

# Perturbation Theory of Large Scale Structure

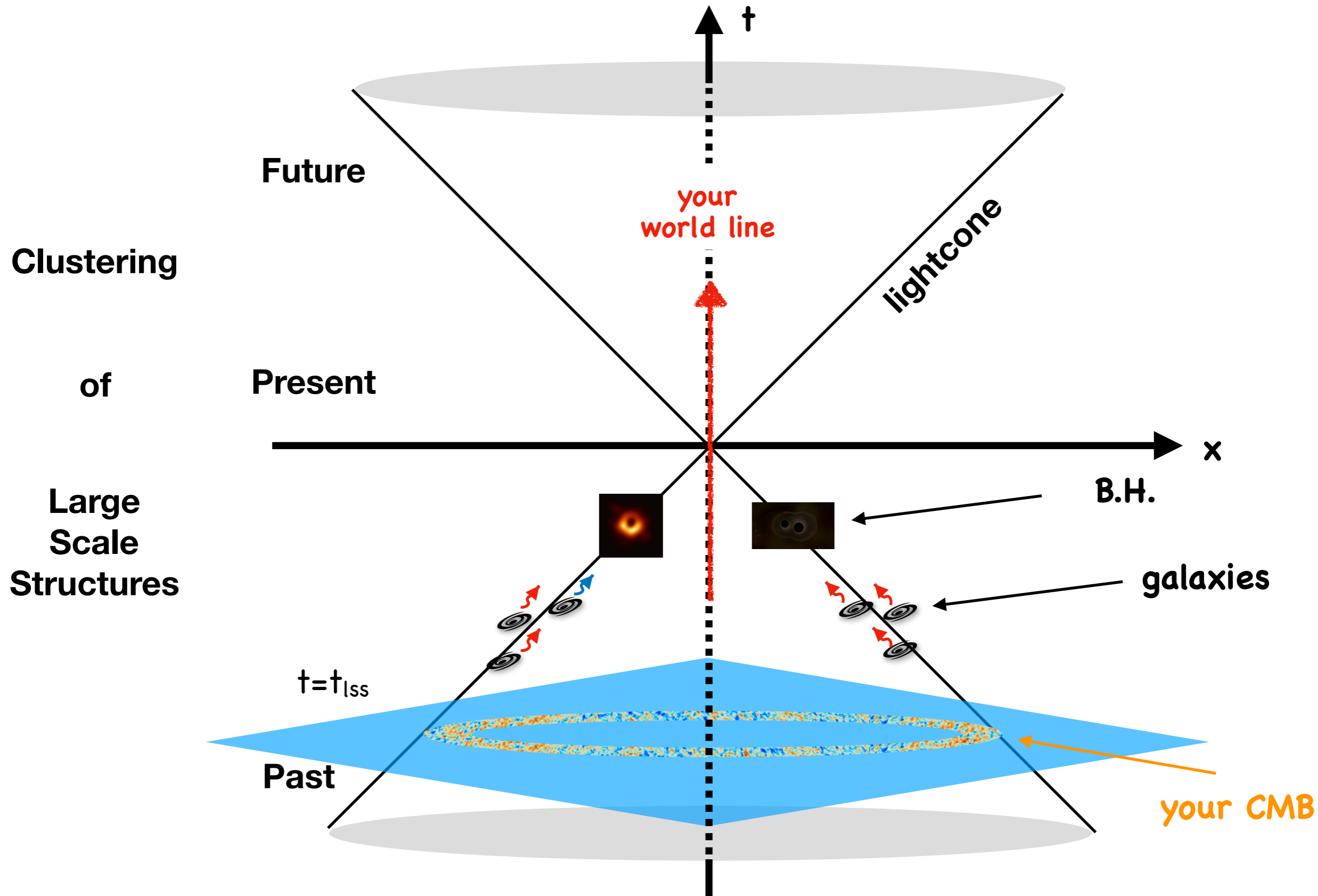
## Numerical Code: CLASS-PT

**Pierros Ntelis**  
**POSTDOC in CPPM**

“ on behalf of the  
**Anton Chudaykin, Mikhail M. Ivanov, Marko Simonović** “  
<https://arxiv.org/pdf/2004.10607.pdf>  
code git forked



# PTofLSS CLASS-PT



**Currently the non-linear and mildly non-linear regime  
of large scale structure is subjective !!!**

**Subjective to what ?**

**To the way we interpret the Large Scale Structures in these regimes !!!**

**Fortunately,**

**theoreticians provide their predictions in num-codes**

**to integrated them and compare those  
models systematically using  
LSS surveys, such as Euclid and DESI, ... !**

# PTofLSS CLASS-PT

## Outline:

- Basics of PToFSS
- Presentation of the code
- IR-Resummation scheme
- CLASS-PT Comparison
- Installation guidelines
- Conclusion and outlook



loops

# PTofLSS CLASS-PT

## Perturbation Theory of Large Scale Structures

PT of LSS  
PTofLSS



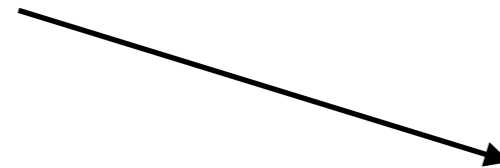
Standard PT (SPTofLSS)

