

Generalized Dark Matter Cosmological Observables & Constraints

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Dark Energy Colloquium – May 2020

Success of CDM ?

Dark matter : Density

$$\rho_{cdm}(a) = \rho_{cdm,0} \cdot a^{-3}$$

Dark matter :
equation of state parameter

$$P_{cdm} = \omega \cdot \rho_{cdm} \cdot c^2$$

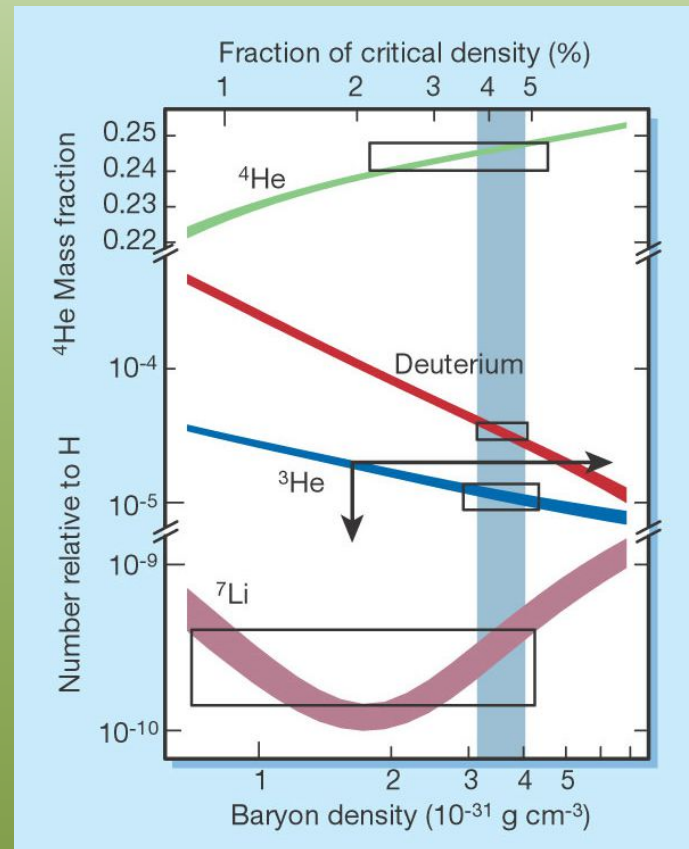
$$\omega = 0$$

Dark matter :
effective sound speed

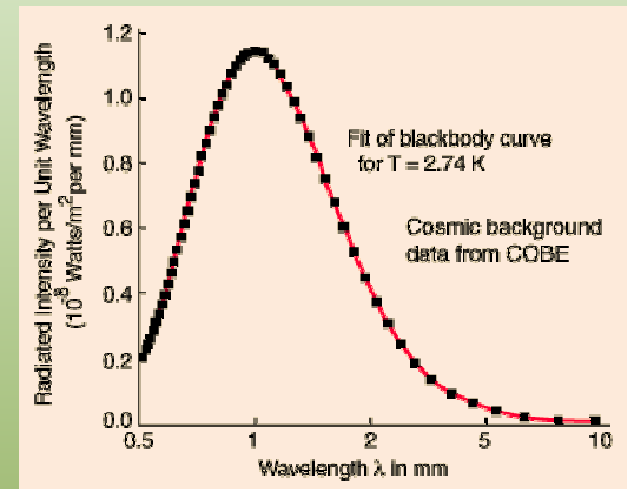
$$c_s^2 = \delta P_{cdm} / \delta \rho_{cdm}$$

$$c_s^2 = 0$$

Success of CDM ?



BBN



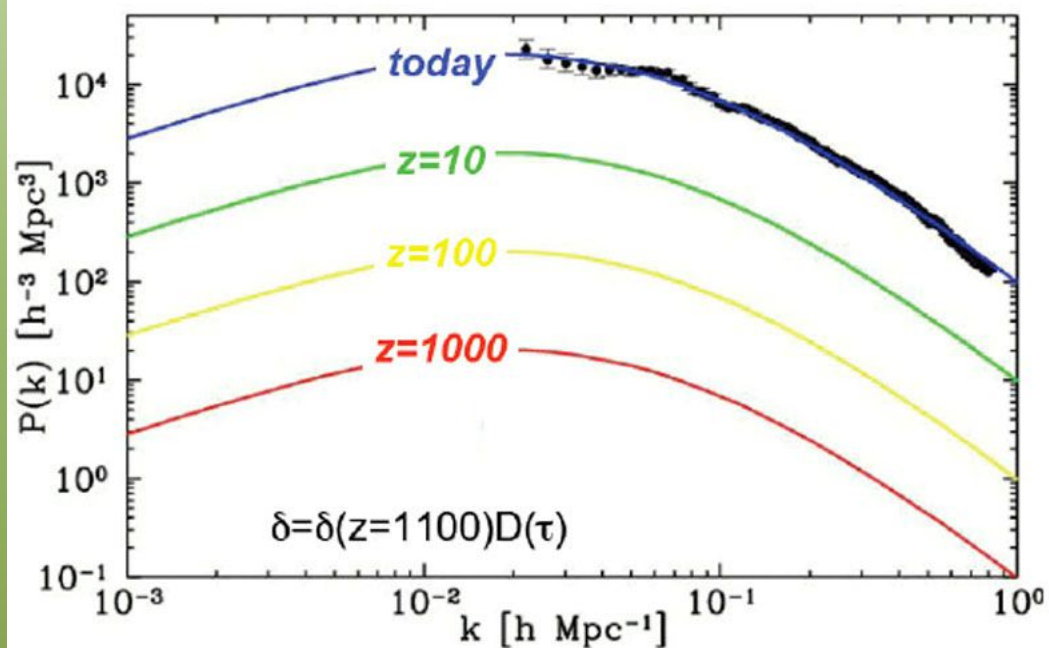
Black body CMB

Mass-To-Light Ratios

Type of Object	Mass-to-Light Ratio
Sun	1
Matter in vicinity of Sun	2
Mass in Milky Way within 80,000 light-years of the center	10
Small groups of galaxies	50-150
Rich clusters of galaxies	250-300

Mass to light ratio

Success of CDM ? : ω null ??

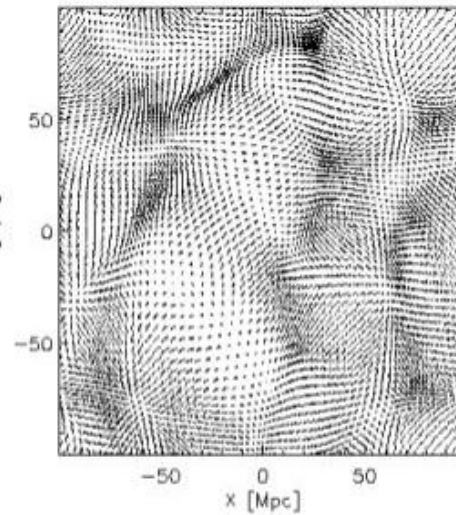
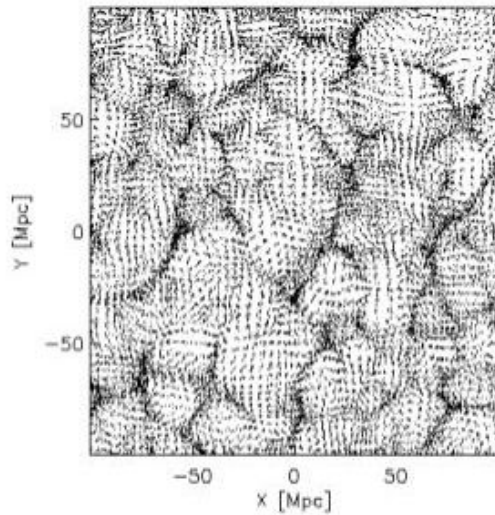
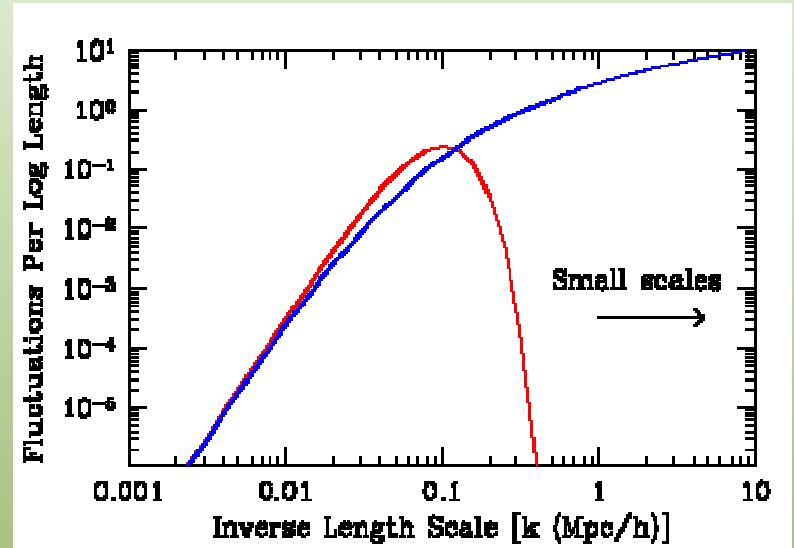


