_P^{J_P}\rangle\right\}_{M_A}^{J_A}.$$

$\uparrow = \sum_m c_m|SD_m\rangle$ from GSM

The wave functions are

$$|\Psi_{M_A}^{J_A}\rangle = \sum_c \int_0^\infty \left(\frac{u_c(r)}{r}\right)|(c,r)\rangle r^2 dr.$$

The Schrödinger equation becomes the coupled-channel equation

$$\sum_c \int_0^\infty r^2 (H_{cc'}(r,r') - EN_{cc'}(r,r') \frac{u_c(r)}{r} = 0.$$

The quantum number $c \to \{A-a, J^T; a, \ell, J_{int}, J_P\}$.

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The quantum number $c \rightarrow \{A-a, J^T; a, \ell, J_{int}, J_P\}$.
One can define entrance and exit channels and use the standard formulas to get the cross-sections.

Coupled channel formulation of GSM (GSM-CC)

One separates the Hamiltonian as

$$\hat{H} = \hat{H}_p + \hat{H}_t + \hat{H}_{tp} \rightarrow \begin{cases} \hat{H}_p[|\Psi_T^{J_T}\rangle \otimes |\Psi_P^{J_P}\rangle] = |\Psi_T^{J_T}\rangle \otimes \hat{H}_p|\Psi_P^{J_P}\rangle \\ \hat{H}_t[|\Psi_T^{J_T}\rangle \otimes |\Psi_P^{J_P}\rangle] = \hat{H}_t|\Psi_T^{J_T}\rangle \otimes |\Psi_P^{J_P}\rangle \end{cases}$$

H_{CM} H_{int}

$$|\Psi_p^{J_p}\rangle = [|K_{CM}, L_{CM}\rangle \otimes |K_{int}, L_{int}\rangle]_{M_p}^{J_p} \rightarrow |\Psi_p^{J_p}\rangle^{HO} = \left[|K_{CM}, L_{CM}\rangle^{HO} \otimes |K_{int}, L_{int}\rangle^{HO}\right]_{M_p}^{J_p}$$

Normalizing $\langle\Psi_p^{J_p}|\Psi_p^{J_p}\rangle = \delta(K_{CM} - K'_{CM})$ is difficult.

$$|\Psi_p^{J_p}\rangle^{HO} = \sum_N C_N^{HO}|SD_N\rangle^{HO} = \sum_n C_n|SD_n\rangle$$

Target SD generated by GSM

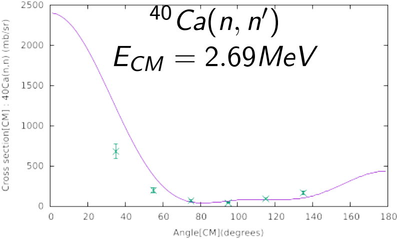
Previous studies using GSM-CC

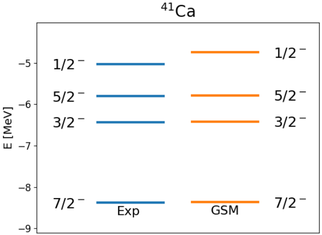
What has been achieved so far with the channel representation:

- $^{18}Ne(p,p')$ Jaganathen, Michel, and Płoszajczak 2014
- $^{7}Be(p,\gamma)^{8}B$ Fossez et al. 2015
- $^{6}Li(p,\gamma)^{7}Be$ and $^{6}Li(n,\gamma)^{7}Li$ Dong et al. 2017
- $^{4}He(d,d)$ Mercenne, Michel, and Płoszajczak 2019
- ...

Cross-sections of ^{40}Ca(d,p)^{41}Ca transfer reaction

- ^{40}Ca core and valence particles in the *fp*-shell.
- The Hamiltonian consist of a Woods-Saxon potential and the FHT interaction fitted using the spectra of ^{41}Ca, ^{41}Sc, ^{42}Ca, ^{42}Ti, and ^{42}Sc.