Effective Field Theory (EFTofLSS)

**Motivations from (EFTofLSS)  
suggest the inclusion on SPTofLSS:  
counterterms  
and  
IR-resummation**

**E.Sefusatti ++**

$$P_{\text{Full}}(z, k) \equiv P_{\text{lin}}(z, k) + P_{1\text{-loop,SPT}}(z, k) + P_{\text{ctr}}(z, k)$$



**effective sound speed  
normally nuisance  
under many authors**

**but it can be modelled IMO**

### 2.3 Power Spectrum of Biased Tracers

In order to calculate the one-loop power spectrum of biased tracers, we have to include all possible operators up to third order in the bias expansion:

$$\delta_g = b_1\delta + \epsilon + \frac{b_2}{2}\delta^2 + b_{\mathcal{G}_2}\mathcal{G}_2 + \frac{b_3}{6}\delta^3 + b_{\delta\mathcal{G}_2}\delta\mathcal{G}_2 + b_{\mathcal{G}_3}\mathcal{G}_3 + b_{\Gamma_3}\Gamma_3 + R_*^2\partial^2\delta. \quad (2.7)$$

Here we have defined

$$\mathcal{G}_2(\Phi_g) \equiv (\partial_i\partial_j\Phi_g)^2 - (\partial_i^2\Phi_g)^2, \quad (2.8)$$

where  $\Phi_g$  is gravitational potential. The only cubic operator that gives nontrivial contribution to the one-loop power spectrum can be written as

$$\Gamma_3 \equiv \mathcal{G}_2(\Phi_g) - \mathcal{G}_2(\Phi_v), \quad (2.9)$$

where  $\Phi_v$  is velocity potential.<sup>7</sup> For the definition of  $\mathcal{G}_3$  and relations of our operators to other equivalent choices of basis, see [58]. The term  $\epsilon$  denotes the stochastic contribution which is uncorrelated with the large-scale density field. Poisson noise,

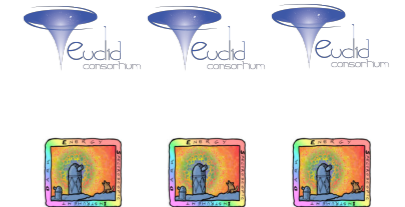
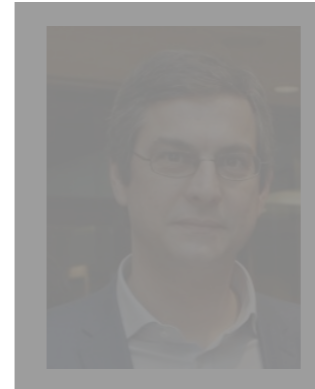
-> **Result to more terms such as:**

$$\begin{aligned} P_{\text{gg}}(z, k) = & b_1^2(z)(P_{\text{lin}}(z, k) + P_{1\text{-loop, SPT}}(z, k)) + b_1(z)b_2(z)\mathcal{I}_{\delta^2}(z, k) \\ & + 2b_1(z)b_{\mathcal{G}_2}(z)\mathcal{I}_{\mathcal{G}_2}(z, k) + (2b_1(z)b_{\mathcal{G}_2}(z) + \frac{4}{5}b_1(z)b_{\Gamma_3}(z))\mathcal{F}_{\mathcal{G}_2}(z, k) \\ & + \frac{1}{4}b_2^2(z)\mathcal{I}_{\delta^2\delta^2}(z, k) + b_{\mathcal{G}_2}^2(z)\mathcal{I}_{\mathcal{G}_2\mathcal{G}_2}(z, k) + b_2(z)b_{\mathcal{G}_2}(z)\mathcal{I}_{\delta^2\mathcal{G}_2}(z, k) \\ & + P_{\nabla^2\delta}(z, k) \\ & + P_{\epsilon\epsilon}(z, k) \longrightarrow \text{stochastic component} \end{aligned}$$

**Gravitational  
Instabilities**

taken from <https://arxiv.org/pdf/2004.10607.pdf>

**E. Sefusatti**

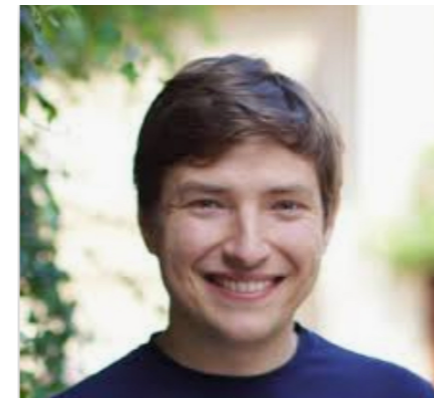


**A. Eggemeier**

What does CLASS-PT do ?