$$\delta \sim (1 + z)^{-1}$$

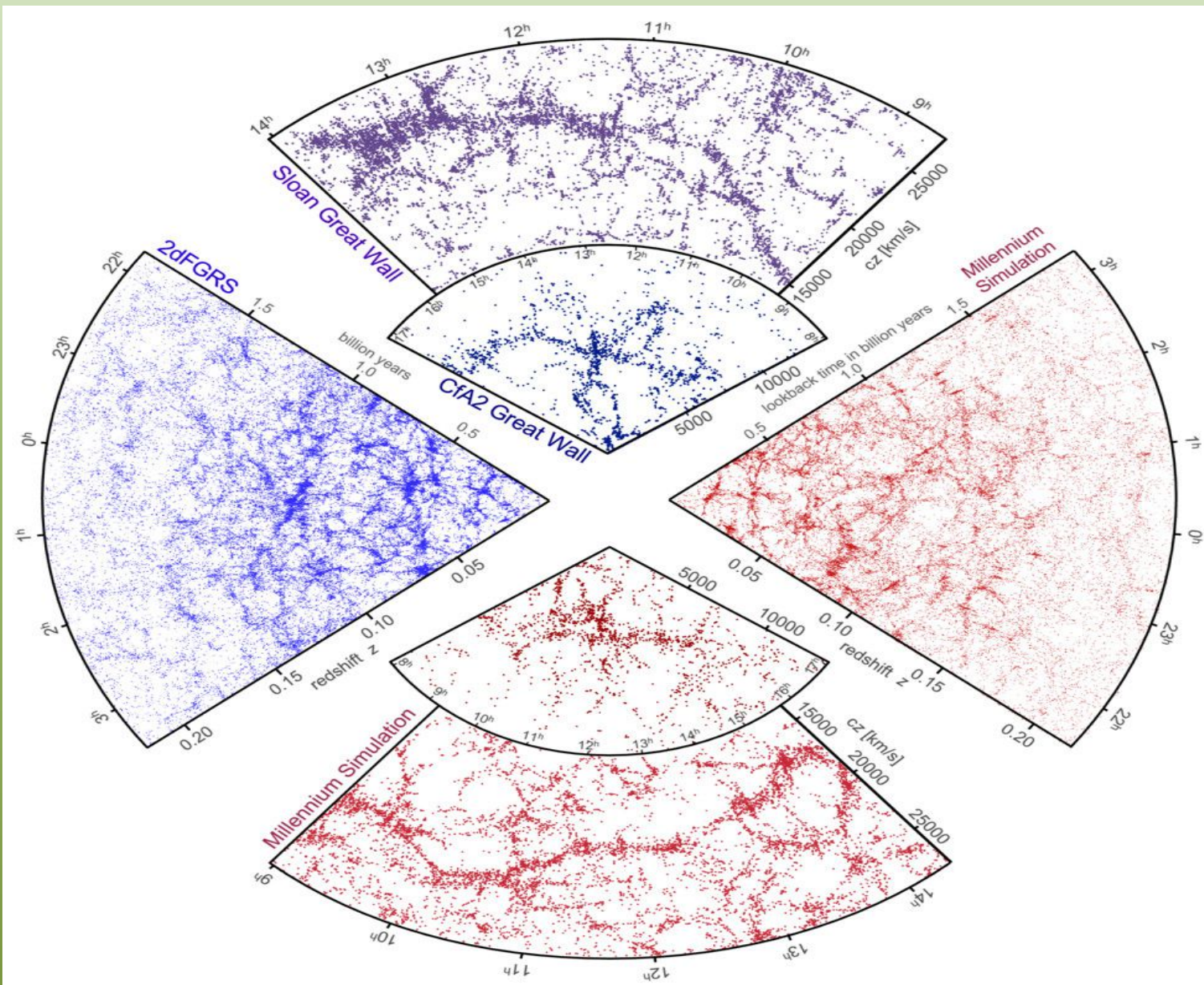
$$\delta_f = 10^{-5} \left(\frac{1 + 1000}{1 + 5} \right) \sim 0.002$$

The C in LCDM ?

Peebles vs Zeldovitch



Simulations



Extensions of CDM ?

Hu 1998

Kopp 2016

$$\omega_{gdm} = P_{gdm} / \rho_{gdm} \neq 0$$

$$c_s^2 = \delta P_{cdm} / \delta \rho_{cdm} \neq 0$$

Warm Dark Matter

Neutrinos

DM Couplings

Chaplygin Gas

Equations of background and perturbations

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$T_i^{\mu\nu} = (\rho_i c^2 + P_i) u^\mu u^\nu + P_i g^{\mu\nu}$$

$$\dot{\bar{\rho}}_i + 3H \left(\bar{\rho}_i + \frac{\bar{P}_i}{c^2} \right) = 0$$

$$\omega_{gdm} = P_{gdm} / \rho_{gdm} \neq 0$$

$$c_s^2 = \delta P_{cdm} / \delta \rho_{cdm} \neq 0$$

$$\begin{aligned} \dot{\delta}_i + 3H (c_{s,i}^2 - w_i) \delta_i = \\ - [1 + w_i + (1 + c_{s,i}^2) \delta_i] \vec{\nabla} \cdot \vec{u}_i \end{aligned}$$

$$\dot{\vec{u}}_i + 2H \vec{u}_i + (\vec{u}_i \cdot \vec{\nabla}) \vec{u}_i + \frac{\vec{\nabla} \phi}{a^2} = 0$$

$$\nabla^2 \phi = 4\pi G a^2 \sum_k \bar{\rho}_k (1 + 3c_{s,k}^2) \delta_k$$

Boltzmann code

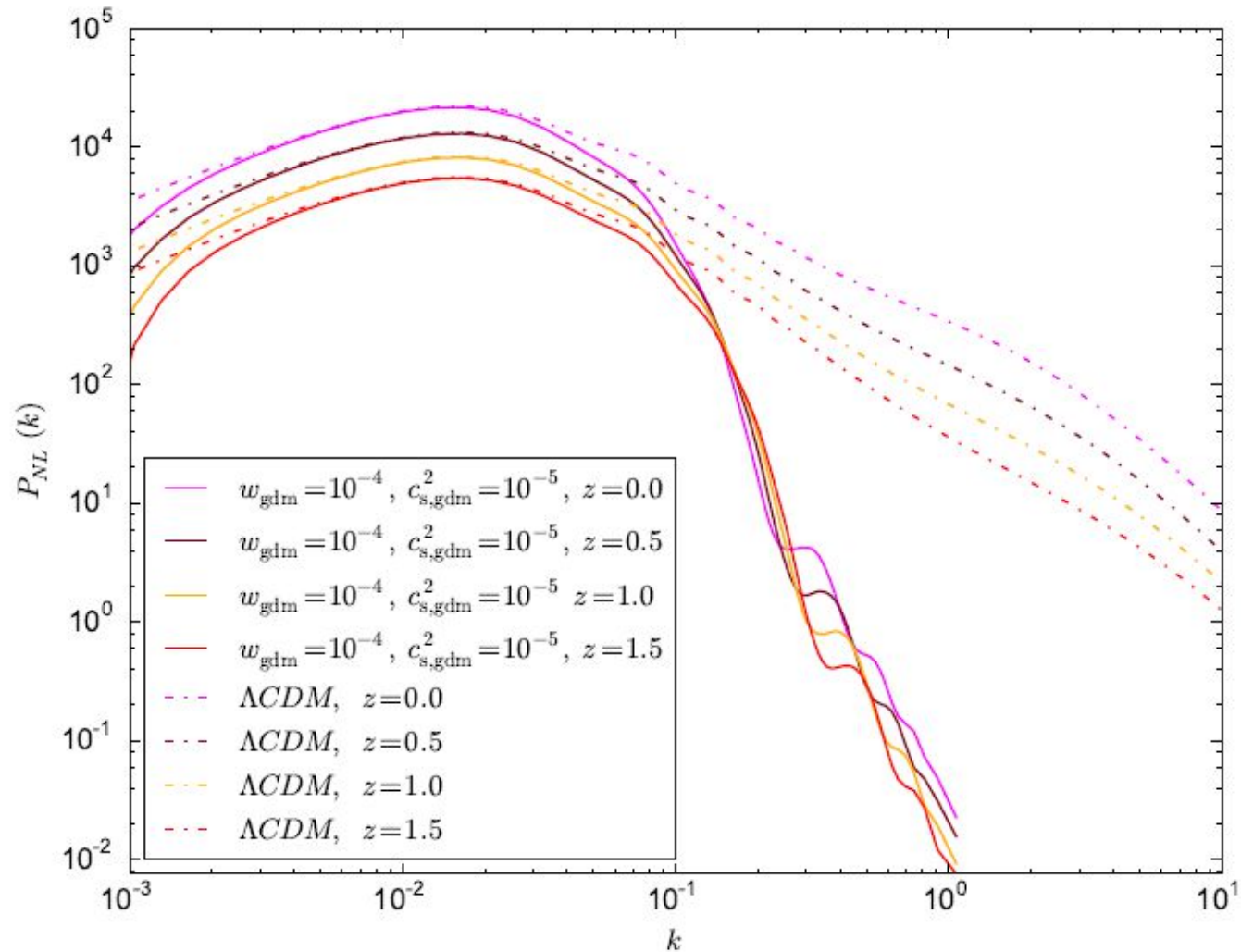
$$\dot{\delta} + (1 + w_{\text{gdm}}) \left(\theta + \frac{\dot{h}}{2} \right) + 3H \left(\frac{\delta p}{\delta \rho} - w_{\text{gdm}} \right) \delta = 0$$

$$\dot{\theta} + H(1 - 3w_{\text{gdm}})\theta + \frac{\dot{w}_{\text{gdm}}}{1 + w_{\text{gdm}}}\theta - \frac{\delta p / \delta \rho}{1 + w_{\text{gdm}}}k^2\delta + k^2\sigma = 0$$

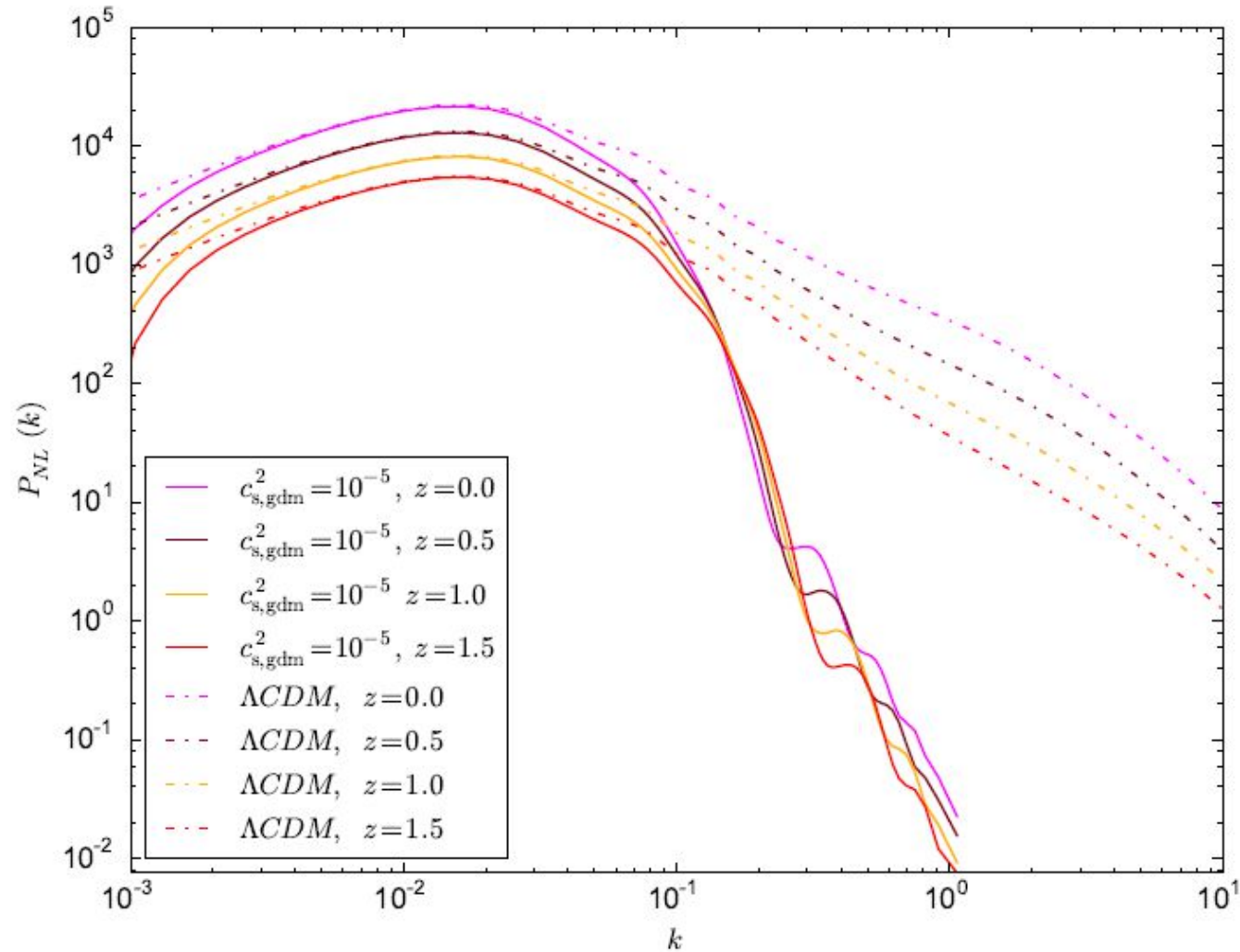
$$\delta p = c_{\text{s,gdm}}^2 \delta \rho - \dot{\rho}(c_{\text{s,gdm}}^2 - c_{\text{a,gdm}}^2)\theta / k^2$$

$$\dot{\sigma} + 3H \frac{c_{\text{a,gdm}}^2}{w_{\text{gdm}}}\sigma = \frac{4}{3} \frac{c_{\text{vis,gdm}}^2}{1 + w_{\text{gdm}}}(2\theta + \dot{h} + 6\dot{\eta})$$