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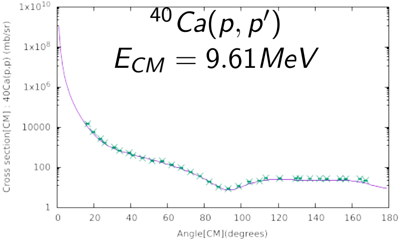
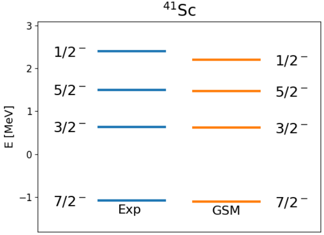
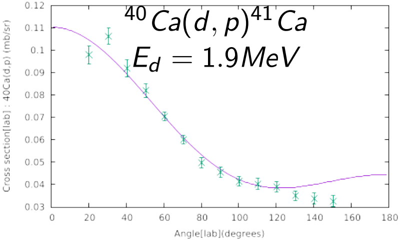
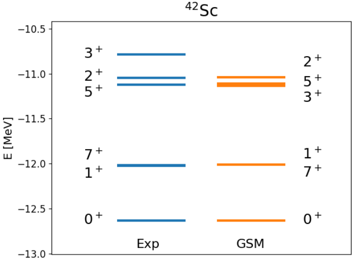






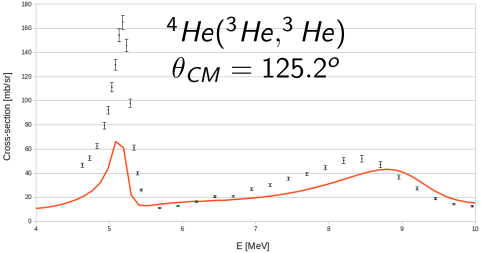


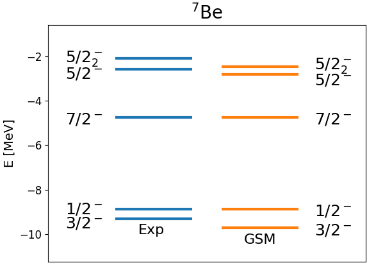
Reactions with clustered wavefuntions

- ^4He core and valence particles in the p-shell.
- Interaction fitted using the spectra of ^6Li, ^7Be and ^7Li.

First reactions to take two different partitions:

- $^6Li + p$
- $^4He + {}^3He$





Conclusions

- Near-threshold phenomena can be studied using an open quantum system formulation of the shell model: the Gamow Shell Model.
- Unification of nuclear structure and nuclear reactions is possible via the shell model treatment of reaction channels.
- First attempts to calculate transfer reactions and reactions in strongly clustered systems in GSM.

Thank you for your attention!

Generating a Berggren basis

We have to solve the radial Schrödinger equation

$$u_\ell''(r) = \left[\frac{\ell(\ell+1)}{r^2} + v_\ell(r) - k^2\right] u_\ell(r),$$

with the following boundary conditions

$$\begin{array}{lcl} u(r) & \overset{r\sim 0}{\sim} & C_0 r^{\ell+1} \\ u(r) & \overset{r\to\infty}{\sim} & C^+(k) H^+_{\ell\eta}(kr) + C^-(k) H^-_{\ell\eta}(kr) \end{array}$$

Bound and resonant states $C^-(k) = 0$.

A few comments on the solution of the GSM

We usually use a Hamiltonian with

- one-body part.
- two-body part.