**Anton Chudaykin**

**Marko Simonović**



**Mikhail M. Ivanov**

All the aforementioned +++



## IR-Resummation

is imperative to properly describe the spread of the BAO peak, which was not implemented in the previous formulas

$$P_{\text{lin}}(k) = P_{\text{nw}}(k) + P_{\text{w}}(k)$$

$$\Sigma^2(z) \equiv \frac{1}{6\pi^2} \int_0^{k_S} dq P_{\text{nw}}(z, q) \left[ 1 - j_0\left(\frac{q}{k_{\text{osc}}}\right) + 2j_2\left(\frac{q}{k_{\text{osc}}}\right) \right]$$

$$P_{\text{mm, LO}}(z, k) = P_{\text{nw}}(z, k) + e^{-k^2 \Sigma^2(z)} P_{\text{w}}(z, k)$$

$$P_{XY} = P_{\text{tree, XY}}[P_{\text{mm, LO}}] + P_{1\text{-loop, XY}}[P_{\text{mm, LO}}],$$

$P_{\text{tree, XY}}$  are given by

$$P_{\text{tree, mm}} = P_{\text{nw}}(z, k) + e^{-k^2 \Sigma^2(z)} P_{\text{w}}(z, k) (1 + k^2 \Sigma^2(z)),$$

$$P_{\text{tree, gm}} = b_1 P_{\text{tree, mm}}, \quad P_{\text{tree, gg}} = b_1^2 P_{\text{tree, mm}}.$$