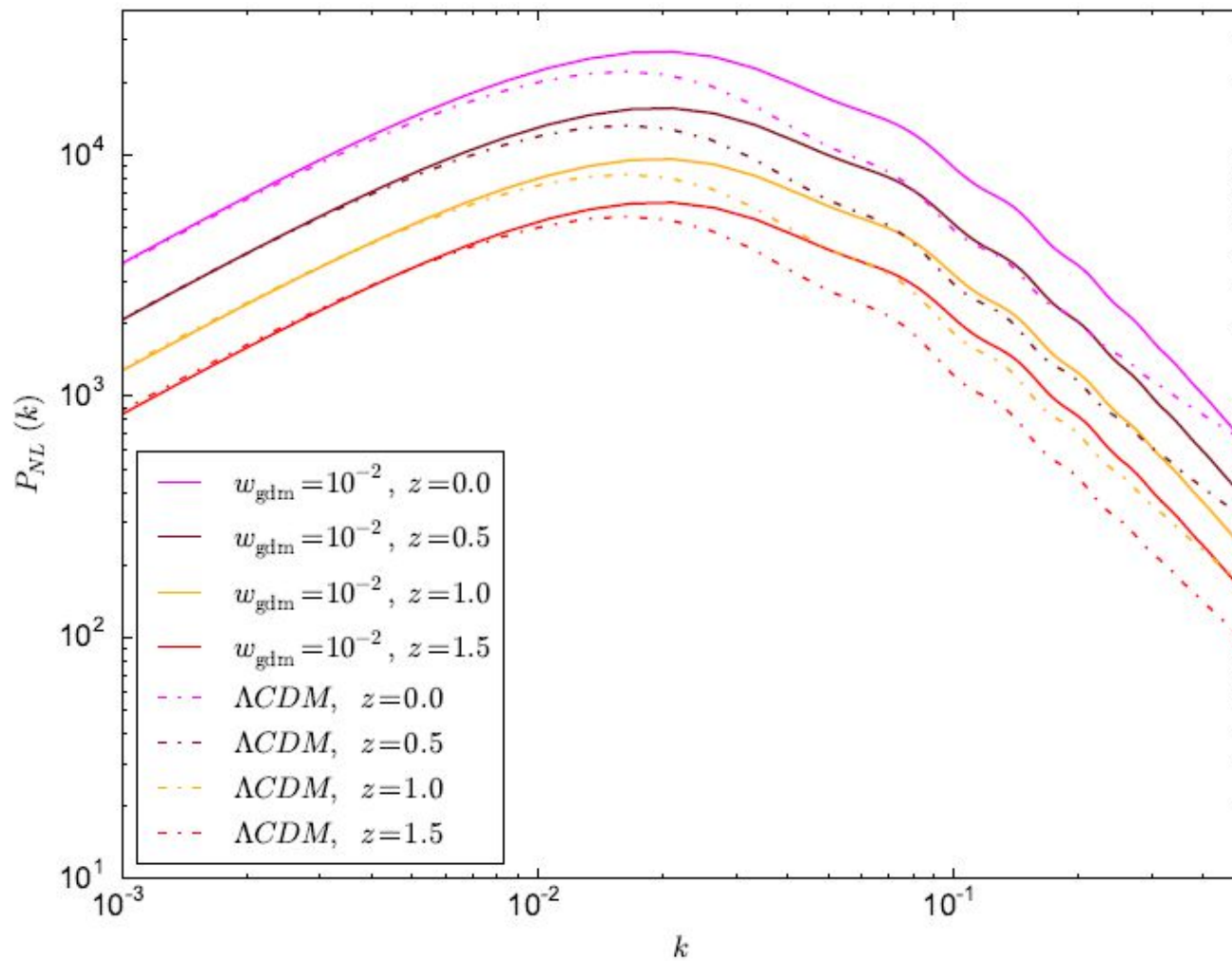
Boltzmann code – power spectrum



Boltzmann code – power spectrum



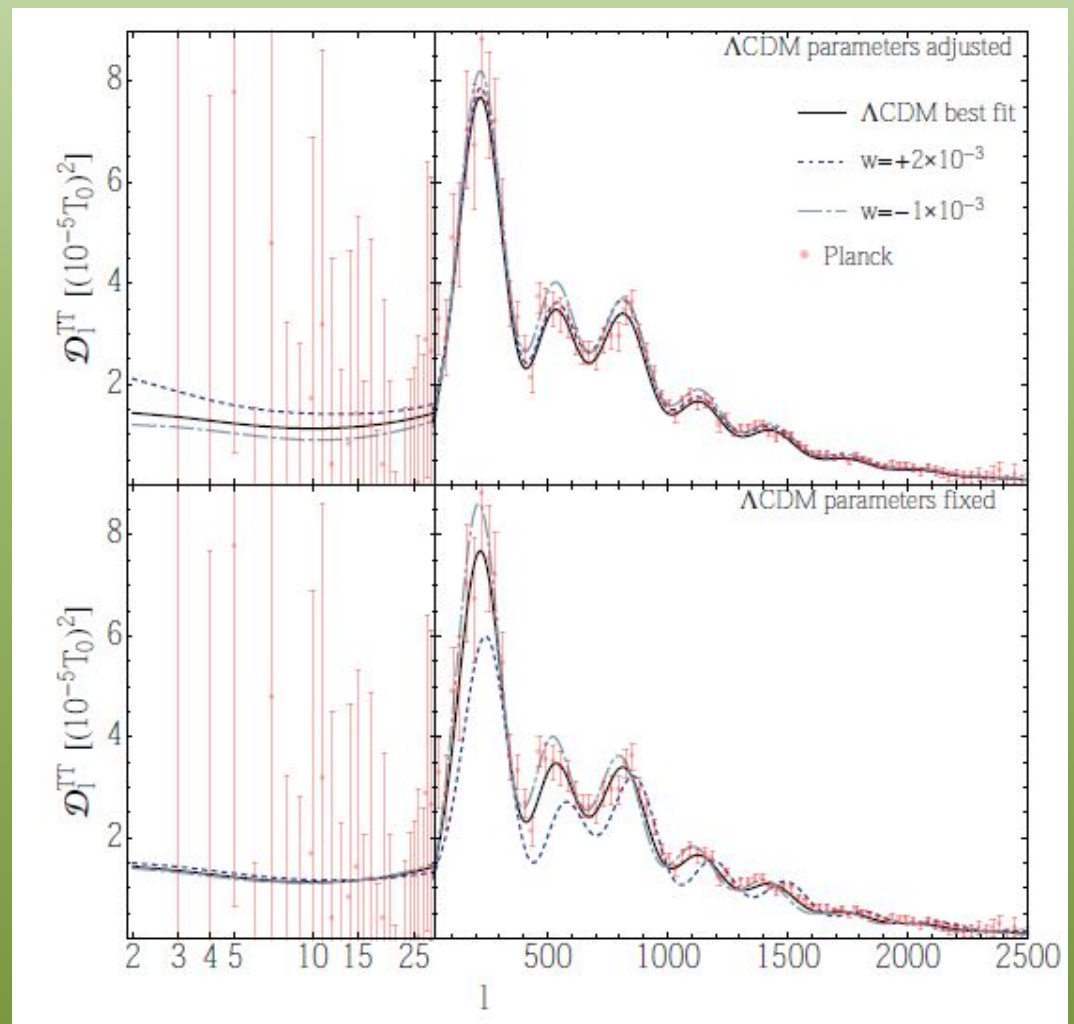
Boltzmann code – power spectrum



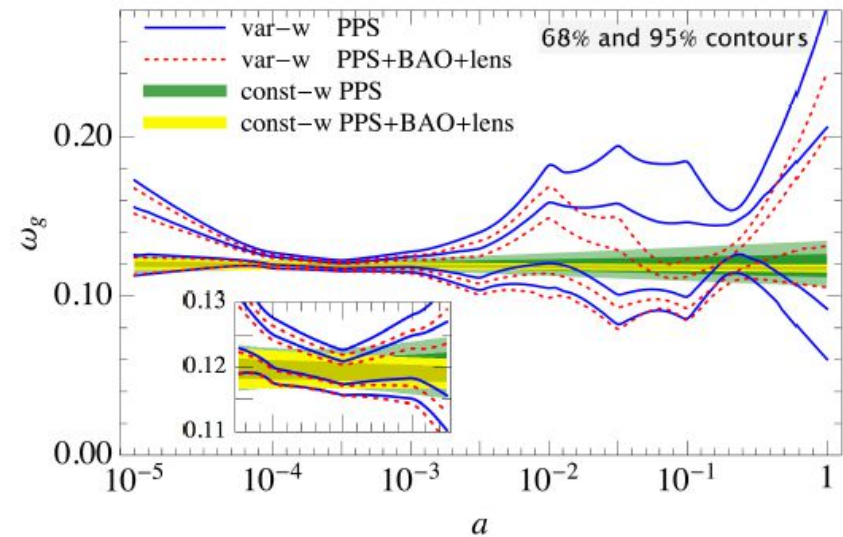
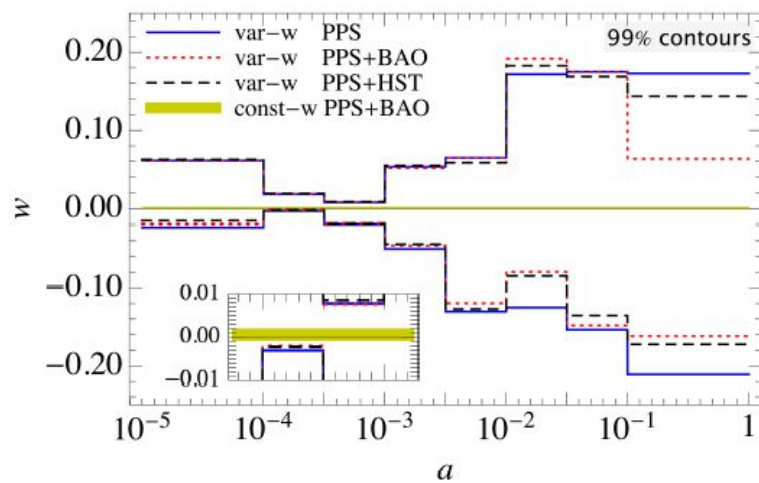
Linear constraints