We can solve in the same way as in standard Shell Model.
We use the so called overlap method.

$$\sum_n \tilde{u}_n(r)u_n(r') + \int_{\mathcal{L}^+} \tilde{u}_k(r)u_k(r')dk = \delta(r - r').$$

We maximize $\langle \Psi_{pivot} | \Psi_i \rangle$

Cross sections

We can use the standard cross-section formulas. The channel wave-function has the asymptotic behavior

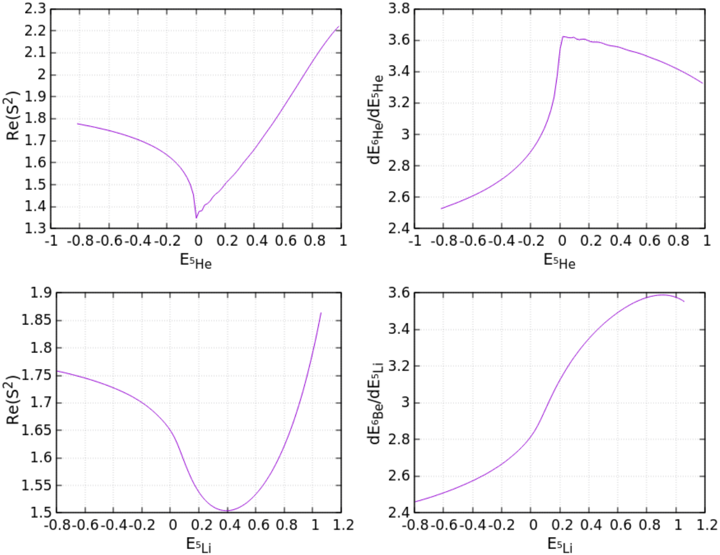
$$u_c^{eJ_A^\pi}(r) \overset{r\to\infty}{\to} \delta_{ce} F_{\ell_e \eta_e}(k_e r) - T_{ec}^{J_A} H^+_{\ell_c \eta_c}(k_c r)$$

The cross-section is

$$\frac{d\sigma_{\tilde{e}\to\tilde{c}}}{d\Omega}(\theta) = \frac{1}{(2J_{int}+1)(2J_T^{\tilde{e}}+1)} \sum_{M_p^{\tilde{e}} M_t^{\tilde{e}} M_p^{\tilde{c}} M_t^{\tilde{c}}} \frac{K_{CM}^{\tilde{c}}}{K_{CM}^{\tilde{e}}} \left| f_{\tilde{e} M_p^{\tilde{e}} M_t^{\tilde{e}} \to \tilde{c} M_p^{\tilde{c}} M_t^{\tilde{c}}}(\theta) \right|^2$$

where the form factor is obtained from the T-matrix.

Observations on the energy derivatives - ^{4}He core



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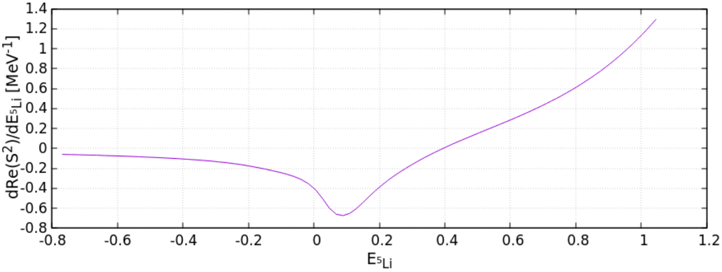
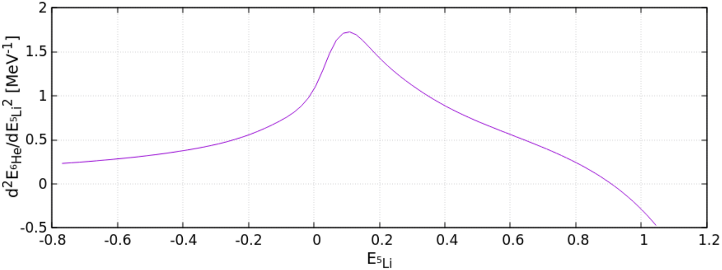
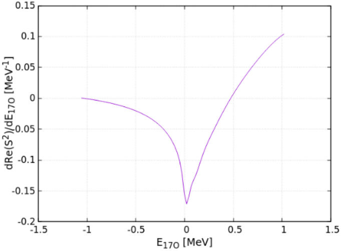
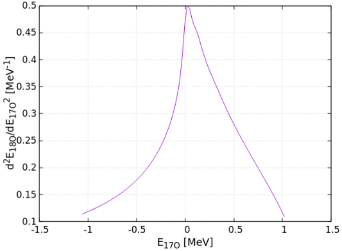