**Simply put it, in Real Space:**

**Wiggle-Non-Wiggle Splitting**

**Sophisticated damping factor**

**Using the Leading Order**

**We map it on**

**Matter Spectra**

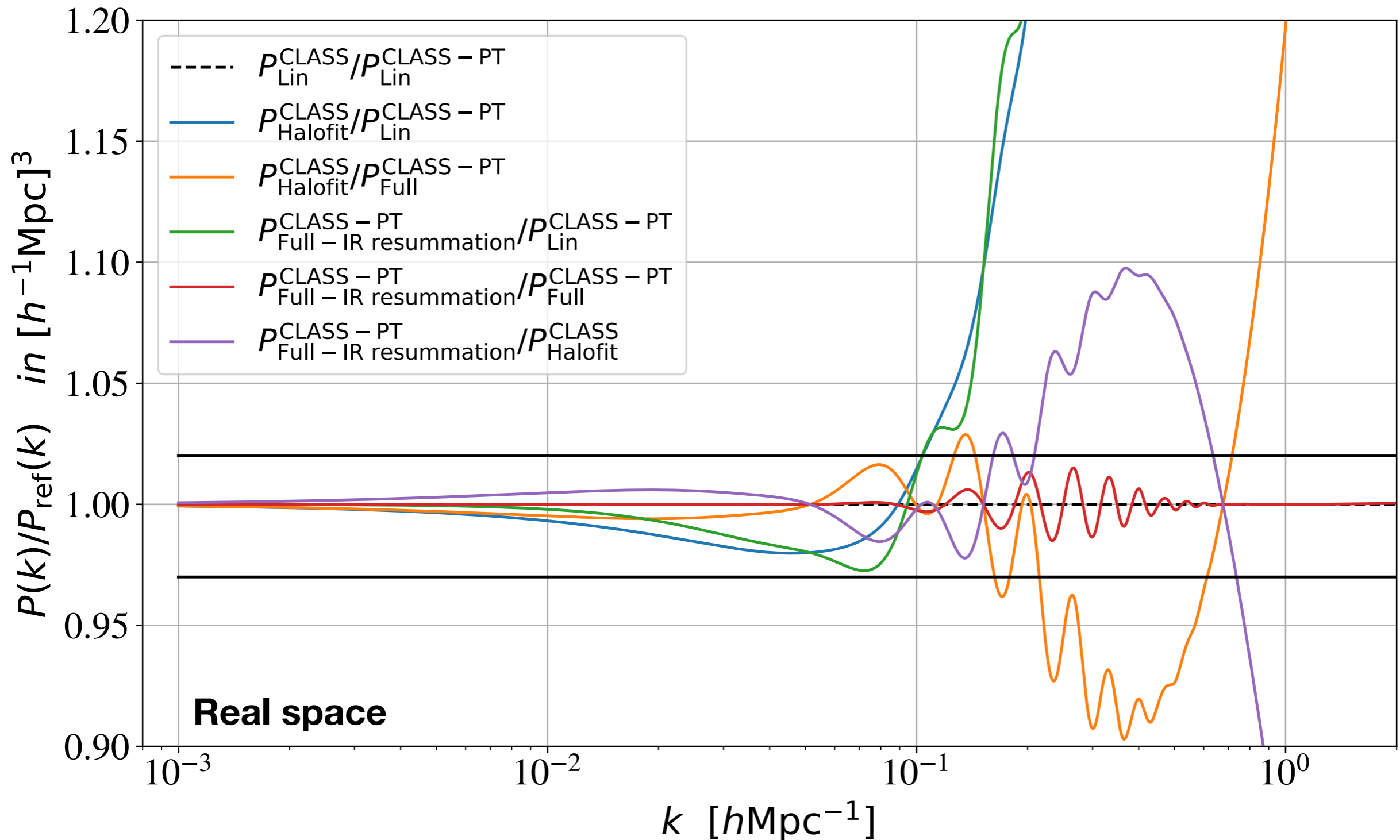
**Galaxy Spectra**

**Based on FFTlog**

taken from <https://arxiv.org/pdf/2004.10607.pdf>

=> For  $k < 0.1 \text{ h/Mpc}$  : LSS NL models agree at  $< 5 \%$

=> For  $k > 0.1 \text{ h/Mpc}$  : LSS NL models models are subjective



**CAVEATS:**

- 1) Note that all those complex concepts are collapsed to simple numerical functions of one argument
- 2) The observables that we use are also collapsed information
- 3) EFTofLSS does the same but with more complex models