Likelihoods	Λ -wDM	
	$10^2 w$	
	95.5%	99.7%
PPS	$0.007^{+0.463}_{-0.466}$	$0.007^{+0.676}_{-0.673}$
PPS + Lens	$0.087^{+0.439}_{-0.448}$	$0.087^{+0.662}_{-0.648}$
PPS + Lens + HST	$0.256^{+0.217}_{-0.217}$	$0.256^{+0.322}_{-0.323}$
PPS + Lens + BAO	$0.063^{+0.108}_{-0.112}$	$0.063^{+0.163}_{-0.164}$

Thomas et al. 1601.05097



Linear constraints



Linear constraints predictions

Model	Parameters	CMB+SNIa+BAO	CMB+SNIa+BAO+WL	Photometric Euclid
Λ CDM	$10^2 \times \Omega_b$	4.834 ± 0.054	4.815 ± 0.053	$(4.834) \pm 0.075$
	Ω_{dm}	0.2562 ± 0.0060	0.2539 ± 0.0057	$(0.2562) \pm 0.0074$
	h	0.6797 ± 0.0049	0.6816 ± 0.0048	$(0.6797) \pm 0.016$
	σ_8	0.8187 ± 0.0089	0.8142 ± 0.0090	$(0.8187) \pm 0.0088$
	n_s	0.9674 ± 0.0043	0.9686 ± 0.0043	$(0.9674) \pm 0.023$
	$10 \times \tau$	0.72 ± 0.13	0.68 ± 0.13	–
Λ GDM	$10^2 \times \Omega_b$	4.71 ± 0.12	4.69 ± 0.12	$(4.834) \pm 0.11$
	Ω_{dm}	0.2491 ± 0.0087	0.2458 ± 0.0088	$(0.2562) \pm 0.014$
	$10^2 \times w$	0.066 ± 0.054	0.055 ± 0.053	$(0.00) \pm 0.21$
	$10^6 \times c_s^2$	< 0.78	< 0.010	< 0.0018
	h	0.6873 ± 0.0082	0.6898 ± 0.0082	$(0.6797) \pm 0.020$
	σ_8	$0.7351^{+0.094}_{-0.041}$	0.8174 ± 0.016	$(0.8187) \pm 0.0096$
	n_s	0.9656 ± 0.0044	0.9682 ± 0.0042	$(0.9674) \pm 0.035$
	$10 \times \tau$	0.73 ± 0.15	0.58 ± 0.16	–

Non Linear constraints ?

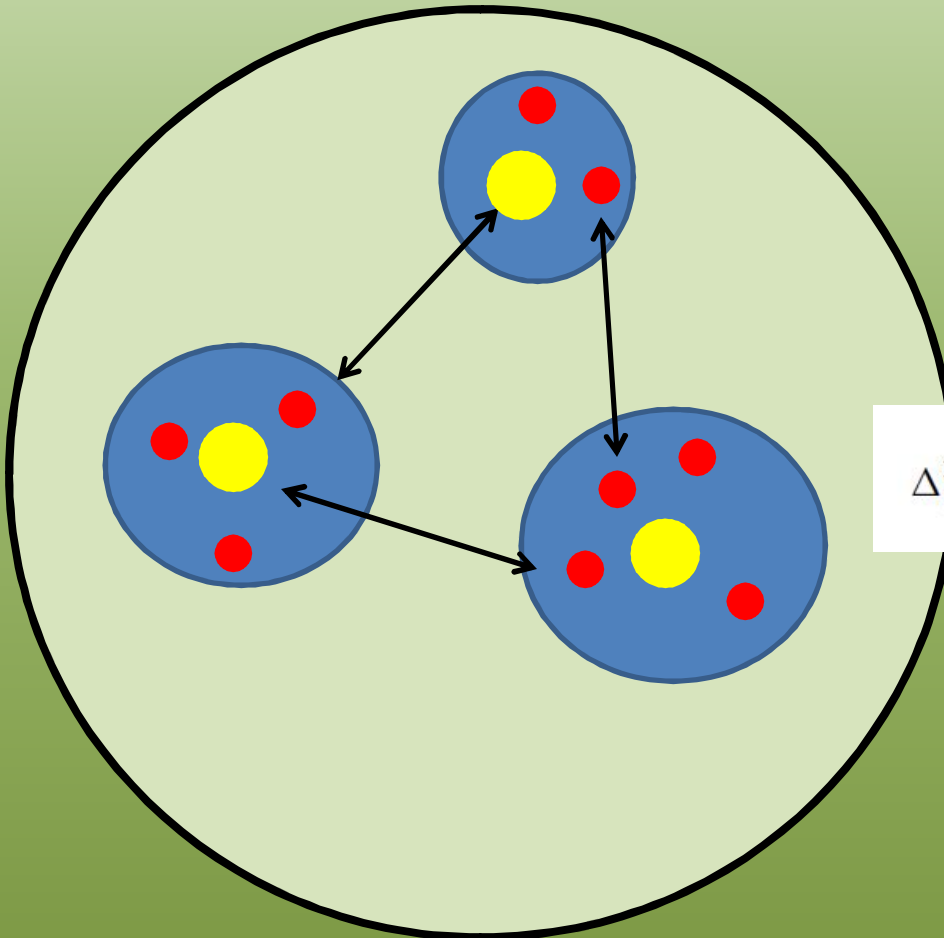
$$\Delta^2(k) = 4\pi V \left(\frac{k}{2\pi} \right)^3 P(k)$$

1 halo term

$$\Delta_{1H}^2(k) = 4\pi \left(\frac{k}{2\pi} \right)^3 \frac{1}{\bar{\rho}^2} \int_0^\infty M^2 W^2(k, M) F(M) dM$$

2 halos term

$$\Delta_{2H}^2(k) = \Delta_{lin}^2(k)$$

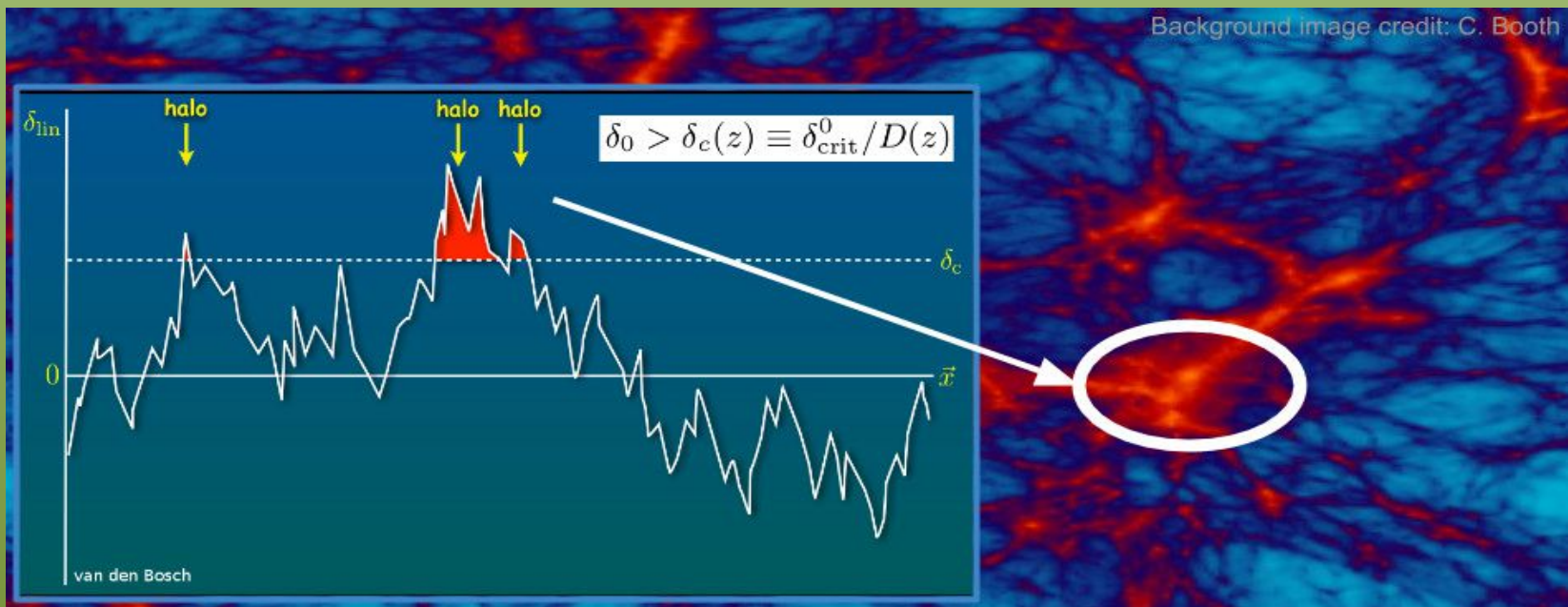


$$p(\delta \geq \delta_c) = \frac{1}{\sqrt{2\pi}\sigma^2} \int_{\delta_c}^{\infty} \exp\left(-\frac{\delta^2}{2\sigma^2}\right) d\delta$$