Observations on the energy derivatives - ^{16}O core



(a)



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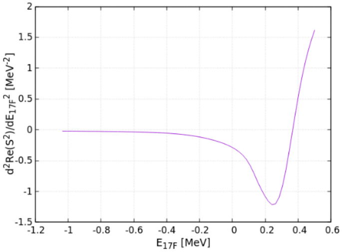

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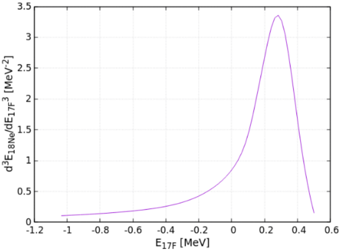
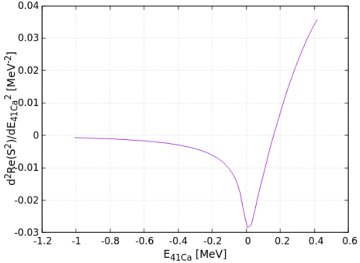
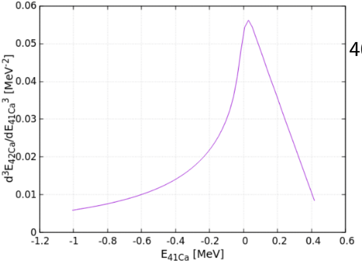


(d)

Observations on the energy derivatives





^{40}Ca core, $\ell = 3$

Interaction

The GSM Hamiltonian is:

$$H = \sum_i \left[\frac{p_i^2}{2m} + V_{WS}(r_i) + V_C(r_i) \right] + V_{res,12}.$$

The Woods-Saxon (WS) potential given by

$$V(r) = -V_{WS}\dot{f}(r) - 4V_{so}(\vec{l} \cdot \vec{s})\frac{1}{r}\left|\frac{df(r)}{dr}\right|,$$

and the WS form factor

$$f(r) = \left[1 + \exp\left(\frac{r - R_0}{d}\right)\right]^{-1}.$$

Interaction

The FHT Furutani, Horiuchi, and Tamagaki 1979 interaction is written in terms of spin-isospin operators Π_{ST} DeShalit and Feshbach 1974:

$$V_c(r) = \sum_{S,T=0,1} V_c^{ST} f_c^{ST}(r) \Pi_{ST}$$

$$V_{LS}(r) = (\vec{L} \cdot \vec{S}) V_{LS}^{11} f_{LS}^{11}(r) \Pi_{11}$$

$$V_T(r) = S_{ij} \sum_{T=0,1} V_T^{1T} f_T^{1T}(r) \Pi_{1T},$$

where $S_{ij} = 3(\vec{\sigma}_i \cdot \hat{r})(\vec{\sigma}_j \cdot \hat{r}) - \vec{\sigma}_i \cdot \vec{\sigma}_j$, $f_i^{ST}(r)$ are radial form factors and V_i^{ST} are parameters to be optimized.

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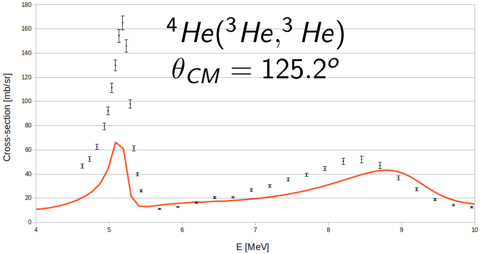

J pi	3He+4He	6Li +p
$3/2^-$	15.24%	84.76%
$1/2^-$	17.41%	82.59%
$7/2^-$	42.62%	57.38%
$5/2^-_1$	50.68%	49.32%
$5/2^-_2$	0%	100%