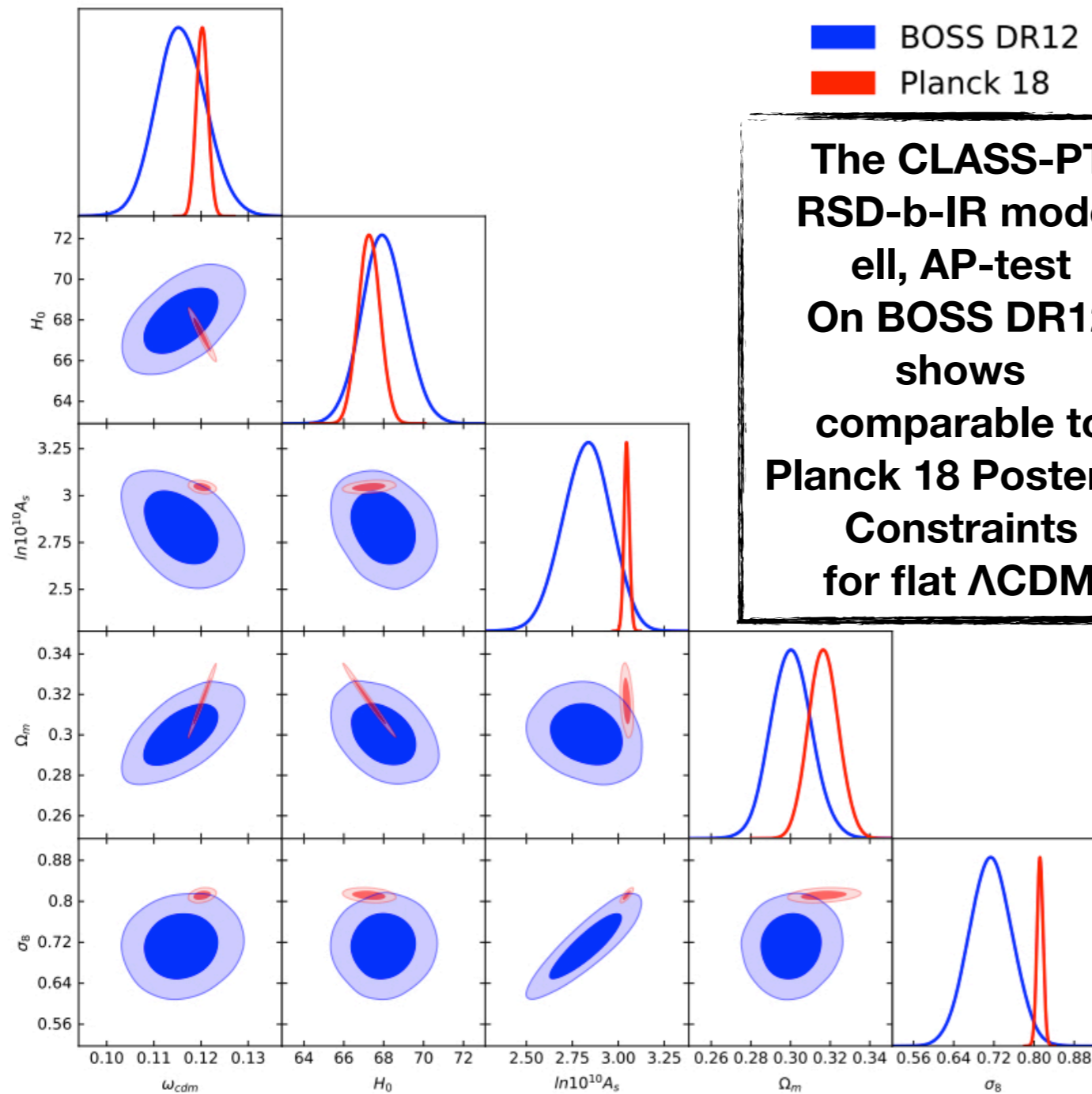
**Systematic treatment:**

To understand better these models:

We should be model the observables in  $\geq 1$  argument functionals

Such as { AP-Test =  $[P_{\text{ell}}(\mathbf{k}), P(\mathbf{k}_{\parallel}, \mathbf{k}_{\text{perp}})]$  , or ... }

taken from <https://arxiv.org/pdf/2004.10607.pdf>



BOSS DR12  
Planck 18

The CLASS-PT RSD-b-IR model ell, AP-test On BOSS DR12 shows comparable to Planck 18 Posterior Constraints for flat  $\Lambda$ CDM

But any NL modelling interprets the LSS under some assumptions

A joint analysis of BOSS-DR12 PLANCK18 With this NL-model to come ! (You) (DESI or Euclid)