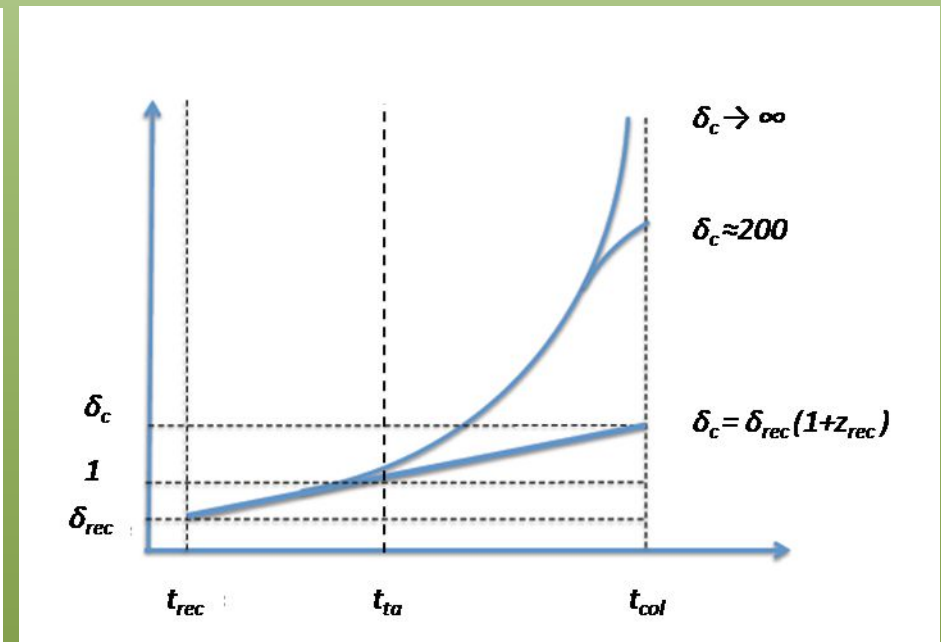
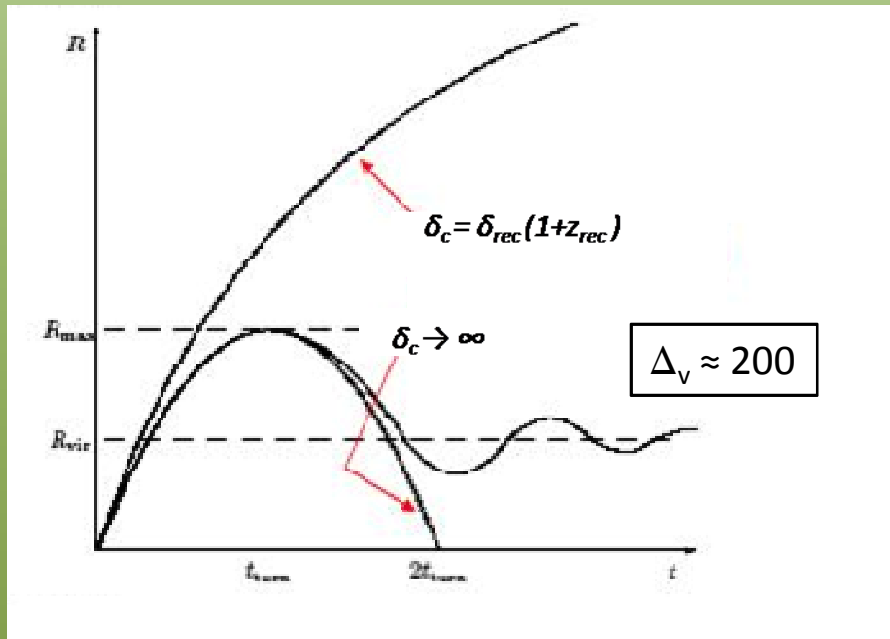
Press - Schechter formalism...

$$F(> M) = 2 \cdot p(> \delta_c)$$

$$\int n(M) dM = \rho/M \cdot F(>M)$$



Spherical collapse model



Spherical collapse model

Derivation and calculation of δ_c

$$A_i \equiv 3H (c_{s,i}^2 - w_i) \delta_i$$

$$B_i \equiv 1 + w_i + (1 + c_{s,i}^2) \delta_i$$

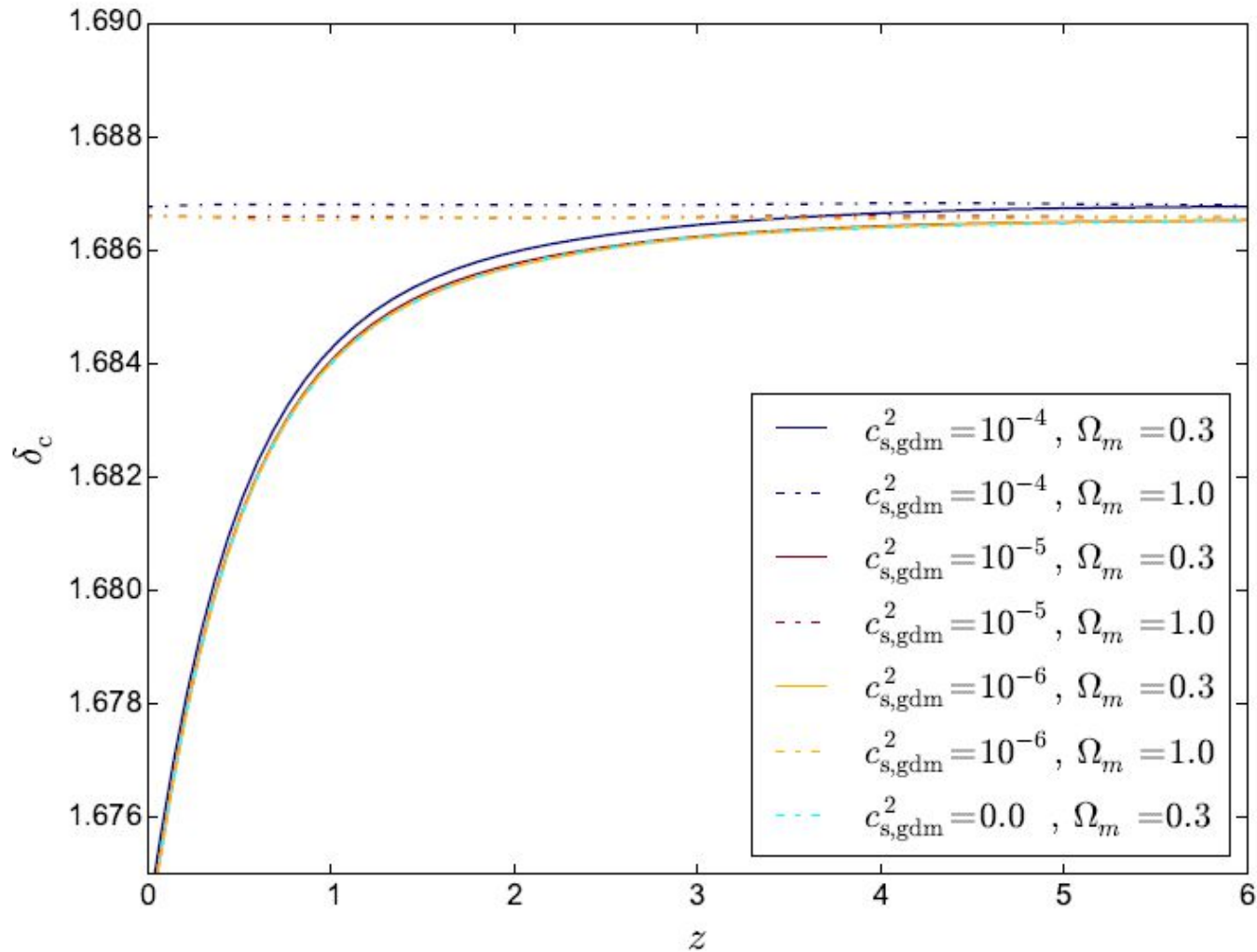
$$\dot{\delta}_i + A_i + B_i \theta_i = 0$$

$$\dot{\theta}_i + 2H\theta_i + \frac{1}{3}\theta_i^2 + \frac{\nabla^2\phi}{a^2} = 0$$

$$\ddot{\delta}_i + \dot{A}_i + \left(2H - \frac{\dot{B}_i}{B_i}\right) (A_i + \dot{\delta}_i) - \frac{1}{3} \frac{(\dot{\delta}_i + A_i)^2}{B_i} - \frac{B_i}{a^2} \nabla^2\phi = 0$$

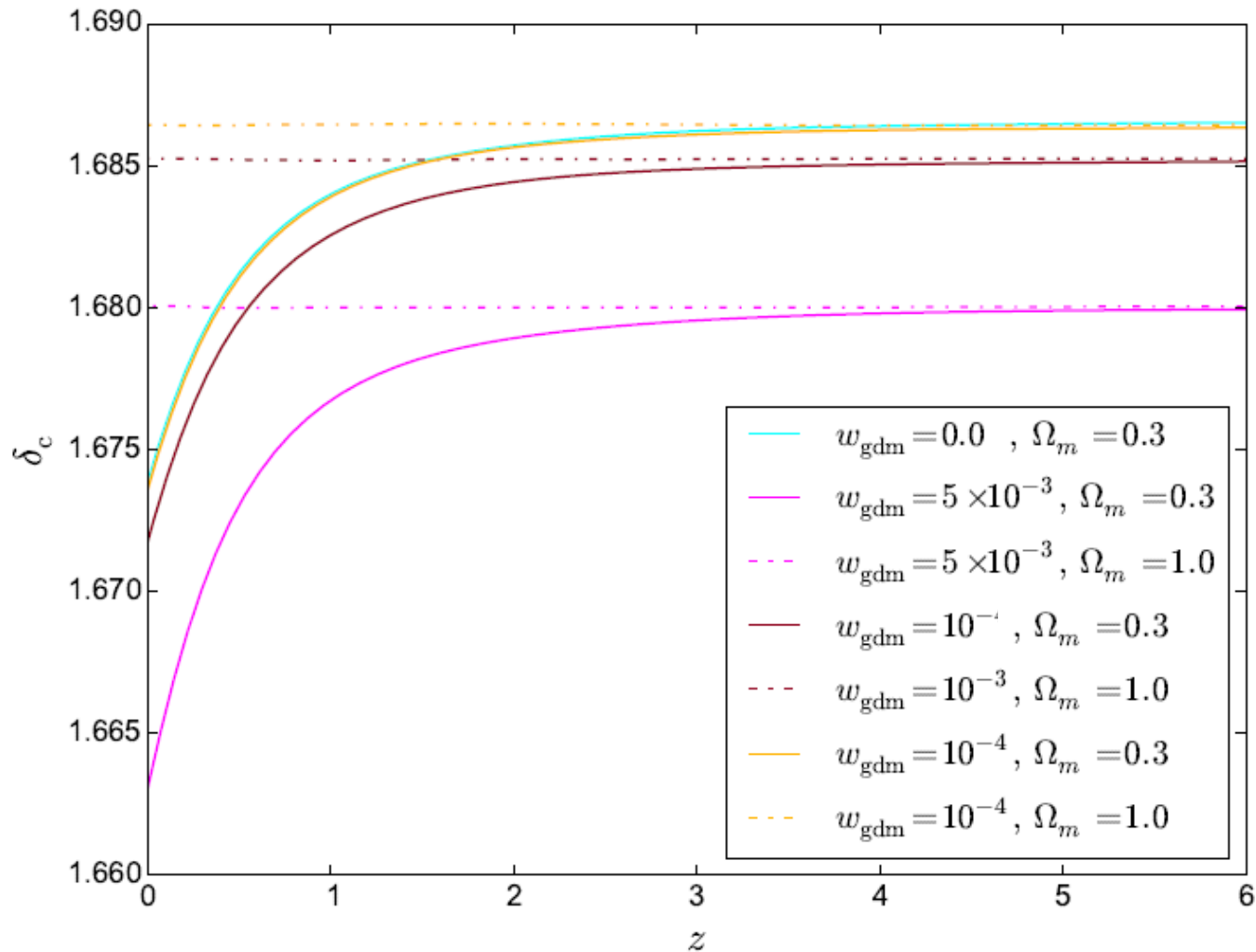
δ_c in GDM

arxiv.1912.12250



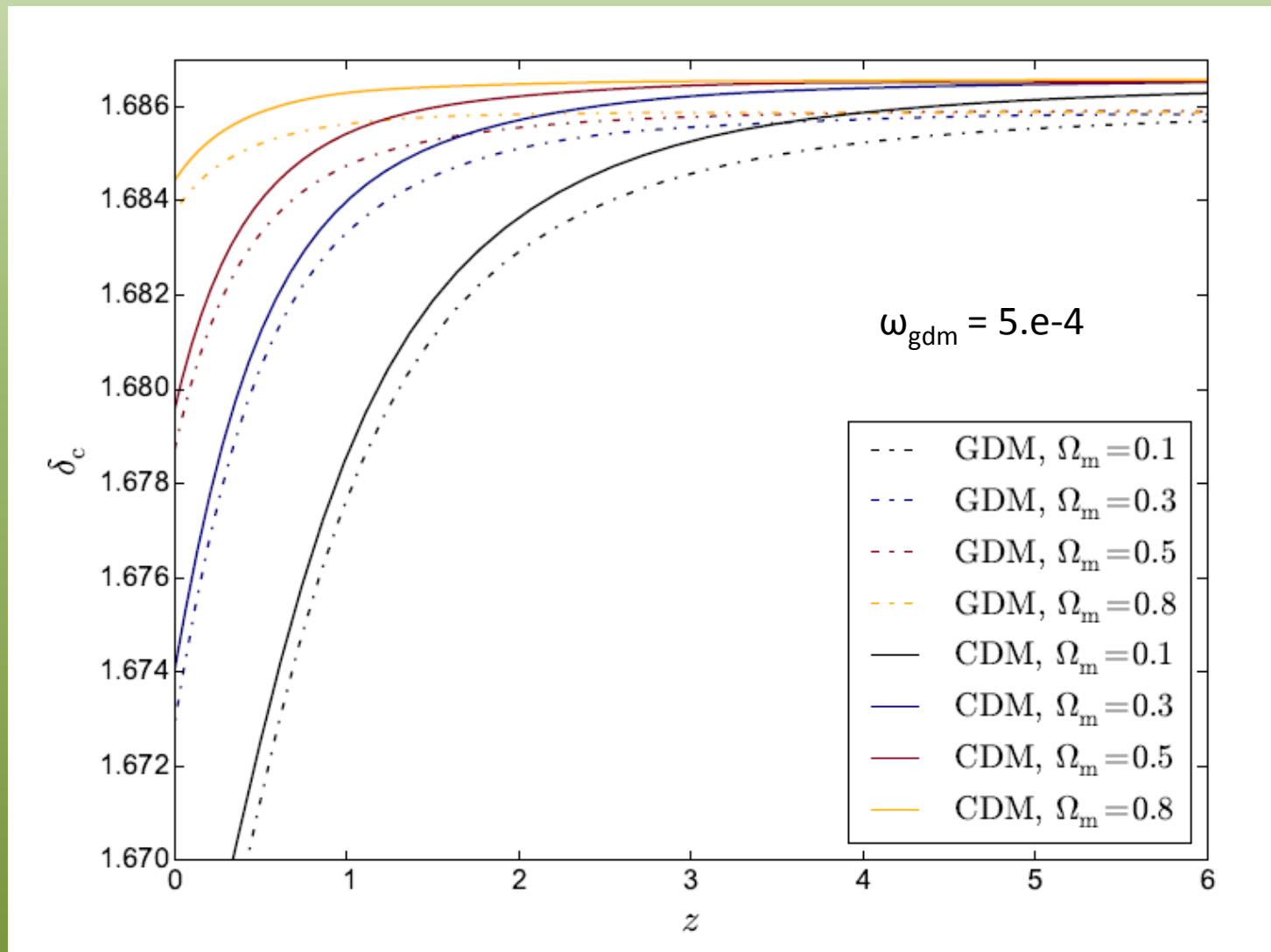
δ_c in GDM

arxiv.1912.12250



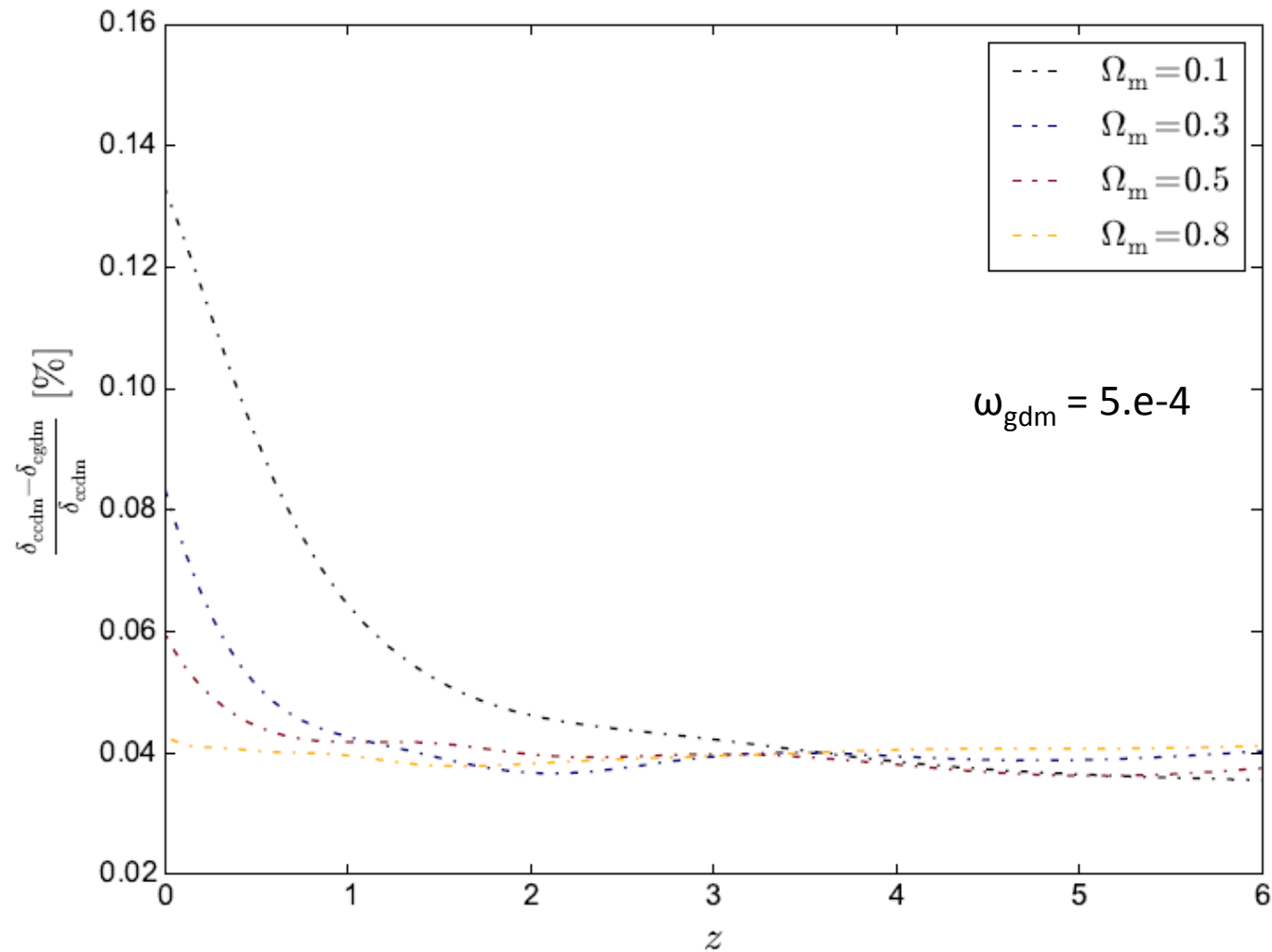
δ_c in GDM, vary Ω_M

arxiv.1912.12250



δ_c in GDM, vary Ω_M

arxiv.1912.12250



Spherical collapse model

Calculation of Δ_{vir}

$$U_{\text{gdm,ta}} + U_{\Lambda,\text{ta}} = U_{\text{gdm,vir}} + T_{\text{gdm,vir}} + U_{\Lambda,\text{vir}} + T_{\Lambda,\text{vir}}$$

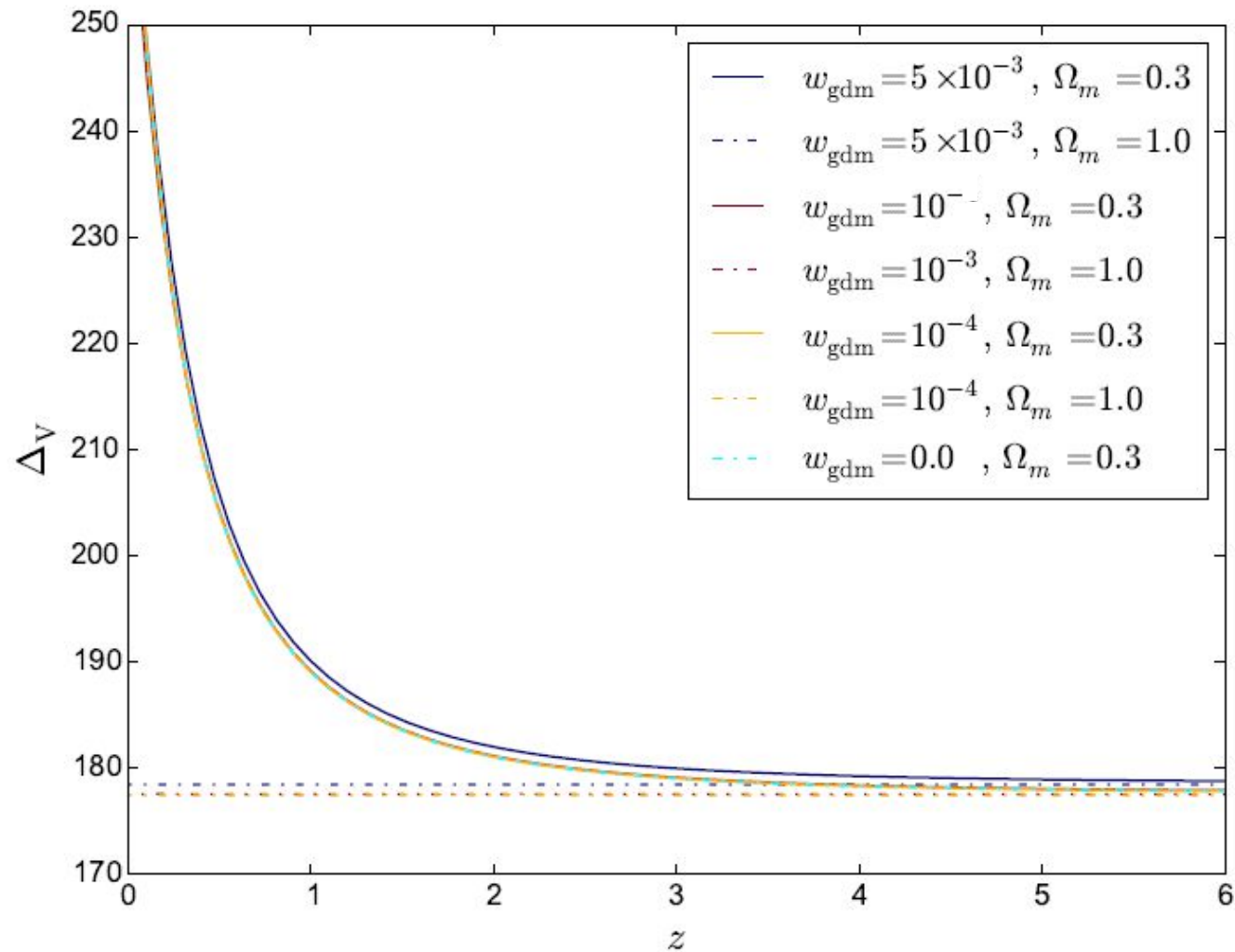
$$U_{\text{gdm,ta}} + U_{\Lambda,\text{ta}} = \frac{1}{2}U_{\text{gdm,vir}} + 2U_{\Lambda,\text{vir}}$$