taken from <https://arxiv.org/pdf/2004.10607.pdf>

To enjoy multiple CLASS implementations which do not conflict

Do as the authors suggest

except since the CLASS-PT wrapper changes the structure of CLASS wrapper

Then

In the  
python/setup.py  
change

the line 33

```
name='classy',
```

=>

```
name='classy_pt',
```

And line 38

```
ext_modules=[Extension("classy", ["classy.pyx"]),
```

=>

```
ext_modules=[Extension("classy_pt", ["classy.pyx"]),
```

And then in their jupyter notebook in line 7 Import it as  
from classy\_pt import Class

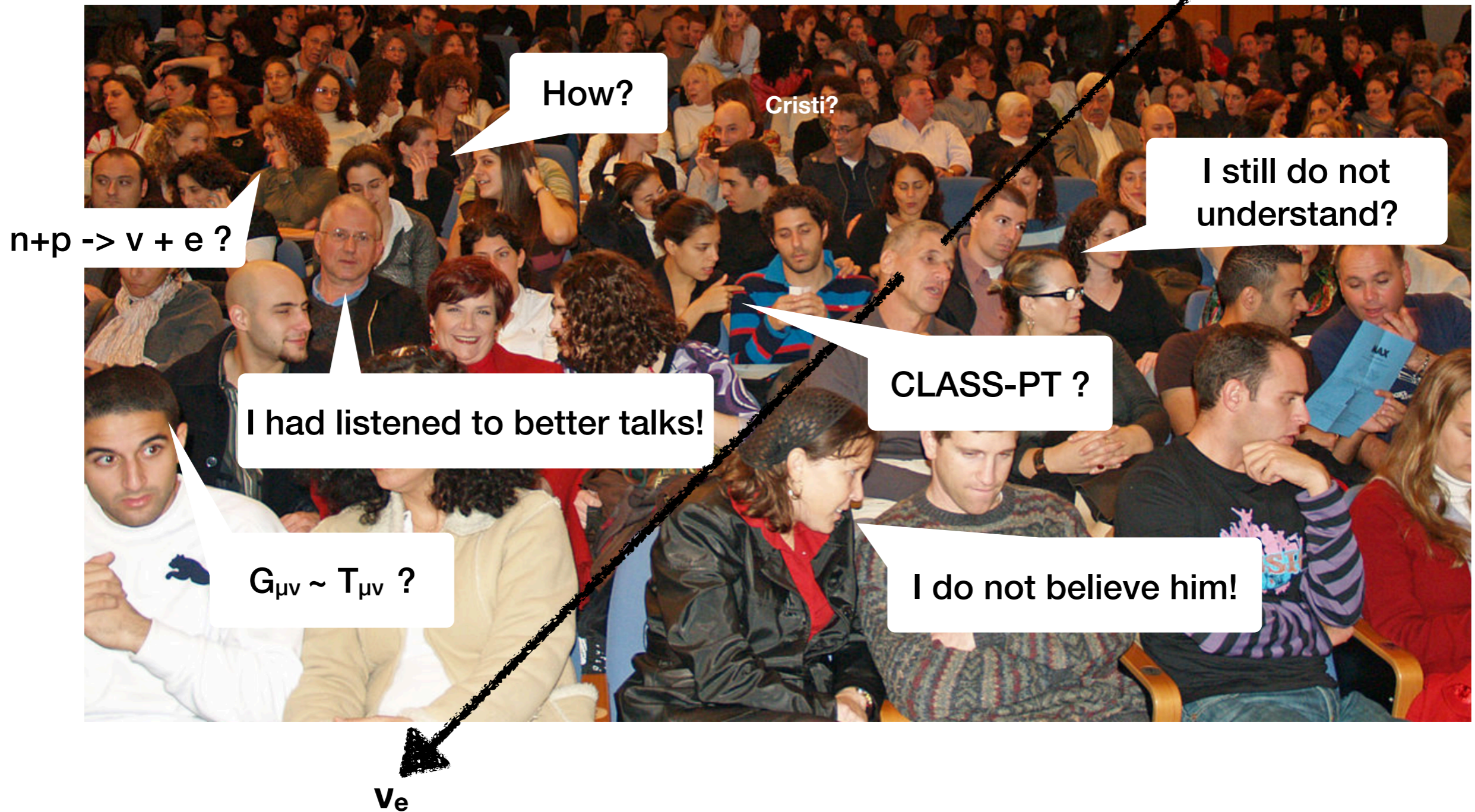
taken from <https://arxiv.org/pdf/2004.10607.pdf>