$$\theta = \frac{\rho_{\Lambda}}{\rho_{\text{gdm}}} \text{ and } \eta = \frac{r_{\text{vir}}}{r_{\text{ta}}}$$

$$\theta\eta^3 + \left(1 + \frac{\theta}{2}\right)\eta - 1/2 = 0$$

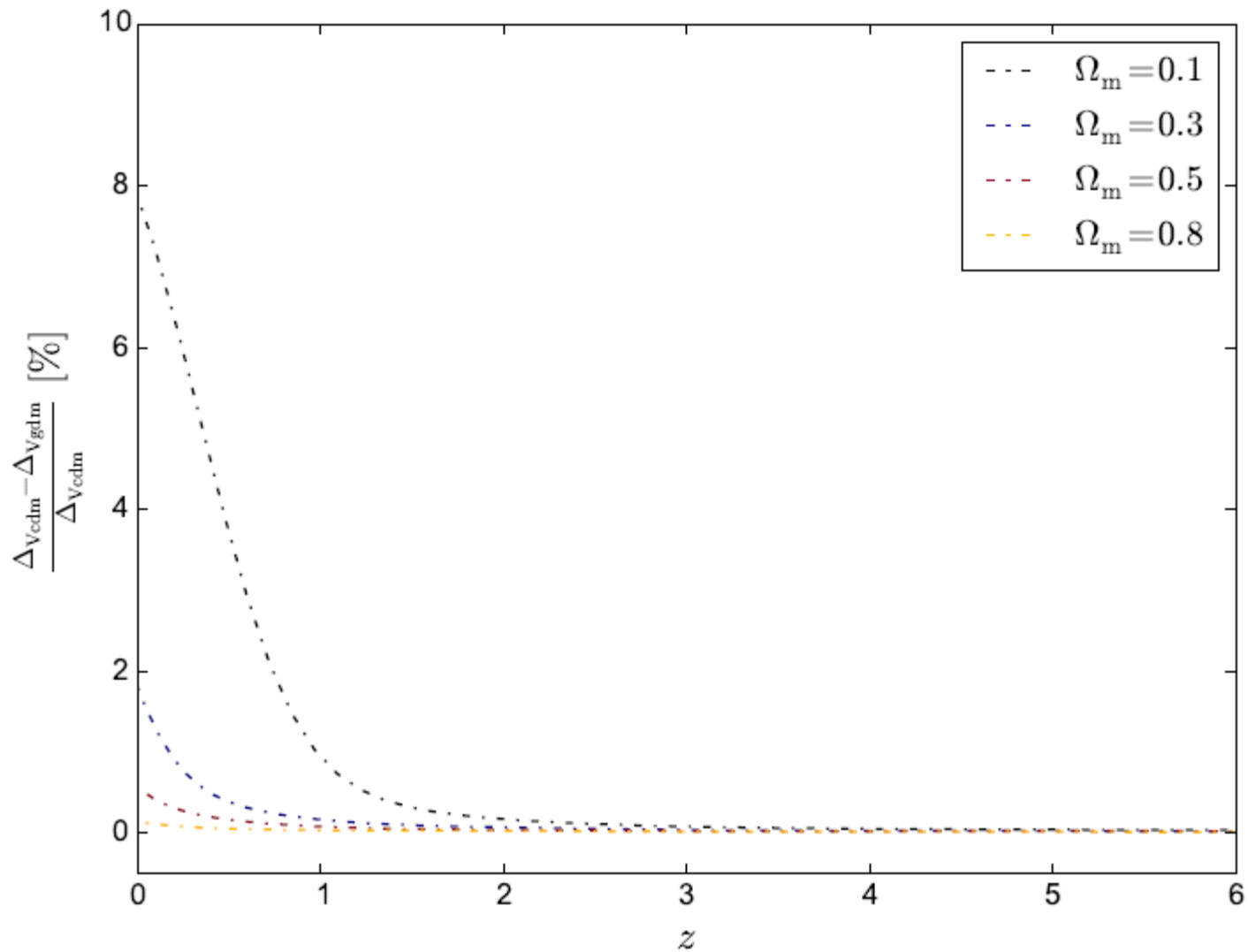
Δ_{vir} in GDM

arxiv.1912.12250



Δ_{vir} in GDM, vary Ω_M

arxiv.1912.12250



Halo mass function

$$\frac{dn(M)}{dM} = -\frac{\bar{\rho}}{M} \frac{d\nu}{dM} \mathcal{F}(\nu)$$

$$\nu \mathcal{F}_{\text{ST}}(\nu) = A \sqrt{\frac{2a}{\pi}} \left[1 + \left(\frac{1}{a\nu^2} \right)^p \right] \nu \exp \left\{ \left[-\frac{a\nu^2}{2} \right] \right\}$$

$$a = 0.4332x^2 + 0.2263x + 0.7665,$$

$$p = -0.1151x^2 + 0.2554x + 0.2488,$$

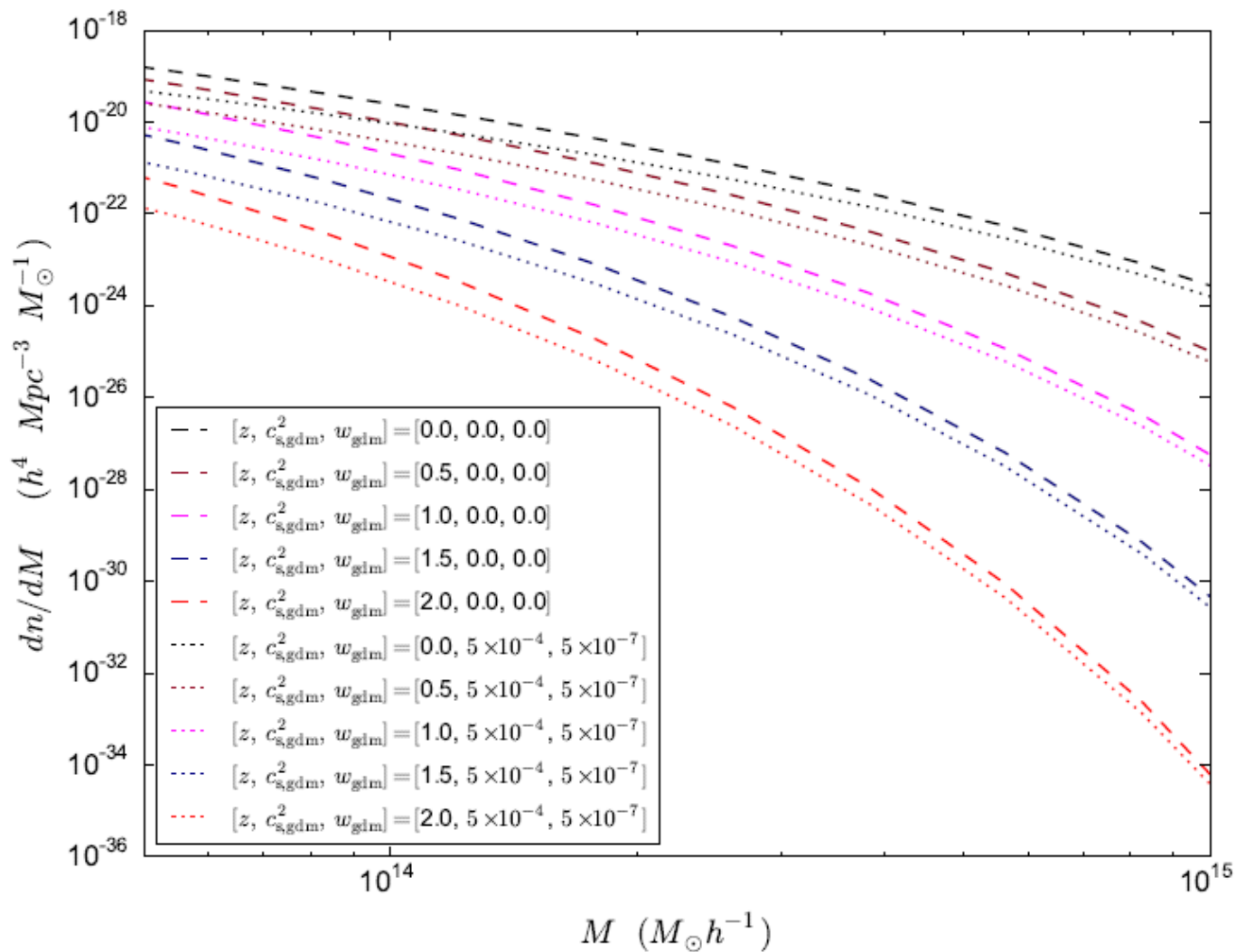
$$A = -0.1362x + 0.3292,$$

where $x = \log(\Delta(z)/\Delta_V(z))$

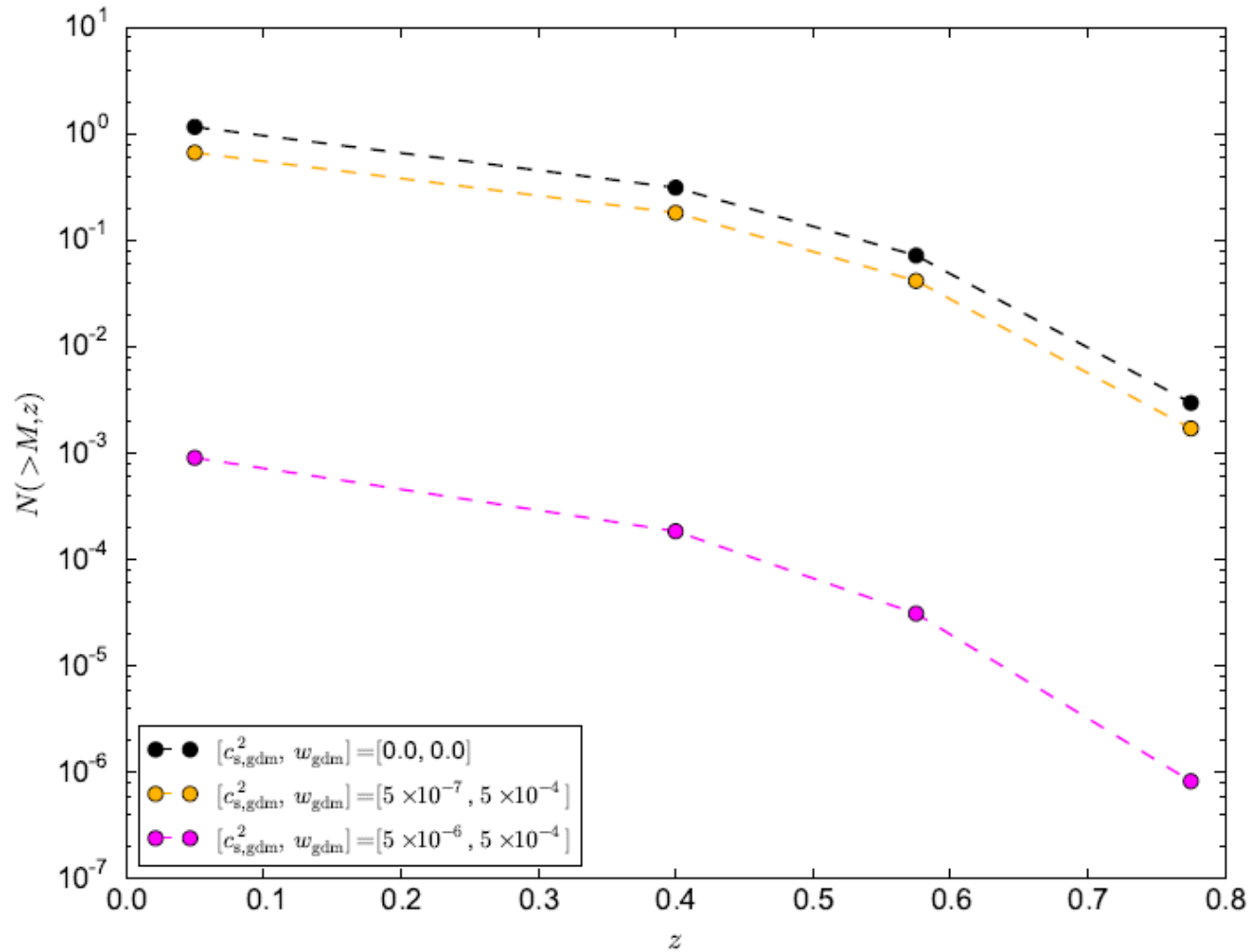
$$\nu = \delta_c / \sigma_R(z)$$

HMF in GDM

arxiv.1912.12250



Application : cluster counts



Application : cluster counts

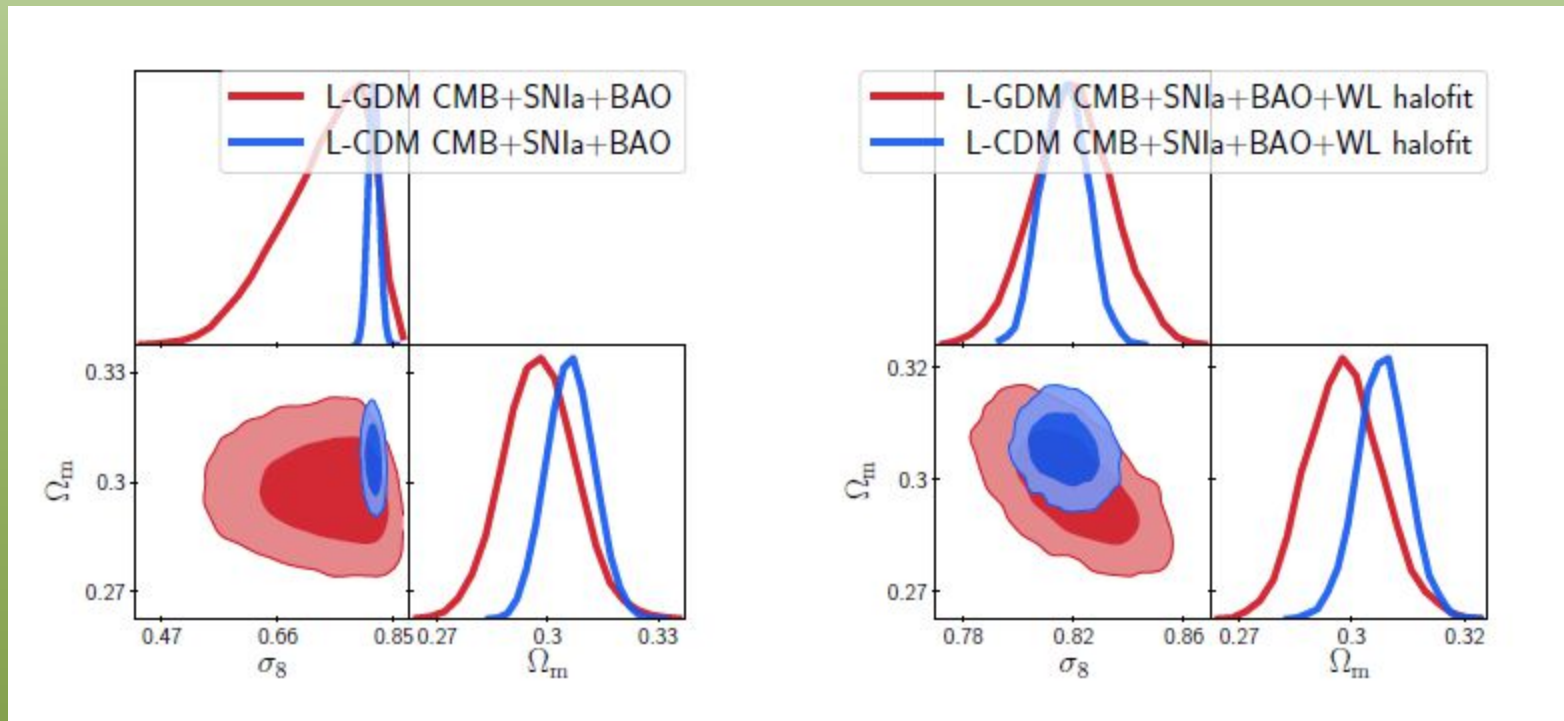
$$\chi_{\text{num}}^2 = 2 \sum_{i=1}^4 \left[N_i - N'_{\text{obs},i} \left(1 + \ln N_i - \ln N'_{\text{obs},i} \right) \right] + \xi^2$$

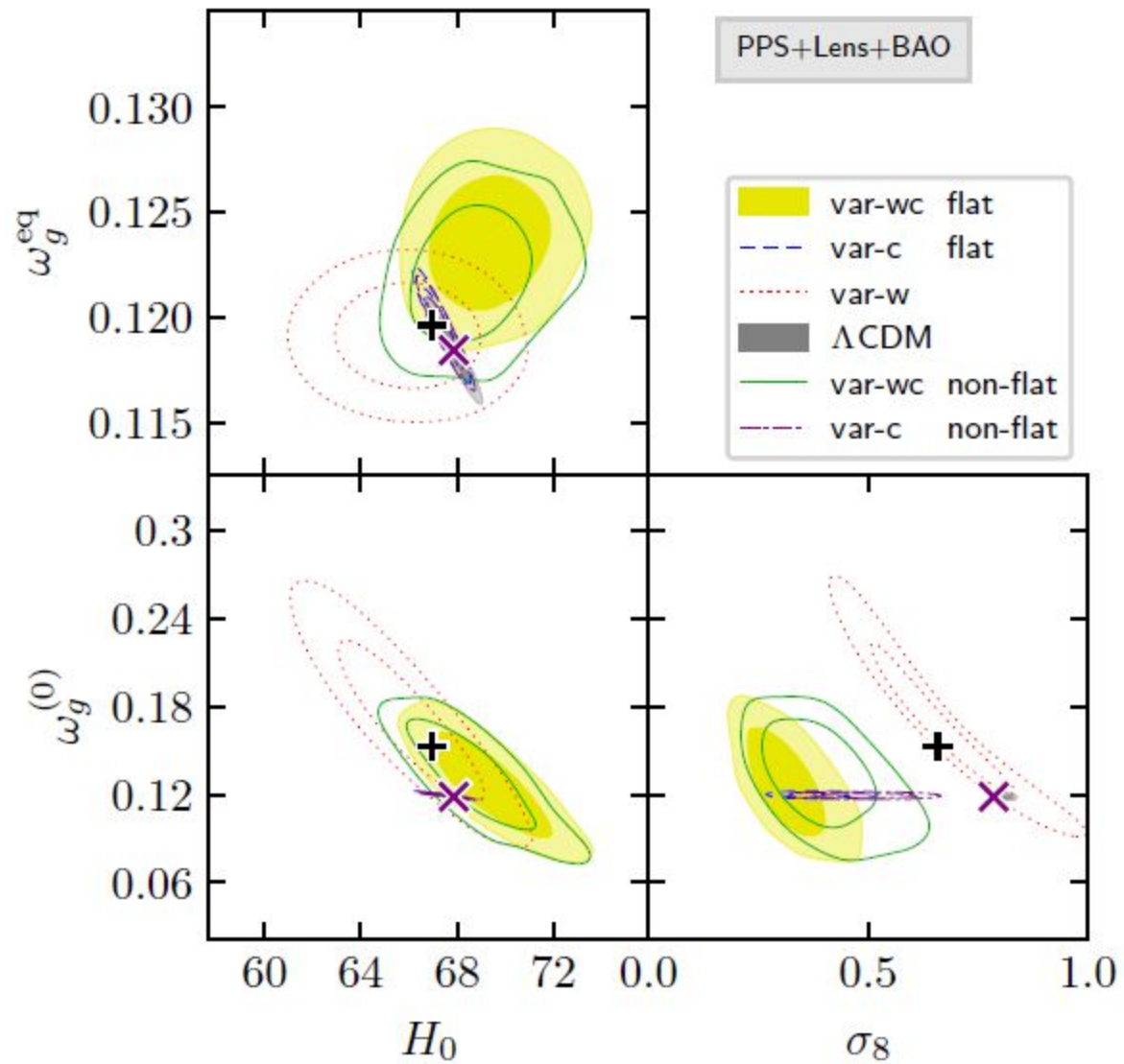
$\chi^2 \approx 11.2$ for Λ CDM

$\chi^2 \approx 15.44$ for Λ GDM with $c_s^2 = 5 \cdot e^{-7}$ and $\omega_{\text{gdm}} = 1 \cdot e^{-4}$

$\chi^2 \approx 79.57$ for Λ GDM with $c_s^2 = 5 \cdot e^{-6}$ and $\omega_{\text{gdm}} = 1 \cdot e^{-4}$

Application σ_8 discrepancy





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