# PTofLSS CLASS-PT

## Conclusions and Outlook:

- **LSS prediction tools are important since they allow to interpret the LSS with the latest understanding of Perturbation Theories**
- **CLASS-PT is already undertaken in Euclid Consortium GC-SWG-modelling challenge**
- **CLASS-PT FULL-IR Resummation agrees with Halofit < 5 % C.L. (at  $k < 0.1 h/\text{Mpc}$ )**
- **The model selection is under development**
  - **New models**
  - **New simulations to validate them**
  - **New likelihood (Gaussian, Non-Gaussian, ... )**
  - **New free-likelihood analysis ( F.Leclercq )**
- **We should compare these codes/interpretations of LSS systematically with current and future survey**

# Thank you for your Attention!



# PTofLSS CLASS-PT

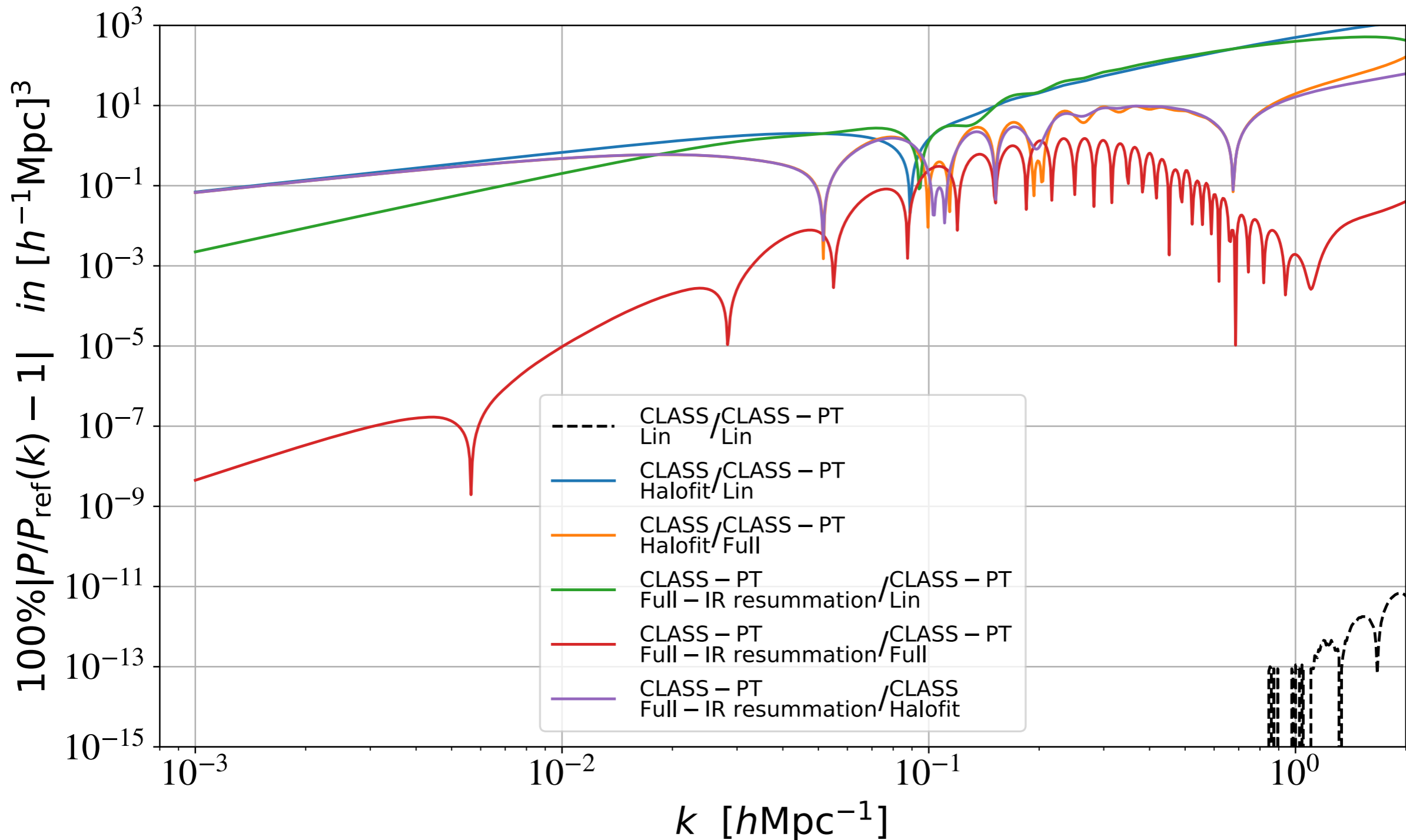
**Back**





=> For  $k < 0.1 \text{ h/Mpc}$  : LSS NL models agree at less than 10% Rel. E.

=> For  $k > 0.1 \text{ h/Mpc}$  : LSS NL models models are subjective



## Caveats of CLASS-PT code

It is important to stress that our implementation of IR resummation at one loop order contains four potential sources of error:

- Imperfectness of wiggly-non-wiggly decomposition,
- Dependence of the damping factor on the separation cutoff,
- Inaccuracy of the factorization prescription,
- One-loop corrections  $\mathcal{O}(P_w^2)$  from two insertions of  $P_w$ .
- **Bispectra are not implemented here, but easily retrievable**

taken from <https://arxiv.org/pdf/2004.10607.pdf>

## EXTRA SLIDES

The FFTLog method is based on the representation of the linear matter power spectrum as a sum of complex power-laws in  $k$ . This is naturally achieved using the discrete Fourier transform with equal spacing in  $\log k$ , hence the name FFTLog [83]. The discrete approximation to the linear power spectrum in a finite momentum interval  $[k_{\min}, k_{\max}]$ , denoted as  $\bar{P}(0, k)$ , can be written as

$$\bar{P}_{\text{lin}}(0, k) = \sum_{m=-N/2}^{m=N/2} c_m k^{\nu+i\eta_m}, \quad (4.1)$$

where the Fourier coefficients  $c_m$  and exponents  $\eta_m$  are given by

$$c_m = \frac{1}{N} \sum_{j=0}^{N-1} P_{\text{lin}}(0, k_j) k_j^{-\nu} k_{\min}^{-i\eta_m} e^{-2\pi i m j / N}, \quad \eta_m = \frac{2\pi m}{\ln(k_{\max}/k_{\min})}. \quad (4.2)$$

**$\nu=-0.3$  (matter) and  $\nu=-1.6$  (biased tracers)**

taken from <https://arxiv.org/pdf/2004.10607.pdf>

## Adding some Redshift Space Distortions:

$$\begin{aligned}
 P_{\text{gg,RSD}}(z, k, \mu) = & Z_1^2(\mathbf{k}) P_{\text{lin}}(z, k) + 2 \int_{\mathbf{q}} Z_2^2(\mathbf{q}, \mathbf{k} - \mathbf{q}) P_{\text{lin}}(z, |\mathbf{k} - \mathbf{q}|) P_{\text{lin}}(z, q) \\
 & + 6 Z_1(\mathbf{k}) P_{\text{lin}}(z, k) \int_{\mathbf{q}} Z_3(\mathbf{q}, -\mathbf{q}, \mathbf{k}) P_{\text{lin}}(z, q) \\
 & + P_{\text{ctr,RSD}}(z, k, \mu) + P_{\epsilon\epsilon,\text{RSD}}(z, k, \mu) .
 \end{aligned} \tag{2.13}$$

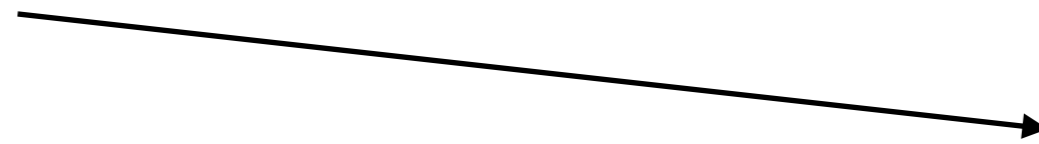
**Kernels become more complicated  
but smartly solvable by the authors !!!**

taken from <https://arxiv.org/pdf/2004.10607.pdf>

$$P_{\text{Full}}(z, k) \equiv P_{\text{lin}}(z, k) + P_{1\text{-loop, SPT}}(z, k) + P_{\text{ctr}}(z, k)$$

$$P_{1\text{-loop, SPT}}(z, k) \equiv D^4(z) |P_{13}(k) + P_{22}(k)|$$

$$P_{\text{ctr}}(z, k) \equiv -2c_s^2(z)k^2 P_{\text{lin}}$$



**Counter term  
Needed for the  
Consistency of  
1-loop result**

**effective sound speed  
normally nuisance  
under many authors**

**but it can be modelled IMO**

$$P_{22}(k) = 2 \int_{\mathbf{q}} F_2^2(\mathbf{q}, \mathbf{k} - \mathbf{q}) P_{\text{lin}}(q) P_{\text{lin}}(|\mathbf{k} - \mathbf{q}|),$$

$$P_{13}(k) = 6P_{\text{lin}}(k) \int_{\mathbf{q}} F_3(\mathbf{k}, -\mathbf{q}, \mathbf{q}) P_{\text{lin}}(q).$$

$F_{i=2,3}(\mathbf{k}, \mathbf{k}, \mathbf{k})$  : Standard Perturbation Theory Kernels of 2nd,3rd order  
see Bernardeau, Gaztañaga, Scoccimaro, ++

## Perturbed Einstein-Boltzmann Equation Solvers

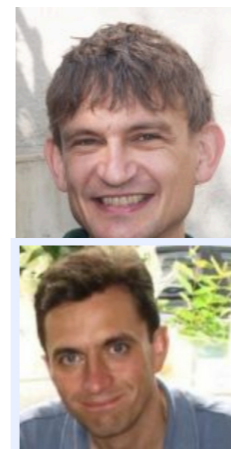
(CLASS)

**C**osmological  
**L**inear  
**A**nisotropy  
**S**olving  
**S**ystem



(CAMB)

**C**ode for the  
**A**nisotropies  
in the cosmic  
**M**icrowave  
**B**ackground



Despite their names, they actually solve numerically the 8 coupled Perturbed Einstein-Boltzmann equations described in their references.

for a short description see

$$\text{Which result to } P_{\text{Lin}}(z,k) = D^2(z) P_{\text{mm}}(k,z=0)$$

2.3 Power Spectrum of Biased Tracers

In order to calculate the one-loop power spectrum of biased tracers, we have to include all possible operators up to third order in the bias expansion:

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Here we have defined

$$\mathcal{G}_2(\Phi_g) \equiv (\partial_i\partial_j\Phi_g)^2 - (\partial_i^2\Phi_g)^2, \quad (2.8)$$

where  $\Phi_g$  is gravitational potential. The only cubic operator that gives nontrivial contribution to the one-loop power spectrum can be written as

$$\Gamma_3 \equiv \mathcal{G}_2(\Phi_g) - \mathcal{G}_2(\Phi_v), \quad (2.9)$$

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Free Parameters:

$b_1 \equiv$  linear bias

$b_2 \equiv$  Quadratic bias

$b_{\mathcal{G}_2} \equiv$  Tidal bias

$b_{\delta\mathcal{G}_2} \equiv$  Perturbations of Tidal bias

$b_{\Gamma_3} \equiv$  non – trivial cubic 1 – loop bias

$R_* \equiv$  Parameter of the 2nd order perturbations of the field  $\delta$

-> Result to more terms such as:

$$P_{gg}(z, k) = b_1^2(z)(P_{lin}(z, k) + P_{1-loop, SPT}(z, k)) + b_1(z)b_2(z)\mathcal{I}_{\delta^2}(z, k) + 2b_1(z)b_{\mathcal{G}_2}(z)\mathcal{I}_{\mathcal{G}_2}(z, k) + (2b_1(z)b_{\mathcal{G}_2}(z) + \frac{4}{5}b_1(z)b_{\Gamma_3}(z))\mathcal{F}_{\mathcal{G}_2}(z, k) + \frac{1}{4}b_2^2(z)\mathcal{I}_{\delta^2\delta^2}(z, k) + b_{\mathcal{G}_2}^2(z)\mathcal{I}_{\mathcal{G}_2\mathcal{G}_2}(z, k) + b_2(z)b_{\mathcal{G}_2}(z)\mathcal{I}_{\delta^2\mathcal{G}_2}(z, k) + P_{\nabla^2\delta}(z, k) + P_{\epsilon\epsilon}(z, k) \longrightarrow \text{stochastic component}$$

Gravitational Instabilities

taken from <https://arxiv.org/pdf/2004.10607.pdf>

# PToF LSS CLASS-PT

## EXTRA SLIDES

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